Practice Test 2

SECTION I: MULTIPLE-CHOICE

Time: 90 minutes

50 questions

> **DIRECTIONS:** Each of the questions or incomplete statements below is followed by four suggested answers or completions. Select the one (or two where indicated) that is best in each case. You have 90 minutes to complete this portion of the test. You may use a calculator and the information sheets provided in the appendix.

1. Two objects are thrown vertically upward from the same initial height. One object has twice the initial velocity of the other. Neglecting any air resistance, the object with the greater initial velocity will rise to a maximum height that is

   (A) twice that of the other object, assuming they have the same mass
   (B) twice that of the other object, independent of their masses
   (C) four times that of the other object, assuming they have the same mass
   (D) four times that of the other object, independent of their masses

2. A 2-kilogram cart has a velocity of 4 meters per second to the right. It collides with a 5-kilogram cart moving to the left at 1 meter per second. After the collision, the two carts stick together. Can the magnitude and the direction of the velocity of the two carts after the collision be determined from the given information?

   (A) No, since the collision is inelastic, we must know the energy lost.
   (B) Yes, the collision is elastic: 3/7 m/s left.
   (C) Yes, the collision is inelastic: 3/7 m/s right.
   (D) Yes, the speed is not 3/7 m/s.

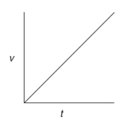
3. Projectile X is launched at a 30-degree angle above the horizon with a speed of 100 m/s. Projectile Y is launched at a 60-degree angle with the same speed. Which of the following correctly compares the horizontal range and maximum altitude obtained by these two projectiles?

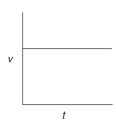
	Range	Altitude
(A)	***X* goes farther**	***X* goes higher**
(B)	***Y* goes farther**	***Y* goes higher**
(C)	***X* goes farther**	***Y* goes higher**
(D)	***X* and *Y* equal**	***Y* goes higher**

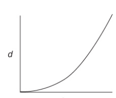
4. A 5-kg mass is sitting at rest on a horizontal surface. A horizontal force of 10 N will start the mass moving. What is the best statement about the coefficient and type of friction between the mass and the surface?

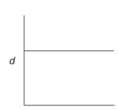
(E) >0.20 static
(F) <0.20 static
(G) <0.20 kinetic
(H) >0.20 kinetic

5. Which of the following graphs represents an object *moving* with no net force acting on it?

(A) 

(B) 

(C) 

(D) 

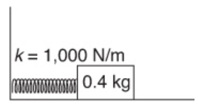
6. A projectile is launched horizontally with an initial velocity $\boldsymbol{v}_0$ from a height h. If it is assumed that there is no air resistance, which of the following expressions represents the vertical position of the projectile? In other words, $y(x) = ?$

(A) $h - gv_0^2/2x^2$
(B) $h - gv/2v_0^2$
(C) $h - gx^2/2v_0^2$

(D) $h - gx^2/v_0^2$

QUESTIONS 7 AND 8 ARE BASED ON THE INFORMATION AND DIAGRAM BELOW:

A 0.4-kilogram mass is oscillating on a spring that has a force constant of k = 1,000 newtons per meter.



7. Which of the following measurements would allow you to determine the maximum velocity experienced by the mass?
   (A) No additional information is required.
   (B) Minimum velocity
   (C) Maximum acceleration
   (D) None of these would allow you to determine maximum velocity.

8. Which of the following statements concerning the oscillatory motion described above is correct? (All statements refer to magnitudes.)
   (A) The maximum velocity and maximum acceleration occur at the same time.
   (B) The maximum velocity occurs when the acceleration is a minimum.
   (C) The velocity is always directly proportional to the displacement.
   (D) The maximum velocity occurs when the displacement is a maximum.

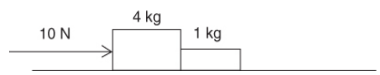
9. A bullet of known mass (m_1) is fired vertically into an initially stationary wood block of known mass (m_2). The resulting wood + bullet combined system is then measured to rise to a maximum height of h. Can the initial speed of the bullet be calculated from this information?
   (A) Yes. Solve $(m_1 + m_2)gh = \frac{1}{2}(m_1)v^2$.
   (B) Yes. Solve the momentum conservation of collision first and the energy conservation of the rising combination second.
   (C) No. We don't know if momentum is conserved during this collision.
   (D) No. We don't know enough details about the mechanical energy lost during the collision.

10. A 10-kg mass is being pushed horizontally by a constant force along a rough surface (μ_k = 0.1) at constant velocity. Which of the following is the best statement regarding the

constant force (f)?

(A) $f = 10$ N
(B) $f > 10$ N
(C) $f < 10$ N
(D) $f > 1$ N

11. A 10-newton force is applied to two masses, 4 kilograms and 1 kilogram, respectively, that are in contact as shown below. The horizontal motion is along a frictionless plane. What is the magnitude of the contact force between the two masses?



(A) 10 N
(B) 8 N
(C) 6 N
(D) 2 N

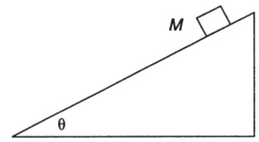
12. An object with mass m is dropped from height h above the ground. While neglecting air resistance, which formula best describes the power generated if the object takes time t to fall?

(A) mgh
(B) $mght$
(C) $mg^2t/2$
(D) mgh/t

13. A 1,500-kilogram car has a velocity of 25 meters per second. If it is brought to a stop by a nonconstant force in 10 to 15 seconds, can the magnitude of the impulse applied be determined?

(A) Yes, it is 37,500 N · s.
(B) No, you need to know the details about the nonconstant force.
(C) No, you must know the exact duration of the impulse.
(D) No, you must know the average force during and the duration of the impulse.

14. A block of mass M rests on a rough incline, as shown below. The angle of elevation of the incline is increased until an angle of θ is reached. At that angle, the mass begins to slide down the incline. Which of the following is an expression for the coefficient of static friction μ?

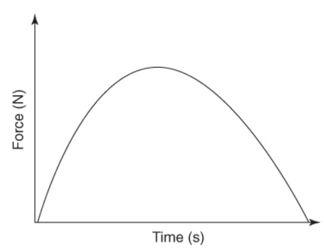


(A) tan θ
(B) sin θ
(C) cos θ
(D) 1/(cos θ)

15. A pendulum of a given length swings back and forth a certain number of times per second. If the pendulum now swings back and forth the same number of times but in twice the time, the length of the pendulum should be

(A) doubled
(B) quartered
(C) quadrupled
(D) halved

16. This graph of force versus time shows how the force acts on an object of mass *m* for a total time of *T* seconds. If the mass begins at rest, which is the correct method to find the final speed of the mass?



