CollegeBoard
Advanced Placement Program

AP® Calculus BC

Practice Exam

The questions contained in this AP® Calculus BC Practice Exam are written to the content specifications of AP Exams for this subject. Taking this practice exam should provide students with an idea of their general areas of strengths and weaknesses in preparing for the actual AP Exam. Because this AP Calculus BC Practice Exam has never been administered as an operational AP Exam, statistical data are not available for calculating potential raw scores or conversions into AP grades.

This AP Calculus BC Practice Exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location. Teachers may not redistribute the files electronically for any reason.

Contents

AP® Calculus BC
Directions for Administration

The AP Calculus BC Exam is 3 hours and 15 minutes in length and consists of a multiple-choice section and a free-response section.

- The 105-minute two-part multiple-choice section contains 45 questions and accounts for 50 percent of the final grade. Part A of the multiple-choice section (28 questions in 55 minutes) does not allow the use of a calculator. Part B of the multiple-choice section (17 questions in 50 minutes) contains some questions for which a graphing calculator is required.

- The 90-minute two-part free-response section contains 6 questions and accounts for 50 percent of the final grade. Part A of the free-response section (3 questions in 45 minutes) contains some questions or parts of questions for which a graphing calculator is required. Part B of the free-response section (3 questions in 45 minutes) does not allow the use of a calculator. During the timed portion for Part B, students are permitted to continue work on questions in Part A, but they are not allowed to use a calculator during this time.

For each of the four parts of the exam, students should be given a warning when 10 minutes remain in that part of the exam. A 10-minute break should be provided after Section I is completed. Students should not have access to their graphing calculators during the break.

The actual AP Exam is administered in one session. Students will have the most realistic experience if a complete morning or afternoon is available to administer this practice exam. If a schedule does not permit one time period for the entire practice exam administration, it would be acceptable to administer Section I one day and Section II on a subsequent day.

Many students wonder whether or not to guess the answers to the multiple-choice questions about which they are not certain. It is improbable that mere guessing will improve a score. However, if a student has some knowledge of the question and is able to eliminate one or more answer choices as wrong, it may be to the student's advantage to answer such a question.

- Graphing calculators are required to answer some of the questions on the AP Calculus BC Exam. Before starting the exam administration, make sure each student has a graphing calculator from the approved list at http://www.collegeboard.com/ap/calculators. During the administration of Section I, Part B, and Section II, Part A, students may have no more than two graphing calculators on their desks; calculators may not be shared. Calculator memories do <u>not</u> need to be cleared before or after the exam. Since graphing calculators can be used to store data, including text, it is important to monitor that students are using their calculators appropriately.

- It is suggested that Section I of the practice exam be completed using a pencil to simulate an actual administration. Students may use a pencil or pen with black or dark blue ink to complete Section II.

- Teachers will need to provide paper for the students to write their free-response answers. Teachers should provide directions to the students indicating how they wish the responses to be labeled so the teacher will be able to associate the response with the question the student intended to answer.

- Instructions for the Section II free-response questions are included. Ask students to read these instructions carefully at the beginning of the administration of Section II. Timing for Section II should begin <u>after</u> you have given students sufficient time to read these instructions.

- Remember that students are not allowed to remove any materials, including scratch work, from the testing site.

Section I

Multiple-Choice Questions

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the student answer sheet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. If $f(x) = \dfrac{3x - 2}{2x + 3}$, then $f'(x) =$

(A) $-\dfrac{13}{(2x + 3)^2}$

(B) $\dfrac{3}{(2x + 3)^2}$

(C) $\dfrac{5}{(2x + 3)^2}$

(D) $\dfrac{13}{(2x + 3)^2}$

(E) $\dfrac{12x + 5}{(2x + 3)^2}$

---

2. A particle moves along the x-axis so that at any time $t \geq 0$, its velocity is given by $v(t) = \sin(2t)$. If the position of the particle at time $t = \dfrac{\pi}{2}$ is $x = 4$, what is the particle's position at time $t = 0$?

(A) $-\dfrac{1}{2}$ (B) 2 (C) 3 (D) 5 (E) 8

GO ON TO THE NEXT PAGE.

3. What is the value of $\sum_{n=0}^{\infty}\left(-\frac{2}{3}\right)^n$?

(A) -2 (B) $-\frac{2}{5}$ (C) $\frac{3}{5}$ (D) 3 (E) The series diverges.

---

4. For values of h very close to 0, which of the following functions best approximates $f(x) = \frac{\tan(x+h) - \tan x}{h}$?

(A) $\sin x$

(B) $\frac{\sin x}{x}$

(C) $\frac{\tan x}{x}$

(D) $\sec x$

(E) $\sec^2 x$

GO ON TO THE NEXT PAGE.

5. The length of the curve $y = x^4$ from $x = 1$ to $x = 5$ is given by

(A) $\int_1^5 \sqrt{1 + 4x^3}\, dx$

(B) $\int_1^5 \sqrt{1 + x^4}\, dx$

(C) $\int_1^5 \sqrt{1 + 4x^6}\, dx$

(D) $\int_1^5 \sqrt{1 + 16x^6}\, dx$

(E) $\int_1^5 \sqrt{1 + x^8}\, dx$

---

6. $\int \frac{e^x}{1 + e^x}\, dx =$

(A) $\ln\left(\frac{1}{e^x} + 1\right) + C$

(B) $\ln\left(1 + e^x\right) + C$

(C) $x - \ln\left(1 + e^x\right) + C$

(D) $e^x + x + C$

(E) $\tan^{-1}\left(e^x\right) + C$

GO ON TO THE NEXT PAGE.

7. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y - 1$ with the initial condition $f(1) = -2$. What is the approximation for $f(1.4)$ if Euler's method is used, starting at $x = 1$ with two steps of equal size?

(A) -2 (B) -1.24 (C) -1.2 (D) -0.64 (E) 0.2

---

x	0	2	4	6
$f(x)$	4	k	8	12

8. The function f is continuous on the closed interval $[0, 6]$ and has the values given in the table above. The trapezoidal approximation for $\int_0^6 f(x)\,dx$ found with 3 subintervals of equal length is 52. What is the value of k ?

(A) 2 (B) 6 (C) 7 (D) 10 (E) 14

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9. The function f is twice differentiable, and the graph of f has no points of inflection. If $f(6) = 3$, $f'(6) = -\frac{1}{2}$, and $f''(6) = -2$, which of the following could be the value of $f(7)$?

(A) 2 (B) 2.5 (C) 2.9 (D) 3 (E) 4

---

10. A function f has Maclaurin series given by $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$. Which of the following is an expression for $f(x)$?

(A) $\cos x$

(B) $e^x - \sin x$

(C) $e^x + \sin x$

(D) $\frac{1}{2}\left(e^x + e^{-x}\right)$

(E) e^{x^2}

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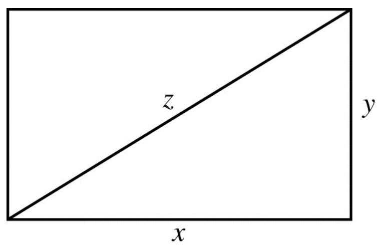

11. The sides and diagonal of the rectangle above are strictly increasing with time. At the instant when $x = 4$ and $y = 3$, $\frac{dx}{dt} = \frac{dz}{dt}$ and $\frac{dy}{dt} = k\frac{dz}{dt}$. What is the value of k at that instant?

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) 3

(D) 4

(E) It cannot be determined from the information given.

---

12. If $f'(x) = \frac{2}{x}$ and $f(\sqrt{e}) = 5$, then $f(e) =$

(A) 2 (B) $\ln 25$ (C) $5 + \frac{2}{e} - \frac{2}{e^2}$ (D) 6 (E) 25

GO ON TO THE NEXT PAGE.

13. For time $t > 0$, the position of a particle moving in the xy-plane is given by the parametric equations $x = 4t + t^2$ and $y = \frac{1}{3t+1}$. What is the acceleration vector of the particle at time $t = 1$?

(A) $\left(2, \frac{1}{32}\right)$

(B) $\left(2, \frac{9}{32}\right)$

(C) $\left(5, \frac{1}{4}\right)$

(D) $\left(6, -\frac{3}{16}\right)$

(E) $\left(6, -\frac{1}{16}\right)$

---

14. $\int \frac{8}{x^2 - 4}\, dx =$

(A) $4\tan^{-1}\left(\frac{x}{2}\right) + C$

(B) $8\ln\left|x^2 - 4\right| + C$

(C) $2\ln\left|\frac{x-2}{x+2}\right| + C$

(D) $2\ln\left|\frac{x+2}{x-2}\right| + C$

(E) $2\ln|x+2| + 2\ln|x-2| + C$

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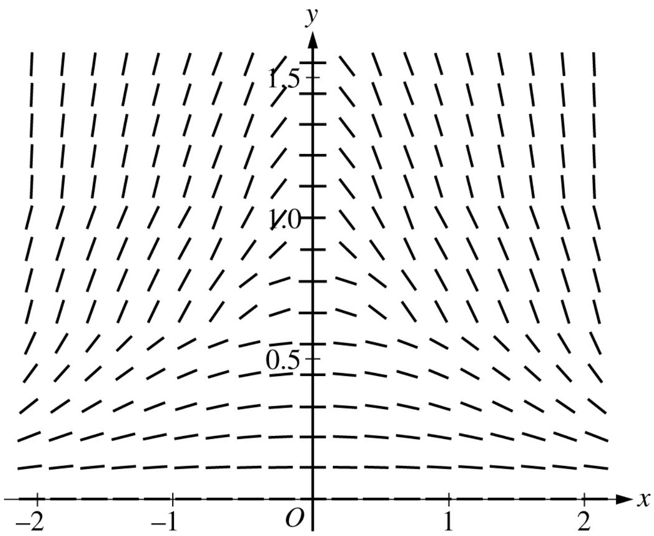


15. The slope field for a certain differential equation is shown above. Which of the following could be a solution to the differential equation with the initial condition $y(0) = 1$?

(A) $y = \cos x$

(B) $y = 1 - x^2$

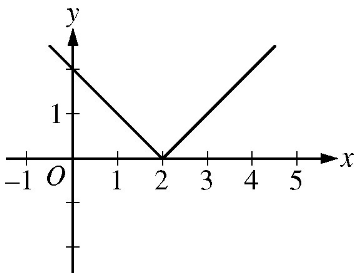
(C) $y = e^x$

(D) $y = \sqrt{1 - x^2}$

(E) $y = \dfrac{1}{1 + x^2}$

16. If $f'(x) = |x - 2|$, which of the following could be the graph of $y = f(x)$?

(A)



