Pre AMC 8 DIGITAL MOCK TEST Solution Book

Complete, easy-to-understand explanations for every question in the Pre AMC 8 Digital Mock Test.

Students get **75 minutes** (more than usual) to solve the test and learn calmly.

Remember that analyzing a test is as important as taking it!

ThrivingScholars 

1. Aunt Anna is 42 years old. Caitlin is 5 years younger than Brianna, and Brianna is half as old as Aunt Anna. How old is Caitlin?

   (A) 15 **(B)** 16 **(C)** 17 **(D)** 21 **(E)** 37

Answer (B): Brianna is half as old as Aunt Anna, so Brianna is 21 years old. Caitlin is 5 years younger than Brianna, so Caitlin is 16 years old.

2. Which of these numbers is less than its reciprocal?

   (A) -2 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2

Answer (A): The number 0 has no reciprocal, and 1 and -1 are their own reciprocals. This leaves only 2 and -2. The reciprocal of 2 is $\frac{1}{2}$, but 2 is not less than $\frac{1}{2}$. The reciprocal of -2 is $-\frac{1}{2}$, and -2 is less than $-\frac{1}{2}$.

3. How many whole numbers lie in the interval between $\frac{5}{3}$ and 2π ?

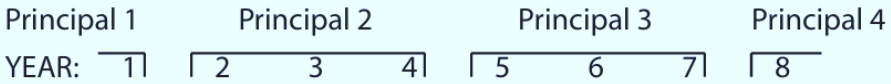
(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** infinitely many

Answer (D): The smallest whole number in the interval is 2 because $\frac{5}{3}$ is more than 1 but less than 2. The largest whole number in the interval is 6 because 2π is more than 6 but less than 7. There are five whole numbers in the interval. They are 2, 3, 4, 5, and 6.

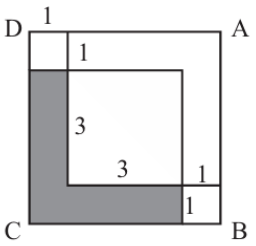
4. Each principal of Lincoln High School serves exactly one 3-year term. What is the maximum number of principals this school could have during an 8-year period?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 8

Answer (C): If the first year of the 8-year period was the final year of a principal's term, then in the next six years two more principals would serve, and the last year of the period would be the first year of the fourth principal's term. Therefore, the maximum number of principals who can serve during an 8-year period is 4.



5. Figure $ABCD$ is a square. Inside this square three smaller squares are drawn with side lengths as labeled. the area of the shaded L-shaped region is



(A) 7 **(B)** 10 **(C)** 12.5 **(D)** 14 **(E)** 15

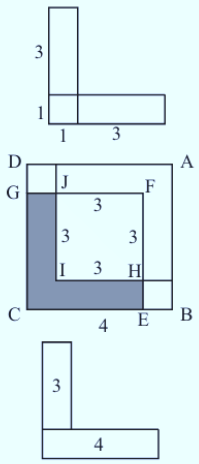
Answer (A): The L-shaped region is made up of two rectangles with area $3 \times 1 = 3$ plus the corner square with area $1 \times 1 = 1$, so the area of the L-shaped figure is $2 \times 3 + 1 = 7$.

OR

Square $FECG-$ square $FHIJ = 4 \times 4 - 3 \times 3 = 16 - 9 = 7$.

OR

The L-shaped region can be decomposed into a 4×1 rectangle and a 3×1 rectangle. So the total area is 7.

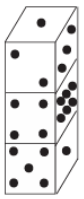


6. What is the minimum possible product of three different numbers of the set $\{-8, -6, -4, 0, 3, 5, 7\}$?

(A) -336 **(B)** -280 **(C)** -210 **(D)** -192 **(E)** 0

Answer (B): The only way to get a negative product using three numbers is to multiply one negative number and two positives or three negatives. Only two reasonable choices exist: $(-8) \times (-6) \times (-4) = (-8) \times (24) = -192$ and $(-8) \times 5 \times 7 = (-8) \times 35 = -280$. The latter is smaller.

7. Three dice with faces numbered 1 through 6 are stacked as shown. Seven of the eighteen faces are visible, leaving eleven faces hidden(back, bottom, between). The total number of dots NOT visible in this view is



(A) 21 **(B)** 22 **(C)** 31 **(D)** 41 **(E)** 53

Answer (D): The numbers on one die total $1+2+3+4+5+6=21$, so the numbers on the three dice total 63. Numbers 1, 1, 2, 3, 4, 5, 6 are visible, and these total 22. This leaves $63-22=41$ not seen.

8. Casey's shop class is making a golf trophy. He has to paint 300 dimples on a golf ball. If it takes him 2 seconds to paint one dimple, how many minutes will he need to do his job?

(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

(D) At 2 seconds per dimple, it takes $300 \times 2 = 600$ seconds to paint them. Since there are 60 seconds in a minute, he will need $600 \div 60 = 10$ minutes.