(A) Average value of this graph times total time divided by mass
(B) Area under this graph divided by mass
(C) Since the final force is zero, the object is at rest after time *T*
(D) Average slope of this graph divided by mass

17. A child of unknown mass is on a swing of unknown length that varies in height from 75 cm at its lowest height above the ground to a maximum height of 225 cm above the ground. Is there enough information to find the speed of the swing at its lowest point?

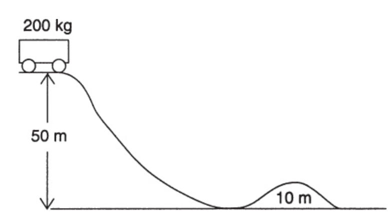
(A) No, the child’s mass must be known.
(B) No, the length of the swing must be known to determine the centripetal acceleration.

(C) Yes, it is 5.5 m/s.
(D) Yes, it is 4 m/s.

18. The gravitational force of attraction between two identical masses is 36 N when the masses are separated by a distance of 3 m. If the distance between them is reduced to 1 m, which of the following is true about the net gravitational field strength due to both masses being at the halfway point?

(A) It is 9 times stronger total.
(B) Not enough information is given to determine net gravitational field strength.
(C) Each mass's gravitation is 9 times stronger, so the net gravitational field strength is 18 times stronger.
(D) It is zero.

19. A 200-kilogram cart rests on top of a frictionless hill as shown below. Can the impulse required to stop the cart when it is at the top of the 10-meter hill be calculated?



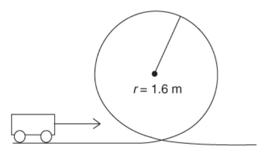
(A) No, more information about friction is required.
(B) No, more information about the impulse is required.
(C) Yes, calculate the velocity from free-fall kinematics and use this velocity in the change in momentum equation.
(D) Yes, calculate the speed from energy conservation and use this speed in the change in momentum equation.

20. An object of mass m starts at a height of H_1 with a speed of v_1. A few minutes later, it is at a height of H_2 and a speed v_2. Which of the following expressions best represents the work done to the mass by nongravitational forces to the object during this time?

(A) $mg(H_2 - H_1) + ½m(v_2^2 - v_1^2)$
(B) $mg(H_2 - H_1) - ½m(v_2^2 - v_1^2)$
(C) $½m(v_2^2 - v_1^2)$
(D) $½m(v_1^2 - v_2^2)$

21. An object with 0.2-kg mass is pushed down vertically onto an elastic spring (k = 20 N/m). The spring is compressed by 20 cm and then released such that the object will fly off. Which of the following will have the largest effect on increasing the maximum height the object will fly? (Assume no air resistance.)

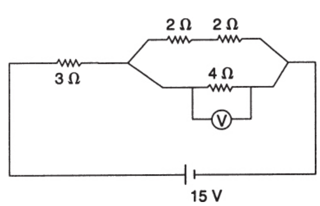
(A) Halving the mass
(B) Doubling the compression distance
(C) Using a spring with a spring constant twice as big
(D) Doing the same experiment on a different planet with half the gravitational field strength



22. A cart with a mass of m needs to complete a loop-the-loop of radius r, as shown above. What is the approximate minimum velocity required to achieve this goal?

(A) $(gr)^{1/2}$
(B) $(5gr)^{1/2}$
(C) $2(gr)^{1/2}$
(D) $(2gr)^{1/2}$

QUESTIONS 23–25 ARE BASED ON THE FOLLOWING CIRCUIT:



23. What is the equivalent resistance of the circuit?

(A) 6 Ω
(B) 8 Ω
(C) 11 Ω
(D) 5 Ω

24. What is the reading of the voltmeter across the 4-ohm resistor?

(A) 3 V

(B) 9 V
(C) 12 V
(D) 6 V

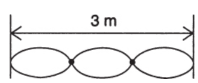
25. What is the current through the leftmost 2-ohm resistor?

(A) 3 A
(B) 1.5 A
(C) 0.75 A
(D) 1 A

26. A wire segment in a circuit with a cross-sectional area of A and length L is replaced by a wire segment made of the same material that has twice the area but half the length. The resistance of the new segment, compared to the original segment, will

(A) be reduced by half
(B) be reduced to one-quarter
(C) quadruple
(D) remain the same

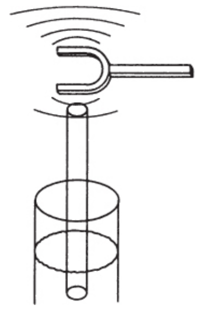
27. In the fixed standing wave shown below (imagine a string of fixed length), what will happen to the wavelength and frequency if the wave speed is raised while the standing wave pictured remains unchanged?



(A) Wavelength increases, while frequency remains the same.
(B) Wavelength remains the same, while frequency increases.
(C) Both wavelength and frequency increase.
(D) Although we know the product of wavelength and frequency increases, we do not know what combination is producing this effect.

28. Bats can find objects in the dark by using echolocation (sending out a high-frequency sound and listening to the echo). They also listen for a change in the pitch of the echo to

(A) confirm the estimated distance of the object
(B) determine the velocity of the object
(C) determine whether the object is moving toward or away from them
(D) estimate the approximate composition of the object



29. A 340-hertz tuning fork sets an air column vibrating in fundamental resonance, as shown above. A hollow tube is inserted into a column of water, and the height of the tube is adjusted until strong resonance is heard. At approximately what length of the air column will this happen?

(A) 100 cm
(B) 75 cm
(C) 50 cm
(D) 25 cm

30. How many different directions can a two-dimensional vector have if its components are of equal magnitude?

(A) One, at 45°
(B) Two
(C) Four
(D) Infinite

31. Two vectors, $\vec{A}$ and $\vec{B}$, have components $A_x = -2$, $A_y = 3$, $B_x = 5$, and $B_y = 1$. What is the approximate magnitude of the vector $\vec{A} + \vec{B}$?

(A) 3
(B) 4
(C) 5
(D) 7

32. As the angle between two vectors increases from 0° to 180°, the magnitude of their resultant

(A) increases, only
(B) increases and then decreases
(C) decreases, only
(D) decreases and then increases

33. At what angle should a projectile be launched in order to achieve the maximum range for a given initial velocity under no air resistance?

(A) 90°
(B) 30°
(C) 45°
(D) 60°

34. An object is dropped from a height of 45 m. Neglecting air resistance, what is the approximate velocity of the object as it hits the ground?

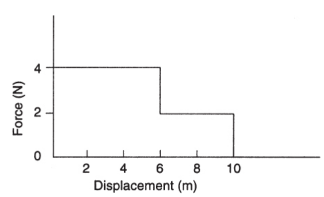
(A) 10 m/s
(B) 15 m/s
(C) 20 m/s
(D) 30 m/s

35. A boat moving due north crosses a river 240 meters wide with a velocity of 8 meters per second relative to the water. The river flows east with a velocity of 6 meters per second. How far downstream will the boat be when it has crossed the river?

