AMC 10 Number Theory Questions - Year wise

American Mathematics contest 10 (AMC 10) - Number Theory problems

AMC 10A, 2021, Problem 10

Which of the following is equivalent to

$$(2+3)\left(2^{2}+3^{2}\right)\left(2^{4}+3^{4}\right)\left(2^{8}+3^{8}\right)\left(2^{16}+3^{16}\right)\left(2^{32}+3^{32}\right)\left(2^{64}+3^{64}\right)?$$

(A) $3^{127}+2^{127}$
(B) $3^{127}+2^{127}+2 \cdot 3^{63}+3 \cdot 2^{63}$
(C) $3^{128}-2^{128}$
(D) $3^{128}+2^{128}$
(E) 5^{127}

AMC 10A, 2021, Problem 11

For which of the following integers b is the base- b number $2021_{b}-221_{b}$ not divisible by 3 ?
(A) 3
(B) 4
(C) 6
(D) 7
(E) 8

AMC 10A, 2021, Problem 16

In the following list of numbers, the integer n appears n times in the list for $1 \leq n \leq 200$.

$$1,2,2,3,3,3,4,4,4,4, \ldots, 200,200, \ldots, 200$$

What is the median of the numbers in this list?

(A) 100.5

(B) 134

(C) 142

(D) 150.5

(E) 167

AMC 10A, 2021, Problem 19

The area of the region bounded by the graph of

$$x^2+y^2=3|x-y|+3|x+y|$$

is $m+n\pi$, where m and n are integers. What is $m+n$?

(A) 18

(B) 27

(C) 36

(D) 45

(E) 54

AMC 10B, 2021, Problem 1

How many integer values of x satisfy $|x|<3\pi$?

(A) 9

(B) 10

(C) 18

(D) 19

(E) 20

AMC 10B, 2021, Problem 13

Let n be a positive integer and d be a digit such that the value of the numeral $32d$ in base n equals 263 , and the value of the numeral 324 in base n equals the value of the numeral $11d1$ in base six. What is $n+d$?

(A) 10

(B) 11

(C) 13

(D) 15

(E) 16

AMC 10B, 2021, Problem 16

Call a positive integer an uphill integer if every digit is strictly greater than the previous digit. For example, 1357, 89 , and 5 are all uphill integers, but 32, 1240, and 466 are not. How many uphill integers are divisible by 15 ?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

AMC 10B, 2021, Problem 19

Suppose that S is a finite set of positive integers. If the greatest integer in S is removed from S, then … (arithmetic mean) of the integers remaining is 32 . If the least integer in S is also removed, then the avera… integers remaining is 35 . If the greatest integer is then returned to the set, the average value of the integers r… The greatest integer in the original set S is 72 greater than the least integer in S. What is the average value of all … integers in the set S?

(A) 36.2

(B) 36.4

(C) 36.6

(D) 36.8

(E) 37

AMC 10A, 2020, Problem 4

A driver travels for 2 hours at 60 miles per hour, during which her car gets 30 miles per gallon of gasoline. She is paid 0.50 per mile, and her only expense is gasoline at 2.00 per gallon. What is her net rate of pay, in dollars per hour, after this expense?

(A) 20

(B) 22

(C) 24

(D)25

(E) 26

AMC 10A, 2020, Problem 6

How many 4-digit positive integers (that is, integers between 1000 and 9999, inclusive) having only even digits are divisible by 5?

(A)80

(B)100

(C)125

(D)200

(E) 500

AMC 10A, 2020, Problem 8

What is the value of

$1+2+3-4+5+6+7-8+\cdots+197+198+199-200?$

(A) $9,800$

(B)$9,900$

(C) $10,000$

(D) $10,100$

(E)$10,200$

AMC 10A, 2020, Problem 9

A single bench section at a school event can hold either 7 adults or 11 children. When N bench sections are co… to end, an equal number of adults and children seated together will occupy all the bench space. What is the least p… positive integer value of N?

(A) 9

(B)18

(C)27

(D)36

(E)77

AMC 10A, 2020, Problem 17

Define $P(x)=(x-1^2)(x-2^2)\cdots(x-100^2)$.How many integers n are there such that $P(n)\leq 0$?

