HL IB Physics 2025 syllabus Revision Notes

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IB Physics HL

YOUR NOTES

Kinematics

CONTENTS

Distance & Displacement
Speed & Velocity
Acceleration
Kinematic Equations
Motion Graphs
Projectile Motion
Fluid Resistance
Terminal Speed

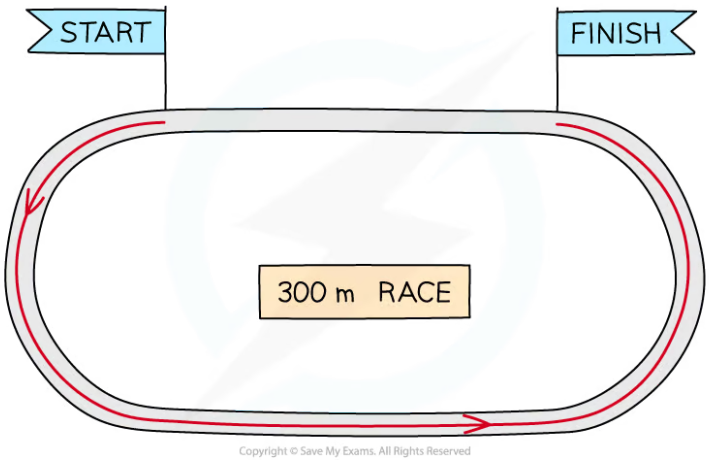
YOUR NOTES

Distance & Displacement

Distance

- **Distance** is a measure of how far an object travels
- It is a scalar quantity - in other words, the direction is not important

Total running distance



The athletes run a total distance of 300 m

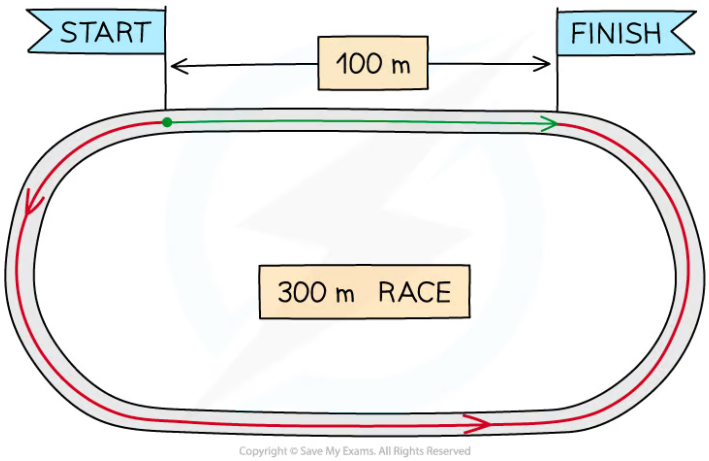
- Consider some athletes running a 300 m race on a 400 m track
- The **distance** travelled by the athletes is **300 m**

Displacement

- **Displacement** is a measure of how far something is from its starting position, along with its direction
  - In other words, it is the **change** in position
- It is a vector quantity - it describes both magnitude and direction

Total distance vs total displacement

YOUR NOTES



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The athletes run a total distance of 300 m, but end up 100 m from where they started

- Distance is a scalar quantity because...

  It describes how far an object has travelled overall, but not the direction it has travelled in

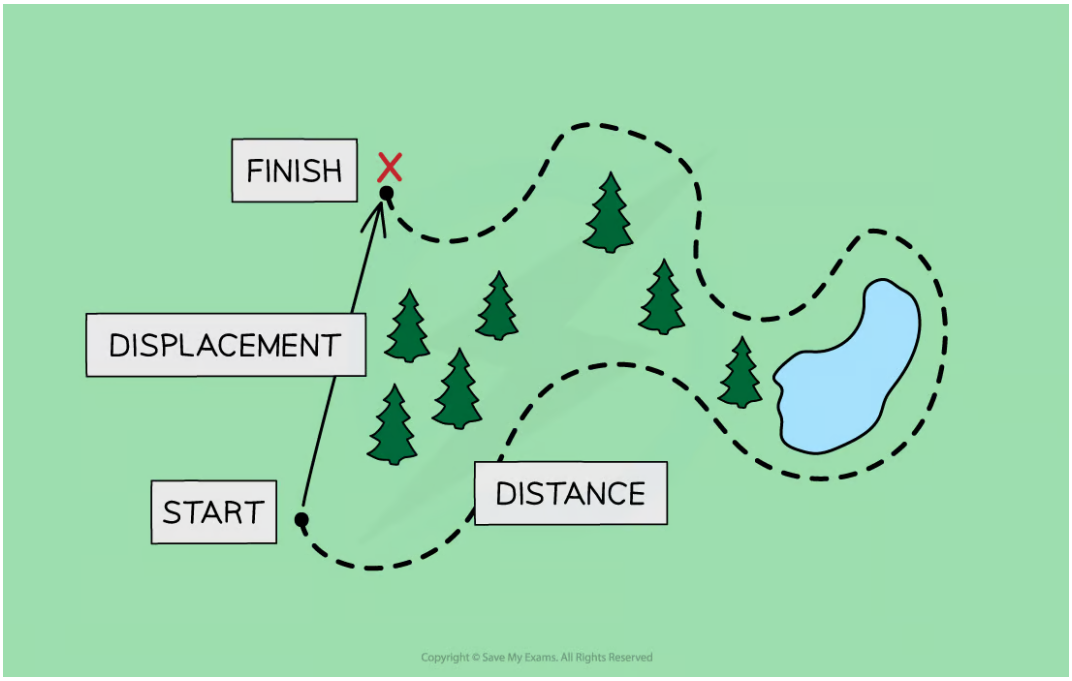
- Displacement is a vector quantity because...

  It describes how far an object is from where it started and in what direction

- For example, if a person goes on a hike in the woods to a location which is a couple of miles from their starting point
  - As the crow flies, their **displacement** will only be a few miles but the **distance** they walked will be much longer

What is the difference between distance and displacement?

YOUR NOTES



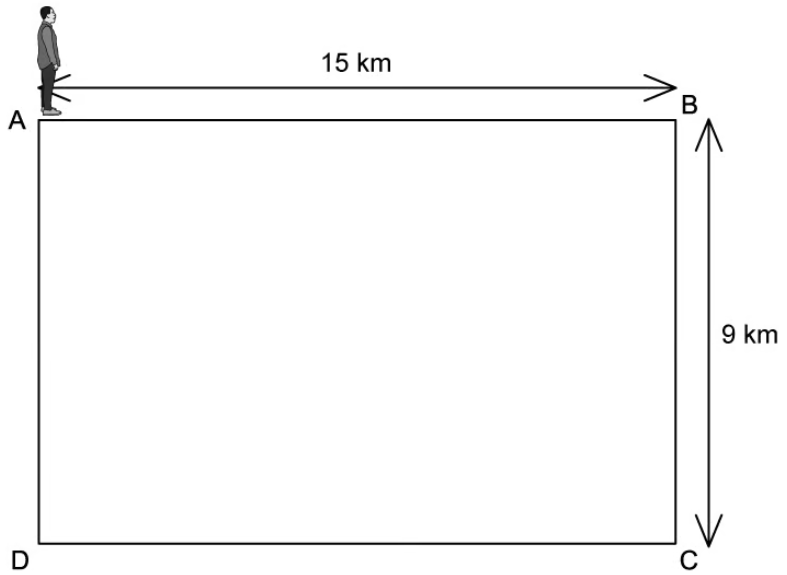
Displacement is a vector while distance is a scalar quantity

- Consider the same 300 m race again
  - The athletes have still run a total **distance** of **300 m** (this is indicated by the arrow in red)
  - However, their **displacement** at the end of the race is **100 m to the right** (this is indicated by the arrow in green)
  - If they had run the full 400 m, their final displacement would be **zero**
- Since the displacement ends **before** the start position, this means it is **–100 m**

YOUR NOTES

Worked Example

A professor walks around her garden following the path ABCDA.



Calculate, at the end of their walk

(a)
the distance the professor travels.
(b)
the displacement of the professor.

