

**DIGITAL
AP CALCULUS
MOCK TEST 1
Solution Book**

**Unit 1: Limits
and Continuity**

ThrivingScholars 

Unit 1: Limits and Continuity

Part A. Directions: Answer Questions *without* using your calculator.

- $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 4}$ is
(A) 1
(B) 0
(C) $-\frac{1}{2}$
(D) -1
(E) ∞
 - $\lim_{x \rightarrow 2} \frac{4 - x^2}{x^2 - 1}$ is
(A) 1
(B) 0
(C) -4
(D) -1
(E) ∞
 - $\lim_{x \rightarrow 3} \frac{x - 3}{x^2 - 2x - 3}$ is
(A) 0
(B) 1
(C) $\frac{1}{4}$
(D) ∞
(E) none of these
 - $\lim_{x \rightarrow 0} \frac{x}{x}$ is
(A) 1
(B) 0
(C) ∞
(D) -1
(E) nonexistent
 - $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$ is
(A) 4
(B) 0
(C) 1
(D) 3
(E) ∞
 - $\lim_{x \rightarrow 2} \frac{4 - x^2}{4x^2 - x - 2}$ is
(A) -2
(B) $-\frac{1}{4}$
(C) 1
(D) 2
(E) nonexistent
 - $\lim_{x \rightarrow 2} \frac{5x^3 + 27}{20x^2 + 10x + 9}$ is
(A) $-\infty$
(B) -1
(C) 0
(D) 3
(E) ∞
 - $\lim_{x \rightarrow 2} \frac{3x^2 + 27}{x^3 - 27}$ is
- (B)** The limit as $x \rightarrow 2$ is $0 \div 8$.
 - (D)** Use the Rational Function Theorem. The degrees of $P(x)$ and $Q(x)$ are the same.
 - (C)** Remove the common factor $x - 3$ from numerator and denominator.
 - (A)** The fraction equals 1 for all nonzero x .
 - (D)** Note that $\frac{x^3 - 8}{x^2 - 4} = \frac{(x - 2)(x^2 + 2x + 4)}{(x - 2)(x + 2)}$.
 - (B)** Use the Rational Function Theorem.
 - (A)** Use the Rational Function Theorem.
 - (E)** Use the Rational Function Theorem.

- (A) 3
- (B) ∞
- (C) 1
- (D) -1
- (E) 0

9. $\lim_{x \rightarrow \infty} \frac{2^{-x}}{2^x}$ is

- (A) -1
- (B) 1
- (C) 0
- (D) ∞
- (E) none of these

10. $\lim_{x \rightarrow -\infty} \frac{2^{-x}}{2^x}$ is

- (A) -1
- (B) 1
- (C) 0
- (D) ∞
- (E) none of these

11. $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$

- (A) = 0
- (B) = $\frac{1}{5}$
- (C) = 1
- (D) = 5
- (E) does not exist

12. $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$

- (A) = 0
- (B) = $\frac{2}{3}$
- (C) = 1
- (D) = $\frac{3}{2}$
- (E) does not exist

13. The graph of $y = \arctan x$ has

- (A) vertical asymptotes at $x = 0$ and $x = \pi$
- (B) horizontal asymptotes at $y = \pm \frac{\pi}{2}$
- (C) horizontal asymptotes at $y = 0$
- (D) vertical asymptotes at $x = \pm \frac{\pi}{2}$
- (E) no asymptotes

14. The graph of $y = \frac{x^2 - 9}{3x - 9}$ has

- (A) a vertical asymptote at $x = 3$
- (B) a horizontal asymptote at $y = \frac{1}{3}$
- (C) a removable discontinuity at $x = 3$
- (D) an infinite discontinuity at $x = 3$
- (E) no asymptotes or discontinuities

15. $\lim_{x \rightarrow 0} \frac{\sin x}{x^2 + 3x}$ is

- (A) 1
- (B) $\frac{1}{3}$
- (C) 3
- (D) ∞
- (E) $\frac{1}{4}$

16. $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ is

- (A) ∞

9. (C) The fraction is equivalent to $\frac{1}{2^x}$; the denominator approaches ∞ .

