ESAT PRACTICE TEST SERIES

ESAT Mock Test 1

Math 1 + Physics + Math 2

- Duration: 2 hours (3 × 40 min)
- All topics

SOLUTION BOOK

ThrivingScholars

MATH 1

Problem 1 If the 5-digit number $1X2X3$ is divisible by 9, what is X?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4 (F) 5 (G) 6

Problem 2 If we define $a \star b = a^2 - b^2$, what is $5 \star 4$?

A) 1 B) 7 C) 9 D) 16 E) 25 F) 41

Problem 3 Jack tosses two fair dice. What is the probability that the sum of two values is no less than 9?

Recall that

$$\text{Probability} = \frac{\text{Number of valid cases}}{\text{Number all possible cases}}.$$

(A) $\frac{1}{4}$ **(B)** $\frac{5}{18}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{5}{12}$ **(E)** $\frac{2}{3}$

Problem 4 How many 4-digit integers $\overline{abcd}$ which satisfy that

(1) The thousands digit a and the tens digit c are even numbers;

(2) The hundreds digit b and unit digit d are odd numbers.

(3) This number is divisible by 5 ?

(A) 80 **(B)** 100 **(C)** 125 **(D)** 250 **(E)** 500

Problem 5 If

$$\frac{m}{10} = \frac{6}{n} = \frac{2}{5},$$

what is $m + n$?

(A) 10 **(B)** 13 **(C)** 14 **(D)** 16 **(E)** 19

MATH 1

Problem 6 Nancy has 100 marbles. She gives 20% of them to her friend Pedro. Then Nancy gives 25% of what is left to another friend, Ben. Finally, Nancy gives 10% of what is now left in the bag to her brother Jimmy. What percentage of her original bag of marbles does Nancy have left for herself?

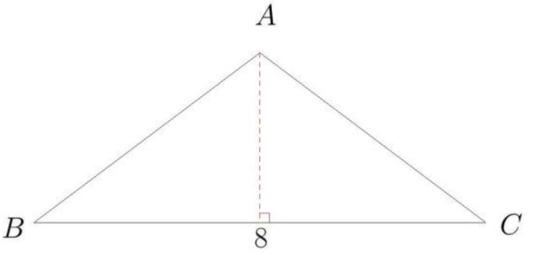
(A) 20 (B) 30 (C) $33\frac{1}{3}$ (D) 38

(E) 45 (F) 50 (G) 54 (H) 60

Problem 7 What is the sum of all positive 2-digit integers have a remainder of 1 when divided by 9 and a remainder of 2 when divided by 5?

(A) 29 **(B)** 56 **(C)** 84 **(D)** 119 **(E)** 164

Problem 8 $\triangle ABC$ is an isosceles triangle. $AB = AC$. If $BC = 8$ and the area of $\triangle ABC$ is 12, what is the perimeter of ABC?



(A) 12 **(B)** 16 **(C)** 18 **(D)** 19 **(E)** 20

Problem 9 What is the smaller angle between the hour hand and minute hand of a clock at 5 : 50 pm?

(A) 100° **(B)** 125° **(C)** 150 **(D)** 155 **(E)** 165

Problem 10 The average age of 5 people in a room is 40 years. An 36-year-old person leaves the room. What is the average age of the four remaining people?

(A) 39 (B) 40 (C) 41 (D) 43 (E) 45

MATH 1

Problem 11 In how many ways can we write 127 as the sum of two prime numbers?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E) 4**

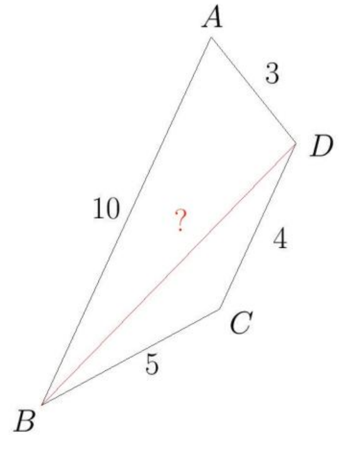
Problem 12 If for some positive integer n,

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{n}\right)=50,$$

what is n?

(A) 20 **(B)** 39 **(C)** 65 **(D)** 99 **(E) 100**

Problem 13 In quadrilateral $ABCD$, $AB=10$, $BC=5$, $CD=4$ and $AD=3$. If the length of BD is an integer, what is BD?



(A) 7 **(B)** 8 **(C)** 9 **(D)** 10

(E) there is no enough information

Problem 14 What is the unit digit of 7^{2020}? Here

$$7^n = \underbrace{7\times 7\times 7\cdots 7}_{\text{n}}.$$

(Hint: check unit digits of 7, 7^2, 7^3, $\cdots$ and find the pattern)

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E) 9**

MATH 1

Problem 15 Here x, y, z and u represent four different numbers.

$$\begin{array}{rcc} & x & y \\ + & z & x \\ \hline & u & x \end{array} \qquad \begin{array}{rcc} & x & y \\ - & z & x \\ \hline & & x \end{array}$$

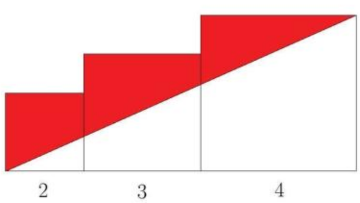
$$x + y + z + u =$$

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E) 20**

Problem 16 Billy, a seasonal worker in the town of Cowra, collected 8 buckets of cherries on his first day. Each day after that he increased the number of buckets he picked by 2 buckets per day. According to this pattern, how many buckets will he collect in the first 20 days?

(A) 308 **(B)** 500 **(C)** 540 **(D)** 580 **(E) 600**

Problem 17 Three squares are lined up horizontally as shown. Their side lengths are 2, 3 and 4 respectively. A straight line is drawn from the top right corner of the largest square to the bottom left hand corner of the smallest square. What is the area of the shaded region?

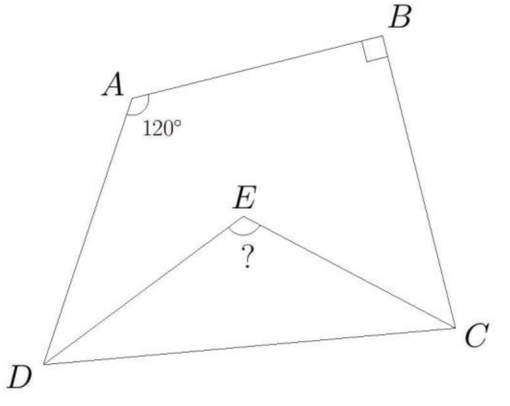


(A) 10 **(B)** 11 **(C)** 13 **(D)** 16 **(E) 18**

Problem 18 In quadrilateral $ABCD$, $\angle A = 120°$ and $\angle B = 90°$. ED and EC bisect $\angle ADC$ and $\angle BCD$ respectively. What is $\angle E$?

(

MATH 1



(A) 90 **(B)** 95 **(C)** 100 **(D)** 105 **(E) 110**

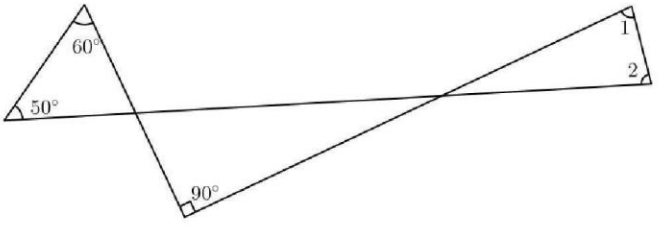
Problem 19 In a mathematics contest with ten problems, a student gains 6 points for a correct answer and loses 1 points for an incorrect answer. If Olivia answered every problem and her score was 32, how many correct answers did she have?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E) 8**

Problem 20 Emily and Owen going to library that is one mile from their house. They leave home simultaneously. Emily rides her bicycle to the library at a constant speed of 12 miles per hour. Owen walks to the library at a constant speed of 5 miles per hour. How many minutes before Owen does Emily arrive?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E) 7**

Problem 21 In the following picture, if $\angle 1 = \angle 2$, what is $\angle 1$?