Answer:

(a) The distance the professor travels is:

- The total distance of each side of the rectangle

$$15 + 9 + 15 + 9 = 48 \text{ km}$$

(b) The displacement of the professor is:

- The displacement is how far the professor is from their **original** position
- As they travel back to point A, the total displacement = **0 km**

Exam Tip

Since distance is a **scalar** quantity it will never have a negative value. Since displacement is a vector, it **can** have a negative value - this means the final position would be in the opposite direction to the starting point **and** the direction of motion

YOUR NOTES

Speed & Velocity

Speed

- The **speed** of an object is the distance it travels every second
- Speed is a **scalar** quantity
  - This is because it only contains a magnitude (without a direction)
- The **average speed** of an object is given by the equation:

$$average\ speed = \frac{total\ distance}{time\ taken}$$

- The SI units for speed are **meters per second (m s^{-1})** but speed can often be measured in alternative units e.g. km h^{-1} or mph, when it is more appropriate for the situation

Velocity

- The **velocity** of a moving object is similar to its speed and also describes the direction of the velocity
- Velocity is defined as:

  The rate of change of position

- Velocity is, therefore, a vector quantity because it describes both **magnitude** and **direction**

The difference between speed and velocity

- Speed is a **scalar** quantity whilst velocity is **vector**
  - Velocity is the **speed** in a given **direction**

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The cars in the diagram above have the same speed (a scalar quantity) but di erent velocities (a vector quantity). Fear not, they are in di erent lanes!

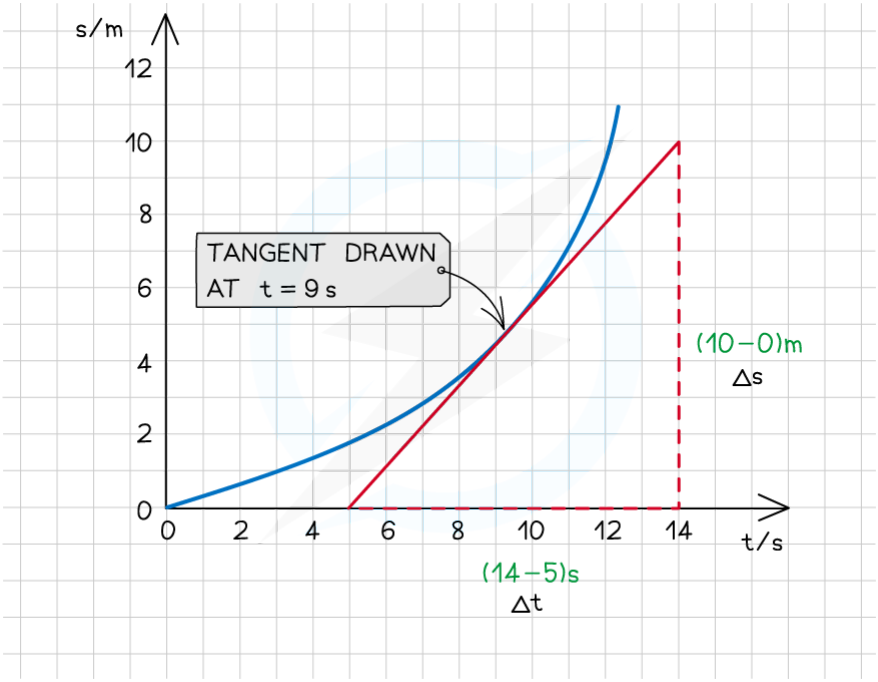
- This means velocity can also have a **negative** value
  - E.g. a ball thrown upwards at a velocity of 3 m s^{-1} comes down at a velocity −5 m s^{-1}, if upwards is considered positive
  - However, their **speeds** are still 3 m s^{-1} and 5 m s^{-1} respectively

Instantaneous Speed & Velocity

YOUR NOTES

- The **instantaneous** speed (or velocity) is the speed (or velocity) of an object **at any given point in time**
- This could be for an object moving at a constant velocity or **accelerating**
  - An object at constant velocity is shown by a **straight line** on a displacement – time graph
  - An object accelerating is shown by a **curved line** on a displacement – time graph
  - An accelerating object will have a **changing velocity**

- To find the instantaneous velocity on a displacement-time graph:
  - Draw a **tangent** at the required time
  - Calculate the **gradient** of that tangent



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The instantaneous velocity is found by drawing a tangent on the displacement time graph

- In the graph above, at t = 9 s, the velocity is:

$$gradient = \frac{10 - 0}{14 - 5} = 1.11 \text{ m s}^{-1}$$

Average Speed & Velocity

- The average velocity $\overline{v}$ of an object can be calculated using

$$\overline{v} = \frac{\Delta x}{\Delta t}$$

- Where:
  - Δx = total displacement, or change in position (m)
  - Δt = total time taken (s)

YOUR NOTES

- If the initial velocity *u* and final velocity *v* are known, the average velocity can also be calculated from

$$\bar{v} = \frac{(u + v)}{2}$$

- To find the average velocity on a displacement-time graph, divide the **total displacement** (on the y-axis) by the **total time** (on the x-axis)
  - This method can be used for both a curved or a straight line on a displacement-time graph

Worked Example

Florence Griffith Joyner set the women's 100 m world record in 1988, with a time of 10.49 s.

Calculate her average speed during the race.

Answer:

- Sprinters typically speed up from rest to a maximum speed
- Because Florence's speed changes over the course of the race, we can calculate her average speed using the equation:

average speed = total distance ÷ time taken

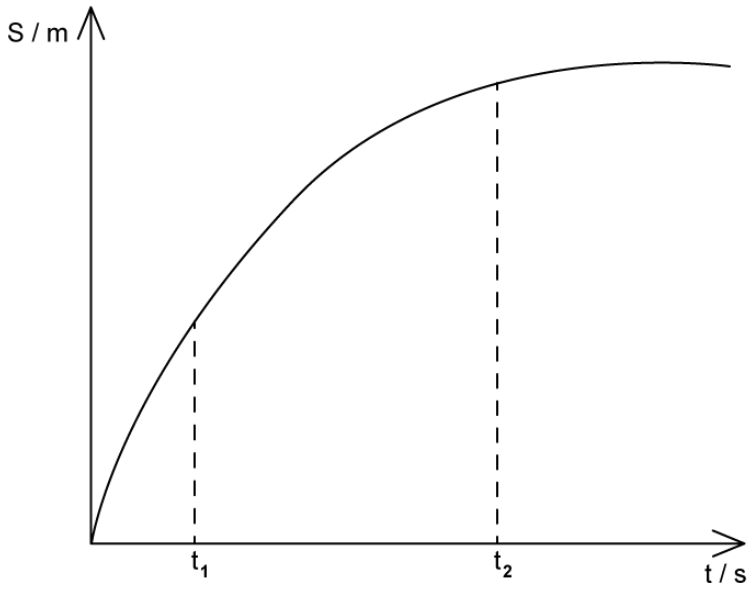
- Where:
  - Total distance, *s* = 100 m
  - Time taken, *t* = 10.49 s

average speed = 100 ÷ 10.49 = 9.5328 = **9.53 m s^{-1}**

YOUR NOTES

Worked Example

The variation of displacement of a box sliding across a rough surface with time *t* is shown on the graph below.



The magnitudes of the instantaneous velocities of the trolley at time t_1 and t_2 are v_1 and v_2 respectively.

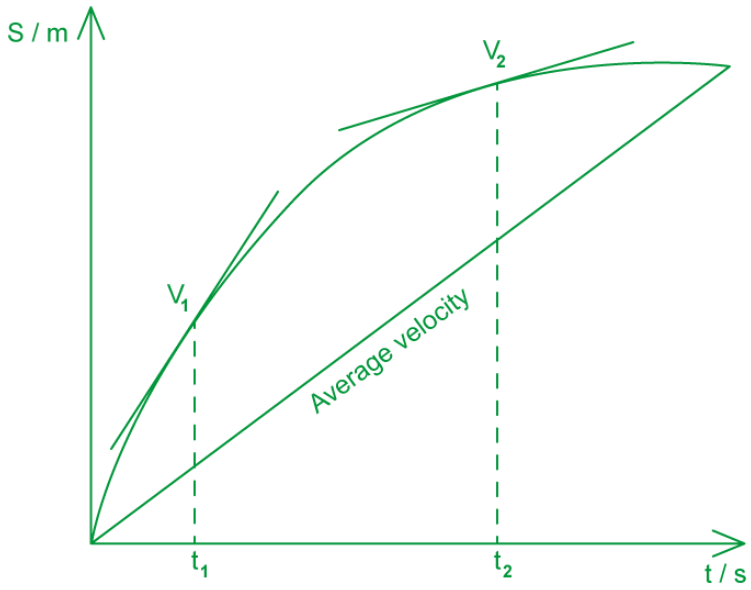
List the following velocities in order from fastest to slowest:

v_1	v_2	average velocity

Answer:

Step 1: Sketch the velocities from the graph

- The instantaneous velocity is the gradient of a tangent at a certain time



- The average velocity is the total displacement over the total time

Step 2: Compare the gradients of each velocity

- The fastest velocity will have the **steepest** gradient and the slowest velocity the **shallowest** gradient
- In order from fastest to slowest:

$$v_1 > \text{average velocity} > v_2$$

Exam Tip

When you draw a tangent to a curve, make sure it **just touches** the point at which you wish to calculate the gradient. The angle between the curve and the tangent line should be roughly equal on both sides of the point.