10. (D) Since $\frac{2^{-x}}{2^x} = 2^{-2x}$, therefore, as $x \rightarrow -\infty$, the fraction $\rightarrow +\infty$.

11. (D) $\lim_{x \rightarrow 0} \frac{\sin 5x}{x} = \lim_{x \rightarrow 0} \frac{\sin 5x}{x} \cdot \frac{5}{5} = 5 \lim_{x \rightarrow 0} \frac{\sin 5x}{5x} = 5$

12. (B) $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x} = \frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin 2x}{x} \cdot \frac{2}{2} = \frac{2}{3} \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = \frac{2}{3}$

13. (B) Because the graph of $y = \tan x$ has vertical asymptotes at $x = \pm \frac{\pi}{2}$, the graph of the inverse function $y = \arctan x$ has horizontal asymptotes at $y = \pm \frac{\pi}{2}$.

14. (C) Since $\frac{x^2 - 9}{3x - 9} = \frac{(x - 3)(x + 3)}{3(x - 3)} = \frac{x + 3}{3}$ (provided $x \neq 3$), y can be defined to be equal to 2 at $x = 3$, removing the discontinuity at that point.

15. (B) Note that $\frac{\sin x}{x^2 + 3x} = \frac{\sin x}{x(x + 3)} = \frac{\sin x}{x} \cdot \frac{1}{x + 3} \rightarrow 1 \cdot \frac{1}{3}$.

16. (C) As $x \rightarrow 0$, $\frac{1}{x}$ takes on varying finite values as it increases. $Y_1 = \sin(1/X)$ exhibits this oscillating discontinuity at $x = 0$.

- (B) 1
- (C) nonexistent
- (D) -1
- (E) none of these

17. Which statement is true about the curve $y = \frac{2x^2 + 4}{2 + 7x - 4x^2}$?
17. (A) $y = \frac{2x^2 + 4}{(2-x)(1+4x)}$, both $x = 2$ and $x = -\frac{1}{4}$ are vertical asymptotes. Also, $y = -\frac{1}{2}$ is a horizontal asymptote.
- (A) The line $x = -\frac{1}{4}$ is a vertical asymptote.
 - (B) The line $x = 1$ is a vertical asymptote.
 - (C) The line $y = -\frac{1}{4}$ is a horizontal asymptote.
 - (D) The graph has no vertical or horizontal asymptote.
 - (E) The line $y = 2$ is a horizontal asymptote.

18. $\lim_{x \rightarrow \infty} \frac{2x^2 + 1}{(2-x)(2+x)}$ is
- (A) -4
 - (B) -2
 - (C) 1
 - (D) 2
 - (E) nonexistent

18. (B) $\frac{2x^2 + 1}{(2-x)(2+x)} = \frac{2x^2 + 1}{4 - x^2}$

Use the Rational Function Theorem.

19. $\lim_{x \rightarrow 0} \frac{|x|}{x}$ is
- (A) 0
 - (B) nonexistent
 - (C) 1
 - (D) -1
 - (E) none of these

19. (B) $|x| = x$ if $x > 0$ but equals $-x$ if

$$x < 0, \lim_{x \rightarrow 0^+} \frac{|x|}{x} = \lim_{x \rightarrow 0^+} \frac{x}{x} = 1$$

$$\text{while } \lim_{x \rightarrow 0^-} \frac{|x|}{x} = \lim_{x \rightarrow 0^-} \frac{-x}{x} = -1.$$

20. $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$ is
- (A) 0
 - (B) ∞
 - (C) nonexistent
 - (D) -1
 - (E) 1

20. (E) Note that $x \sin \frac{1}{x}$ can be rewritten as

$$\frac{\sin \frac{1}{x}}{\frac{1}{x}} \text{ and that, as } x \rightarrow \infty, \frac{1}{x} \rightarrow 0.$$

21. $\lim_{x \rightarrow \pi} \frac{\sin(\pi - x)}{(\pi - x)}$ is
- (A) 1
 - (B) 0
 - (C) ∞
 - (D) π
 - (E) nonexistent

21. (A) As $x \rightarrow \pi$, $(\pi - x) \rightarrow 0$.