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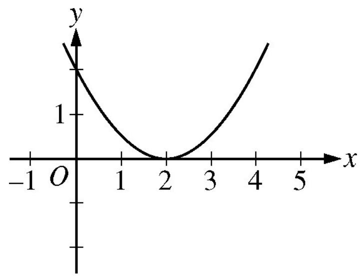
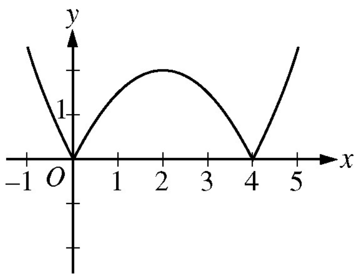


(C)



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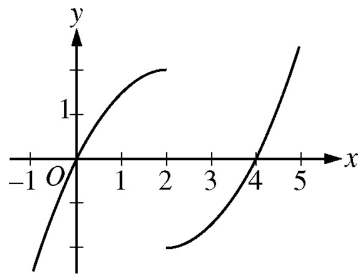
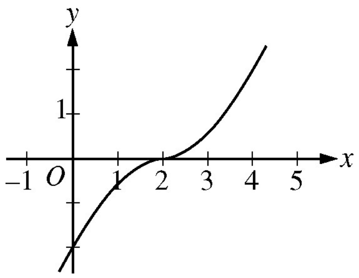


(E)



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17. The radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{(x-3)^{2n}}{n}$ is equal to 1. What is the interval of convergence?

(A) $-4 \leq x < -2$

(B) $-1 < x < 1$

(C) $-1 \leq x < 1$

(D) $2 < x < 4$

(E) $2 \leq x < 4$

---

18. If $f(x) = \arccos\left(x^2\right)$, then $f'(x) =$

(A) $\frac{1}{\sqrt{1-x^4}}$

(B) $\frac{-2x}{\sqrt{1-x^4}}$

(C) $\frac{2x}{\sqrt{1-x^4}}$

(D) $\frac{-4x^3}{\sqrt{1-x^4}}$

(E) $\frac{4x^3}{\sqrt{1-x^4}}$

19. What is the slope of the line tangent to the curve $y + 2 = \frac{x^2}{2} - 2\sin y$ at the point $(2, 0)$?

(A) -2 (B) 0 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) 2

---

20. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

II. $\sum_{n=1}^{\infty} \frac{3^n}{n!}$

III. $\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) II and III only

GO ON TO THE NEXT PAGE.

21. The function f given by $f(x) = 9x^{2/3} + 3x - 6$ has a relative minimum at $x =$

(A) -8 (B) $-\sqrt[3]{2}$ (C) -1 (D) $-\frac{1}{8}$ (E) 0

---

x	$f(x)$	$f'(x)$
0	2	5
4	−3	11

22. The function f has a continuous derivative. The table above gives values of f and its derivative for $x = 0$ and $x = 4$. If $\int_0^4 f(x)\,dx = 8$, what is the value of $\int_0^4 x f'(x)\,dx$?

(A) -20 (B) -13 (C) -12 (D) -7 (E) 36

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23. What is the slope of the line tangent to the polar curve $r = 2\theta$ at the point $\theta = \frac{\pi}{2}$?

(A) $-\frac{\pi}{2}$ (B) $-\frac{2}{\pi}$ (C) 0 (D) $\frac{\pi}{2}$ (E) 2

---

24. The radius of a circle is increasing. At a certain instant, the rate of increase in the area of the circle is numerically equal to twice the rate of increase in its circumference. What is the radius of the circle at that instant?

(A) $\frac{1}{2}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

GO ON TO THE NEXT PAGE.

n	$\sum_{k=1}^{n} \left(\frac{1}{x_k}\right)\left(\frac{1}{n}\right)$
100	5.19
200	5.88
300	6.28
400	6.57
500	6.79

25. The table above shows several Riemann sum approximations to $\int_0^1 \frac{1}{x}\,dx$ using right-hand endpoints of n subintervals of equal length of the interval $[0, 1]$. Which of the following statements best describes the limit of the Riemann sums as n approaches infinity?

(A) The limit of the Riemann sums is a finite number less than 10.

(B) The limit of the Riemann sums is a finite number greater than 10.

(C) The limit of the Riemann sums does not exist because $\left(\frac{1}{x_n}\right)\left(\frac{1}{n}\right)$ does not approach 0.

(D) The limit of the Riemann sums does not exist because it is a sum of infinitely many positive numbers.

(E) The limit of the Riemann sums does not exist because $\int_0^1 \frac{1}{x}\,dx$ does not exist.

26. The coefficients of the power series $\sum_{n=0}^{\infty} a_n(x-2)^n$ satisfy $a_0 = 5$ and $a_n = \left(\frac{2n+1}{3n-1}\right)a_{n-1}$ for all $n \geq 1$. The radius of convergence of the series is

(A) 0 (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) 2 (E) infinite

---

27. If f is the function given by $f(x) = \int_4^{2x} \sqrt{t^2 - t}\, dt$, then $f'(2) =$

(A) 0 (B) $\frac{7}{2\sqrt{12}}$ (C) $\sqrt{2}$ (D) $\sqrt{12}$ (E) $2\sqrt{12}$

28. The function f is given by $f(x) = \sin\left(\frac{x+1}{x^2}\right)$. Which of the following statements are true?

I. The graph of f has a horizontal asymptote at $y = 0$.

II. The graph of f has a horizontal asymptote at $y = 1$.

III. The graph of f has a vertical asymptote at $x = 0$.

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and place the letter of your choice in the corresponding box on the student answer sheet. Do not spend too much time on any one problem.