If you are asked to find the instantaneous velocity from a graph, you will be told the **time** at which they want this velocity for.

YOUR NOTES

Acceleration

- Acceleration is defined as:

  The rate of change of velocity

- Acceleration is a vector quantity and is measured in metres per second squared (m s^{-2})
  - It describes how much an object's velocity **changes** every **second**
- The **average acceleration** of an object can be calculated using:

$$average\ acceleration = \frac{change\ in\ velocity}{time\ taken}$$

$$a = \frac{\Delta v}{\Delta t}$$

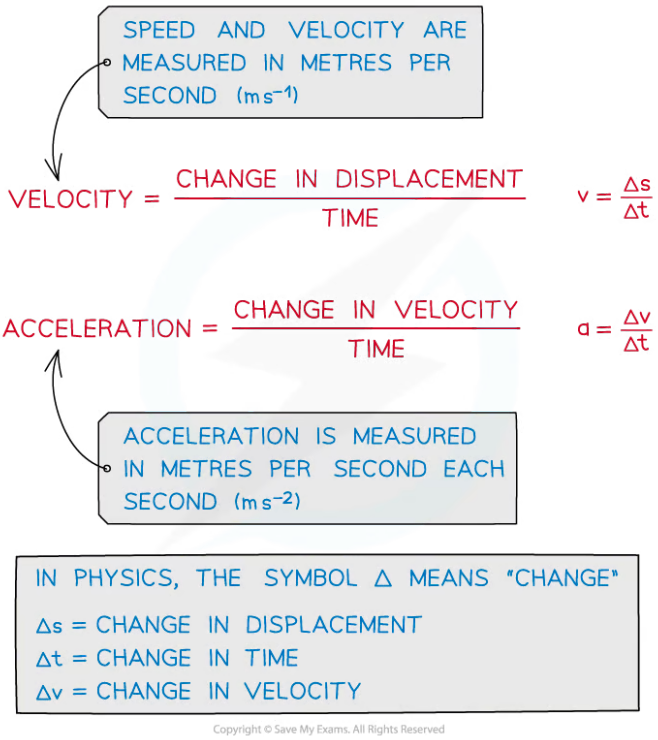
- Where:
  - a = average acceleration (m s^{-2})
  - Δv = change in velocity (m s^{-1})
  - Δt = total time taken (s)
- The **change in velocity** is the **difference** between the initial and final velocity, as written below:

change in velocity = final velocity – initial velocity

$$\Delta v = (v - u)$$

Equations linking displacement, velocity, and acceleration

YOUR NOTES



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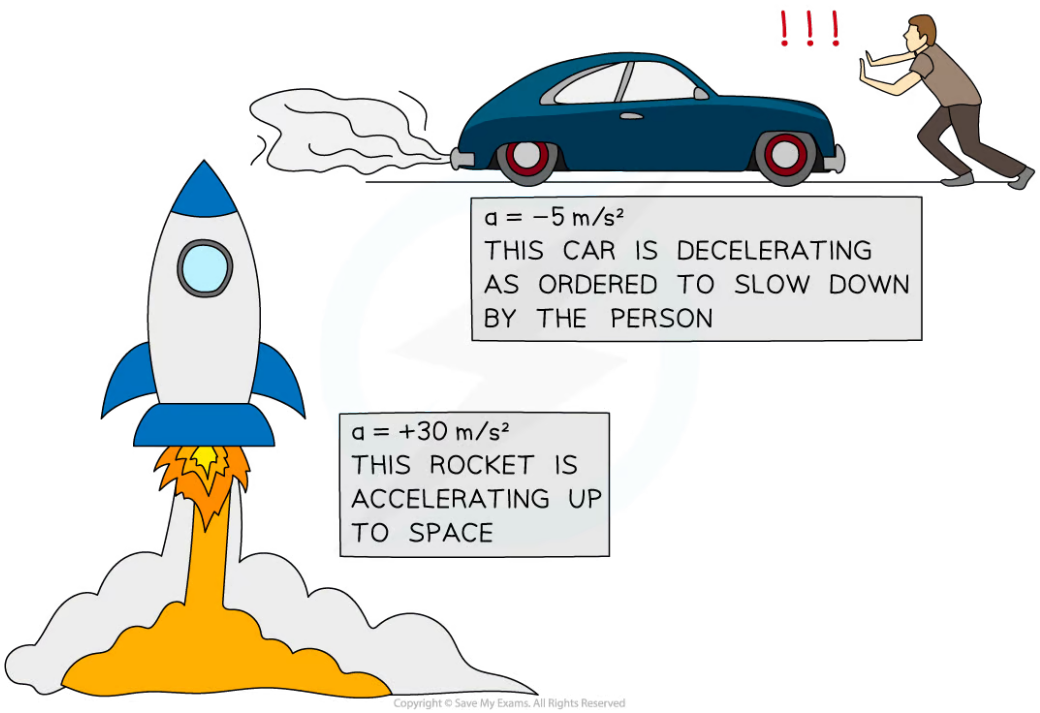
Instantaneous Acceleration

- The **instantaneous** acceleration is the velocity of an object **at any given point in time**
- This could be for an object with a constantly changing acceleration
  - An object accelerating is shown by a **curved line** on a velocity-time graph
- This is calculated in the same way as instantaneous velocity but on a velocity-time graph instead of a displacement-time graph

What is a negative acceleration called?

- The **acceleration** of an object can be **positive** or **negative**, depending on whether the object is speeding up or slowing down
  - If an object is **speeding up**, its acceleration is **positive**
  - If an object is **slowing down**, its acceleration is **negative** (deceleration)
- However, acceleration can also be negative if it is accelerating in the **negative direction**

YOUR NOTES



A rocket speeding up (accelerating) and a car slowing down (decelerating)

YOUR NOTES

Worked Example

A Japanese bullet train decelerates at a constant rate in a straight line.

The velocity of the train decreases from an initial velocity of 50 m s^{-1} to a final velocity of 42 m s^{-1} in 30 seconds.

(a)
Calculate the change in velocity of the train.
(b)
Calculate the deceleration of the train, and explain how your answer shows the train is slowing down.

Answer:

(a)

- The change in velocity is equal to

$$\Delta v = v - u$$

- Where:
  - Initial velocity, $u = 50$ m s^{-1}
  - Final velocity, $v = 42$ m s^{-1}

$$\Delta v = 42 - 50 = -8 \text{ m s}^{-1}$$

(b)

- Acceleration is equal to

$$a = \frac{\Delta v}{\Delta t}$$

- Where the time taken is $\Delta t = 30$ s

$$a = \frac{-8}{30} = -0.27 \text{ m s}^{-1}$$

- The answer is **negative**, which indicates the train is **slowing down**

Exam Tip

Remember the units for acceleration are **metres per second squared**, m s^{-2}. In other words, acceleration measures how much the velocity (in m s^{-1}) changes every second, (m s^{-1}) s^{-1}

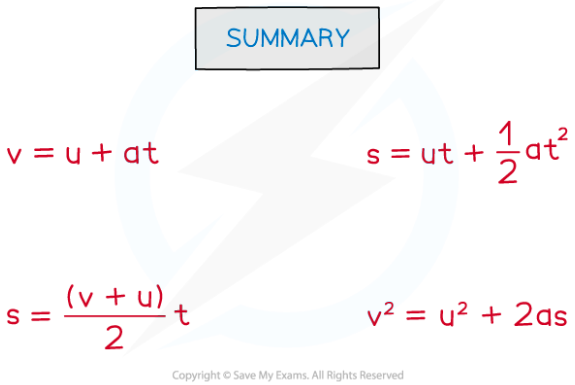
YOUR NOTES

Kinematic Equations

- The kinematic equations of motion are a set of four equations that can describe any object moving with **constant** or **uniform** acceleration
- They relate the five variables:
  - *s* = **displacement**
  - *u* = **initial velocity**
  - *v* = **final velocity**
  - *a* = **acceleration**
  - *t* = **time interval**

Kinematic Equations of Motion

- There are four kinematic equations:



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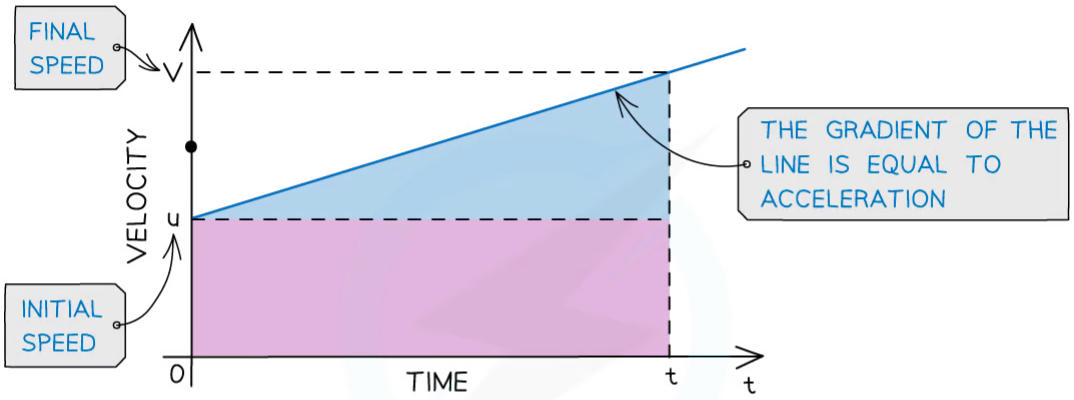
How to derive the kinematic equations

YOUR NOTES

Deriving $v = u + at$



THE VELOCITY–TIME GRAPH SHOWS A STRAIGHT LINE, THEREFORE, THE OBJECT'S ACCELERATION IS CONSTANT

FROM THE GRADIENT WE CAN DEDUCE ACCELERATION IS EQUAL TO

- $\frac{\Delta y}{\Delta x} = \frac{\Delta v}{\Delta t} = \frac{(v - u)}{t}$
- $a = \frac{(v - u)}{t}$

MULTIPLY BOTH SIDES BY t

- $at = (v - u)$

REARRANGING LEADS TO

- $v = u + at$

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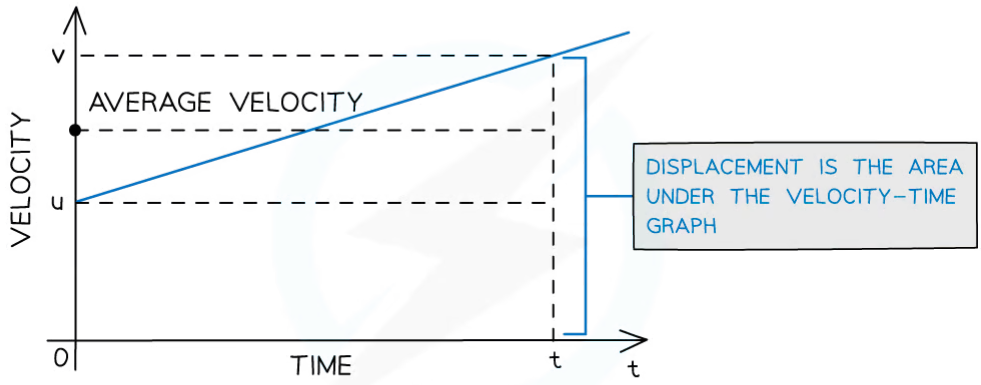
Derivation of v = u + at

YOUR NOTES

Deriving $s = \frac{(u + v)}{2} t$



THE OBJECT'S AVERAGE VELOCITY IS HALF-WAY BETWEEN u AND v:

$$\frac{(v + u)}{2}$$

DISPLACEMENT IS EQUAL TO AVERAGE VELOCITY × TIME SO:

$$s = \frac{(v + u)}{2} t$$

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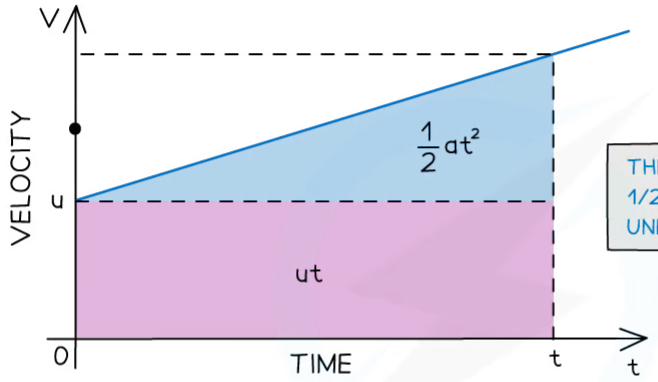
YOUR NOTES

Deriving $s = ut + \frac{1}{2}at^2$



THE TWO TERMS ut AND $1/2at^2$ MAKE UP THE AREA UNDER THE GRAPH

TAKING THE EQUATIONS WE DERIVED ABOVE

- $v = u + at$ (1)
- $s = \frac{(v + u)}{2}t$ (2)

SUBSTITUTING EQUATION (1) AND (2)

MULTIPLY EVERYTHING IN THE BRACKET BY t

- $s = \frac{(u + u + at)}{2}t$
- $s = \frac{2ut}{2} + \frac{at^2}{2}$

SEPARATE THE t AND t^2 TERMS

- $s = ut + \frac{1}{2}at^2$

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Deriving $v^2 = u^2 + 2as$

TAKING THE EQUATIONS WE DERIVED ABOVE

- $v = u + at \longrightarrow t = \frac{v - u}{a}$ (1)

$s = \frac{(v + u)}{2}t$ (2)

SUBSTITUTING (1) INTO (2)

- $s = \frac{(v + u)}{2} \times \frac{(v - u)}{a}$
- $s = \frac{v^2 - u^2}{2a}$

MULTIPLY BOTH SIDES BY 2a

- $v^2 = u^2 + 2as$

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This final equation can be derived from two of the others

YOUR NOTES

Key Takeaways

- The key terms to look out for are:

  'Starts from rest'

- This means $\boldsymbol{u = 0}$ and $\boldsymbol{t = 0}$
- This can also be assumed if the initial velocity is not mentioned

  'Falling due to gravity'

- This means $\boldsymbol{a = g = 9.81\ m\ s^{-2}}$
  - It doesn't matter which way is positive or negative for the scenario, as long as it is consistent for all the vector quantities
  - If downwards is considered positive, this is $9.81\ m\ s^{-2}$, otherwise, it is $-9.81\ m\ s^{-2}$
- For example, if downwards is negative then for a ball travelling upwards, ***s*** must be positive and ***a*** must be negative

  'Constant acceleration in a straight line'

- This is a key indication for the kinematic equations are intended to be used
  - For example, an object **falling** in a **uniform** gravitational field **without** air resistance

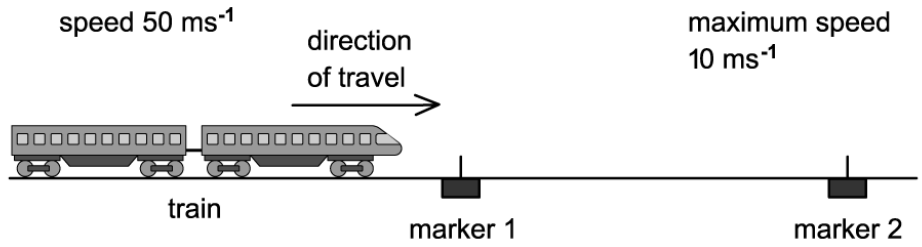
How to use the kinematic formulae

- **Step 1:** Write out the variables that are given in the question, both known and unknown, and use the context of the question to deduce any quantities that aren't explicitly given
  - e.g. for vertical motion $a = \pm 9.81\ m\ s^{-2}$, an object which starts or finishes at rest will have $u = 0$ or $v = 0$
- **Step 2:** Choose the equation which contains the quantities you have listed
  - e.g. the equation that links s, u, a and t is $s = ut + \frac{1}{2}at^2$
- **Step 3:** Convert any units to SI units and then insert the quantities into the equation and rearrange algebraically to determine the answer

YOUR NOTES

Worked Example

The diagram shows an arrangement to stop trains that are travelling too fast.



At marker 1, the driver must apply the brakes so that the train decelerates uniformly in order to pass marker 2 at no more than 10 m s^{-1}.

The train carries a detector that notes the times when the train passes each marker and will apply an emergency brake if the time between passing marker 1 and marker 2 is less than 20 s.

Trains coming from the left travel at a speed of 50 m s^{-1}.

Determine how far marker 1 should be placed from marker 2.