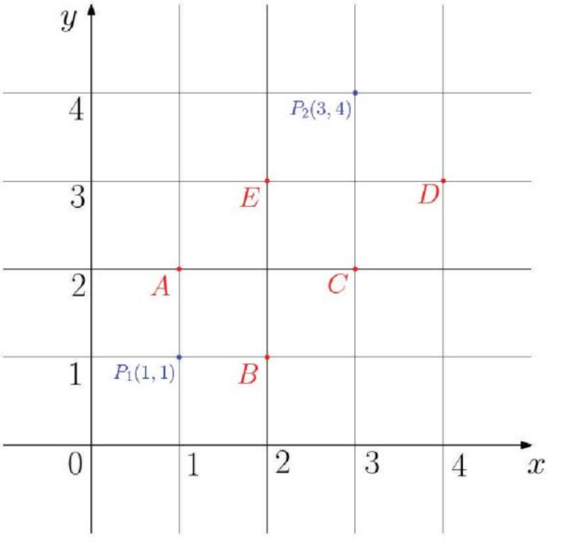


(A) 60° **(B)** 65° **(C)** 70° **(D)** 75° **(E)** 80°

Problem 22 Jack, a zoo keeper, is going to give N peaches to a group of monkey. If he gives 10 peaches to each monkey, two monkeys will receive no peach. Or, the peaches are exactly enough to give 8 to each monkey. What is the sum of digits of N?

(A) 5 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 10

Problem 23 Every point on the plane can be assigned with a coordinate that has two numbers. For example, the coordinate of point P_1 in the following picture is $(1, 1)$ and the coordinate of point P_2 is $(3, 4)$. Which of the following points has the coordinate $(2, 3)$?



(A) A **(B)** B **(C)** C **(D)** D **(E)** E

Problem 24 A pizza-shop offers a basic version of pizza with mozzarella and tomatoes. Two different toppings from the following list must be added: anchovies, artichokes, mushrooms, capers.
Moreover, for each pizza three different sizes are available: small, medium, large. How many different types of pizza are available at all?

(A) 11 **(B)** 12 **(C)** 15 **(D)** 18 **(E)** 36

Problem 25 Bag A : 1 Snickers bar + 1 Butterfinger bar.
Bag B : 1 Butterfinger bar + 3 Reese's Peanut Butter Cups.
Bag C : 1 Snickers bar + 2 Reese's Peanut Butter Cups.
If each bag costs 50 cents, what is the price of
1 Snickers bar+1 Butterfinger bar+1 Reese's Peanut Butter Cups, in cents ?

(A) 40 **(B)** 45 **(C)** 50 **(D)** 55 **(E)** 60

Problem 26 For how many positive integer x,

$$(x-1)(x-4)(x-9)$$

is negtive?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Problem 27 A gumball machine contains 9 red, 7 white, and 8 blue gumballs. The least number of gumballs a person must buy to be sure of getting four gumballs of the same color is

(A) 8 **(B)** 9 **(C)** 10 **(D)** 12 **(E)** 18

MATH 1

Answers

1 G 2. 9 3 C 4. B 5. E

6. E

7. D (Hint: You may first list all positive 2-digit integers have a remainder of 1 when divided by 9. Then pick those that have a remainder of 2 when divided by 5.)

8. C (Hint: Use are to find the height on BC. Then apply Pythagorean Theorem to find AB and AC-)

9. B 10. C

11. A (Hint: Consider parity of prime numbers.)

12. D (Hint: $1 + \frac{1}{2} = \frac{3}{2}$, $1 + \frac{1}{3} = \frac{4}{3}$, $\cdots$ Then consider cancellation.)

13. B (Hint: Recall that sum of two sides of a triangle is always greater than the third one.)

14. A (HInt: Check unit digits of 7, 7^2, 7^3,..., and find the pattern.)

15. D 16. C (Hint: Use Gauss's formula.)

17. B (Hint: Use cut and paste technique to find the area.)

18. D (Hint: What is $\angle ADC + \angle BCD$?)

19. C 20. E

21. E (Hint: The sum of three angles of a triangle is 180°.)

22. C (Hint: You may let x be the number of monkeys and then set up an equation based on the number of peaches.)

23. E

24. D (Hint: Use multiplication principle.)

25. E 26. C

27. C (Hint: What is the maximum number of gumballs he could buy if there is no four gumballs of the same color?)

PHYSICS

Problem 1

Which of the following statements are **FALSE**?

A. Electromagnetic waves cause things to heat up.
B. X-rays and gamma rays can knock electrons out of their orbits.
C. Loud sounds can make objects vibrate.
D. Wave power can be used to generate electricity.
E. Since waves carry energy away, the source of a wave loses energy.
F. The amplitude of a wave determines its mass.

Problem 2

A spacecraft is analysing a newly discovered exoplanet. A rock of unknown mass falls on the planet from a height of 30 m. Given that g = 5.4 ms^{-2} on the planet, calculate the speed of the rock when it hits the ground and the time it took to fall.

	Speed (ms^{-1})	Time (s)
A	18	3.3
B	18	3.1
C	12	3.3
D	10	3.7
E	9	2.3
F	1	0.3

Problem 3

A canoe floating on the sea rises and falls 7 times in 49 seconds. The waves pass it at a speed of 5 ms^{-1}. How long are the waves?

A. 12 m
B. 22 m
C. 25 m
D. 35 m
E. 57 m
F. 75 m

PHYSICS QUESTIONS

Problem 4

Miss Orrell lifts her 37.5 kg bike for a distance of 1.3 m in 5 s. The acceleration of free fall is 10 ms^{-2}. What is the average power that she develops?

A. 9.8 W
B. 12.9 W
C. 57.9 W
D. 79.5 W
E. 97.5W
F. 98.0 W

Problem 5

A truck accelerates at 5.6 ms^{-2} from rest for 8 seconds. Calculate the final speed and the distance travelled in 8 seconds.

	Final Speed (ms^{-1})	Distance (m)
A	40.8	119.2
B	40.8	129.6
C	42.8	179.2
D	44.1	139.2
E	44.1	179.7
F	44.2	129.2
G	44.8	179.2
H	44.8	179.7

PHYSICS QUESTIONS

Problem 6

Which of the following statements is true when a sky diver jumps out of a plane?

A. The sky diver leaves the plane and will accelerate until the air resistance is greater than their weight.
B. The sky diver leaves the plane and will accelerate until the air resistance is less than their weight.
C. The sky diver leaves the plane and will accelerate until the air resistance equals their weight.
D. The sky diver leaves the plane and will accelerate until the air resistance equals their weight squared.
E. The sky diver will travel at a constant velocity after leaving the plane.

Problem 7

A 100 g apple falls on Isaac's head from a height of 20 m. Calculate the apple's momentum before the point of impact. Take $g = 10\ ms^{-2}$

A. $0.1\ kgms^{-1}$
B. $0.2\ kgms^{-1}$
C. $1\ kgms^{-1}$
D. $2\ kgms^{-1}$
E. $10\ kgms^{-1}$
F. $20\ kgms^{-1}$

PHYSICS QUESTIONS

Problem 8

Which of the following do all electromagnetic waves all have in common?

1. They can travel through a vacuum.
2. They can be reflected.
3. They are the same length.
4. They have the same amount of energy.
5. They can be polarised.

A. 1, 2 and 3 only
B. 1, 2, 3 and 4 only
C. 4 and 5 only
D. 3 and 4 only
E. 1, 2 and 5 only
F. 1 and 5 only

Problem 9

A battery with an internal resistance of 0.8 Ω and e.m.f of 36 V is used to power a drill with resistance 1 Ω. What is the current in the circuit when the drill is connected to the power supply?