(A) 240 m
(B) 180 m
(C) 420 m
(D) 300 m

QUESTIONS 36 AND 37 ARE BASED ON THE FOLLOWING INFORMATION:

A variable force acts on a 2-kilogram mass according to the graph below.



36. How much work was done while displacing the mass 10 meters?

(A) 40 J
(B) 38 J
(C) 32 J
(D) 30 J

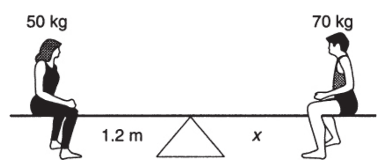
37. What was the average force supplied to the mass for the entire 10-meter displacement?

(A) 3.2 N
(B) 1.2 N
(C) 4.4 N
(D) 4 N

38. A man weighing himself is standing on a bathroom scale in an elevator that is accelerating upward at a rate of 0.5 meter per second squared. By what percentage is the reading of the scale off from the person's true weight?

(A) 0% (accurate)
(B) 5% too high
(C) 5% too low
(D) 0.5% too high

39. A 50-kilogram person is sitting on a seesaw 1.2 meters from the balance point. On the other side, a 70-kilogram person is balanced. How far from the balance point is the second person sitting?



(A) 0.57 m
(B) 0.75 m
(C) 0.63 m
(D) 0.86 m

40. An object rolls down a steep incline with very little friction. At the same time, an object of equal mass slides down a similar but frictionless incline. Which one takes less time to get to the bottom?

(A) They take the same time.
(B) The rolling object takes less time.
(C) The sliding object takes less time.
(D) The answer depends on the rolling object's moment of inertia.

41. What is the value of g at a position above Earth's surface equal to Earth's radius?

(A) 9.8 N/kg
(B) 4.9 N/kg
(C) 2.45 N/kg

(D) 1.6 N/kg

42. If it takes a minimum of 5 N to move 2 C of charge away from some other charge, how much energy is needed to move 3 C of charge 40 cm in the same situation?

(A) 2.5 J
(B) 7.5 J
(C) 2.0 J
(D) 3.0 J

43. If an object is spinning at 150 RPM (revolutions per minute) and comes to stop in 2 seconds, what is its average acceleration in radians/s^2?

(A) -2.5π
(B) -2π
(C) -5π
(D) -10π

44. What is the work done by a horizontal spring (spring constant k) expanding from a compression distance x to an extension distance x to an attached mass?

(A) $2kx^2$
(B) $\frac{1}{2}kx^2$
(C) kx^2
(D) 0

45. If an isolated spinning object's moment of inertia is reduced by a factor of 3 by internal forces, how will its angular momentum change?

(A) Angular momentum will be 3 times its previous value.
(B) Angular momentum will be reduced to 1/3 its previous value.
(C) Angular momentum will be reduced to 1/9 its previous value.
(D) Angular momentum will remain unchanged.

46. If the tension in a taut string is increased, which of the following will also be increased when the fundamental frequency is struck? Select two correct answers.

(A) The velocity of propagation
(B) The frequency of the fundamental
(C) The wavelength of the fundamental
(D) The amplitude of the wave

47. A friend is balancing a fork on one finger. Which of the following are correct explanations of how he accomplishes this? Select two correct answers.

(A) Total energy is conserved.
(B) The fork's moment of inertia is zero.
(C) The fork's center of mass is above his finger.
(D) The fork's clockwise torque is equal to its counterclockwise torque.

48. Two tuning forks are struck at the same time. A beat frequency of 12 beats per second is observed. If one tuning fork has a frequency of 384 Hz, what could be the frequency of the second tuning fork? Select two correct answers.

(A) 260 Hz
(B) 372 Hz
(C) 396 Hz
(D) 408 Hz

49. A book rests on top of a table. Which of the following are action-reaction pairs described by Newton's third law? Select two correct answers.

(A) The weight of the book and the normal force of the table upon the book
(B) The weight of the book and the weight of the table
(C) The weight of the table and the upward pull of the table on Earth
(D) The normal force upon the book from the table and the downward force on the table due to the book

50. Consider the impulse received by the first car in each of the following cases. In each case, the cars are at rest after the collision. In which two of the following cases will the car receive the same impulse? Select two correct answers.

(A) A 5,000-kg car traveling at 10 m/s has a head-on collision with an equal and oppositely directed second car.
(B) A 5,000-kg car traveling at 10 m/s has a head-on collision with a large building.
(C) A 2,500-kg car traveling at 10 m/s has a head-on collision with an equal and oppositely directed second car.
(D) A 2,500-kg car traveling at 5 m/s has a head-on collision with an equal and oppositely directed second car.

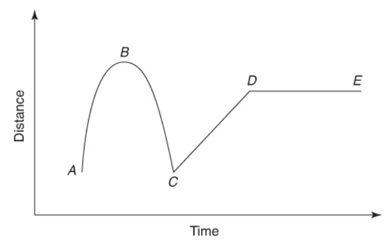
SECTION II: FREE-RESPONSE

Time: 90 minutes

5 questions

DIRECTIONS: You have 90 minutes to complete this portion of the test. You may use a calculator and the information sheets provided in the appendix.

1. (7 points; ~13 minutes) Use the sketch to answer the following questions.



(a) Use the qualitative keywords below to fill in the chart beneath with a brief description of the motion:

List 1: constant, increasing, decreasing

List 2: negative, positive, zero

Time Interval	Velocity (One Word from List 2)	Speed (One Word from List 1)	Acceleration (One Word from List 2)
$A \rightarrow B$			
$B \rightarrow C$			
$C \rightarrow D$			
$D \rightarrow E$			

(b) Which is faster, the average velocity from *C* to *D* or the average velocity from *C* to *E*? Explain your reasoning.

(c) What is the instantaneous velocity at point *B*? Explain your reasoning.

(d) Pick a couple of points on this graph that are not a realistic representation of a real-world object's motion. Explain what is problematic about these two points.

(e) Sketch a velocity versus time graph from these data. Label points *A, B, C, D*, and *E*.

2. (12 points; ~25 minutes) A group of students are given the following supplies: a stopwatch, a long string, various metersticks and protractors, and a large supply of various styles of predetermined masses.