(A)4900

(B)4950

(C)5000

(D)5050

(E) 5100

AMC 10A, 2020, Problem 21

There exists a unique strictly increasing sequence of nonnegative integers $a_1<a_2<\ldots<a_k$ such that $\frac{2^{289}+1}{2^{17}+1}=2^{a_1}+2^{a_2}+\ldots+2^{a_k}$ .What is k?

(A) 117

(B)136

(C)137

(D)273

(E)306

AMC 10A, 2020, Problem 22

For how many positive integers $n\leq 1000$ is $\left\lfloor\frac{998}{n}\right\rfloor+\left\lfloor\frac{999}{n}\right\rfloor+\left\lfloor\frac{1000}{n}\right\rfloor$ not divisible by 3? (Recall that $\lfloor x\rfloor$ is the greatest integer less than or equal to x.)

(A) 22

(B)23

(C)24

(D)25

(D)25

(E)26

AMC 10A, 2020, Problem 24

Let n be the least positive integer greater than 1000 for which $\operatorname{gcd}(63, n+120)=21 \quad$ and $\quad \operatorname{gcd}(n+63,120,$
.What is the sum of the digits of n?

(A) 12

(B)15

(C)18

(D)21

(E)24

AMC 10B, 2020, Problem 24

How many positive integers n satisfy

$$\frac{n+1000}{70}=\lfloor\sqrt{n}\rfloor ?$$

(Recall that $\lfloor x\rfloor$ is the greatest integer not exceeding x.)
(A) 2
(B) 4
(C) 6
(D) 30
(E) 32

AMC 10B, 2020, Problem 25

Let $D(n)$ denote the number of ways of writing the positive integer n as a product $n=f_1 \cdot f_2 \ldots f_k$ where $k \geq 1$, the f_i are integers strictly greater than 1 , and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number 6 can be written as $6, 2 \cdot 3$, and $3 \cdot 2$, so $D(6)=3$. What is $D(96)$?
(A) 112
(B) 128
(C) 144
(D) 172
(E) 184

AMC 10A, 2019, Problem 5

What is the greatest number of consecutive integers whose sum is 45 ?
(A) 9
(B) 25
(C) 45
(D) 90
(E) 120

AMC 10A, 2019, Problem 9

What is the greatest three-digit positive integer n for which the sum of the first n positive integers i… product of the first n positive integers?

(A) 995

(B) 996

(C) 997

(D) 998

(E) 999

AMC 10A, 2019, Problem 15

A sequence of numbers is defined recursively by $a_1 = 1, a_2 = \frac{3}{7}$, and

$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

for all $n \geq 3$ Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p+q$?

(A) 2020

(B) 40394(C)46057

(D) 6061

(E) 8078

AMC 10A, 2019, Problem 18

For some positive integer k, the repeating base- k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323\ldots k$. What is k?

(A) 13

(B) 14

(C) 15

(D) 16

(E) 17

AMC 10A, 2019, Problem 19

What is the least possible value of

$$(x+1)(x+2)(x+3)(x+4) + 2019$$

where x is a real number?

(A) 2017

(B) 2018

(C) 2019

(D) 2020(E)2021$

AMC 10A, 2019, Problem 25

For how many integers n between 1 and 50 , inclusive, is

$$\frac{(n^2-1)!}{(n!)^n}$$

an integer? (Recall that $0! = 1$.)

an integer? (Recall that $0! = 1$.)
(A) 31
(B) 32
(C) 33
(D) 34
(E) 35

AMC 10B, 2019, Problem 6

There is a positive integer n such that $(n+1)!+(n+2)!=n! \cdot 440$. What is the sum of the digits of n ?
(A) 3
(B) 8
(C) 10
(D) 11
(E) 12

AMC 10B, 2019, Problem 9

The function f is defined by

$$f(x)=\lfloor|x|\rfloor-|\lfloor x\rfloor|$$

for all real numbers x, where $\lfloor r\rfloor$ denotes the greatest integer less than or equal to the real number r. What is the range of f?
(A) $-1,0$
(B) The set of nonpositive integers
(C) $-1,0,1$
(D) 0
(E) The set of nonnegative integers

AMC 10B, 2019, Problem 12

What is the greatest possible sum of the digits in the base-seven representation of a positive integer less than 2019 ?
(A) 11
(B) 14
(C) 22
(D) 23
(E) 27

AMC 10B, 2019, Problem 19

Let S be the set of all positive integer divisors of $100,000$. How many numbers are the product of two distinct elements of S?
(A) 98
(B) 100
(C) 117
(D) 119
(E) 121

AMC 10B, 2019, Problem 24

Define a sequence recursively by $x_0=5$ and

$$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$$

for all nonnegative integers n. Let m be the least positive integer such that

$$x_m \leq 4 + \frac{1}{2^{20}}$$

In which of the following intervals does m lie?