In this exam:

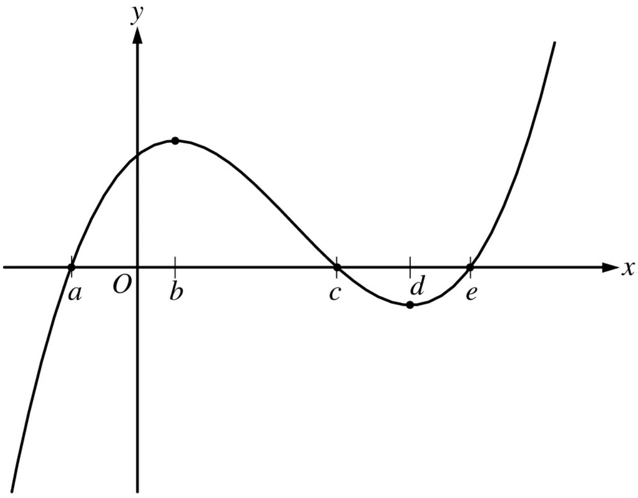
(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

76. If $\int_0^1 f(x)\,dx = 2$ and $\int_0^4 f(x)\,dx = -3$, then $\int_1^4 (3f(x) + 2)\,dx =$

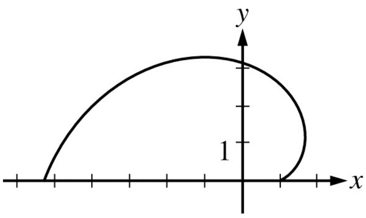
(A) -13 (B) -9 (C) -7 (D) 3 (E) 21



Graph of f

77. The figure above shows the graph of the polynomial function f. For which value of x is it true that $f''(x) < f'(x) < f(x)$?

(A) a (B) b (C) c (D) d (E) e



78. The graph above shows the polar curve $r = 2\theta + \cos\theta$ for $0 \le \theta \le \pi$. What is the area of the region bounded by the curve and the x-axis?

(A) 3.069 (B) 4.935 (C) 9.870 (D) 17.456 (E) 34.912

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79. If f is a differentiable function such that $f(3) = 8$ and $f'(3) = 5$, which of the following statements could be false?

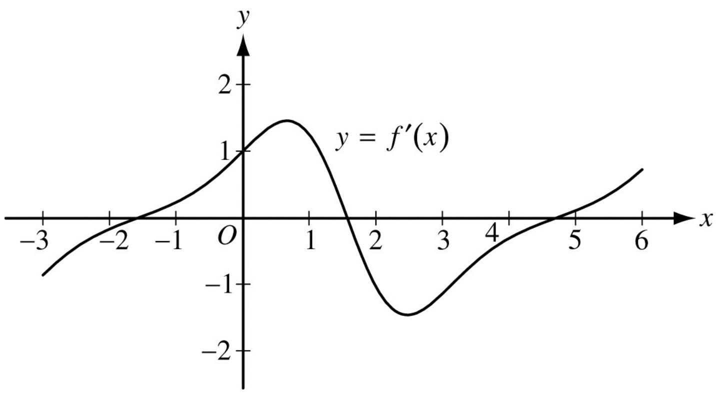
(A) $\lim_{x \to 3} f(x) = 8$

(B) $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$

(C) $\lim_{x \to 3} \frac{f(x) - 8}{x - 3} = 5$

(D) $\lim_{h \to 0} \frac{f(3 + h) - 8}{h} = 5$

(E) $\lim_{x \to 3} f'(x) = 5$



80. The figure above shows the graph of f', the derivative of the function f, on the interval $[-3, 6]$. If the derivative of the function h is given by $h'(x) = 2f'(x)$, how many points of inflection does the graph of h have on the interval $[-3, 6]$?

(A) One (B) Two (C) Three (D) Four (E) Five

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81. Let R be the region in the first quadrant bounded by the y-axis, the x-axis, the graph of $y = e^{-x^2/2}$, and the line $x = 3$. The region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) 0.886 (B) 0.906 (C) 1.078 (D) 1.245 (E) 2.784

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82. If f is a continuous function on the closed interval $[a, b]$, which of the following must be true?

(A) There is a number c in the open interval (a, b) such that $f(c) = 0$.

(B) There is a number c in the open interval (a, b) such that $f(a) < f(c) < f(b)$.

(C) There is a number c in the closed interval $[a, b]$ such that $f(c) \geq f(x)$ for all x in $[a, b]$.

(D) There is a number c in the open interval (a, b) such that $f'(c) = 0$.

(E) There is a number c in the open interval (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

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x	2.5	2.8	3.0	3.1
$f(x)$	31.25	39.20	45	48.05

83. The function f is differentiable and has values as shown in the table above. Both f and f' are strictly increasing on the interval $0 \leq x \leq 5$. Which of the following could be the value of $f'(3)$?

(A) 20 (B) 27.5 (C) 29 (D) 30 (E) 30.5

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84. The rate of change, $\frac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1200. At 10 A.M., the number of people on the beach is 200 and is increasing at the rate of 400 people per hour. Which of the following differential equations describes the situation?

(A) $\frac{dP}{dt} = \frac{1}{400}(1200 - P) + 200$

(B) $\frac{dP}{dt} = \frac{2}{5}(1200 - P)$

(C) $\frac{dP}{dt} = \frac{1}{500}P(1200 - P)$

(D) $\frac{dP}{dt} = \frac{1}{400}P(1200 - P)$

(E) $\frac{dP}{dt} = 400P(1200 - P)$

---

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
2	0	0	5	7

85. The third derivative of the function f is continuous on the interval $(0, 4)$. Values for f and its first three derivatives at $x = 2$ are given in the table above. What is $\lim_{x \to 2} \frac{f(x)}{(x-2)^2}$?