A. 5 A
B. 10 A
C. 15 A
D. 20 A
E. 25 A
F. 30 A

Problem 10

Officer Bailey throws a 20 g dart at a speed of 100 ms^{-1}. It strikes the dartboard and is brought to rest in 10 milliseconds. Calculate the average force exerted on the dart by the dartboard.

A. 0.2 N
B. 2 N
C. 20 N
D. 200 N
E. 2,000 N
F. 20,000 N

Problem 11

Professor Huang lifts a 50 kg bag through a distance of 0.7 m in 3 s. What average power does she develop to 3 significant figures? Take $g = 10ms^{-2}$

A. 112 W
B. 113 W
C. 114 W
D. 115 W
E. 116 W
F. 117 W

PHYSICS QUESTIONS

Problem 12

An electric scooter is travelling at a speed of 30 ms^{-1} and is kept going against a 50 N frictional force by a driving force of 300 N in the direction of motion. Given that the engine runs at 200 V, calculate the current in the scooter.

A. 4.5 A
B. 45 A
C. 450 A
D. 4,500 A
E. 45,000 A
F. More information needed.

Problem 13

Which of the following statements about the physical definition of work are correct?

1. $Work\ done = \frac{Force}{distance}$
2. The unit of work is equivalent to $Kgms^{-2}$.
3. Work is defined as a force causing displacement of the body upon which it acts.

A. Only 1
B. Only 2
C. Only 3
D. 1 and 2
E. 2 and 3
F. 1 and 3

Problem 14

Which of the following statements about kinetic energy are correct?

1. It is defined as $E_k = \frac{mv^2}{2}$
2. The unit of kinetic energy is equivalent to Pa x m^3.
3. Kinetic energy is equal to the amount of energy needed to decelerate the body in question from its current speed.

A. Only 1
B. Only 2
C. Only 3
D. 1 and 2
E. 2 and 3
F. 1 and 3
G. 1, 2 and 3

Problem 15

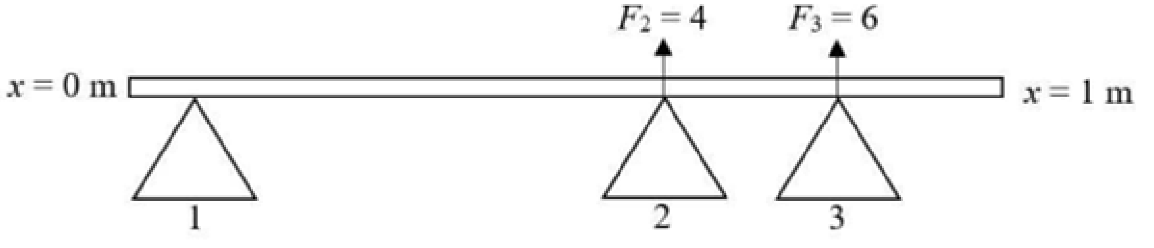
In relation to radiation, which of the following statements is **FALSE**?

A. Radiation is the emission of energy in the form of waves or particles.
B. Radiation can be either ionizing or non-ionizing.
C. Gamma radiation has very high energy.
D. Alpha radiation is of higher energy than beta radiation.
E. X-rays are an example of wave radiation.

PHYSICS QUESTIONS

Problem 16

A uniform rod of length 1 m is balanced on 3 supports so that it is in equilibrium. The position of the first support is at $x_1 = 0.1$ m, where x is the distance along the rod. The second support is at $x_2 = 0.6$ m and applies a force, F_2, of 4 N to the rod. The third support is at $x_3 = 0.8$ m and applies a force, F_3, of 6 N to the rod.

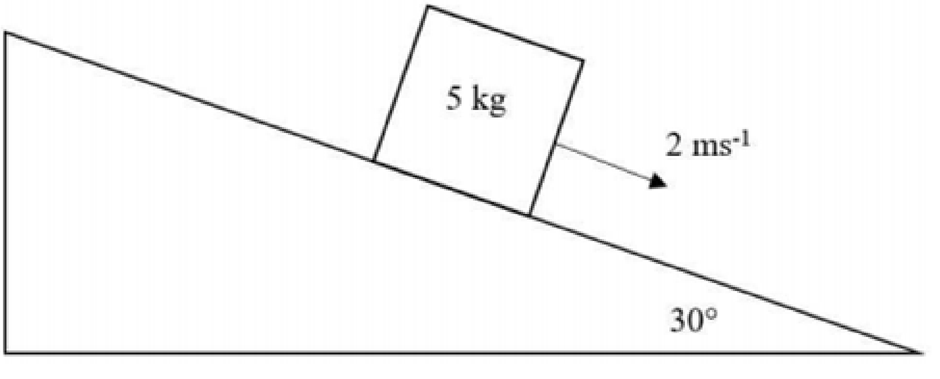


What is the force applied by the first support, F_1?

A. 4.5 N
B. 5.0 N
C. 5.3 N
D. 6.7 N
E. 5.5 N
F. 11.0 N
G. 10.5 N
H. 7.3 N

Problem 17

A block of mass 5 kg slides down a rough slope angled at 30° to the horizontal. The block moves at a constant velocity of 2 ms^{-1}. (Take g = 10 ms^{-2}).



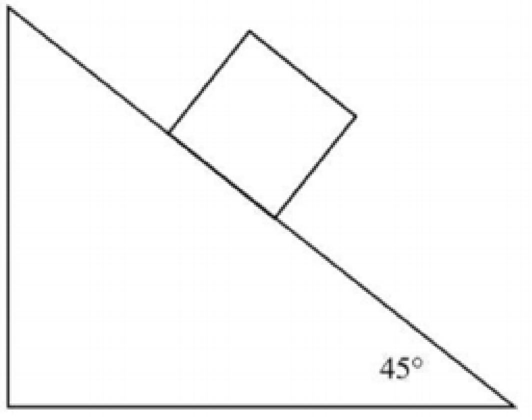
What is the frictional force on the block?

A. 14 N
B. 25 N
C. $\frac{50}{\sqrt{2}}$ N
D. $50\sqrt{2}$ N
E. $25\sqrt{3}$ N
F. 55 N
G. $50\sqrt{3}$ N
H. 50 N

PHYSICS QUESTIONS

Problem 18

A block slides down a smooth slope angled at 45° to the horizontal. The block is initially stationary. (Take g = 10 ms^{-2}).



What is the velocity of the block after 10 s?

A. $50\ ms^{-1}$
B. $40\ ms^{-1}$
C. $\frac{100}{\sqrt{2}}\ ms^{-1}$
D. $\frac{200}{\sqrt{3}}\ ms^{-1}$
E. $\frac{100}{\sqrt{3}}\ ms^{-1}$
F. $75\ ms^{-1}$
G. $\frac{200}{3}\ ms^{-1}$
H. $\frac{200}{\sqrt{2}}\ ms^{-1}$

Problem 19

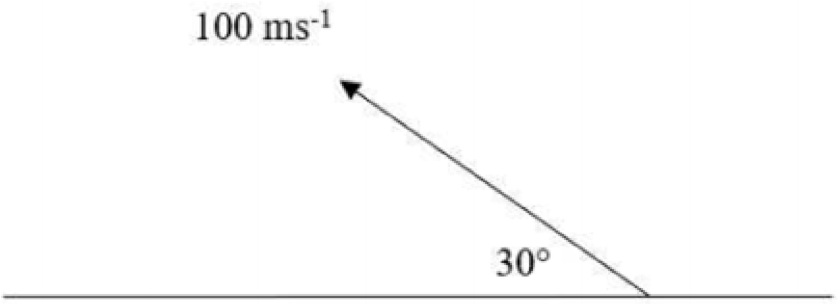
An object of mass 10 kg falls from a great height. If the force due to air resistance can be modelled as F_a = 0.5u N where u is the velocity of the free-falling object, what is the terminal velocity of the object? (Take g = 10 Nkg^{-1}).

