

ESAT PRACTICE TEST SERIES

ESAT Mock Test 2

Math 1 + Physics + Math 2

- Duration: 2 hours (3 × 40 min)
- All topics

SOLUTION BOOK

ThrivingScholars 

MATH 1

Problem 1 If $\frac{a}{c} \Big| \frac{b}{d} = a \cdot d - b \cdot c$, what is the value of $\frac{3}{6} \Big| \frac{4}{8}$?

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Problem 2 The stock price of a company was \$100 in 2005. It went up by 20% in 2006. However, due to the financial crisis, the stock price went down by 20% in 2007. What was the stock price at the end of 2007?

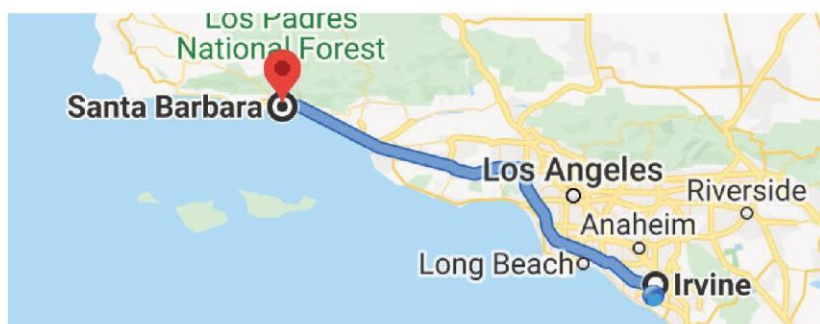
- (A) \$80 (B) \$96 (C) \$98 (D) \$100 (E) \$120

Problem 3 Irvine (CA) and Santa Barbara are approximate 120 miles apart. Austin drove from Irvine to visit his brother Benjamin in Santa Barbara, averaging 60 miles per hour. On his return trip along the same route, due to some heavy traffic on I-5, he averaged only 40 miles per hour. What was the average speed for the round trip, in miles per hour?

(Recall that

$$\text{Average speed} = \frac{\text{Total travel distance}}{\text{Total amount of time}}.$$

)



- (A) 45 (B) 46 (C) 48 (D) 50 (E) 54

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Problem 4 Find the number a such that

$$\frac{7}{8} = \frac{1}{1 + \frac{1}{a}}.$$

Problem 5 [AMC8 Problem] Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, 'I didn't get the lowest score in our class,' and Bridget adds, 'I didn't get the highest score.' What is the ranking of the three girls from highest to lowest?

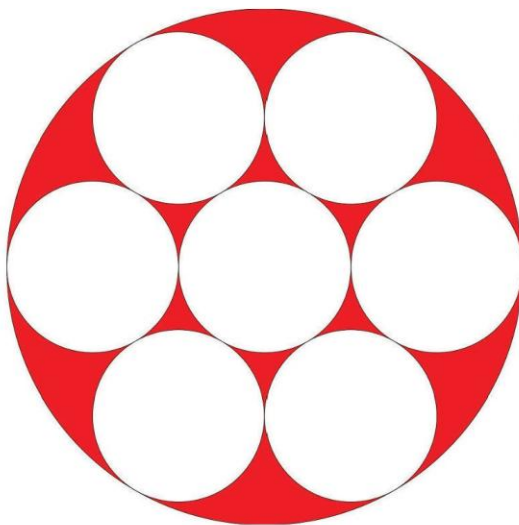
- (A) Hannah, Cassie, Bridget (B) Hannah, Bridget, Cassie
(C) Cassie, Bridget, Hannah (D) Cassie, Hannah, Bridget
(E) Bridget, Cassie, Hannah

Problem 6 The following is an arithmetic sequence that has 10 numbers

$$1, a_2, a_3, a_4, \dots, a_{10}.$$

If these 10 numbers add up to 100, what is the second number a_2 ?

Problem 7 Seven small circles with radius 1 are circumscribed by a large circle. Adjacent circles are tangent (i.e., they touch at only one point). Six small circles are externally tangent to center one and internally tangent to the big one. What fraction of the large circle's area is shaded?



MATH 1 QUESTIONS

- (A)
- $\frac{2}{9}$
- (B)
- $\frac{1}{4}$
- (C)
- $\frac{1}{3}$
- (D)
- $\frac{1}{2}$
- (E)
- $\frac{5}{9}$

Problem 8 x , y and z are three numbers satisfying that

$$\begin{cases} x + 2y + 3z = 25 \\ 3x + 2y + z = 35. \end{cases}$$

What is $x + y + z$?

- (A) 10 (B) 12 (C) 15 (D) 16
-
- (E) There are more than one possible values

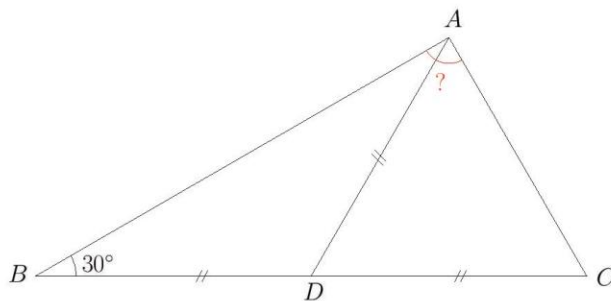
Problem 9 A bag contains 5 balls (three red and 2 blue). Leo picks two balls at random. What is the probability that he gets two balls of the same color?

Recall that

$$\text{Probability} = \frac{\text{Number of valid cases}}{\text{Number of all possible cases}}.$$

- (A)
- $\frac{1}{5}$
- (B)
- $\frac{3}{10}$
- (C)
- $\frac{2}{5}$
- (D)
- $\frac{1}{2}$
- (E)
- $\frac{3}{5}$

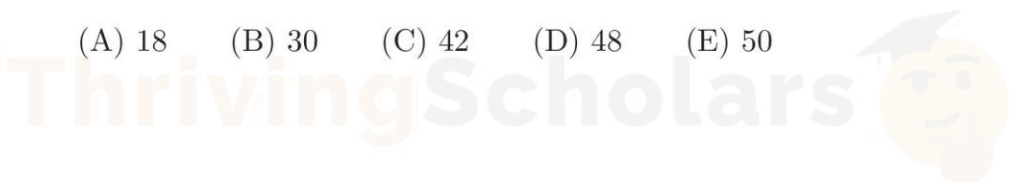
Problem 10 In $\triangle ABC$, D is the midpoint on BC . If $AD = BD = CD$ and $\angle B = 30^\circ$, what is $\angle BAC$?



- (A)
- 60°
- (B)
- 80°
- (C)
- 86°
- (D)
- 90°
- (E)
- 95°

Problem 11 How many 2-digit positive integers are divisible by 3 or 5 or both?

- (A) 18 (B) 30 (C) 42 (D) 48 (E) 50



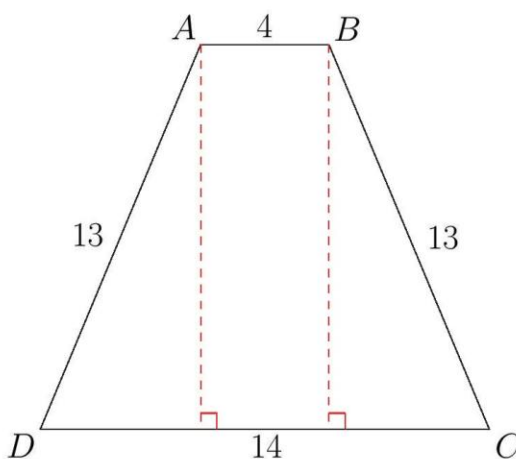
Problem 12 If a is the smallest two-digit number such that

$$\text{lcm}(a, 12) = 60,$$

how many different divisors does a have? Recall that “ lcm ” stands for the *least common multiple*.

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12

Problem 13 $ABCD$ is an isosceles trapezoid. AB is parallel with CD . If $AB = 4$, $BC = AD = 13$ and $CD = 14$, what is the area of the trapezoid?



- (A) 48 (B) 78 (C) 90 (D) 108 (E) 110

Problem 14 A water tank weighs 190 kg when it is full of water and 120 kg when half full. How much does it weigh, in kilograms, when empty?

- (A) 30 (B) 40 (C) 50 (D) 60 (E) 70

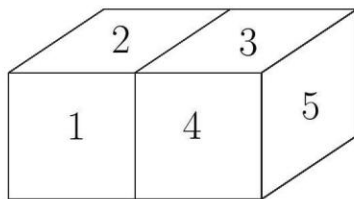
Problem 15 If it is 8 am now, what time will it be 2020 hours later?

- (A) 7am (B) 8am (C) 11am (D) 12pm (E) 7pm

Problem 16 Two cubes with numbers on their faces are put next to each other as follows. For each cube, the sum of numbers on every two opposite faces is 15, what is the sum of numbers on those 7 faces that are hidden in the following picture? (Chinese Hope Cup Math Competition)



MATH 1 QUESTIONS

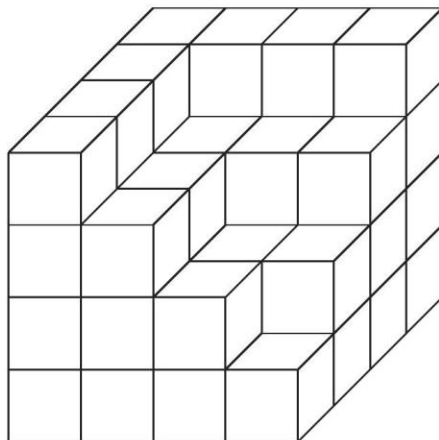


- (A) 60 (B) 65 (C) 75 (D) 76
 (E) 80

Problem 17 A positive integer is called 13-special if the quotient and remainder are the same when this integer is divided by 13. How many positive 13-special integers are between 1 and 100 (inclusive)?