(A) 0 (B) $\frac{5}{2}$ (C) 5 (D) 7 (E) The limit does not exist.

86. The Taylor polynomial of degree 100 for the function f about $x = 3$ is given by
$P(x) = (x-3)^2 - \frac{(x-3)^4}{2!} + \frac{(x-3)^6}{3!} + \cdots + (-1)^{n+1}\frac{(x-3)^{2n}}{n!} + \cdots - \frac{(x-3)^{100}}{50!}$. What is the value of $f^{(30)}(3)$?

(A) $-\frac{30!}{15!}$ (B) $-\frac{1}{30!}$ (C) $\frac{1}{30!}$ (D) $\frac{1}{15!}$ (E) $\frac{30!}{15!}$

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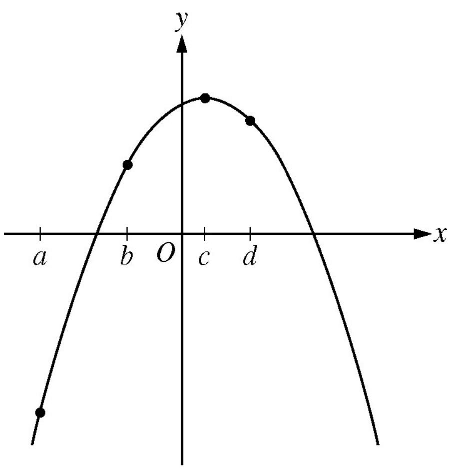
87. The function f has derivatives of all orders for all real numbers, and $f^{(4)}(x) = e^{\sin x}$. If the third-degree Taylor polynomial for f about $x = 0$ is used to approximate f on the interval $[0, 1]$, what is the Lagrange error bound for the maximum error on the interval $[0, 1]$?

(A) 0.019 (B) 0.097 (C) 0.113 (D) 0.399 (E) 0.417

88. The rate at which water is sprayed on a field of vegetables is given by $R(t) = 2\sqrt{1 + 5t^3}$, where t is in minutes and $R(t)$ is in gallons per minute. During the time interval $0 \le t \le 4$, what is the average rate of water flow, in gallons per minute?

(A) 8.458 (B) 13.395 (C) 14.691 (D) 18.916 (E) 35.833



Graph of f

89. The figure above shows the graph of a function f. Which of the following has the greatest value?

(A) $f(a)$ (B) $f'(a)$ (C) $f'(c)$ (D) $f(c) - f(d)$ (E) $\dfrac{f(b) - f(a)}{b - a}$

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90. The nth derivative of a function f at $x = 0$ is given by $f^{(n)}(0) = (-1)^n \dfrac{n+1}{(n+2)2^n}$ for all $n \geq 0$. Which of the following is the Maclaurin series for f?

(A) $-\frac{1}{2} + \frac{1}{3}x - \frac{3}{32}x^2 + \frac{1}{60}x^3 - \cdots$

(B) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{16}x^2 - \frac{1}{10}x^3 + \cdots$

(C) $\frac{1}{2} + \frac{1}{3}x + \frac{3}{32}x^2 + \frac{1}{60}x^3 + \cdots$

(D) $\frac{1}{2} - \frac{1}{3}x + \frac{3}{32}x^2 - \frac{1}{60}x^3 + \cdots$

(E) $\frac{1}{2} - 3x + \frac{32}{3}x^2 - 60x^3 + \cdots$

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x	$f(x)$	$f'(x)$
0	1	1
1	3	4
2	11	13

91. The table above gives selected values for a differentiable and increasing function f and its derivative. If g is the inverse function of f, what is the value of $g'(3)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) 1 (D) 4 (E) 13

---

92. Let f be the function with first derivative defined by $f'(x) = \sin\left(x^3\right)$ for $0 \le x \le 2$. At what value of x does f attain its maximum value on the closed interval $0 \le x \le 2$?

(A) 0 (B) 1.162 (C) 1.465 (D) 1.845 (E) 2

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

Section II

Free-Response Questions

AP® Calculus
Instructions for Section II Free-Response Questions

Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be graded.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be graded on the correctness and completeness of your methods as well as your answers. Answers without supporting work may not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

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1. The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

   (a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

   (b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

   (c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

GO ON TO THE NEXT PAGE.

2. A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t > 0$, where

$$\frac{dx}{dt} = \left(\frac{6}{t} - 3\right)^{1/3} \text{ and } \frac{dy}{dt} = te^{-t}.$$

At time $t = 3$, the particle is at the point $(5, 4)$.

(a) Find the speed of the particle at time $t = 3$.

(b) Write an equation for the line tangent to the path of the particle at time $t = 3$.

(c) Is there a time t at which the particle is farthest to the right? If yes, explain why and give the value of t and the x-coordinate of the position of the particle at that time. If no, explain why not.

(d) Describe the behavior of the path of the particle as t increases without bound.