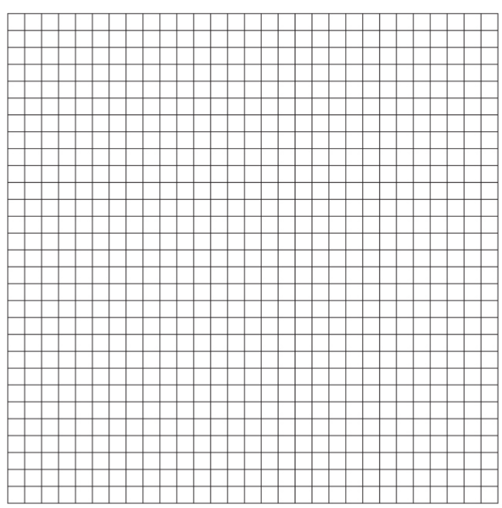
(a) Describe three short experimental procedures to determine the dependency of a simple pendulum's period of oscillation on amplitude, mass, and length. You may include a labeled diagram of your setup to help in your description. Indicate what measurements you would take and how you would take them. Include enough detail so that another student could carry out your procedure.

(b) What are the expected results of each investigation? Sketch out what the data will look like in each of the three investigations (amplitude, mass, and length).

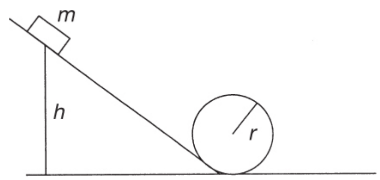
(c) What are the common sources of error or expected deviations from ideal results that might happen during this investigation? Which of the three investigations might you expect to deviate the most from the ideal results and why?

(d) Here are some raw data taken from the length vs. period investigation by a student who suspects there is a correlation between the two. Determine which variables to graph to produce a linear function. Graph the new data below, and interpret the slope of your best-fit line.

Length (cm)	Period (s)	?
10	0.62	
20	0.90	
30	1.09	
40	1.28	
55	1.48	
75	1.75	
85	1.85	



3. (12 points; ~25 minutes) A mass m is resting at a height h above the ground. When released, the mass can slide down a frictionless track to a loop-the-loop of radius r as shown below.



(a) As the mass slides down the incline, it gains speed. However, the mass may or may not "make it" around the loop without falling out. Why and under what conditions will the mass travel around the loop without falling out? Why and under what conditions will it not complete the loop? Explain your reasoning qualitatively, making sure you address the normal force experienced by the mass.

(b) Two fellow students are arguing about whether the maximum possible r of a loop that the mass can make it around will be linearly dependent on height h or if the relationship will be some power law. Without solving for the relationship explicitly, indicate a line of reasoning that would settle this argument.

(c) Within the loop, the normal force is always acting in a centripetal direction. Explain why the normal force is not always equal to mv^2/r at each point. Are there any points along the loop where the normal force is equal to mv^2/r? Support your argument with free-body diagrams.

(d) Using quantitative reasoning, derive an expression for the normal force in terms of mg when the mass is halfway up the loop. Assume that the minimum height, h, that enables a complete loop has been chosen.

(e) If the minimum height h has been found for a fixed loop of size r, but a rolling ball of mass m is substituted for the sliding mass, the ball will (indicate your choice):

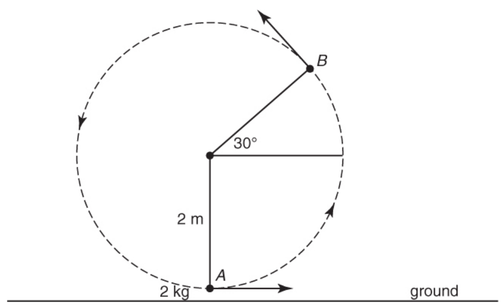
_____ make it through the loop more easily

_____ pass through the loop the same as before

_____ fall out of the loop

Justify your answer qualitatively, with no equations or calculations.

4. (7 points; ~13 minutes) A 2-kilogram mass is twirled in a vertical circle as shown. It is attached to a 2-meter rope. As the mass just clears the ground (point A), its velocity is 10 m/s to the right. Neglect any air resistance. When the mass reaches point B, it makes a 30-degree angle to the horizontal as shown.



(a) Describe qualitatively, using words and diagrams, the difference in the object's speed at point *B* if the twirling is done (i) at constant speed or (ii) at constant tension. Which would result in the greater speed at point *B*? Include an analysis of how the tension must change in part (i).

(b) At point *B*, the rope is released and the mass becomes a projectile. Assume the rope has a uniform distribution of mass and remains attached to the flying object. Which of the following options best describes the projectile's range as compared with a launch without the rope?

_____ addition of rope makes no difference to the range

_____ addition of rope increases the range

_____ addition of rope decreases the range

Explain your reasoning.

(c) When twirling the object at constant speed, explain in terms of work and energy how the constant speed is maintained. Is the path truly circular when the object's speed is maintained? In your explanation, be explicit about which forces are doing positive work and which are doing negative work. Your answer should be in a clear, coherent paragraph.

5. (7 points; ~13 minutes) You are given three 2-ohm resistors, some wire, a variable DC voltage supply, a voltmeter, and an ammeter.

(a) Draw a schematic diagram of a circuit that will produce an equivalent resistance of 3 ohms, including ammeter and voltmeter, as appropriate, to be able to measure the circuit current and source voltage.

(b) Which, if any, of these components are assumed to have zero internal resistance? Which, if any, of these components are assumed to have infinite resistance? Justify your choices.

(c) Trace out one complete loop in your circuit, and prove Kirchhoff's loop rule. Once you set up your rule, assume a setting of 6 volts on the power supply. Verify your rule numerically.

(d) A student wishes to measure the voltage and current in a simple circuit using small lightbulbs instead of commercially manufactured resistors. She finds that after a short while, the current in the ammeter is decreasing. How might she account for this?

ANSWER KEY

1. **D**
2. **C**
3. **D**
4. **B**
5. **B**
6. **C**
7. **C**
8. **B**
9. **B**
10. **A**
11. **D**
12. **D**
13. **A**
14. **A**
15. **C**
16. **B**
17. **C**
18. **D**
19. **D**
20. **A**
21. **B**
22. **B**
23. **D**
24. **D**

25. B
26. B
27. B
28. C
29. D
30. C
31. C
32. C
33. C
34. D
35. B
36. C
37. A
38. B
39. D
40. C
41. C
42. D
43. A
44. D
45. D
46 A, B
47. C, D
48. B, C
49. C, D
50. A, B

ANSWERS EXPLAINED

Section I: Multiple-Choice

1. **(D)** Twice the initial vertical velocity will give twice the time in flight. Average vertical velocity will also be doubled. Displacement is the product of these two: 2 × 2 = 4.

   Alternatively, one could use $v^2 = v_0^2 + 2ad$, since v at the maximum height is zero. Since v_0 is doubled and then squared, the vertical displacement d must be 4 times bigger.

   Free-fall problems are independent of mass.

2. **(C)** Conservation of momentum:

$$\begin{aligned} p_{\text{tot}} &= 2(+4) + 5(-1) = +3 \text{ kg m/s} = mv = (7 \text{ kg})v \\ v &= +3/7 \text{ m/s} \end{aligned}$$

   The collision is inelastic since the carts stick together.