Answer:

STEP 1

OUR KNOWN VARIABLES ARE

- $u = 50\ ms^{-1}$
- $v = 10\ ms^{-1}$
- $t = 20\ s$

AND WE ARE ASKED TO FIND DISTANCE, s.

STEP 2

SO THE EQUATION THAT LINKS u, v, t AND s IS

$$s = \frac{(u + v)}{2} t$$

STEP 3

NO REARRANGING IS REQUIRED SO WE SIMPLY PLUG IN THE VARIABLES:

$$s = \frac{(50 + 10)}{2} \times 20 = 30 \times 20 = 600\ m$$

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YOUR NOTES

Worked Example

A cyclist is travelling directly east through a village, which is completely flat, at a velocity of 6 m s^{-1}. They then start to constantly accelerate at 2 m s^{-2} for 4 seconds.

(a)
Calculate the distance that the cyclist covers in the 4 s acceleration period.
(b)
Calculate the cyclist's final velocity after the 4 s interval of acceleration.

Later on in their journey, cyclist **A** is now cycling through a different village, at a constant velocity of 18 m s^{-1}. Cyclist **A** passes a friend, Cyclist **B** who begins accelerating from rest at a constant acceleration of 1.5 m s^{-2} in the same direction as Cyclist **A** at the moment they pass.

(c)
Calculate how long it takes for Cyclist **B** to catch up to Cyclist **A**.

Answer:

(a) Calculate the displacement, *s*

Step 1: List the known quantities

- Initial velocity, $u = 6$ m s^{-1}
- Acceleration, $a = 2$ m s^{-2}
- Time, $t = 4$ s
- Displacement = *s* (this needs to be calculated)

Step 2: Identify the best SUVAT equation to use

- Since the question states **constant acceleration**, the kinematic equations can be used
- In this problem, the equation that links *s*, *u*, *a*, and *t* is

$$s = ut + \frac{1}{2}at^2$$

Step 3: Substitute the known quantities into the equation

$$s = (6 \times 4) + (0.5 \times 2 \times 4^2) = 24 + 16$$

Displacement: **s = 40 m**

(b) Calculate the final velocity, *v*

Step 1: List the known quantities

- Initial velocity, $u = 6$ m s^{-1}
- Acceleration, $a = 2$ m s^{-2}
- Time, $t = 4$ s

YOUR NOTES

- Final velocity = v (this needs to be calculated)

Step 2: Identify and write down the equation to use

- Since the question states constant acceleration - SUVAT equation(s) - can be used
- In this problem, the equation that links v, u, a, and t is:

$$v = u + at$$

Step 3: Substitute the known quantities into the equation

$$v = 6 + (2 \times 4)$$

Final velocity: $\boldsymbol{v = 14\ m\ s^{-1}}$

(c) Calculate the time t for **B** to catch up to **A**

Step 1: List the known quantities for cyclist A

- Initial velocity, $u_A = 18\ m\ s^{-1}$
- Acceleration, $a_A = 0\ m\ s^{-2}$
- Time = t
- Displacement = s_A

Step 2: List the known quantities for cyclist B

- Initial velocity, $u = 0\ m\ s^{-1}$
- Acceleration, $a = 1.5\ m\ s^{-2}$
- Time = t
- Displacement = s_B

Step 3: Write expressions for Cyclist A and Cyclist B in terms of their displacement

- Cyclist **A**'s motion can be expressed by:

$$s_A = u_A t + \frac{1}{2} a_A t^2$$

$$s_A = 18t + 0 = 18t$$

- Cyclist **B**'s motion can be expressed by:

$$s_B = u_B t + \frac{1}{2} a_B t^2$$

$$s_B = 0 + \left(\frac{1}{2} \times 1.5 \times t^2\right) = \frac{3}{4} t^2$$

Step 4: Equate the two equations and solve for t

- The two equations describe the displacement of each cyclist respectively
- When equating them, this will find the time when the cyclists are at the same location

$$s_A = s_B$$

$$18t = \frac{3}{4}t^2 \quad \Rightarrow \quad \frac{3}{4}t^2 - 18t = 0$$

$$(t^2 - 24t) = 0$$

- Therefore, solving for *t*, it can be two possible answers:

$t = 0$ s **or** $t = 24$ s

- Since the question is seeking the time when the two cyclists meet after first passing each other, the final answer is **24 s**

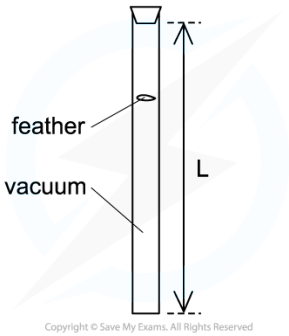
YOUR NOTES

YOUR NOTES

Worked Example

A science museum designed an experiment to show the fall of a feather in a vertical glass vacuum tube.

The time of fall from rest is 0.5 s.



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Use an appropriate SUVAT equation to calculate the length *L* of the tube.

Answer:

IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION. FIRST WE MUST LIST THE KNOWN VARIABLES.

$a = 9.81\ ms^{-2}$ $u = 0$ $t = 0.5\ s$ $L = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$L = \frac{1}{2}gt^2$$

$$L = \frac{1}{2} \times 9.81 \times 0.5^2 = 1.2\ m$$

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Exam Tip

This is one of the most important sections of this topic - usually, there will be one, or more, questions in the exam about solving problems with the kinematic equations equations. The best way to master this section is to practice as many questions as possible!

Watch out for the direction of vectors: displacement, acceleration and velocity. Take a single direction as positive (and hence the opposite direction is negative) and **stick with it** throughout the question, this is the most common pitfall.

Don't worry, you won't have to memorise these, they are give in your data booklet in the exam.

You may sometimes see these equations referred to as 'SUVAT' equations.

YOUR NOTES

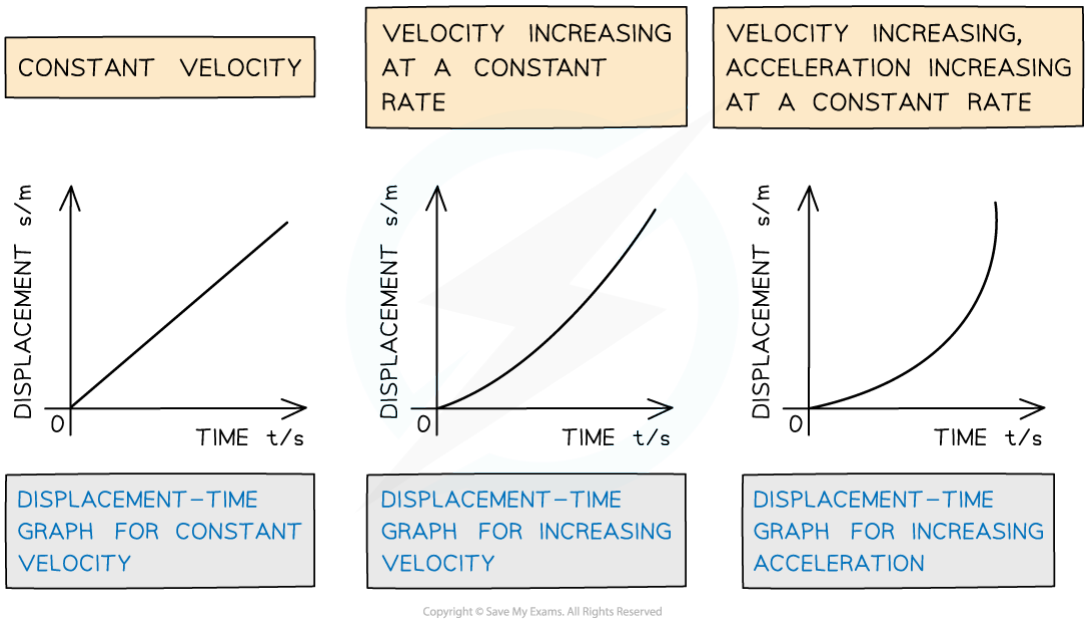
YOUR NOTES

Motion Graphs

- The motion of objects can be analysed in terms of position, velocity and acceleration
  - These are all related to each other by **gradients** and **areas** under curves
- Three types of graphs that can represent the motion of an object are:
  - **Displacement-time** graphs
  - **Velocity-time** graphs
  - **Acceleration-time** graphs

Displacement-Time Graphs

- On a **displacement-time** graph:
  - **Slope** equals **velocity**
  - The **y-intercept** equals the **initial displacement**
  - A **straight** (diagonal) line represents a **constant** velocity
  - A **curved** line represents an **acceleration**
  - A **positive slope** represents motion in the **positive direction**
  - A **negative slope** represents motion in the **negative direction**
  - A **zero** slope (horizontal line) represents a state of **rest**
  - The area under the curve is meaningless