22. Let $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1 \end{cases}$.

22. (C) Since $f(x) = x + 1$ if $x \neq 1$, $\lim_{x \rightarrow 1} f(x)$ exists (and is equal to 2).

Which of the following statements is (are) true?

- I. $\lim_{x \rightarrow 1} f(x)$ exists
 - II. $f(1)$ exists
 - III. f is continuous at $x = 1$
- (A) I only
 - (B) II only
 - (C) I and II
 - (D) none of I, II, or III
 - (E) all of I, II, or III

23. If $\begin{cases} f(x) = \frac{x^2 - x}{2x} & \text{for } x \neq 0, \\ f(0) = k, \end{cases}$

23. (B) $f(x) = \frac{x(x-1)}{2x} = \frac{(x-1)}{2}$

, for all $x \neq 0$. For f to be continuous at

and if f is continuous at $x = 0$, then $k =$

$$x = 0, \lim_{x \rightarrow 0} f(x) \text{ must equal } f(0). \lim_{x \rightarrow 0} f(x) = -\frac{1}{2}.$$

- (A) -1

- (B) $-\frac{1}{2}$
- (C) 0
- (D) $\frac{1}{2}$
- (E) 1

24. Suppose
$$f(x) = \begin{cases} \frac{3x(x-1)}{x^2-3x+2} & \text{for } x \neq 1, 2, \\ f(1) = -3, \\ f(2) = 4. \end{cases}$$

Then $f(x)$ is continuous

- (A) except at $x = 1$
- (B) except at $x = 2$
- (C) except at $x = 1$ or 2
- (D) except at $x = 0, 1,$ or 2
- (E) at each real number

25. The graph of $f(x) = \frac{4}{x^2 - 1}$ has

- (A) one vertical asymptote, at $x = 1$
- (B) the y -axis as vertical asymptote
- (C) the x -axis as horizontal asymptote and $x = \pm 1$ as vertical asymptotes
- (D) two vertical asymptotes, at $x = \pm 1,$ but no horizontal asymptote
- (E) no asymptote

26. The graph of $y = \frac{2x^2 + 2x + 3}{4x^2 - 4x}$ has

- (A) a horizontal asymptote at $y = +\frac{1}{2}$ but no vertical asymptote
- (B) no horizontal asymptote but two vertical asymptotes, at $x = 0$ and $x = 1$
- (C) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$
- (D) a horizontal asymptote at $x = 2$ but no vertical asymptote
- (E) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$

27. Let
$$f(x) = \begin{cases} \frac{x^2 + x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Which of the following statements is (are) true?

- I. $f(0)$ exists
- II. $\lim_{x \rightarrow 0} f(x)$ exists
- III. f is continuous at $x = 0$
- (A) I only
- (B) II only
- (C) I and II only
- (D) I, II, and III
- (E) none of I, II, or III

28. If $[x]$ is the greatest integer not greater than x , then $\lim_{x \rightarrow 1/2} [x]$ is

- (A) $\frac{1}{2}$
- (B) 1
- (C) nonexistent
- (D) 0
- (E) none of these

29. (With the same notation) $\lim_{x \rightarrow -2} [x]$ is

- (A) -3
- (B) -2
- (C) -1
- (D) 0
- (E) nonexistent

24. (B) Only $x = 1$ and $x = 2$ need be checked. Since $f(x) = \frac{3x}{x-2}$ for $x \neq 1, 2,$ and $\lim_{x \rightarrow 1} f(x) = -3 = f(1),$ f is continuous at $x = 1.$ Since $\lim_{x \rightarrow 2} f(x)$

does not exist, f is not continuous at $x = 2.$

25. (C) As $x \rightarrow \pm\infty, y = f(x) \rightarrow 0,$ so the x -axis is a horizontal asy.

Also, as $x \rightarrow \pm 1, y \rightarrow \infty,$ so $x = \pm 1$ are vertical asymptotes.