- (A) 5 (B) 6 (C) 7 (D) 10 (E) 12

Problem 18 The following three-dimensional figure is made of small cubes with equal side length. How many small cubes are there?



- (A) 48 (B) 49 (C) 50 (D) 51 (E) 52

Problem 19 There are 7 cards with seven different numbers 1-7 written on them. Austen, Sam and Stephanie each takes 2 cards. (Chinese Hope Cup Math Competition)

Austen said: “Numbers on my two cars add up to 7”.

MATH 1 QUESTIONS

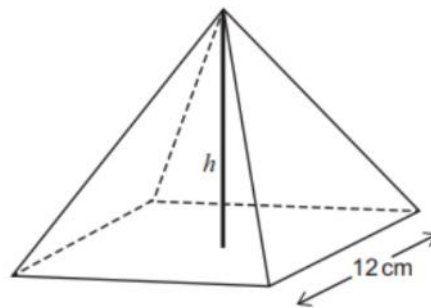
Sam said: ‘The difference between my two numbers is 1’.

Stephanie said: ‘The product of my two numbers is 12.’

What is the number on the card that is left?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 7

Problem 20



[diagram not to scale]

A solid pyramid has a square base of side length 12 cm and a vertical height of h cm.

The volume of the pyramid, in cm^3 , is equal to the total surface area of the pyramid, in cm^2 .

What is the value of h ?

(volume of pyramid = $\frac{1}{3} \times$ area of base \times vertical height)

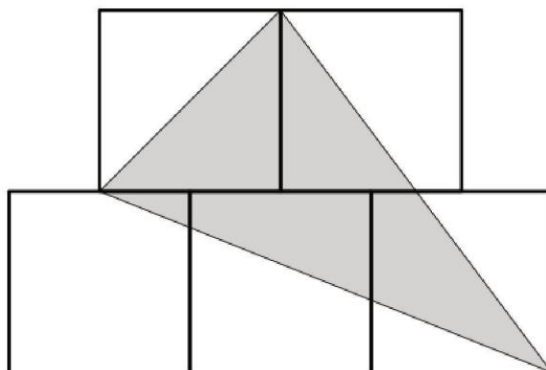
- A $\frac{72}{35}$
 B $2\sqrt{3}$
 C 6
 D $\frac{144}{23}$
 E 8
 F $2\sqrt{21}$

MATH 1 QUESTIONS

Problem 21 The average score of students in team A and team B altogether is 18 in an AMC 8 contest. Team A has 20 students with an average score of 14. If the average score of students in class B is 23, how many students in team B?

- (A) 12 (B) 14 (C) 15 (D) 16 (E) 18

Problem 22 The diagram shows 5 identical squares of side length 2 arranged symmetrically. What fraction of the diagram's area is shaded?



- (A) $\frac{3}{5}$ (B) $\frac{7}{20}$ (C) $\frac{7}{10}$ (D) $\frac{3}{4}$ (E) $\frac{4}{5}$

Problem 23 How many 7-digit palindromes (numbers that read the same backward as forward) can be formed using the digits 2, 2, 3, 3, 5, 5, 5? For example, one such number is 2355532. (AMC 10 Problem)

- (A) 6 (B) 12 (C) 24 (D) 36 (E) 48

Problem 24 A standard, six-sided die is rolled three times. What is the probability that the three rolls are either all the same or all different?

- (A) $\frac{1}{6}$ (B) $\frac{3}{8}$ (C) $\frac{5}{12}$ (D) $\frac{7}{12}$ (E) $\frac{11}{36}$



Answers

1. C
2. B (Hint: Up and down percentages are based on the previous stock price instead of the stock price at the very beginning.)
3. C 4. 7 5. D
6. 3 (Hint: According to Gauss's trick, what is the relation between 100 and $1 + a_{10}^2$?)
7. A 8. C 9. C 10. D
11. C 12. A
13. D (Hint: Draw the height of the trapezoid (the red line in the picture) and employ Pythagorean theorem to find the height. The use "cut and pastes" to find the area.)
14. C 15. D
16. C 17. C 18. C 19. A
20. $V = \frac{1}{3} \cdot 12^2 \cdot h = 48h$, and $SA = 12^2 + 4 \left(\frac{1}{2} \cdot 12 \cdot \sqrt{h^2 + 6^2} \right) = 144 + 24\sqrt{h^2 + 36}$.
 E Set $48h = 144 + 24\sqrt{h^2 + 36} \Rightarrow 2h = 6 + \sqrt{h^2 + 36} \Rightarrow h = 8$ Answer E
21. D (Hint: You may let x be the number of students in team B and set up an equation)
22. B (Hint: You may use cut-and-paste technique. For example, put two 2×1 rectangles on two upper corners to form a 6×4 big rectangle. Here the side length = 2 is not really needed. You can use any number and will get the same fraction.)
23. A (Hint: 5 must be in the middle. You just need to arrange 2,3,5 on one side and the other side is symmetric.)
24. D 25. E 26. D
27. C (Hint: Note that each cycle (1 white, 1 black, 1 white, 2 black, 1 white, 3 black) has 9 beads.)



PHYSICS

Question 01:

A 1,000 kg car accelerates from rest at 5 ms^{-2} for 10 s. Then, a braking force is applied to bring it to rest within 20 seconds. What distance has the car travelled?

- A. 125 m
- B. 250 m
- C. 650 m
- D. 750 m
- E. 1,200 m
- F. More information needed

Question 02:

An electric heater is connected to 120 V mains by a copper wire that has a resistance of 8 ohms. What is the power of the heater?

- A. 90 W
- B. 180 W
- C. 900 W
- D. 1800 W
- E. 9,000W
- F. 18,000 W
- G. More information needed

Question 03:

In a particle accelerator electrons are accelerated through a potential difference of 40 MV and emerge with an energy of 40MeV ($1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$). Each pulse contains 5,000 electrons. The current is zero between pulses. Assuming that the electrons have zero energy prior to being accelerated what is the power delivered by the electron beam?

- A. 1 kW
- B. 10 kW
- C. 100 kW
- D. 1,000 kW
- E. 10,000 kW
- F. More information needed

PHYSICS QUESTIONS**Question 04:**

Which of the following statements is true?

- A. When an object is in equilibrium with its surroundings, there is no energy transferred to or from the object and so its temperature remains constant.
- B. When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate and so its temperature remains constant.
- C. Radiation is faster than convection but slower than conduction.
- D. Radiation is faster than conduction but slower than convection.
- E. None of the above.

Question 05:

A 6kg block is pulled from rest along a horizontal frictionless surface by a constant horizontal force of 12 N. Calculate the speed of the block after it has moved 300 cm.

- A. $2\sqrt{3} \text{ ms}^{-1}$
- B. $4\sqrt{3} \text{ ms}^{-1}$
- C. $4\sqrt{3} \text{ ms}^{-1}$
- D. 12 ms^{-1}
- E. $\sqrt{\frac{3}{2}} \text{ ms}^{-1}$

Question 06:

A 100 V heater heats 1.5 litres of pure water from 10°C to 50°C in 50 minutes. Given that 1 kg of pure water requires 4,000 J to raise its temperature by 1°C, calculate the resistance of the heater.

- A. 12.5 ohms
- B. 25 ohms
- C. 125 ohms
- D. 250 ohms
- E. 500 ohms
- F. 850 ohms

PHYSICS QUESTIONS**Question 07:**

Which of the following statements are true?

1. Nuclear fission is the basis of nuclear energy.
2. Following fission, the resulting atoms are a different element to the original one.
3. Nuclear fission often results in the production of free neutrons and photons.

- | | | |
|------------|---------------|----------------|
| A. Only 1 | E. 2 and 3 | H. None of the |
| B. Only 2 | F. 1 and 3 | statements are |
| C. Only 3 | G. 1, 2 and 3 | true |
| D. 1 and 2 | | |

Question 08:

Which of the following statements are true? Assume $g = 10 \text{ ms}^{-2}$.

1. Gravitational potential energy is defined as $\Delta E_p = m \times g \times \Delta h$.
2. Gravitational potential energy is a measure of the work done against gravity.
3. A reservoir situated 1 km above ground level with 10^6 litres of water has a potential energy of 1 Giga Joule.

- | | |
|------------|------------------------------------|
| A. Only 1 | E. 2 and 3 |
| B. Only 2 | F. 1 and 3 |
| C. Only 3 | G. 1, 2 and 3 |
| D. 1 and 2 | H. None of the statements are true |



PHYSICS QUESTIONS**Question 9:**

A stone is thrown horizontally from the top of a cliff at 10 ms^{-1} . The stone falls 50 m before it hits the ground. Take g as 10 Nkg^{-1} and neglect air resistance. What is the speed of the stone when it hits the ground?

- | | | |
|----------------------------------|----------------------------------|---------------------------------|
| A. $10\sqrt{11} \text{ ms}^{-1}$ | D. $10\sqrt{5} \text{ ms}^{-1}$ | G. $5\sqrt{10} \text{ ms}^{-1}$ |
| B. $11\sqrt{10} \text{ ms}^{-1}$ | E. $15\sqrt{11} \text{ ms}^{-1}$ | H. $11\sqrt{5} \text{ ms}^{-1}$ |
| C. $50\sqrt{10} \text{ ms}^{-1}$ | F. $50\sqrt{5} \text{ ms}^{-1}$ | |

Question 10:

A sailing boat has a mass of 200 kg. When the boat travels at 30 ms^{-1} the resistive force exerted on the hull by the water is 300 N. If the pressure exerted on the sail by the wind is 40 Nm^{-2} , and the boat is still accelerating at 1 ms^{-2} when it travels at 30 ms^{-1} , how big is the sail?