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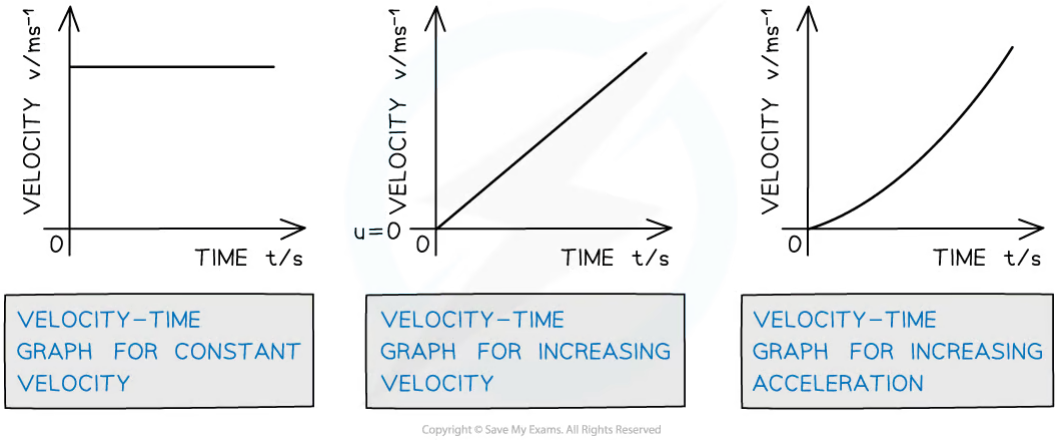
Displacement-time graphs displacing different values of velocity

Velocity-Time Graphs

- On a **velocity-time** graph:
  - **Slope** equals **acceleration**
  - The **y-intercept** equals the **initial velocity**

YOUR NOTES

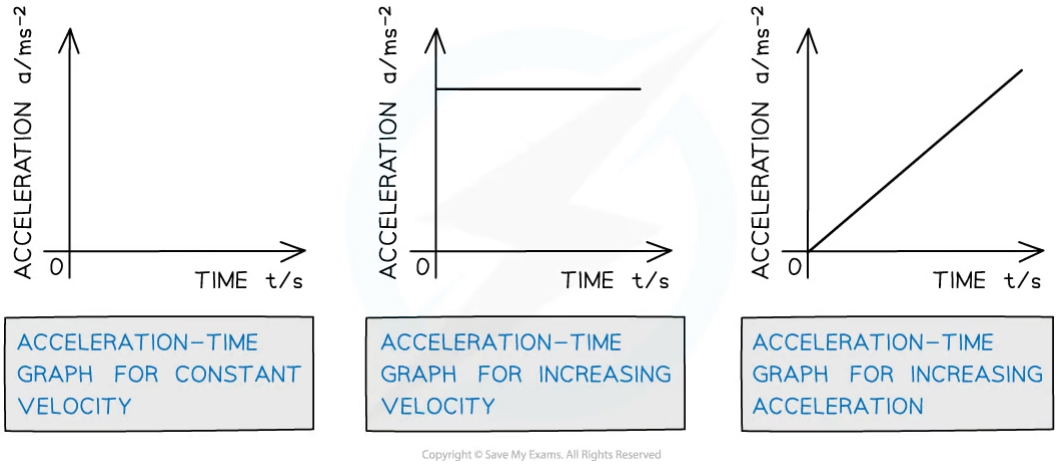
- A **straight** (diagonal) line represents **uniform acceleration**
- A **curved** line represents **non-uniform acceleration**
- A **positive** slope represents **acceleration** in the **positive direction**
- A **negative** slope represents **acceleration** in the **negative direction**
- A **zero** slope (horizontal line) represents motion with **constant velocity**
- The **area** under the curve equals the **change** in **displacement**



Velocity-time graphs displacing different values of acceleration

Acceleration-Time Graphs

- On an acceleration-time graph:
  - Slope is meaningless
  - The **y-intercept** equals the **initial acceleration**
  - A **zero slope** (horizontal line) represents an object undergoing **constant acceleration**
  - The **area** under the curve equals the **change** in **velocity**



How displacement, velocity and acceleration graphs relate to each other

- Acceleration can either be
  - **Uniform** i.e. a constant value. For example, acceleration due to gravity on Earth

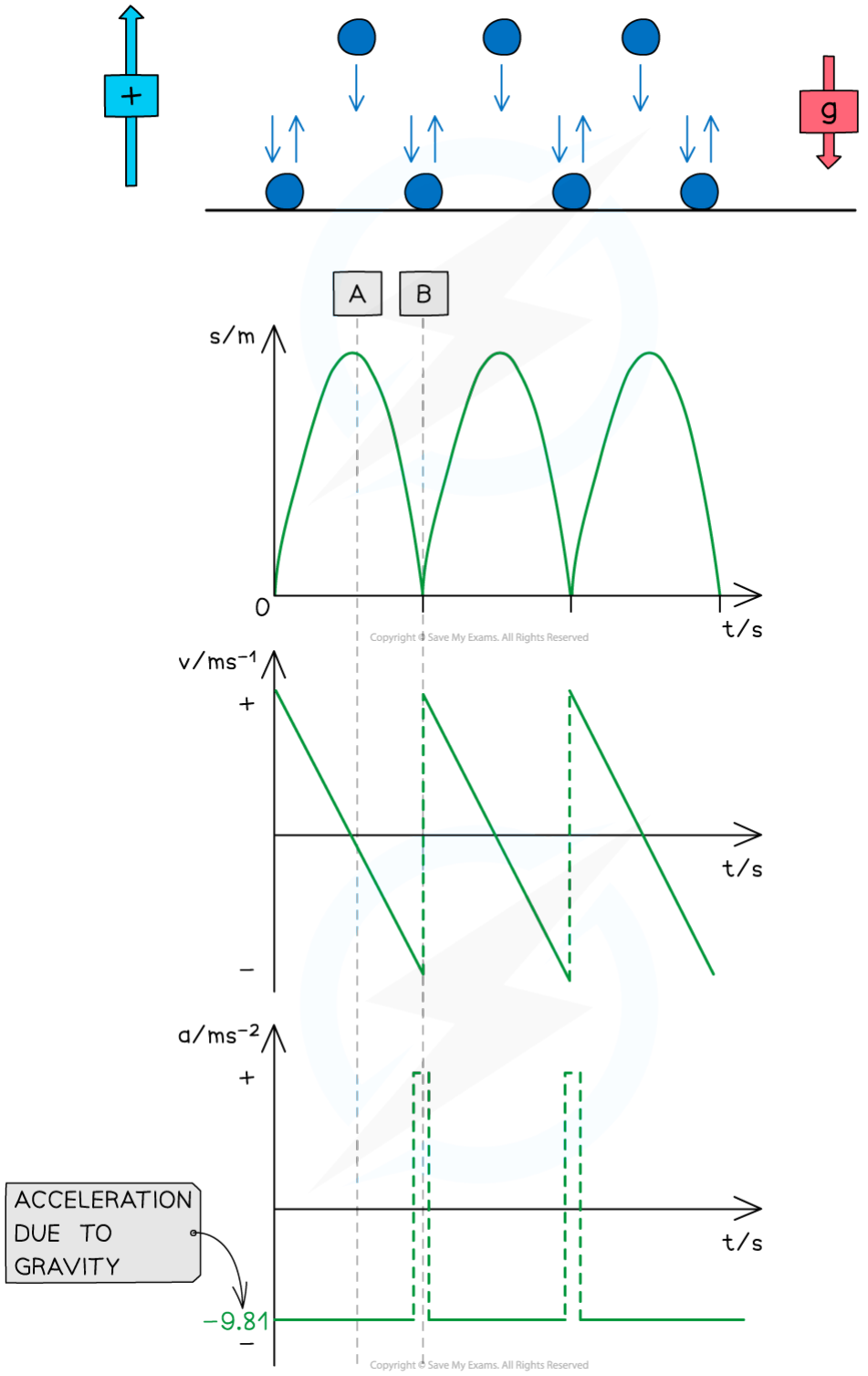
- **Non-uniform** i.e. a changing value. For example, an object with increasing acceleration