A. $700\ ms^{-1}$
B. $150\ ms^{-1}$
C. $50\ ms^{-1}$
D. $100\ ms^{-1}$
E. $800\ ms^{-1}$
F. $400\ ms^{-1}$
G. $200\ ms^{-1}$
H. $500\ ms^{-1}$

PHYSICS QUESTIONS

Problem 20

A cannonball is fired from ground level at 30° to the horizontal with an initial velocity of 100 ms^{-1}. The ground is flat and air resistance is negligible. (Take g =10 Nkg^{-1}).

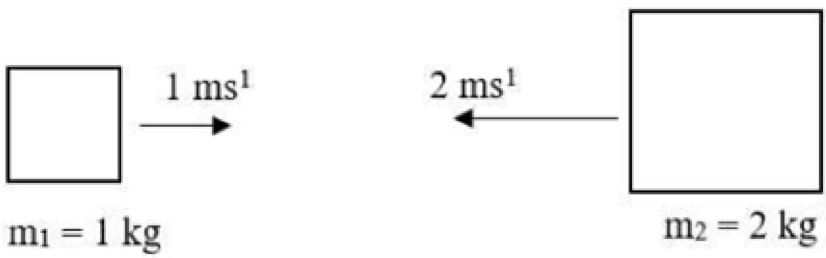


How far does the cannon ball travel horizontally before it hits the ground?

A. $250\, m$
B. $250\sqrt{2}\, m$
C. $300\, m$
D. $500\sqrt{3}\, m$
E. $500\, m$
F. $\frac{500}{\sqrt{2}}\, m$
G. $600\sqrt{3}\, m$
H. $600\sqrt{2}\, m$

Problem 21

Two blocks, one of mass 1 kg and one of mass 2 kg, move directly towards each other and collide inelastically. Before collision, their velocities have magnitudes of 1 ms^{-1} and 2 ms^{-1} respectively.



Assuming the 2 blocks coalesce on impact, what fraction of the kinetic energy is lost e.g. to heat?

A. 1
B. 0
C. $\frac{1}{3}$
D. $\frac{1}{2}$
E. $\frac{2}{3}$
F. $\frac{1}{4}$
G. $\frac{3}{4}$
H. $\frac{2}{5}$

PHYSICS QUESTIONS

Problem 22

An object of mass 20 kg falls 100 m and experiences an average resistive force of 40 N due to air resistance. If the object is initially at rest, what is its final velocity? (Take g = 10 Nkg^{-1}).

A. $60\ ms^{-1}$
B. $420\ ms^{-1}$
C. $360\ ms^{-1}$
D. $200\ ms^{-1}$
E. $80\ ms^{-1}$
F. $500\ ms^{-1}$
G. $40\ ms^{-1}$
H. $800\ ms^{-1}$

Problem 23

A velocity profile is given by $v = \frac{(5t+3)^2}{t^{\frac{1}{2}}}$ (SI units). Find the total distance travelled from t = 1 to t = 4 s.

A. $490\ m$
B. $955\ m$
C. $1082\ m$
D. $683\ m$
E. $575\ m$
F. $23\ m$
G. $1820\ m$
H. $468\ m$

Problem 24

A spring is used to lift 20 cm^3 of loosely packed sand in a small bucket. The spring constant, k, is 4 Nm^{-1} and the sand causes the spring to extend by 10 cm. What is the density of the sand at this packing fraction? (Take g = 10 Nkg^{-1}).

A. $3200\ kgm^{-3}$
B. $32\,000\ kgm^{-3}$
C. $2000\ kgm^{-3}$
D. $1000\ kgm^{-3}$
E. $20\,000\ kgm^{-3}$
F. $2500\ kgm^{-3}$
G. $4500\ kgm^{-3}$
H. $800\ kgm^{-3}$

PHYSICS QUESTIONS

Problem 25

A cylindrical wire of radius 1 mm, and length 10 cm is extended by applying a tension of 20 N to it. Resulting in an elastic extension of 2 mm..

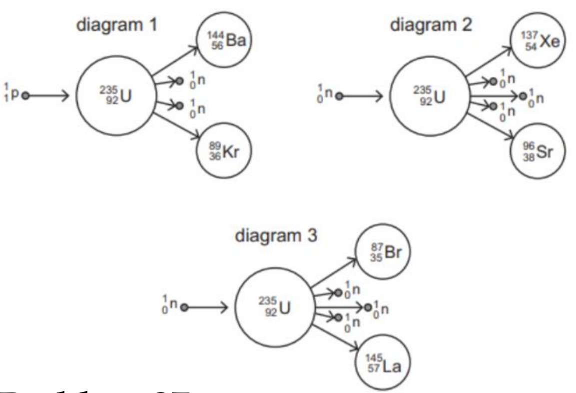
What is the Young's modulus of the material?

A. $2\pi\ GPa$
B. $\frac{2}{\pi}\ GPa$
C. $\pi\ GPa$
D. $\frac{1}{\pi}\ GPa$
E. $20\pi\ MPa$
F. $\frac{20}{\pi}\ MPa$
G. $10\ \pi\ GPa$
H. $\frac{10}{\pi}\ GPa$

Problem 26

A uranium-235 nucleus can undergo fission to produce two smaller nuclei.

Which of the diagrams, if any, could represent this process?



A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

Problem 27

A wave travels at 30 m in 10 seconds and has a frequency, *f*, of 2 Hz. How many complete wave cycles occur if the wave travels 120 m.

A. 100
B. 80
C. 50
D. 40
E. 120
F. 20
G. 30
H. 240

PHYSICS ANSWERS

Problem 1: F

That the amplitude of a wave determines its mass is false. Waves are not objects and do not have mass.

Problem 2: A

We know that displacement s = 30 m, initial speed u = 0 ms^{-1}, acceleration a = 5.4 ms^{-2}, final speed v = ?, time t = ?

And that $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 5.4 \times 30$

$v^2 = 324$ so v = 18 ms^{-1}

and $s = ut + 1/2\ at^2$

so $30 = 1/2 \times 5.4 \times t^2$

$t^2 = 30/2.7$ so t = 3.3 s

Problem 3: D

The wavelength is given by: velocity $v = \lambda f$ and frequency $f = 1/T$ so $v = \lambda/T$ giving wavelength $\lambda = vT$

The period T = 49 s/7 so λ = 5 ms^{-1} x 7 s = 35 m

PHYSICS ANSWERS

Problem 4: E

This is a straightforward question as you only have to put the numbers into the equation (made harder by the numbers being hard to work with).

$$Power = \frac{Force\ x\ Distance}{Time} = \frac{375\ N\ x\ 1.3\ m}{5\ s}$$
$$= 75\ x\ 1.3 = 97.5\ W$$

Problem 5: G

v = u + at

v = 0 + 5.6 x 8 = 44.8 ms^{-1}

And $s = ut + \frac{at^2}{2} = 0 + 5.6\,\text{x}\frac{8^2}{2} = 179.2$

Problem 6: C

The sky diver leaves the plane and will accelerate until the air resistance equals their weight – this is their terminal velocity. The sky diver will accelerate under the force of gravity. If the air resistance force exceeded the force of gravity the sky diver would accelerate away from the ground, and if it was less than the force of gravity they would continue to accelerate toward the ground.

Problem 7: D

s = 20 m, u = 0 ms^{-1}, a = 10 ms^{-2}

and $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 10 \times 20$

$v^2 = 400$

v = 20 ms^{-1}

Momentum = Mass x velocity = 20 x 0.1 = 2 $kgms^{-1}$

Problem 8: E

Electromagnetic waves have varying wavelengths and frequencies and their energy is proportional to their frequency. Therefore, only affirmatives 1, 2 and 5 are correct.