3. **(D)** Higher altitude is strictly a function of V_Y (Projectile Y). Range is a function of both V_X and V_Y such that the angles equally above and below 45 degrees (the max angle for range) will result in equal horizontal displacement. Note that 30 degrees and 60 degrees are both 15 degrees off from 45 degrees.

4. **(B)** Since the 10 N force will start the mass moving, this must be greater than the static friction force holding the mass in place. The maximum *static* friction must be 10 N or less.

$$\begin{aligned} F_s &< 10 \text{ N} \\ \mu N &< 10 \text{ N} \\ \mu mg &< 10 \text{ N} \\ \mu(5)(10) &< 10 \\ \mu &< 1/5 \end{aligned}$$

5. **(B)** The object is moving, so the velocity is not zero. The object is not accelerating, so velocity is constant.

6. **(C)** In general:

$$y(t) = y_0 - v_{y0} - \tfrac{1}{2}gt^2$$
$$x(t) = v_{x0}t$$

Solve for t in the last equation. Then plug back into the first equation and substitute in $y_0 = h$, $v_{y0} = 0$, and $v_{x0} = v_0$:

$$y\left(x\right) = h - \left(0\right)\left(x/v_0\right) - \frac{1}{2}g(x/v_0)^2 = h - gx^2/\left(2v_0{}^2\right)$$

7. **(C)** Maximum acceleration allows you to determine maximum displacement:

$$ma = kA$$

Knowing the amplitude allows you to determine easily the maximum speed via energy conservation:

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2$$

8. **(B)** Maximum velocities happen when going through the equilibrium point (zero acceleration and zero displacement): all kinetic energy (*KE*) and no potential energy (*PE*).

9. **(B)** Momentum is conserved during the collision, which enables us to solve for an initial upward velocity of the combination. Then energy conservation can be used to relate the height to that initial upward velocity. Choice (A) cannot be used because some unknown amount of mechanical energy will be lost by the bullet embedding itself in the wood.

10. **(A)** Constant velocity means no acceleration, so $\overrightarrow{F}_{\text{net}} = 0$.

$$\begin{aligned} F_{\text{push}} - F_{\text{friction}} &= 0 \\ F_{\text{push}} - \mu_k N = F_{\text{push}} - \mu_k mg &= 0 \\ F_{\text{push}} = \mu_k mg = \left(0.1\right)\left(10 \text{ kg}\right)\left(10 \text{ m/s}^2\right) &= 10 \text{ N} \end{aligned}$$

11. **(D)** First find the acceleration of the system:

$$\begin{aligned} \vec{F}_{\text{net}} &= m\vec{a} \\ 10\text{ N} &= (4+1\text{ kg})a \\ a &= 2\text{ m/s}^2 \end{aligned}$$

The contact force, P, is the only force felt by the 1-kg mass:

$$\vec{F}_{\text{net}} = m\vec{a}$$
$$P = \left(1\text{ kg}\right)\left(2\text{ m/s}^2\right) = 2\text{ N}$$

12. **(D)** $P = \text{work}/t = Fd/t = mgh/t$

13. **(A)** Impulse can be found by Ft if the details of the force are known or, alternatively, by:

$$\text{Impulse} = \Delta p = (1{,}500\text{ kg})(0\text{ m/s} - 25\text{ m/s}) = -37{,}500\text{ N}\cdot\text{s}$$

Ignore the minus sign as the question asks about magnitude.

14. **(A)** When the block is at the maximum height, static friction is obtained:

$$\begin{aligned} \vec{F}_{\text{net}} = mg\sin\theta - \mu N &= 0 \\ mg\sin\theta - \mu\left(mg\cos\theta\right) &= 0 \\ \mu = (\sin\theta)/(\cos\theta) &= \tan\theta \end{aligned}$$

15. **(C)** Doubling the period requires a quadrupling of length:

$$T = 2\pi(\ell/g)^{½}$$

16. **(B)** Impulse = area of F vs. T graph = $\Delta p = mv_f - 0$

Solving for v_f = (area under graph)/m

17. **(C)** Conservation of energy:

$$\begin{aligned} \tfrac{1}{2}mv^2 &= mg\Delta h \\ \tfrac{1}{2}v^2 &= g\Delta h = \left(10\ \text{m/s}^2\right)(2.25\ \text{m} - 0.75\ \text{m}) \\ v &= (30)^{1/2} = 5.5\ \text{m/s} \end{aligned}$$

18. **(D)** Gravitational field strength halfway between any two equal masses is always zero as each contributes oppositely directed gravitational fields.

19. **(D)** Conservation of energy while on the frictionless hill can give the speed at the top of the 10-m hill:

$$\frac{1}{2}mv^2 = KE = mg\Delta h$$

Then impulse equals change in momentum can be used since the final momentum must be zero.

20. **(A)**

$$\begin{aligned} \text{Mechanical energy} &= mgh + \tfrac{1}{2}mv^2 \\ \text{Energy "lost" or "gained"} &= mgH_2 + \tfrac{1}{2}mv_2{}^2 - \left(mgH_1 + \tfrac{1}{2}mv_1{}^2\right) \end{aligned}$$

Work by forces other than gravity = change in energy

21. **(B)** Energy conservation:

$$\frac{1}{2}kx^2 = mgh$$

Solving for h:

$$h = kx^2/2mg$$

Doubling x will quadruple the height, whereas all other factors will only double the height.

22. **(B)** Circular motion at the top:

$$F_{\text{net}} = mg + N = mv^2/r$$

The lowest speed will be when there is no normal force ($N = 0$):

$$mg = mv^2/r$$
$$v_{\text{top}} = (gr)^{1/2}$$

Kinetic energy at the bottom must give both this speed and potential energy to gain $2r$ in height:

$$\begin{aligned} E_{\text{bottom}} &= E_{\text{top}} \\ \tfrac{1}{2}mv^2 &= gm(2r) + \tfrac{1}{2}mv_{\text{top}}{}^2 \\ \tfrac{1}{2}mv^2 &= gm(2r) + \tfrac{1}{2}m\,(gr) \end{aligned}$$

Solving for v:

$$v = (5gr)^{1/2}$$

23. **(D)** The two 2-Ω resistors are in series with each other: 2 Ω + 2 Ω = 4 Ω.

This 4-Ω equivalent resistance is in parallel with the existing 4-Ω resistor: ¼ Ω + ¼ Ω = ½ Ω. So the equivalent resistance for that section of the circuit is 2 Ω. This 2-Ω equivalent resistance is in series with the 3-Ω resistor: 2 Ω + 3 Ω = 5 Ω.