- | | | | |
|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| A. 10 m^2 | C. $\frac{10}{4} \text{ m}^2$ | F. $\frac{25}{4} \text{ m}^2$ | H. $\frac{16}{3} \text{ m}^2$ |
| B. $\frac{25}{2} \text{ m}^2$ | D. 12 m^2 | G. $\frac{15}{2} \text{ m}^2$ | |
| | E. 24 m^2 | | |

Question 11:

A watch, when submerged in water, can withstand an average force of 10 kN on its surface. The total surface area of the watch is 50 cm^2 . Given that atmospheric pressure is 1 bara = 10^5 Pa , the density of water is 1000 kgm^{-3} , and g is 10 Nkg^{-1} , what is the deepest the watch can be taken underwater?

- | | | | |
|----------|----------|----------|----------|
| A. 120 m | C. 470 m | E. 530 m | G. 50 m |
| B. 30 m | D. 200 m | F. 500 m | H. 190 m |

Question 12:

A plane of mass 10,000 kg coming into land hits the runway travelling at 100 ms^{-1} . It decelerates uniformly over a distance of 500 m, until it reaches a taxi velocity of 10 ms^{-1} . What is the resultant force on the aircraft?

- | | | | |
|------------|------------|------------|-----------|
| A. 1.00 kN | C. 9.09 kN | E. 99.0 kN | G. 101 kN |
| B. 1.80 kN | D. 10.0 kN | F. 100 kN | H. 200 kN |



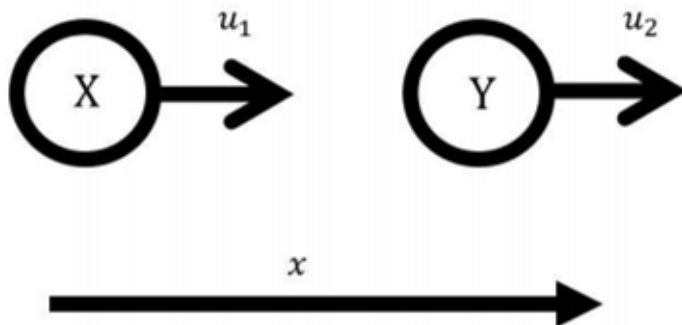
PHYSICS QUESTIONS**Question 13:**

Two cups in different buildings are connected by a wire, which is 1000 m long. If one person sings a note with a frequency of 250 Hz into the cup, a 0.2 s delay is recorded until it is first heard in the other cup. What is the wavelength of the sound wave travelling through the wire?

- | | | | |
|--------------------|-------------------|-------------------|-------------------|
| A. 0.20 m | C. 1.0 m | E. 5.0 m | G. 20 m |
| B. 0.80 m | D. 4.0 m | F. 10 m | H. 200 m |

Question 14:

Two identical snooker balls (X and Y) of mass 4 kg , with velocities u_1 and u_2 respectively (in the positive x direction, $u_1 > u_2$) collide elastically. After the collision, ball X comes to rest, while ball Y moves off with velocity v . The diagram below shows the state of the balls before the collision.



If $u_1 = 1\text{ ms}^{-1}$, what is the value of u_2 ?

- | | | | |
|---------------------------|---------------------------|---------------------------|--------------------------|
| A. 0.000 ms^{-1} | C. 0.125 ms^{-1} | E. 0.375 ms^{-1} | G. 1.00 ms^{-1} |
| B. 0.100 ms^{-1} | D. 0.250 ms^{-1} | F. 0.500 ms^{-1} | H. 2.00 ms^{-1} |

Question 15:

A car of mass 500 kg is climbing a slope inclined at 30° to the horizontal. The car is travelling at a speed of 20 ms^{-1} , when the engine breaks down, so that the only forces acting on the car are gravity and a constant friction force of 7500 N . The lorry travels a distance x parallel to the slope before coming to rest instantaneously. How far is x ?

- | | | | |
|-------------------|-------------------|-------------------|-------------------|
| A. 1.0 m | C. 5.0 m | E. 13 m | G. 130 m |
| B. 2.0 m | D. 10 m | F. 100 m | H. 200 m |

PHYSICS QUESTIONS

Question 16:

A hydroelectric dam stores energy by pumping water from a lower reservoir to a higher reservoir using an electric pump (supplied with 100V and 10A). This power can then be harnessed again by allowing the water to fall down to the lower reservoir, driving a turbine, generating electricity. It takes 1000s for the pump to lift 1000 kg of water to a height of 75 m above the reservoir. This mass of water is then used to drive a turbine, which converts the gravitational potential energy of the water into electrical energy with a 33.3% efficiency. What percentage of energy is lost in the act of storing energy using this hydroelectric dam?

- A. 0% C. 33.3% E. 66.7% G. 80%
- B. 25% D. 50% F. 75%

Question 17:

An object falling through the air in a straight-line experiences quadratic air resistance, such that the acceleration, a of the object as a function of its velocity, v is given by:

$$a(v) = g - bv^2$$

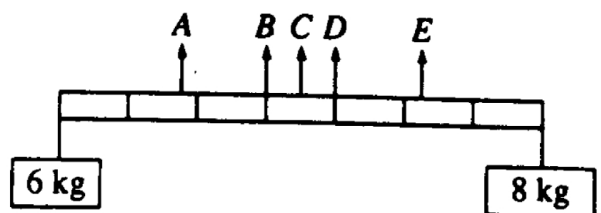
Where b is a non-negative constant and g is acceleration due to gravity. Which of the following is an expression for the terminal velocity, v_0 of the object?

- A. $g - b$ D. $g - \sqrt{b}$ G. $-2bv$
- B. $\sqrt{\frac{g}{b}}$ E. $b - \sqrt{g}$ H. $gv - \frac{1}{3}bv^3$
- C. $\frac{g}{b}$ F. $\sqrt{\frac{b}{g}}$

Question 18:

Two objects, of masses 6 and 8 kilograms, are hung from the ends of a stick that is 70 cm long and has marks every 10 cm, as shown. If the mass of the stick is negligible, at which of the points indicated should a cord be attached if the stick is to remain horizontal when suspended from the cord?

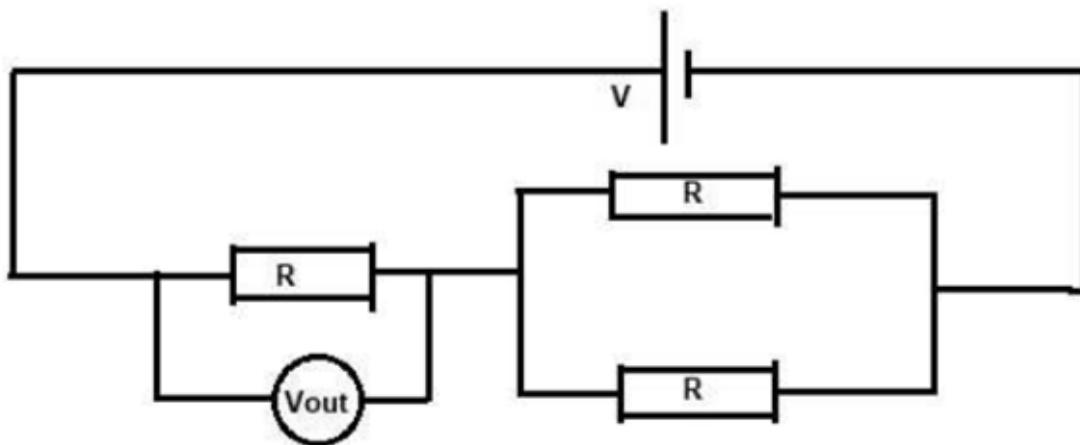
- (A) A (B) B (C) C (D) D



PHYSICS QUESTIONS

Question 19:

In the below circuit, a cell with an e.m.f of $V = 1V$ drives a current through three identical resistors R .



What is the potential difference across the voltmeter marked V_{out} ?

- | | | |
|-------------------|-------------------|-------------------|
| A. $\frac{1}{6}V$ | C. $\frac{1}{3}V$ | E. $\frac{2}{3}V$ |
| B. $\frac{1}{4}V$ | D. $\frac{1}{2}V$ | F. $\frac{3}{4}V$ |

Question 20:

A particle's velocity, v as a function of time, t is given by the function:

$$v(t) = t^5 - 5t^3 + 4t$$

In the entire range of t ($-\infty < t < \infty$), how many times does the particle instantaneously come to rest?