Problem 9: D

The total resistance = R + r = 0.8 + 1 = 1.8 Ω

and $I = \frac{e.m.f}{total\ resistance} = \frac{36}{1.8} = 20\ A$

PHYSICS ANSWERS

Problem 10: D

Use Newton's second law and remember to work in SI units:

So $Force = mass \times accelaration = mass \times \frac{\Delta velocity}{time}$

$$= 20 \times 10^{-3} \times \frac{100 - 0}{10 \times 10^{-3}}$$

$$= 200\ N$$

Problem 11: F

In this case, the work being done is moving the bag 0.7 m

i.e. $Work\ Done = Bag's\ Weight\ x\ Distance = 50 \times 10 \times 0.7 = 350\ N$

$Power = \frac{Work}{Time} = \frac{350}{3} = 116.7$ W

= 117 W to 3 significant figures

Problem 12: B

Firstly, use P = Fv to calculate the power [Ignore the frictional force as we are not concerned with the resultant force here].

So P = 300 x 30 = 9000 W

Then, use P = IV to calculate the current.

I = P/V = 9000/200 = 45 A

Problem 13: C

Work is defined as W = F x s. Work can also be defined as work = force x distance moved in the direction of force.Work is measured in joules and 1 Joule = 1 Newton x 1 Metre, and 1 Newton = 1 Kg x ms^{-2} [F = ma].

Thus, 1 Joule = Kgm^2s^{-2}

Problem 14: G

Joules are the unit of energy (and also Work = Force x Distance). Thus, 1 Joule = 1 N x 1 m.

Pa is the unit of Pressure (= Force/Area). Thus, Pa = N x m^{-2}. So, J = Nm^{-2} x m^3 = Pa x m^3. Newton's third law describes that every action produces an equal and opposite reaction. For this reason, the energy required to decelerate a body is equal to the amount of energy it possesses during movement, i.e. its kinetic energy, which is defined as in statement 1.

Problem 15: D

Alpha radiation is of the lower energy, as it represents the movement of a fairly large particle consisting of 2 neutrons and 2 protons. Beta radiation consists of high-energy, high-speed electrons or positrons.

PHYSICS ANSWERS

Problem 16: E

The mass of the rod is unknown so take moments about the centre of mass at x_c = 0.5 m.

$F_1(x_c - x_1) = F_2(x_2 - x_c) + F_3(x_3 - x_c)$

$F_1(0.5 - 0.1) = 4(0.6 - 0.5) + 6(0.8 - 0.5)$

$0.4F_1 = 0.4 + 1.8 = 2.2$

$\therefore F_1 = \frac{22}{4} = 5.5\ N$

Problem 17: B

The value of the velocity is not important, but a constant velocity means that the forces on the block are balanced. Therefore, the friction is equal to the component of the weight in the direction of the slope.

$F = mg\ sin\theta = 5 \times 10 \times sin30$

Remember: $sin30 = 0.5$

$\therefore F = 5 \times 10 \times 0.5 = 25\ N$

Problem 18: C

The slope is smooth so there is no frictional force. Therefore, the acceleration is the component of the acceleration due to gravity in the direction of the slope. (You can also first write it in terms of force by using a variable m for mass, but this just cancels out as $a = \frac{F}{m}$).

$a = g\ sin\theta = 10 \times sin45$

Remember: $sin45 = \frac{1}{\sqrt{2}}$

$\therefore a = \frac{10}{\sqrt{2}}$

$v = u + at = 0 + \frac{10}{\sqrt{2}} \times 10 = \frac{100}{\sqrt{2}}\ ms^{-1}$

PHYSICS ANSWERS

Problem 19: G

At terminal velocity the velocity is constant and the forces due to weight and air resistance are equal.

$W = F_a$

$mg = 0.5u$

$10 \times 10 = 0.5u$

$u = 200\ ms^{-1}$

Note: $F_a = 0.5u$ N is a very simple model for air resistance and is unlikely to be very accurate for real life modelling. However, note that as the velocity of the object increases, the force of air resistance also increases, and when velocity is zero the air resistance is also zero.

PHYSICS ANSWERS

Problem 20: D

Split the initial velocity into horizontal and vertical components and use the vertical component to find out how long the ball is in the air. The horizontal velocity component is constant as air resistance is negligible.

Initial vertical component:

$u_v = u_0\, sin\theta = 100\, sin30 = 100 \times 0.5 = 50\, ms^{-1}$

Initial horizontal component:

$u_h = u_0\, cos\theta = 100\, cos30 = 100 \times \frac{\sqrt{3}}{2} = 50\sqrt{3}\, ms^{-1}$

Use equation of motion for vertical direction:

$s_v = ut + \frac{1}{2}at^2 = u_v t - \frac{1}{2}gt^2 = 50t - \frac{1}{2} \times 10t^2$

Note: positive direction has been taken as upwards so a = - g as gravity acts downwards.

Remember vertical displacement will be zero when the ball returns to original height (ground level).

$0 = 50t - 5t^2 = 5t(10 - t)$

$0 = t(10 - t)$

Ball is at ground level at t = 0 and t = 10 s so ball travels for 10 s. Calculate horizontal distance travelled in that time:

$s_h = u_h t = 50\sqrt{3} \times 10 = 500\sqrt{3}\, m$

PHYSICS ANSWERS

Problem 21: E

Do a momentum balance where after the blocks coalesce, they act like one mass with velocity u_3. The positive direction has been taken as the initial velocity direction of m_2.

Momentum balance:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)u_3$$
$$-1 \times 1 + 2 \times 2 = (1 + 2)u_3$$
$$3 = 3u_3$$
$$u_3 = 1\, ms^{-1}$$

Note: This velocity is in the same direction as m_2 initially.

Initial KE:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2} \times 1 \times 1^2 + \frac{1}{2} \times 2 \times 2^2 = 4.5\, J$$

Final KE:

$$\frac{1}{2}(m_1 + m_2)u_3^2 = \frac{1}{2} \times 3 \times 1^2 = 1.5\, J$$

Therefore, fraction of KE lost:

$$\frac{4.5 - 1.5}{4.5} = \frac{3}{4.5} = \frac{2}{3}$$

Problem 22: G

Do an energy balance where the gravitational potential energy lost is equal to the kinetic energy gained and the work done against air resistance.

Energy balance:

$$GPE = KE + WD$$
$$mgh = \frac{1}{2}mv^2 + F_f h$$
$$20 \times 10 \times 100 = \frac{1}{2} \times 20 \times v^2 + 40 \times 100$$
$$20\,000 = 10v^2 + 4000$$
$$v^2 = 1600$$
$$v = 40\, ms^{-1}$$

PHYSICS ANSWERS

Problem 23: H

Distance travelled is area under the curve.

$$v = \frac{(5t+3)^2}{t^{\frac{1}{2}}} = \frac{25t^2 + 30t + 9}{t^{\frac{1}{2}}} = 25t^{\frac{3}{2}} + 30t^{\frac{1}{2}} + 9t^{-\frac{1}{2}}$$

$$s = \int_1^4 v\,dt = \int_1^4 25t^{\frac{3}{2}} + 30t^{\frac{1}{2}} + 9t^{-\frac{1}{2}}\,dt$$

$$\left[10t^{\frac{5}{2}} + 20t^{\frac{3}{2}} + 18t^{\frac{1}{2}}\right]_1^4$$

$$10 \times 4^{\frac{5}{2}} + 20 \times 4^{\frac{3}{2}} + 18 \times 4^{\frac{1}{2}} - (10 + 20 + 18)$$

$$10 \times 2^5 + 20 \times 2^3 + 18 \times 2 - 48$$

$$10 \times 32 + 20 \times 8 + 36 - 48$$

$$\therefore s = 468\,m$$

Problem 24: C

Work out mass of sand by using Hooke's law for a spring to obtain the weight. Then find density.