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t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

3. The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

(a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

(b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

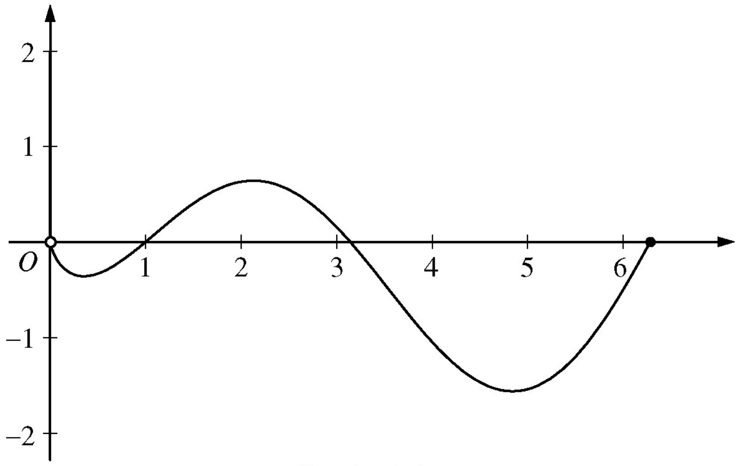
(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



Graph of f

4. Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

   (a) Find $g(1)$ and $g'(1)$.

   (b) On what intervals, if any, is g increasing? Justify your answer.

   (c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

   (d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

GO ON TO THE NEXT PAGE.

5. Let f be the function satisfying $f'(x) = 4x - 2xf(x)$ for all real numbers x, with $f(0) = 5$ and $\lim_{x \to \infty} f(x) = 2$.

(a) Find the value of $\int_0^{\infty} (4x - 2xf(x))\, dx$. Show the work that leads to your answer.

(b) Use Euler's method to approximate $f(-1)$, starting at $x = 0$, with two steps of equal size.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 4x - 2xy$ with the initial condition $f(0) = 5$.

---

6. The function g is continuous for all real numbers x and is defined by

$$g(x) = \frac{\cos(2x) - 1}{x^2} \text{ for } x \neq 0.$$

(a) Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

(b) Let f be the function given by $f(x) = \cos(2x)$. Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(c) Use your answer from part (b) to write the first three nonzero terms and the general term of the Taylor series for g about $x = 0$.

(d) Determine whether g has a relative minimum, a relative maximum, or neither at $x = 0$. Justify your answer.

STOP

END OF EXAM

Name: ______________________________

AP® Calculus BC
Student Answer Sheet for Multiple-Choice Section

No.	Answer
1	
2	
3	
4	
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6	
7	
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14	
15	
16	
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21	
22	
23	
24	
25	
26	
27	
28	

No.	Answer
76	
77	
78	
79	
80	
81	
82	
83	
84	
85	
86	
87	
88	
89	
90	
91	
92	

AP® Calculus BC
Multiple-Choice Answer Key

No.	Correct Answer
1	D
2	C
3	C
4	E
5	D
6	B
7	B
8	D
9	A
10	D
11	B
12	D
13	B
14	C
15	E
16	E
17	D
18	B
19	D
20	E
21	E
22	A
23	B
24	D
25	E
26	C
27	E
28	A

No.	Correct Answer
76	B
77	B
78	D
79	E
80	B
81	A
82	C
83	D
84	C
85	B
86	E
87	B
88	C
89	B
90	D
91	B
92	C

AP® Calculus BC
Free-Response Scoring Guidelines

Question 1

The rate at which raw sewage enters a treatment tank is given by $E(t) = 850 + 715\cos\left(\frac{\pi t^2}{9}\right)$ gallons per hour for $0 \le t \le 4$ hours. Treated sewage is removed from the tank at the constant rate of 645 gallons per hour. The treatment tank is empty at time $t = 0$.

(a) How many gallons of sewage enter the treatment tank during the time interval $0 \le t \le 4$? Round your answer to the nearest gallon.

(b) For $0 \le t \le 4$, at what time t is the amount of sewage in the treatment tank greatest? To the nearest gallon, what is the maximum amount of sewage in the tank? Justify your answers.

(c) For $0 \le t \le 4$, the cost of treating the raw sewage that enters the tank at time t is $(0.15 - 0.02t)$ dollars per gallon. To the nearest dollar, what is the total cost of treating all the sewage that enters the tank during the time interval $0 \le t \le 4$?

(a) $\int_0^4 E(t)\,dt \approx 3981$ gallons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Let $S(t)$ be the amount of sewage in the treatment tank at time t. Then $S'(t) = E(t) - 645$ and $S'(t) = 0$ when $E(t) = 645$. On the interval $0 \le t \le 4$, $E(t) = 645$ when $t = 2.309$ and $t = 3.559$.

t (hours)	amount of sewage in treatment tank
0	0
2.309	$\int_0^{2.309} E(t)\,dt - 645(2.309) = 1637.178$
3.559	$\int_0^{3.559} E(t)\,dt - 645(3.559) = 1228.520$
4	$3981.022 - 645(4) = 1401.022$

The amount of sewage in the treatment tank is greatest at $t = 2.309$ hours. At that time, the amount of sewage in the tank, rounded to the nearest gallon, is 1637 gallons.

$4 : \begin{cases} 1 : \text{sets } E(t) = 645 \\ 1 : \text{identifies } t = 2.309 \text{ as a candidate} \\ 1 : \text{amount of sewage at } t = 2.309 \\ 1 : \text{conclusion} \end{cases}$

(c) Total cost $= \int_0^4 (0.15 - 0.02t)E(t)\,dt = 474.320$

The total cost of treating the sewage that enters the tank during the time interval $0 \le t \le 4$, to the nearest dollar, is $474.

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

AP® Calculus BC
Free-Response Scoring Guidelines

Question 2

A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t > 0$, where

$$\frac{dx}{dt} = \left(\frac{6}{t} - 3\right)^{1/3} \text{ and } \frac{dy}{dt} = te^{-t}.$$

At time $t = 3$, the particle is at the point $(5, 4)$.