9. I'm thinking of two whole numbers. Their product is 24 and their sum is 11. What is the larger number?

(A) 3 (B) 4 (C) 6 (D) 8 (E) 12

(D) Since their sum is to be 11, only positive factors need to be considered. Number pairs whose product is 24 are $(1, 24)$, $(2, 12)$, $(3, 8)$ and $(4, 6)$. The sum of the third pair is 11, so the numbers are 3 and 8. The larger one is 8.

10. Granny Smith has \$63. Elberta has \$2 more than Anjou and Anjou has one-third as much as Granny Smith. How many dollars does Elberta have?

(A) 17 (B) 18 (C) 19 (D) 21 (E) 23

(E) Anjou has one-third as much money as Granny Smith, so Anjou has \$21. Elberta has \$2 more than Anjou, and $\$21 + \$2 = \$23$.

11. The digits 1, 2, 3, 4 and 9 are each used once to form the smallest possible **even** five-digit number. The digit in the tens place is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 9

(E) To make the number as small as possible, the smaller digits are placed in the higher-value positions. To make the number even, the larger even digit 4 must be the units digit. The smallest possible even number is 12394 and 9 is in the tens place.

12. On a dark and stormy night Snoopy suddenly saw a flash of lightning. Ten seconds later he heard the sound of thunder. The speed of sound is 1088 feet per second and one mile is 5280 feet. Estimate, to the nearest half-mile, how far Snoopy was from the flash of lightning.

(A) 1 (B) $1\frac{1}{2}$ (C) 2 (D) $2\frac{1}{2}$ (E) 3

(C) Use the formula $d = rt$ (distance equals rate times time): 1088 feet per second $\times 10$ seconds $= 10880$ feet, which is just 320 feet more than two miles. Therefore, Snoopy is just about two miles from the flash of lightning.

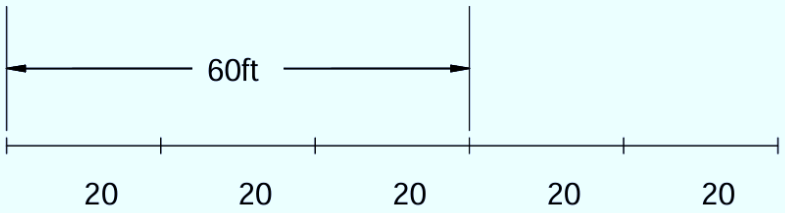
OR

Since this is an estimate, round the speed of sound down to 1000 feet per second and the length of a mile down to 5000 feet. Then $5000 \div 1000 = 5$ seconds per mile, so in 10 seconds the sound will travel about 2 miles.

13. Six trees are equally spaced along one side of a straight road. The distance from the first tree to the fourth is 60 feet. What is the distance in feet between the first and last trees?

(A) 90 (B) 100 (C) 105 (D) 120 (E) 140

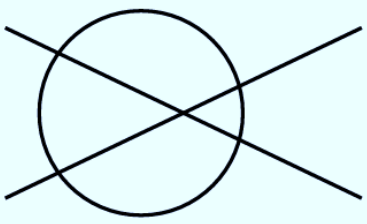
(B) There are three spaces between the first tree and the fourth tree, so the distance between adjacent trees is 20 feet. There are 6 trees with five of these 20-foot spaces, so the distance between the first and last trees is 100 feet.



14. A circle and two distinct lines are drawn on a sheet of paper. What is the largest possible number of points of intersection of these figures?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

(D) Two distinct lines can intersect in one point whereas a line can intersect a circle in two points. The maximum number 5 can be achieved if the lines and circle are arranged as shown. Note that the lines could also meet outside the circle for the same result. (Other arrangements of the lines and circle can produce 0, 1, 2, 3, or 4 points of intersection.)



15. The average of the five numbers in a list is 54. The average of the first two numbers is 48. What is the average of the last three numbers?

(A) 55 **(B)** 56 **(C)** 57 **(D)** 58 **(E)** 59

(D) The sum of all five numbers is $5 \times 54 = 270$. The sum of the first two numbers is $2 \times 48 = 96$, so the sum of the last three numbers is $270 - 96 = 174$. The average of the last three numbers is $\frac{174}{3} = 58$.

16. What is the smallest possible average of four distinct positive even integers?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

(C) The smallest average will occur when the numbers are as small as possible. The four smallest distinct positive even integers are 2, 4, 6, and 8 and their average is 5.

Note: These numbers form an arithmetic sequence. The average of the numbers in any arithmetic sequence is the average of the first and last terms.

17. How many different combinations of \$5 bills and \$2 bills can be used to make a total of \$17? Order does not matter in this problem.

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

(A) Since the total \$17 is odd, there must be an odd number of \$5 bills. One \$5 bill plus six \$2 bills is a solution, as is three \$5 bills plus one \$2 bill. Five \$5 bills exceeds \$17, so these are the only two combinations that work.

18. The year 2002 is a palindrome (a number that reads the same from left to right as it does from right to left). What is the product of the digits of the next year after 2002 that is a palindrome?

(A) 0 **(B)** 4 **(C)** 9 **(D)** 16 **(E)** 25

(B) The next palindrome is 2112. The product of its digits is $2 \cdot 1 \cdot 1 \cdot 2 = 4$.