- A. 0
- B. 1
- C. 2
- D. 3
- E. 4
- F. 5
- G. 6

PHYSICS QUESTIONS

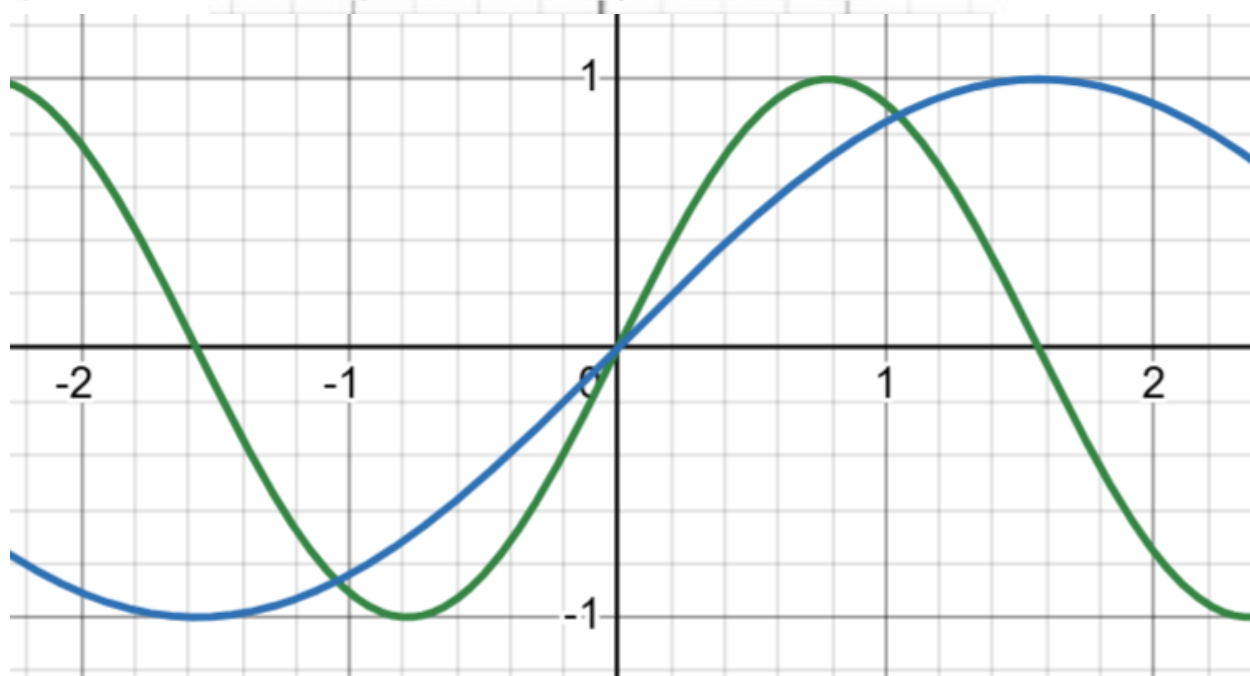
Question 21:

Velocity, v is the rate of change of displacement, s . For the same particle as in question 8 (with the same velocity function), what is the distance travelled by the particle between $t = -1$ s and $t = 1$ s?

- A. 0 m
- B. $\frac{1}{12}$ m
- C. $\frac{1}{8}$ m
- D. $\frac{11}{12}$ m
- E. $\frac{11}{6}$ m
- F. $\frac{13}{6}$ m
- G. 4 m

Question 22:

An oscilloscope measures an electrical signal from an alternating current power supply. The voltage, V measured by the oscilloscope has the form $V = V_0 \sin(\omega t + \phi)$, where V_0 , ω and ϕ are all positive constants.



If the blue curve is the original signal, which of the following operations would give rise to the signal shown by the green curve?

- A. Halve ϕ
- B. Double ϕ
- C. Halve ω
- D. Double ω
- E. Halve V_0
- F. Double V_0
- G. Increase ϕ by 180°



PHYSICS QUESTIONS**Question 23:**

An object of mass 0.1 kg sits on a rough plane, inclined at θ to the horizontal. If the coefficient of friction of the plane is $\frac{1}{\sqrt{3}}$, what is the maximum value of θ (in degrees) without the object slipping?

- A. 0°
- B. 15°
- C. 30°
- D. 45°
- E. 60°
- F. 75°

Question 24:

The Tsiolkovsky rocket equation (below) is used to calculate the change in velocity, Δv , experienced by a spacecraft during a manoeuvre in terms of its exhaust velocity, u , and the ratio of its initial mass, m_0 , and final mass, m_f .

$$\Delta v = u \log_e \left(\frac{m_0}{m_f} \right)$$

In order to achieve place a payload of mass 1 kg in a circular orbit, a Δv of 5000 ms^{-1} must be achieved. If using a rocket with an exhaust velocity of 2500 ms^{-1} , what is the minimum mass of fuel that can be used in order to place the payload in circular orbit?

- | | | |
|---------------------------------|-------------------------------------|------------------------------|
| A. $e^2 \text{ kg}$ | D. $e^{\frac{1}{2}} - 1 \text{ kg}$ | G. $\log_e 2 - 1 \text{ kg}$ |
| B. $e^{\frac{1}{2}} \text{ kg}$ | E. $e^2 + 1 \text{ kg}$ | H. $\log_e 2 + 1 \text{ kg}$ |
| C. $e^2 - 1 \text{ kg}$ | F. $\log_e 2 \text{ kg}$ | |

Question 25:

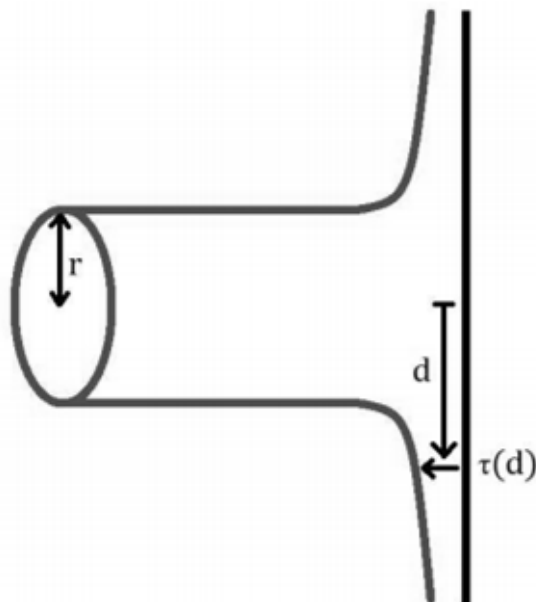
Stationary waves can be set up in tube with one closed end and one open end, such that there is a *node* at the closed end and an *antinode* at the open end. What is the **second** lowest frequency note that can be produced by a tube which is 0.6 m long, given that the speed of sound is 340 ms^{-1} ?

- | | | |
|---------------------|---------------------|----------------------|
| A. 283 Hz | C. 425 Hz | E. 850 Hz |
| B. 378 Hz | D. 567 Hz | F. 1700 Hz |

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PHYSICS QUESTIONS**Question 26:**

A cylindrical jet of water, with a radius, r and velocity, v strikes a wall and spreads out into a plane.



The velocity and density of the water remains constant. Which of the following is an expression for the thickness of the plane, τ as a function of the distance from the point of impact of the jet, d ?

Hint: Mass is conserved. Consider the mass coming in and out in a certain amount of time.

- | | | |
|---------------------|--------------------|--------------------|
| A. $\frac{r^2}{2d}$ | D. $\frac{d}{r^2}$ | F. $\frac{2r}{dv}$ |
| B. $\frac{v^2}{rd}$ | E. $\frac{2d}{v}$ | |
| C. dr^{-2} | | |

Question 27:

A canon on top of a 20 m castle wall is fired horizontally at 100 ms^{-1} . How far away from the foot of the castle wall does the canon ball land (you may ignore the effect of air resistance)?

- | | |
|----------|----------|
| A. 100 m | H. 800 m |
| B. 200 m | |
| C. 300 m | |
| D. 400 m | |
| E. 500 m | |
| F. 600 m | |
| G. 700 m | |



PHYSICS ANSWERS**Question 01: D**

Total Distance = Distance during acceleration phase + Distance during braking phase

Distance during acceleration phase is given by:

$$s = ut + \frac{at^2}{2} = 0 + \frac{5 \times 10^2}{2} = 250 \text{ m}$$

$$v = u + at = 0 + 5 \times 10 = 50 \text{ ms}^{-1}$$

And use $a = \frac{v-u}{t}$ to calculate the deceleration: $a = \frac{0-50}{20} = -2.5 \text{ ms}^{-2}$

Distance during the deceleration phase is given by:

$$s = ut + \frac{at^2}{2} = 50 \times 20 + \frac{-2.5 \times 20^2}{2} = 1000 - \frac{2.5 \times 400}{2}$$

$$s = 1000 - 500 = 500 \text{ m}$$

Thus, *Total Distance* = 250 + 500 = 750 m

Question 02: G

It is not possible to calculate the power of the heater as we don't know the current that flows through it or its internal resistance. The 8 ohms refers to the external copper wire and not the heater. Whilst it's important that you know how to use equations like $P = IV$, it's more important that you know when you **can't** use them!



PHYSICS ANSWERS**Question 03: F**

This question has a lot of numbers but not any information on time, which is necessary to calculate power. You cannot calculate power by using $P = IV$ as you don't know how many electrons are accelerated through the potential difference per unit time. Thus, more information is required to calculate the power.

Question 04: B

When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate and so its temperature remains constant i.e. there is no *net* energy transfer. Radiation is slower than conduction and convection.

Question 05: A

The work done by the force is given by: $Work\ Done = Force \times Distance = 12\ N \times 3\ m = 36\ J$

Since the surface is frictionless, $Work\ Done = Kinetic\ Energy$.

$$E_k = \frac{mv^2}{2} = \frac{6v^2}{2}$$

$$\text{Thus, } 36 = 3v^2$$

$$v = \sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}\ ms^{-1}$$

Question 06: C

Total energy supplied to water

$$= \text{Change in temperature} \times \text{Mass of water} \times 4,000\ J$$

$$= 40 \times 1.5 \times 4,000 = 240,000\ J$$

$$\text{Power of the heater} = \frac{\text{Work Done}}{\text{time}} = \frac{240,000}{50 \times 60} = \frac{240,000}{3,000} = 80\ W$$

Using $P = IV = \frac{V^2}{R}$:

$$R = \frac{V^2}{P} = \frac{100^2}{80} = \frac{10,000}{80} = 125\ ohms$$



PHYSICS ANSWERS**Question 07: G**

The large amount of energy released during atomic fission is the basis underlying nuclear power plants. Splitting an atom into two or more parts will by definition produce molecules of different sizes than the original atom; therefore, it produces two new atoms. The free neutrons and photons produced by the splitting of atoms form the basis of the energy release.