(a) Find the speed of the particle at time $t = 3$.

(b) Write an equation for the line tangent to the path of the particle at time $t = 3$.

(c) Is there a time t at which the particle is farthest to the right? If yes, explain why and give the value of t and the x-coordinate of the position of the particle at that time. If no, explain why not.

(d) Describe the behavior of the path of the particle as t increases without bound.

(a) Speed $= \sqrt{(x'(3))^2 + (y'(3))^2} = 1.011$	1 : answer
(b) At $t = 3$, $\frac{dy}{dx} = \frac{y'(3)}{x'(3)} = \frac{3e^{-3}}{-1} = -\frac{3}{e^3} = -0.149$ An equation for the line tangent to the path at the point $(5, 4)$ is $y - 4 = -\frac{3}{e^3}(x - 5)$.	2 : { 1 : slope 1 : equation of tangent line
(c) Since $\frac{dx}{dt} > 0$ for $0 < t < 2$ and $\frac{dx}{dt} < 0$ for $t > 2$, the particle is farthest to the right at $t = 2$. The x-coordinate of the position is $x(2) = 5 + \int_3^2 \left(\frac{6}{t} - 3\right)^{1/3} dt = 5.791$ or 5.792	4 : { 1 : "yes" with explanation 1 : $t = 2$ 2 : { 1 : integral for $x(2)$ 1 : x-coordinate
(d) Since $\lim_{t\to\infty} \frac{dx}{dt} = (-3)^{1/3}$ and $\lim_{t\to\infty} \frac{dy}{dt} = 0$, the particle will continue moving farther to the left while approaching a horizontal asymptote.	2 : { 1 : behavior of x 1 : behavior of y

AP® Calculus BC
Free-Response Scoring Guidelines

Question 3

t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

(a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

(b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

(c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

(d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

(a) $H'(10) \approx \dfrac{H(12) - H(8)}{12 - 8} = \dfrac{80 - 73}{4} = \dfrac{7}{4}\,°\text{C/min}$

2 : { 1 : difference quotient; 1 : answer with units }

(b) Average temperature is $\dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt$

$$\int_0^{16} H(t)\,dt \approx 4 \cdot (65 + 68 + 73 + 80)$$

Average temperature $\approx \dfrac{4 \cdot 286}{16} = 71.5°\text{C}$

3 : { 1 : $\dfrac{1}{16}\displaystyle\int_0^{16} H(t)\,dt$; 1 : left Riemann sum; 1 : answer }

(c) The left Riemann sum approximation is an underestimate of the integral because the graph of H is increasing. Dividing by 16 will not change the inequality, so 71.5°C is an underestimate of the average temperature.

1 : answer with reason

(d) If a continuous function is increasing at an increasing rate, then the slopes of the secant lines of the graph of the function are increasing. The slopes of the secant lines for the four intervals in the table are $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$, and $\frac{10}{4}$, respectively.

Since the slopes are increasing, the data are consistent with the claim.

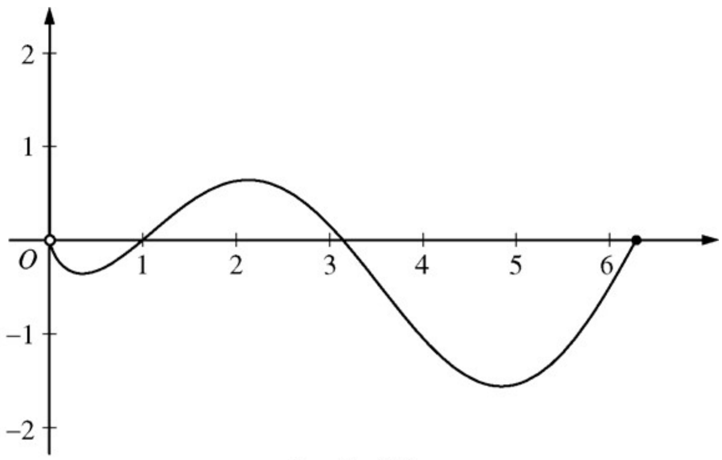
OR

By the Mean Value Theorem, the slopes are also the values of $H'(c_k)$ for some times $c_1 < c_2 < c_3 < c_4$, respectively. Since these derivative values are positive and increasing, the data are consistent with the claim.

3 : { 1 : considers slopes of four secant lines; 1 : explanation; 1 : conclusion consistent with explanation }

AP® Calculus BC
Free-Response Scoring Guidelines

Question 4



Graph of f

Let f be the function given by $f(x) = (\ln x)(\sin x)$. The figure above shows the graph of f for $0 < x \le 2\pi$. The function g is defined by $g(x) = \int_1^x f(t)\,dt$ for $0 < x \le 2\pi$.

(a) Find $g(1)$ and $g'(1)$.

(b) On what intervals, if any, is g increasing? Justify your answer.

(c) For $0 < x \le 2\pi$, find the value of x at which g has an absolute minimum. Justify your answer.

(d) For $0 < x < 2\pi$, is there a value of x at which the graph of g is tangent to the x-axis? Explain why or why not.