(A) $[9, 26]$

(B) $[27, 80]$

(C) $[81, 242]$

(D) $[243, 728]$

(E) $[729, \infty)$

AMC 10A, 2018, Problem 7

For how many (not necessarily positive) integer values of n is the value of $4000 \cdot \left(\frac{2}{5}\right)^n$ an integer?

(A) 3

(B) 4

(C) 6

(D) 8

(E) 9

AMC 10A, 2018, Problem 14

What is the greatest integer less than or equal to

$$\frac{3^{100} + 2^{100}}{3^{96} + 2^{96}}?$$

(A) 80

(B) 81

(C) 96

(D) 97

(E) 625

AMC 10A, 2018, Problem 18

How many nonnegative integers can be written in the form

$$a_7 \cdot 3^7 + a_6 \cdot 3^6 + a_5 \cdot 3^5 + a_4 \cdot 3^4 + a_3 \cdot 3^3 + a_2 \cdot 3^2 + a_1 \cdot 3^1 + a_0 \cdot 3^0,$$

where $a_i \in -1, 0, 1$ for $0 \leq i \leq 7$?

(A) 512

(B) 729

(C) 1094

(D) 3281

(E) $59,048$

AMC 10A, 2018, Problem 22

AMC 10A, 2018, Problem 22

Let $a, b, c,$ and d be positive integers such that $gcd(a, b) = 24$, $gcd(b, c) = 36$, $gcd(c, d) = 54$, and .
Which of the following must be a divisor of a ?

(A) 5

(B) 7

(C) 11

(D) 13

(E) 17

AMC 10A, 2018, Problem 25

For a positive integer n and nonzero digits $a, b,$ and c, let A_n be the n -digit integer each of whose digits is equal to a; let B_n be the n -digit integer each of whose digits is equal to b, and let C_n be the $2n$ -digit (not n -digit) integer each of whose digits is equal to c. What is the greatest possible value of $a + b + c$ for which there are at least two values of n such that $C_n - B_n = A_n^2$?

(A) 12

(B) $144 (C) 16 (D) 18 (E) 20$$

AMC 10B, 2018, Problem 5

How many subsets of $2, 3, 4, 5, 6, 7, 8, 9$ contain at least one prime number?

(A) 128

(B) 192

(C) 224

(D) 240

(E) 256

AMC 10B, 2018, Problem 11

Which of the following expressions is never a prime number when p is a prime number?

(A) $p^2 + 16$

(B) $p^2 + 24$

(C) $p^2 + 26$

(D) $p^2 + 46$

(E) $p^2 + 96$

AMC 10B, 2018, Problem 13

How many of the first 2018 numbers in the sequence $101, 1001, 10001, 100001, \ldots$ are divisible by 101 ?

(A) 253

(B) 504

(C) 505

(D) 506

(E) 1009

AMC 10B, 2018, Problem 14

A list of 2018 positive integers has a unique mode, which occurs exactly 10 times. What is the least number of distinct values that can occur in the list?

(A) 202

(B) 223

(C) 224

(D) 225

(E) 234

AMC 10B, 2018, Problem 16

Let $a_1, a_2, \ldots, a_{2018}$ be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \cdots + a_{2018} = 2018^{2018}$$

What is the remainder when $a_1^3 + a_2^3 + \cdots + a_{2018}^3$ is divided by 6 ?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

AMC 10B, 2018, Problem 20

A function f is defined recursively by $f(1) = f(2) = 1$ and

$$f(n) = f(n-1) - f(n-2) + n$$

for all integers $n \geq 3$. What is $f(2018)$?