26. (C) As $x \rightarrow \infty, y \rightarrow \frac{1}{2}$

27. (D) The function is defined at 0 to be 1,

which is also

$$\lim_{x \rightarrow 0} \frac{x^2 + x}{x} = \lim_{x \rightarrow 0} (x + 1).$$

28. (D)

29. (E) $\lim_{x \rightarrow -2^-} [x] = -3$ but $\lim_{x \rightarrow -2^+} [x] = -2.$

30. $\lim_{x \rightarrow \infty} \sin x$
 (A) is -1
 (B) is infinity
 (C) oscillates between -1 and 1
 (D) is zero
 (E) does not exist

30. (E) As $x \rightarrow \infty$, the function $\sin x$ oscillates between -1 and 1 ; hence the limit does not exist.

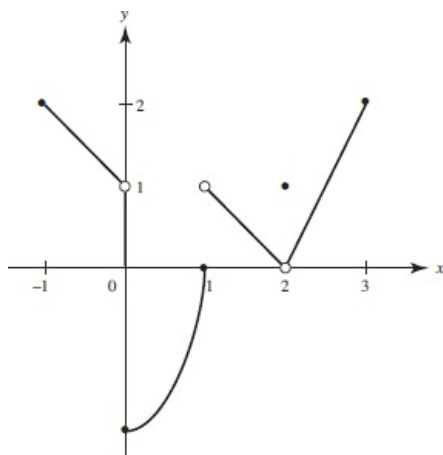
Part B. Directions: Some of the following questions require the use of a graphing calculator.

31. The function $f(x) = \begin{cases} \frac{x^2}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$
 (A) is continuous everywhere
 (B) is continuous except at $x = 0$
 (C) has a removable discontinuity at $x = 0$
 (D) has an infinite discontinuity at $x = 0$
 (E) has $x = 0$ as a vertical asymptote

31. (A) Note that $\frac{x^2}{x} = x$ if $x \neq 0$ and that $\lim_{x \rightarrow 0} f = 0$.

Questions 32-36 are based on the function f shown in the graph and defined below:

$$f(x) = \begin{cases} 1 - x & (-1 \leq x < 0) \\ 2x^2 - 2 & (0 \leq x \leq 1) \\ -x + 2 & (1 < x < 2) \\ 1 & (x = 2) \\ 2x - 4 & (2 < x \leq 3) \end{cases}$$



32. $\lim_{x \rightarrow 2} f(x)$
 (A) equals 0
 (B) equals 1
 (C) equals 2
 (D) does not exist
 (E) none of these

32. (A) $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2^+} f(x) = 0$.

33. The function f is defined on $[-1, 3]$
 (A) if $x \neq 0$
 (B) if $x \neq 1$
 (C) if $x \neq 2$
 (D) if $x \neq 3$
 (E) at each x in $[-1, 3]$

33. (E) Verify that f is defined at $x = 0, 1, 2,$ and 3 (as well as at all other points in $[-1, 3]$).

34. The function f has a removable discontinuity at
 (A) $x = 0$
 (B) $x = 1$
 (C) $x = 2$
 (D) $x = 3$
 (E) none of these

34. (C) Note that $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2^+} f(x) = 0$. However, $f(2) = 1$. Redefining $f(2)$ as 0 removes the discontinuity.

35. On which of the following intervals is f continuous?
 (A) $-1 \leq x \leq 0$

35. (B) The function is not continuous at $x = 0, 1,$ or 2 .

- (B) $0 < x < 1$
- (C) $1 \leq x \leq 2$
- (D) $2 \leq x \leq 3$
- (E) none of these

36. The function f has a jump discontinuity at

- (A) $x = -1$
- (B) $x = 1$
- (C) $x = 2$
- (D) $x = 3$
- (E) none of these

36. (B) $\lim_{x \rightarrow 1^-} f(x) = 0 \neq \lim_{x \rightarrow 1^+} f(x) = 1$.

37. $\lim_{x \rightarrow 0} \sqrt{3 + \arctan \frac{1}{x}}$ is

- (A) $-\infty$
- (B) $\sqrt{3 - \frac{\pi}{2}}$
- (C) $\sqrt{3 + \frac{\pi}{2}}$
- (D) ∞
- (E) nonexistent

37. (E) As $x \rightarrow 0^-$, $\arctan \frac{1}{x} \rightarrow -\frac{\pi}{2}$, so $y \rightarrow \sqrt{3 - \frac{\pi}{2}}$. As $x \rightarrow 0^+$, $y \rightarrow \sqrt{3 + \frac{\pi}{2}}$

The graph has a jump discontinuity at $x = 0$. (Verify with a calculator.)