24. **(D)** The 15 volts must be split between the 3-ohm resistor and the 2-ohm equivalent resistance of the right-hand side (see answer 23). Voltage for the 4-ohm resistor is the same as the voltage across the 2-ohm equivalent resistance of the right-hand side:

$$V = (2/5)(15\text{ V}) = 6\text{ V}$$

25. **(B)** The 15-volt battery will supply 3 amps of current for the 5-ohm circuit:

$$\begin{aligned} V &= IR \\ 15\text{ V} &= (3\text{A})\,(5\text{ ohms}) \end{aligned}$$

This 3-amp current will split as it comes to the branching point before the 4-ohm resistor. Since both pathways have equal resistance (4 ohms), the current will split evenly: 1.5 amps.

26. **(B)** Resistance is determined by:

$$\rho L/A$$

The same material means the resistivity, ρ, is the same. Letting $L \rightarrow L/2$ and $A \rightarrow 2A$:

$$\rho L/A \rightarrow \rho(L/2)/(2A) = (\rho L/A)/4$$

27. **(B)** The wavelength is set by the length of the standing wave and the number of the harmonic. Therefore, the wavelength remains the same. The frequency of the wave must have been raised to correspond to the higher wave speed: $\lambda f = v$.

28. **(C)** The Doppler effect can be used to determine relative speed toward or away from the source by looking at frequency shift in the reflected wave. Note that the velocity vector cannot be determined. Components of velocity not directed toward or away from the receiver do not contribute to the Doppler shift.

29. **(D)** The fundamental frequency in a half-pipe is ¼ wavelength:

$$\text{Wavelength} = \text{wave speed/frequency} = 340/340 = 1 \text{ m}$$

The pipe needs to be ¼ of this: 0.25 m = 25 cm.

30. **(C)** Any 45-45-90 triangle has sides of equal length. Therefore, a 45° vector would have equal length components. So would 135°, 225°, and 315°, as these vectors all form 45-45-90 triangles with their components.

31. **(C)** Add the components of the vectors:

$$A_x + B_x = -2 + 5 = 3$$
$$A_y + B_y = 3 + 1 = 4$$

Use the Pythagorean theorem on these net components:

$$3^2 + 4^2 = 5^2$$

32. **(C)** Maximum vector addition is when the two vectors are aligned (0°) and minimum is when they are opposite (180°). All values in between will steadily decrease the resultant from maximum to minimum.

33. **(C)** Maximum range is at 45° since this gives decent time in flight and decent horizontal velocity.

34. **(D)** First, find the time in flight:

$$\begin{aligned} h &= \tfrac{1}{2}(10)t^2 \\ 45 &= 5t^2 \\ t &= 3 \text{ s} \end{aligned}$$

Now use this time to find the final speed:

$$v_f = at = (10 \text{ m/s}^2)(3 \text{ s}) = 30 \text{ m/s}$$

35. **(B)** Find time to cross the river using only perpendicular components:

$$\begin{aligned} D &= V_y t \\ 240 &= 8t \\ t &= 30 \text{ s} \end{aligned}$$

Next find the downstream distance using the parallel component:

$$\begin{aligned} D &= V_x t \\ D &= (6 \text{ m/s})(30 \text{ s}) = 180 \text{ m} \end{aligned}$$

36. **(C)** Work is the area under the curve:

$$(4 \text{ N})(6 \text{ m}) + (2 \text{ N})(4 \text{ m}) = 24 \text{ J} + 8 \text{ J} = 32 \text{ J}$$

37. **(A)** work $= F_{\text{average}} \cdot$ displacement

$$\begin{aligned} 32 \text{ J} &= (F_{\text{average}})(10 \text{ m}) \\ F_{\text{average}} &= 3.2 \text{ N} \end{aligned}$$

38. **(B)** The bathroom scale reads the normal force:

$$\begin{aligned} F_{\text{net}} = N - mg &= ma \\ 0.5 \text{ m/s}^2 &= 0.05g \\ N - mg &= m(0.05g) \\ N &= m(1.05g) \end{aligned}$$

The reading will be 5% too high.

39. **(D)** Balanced means the torques are equal and opposite:

$$\begin{aligned} 50 \text{ kg} \cdot g \cdot 1.2 &= 70 \text{ kg} \cdot g \cdot x \\ x &= 6/7 = 0.86 \text{ m} \end{aligned}$$

40. **(C)** Both objects start with the same potential energy. However, the rolling object must use some of that potential energy for rotational energy, leaving less for linear kinetic energy. Therefore, the rolling object moves more slowly down the hill.

41. **(C)** Universal gravity (and the gravitational field) are $1/r^2$ laws; doubling r will quarter the field. (1/4)g = (1/4)(9.8 N/kg) Note that N/kg = m/s^2.

42. **(D)** First, both situations will have the same force to charge ratio:

$$F/q = 5/2 \text{ N/C}$$

Next, find the force for the new charge:

$$(5/2)(3) = 7.5 \text{ N}$$

Now, find the work done:

$$W = F \cdot D = (7.5 \text{ N})(0.4 \text{ m}) = 3 \text{ J}$$

43. **(A)** First, convert RPMs to rad/s:

$$150 \text{ rev/min} \times (2\pi/1 \text{ rev}) \times (1 \text{ min}/60 \text{ s}) = 5\pi \text{ rad/s}$$

$$\text{acceleration} = \text{change in velocity/time} = (0 - 5\pi \text{ rad/s})/(2 \text{ s}) = -2.5\pi \text{ rad/s}^2$$

44. **(D)** $\frac{1}{2}kx^2$ of work is delivered to the mass while uncompressing, followed by $-\frac{1}{2}kx^2$ done while the mass extends the spring outward, totaling to 0 net work for the entire expansion. Alternatively, think about the velocity being zero at the beginning and end of that single oscillation. No change in kinetic energy occurs; therefore, no net work is done.

45. **(D)** No external torque means momentum must be conserved.

46. **(A) and (B)** Higher tension means waves will propagate at higher speeds. Since the wavelength of the standing wave is fixed by the length of the string, the frequency of the fundamental must also increase.

47. **(C) and (D)** An object's center of mass is the balancing point by definition because the moment arm on one side is the same as on the other. Therefore, the torque caused by gravity on either side will cancel.

48. **(B) and (C)** The beat frequency is caused by the difference between the two frequencies: 384 ± 12 = 396 or 372 Hz.

49. **(C) and (D)** Action-reaction pairs must be the same type of force and found on each partner of the force exchanging objects. The weight of an object is caused by the entire planet. Therefore, that reaction force is on the planet itself. Likewise, the normal forces exchanged between the bottom surface of the book and top surface of the table are an action-reaction pair.

50. **(A) and (B)** The cars will both experience an impulse of 50,000 N · s. The other two cases result in smaller impulses.