Question 08: D

Gravitational potential energy is just an extension of the equation work done = force \times distance (force is the weight of the object, mg , and distance is the height, h). The reservoir in statement 3 would have a potential energy of 10^{10} Joules i.e. 10 Giga Joules ($E_p = 10^6 \text{ kg} \times 10 \text{ N} \times 10^3 \text{ m}$).

Question 09: A

Work out the vertical component of the velocity due to the acceleration of gravity and combine it with the horizontal velocity component, which does not change.

$$v^2 = u^2 + 2as$$

$$\therefore v_v^2 = 0^2 + 2 \times 10 \times 50 = 1000 \text{ m}^2\text{s}^{-2}$$

$$v_v = \sqrt{1000} \text{ ms}^{-1}$$

$$v_h = 10 \text{ ms}^{-1}$$

$$v_{tot} = \sqrt{v_v^2 + v_h^2} = \sqrt{1000 + 10^2} = \sqrt{1100}$$

$$v_{tot} = 10\sqrt{11} \text{ ms}^{-1}$$

Question 29 10: B

Work out the force that needs to be supplied by the sail and use this to calculate its area. The force can be found from a force balance.

$$F_{sail} - F_{drag} = ma$$

$$F_{sail} - 300 = 200 \times 1$$

$$F_{sail} = 500 \text{ N}$$

Using equation for pressure:

$$P = \frac{F}{A}$$

$$\Rightarrow A = \frac{F}{P}$$

$$A = \frac{500}{40} = \frac{25}{2} \text{ m}^2$$



PHYSICS ANSWERS**Question 11: H**

Work out the maximum pressure the watch can withstand then find the depth of water such that the hydrostatic pressure and atmospheric pressure sum to this value.

$$P_{max} = \frac{F_{max}}{A}$$

$$P_{max} = \frac{10000}{50 \times 10^{-4}} = 2 \times 10^6 \text{ Pa}$$

hydrostatic pressure, $P_h = \rho gh$

At the maximum depth:

$$P_{max} = P_h + P_{atm}$$

$$P_{max} = \rho gh + P_{atm}$$

$$2 \times 10^6 = 1000 \times 10 \times h + 10^5$$

$$2 \times 10^6 = 10^4 h + 10^5$$

$$200 = h + 10$$

$$h = 190 \text{ m}$$

Question 12: E

The relevant SUVAT equation is $v^2 = u^2 + 2as$, where $u = 100$ (initial speed), $v = 10$ (final speed) and $s = 500$ (displacement). Plugging these numbers into the equation gives $-9900 = 1000a$, and hence $a = -9.9 \text{ ms}^{-2}$. You then use $F = ma$ to find the resultant force on the aircraft, $9.9 \times 10000 = 99000 \text{ N}$.

Question 13: G

Relevant equations are speed = distance / time, and for waves, speed = frequency \times wavelength. We know the distance travelled by the wave (1000m), the time taken for it travel (0.2s), as well as the frequency (250Hz), so we just rearrange for the wavelength:

$$\text{wavelength} = \frac{\text{distance}}{\text{time} \times \text{frequency}} = \frac{1000}{0.2 \times 250} = 20\text{m}$$



PHYSICS ANSWERS**Question 14: A**

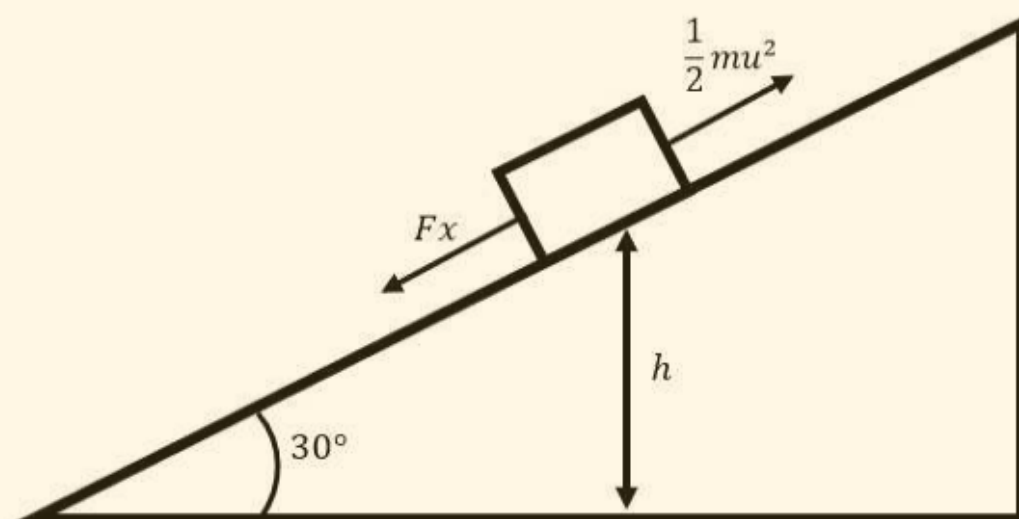
The key idea of the question is to identify that the collision is elastic, and hence energy is conserved. Also, momentum is conserved in any collision. Given that the balls are identical, if the first ball strikes the second ball and comes to rest, it has transferred all its momentum and energy to the second ball.

The second ball after the collision hence has momentum $mv = m(u_1 + u_2)$ (conservation of momentum) and energy $\frac{1}{2}mv^2 = \frac{1}{2}m(u_1 + u_2)^2$ (using previous result). The initial energy is however $\frac{1}{2}m(u_1^2 + u_2^2)$. The initial and final energy can hence only be equal when $u_2 = 0$. It is worth noting that this result holds regardless of the value of u_1 .

Question 15: D

The best way to approach this problem is with conservation of energy. The car has initial kinetic energy $\frac{1}{2}mu^2$ ($u = 20 \text{ ms}^{-1}$). In order that energy is conserved, when the kinetic energy goes to zero (when the car is at rest, $v = 0$) it has been converted into other forms – namely gravitational potential energy, and work done against friction. We hence write down the energy equation:

$$\frac{1}{2}mu^2 = mgh + Fx$$



As gravitational potential energy is dependent on the vertical height, you use trigonometry to show that $h = x \sin(30^\circ) = \frac{x}{2}$, plugging in $m = 500 \text{ kg}$, $F = 7500 \text{ N}$ and taking $g = 10 \text{ ms}^{-2}$, we can rearrange to show: $100000 = 2500x + 7500x$ and hence $x = 10 \text{ m}$.

PHYSICS ANSWERS**Question 16: F**

This question is very wordy, but the crucial step is noticing that the electrical energy into the pump (given by the equation $E = VIt$) is 1MJ but the gravitational potential energy of the water (given by the equation $E = mgh$) is 0.75MJ. You therefore multiply the efficiencies of the two processes for an overall efficiency of $75\% \times 33.3\% = 25\%$, so 75% of the energy is lost.

Question 17: B

The equation might confuse you, but the key thing is to remember the physical meaning of terminal velocity - is the value of v such that the acceleration is 0. We simply solve for $a(v_0) = 0$, setting $a = 0$ and hence rearranging $bv_0^2 = g$, which

leads to $v_0 = \sqrt{\frac{g}{b}}$.

Question 18: D

To balance the torques on each side, we obviously need to be closer to the heavier mass.

Trying point D as a pivot point we have:

$$(m_1g) \cdot r_1 = (m_2g) \cdot r_2$$

$$(6\text{kg})(40\text{ cm}) = (8\text{kg})(30\text{ cm}) \quad \text{and we see it works.}$$

Question 19: E

We notice that this a potential divider circuit, therefore we can use the equation:

$$V_{out} = V_{in} \left(\frac{R_1}{R_1 + R_2} \right)$$

Where R_1 is the resistance of the series resistor, and R_2 is the remaining resistance of the circuit. We note that the resistance of two resistors in parallel is half that of single resistor, hence $R_2 = \frac{R}{2}$ (this results can be derived from Kirchhoff's laws, as each resistor carries the same current, so overall twice as much current flows for the same voltage, hence half the resistance). Plugging in R_1 and R_2 , and using $V_{in} = 1$:

$$V_{out} = \frac{1}{1 + 0.5} V = \frac{2}{3} V$$

PHYSICS ANSWERS**Question 20: F**

Here we are being asked to find the solutions to $v(t) = 0$ (the meaning of *at rest*). This is a quintic function, which would usually be unsolvable (we can also instantly discount 0, 2, 4 and 6 as potential solutions, as a polynomial can only have as many solutions as the order of the polynomial, and odd polynomials must have an odd number of solutions). However, we can instantly factor out a power of t :

$$t^5 - 5t^3 + 4t = t(t^4 - 5t^2 + 4)$$

In the brackets is a quartic, but if we substitute $x = t^2$ it becomes a quadratic, allowing us to factorise:

$$t^5 - 5t^3 + 4t = t(t^2 - 4)(t^2 - 1)$$

Each of the remaining two quadratics are the difference of two squares, and hence have two real solutions, giving us five solutions ($t = 0, t = 1, t = 2, t = -1, t = -2$):

$$t^5 - 5t^3 + 4t = t(t - 2)(t - 1)(t + 1)(t + 2)$$

Question 21: E

We are told that velocity is the rate of change of displacement, and hence the derivative of displacement with respect to time: $v = \frac{ds}{dt}$. To find the displacement, we hence integrate the velocity function to find: $s(t) = \frac{1}{6}t^6 - \frac{5}{4}t^4 + 2t^2$. However, we cannot simply plug in the limits of $t = -1$ and $t = 1$, as the *distance* was requested. We know there is a solution at $t = 0$, so must compute the integrals in the limits $t = -1$ and $t = 0$, and $t = 0$ and $t = 1$, and find add the absolute values. Each integral comes to $\pm \frac{11}{12}$ so the final answer is $\frac{11}{6}$.