38. Suppose $\lim_{x \rightarrow -3^-} f(x) = -1$, $\lim_{x \rightarrow -3^+} f(x) = -1$ and $f(-3)$ is not defined. Which of the following statements is (are) true?

- I. $\lim_{x \rightarrow -3} f(x) = -1$
- II. f is continuous everywhere except at $x = -3$.
- III. f has a removable discontinuity at $x = -3$.
- (A) None of I, II, or III
- (B) I only
- (C) III only
- (D) I and III only
- (E) I, II, and III

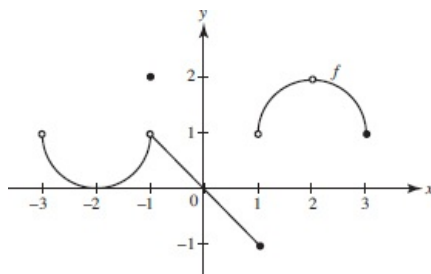
38. (D) No information is given about the domain of f except in the neighborhood of $x = -3$.

39. If $y = \frac{1}{2 + 10^{1/x}}$, then $\lim_{x \rightarrow 0} y$ is

- (A) 0
- (B) $\frac{1}{12}$
- (C) $\frac{1}{2}$
- (D) $\frac{1}{3}$
- (E) nonexistent

39. (E) As $x \rightarrow 0^+$, $10^{1/x} \rightarrow \infty$ and therefore $y \rightarrow 0$. As $x \rightarrow 0^-$, $\frac{1}{x} \rightarrow -\infty$, so $10^{1/x} \rightarrow 0$ and therefore $y \rightarrow \frac{1}{2}$. Because the two one-sided limits are not equal, the limit does not exist. (Verify with a calculator.)

Questions 40-42 are based on the function f shown in the graph.



40. For what value(s) of a is it true that $\lim_{x \rightarrow a} f(x)$ exists and $f(a)$ exists, but $\lim_{x \rightarrow a} f(x) \neq f(a)$? It is possible that $a =$

- (A) -1 only
- (B) 1 only
- (C) 2 only
- (D) -1 or 1 only
- (E) -1 or 2 only

40. (A) $\lim_{x \rightarrow -1} f(x) = 1$, but $f(-1) = 2$. The limit does not exist at $a = 1$ and $f(2)$ does not exist.

41. $\lim_{x \rightarrow a} f(x)$ does not exist for $a =$

41. (B) $\lim_{x \rightarrow -1} f(x) = 1$ and $\lim_{x \rightarrow 2} f(x) = 2$.

- (A) -1 only
- (B) 1 only
- (C) 2 only
- (D) 1 and 2 only
- (E) -1, 1, and 2

42. Which statements about limits at $x = 1$ are true?

- I. $\lim_{x \rightarrow 1} f(x)$ exists.
- II. $\lim_{x \rightarrow 1^+} f(x)$ exists.
- III. $\lim_{x \rightarrow 1^-} f(x)$ exists.

- (A) none of I, II, or III
- (B) I only
- (C) II only
- (D) I and II only
- (E) I, II, and III

42. **(D)** $\lim_{x \rightarrow 1^-} f(x) = -1$ and $\lim_{x \rightarrow 1^+} f(x) = 1$

but since these two limits are not the same,

$\lim_{x \rightarrow 1} f(x)$ does not exist.

43. For the function $f(x) = \begin{cases} \frac{28x^2 - 13x - 6}{7x + 2}, & x \neq -\frac{7}{2} \\ k, & x = -\frac{7}{2} \end{cases}$ what must be the

value of k for the function to be continuous at $x = -\frac{7}{2}$?

- (A) -17
- (B) $-\frac{7}{2}$
- (C) 0
- (D) $\frac{23}{3}$

44. Find the limit: $\lim_{x \rightarrow 5} \frac{x^2 - 24}{5 - x}$.

- (A) $-\frac{24}{5}$
- (B) 0
- (C) 1
- (D) The limit does not exist.