Section II: Free-Response

1. (a) (3 points; 1 point for each column in the table)

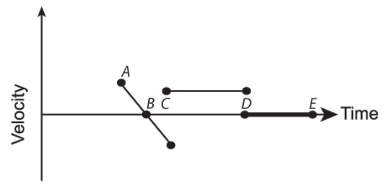
Time Interval	Velocity (One Word from List 2)	Speed (One Word from List 1)	Acceleration (One Word from List 2)
$A \rightarrow B$	positive	decreasing	negative
$B \rightarrow C$	negative	increasing	negative
$C \rightarrow D$	positive	constant	zero
$D \rightarrow E$	zero	constant	zero

(b) (1 point) The slope from *C* to *D* is steeper than the slope drawn from *C* to *E*. Therefore, the average velocity is greater from *C* to *D*.

(c) (1 point) Point *B* is a turnaround point. The slope of a tangent line drawn at that point would be horizontal. For these reasons, the instantaneous velocity at this point is zero.

(d) (1 point) Points *C* and *D* are not continuous, which means the slope at these points is undefined. Therefore, the instantaneous velocity at these two points does not exist. Real-world motion is always continuous.

(e) (1 point)

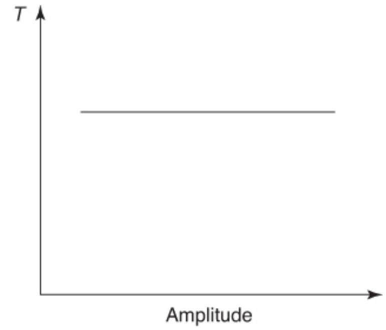


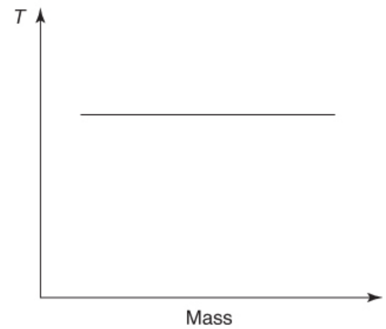
2. (a) (3 points) This is really three separate investigations. In each case, students should measure the period repeatedly and then take the mean value.

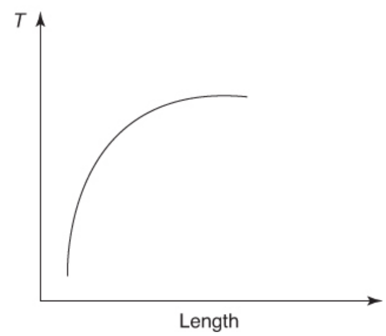
1. (1 point) Amplitude versus period. Take a period measurement for 5 to 10 different amplitudes while keeping the mass and length the same. The amplitude can be controlled by pulling the string-mass combo out to a certain angle as measured by the protractor.
2. (1 point) Mass versus period. Take a period measurement for 5 to 10 different masses while keeping the amplitude and length constant.
3. (1 point) Length versus period. Take a period measurement for 5 to 10 different lengths while keeping the mass and amplitude constant. Length is measured from the pivot point to the center of mass.

(b) (3 points; 1 point for each graph) No correlation is expected for amplitude and mass variations. A nonlinear relationship is expected for length and period:

$$T_p = 2\pi(L/g)^{1/2}$$







(c) (2 points) Beyond the usual random errors of measurement (especially when using a timer but minimized by taking the median value of a few trials each time), one predictable deviation is in the amplitude investigation. Pendulums actually behave as simple harmonic oscillators only under the conditions of small angles (small enough that sin θ is approximately θ). A large enough amplitude will require the pendulum to oscillate at larger angles. This means that at large amplitudes, one can predict the results to deviate from the expected as the gravitational force no longer acts as a simple restorative force. (1 point)

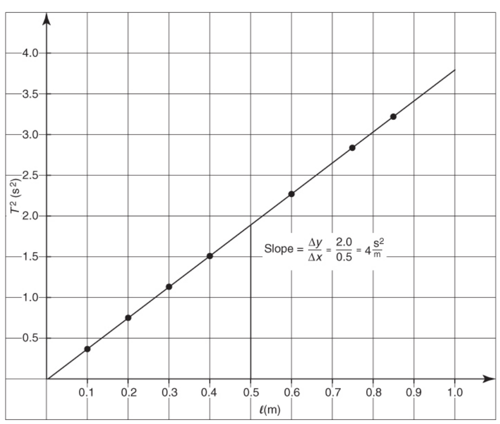
One other possible source of systematic error would be in the mass investigation. As various masses are swapped out on a fixed length of string, the students may inadvertently be changing the length of the string when adding on different-sized masses. The length of the pendulum is from the pivot point to the *center of mass*. If the students did not compensate for this when adding larger masses by shortening the string, they may see an artificial relationship at higher masses in their graph of mass versus period. (1 point)

(d) (4 points) To obtain a linear graph, one must plot T^2 versus L:

$$T^2 = (4\pi^2/g)L$$

The slope of this plot would then be equal to $4\pi^2/g$. Alternatively, one could plot T versus $\sqrt{L}$. (1 point for the table; 2 points for the graph)

Length (cm)	Period (s)	T^2 (s^2)
10	0.62	0.38
20	0.90	0.81
30	1.09	1.19
40	1.28	1.64
55	1.48	2.19
75	1.75	3.06
85	1.85	3.42



The measured slope is 4 s^2/m. It can be compared to the theoretical prediction of slope:

$$4\pi^2/g = 4.02 \text{ s}^2/\text{m} \quad \text{(1 point)}$$

3. (a) (2 points) As the value h is raised, the mass gains velocity at the bottom of the incline. As the mass slides around the inside of the loop, it will lose speed as it goes back up. The entire time the mass is inside the loop, it must be undergoing centripetal acceleration (specifically, its v^2/r must equal its inward acceleration). If the mass is moving too slowly to maintain its circular motion, it will fail to complete the loop. The centripetal force is supplied by the normal force throughout the trajectory (sometimes also with or against the force of gravity). If the normal force goes to zero, this indicates a loss of contact between the sliding mass and the track. The slowest speed will occur at the top of the loop. Therefore, this is the most likely place for the mass to fall. At the top, gravity is acting centripetally along with the normal force, so Newton's second law gives us:

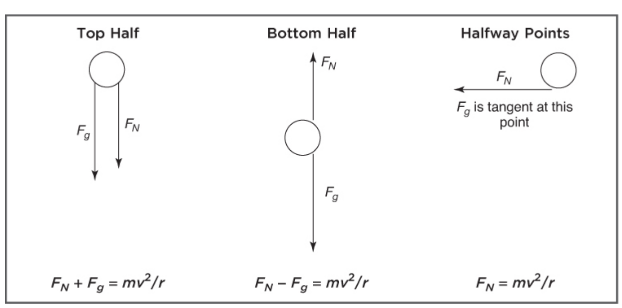
$$N + mg = mv^2/r \quad \text{(1 point)}$$

As h gets smaller, the speed at the top will get smaller, which will decrease the right-hand side of the equation above. At some point, mv^2/r will get as small as mg, at which point the normal force will be at zero. Any speed greater than this will let the mass complete the loop. Any speed lower than this will cause the mass to fail to complete the loop. (1 point)

(b) (2 points) The object has gravitational potential energy in the beginning, which is linearly proportional to h. Part of the potential energy is transformed into kinetic energy at the top of the loop. Kinetic energy is proportional to v^2. However, centripetal force is also proportional to v^2, and the centripetal force is linear with respect to the normal force. It is this normal force going to zero that determines the critical speed. So the student who claims there is a linear dependence between h and r is correct.