$$Weight = Force\ on\ spring$$

$$mg = kx$$

$$m = \frac{kx}{g} = \frac{4 \times 0.1}{10} = 0.04\,kg$$

$$V = 20\,cm^3 = 20 \times 10^{-6}\,m^3$$

$$Density, \rho = \frac{m}{V} = \frac{0.04}{20 \times 10^{-6}} = 2000\,kgm^{-3}$$

PHYSICS ANSWERS

Problem 25: D

Use stress and strain to find Young's modulus.

$$Young's\ modulus = \frac{stress}{strain}$$

$$stress = \frac{Force}{Cross\ sectional\ area} = \frac{20}{\pi \times 0.001^2} = \frac{20 \times 10^6}{\pi}\ Pa$$

$$strain = \frac{extension}{unstretched\ length} = \frac{2\ mm}{10\ cm} = \frac{2}{100} = 0.02$$

$$Young's\ modulus = \frac{20 \times 10^6}{\pi}\frac{1}{0.02} = \frac{20 \times 10^6}{\pi}\frac{1}{20 \times 10^{-3}}$$

$$\therefore Young's\ modulus = \frac{10^9}{\pi}\ Pa = \frac{1}{\pi}\ GPa$$

PHYSICS ANSWERS

Problem 26: C

Check that the mass and the charge is conserved for each of the fission processes.

Diagram 1 cannot be true as the mass of a proton and uranium adds up to 236. This is not equal to the mass of the products which is 144 + 1 + 1 + 89 = 235

Diagram 3 cannot be true using a a similar argument; mass of the incoming neutron and Uranium adds up to 236 but the mass of the products is 87 + 1 + 1 + 1 + 145 = 235

Problem 27: B

One method is to find the wavelength and see how many fits into 120 m.

$$wave\ speed, v = \frac{30}{10} = 3\ ms^{-1}$$

$$v = f\lambda$$

$$\lambda = \frac{v}{f} = \frac{3}{2} = 1.5\ m$$

$$number\ of\ cycles = \frac{120}{1.5} = 80$$

MATH 2

Problem 1:

Rearrange $\frac{(7x+10)}{(9x+5)} = 3y^2 + 2$, to make 4x the subject.

A. $\frac{15\,y^2}{7-9(3y^2+2)}$

B. $\frac{15\,y^2}{7+9(3y^2+2)}$

C. $-\frac{15\,y^2}{7-9(3y^2+2)}$

D. $-\frac{15\,y^2}{7+9(3y^2+2)}$

E. $-\frac{5\,y^2}{7+9(3y^2+2)}$

F. $\frac{5\,y^2}{7+9(3y^2+2)}$

Problem 2:

Simplify $3x\left(\frac{3x^7}{x^{\frac{1}{3}}}\right)^3$

A. $9x^{20}$

B. $27x^{20}$

C. $87x^{20}$

D. $9x^{21}$

E. $27x^{21}$

F. $81x^{21}$

Problem 3:

Simplify $2x[(2x)^7]^{\frac{1}{14}}$

A. $2x\sqrt{2\,x^4}$

B. $2x\sqrt{2x^3}$

C. $2\sqrt{2\,x^4}$

D. $2\sqrt{2x^3}$

E. $8x^3$

F. $8x$

Problem 4:

What is the circumference of a circle with an area of 10π?

A. $2\pi\sqrt{10}$

B. $\pi\sqrt{10}$

C. 10π

D. 20π

E. $\sqrt{10}$

F. More information needed.

MATH 2 QUESTIONS

Problem 5:

If $a.b = (ab) + (a + b)$, then calculate the value of $(3.4).5$

A. 19 B. 54 C. 100 D. 119 E. 132

Problem 6:

If $a.b = \frac{a^b}{a}$, calculate $(2.3).2$

A. $\frac{16}{3}$ B. 1 C. 2 D. 4 E. 8

Problem 7:

Solve $x^2 + 3x - 5 = 0$

A. $x = -\frac{3}{2} \pm \frac{\sqrt{11}}{2}$

C. $x = -\frac{3}{2} \pm \frac{\sqrt{11}}{4}$

E. $x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$

B. $x = \frac{3}{2} \pm \frac{\sqrt{11}}{2}$

D. $x = \frac{3}{2} \pm \frac{\sqrt{11}}{4}$

F. $x = -\frac{3}{2} \pm \frac{\sqrt{29}}{2}$

Problem 8:

How many times do the curves $y = x^3$ and $y = x^2 + 4x + 14$ intersect?

A. 0 B. 1 C. 2 D. 3 E. 4

MATH 2 QUESTIONS

Problem 9:

Solve for *x*, *y*, and *z*.

1. $x + y - z = -1$
2. $2x - 2y + 3z = 8$
3. $2x - y + 2z = 9$

	x	y	z
A	2	-15	-14
B	15	2	14
C	14	15	-2
D	-2	15	14
E	2	-15	14
F	No solutions possible		

Problem 10:

Fully factorise: $3a^3 - 30a^2 + 75a$

A. $3a(a - 3)^3$

B. $a(3a - 5)^2$

C. $3a(a^2 - 10a + 25)$

D. $3a(a - 5)^2$

E. $3a(a + 5)^2$

Problem 11:

Solve for x and y:

$$4x + 3y = 48$$

$$3x + 2y = 34$$

	x	y
A	8	6
B	6	8
C	3	4
D	4	3
E	30	12
F	12	30
G	No solutions possible	

Problem 12:

Evaluate: $\frac{-(5^2-4 \times 7)^2}{-6^2+2 \times 7}$

A. $-\frac{3}{50}$

B. $\frac{11}{22}$

C. $-\frac{3}{22}$

D. $\frac{9}{50}$

E. $\frac{9}{22}$

F. 0

Problem 13:

All license plates are 6 characters long. The first 3 characters consist of letters and the next 3 characters of numbers. How many unique license plates are possible?

A. 676,000

B. 6,760,000

C. 67,600,000

D. 1,757,600

E. 17,576,000

F. 175,760,000

MATH 2 QUESTIONS

Problem 14:

How many solutions are there for: $2(2(x^2 - 3x)) = -9$?

A. 0

B. 1

C. 2

D. 3

E. Infinite solutions.

Problem 15:

Evaluate: $\left(x^{\frac{1}{2}} y^{-3}\right)^{\frac{1}{2}}$

A. $\frac{x^{\frac{1}{2}}}{y}$

B. $\frac{x}{y^{\frac{3}{2}}}$

C. $\frac{x^{\frac{1}{4}}}{y^{\frac{3}{2}}}$

D. $\frac{y^{\frac{1}{4}}}{x^{\frac{3}{2}}}$

Problem 16:

Bryan earned a total of £ 1,240 last week from renting out three flats. From this, he had to pay 10% of the rent from the 1-bedroom flat for repairs, 20% of the rent from the 2-bedroom flat for repairs, and 30% from the 3-bedroom flat for repairs. The 3-bedroom flat costs twice as much as the 1-bedroom flat. Given that the total repair bill was £ 276 calculate the rent for each apartment.

	1 Bedroom	2 Bedrooms	3 Bedrooms
A	280	400	560
B	140	200	280
C	420	600	840
D	250	300	500

MATH 2 QUESTIONS

Problem 17:

At a Pizza Parlour, you can order single, double or triple cheese in the crust. You also have the option to include ham, olives, pepperoni, bell pepper, meat balls, tomato slices, and pineapples. How many different types of pizza are available at the Pizza Parlour?

A. 10
B. 96
C. 192
D. 384
E. 768
F. None of the above

Problem 18:

Solve the simultaneous equations $x^2 + y^2 = 1$ and $x + y = \sqrt{2}$, for $x, y > 0$.