(A) 2016

(B) 2017

(C) 2018

(D) 2019

(E) 2020

AMC 10B, 2018, Problem 21

Mary chose an even 4 -digit number n. She wrote down all the divisors of n in increasing order from left to right: $1, 2, \ldots, \frac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323 ?

(A) 324

(B) 330

(C) 340

(D) 361

(E) 646

AMC 10B, 2018, Problem 23

How many ordered pairs (a, b) of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot lcm(a, b) + 12 \cdot gcd(a, b)$$

where $gcd(a, b)$ denotes the greatest common divisor of a and b, and $lcm(a, b)$ denotes their least common multiple?

(A) 0

(B) 2

(C) 4

(D) 6

(E) 8

AMC 10B, 2018, Problem 25

Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. How many real numbers x satisfy the equation $x^2 + 10,000\lfloor x \rfloor = 10,000x$?

(A) 197

(B) 198

(C) 199

(D) 200

(E) 201

AMC 10B, 2018, Problem 12

Let S be a set of points (x, y) in the coordinate plane such that two of the three quantities $3, x+2$, and $y-4$ are equal and the third of the three quantities is no greater than this common value. Which of the following is a correct description for S ?

(A) a single point

(B) two intersecting lines

(C) three lines whose pairwise intersections are three distinct points

(D) a triangle

(E) three rays with a common endpoint

AMC 10A, 2017, Problem 13

Define a sequence recursively by $F_0 = 0, F_1 = 1$, and $F_n =$ the remainder when $F_{n-1} + F_{n-2}$ is divided by 3, for all $n \geq 2$. Thus the sequence starts $0, 1, 1, 2, 0, 2, \ldots$ What is $F_{2017} + F_{2018} + F_{2019} + F_{2020} + F_{2021} + F_{2022} + F_{2023} + F_{2024}$?

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

AMC 10A, 2017, Problem 17

Distinct points P, Q, R, S lie on the circle $x^2 + y^2 = 25$ and have integer coordinates. The distances PQ and RS are irrational numbers. What is the greatest possible value of the ratio $\frac{PQ}{RS}$?

(A) 3

(B) 5

(C) $3\sqrt{5}$

(D) 7

(E) $5\sqrt{2}$

AMC 10A, 2017, Problem 20

Let $S(n)$ equal the sum of the digits of positive integer n. For example, $S(1507) = 13$. For a particular positive integer n, $S(n) = 1274$. Which of the following could be the value of $S(n+1)$?

(A) 1

(B) 3

(C) 12

(D) 1239

(E) 1265

AMC 10A, 2017, Problem 23

How many triangles with positive area have all their vertices at points (i, j) in the coordinate plane, where i a
integers between 1 and 5, inclusive?
(A) 2128
(B) 2148
(C) 2160
(D) 2200
(E) 2300

AMC 10A, 2017, Problem 25

How many integers between 100 and 999 , inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and 999? For example, both 121 and 211 have this property.
(A) 226
(B) 243
(C) 270
(D) 469
(E) 486

AMC 10B, 2017, Problem 14

An integer N is selected at random in the range $1 \leq N \leq 2020$. What is the probability that the remainder when N^{16} is divided by 5 is 1 ?
(A) $\frac{1}{5}$
(B) $\frac{2}{5}$
(C) $\frac{3}{5}$
(D) $\frac{4}{5}$
(E) 1

AMC 10B, 2017, Problem 16

How many of the base-ten numerals for the positive integers less than or equal to 2017 contain the digit 0 ?
(A) 469
(B) 471
(C) 475
(D) 478
(E) 481

AMC 10B, 2017, Problem 23

Let $N = 123456789101112\ldots4344$ be the 79 -digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45 ?
(A) 1
(B) 4
(C) 9
(D) 18
(E) 44

AMC 10B, 2017, Problem 25

Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 a
each test she noticed that the average of her test scores was an integer. Her score on the seventh test
her score on the sixth test?

(A) 92

(B) 94

(C) 96

(D) 98

(E) 100

AMC 10A, 2016, Problem 4

The remainder can be defined for all real numbers x and y with $y \neq 0$ by

$$rem(x, y) = x - y \left\lfloor \frac{x}{y} \right\rfloor$$

where $\left\lfloor \frac{x}{y} \right\rfloor$ denotes the greatest integer less than or equal to $\frac{x}{y}$. What is the value of $rem\left(\frac{3}{8}, -\frac{2}{5}\right)$?