45. Find the limit: $\lim_{x \rightarrow \infty} \frac{\sqrt{x} - 7}{6 - 5\sqrt{x}}$.

- (A) $-\frac{7}{6}$
- (B) $-\frac{1}{5}$
- (C) 0
- (D) ∞

Free Response Questions

Part C. Directions: Some of the following questions require the use of a graphing calculator.

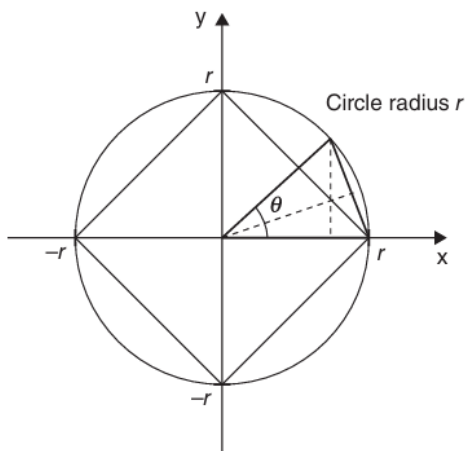
46. (A) Find the limit: $\lim_{x \rightarrow \frac{\pi}{4}} \frac{x(1 - \tan x)}{\cos x - \sin x}$.

(B) Use the result of (a) to derive an approximation of $\sec x$ in terms of x for values of x near $\frac{\pi}{4}$.

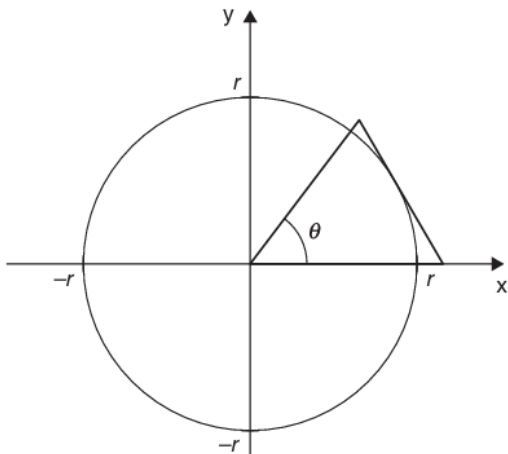
46. (A) $\lim_{x \rightarrow \pi/4} \frac{x(1 - \tan x)}{\cos x - \sin x} = \lim_{x \rightarrow \pi/4} \frac{x \left(\frac{\cos x - \sin x}{\cos x} \right)}{\cos x - \sin x} = \lim_{x \rightarrow \pi/4} \frac{x}{\cos x} = \frac{2\pi}{4\sqrt{2}} = \frac{\pi\sqrt{2}}{4}$

(B) Near $x = \frac{\pi}{4}$, $x \cdot \sec x = \frac{\pi\sqrt{2}}{4}$. This implies that $\sec x = \frac{\pi\sqrt{2}}{4x}$.

47. (A) Use a limit to verify the formula for the area of a circle using inscribed n-gons.



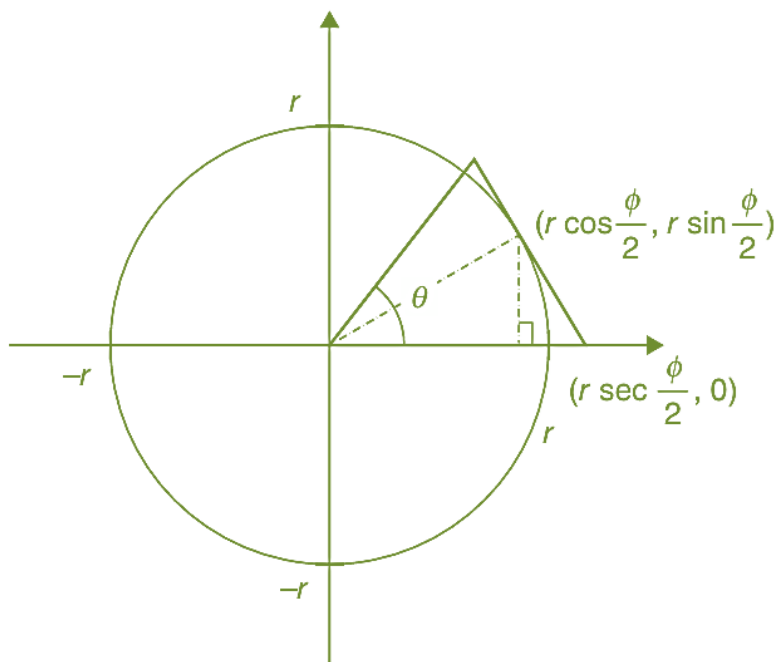
(B) Use a limit to verify the formula for the area of a circle using circumscribed n-gons.