(c) (3 points) Because the normal force is not always the only centripetal force, it alone is not equal to mv^2/r. Generally speaking, a component of the force due to gravity is acting centripetally (top half of the loop) or centrifugally (bottom half of the loop). However, at the points halfway between the top and bottom, the normal force indeed is acting alone centripetally and is equal to mv^2/r.

The following drawings show only the radial component of the two forces (F_N, F_g).



Tangent components of gravity speed up or slow down the object but do not participate in the centripetal acceleration.

(d) (3 points) First, determine the velocity at the top of the loop by setting $N = 0$ in the expression for net force at the top of the loop from part (a):

$$v_{\text{top}}{}^2 = gr \quad (1\text{ point})$$

The mechanical energy is conserved. Set the expression at the top of the loop equal to that at midpoint:

$$mg\left(2r\right) + \frac{1}{2}mv_{\text{top}}{}^2 = mg\left(r\right) + \frac{1}{2}mv^2$$

Substitute in our expression for v_{top}:

$$mg\left(2r\right) + \frac{1}{2}m\left(gr\right) = mgr + \frac{1}{2}mv^2$$

Solving the above for the velocity at the halfway point:

$$v^2 = 3gr \quad (1\text{ point})$$

Using our expression from part (c), the normal force is the centripetal force at this position:

$$F_N = \frac{mv^2}{r} = m \times \frac{3gr}{r} = 3mg \quad (1\text{ point})$$

(e) (2 points) __X__ fall out of the loop

Since the original gravitational potential energy must now be going into rotational kinetic energy, less energy will be available for the tangential speed needed to make it through the loop. Since the height, h, was set minimally for the sliding block, the rolling object will not have enough linear kinetic energy for its tangential speed to clear the top of the loop.

4. (a) (4 points)

(i) (2 points) To maintain constant speed, the tension must be constantly adjusted as the object undergoes its centripetal motion. The tension will be lower at point B than it is at the bottom as there is a component of gravity “helping” the tension in the centripetal direction (as opposed to gravity acting centrifugally at point A):

$$
\begin{aligned}
F_{cA} &= F_{cB} = mv^2/r \\
T_A - mg &= T_B + mg\sin 30^\circ = mv^2/r \\
T_B &= T_A - mg\,(1 + \sin 30^\circ)
\end{aligned}
$$

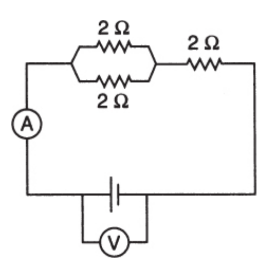
(ii) (2 points) Constant tension will result in a much higher speed at point *B* as the tension will be locked in at T_A above. Since the tension at point *B* will be higher than in case (i), mv^2/r must be greater as well. Constants *m* and *r* mean the speed must be increased.

(b) (1 point) __X__ addition of rope decreases the range

Although we are ignoring air friction in this problem, we must consider the effect of the additional mass. Although free-fall problems generally ignore mass, the rope will have an effect on the effective launch velocity. The end of the rope directly attached to the twirling mass has the same velocity as the object. However, the rest of the distributed mass is moving at a lower velocity (down to zero at the center of the circle). Therefore, the net effect of the added mass will be to decrease the launch velocity. In addition, recall that the equations of motion actually track the center of mass of an object. Since the center of mass for the combined rope-plus-object system is lower (the mass of the rope extends below the 2-kg mass), the combined rope-plus-object system is effectively being launched from a lower height. A lower launch velocity from a lower height will result in a shorter range for the projectile.

(c) (2 points) While on the way up during the circular motion, the force due to gravity is doing negative work. The component of weight that is tangent does this negative work. Therefore, to maintain the same kinetic energy, the tension must do some positive work. In order to accomplish this, the tension must supply an upward tangent component as well; that is, the circle cannot be perfect. On the downward half of the circular motion, the positive and negative work roles of the tangent components of gravity and tension must reverse.

5. (a) (2 points)



(b) (2 points) Ammeters, wires, and power supplies are assumed to have zero internal resistance as they have no resistance indicated in the circuit diagram. If there were any resistance, unaccounted losses of voltage would occur in our circuit. When modeling actual resistance inside of components, an “internal” resistor is added to the circuit diagram. (1 point)

Voltmeters would create a parallel path if they did not have infinite resistance. To the extent they have less than infinite resistance, the addition of voltmeters to the circuit would effectively lower the resistance, thereby lowering the voltage reading. (1 point)

(c) (2 points) Kirchhoff’s loop rule states that the sum of voltages changing around the loop must be zero. As an example of this, start to the right of the power supply and trace a loop clockwise through the top parallel resistor and then the resistor to the right and back to our starting point. Tracing our changes in voltage:

$$+V - (I/2)R - IR = 0 \quad \text{(1 point)}$$

For the entire circuit:

$$\begin{aligned} V &= IR \\ 6 &= I(3) \\ I &= 2 \text{ amps} \quad \text{(1 point)} \end{aligned}$$

Loop rule:

$$+6 - (1)(2) - (2)(2) = 6 - 2 - 4 = 0$$

(d) (1 point) The lightbulbs get hot very quickly, which increases their electrical resistance. This, in turn, reduces the current measured by the ammeter.

TEST ANALYSIS

PRACTICE TEST 1

Section 1: Multiple-Choice

Note that the questions requiring two answers are to be graded as completely correct (1 point) or incorrect (0 points, even if you have one of the two answers correct).

$$\text{Number correct (out of 50)} = \text{Multiple-Choice Score}$$

Section 2: Free-Response

Partial credit is awarded for any correct responses within an individual free-response question.

Question 1 = (out of 7)

Question 2 = (out of 12)

Question 3 = (out of 12)

Question 4 = (out of 7)

Question 5 = (out of 7)

Total 4 = (out of 45)

Final Score

$$\text{Multiple-Choice Score} + \left(1.11 \times \text{Free-Response Score}\right) = \text{Total (out of 100)}$$

Final Score Range*

Final Score Range	AP Score
71–100	**5**
56–70	**4**
41–55	**3**
26–40	**2**
0–25	**1**

***Note: The guidelines above are based on the released scores for past AP Physics 1 exams. Actual score ranges vary from year to year and are determined by the College Board each year. Thus, the ranges shown are approximate.**