(A) $-\frac{3}{8}$

(B) $-\frac{1}{40}$

(C) 0

(D) $\frac{3}{8}$

(E) $\frac{31}{40}$

AMC 10A, 2016, Problem 9

A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row, and so on up to N coins in the N th row. What is the sum of the digits of N ?

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

AMC 10A, 2016, Problem 17

Let N be a positive multiple of 5 . One red ball and N green balls are arranged in a line in random order. Let $P(N)$ be the probability that at least $\frac{3}{5}$ of the green balls are on the same side of the red ball. Observe that $P(5) = 1$ and that $P(N)$ approaches $\frac{4}{5}$ as N grows large. What is the sum of the digits of the least value of N such that $P(N) < \frac{321}{400}$?

(A) 12

(B) 14

(C) 16

(D) 18

(E) 20

AMC 10A, 2016, Problem 20

For some particular value of N, when $(a + b + c + d + 1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables a, b, c, and d, each to some positive power. What is N ?

(A) 9

(B) 14

(C) 16

(D) 17
(E) 19

AMC 10A, 2016, Problem 22

For some positive integer n, the number $110\mathrm{n}^3$ has 110 positive integer divisors, including 1 and the number 11
many positive integer divisors does the number $81\mathrm{n}^4$ have?
(A) 110
(B) 191
(C) 261
(D) 325
(E) 425

AMC 10A, 2016, Problem 25

How many ordered triples (x, y, z) of positive integers satisfy $lcm(x, y) = 72$, $lcm(x, z) = 600$ and $lcm(y, z) = 900$?
(A) 15
(B) 16
(C) 24
(D) 27
(E) 64

AMC 10B, 2016, Problem 6

Laura added two three-digit positive integers. All six digits in these numbers are different. Laura's sum is a three-digit number S. What is the smallest possible value for the sum of the digits of S ?
(A) 1
(B) 4
(C) 5
(D) 15
(E) 20

AMC 10B, 2016, Problem 13

At Megapolis Hospital one year, multiple-birth statistics were as follows: Sets of twins, triplets, and quadruplets accounted for 1000 of the babies born. There were four times as many sets of triplets as sets of quadruplets, and there was three times as many sets of twins as sets of triplets. How many of these 1000 babies were in sets of quadruplets?
(A) 25
(B) 40
(C) 64
(D) 100
(E) 160

AMC 10B, 2016, Problem 24

How many four-digit integers $abcd$, with $a \neq 0$, have the property that the three two-digit integers $ab < bc < cd$ form an increasing arithmetic sequence? One such number is 4692 , where $a = 4, b = 6, c = 9$, and $d = 2$.
(A) 9
(B) 15
(C) 16
(D) 17
(E) 20

AMC 10B, 2016, Problem 25

Let $f(x)=\sum_{k=2}^{10}(\lfloor kx\rfloor-k\lfloor x\rfloor)$, where $\lfloor r\rfloor$ denotes the greatest integer less than or equal to r.

How many distinct values does $f(x)$ assume for $x\geq 0$?

(A) 32
(B) 36
(C) 45
(D) 46
(E) infinitely many

AMC 10A, 2015, Problem 18

Hexadecimal (base-16) numbers are written using numeric digits 0 through 9 as well as the letters A through F to represent 10 through 15. Among the first 1000 positive integers, there are n whose hexadecimal representation contains only numeric digits. What is the sum of the digits of n ?
(A) 17
(B) 18
(C) 19
(D) 20
(E) 21

AMC 10A, 2015, Problem 23

The zeroes of the function $f(x)=x^2-ax+2a$ are integers. What is the sum of the possible values of a?
(A) 7
(B) 8
(C) 16
(D) 17
(E) 18

AMC 10A, 2015, Problem 25

Let S be a square of side length 1 . Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a-b\pi}{c}$, where a, b, and c are positive integers with $gcd(a,b,c)=1$. What is $a+b+c$?
(A) 59
(B) 60
(C) 61
(D) 62
(E) 63

AMC 10B, 2015, Problem 10

What are the sign and units digit of the product of all the odd negative integers strictly greater than -2015 ?
(A) It is a negative number ending with a 1.
(B) It is a positive number ending with a 1 .
(C) It is a negative number ending with a 5 .
(D) It is a positive number ending with a 5.
(E) It is a negative number ending with a 0 .