(a) $g(1) = \int_1^1 f(t)\,dt = 0$ and $g'(1) = f(1) = 0$	$2 : \begin{cases} 1 : g(1) \\ 1 : g'(1) \end{cases}$
(b) Since $g'(x) = f(x)$, g is increasing on the interval $1 \le x \le \pi$ because $f(x) > 0$ for $1 < x < \pi$.	$2 : \begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$
(c) For $0 < x < 2\pi$, $g'(x) = f(x) = 0$ when $x = 1, \pi$. $g' = f$ changes from negative to positive only at $x = 1$. The absolute minimum must occur at $x = 1$ or at the right endpoint. Since $g(1) = 0$ and $g(2\pi) = \int_1^{2\pi} f(t)\,dt = \int_1^{\pi} f(t)\,dt + \int_{\pi}^{2\pi} f(t)\,dt < 0$ by comparison of the two areas, the absolute minimum occurs at $x = 2\pi$.	3 : 1 : identifies 1 and 2π as candidates - or - indicates that the graph of g decreases, increases, then decreases 1 : justifies $g(2\pi) < g(1)$ 1 : answer
(d) Yes, the graph of g is tangent to the x-axis at $x = 1$ since $g(1) = 0$ and $g'(1) = 0$.	$2 : \begin{cases} 1 : \text{answer of “yes” with } x = 1 \\ 1 : \text{explanation} \end{cases}$

AP® Calculus BC
Free-Response Scoring Guidelines

Question 5

Let f be the function satisfying $f'(x) = 4x - 2xf(x)$ for all real numbers x, with $f(0) = 5$ and $\lim_{x\to\infty} f(x) = 2$.

(a) Find the value of $\int_0^\infty (4x - 2xf(x))\,dx$. Show the work that leads to your answer.

(b) Use Euler's method to approximate $f(-1)$, starting at $x = 0$, with two steps of equal size.

(c) Find the particular solution $y = f(x)$ to the differential equation $\frac{dy}{dx} = 4x - 2xy$ with the initial condition $f(0) = 5$.

(a) $\int_0^\infty (4x - 2xf(x))\,dx$

$= \int_0^\infty f'(x)\,dx = \lim_{b\to\infty} \int_0^b f'(x)\,dx = \lim_{b\to\infty}\left[f(x)\Big|_0^b \right]$

$= \lim_{b\to\infty} (f(b) - f(0)) = 2 - 5 = -3$

$2 : \begin{cases} 1 : \text{use of FTC} \\ 1 : \text{answer from limiting process} \end{cases}$

(b) $f\left(-\frac{1}{2}\right) \approx f(0) + f'(0)\cdot\left(-\frac{1}{2}\right) = 5 + 0\cdot\left(-\frac{1}{2}\right) = 5$

$f(-1) \approx f\left(-\frac{1}{2}\right) + f'\left(-\frac{1}{2}\right)\cdot\left(-\frac{1}{2}\right) = 5 + 3\cdot\left(-\frac{1}{2}\right) = \frac{7}{2}$

$2 : \begin{cases} 1 : \text{Euler's method with two steps} \\ 1 : \text{Euler's approximation to } f(-1) \end{cases}$

(c) $\frac{1}{4-2y}\,dy = x\,dx$

$-\frac{1}{2}\ln|4 - 2y| = \frac{1}{2}x^2 + A$

$\ln|4 - 2y| = -x^2 + B$

$2y - 4 = Ce^{-x^2}$

$C = 6$

$y = 2 + 3e^{-x^2}$ for all real numbers x

$5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $2/5$ $[1\text{-}1\text{-}0\text{-}0\text{-}0]$ if no constant of integration

AP® Calculus BC
Free-Response Scoring Guidelines

Question 6

The function g is continuous for all real numbers x and is defined by

$$g(x) = \frac{\cos(2x) - 1}{x^2} \text{ for } x \neq 0.$$

(a) Use L'Hospital's Rule to find the value of $g(0)$. Show the work that leads to your answer.

(b) Let f be the function given by $f(x) = \cos(2x)$. Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(c) Use your answer from part (b) to write the first three nonzero terms and the general term of the Taylor series for g about $x = 0$.

(d) Determine whether g has a relative minimum, a relative maximum, or neither at $x = 0$. Justify your answer.

(a) Since g is continuous,

$$g(0) = \lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{\cos(2x) - 1}{x^2}$$
$$= \lim_{x \to 0} \frac{-2\sin(2x)}{2x}$$
$$= \lim_{x \to 0} \frac{-4\cos(2x)}{2} = -2$$

2 : { 1 : uses L'Hospital's Rule correctly at least once; 1 : answer }

(b) $$\cos(2x) = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots + (-1)^n \frac{(2x)^{2n}}{(2n)!} + \cdots$$
$$= 1 - \frac{4}{2!}x^2 + \frac{16}{4!}x^4 - \frac{64}{6!}x^6 + \cdots + (-1)^n \frac{2^{2n}}{(2n)!}x^{2n} + \cdots$$

3 : { 1 : first two terms; 1 : next two terms; 1 : general term }

(c) $$g(x) = -\frac{4}{2!} + \frac{16}{4!}x^2 - \frac{64}{6!}x^4 + \cdots + (-1)^n \frac{2^{2n}}{(2n)!}x^{2n-2} + \cdots$$

2 : { 1 : first three terms; 1 : general term }

(d) $g'(x) = \frac{2 \cdot 16}{4!}x - \frac{4 \cdot 64}{6!}x^3 + \cdots$, so $g'(0) = 0$.

$g''(x) = \frac{2 \cdot 16}{4!} - \frac{3 \cdot 4 \cdot 64}{6!}x^2 + \cdots$, so $g''(0) > 0$.

Therefore, g has a relative minimum at $x = 0$ by the Second Derivative Test.

2 : { 1 : answer; 1 : justification }