A. $(x, y) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
B. $(x, y) = (½, \frac{\sqrt{3}}{2})$
C. $(x, y) = (\sqrt{2} - 1, 1)$
D. $(x, y) = (\sqrt{2}, ½)$

Problem 19:

Which of the following statements is **FALSE**?

A. Congruent objects always have the same dimensions and shape.
B. Congruent objects can be mirror images of each other.
C. Congruent objects do not always have the same angles.
D. Congruent objects can be rotations of each other.
E. Two triangles are congruent if they have two sides and one angle of the same magnitude.

Problem 20:

Solve the inequality $x^2 \geq 6 - x$

A. $x \leq -3$ and $x \leq 2$
B. $x \leq -3$ and $x \geq 2$
C. $x \geq -3$ and $x \leq 2$
D. $x \geq -3$ and $x \geq 2$
E. $x \geq 2$ only
F. $x \geq -3$ only

Problem 21:

MATH 2 QUESTIONS

The hypotenuse of an isosceles right-angled triangle is x cm. What is the area of the triangle in terms of x?

A. $\frac{x^2}{4}$ B. $\frac{x}{4}$ C. $\frac{3x^2}{4}$ D. $\frac{x^2}{10}$

Problem 22:

Mr Heard derives a formula: $Q = \frac{(X+Y)^2 A}{3B}$. He doubles the values of X and Y, halves the value of A and triples the value of B. What happens to value of Q?

A. Decreases by $\frac{1}{3}$

B. Increases by $\frac{1}{3}$

C. Decreases by $\frac{2}{3}$

D. Increases by $\frac{2}{3}$

E. Increases by $\frac{4}{3}$

F. Decreases by $\frac{4}{3}$

Problem 23:

Consider the graphs $y = x^2 - 2x + 3$, and $y = x^2 - 6x - 10$. Which of the following is true?

A. Both equations intersect the x-axis.

B. Neither equation intersects the x-axis.

C. The first equation does not intersect the x-axis; the second equation intersects the x-axis.

D. The first equation intersects the x-axis; the second equation does not intersect the x-axis.

Problem 24:

Evaluate

$$\log_2\left(\frac{5}{4}\right) + \log_2\left(\frac{6}{5}\right) + \log_2\left(\frac{7}{6}\right) + \cdots + \log_2\left(\frac{64}{63}\right)$$

A -2

B 3

C 4

D 6

Problem 25:

MATH 2 QUESTIONS

Given that $7\cos\theta - 3\tan\theta\sin\theta = 1$, which one of the following is true?

A $\cos\theta = -\frac{3}{5}$ or $-\frac{1}{2}$

B $\cos\theta = -\frac{3}{5}$ or $\frac{1}{2}$

C $\cos\theta = \frac{3}{5}$ or $\frac{1}{2}$

D $\cos\theta = \frac{3}{5}$ or $-\frac{1}{2}$

Problem 26:

k is the smallest positive value of x which is a solution to **both** the equations $2\sin x + 1 = 0$ **and** $2\cos 2x = 1$

How many values of x in the range $0 \le x \le k$ are solutions to at least one of these equations?

A 0

B 2

C 3

D 4

E 8

Problem 27:

How many solutions of the equation $2\sin^3\theta = \sin\theta$ lie in the interval $-\frac{\pi}{2} \le \theta \le \pi$?

A 2

B 3

C 4

D 5

E 6

F 7

Problem 1: A

Multiply by the denominator to give: $(7x + 10) = (3y^2 + 2)(9x + 5)$

Partially expand brackets on right side: $(7x + 10) = 9x(3y^2 + 2) + 5(3y^2 + 2)$

Take x terms across to left side: $7x - 9x(3y^2 + 2) = 5(3y^2 + 2) - 10$

Take x outside the brackets: $x[7 - 9(3y^2 + 2)] = 5(3y^2 + 2) - 10$

Thus: $x = \frac{5(3y^2+2)-10}{7-9(3y^2+2)}$

Simplify to give: $x = \frac{(15y^2)}{(7-9(3y^2+2))}$

Problem 2: F

$$3x\left(\frac{3x^7}{x^{\frac{1}{3}}}\right)^3 = 3x\left(\frac{3^3x^{21}}{x^{\frac{3}{3}}}\right)$$

$$= 3x\,\frac{27x^{21}}{x} = 81x^{21}$$

Problem 3: D

$$2x[2^{\frac{7}{14}}\,x^{\frac{7}{14}}] = 2x[2^{\frac{1}{2}}\,x^{\frac{1}{2}}]$$

$$= 2x(\sqrt{2}\,\sqrt{x}) = 2\,[\sqrt{x}\sqrt{x}][\sqrt{2}\,\sqrt{x}]$$

$$= 2\sqrt{2x^3}$$

Problem 4: A

$A = \pi r^2$, therefore $10\pi = \pi r^2$

Thus, $r = \sqrt{10}$

Therefore, the circumference is $2\pi\sqrt{10}$

Problem 5: B

$3.4 = 12 + (3 + 4) = 19$

$19.5 = 95 + (19 + 5) = 119$

Problem 6: D

$2.3 = \frac{2^3}{2} = 4$

$4.2 = \frac{4^2}{4} = 4$

Problem 7: F

This is a tricky question that requires you to know how to 'complete the square':

$(x + 1.5)(x + 1.5) = x^2 + 3x + 2.25$

Thus, $(x + 1.5)^2 - 7.25 = x^2 + 3x - 5 = 0$

Therefore, $(x + 1.5)^2 = 7.25 = \frac{29}{4}$

Thus, $x + 1.5 = \sqrt{\frac{29}{4}}$

Thus $x = -\frac{3}{2} \pm \sqrt{\frac{29}{4}} = -\frac{3}{2} \pm \frac{\sqrt{29}}{2}$

Problem 8: B

Whilst you definitely need to solve this graphically, it is necessary to complete the square for the first equation to allow you to draw it more easily:

$(x + 2)^2 = x^2 + 4x + 4$

Thus, $y = (x + 2)^2 + 10 = x^2 + 4x + 14$

This is now an easy curve to draw ($y = x^2$ that has moved 2 units left and 10 units up). The turning point of this quadratic is to the left and well above anything in x^3, so the only solution is the first intersection of the two curves in the upper right quadrant around (3.4, 39).

Problem 9: D

We can eliminate z from equation (1) and (2) by multiplying equation (1) by 3 and adding it to equation (2):

$3x + 3y - 3z = -3$ Equation (1) multiplied by 3

$\underline{2x - 2y + 3z = 8}$ Equation (2) then add both equations

$5x + y = 5$ We label this as equation (4)

Now we must eliminate the same variable z from another pair of equations by using equation (1) and (3):

$2x + 2y - 2z = -2$ Equation (1) multiplied by 2

$\underline{2x - y + 2z = 9}$ Equation (3) then add both equations

$4x + y = 7$ We label this as equation (5)

We now use both equations (4) and (5) to obtain the value of x:

$5x + y = 5$ Equation (4)

$\underline{-4x - y = -7}$ Equation (5) multiplied by -1

$x = -2$

Substitute x back in to calculate y:

$4x + y = 7$

$4(-2) + y = 7$

$-8 + y = 7$

$y = 15$

Substitute x and y back in to calculate z:

$x + y - z = -1$

$-2 + 15 - z = -1$

$13 - z = -1$

$-z = -14$

$z = 14$

Thus: $x = -2$, $y = 15$, $z = 14$

MATH 2 QUESTIONS

Problem 10: D

This is one of the easier maths questions. Take $3a$ as a factor to give:

$3a(a^2 - 10a + 25) \quad = 3a(a-5)(a-5) = 3a(a-5)^2$

Problem 11: B

Note that 12 is the Lowest Common Multiple of 3 and 4. Thus:

$-3(4x + 3y) = -3(48)$ Multiply each side by -3

$4(3x + 2y) = 4(34)$ Multiply each side by 4

$-12x - 9y = -144$

$\underline{12x + 8y = 136}$ Add together

$-y = -8$

$y = 8$

Substitute y back in: $4x + 3y = 48$

$4x + 3(8) = 48$

$4x + 24 = 48$

$4x = 24$

$x = 6$

Problem 12: E

Don't be fooled, this is an easy question, just obey BODMAS and don't skip steps.

$$\frac{-(25-28)^2}{-36+14} = \frac{-(-3)^2}{-22}$$

This gives: $\frac{-(9)}{-22} = \frac{9}{22}$

Problem 13: E

MATH 2 QUESTIONS

Since there are 26 possible letters for each of the 3 letters in the license plate, and there are 10 possible numbers (0-9) for each of the 3 numbers in the same plate, then the number of license plates would be:

$$(26) \times (26) \times (26) \times (10) \times (10) \times (10) = 17{,}576{,}000$$

Problem 14: B

Expand the brackets to give: $4x^2 - 12x + 9 = 0$.