AMC 10B, 2015, Problem 14

Let a, b, and c be three distinct one-digit numbers. What is the maximum value of the sum of the root
$(x-a)(x-b)+(x-b)(x-c)=0$?
(A) 15
(B) 15.5
(C) 16
(D) 16.5
(E) 17

AMC 10B, 2015, Problem 21

Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump. Cozy goes two steps up with each jump (though if necessary, he will just jump the last step). Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left). Suppose Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s ?
(A) 9
(B) 11
(C) 12
(D) 13
(E) 15

AMC 10B, 2015, Problem 23

Let n be a positive integer greater than 4 such that the decimal representation of $n!$ ends in k zeros and the decimal representation of $(2n)$. ends in $3k$ zeros. Let s denote the sum of the four least possible values of n. What is the sum of the digits of s ?
(A) 7
(B) 8
(C) 9
(D) 10
(E) 11

AMC 10A, 2014, Problem 20

The product $(8)(888\ldots8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000 . What is k ?
(A) 901
(B) 911
(C) 919
(D) 991
(E) 999

AMC 10A, 2014, Problem 24

A sequence of natural numbers is constructed by listing the first 4 , then skipping one, listing the next 5 , skipping 2 , listing 6 , skipping 3 , and, on the n th iteration, listing $n+3$ and skipping n. The sequence begins $1,2,3,4,6,7,8,9,10,13$. What is the 500,000 th number in the sequence?
(A) 996,506
(B) 996,507
(C) 996,508
(D) 996,509

(D) 996,509

(E) 996,510

AMC 10A, 2014, Problem 25

The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m, n) are there such that $1 \leq m \leq$.

$$5^n < 2^m < 2^{m+2} < 5^{n+1}?$$

(A) 278

(B) 279

(C) 280

(D) 281

(E) 282

AMC 10B, 2014, Problem 12

The largest divisor of $2,014,000,000$ is itself. What is its fifth-largest divisor?

(A) $125,875,000$

(B) $201,400,000$

(C) $251,750,000$

(D) $402,800,000$

(E) $503,500,000$

AMC 10B, 2014, Problem 14

Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abe miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a+b+c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2+b^2+c^2$?

(A) 26

(B) 27

(C) 36

(D) 37

(E) 41

AMC 10B, 2014, Problem 17

What is the greatest power of 2 that is a factor of $10^{1002}-4^{501}$?

(A) 2^{1002}

(B) 2^{1003}

(C) 2^{1004}

(D) 2^{1005}

(E) 2^{1006}

AMC 10B, 2014, Problem 20

For how many integers x is the number x^4-51x^2+50 negative?

(A) 8

(B) 10

(C) 12

(D) 14

(E) 16

AMC 10A, 2013, Problem 13

How many three-digit numbers are not divisible by 5 , have digits that sum to less than 20 , and have t
the third digit?

(A) 52

(B) 60

(C) 66

(D) 68

(E) 70

AMC 10A, 2013, Problem 19

In base 10, the number 2013 ends in the digit 3 . In base 9 , on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6 . For how many positive integers b does the base- b -representation of 2013 end in the digit 3 ?

(A) 6

(B) 9

(C) 13

(D) 16

(E) 18

AMC 10B, 2013, Problem 4

When counting from 3 to 201,53 is the 51^{nt} number counted. When counting backwards from 201 to 3,53 is the n^{th} number counted. What is n ?

(A) 146

(B) 147

(C) 148

(D) 149

(E) 150

AMC 10B, 2013, Problem 5

Positive integers a and b are each less than 6 . What is the smallest possible value for $2 \cdot a - a \cdot b$?

(A) -20

(B) -15

(C) -10

(D) 0

(E) 2

AMC 10B, 2013, Problem 9

Three positive integers are each greater than 1 , have a product of 27000 , and are pairwise relatively prime. What is their sum?