19. Carlos Montado was born on Saturday, November 9, 2002. On what day of the week will Carlos be 706 days old?

(A) Monday **(B)** Wednesday **(C)** Friday **(D)** Saturday **(E)** Sunday

(C) Since 706 days is 700 plus 6 days, it is 100 weeks plus 6 days. Friday is 6 days after Saturday.

20. Which of the following numbers has the smallest prime factor?

(A) 55 **(B)** 57 **(C)** 58 **(D)** 59 **(E)** 61

(C) The smallest prime is 2, which is a factor of every even number. Because 58 is the only even number, it has the smallest prime factor.

21. If 20% of a number is 12, what is 30% of the same number?

(A) 15 **(B)** 18 **(C)** 20 **(D)** 24 **(E)** 30

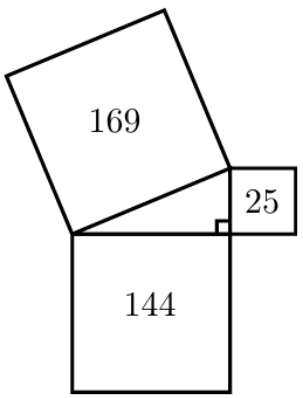
(B) If 20% of the number is 12, the number must be 60. Then 30% of 60 is $0.30 \times 60 = 18$.

OR

Since 20% of the number is 12, it follows that 10% of the number is 6. So 30% of the number is 18.

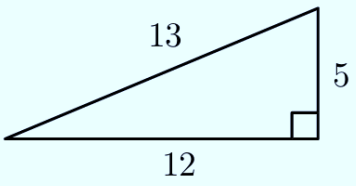
22. Given the areas of the three squares in the figure, what is the area of the interior triangle?

(A) 13 **(B)** 30 **(C)** 60 **(D)** 300 **(E)** 1800

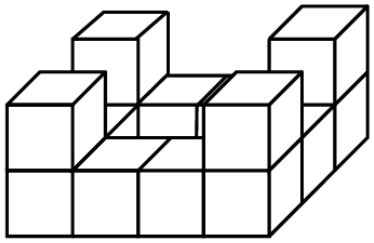


(B)

$$A = \frac{1}{2}(\sqrt{144})(\sqrt{25})$$
$$A = \frac{1}{2} \cdot 12 \cdot 5$$
$$A = 30 \text{ square units}$$



23. Fourteen white cubes are put together to form the figure on the right. The complete surface of the figure, including the bottom, is painted red. The figure is then separated into individual cubes. How many of the individual cubes have exactly four red faces?



(A) 4 **(B)** 6 **(C)** 8 **(D)** 10 **(E)** 12

(B) A cube has four red faces if it is attached to exactly two other cubes. The four top cubes are each attached to only one other cube, so they have five red faces. The four bottom corner cubes are each attached to three others, so they have three red faces. The remaining six each have four red faces.

24. In this addition problem, each letter stands for a different digit.

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

If $T = 7$ and the letter O represents an even number, what is the only possible value for W?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

(D) As given, $T = 7$. This implies that $F = 1$ and that O equals either 4 or 5. Since O is even, $O = 4$. Therefore, $R = 8$. Replacing letters with numerals gives

$$\begin{array}{cccc} & 7 & W & 4 \\ + & 7 & W & 4 \\ \hline 1 & 4 & U & 8 \end{array}$$

$W + W$ must be less than 10; otherwise, a 1 would be carried to the next column, and O would be 5. So $W < 5$. $W \neq 0$ because $W \neq U$, $W \neq 1$ because $F = 1$, $W \neq 2$ because if $W = 2$ then $U = 4 = O$, and $W \neq 4$ because $O = 4$. So $W = 3$.

The addition problem is

$$\begin{array}{cccc} & 7 & 3 & 4 \\ + & 7 & 3 & 4 \\ \hline 1 & 4 & 6 & 8 \end{array}$$

25. The six children listed below are from two families of three siblings each. Each child has blue or brown eyes and black or blond hair. Children from the same family have at least one of these characteristics in common. Which two children are Jim's siblings?

Child	Eye Color	Hair Color
Benjamin	Blue	Black
Jim	Brown	Blond
Nadeen	Brown	Black
Austin	Blue	Blond
Tevyn	Blue	Black
Sue	Blue	Blond

(A) Nadeen and Austin
(B) Benjamin and Sue
(C) Benjamin and Austin
(D) Nadeen and Tevyn
(E) Austin and Sue

(E) Because Jim has brown eyes and blond hair, none of his siblings can have both blue eyes and black hair. Therefore, neither Benjamin nor Tevyn can be Jim's sibling. Consequently, there are only three possible pairs for Jim's siblings – Nadeen and Austin, Nadeen and Sue, or Austin and Sue. Since Nadeen has different hair color and eye color from both Austin and Sue, neither can be Nadeen's sibling. So Austin and Sue are Jim's siblings. Benjamin, Nadeen and Tevyn are siblings in the other family.