Question 22: D

To answer this question, you need to know the effects of transformations on the function. This is effectively halving the effect on the x -axis, so we expect the value of x to be double (and hence ω is doubled). ω is the frequency of the signal, so we expect that doubling it halves the period of oscillation (V_0 is the amplitude and φ is the phase difference).



PHYSICS ANSWERS**Question 23: C**

This is a mechanics problem, so we resolve parallel and perpendicular to the plane.

Expressing the normal force as N and the friction force as μN :

Parallel to the plane: $N = mg \sin \theta$

Perpendicular to the plane: $\mu N = \frac{1}{\sqrt{3}} mg \cos \theta$

For the maximum value of θ with the object remaining stationary:

$$\frac{1}{\sqrt{3}} mg \cos \theta = mg \sin \theta$$

$$\frac{1}{\sqrt{3}} = \tan \theta$$

It should therefore be recognised that $\tan 30 = \frac{1}{\sqrt{3}}$, hence the answer is 30° .

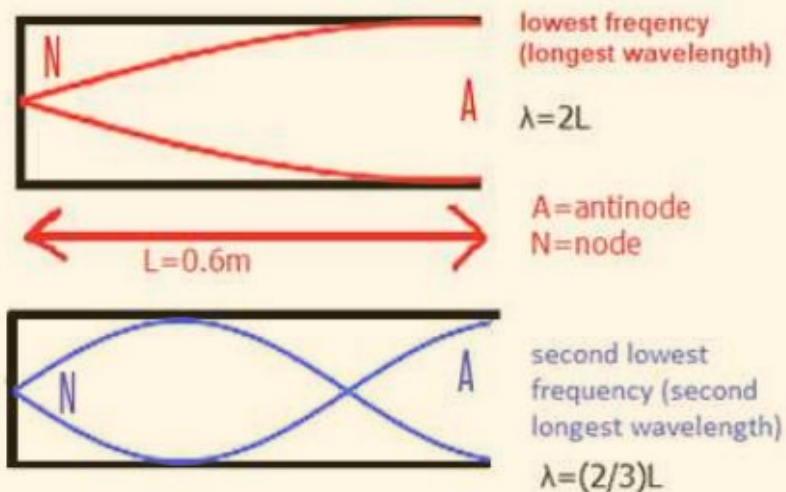
Question 24: C

Although the question is very wordy, it is actually simple, as you just need to use the rules for exponentials to rearrange the equation for m_0 . The only other step is to subtract the payload mass from the final answer, to find the amount of *fuel* burned. The equation rearranges as:

$$\Delta v = u \log_e \left(\frac{m_0}{m_f} \right) \therefore \frac{m_0}{m_f} = e^{\frac{\Delta v}{u}}$$

Plugging in the numbers give $m_0 = e^2$ and hence the mass of fuel is $e^2 - 1$ kg.



PHYSICS ANSWERS**Question 25: E**

As the diagram shows, the second lowest frequency corresponds to a wavelength of 0.4 m. We now just use the speed of sound in the equation, $v = f\lambda$ to rearrange to find:

$$f = \frac{340}{0.4} = 850 \text{ Hz}$$

PHYSICS ANSWERS**Question 26: A**

This question is very difficult if you are not used to thinking about volumes and areas in physics. The key idea is to think geometrically. The density of the water is constant, as is the flow velocity, so we need to consider the cross-sectional area of water coming in, and the cross-sectional area of water coming out.

Mass conservation with constant density = volume conservation i.e. vol of water in = vol of water out

Volume flow rate = cross sectional area \times flow velocity, so for a constant flow velocity and volume conservation, we arrive at area conservation i.e. cross-sectional area of incoming water = cross-sectional area of outgoing water.

The incoming water is a circle, with area πr^2 . The outgoing water is the surface area of a cylinder with radius d from the centre where the incoming jet hits the wall, so the total area of outgoing water is $2\pi d \times \tau$ where τ is the thickness. By equating these areas, we arrive at the result: $\tau = \frac{r^2}{2d}$

It is also possible to deduce the answer by looking at the dimensions of the other answers. Only **A** has dimensions of length (B has dimensions of frequency, C has dimension of volume, D of inverse length, E is dimensionless, and F has dimensions of time). If you are not familiar with this technique, do not worry, but one way of thinking about it is to substitute in the units. If your answer for a thickness is not in metres, you have a problem.

Question 27: B

This is a classic projectile motion problem, so we must deal with motion parallel and perpendicular to the ground separately. To find when the cannonball hits the ground, we use the relevant SUVAT equation using $s = 20$, $a = 10$ and $u = 0$ as the cannonball accelerates towards the ground. Plugging this into $s = ut + \frac{1}{2}at^2$ gives $t = 2s$.

We now consider the movement parallel to the ground – the cannonball travels constantly at $v = 100 \text{ ms}^{-1}$ for 2 s, so it travels 200 m.



MATH 2

Question 1:

Which of the following graphs **do not** intersect?

1. $y = x$

2. $y = x^2$

3. $y = 1 - x^2$

4. $y = 2$

A. 1 and 2

C. 3 and 4

E. 1 and 4

B. 2 and 3

D. 1 and 3

F. 2 and 4

Question 2:

Calculate the product of 897,653 and 0.009764.

A. 87646.8

C. 876.468

E. 8.76468

B. 8764.68

D. 87.6468

F. 0.876468

Question 3:

Solve for x : $\frac{7x+3}{10} + \frac{3x+1}{7} = 14$

A. $\frac{929}{51}$

B. $\frac{949}{47}$

C. $\frac{949}{79}$

D. $\frac{980}{79}$

Question 4:

What is the area of an equilateral triangle with side length x ?

A. $\frac{x^2\sqrt{3}}{4}$

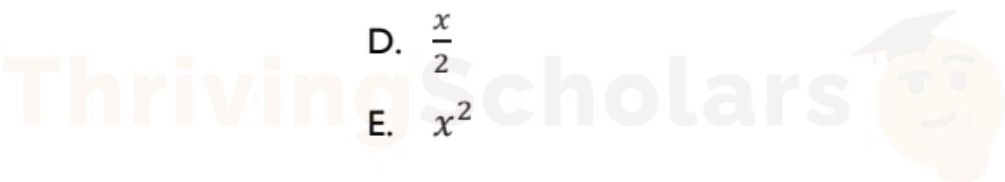
C. $\frac{x^2}{2}$

F. x

B. $\frac{x\sqrt{3}}{4}$

D. $\frac{x}{2}$

E. x^2



MATH 2 QUESTIONS

Question 5:

Simplify $3 - \frac{7x(25x^2 - 1)}{49x^2(5x+1)}$

A. $3 - \frac{5x-1}{7x}$

C. $3 + \frac{5x-1}{7x}$

E. $3 - \frac{5x^2}{49}$

B. $3 - \frac{5x+1}{7x}$

D. $3 + \frac{5x+1}{7x}$

F. $3 + \frac{5x^2}{49}$

Question 6:

Solve the equation $x^2 - 10x - 100 = 0$

A. $-5 \pm 5\sqrt{5}$

C. $5 \pm 5\sqrt{5}$

E. $5 \pm 5\sqrt{125}$

B. $-5 \pm \sqrt{5}$

D. $5 \pm \sqrt{5}$

F. $-5 \pm \sqrt{125}$

Question 7:

Rearrange $x^2 - 4x + 7 = y^3 + 2$ to make x the subject.

A. $x = 2 \pm \sqrt{y^3 + 1}$

C. $x = -2 \pm \sqrt{y^3 - 1}$

E. x cannot be made the subject for this equation.

B. $x = 2 \pm \sqrt{y^3 - 1}$

D. $x = -2 \pm \sqrt{y^3 + 1}$

Question 8:

Rearrange $3x + 2 = \sqrt{7x^2 + 2x + y}$ to make y the subject.

A. $y = 4x^2 + 8x + 2$

C. $y = 2x^2 + 10x + 2$

E. $y = x^2 + 10x + 2$

B. $y = 4x^2 + 8x + 4$

D. $y = 2x^2 + 10x + 4$

F. $y = x^2 + 10x + 4$

Question 14:

Differentiate:

$$y = x^2 \ln x.$$

- (A) $2x \ln x$
- (B) $2x \ln x + x$
- (C) $x \ln x + 2x$
- (D) $x^2 \cdot \frac{1}{x}$
- (E) $2x^2 \ln x$

Question 15:Find the range of $y = \sin x + 2$.

- (A) $[-1, 1]$
- (B) $[0, 2]$
- (C) $[1, 3]$
- (D) $[2, 4]$
- (E) $(-\infty, \infty)$

Question 16:On the interval $1 \leq x \leq e$, what is the area of the region between the curves $y = \ln x$ and $y = 1 - x$?