(A) 100

(B) 137

(C) 156

(D) 160

(E) 165

AMC 10B, 2013, Problem 18

The number 2013 has the property that its units digit is the sum of its other digits, that is $2 + 0 + 1 = 3$. How many

integers less than 2013 but greater than 1000 have this property?
(A) 33
(B) 34
(C) 45
(D) 46
(E) 58

AMC 10B, 2013, Problem 20

The number 2013 is expressed in the form

$$2013=\frac{a_1!a_2!\ldots a_m!}{b_1!b_2!\ldots b_n!}$$

where $a_1 \geq a_2 \geq \cdots \geq a_m$ and $b_1 \geq b_2 \geq \cdots \geq b_n$ are positive integers and a_1+b_1 is as small as possible. What is $|a_1-b_1|$?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

AMC 10B, 2013, Problem 14

Two non-decreasing sequences of nonnegative integers have different first terms. Each sequence has the property that each term beginning with the third is the sum of the previous two terms, and the seventh term of each sequence is N. What is the smallest possible value of N ?
(A) 55
(B) 89
(C) 104
(D) 144
(E) 273

AMC 10B, 2013, Problem 18

The number 2013 has the property that its units digit is the sum of its other digits, that is $2+0+1=3$. How many integers less than 2013 but greater than 1000 have this property?
(A) 33
(B) 34
(C) 45
(D) 46
(E) 58

AMC 10B, 2013, Problem 24

A positive integer n is nice if there is a positive integer m with exactly four positive divisors (including 1 and m) such that the sum of the four divisors is equal to n. How many numbers in the set $2010, 2011, 2012, \ldots, 2019$ are nice?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

(E) 5

AMC 10B, 2013, Problem 25

Bernardo chooses a three-digit positive integer N and writes both its base- 5 and base-6 representations o Later LeRoy sees the two numbers Bernardo has written. Treating the two numbers as base-10 integers, he add obtain an integer S. For example, if $N=749$, Bernardo writes the numbers 10,444 and 3,245 , and LeRoy obtains $S=13,689$. For how many choices of N are the two rightmost digits of S, in order, the same as those of $2N$?

(A) 5

(B) 10

(C) 15

(D) 20

(E) 25

AMC 10A, 2012, Problem 24

Let a, b, and c be positive integers with $a \geq b \geq c$ such that $a^2-b^2-c^2+ab=2011$ and
$a^2+3b^2+3c^2-3ab-2ac-2bc=-1997$.
What is a?

(A) 249

(B) 250

(C) 251

(D) 252

(E) 253

AMC 10B, 2012, Problem 8

What is the sum of all integer solutions to $1<(x-2)^2<25$?

(A) 10

(B) 12

(C) 15

(D) 19

(E) 25

AMC 10B, 2012, Problem 10

How many ordered pairs of positive integers (M, N) satisfy the equation $\frac{M}{6}=\frac{6}{N}$?

(A) 6

(B) 7

(C) 8

(D) 9

(E) 10

AMC 10B, 2012, Problem 20

Bernardo and Silvia play the following game. An integer between 0 and 999 inclusive is selected and given to Bernardo. Whenever Bernardo receives a number, he doubles it and passes the result to Silvia. Whenever Silvia receives a number, she adds 50 to it and passes the result to Bernardo. The winner is the last person who produces a number less than 1000 . Let N be the smallest initial number that results in a win for Bernardo. What is the sum of the digits of N ?

(A) 7

(B) 8

(C) 9

(D) 10

(D) 10
(E) 11

AMC 10A, 2011, Problem 13

How many even integers are there between 200 and 700 whose digits are all different and come from the set 1,
(A) 12
(B) 20
(C) 72
(D) 120
(E) 200

AMC 10A, 2011, Problem 17

In the eight term sequence A, B, C, D, E, F, G, H, the value of C is 5 and the sum of any three consecutive terms is 30 . What is $A+H$?
(A) 17
(B) 18
(C) 25
(D) 26
(E) 43

AMC 10A, 2011, Problem 19

In 1991 the population of a town was a perfect square. Ten years later, after an increase of 150 people, the population was 9 more than a perfect square. Now, in 2011 , with an increase of another 150 people, the population is once again a perfect square. Which of the following is closest to the percent growth of the town's population during this twenty-year period?
(A) 42
(B) 47
(C) 52
(D) 57
(E) 62