YOUR NOTES

Motion of a Bouncing Ball

- For a bouncing ball, the acceleration due to gravity is **always** in the same direction (in a uniform gravitational field such as the Earth's surface)
  - This is assuming there are **no** other forces on the ball, such as **air resistance**
- Since the ball changes its direction when it reaches its highest and lowest point, the direction of the velocity will change at these points
- The vector nature of velocity means the ball will sometimes have a:
  - **Positive velocity** if it is travelling in the positive direction
  - **Negative velocity** if it is traveling in the negative direction
- An example could be a ball bouncing from the ground back upwards and back down again
  - The positive direction is taken as upwards
  - This will be either stated in the question or can be chosen, as long as the direction is consistent throughout
- Ignoring the effect of air resistance, the ball will reach the same height every time before bouncing from the ground again
- When the ball is traveling upwards, it has a positive velocity which slowly decreases (decelerates) until it reaches its highest point

YOUR NOTES



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- At point **A** (the highest point):
  - The ball is at its **maximum displacement**
  - The ball momentarily has **zero velocity**
  - The **velocity** changes from **positive** to **negative** as the ball changes direction

- The **acceleration**, *g*, is still **constant** and directed vertically downwards

- At point **B** (the lowest point):
  - The ball is at its **minimum displacement** (on the ground)
  - Its **velocity** changes instantaneously from **negative** to **positive**, but its **speed** (magnitude) **remains the same**
  - The **change** in direction causes a **momentary acceleration** (since acceleration = change in velocity / time)

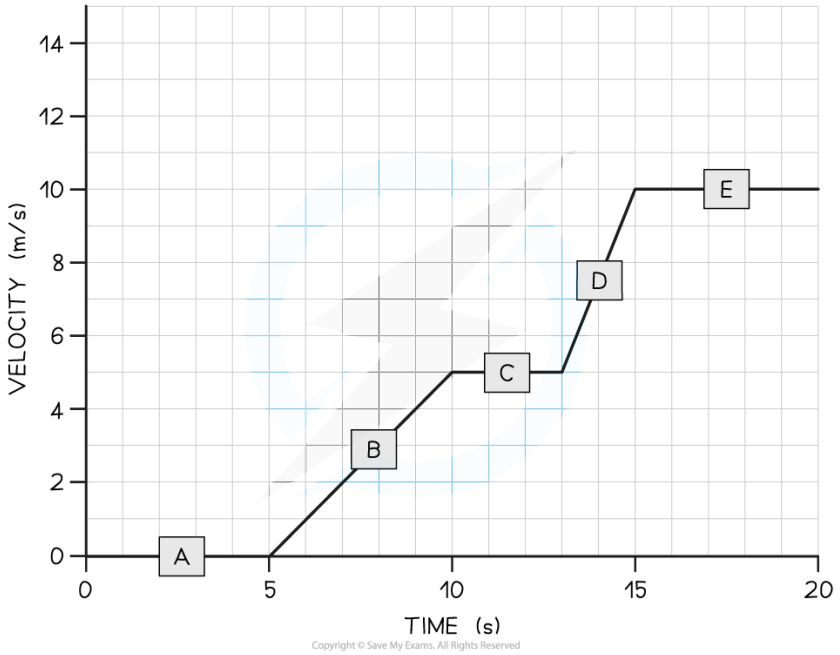
YOUR NOTES

YOUR NOTES

Worked Example

Tora is training for a cycling tournament.

The velocity-time graph below shows her motion as she cycles along a flat, straight road.



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(a)
In which section (A, B, C, D, or E) of the velocity-time graph is Tora's acceleration the largest?
(b)
Calculate Tora's acceleration between 5 and 10 seconds.

Answer:

(a)

Step 1: Recall that the slope of a velocity-time graph represents the magnitude of acceleration

- The slope of a velocity-time graph indicates the magnitude of acceleration Therefore, the only sections of the graph where Tora is accelerating is section B and section D
- Sections A, C, and E are flat – in other words, Tora is moving at a constant velocity (i.e. not accelerating)

Step 2: Identify the section with the steepest slope

- Section D of the graph has the steepest slope
- Hence, the largest acceleration is shown in **section D**

YOUR NOTES

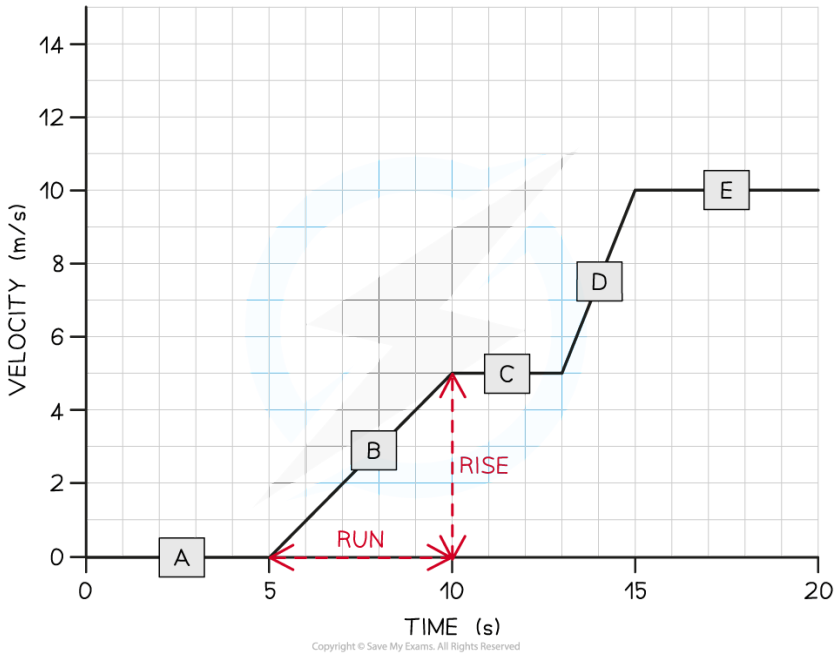
(b)

Step 1: Recall that the gradient of a velocity-time graph gives the acceleration

- Calculating the gradient of a slope on a velocity-time graph gives the acceleration for that time period

Step 2: Draw a large gradient triangle at the appropriate section of the graph

- A gradient triangle is drawn for the time period between 5 and 10 seconds below:



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Step 3: Calculate the size of the gradient and state this as the acceleration

- The acceleration is given by the gradient, which can be calculated using:

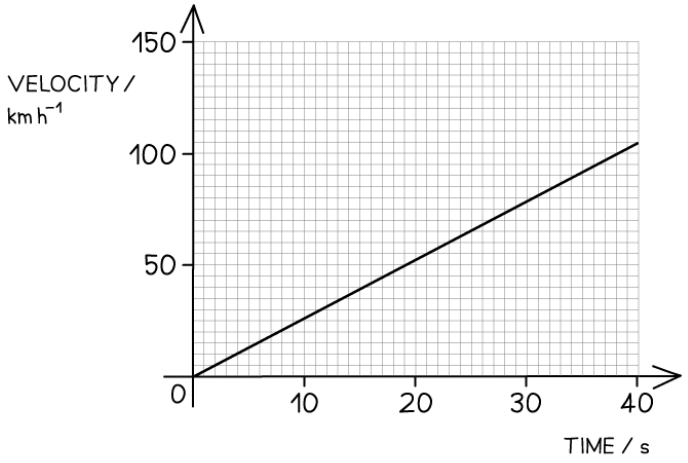
$$acceleration = gradient = \frac{5}{5} = 1 \text{ m s}^{-2}$$

- Therefore, Tora accelerated at 1 m s^{-2} between 5 and 10 seconds

YOUR NOTES

Worked Example

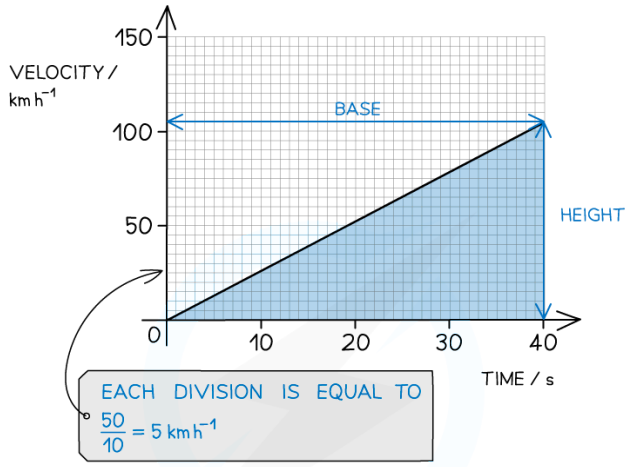
The velocity-time graph of a vehicle travelling with uniform acceleration is shown in the diagram below.



Calculate the displacement of the vehicle at 40 s.