Factorise: $(2x - 3)(2x - 3) = 0$.

Thus, only one solution exists, x = 1.5.

Note that you could also use the fact that the discriminant, $b^2 - 4ac = 0$ to get the answer.

Problem 15: C

$$= \left(x^{\frac{1}{2}}\right)^{\frac{1}{2}} (y^{-3})^{\frac{1}{2}}$$

$$= x^{\frac{1}{4}} y^{-\frac{3}{2}} = \frac{x^{\frac{1}{4}}}{y^{\frac{3}{2}}}$$

Problem 16: A

- 1-bed: x
- 2-bed: y
- 3-bed: z

Given $z = 2x$ and total rent $x + y + z = 1240$:

$$x + y + 2x = 1240 \Rightarrow 3x + y = 1240 \Rightarrow y = 1240 - 3x$$

Total repairs:

$$0.1x + 0.2y + 0.3z = 276$$

Substitute $y = 1240 - 3x$ and $z = 2x$:

$$0.1x + 0.2(1240 - 3x) + 0.3(2x) = 276$$

$$0.1x + 248 - 0.6x + 0.6x = 276 \Rightarrow 0.1x + 248 = 276$$

$$0.1x = 28 \Rightarrow x = 280$$

So $z = 2x = 560$, and $y = 1240 - 3(280) = 400$.

Problem 17: D

There are three outcomes from choosing the type of cheese in the crust. For each of the additional toppings to possibly add, there are 2 outcomes: 1 to include and another not to include a certain topping, for each of the 7 toppings

Thus, the number of different kinds of pizza is: $3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 3 \times 2^7$ $= 3 \times 128 = 384$

Problem 18: A

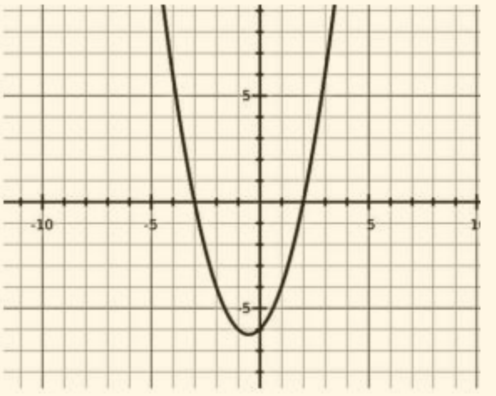
Although it is possible to do this algebraically, by far the easiest way is via trial and error. The clue that you shouldn't attempt it algebraically is the fact that rearranging the first equation to make x or y the subject leaves you with a difficult equation to work with (e.g. $x = \sqrt{1 - y^2}$) when you try to substitute in the second.

An exceptionally good student might notice that the equations are symmetric in x and y, i.e. the solution is when $x = y$. Thus $2x^2 = 1$ and $2x = \sqrt{2}$ which gives $\frac{\sqrt{2}}{2}$ as the answer.

Problem 19: C

If two shapes are congruent, then they are the same size and shape. Thus, congruent objects can be rotations and mirror images of each other. The two triangles in E are indeed congruent (SAS). Congruent objects must, by definition, have the same angles.

Problem 20: B



Rearrange the equation: $x^2 + x - 6 \geq 0$

Factorise: $(x + 3)(x - 2) \geq 0$

Remember that this is a quadratic inequality so requires a quick sketch to ensure you don't make a silly mistake with which way the sign is.

Thus, $y = 0$ when $x = 2$ and $x = -3$. $y > 0$ when $x > 2$ or $x < -3$.

Thus, the solution is: $x \leq -3$ *and* $x \geq 2$.

Problem 21: B

Using Pythagoras: $a^2 + b^2 = x^2$

Since the triangle is isosceles: $a = b$, $so\ 2a^2 = x^2$

Area $= \frac{1}{2} base\ x\ height = \frac{1}{2}a^2$. From above, $a^2 = \frac{x^2}{2}$

Thus the area = $\frac{1}{2} x \frac{x^2}{2} = \frac{x^2}{4}$

Problem 22: A

If X and Y are doubled, the value of Q increases by 4. Halving the value of A reduces this to 2. Finally, tripling the value of B reduces this to $\frac{2}{3}$, i.e. the value decreases by $\frac{1}{3}$.

Problem 23: C

Check the discriminant $b^2 - 4ac$ (positive ⇒ intersects the x-axis; negative ⇒ no intersection).

- For $y = x^2 - 2x + 3$: $b^2 - 4ac = (-2)^2 - 4(1)(3) = 4 - 12 = -8 < 0$ ⇒ **does not** intersect the x-axis.
- For $y = x^2 - 6x - 10$: $b^2 - 4ac = (-6)^2 - 4(1)(-10) = 36 + 40 = 76 > 0$ ⇒ **does** intersect the x-axis.

Problem 24: C

log2(64) - log2(4) = log2(64/4)

= log2(16)

= log2(2^4) = 4

Problem 25: D

7 cos θ - 3 tan θ sin θ = 1

7 - 3 sin^2 θ/cos^2 θ = 1/cos θ

7 - 3 tan^2 θ =sec θ

7 - 3 (sec^2 θ - 1) = sec θ

7 - 3 sec^2 θ + 3 = sec θ

3 sec^2 θ + sec θ - 10 = 0

(3 sec θ - 5) (sec θ + 2) = 0

sec θ = +5/3 or -2 --> cos θ = +3/5 or -1/2

Problem 26: C

2 sin x + 1 = 0

sin x = -1/2

x = 210° (by symmetry because we are using the results that sin 30° = 1/2 and sin 180° = 0)

2 cos (420°) = 2 * cos 60° = 1 , therefore 210° is K

No other solutions to 2 sin x + 1 = 0 is range 0 ≤ x ≤ 210° as we just found that 210°is the first.

But for 2 cos 2x = 1 we can have x = 30°, 150°

Therefore three values of x being 30°,150°,210°

Problem 27: D

$\sin sin\, \theta\left(2sin^2\theta - 1\right) = 0$

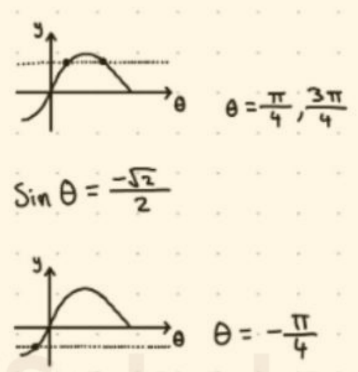
$\sin sin\, \theta = 0$

$\theta = 0\,,\ \pi$

$2sin^2\theta - 1 = 0$

$\sin sin\, \theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$\sin sin\, \theta = \frac{\sqrt{2}}{2}$



5 Solutions