AMC 10A, 2011, Problem 23

Seven students count from 1 to 1000 as follows:
Alice says all the numbers, except she skips the middle number in each consecutive group of three numbers. That is, Alice says 1, 3, 4, 6, 7, 9, . . . , 997, 999, 1000
Barbara says all of the numbers that Alice doesn't say, except she also skips the middle number in each consecutive group of three numbers.
Candice says all of the numbers that neither Alice nor Barbara says, except she also skips the middle number in each consecutive group of three numbers.
Debbie, Eliza, and Fatima say all of the numbers that none of the students with the first names beginning before theirs in the alphabet say, except each also skips the middle number in each of her consecutive groups of three numbers.
Finally, George says the only number that no one else says.
What number does George say?
(A) 37
(B) 242
(C) 365
(D) 728
(E) 998

AMC 10A, 2011, Problem 25

Let R be a unit square region and $n \geq 4$ an integer. A point X in the interior of R is called n -ray parti
rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitio
ray partitional?

(A) 1500

(B) 1560

(C) 2320

(D) 2480

(E) 2500

AMC 10B, 2011, Problem 10

Consider the set of numbers $1, 10, 10^2, 10^3, \ldots, 10^{10}$. The ratio of the largest element of the set to the sum of the other ten elements of the set is closest to which integer?

(A) 1

(B) 9

(C) 10

(D) 11

(E) 101

AMC 10B, 2011, Problem 21

Brian writes down four integers $w > x > y > z$ whose sum is 44 . The pairwise positive differences of these numbers are $1, 3, 4, 5, 6,$ and 9 What is the sum of the possible values for w ?

(A) 16

(B) 31

(C) 48

(D) 62

(E) 93

AMC 10B, 2011, Problem 23

What is the hundreds digit of 2011^{2011} ?

(A) 1

(B) 4

(C) 5

(D) 6

(E) 9

AMC 10B, 2011, Problem 24

A lattice point in an xy -coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \leq 100$ for all m such that $\frac{1}{2} < m < a$. What is the maximum possible value of a ?

(A) $\frac{51}{101}$

(B) $\frac{50}{99}$

(C) $\frac{51}{100}$

(D) $\frac{52}{101}$

(E) $\frac{13}{25}$

AMC 10A, 2010, Problem 9

A palindrome, such as 83438 , is a number that remains the same when its digits are reversed. The n
are three-digit and four-digit palindromes, respectively. What is the sum of the digits of x?
(A) 20
(B) 21
(C) 22
(D) 23
(E) 24

AMC 10A, 2010, Problem 25

Jim starts with a positive integer n and creates a sequence of numbers. Each successive number is obtained by subtracting the largest possible integer square less than or equal to the current number until zero is reached. For example, if Jim starts with $n = 55$, then his sequence contains 5 numbers:

$$55 \; 55-7^2 \qquad =6 \; 6-2^2=2 \; 2-1^2 \qquad =1 \; 1-1^2=0$$

Let N be the smallest number for which Jim's sequence has 8 numbers. What is the units digit of N ?
(A) 1
(B) 3
(C) 5
(D) 7
(E) 9

AMC 10A, 2009, Problem 5

What is the sum of the digits of the square of 111111111 ?
(A) 18
(B) 27
(C) 45
(D) 63
(E) 81

AMC 10A, 2009, Problem 13

Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m, n)?
(A) P^2Q
(B) P^nQ^m
(C) P^nQ^{2m}
(D) $P^{2m}Q^n$
(E) $P^{2n}Q^m$

AMC 10A, 2009, Problem 25

For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6 . Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?
(A) 6
(B) 7
(C) 8
(D) 9
(E) 10

AMC 10A, 2009, Problem 21

What is the remainder when $3^0+3^1+3^2+\cdots+3^{2009}$ is divided by 8 ?

(A) 0

(B) 1

(C) 2

(D) 4

(E) 6

AMC 10A, 2008, Problem 24

Let $k=2008^2+2^{2008}$. What is the units digit of k^2+2^k ?

(A) 0

(B) 2

(C) 4

(D) 6

(E) 8

AMC 10B, 2008, Problem 13

For each positive integer n, the mean of the first n terms of a sequence is n. What is the 2008^{th} term of the sequence?

(A) 2008

(B) 4015

(C) 4016

(D) $4,030,056$

(E) $4,032,064$