- (A) $\frac{e^2 - 1}{2}$
- (B) $\frac{e^2 - 2e + 1}{2}$
- (C) $\frac{e^2 - 2e + 3}{2}$
- (D) $e - \frac{1}{e}$
- (E) $e^2 - e + 1$

MATH 2 QUESTIONS

Question 17:

For $x > 0$, define $f(x) = x + \frac{1}{x} + \ln x$. The minimum value of $f(x)$ is

- (A) $2 + \ln\left(\frac{1}{2}\right)$
 (B) $\sqrt{5} + \ln\left(\frac{\sqrt{5}-1}{2}\right)$
 (C) $\sqrt{5} + \ln\left(\frac{\sqrt{5}+1}{2}\right)$
 (D) $2\sqrt{5} + \ln\left(\frac{\sqrt{5}-1}{2}\right)$
 (E) $\sqrt{5} - \ln\left(\frac{\sqrt{5}-1}{2}\right)$

Question 18:

For $0 < x < \frac{\pi}{2}$, the maximum value of

$$\ln(\sin x) + 2 \ln(\cos x)$$

is

- (A) $\ln\left(\frac{2}{3}\right)$
 (B) $\ln\left(\frac{2}{3\sqrt{3}}\right)$
 (C) $\ln\left(\frac{1}{3}\right)$
 (D) $\ln\left(\frac{4}{9}\right)$
 (E) $\ln\left(\frac{1}{3\sqrt{3}}\right)$

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Question 19:**MATH 2 QUESTIONS**

Given that $y = (2 + 3x)^6$, what is the coefficient of x^3 in $\frac{dy}{dx}$?

- A 240
- B 4320
- C 4860
- D 12960
- E 19440

Question 20:

Given that

$$\int_0^2 x^m dx = \frac{16\sqrt{2}}{7}$$

and

$$\int_0^2 x^{m+1} dx = \frac{32\sqrt{2}}{9}$$

what is the value of m ?

- A $-\frac{11}{2}$
- B $-\frac{9}{2}$
- C $-\frac{22}{29}$
- D $\frac{7}{22}$
- E $\frac{5}{2}$
- F $\frac{7}{2}$

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MATH 2 QUESTIONS

Question 21:

The complete set of values of a for which the equation $3x^2 = (a+2)x - 3$ has two real distinct roots is

- A no values of a
- B $-4\sqrt{2} < a < 4\sqrt{2}$
- C $a < -4\sqrt{2}, a > 4\sqrt{2}$
- D $-4 < a < 8$
- E $a < -4, a > 8$
- F $-8 < a < 4$
- G $a < -8, a > 4$
- H all values of a

Question 22:

$f(x) = x^3 - a^2x$ where a is a positive constant.

Find the complete set of values of x for which $f(x)$ is an increasing function.

- A $x \leq -a, x \geq a$
- B $-a \leq x \leq a$
- C $x \leq -a, 0 \leq x \leq a$
- D $-a \leq x \leq 0, x \geq a$
- E $x \leq -\frac{a}{3}, x \geq \frac{a}{3}$
- F $-\frac{a}{3} \leq x \leq \frac{a}{3}$
- G $x \leq -\frac{a}{\sqrt{3}}, x \geq \frac{a}{\sqrt{3}}$
- H $-\frac{a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$



MATH 2 QUESTIONS

Question 23:

What is the coefficient of x^3 in the expansion of $(1 - 2x)^5(1 + 2x)^5$?

- A -6400
- B -640
- C -80
- D 0
- E 80
- F 800
- G 960

Question 24:

What is the equation of the line that is normal to the following function at $x = 4$?

$$y = f(x) = x^3 - 3x^{\frac{3}{2}} + 5x - 2$$

- A. $y = -\frac{1}{8}x + \frac{11}{2}$
- B. $y = -\frac{1}{8}x + \frac{21}{2}$
- C. $y = -\frac{1}{8}x + \frac{7}{2}$

- D. $y = 8x + \frac{2}{3}$
- E. $y = 8x - \frac{16}{3}$
- F. $y = -\frac{1}{5}x + \frac{3}{2}$

- G. $y = -\frac{1}{5}x + \frac{7}{2}$
- H. $y = 5x - \frac{16}{3}$



Question 25:**MATH 2 QUESTIONS**

The curve $y = x^2$ is translated by the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and then reflected in the line $y = -1$

Which one of the following is an equation of the resulting curve?

- A $y = -3 - (x - 4)^2$
- B $y = -3 + (x + 4)^2$
- C $y = 3 - (x + 4)^2$
- D $y = 3 + (x - 4)^2$
- E $y = -5 - (x - 4)^2$
- F $y = -5 + (x + 4)^2$
- G $y = 5 - (x + 4)^2$
- H $y = 5 + (x - 4)^2$

Question 26:

The graph of the function $y = x^3 + px^2 + qx + 6$, where p and q are real constants, has a local maximum when $x = 2$ and a local minimum when $x = 4$.

What are the values of p and q ?

- A $p = -3$ and $q = -8$
- B $p = -3$ and $q = 8$
- C $p = 3$ and $q = -8$
- D $p = -9$ and $q = 24$
- E $p = 9$ and $q = 24$
- F $p = 9$ and $q = -24$



Question 27:**MATH 2 QUESTIONS**

A square PQRS is drawn above the x -axis with the side PQ on the x -axis.

P is the point $(-5, 0)$ and Q is the point $(1, 0)$.

A circle is drawn inside the square with diameter equal in length to the side of the square.

Which one of the following is an equation of the circle?

A $x^2 + y^2 - 4x + 6y + 4 = 0$

B $x^2 + y^2 - 4x + 6y + 9 = 0$

C $x^2 + y^2 + 4x - 6y + 4 = 0$

D $x^2 + y^2 + 4x - 6y + 9 = 0$

E $x^2 + y^2 - 6x - 4y + 9 = 0$

F $x^2 + y^2 - 6x + 4y + 4 = 0$

G $x^2 + y^2 + 6x - 4y + 4 = 0$

H $x^2 + y^2 + 6x + 4y + 9 = 0$



Question 1: C

By far the easiest way to solve this is to sketch them (don't waste time solving them algebraically). As soon as you've done this, it'll be very obvious that $y = 2$ and $y = 1 - x^2$ don't intersect, since the latter has its turning point at $(0, 1)$ and zero points at $x = -1$ and 1 . $y = x$ and $y = x^2$ intersect at the origin and $(1, 1)$, and $y = 2$ runs through both.

Question 2: B

Notice that you're not required to get the actual values – just the number's magnitude. Thus, 897.653 can be approximated to 900,000 and 0.009764 to 0.01. Therefore, $900,000 \times 0.01 = 9,000$, which represents the same magnitude of 8764.68.

Question 3: C

Multiply through by 70: $7(7x + 3) + 10(3x + 1) = 14 \times 70$

Simplify: $49x + 21 + 30x + 10 = 980$

$79x + 31 = 980$

$$x = \frac{949}{79}$$

Question 4: A

Split the equilateral triangle into 2 right-angled triangles and apply Pythagoras' theorem:

$$x^2 = \left(\frac{x}{2}\right)^2 + h^2. \text{ Thus } h^2 = \frac{3}{4}x^2$$

$$h = \sqrt{\frac{3x^2}{4}} = \frac{\sqrt{3x^2}}{2}$$

$$\text{The area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times x \times \frac{\sqrt{3x^2}}{2}$$

$$\text{Simplifying gives: } x \frac{\sqrt{3x^2}}{4} = x \frac{\sqrt{3} \sqrt{x^2}}{4} = \frac{x^2 \sqrt{3}}{4}$$

Question 5: A

This is a question testing your ability to spot 'the difference between two squares'.

Factorise to give: $3 - \frac{7x(5x-1)(5x+1)}{(7x)^2(5x+1)}$

Cancel out: $3 - \frac{(5x-1)}{7x}$

Question 6: C

The easiest way to do this is to 'complete the square':

$$(x - 5)^2 = x^2 - 10x + 25$$

Thus, $(x - 5)^2 - 125 = x^2 - 10x - 100 = 0$

Therefore, $(x - 5)^2 = 125$

$$x - 5 = \pm\sqrt{125} = \pm\sqrt{25} \sqrt{5} = \pm 5\sqrt{5}$$

$$x = 5 \pm 5\sqrt{5}$$

Question 7: B

Factorise by completing the square:

$$x^2 - 4x + 7 = (x - 2)^2 + 3$$

Simplify: $(x - 2)^2 = y^3 + 2 - 3$

$$x - 2 = \pm\sqrt{y^3 - 1}$$

$$x = 2 \pm \sqrt{y^3 - 1}$$

Question 8: D

Square both sides to give: $(3x + 2)^2 = 7x^2 + 2x + y$

Thus: $y = (3x + 2)^2 - 7x^2 - 2x = (9x^2 + 12x + 4) - 7x^2 - 2x$

$$y = 2x^2 + 10x + 4$$

Question 9: C

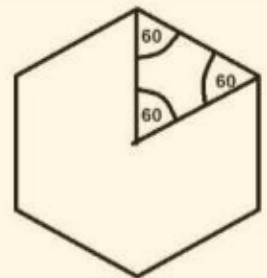
MATH 2 ANSWERS

Following BODMAS:

$$\begin{aligned}
 &= 5 \left[5(6^2 - 5 \times 3) + 400^{\frac{1}{2}} \right]^{1/3} + 7 \\
 &= 5 [5(36 - 15) + 20]^{\frac{1}{3}} + 7 \\
 &= 5 [5(21) + 20]^{\frac{1}{3}} + 7 \\
 &= 5 (105 + 20)^{\frac{1}{3}} + 7 \\
 &= 5 (125)^{\frac{1}{3}} + 7 \\
 &= 5 (5) + 7 \\
 &= 25 + 7 = 32
 \end{aligned}$$

Question 10: B

Consider a triangle formed by joining the centre to two adjacent vertices. Six similar triangles can be made around the centre – thus, the central angle is 60 degrees. Since the two lines forming the triangle are of equal length, we have 6 identical equilateral triangles in the hexagon.