47. (A) The area of a triangle inscribed as shown is given by $\frac{1}{2}bh = \frac{1}{2}r^2 \sin \theta$. Since there can be $\frac{2\pi}{\theta}$ triangles of the same size inscribed in the circle, the total area of the n-gon is given by $\frac{2\pi}{\theta} \cdot \frac{r^2 \sin \theta}{2}$. As $\theta \rightarrow 0$, the area of this polygon will more closely match the area of the circle.

$$\lim_{\theta \rightarrow 0} \frac{\pi r^2 \sin \theta}{\theta} = \pi r^2 \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = \pi r^2$$

(B)



The area of a triangle circumscribed as shown is given by $2 \left(\frac{1}{2} \right) \left(r \sec \frac{\phi}{2} \right) \left(r \sin \frac{\phi}{2} \right) = r^2 \tan \frac{\phi}{2}$. Since there can be $\frac{2\pi}{\phi}$ such triangles circumscribed, the total area of an n-gon composed of such triangles is given by $\frac{2\pi}{\phi} \cdot r^2 \tan \frac{\phi}{2}$. As $\phi \rightarrow 0$, the area of this polygon will more closely match the area of the circle.

$$\lim_{\phi \rightarrow 0} \frac{2\pi r^2 \tan \frac{\phi}{2}}{\phi} = \pi r^2 \cdot \lim_{\phi \rightarrow 0} \frac{\sin \frac{\phi}{2}}{\frac{\phi}{2}} \cdot \lim_{\phi \rightarrow 0} \sec \frac{\phi}{2} = \pi r^2$$

48. (A) Let $f(x) = \frac{p(x)}{q(x)}$, where p and q are both fourth-degree polynomial functions. Discuss the possible number of discontinuities, removable and nonremovable.
- (B) How does the situation change if q is a fifth-degree polynomial?
- (C) What is the situation regarding discontinuities if p is a fifth-degree polynomial, but q is a fourth-degree polynomial?

49. Many junior high students have noticed that $1 = \frac{1}{3} + \frac{2}{3} = 0.\overline{33} + 0.\overline{66} = 0.\overline{99}$.
How would you use a limit to prove the same result?

50. Find the limit: $\lim_{x \rightarrow 0} \frac{4 - \sqrt{16 - x}}{x}$.

48. (A) Polynomials of even degree do not necessarily have a root, so $f(x)$ need not have any discontinuities. Also, $f(x)$ may have as many as four discontinuities. These discontinuities will only be removable where p and q have roots in common.

(B) Since polynomials of odd degree will have at least one root, $f(x)$ will have at least one discontinuity, and as many as five. But the polynomial need not have any non-removable discontinuities due to multiplicity of roots.

(C) This situation is the same as the first. Roots of the numerator do not produce discontinuities.

$$49. 0.\overline{99} = \lim_{x \rightarrow \infty} (1 - 10^{-x}) = 1 - 0 = 1$$

$$\begin{aligned} 50. \lim_{x \rightarrow 0} \frac{4 - \sqrt{16 - x}}{x} &= \lim_{x \rightarrow 0} \frac{4 - \sqrt{16 - x}}{x} \cdot \frac{4 + \sqrt{16 - x}}{4 + \sqrt{16 - x}} = \lim_{x \rightarrow 0} \frac{x}{x(4 + \sqrt{16 - x})} \\ &= \lim_{x \rightarrow 0} \frac{1}{4 + \sqrt{16 - x}} = \frac{1}{8} \end{aligned}$$