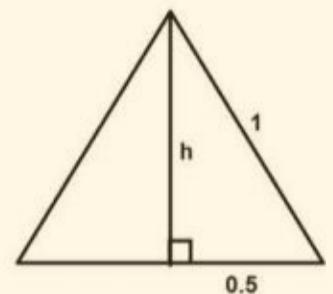
Now split the triangle in half and apply Pythagoras' theorem:

$$1^2 = 0.5^2 + h^2$$

Thus, $h = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Thus, the area of the triangle is: $\frac{1}{2}bh = \frac{1}{2} \times 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

Therefore, the area of the hexagon is: $\frac{\sqrt{3}}{4} \times 6 = \frac{3\sqrt{3}}{2}$



Question 11: B**MATH 2 ANSWERS**

Let x be the width and $x+19$ be the length.

Thus, the area of a rectangle is $x(x + 19) = 780$.

Therefore:

$$x^2 + 19x - 780 = 0$$

$$(x - 20)(x + 39) = 0$$

$$x - 20 = 0 \text{ or } x + 39 = 0$$

$$x = 20 \text{ or } x = -39$$

Since length can never be a negative number, we disregard $x = -39$ and use $x = 20$ instead.

Thus, the width is 20 metres and the length is 39 metres.

Question 12: B

The quickest way to solve is by trial and error, substituting the provided options.

However, if you're keen to do this algebraically, you can do the following:

Start by setting up the equations: Perimeter = $2L + 2W = 34$

$$\text{Thus: } L + W = 17$$

$$\text{Using Pythagoras: } L^2 + W^2 = 13^2$$

$$\text{Since } L + W = 17, W = 17 - L$$

$$\text{Therefore: } L^2 + (17 - L)^2 = 169$$

$$L^2 + 289 - 34L + L^2 = 169$$

$$2L^2 - 34L + 120 = 0$$

$$L^2 - 17L + 60 = 0$$

$$(L - 5)(L - 12) = 0$$

$$\text{Thus: } L = 5 \text{ and } L = 12$$

$$\text{And: } W = 12 \text{ and } W = 5$$

Question 13: F**MATH 2 ANSWERS**

$$\text{Solve: } \frac{5x - 5}{2} + \frac{x + 5}{4} = x + 1$$

$$\text{Multiply by 4: } 2(5x - 5) + (x + 5) = 4x + 4$$

$$10x - 10 + x + 5 = 4x + 4 \Rightarrow 11x - 5 = 4x + 4 \Rightarrow 7x = 9 \Rightarrow x = \frac{9}{7} \approx 1.286$$

Not in the options \Rightarrow F (None of the above)

Question 14: B

$$y = x^2 \ln x$$

$$y' = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x$$

Question 15: C

$$\sin x \in [-1, 1] \Rightarrow \sin x + 2 \in [1, 3]$$

Question 16: C

$$\text{On } 1 \leq x \leq e, \ln x \geq 1 - x.$$

$$\begin{aligned} \text{Area} &= \int_1^e (\ln x - (1 - x)) dx = \int_1^e (\ln x - 1 + x) dx \\ &= \left(x \ln x - 2x + \frac{x^2}{2} \right)_1^e = \frac{e^2 - 2e + 3}{2} \end{aligned}$$

Question 17: B

$$f(x) = x + \frac{1}{x} + \ln x$$

$$f'(x) = 1 - \frac{1}{x^2} + \frac{1}{x} = 0 \Rightarrow x^2 + x - 1 = 0 \Rightarrow x = \frac{\sqrt{5} - 1}{2}$$

$$\text{From } x^2 + x = 1 \Rightarrow x + \frac{1}{x} = \sqrt{5}.$$

$$f_{\min} = \sqrt{5} + \ln \left(\frac{\sqrt{5} - 1}{2} \right)$$

Question 18: B

Maximise $\ln(\sin x) + 2 \ln(\cos x) = \ln(\sin x \cos^2 x)$.

$$\frac{d}{dx} (\ln(\sin x \cos^2 x)) = \cot x - 2 \tan x = 0 \Rightarrow \tan^2 x = \frac{1}{2}$$

So $\sin x = \frac{1}{\sqrt{3}}$, $\cos x = \frac{\sqrt{2}}{\sqrt{3}}$.

$$\sin x \cos^2 x = \frac{1}{\sqrt{3}} \cdot \frac{2}{3} = \frac{2}{3\sqrt{3}} \Rightarrow \max = \ln\left(\frac{2}{3\sqrt{3}}\right)$$

Question 19: E

$$dy/dx = 6(2 + 3x)^5 \cdot 3$$

$$= 18(3x + 2)$$

$$\text{Coefficient of } x^3: 18(3x)^3 \cdot (2)^2 \cdot 5C3$$

$$= 10 \cdot 4 \cdot 18 \cdot 27$$

$$= 40 \cdot 18 \cdot 27$$

$$= 40 \cdot (540 - 54)$$

$$= 40 \cdot 486$$

$$= 19440$$

Question 20: E**MATH 2 ANSWERS**

$$= [x^{m+1}/m+1]^2 = 2^{m+1}/m+1 \rightarrow (1)$$

$$= [x^{m+2}/m+2]^2 = 2^{m+2}/m+2 \rightarrow (2)$$

$$(2)/(1) : 32\sqrt{2}/9 * 7/16\sqrt{2} = 2^{m+2}/m+2 * m+1/2^{m+1}$$

$$32\sqrt{2}/9 * 7/16\sqrt{2} = 2(m+1)/m+2$$

$$m+1/m+2 = 7/9$$

$$9m + 9 = 7m + 14$$

$$m = 5/2$$

Question 21: E

$$3x^2 = (a + 2)x - 3$$

$$3x^2 - (a + 2)x + 3 = 0$$

For two distinct real roots $b^2 - 4ac > 0$:

$$(a + 2)^2 - 4(3)(3) > 0$$

$$(a + 2)^2 > 36$$

$$a + 2 > 6 \text{ or } a + 2 < -6$$

$$a > 4 \text{ or } a < -8$$

Question 22: G

$$f(x) = x^3 - a^2x$$

$$df/dx = 3x^2 - a^2$$

To be increasing, $df/dx \geq 0$

$$3x^2 - a^2 \geq 0$$

$$a^2 \leq 3x^2$$

$$x^2 \geq a^2/3$$

$$x \geq a/\sqrt{3} \text{ or } x \leq -a/\sqrt{3}$$

Question 23: D

$$(a - b)(a + b) \equiv a^2 - b^2$$

$$(1 - 2x)^5(1 + 2x)^5 = (1 - 2x)(1 + 2x)^5$$

$$= (1 - 4x^2)^5$$

Expanding this only gives even powers of x :

$$(4x^2)^0, (4x^2)^1, (4x^2)^2, \dots, (4x^2)^5$$

Question 24: D

Differentiate $f(x)$ to find the gradient at $x = 4$. Then find the gradient of the normal and deduce the equation of the straight line from a point on the line.

Need a point on the straight line (use $f(x)$ at $x = 4$):

$$y = f(x) = \frac{1}{4}x^3 - 3x^{\frac{3}{2}} + 5x - 2$$

$$f(4) = \frac{1}{4} \times 4^3 - 3 \times 4^{\frac{3}{2}} + 5 \times 4 - 2 = 10$$

point $(4, 10)$

Get gradient, m :

$$f'(x) = \frac{3}{4}x^2 - \frac{9}{2}x^{\frac{1}{2}} + 5$$

$$m = f'(4) = \frac{3}{4} \times 4^2 - \frac{9}{2} \times 4^{\frac{1}{2}} + 5 = 8$$

$$\text{gradient of normal} = m' = -\frac{1}{m} = -\frac{1}{8}$$

Get equation of normal:

$$y - y_1 = m'(x - x_1)$$

$$y - 10 = -\frac{1}{8}(x - 4)$$

$$y = -\frac{1}{8}x + \frac{21}{2}$$

Question 25: E

Translation by vector (4, 3): $y = (x - 4)^2 + 3$

Reflection in line $y = -1$: This is equivalent to a translation by $2(y + 1)$ units downwards for each point on the curve.

$$2(y + 1)$$

$$= 2((x - 4)^2 + 3 + 1)$$

$$= 2(x - 4)^2 + 8$$

$$\Rightarrow \text{new curve: } y = (x - 4)^2 + 3 - (2(x - 4)^2 + 8)$$

$$= -5 - (x - 4)^2$$

Question 26: D

$$y = x^3 + px^2 + qx + 6$$

$$dy/dx = 3x^2 + 2px + q$$

$$dy/dx|_{x=2} = 3(2)^2 + 4p + q = 0 \quad dy/dx|_{x=4} = 3(4)^2 + 8p + q = 0$$

$$\Rightarrow 4p + q = -12 \Rightarrow 8p + q = -48$$

$$\Rightarrow (8p + q) - (4p + q) = -48 + 12$$

$$4p = -36$$

$$p = -9$$

$$q = -48 + 8(9)$$

$$= 24$$

$$\text{Check } d^2y/dx^2 = 6x + 2p \text{ when } x = 2, d^2y/dx^2 = -18 + 12 = -6 < 0$$

Hence a maximum

when $x = 4$, $d^2y/dx^2 = -18 + 24 = 6$ so hence a minimum.

Question 27: C

Graph drawn correctly with:

Length of the side of the square = 6

Hence centre of circle is at: (-2, 3)

Has a radius of 3.

$$(x + 2)^2 + (y - 3)^2 = 3^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 9$$

$$x^2 + y^2 + 4x - 6y + 4 = 0$$