Answer:

THE DISPLACEMENT IS EQUAL TO THE AREA UNDER A VELOCITY–TIME GRAPH



EACH DIVISION IS EQUAL TO $\frac{50}{10} = 5\ km\ h^{-1}$

CONVERT $km\ h^{-1}$ TO $km\ s^{-1}$

BASE = TIME = 40s

HEIGHT = VELOCITY = $105\ km\ h^{-1}$

$\frac{105}{60 \times 60} = 0.0292\ km\ s^{-1}$

AREA OF A TRIANGLE = $\frac{1}{2}$ × BASE × HEIGHT

WORK OUT THE DISPLACEMENT

DISPLACEMENT = VELOCITY × TIME = $\frac{1}{2} \times 40 \times 0.0292 = 0.6\ km$ OR $600\ m$

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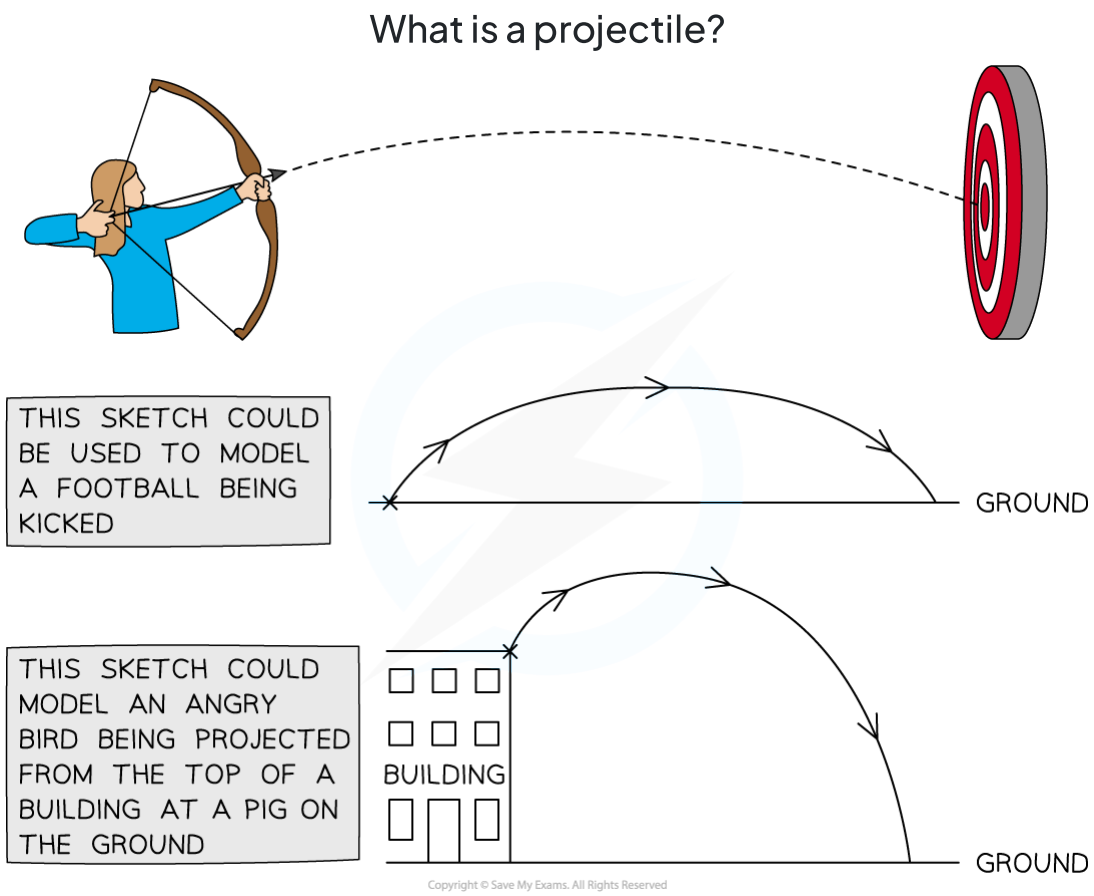
YOUR NOTES

Projectile Motion

- A **projectile** is a particle moving freely, under gravity, in a two-dimensional plane
- Examples of projectile motion include:
  - Sports such as basketball, javelin, archery, etc
  - Computer games such as Angry Birds



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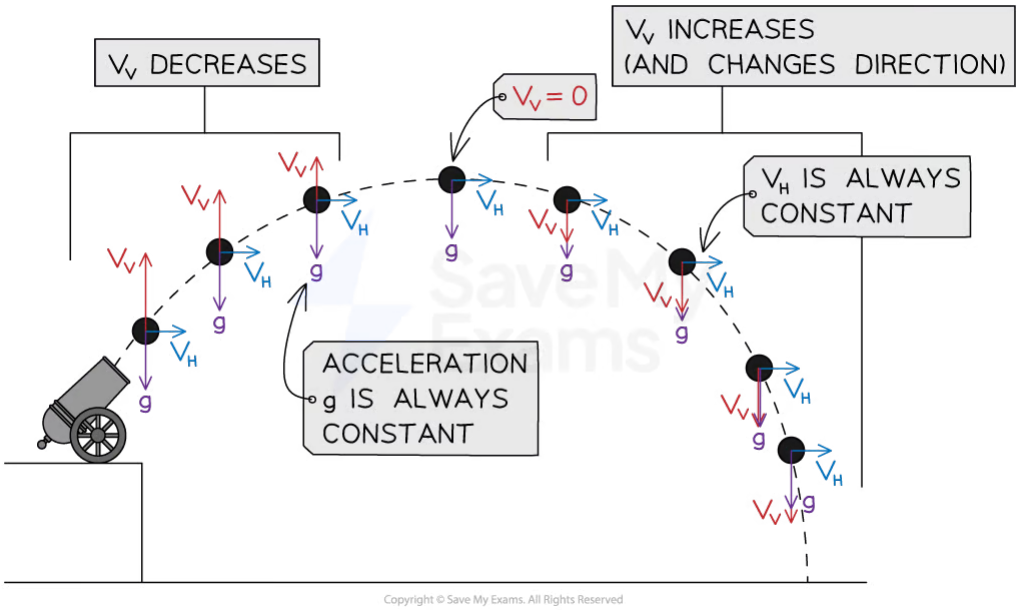
Projectile Motion

- The trajectory of an object undergoing projectile motion consists of a **vertical** component and a **horizontal** component
  - These need to be evaluated **separately**
- The key parts of the motion are:
  - The acceleration due to gravity is g and is **constant** and **downwards** throughout the motion
  - The horizontal velocity, v_H is **constant** and in the direction of the object throughout the motion
  - The vertical velocity, v_V **changes** in magnitude **and** direction throughout the motion e.g. at the top, $v_V = 0$

YOUR NOTES

Velocity & acceleration of a projectile



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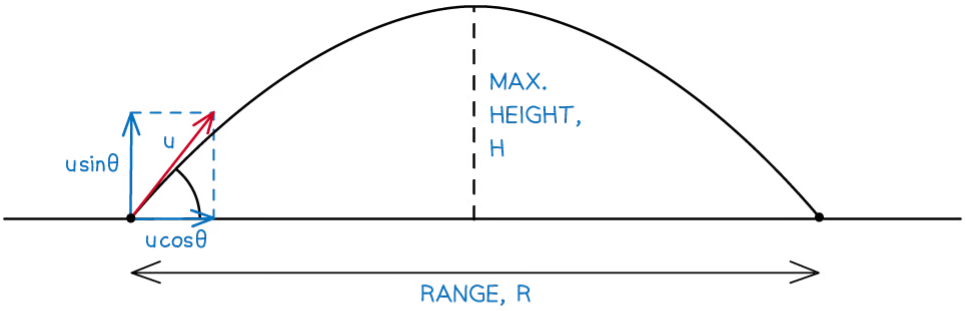
Acceleration and horizontal velocity are always constant whilst vertical velocity changes

- Some key terms to know and calculate are:
  - **Time of flight:** how long the projectile is in the air
  - **Maximum height attained:** the height at which the projectile is momentarily at rest
  - **Range:** the horizontal distance travelled by the projectile
- Some key terms to know, and how to calculate them, are:

YOUR NOTES



VERTICAL MOTION (↑)

INITIAL SPEED, $u = u\sin\theta$

ACCELERATION, $a = 9.81\ ms^{-2}$

DISPLACEMENT = 0

TIME OF FLIGHT

$u = u\sin\theta \quad v = 0 \quad a = -g \quad t = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v = u + at$

$0 = u\sin\theta - gt$

$t = \frac{u\sin\theta}{g}$

$2t = \frac{2u\sin\theta}{g}$

IF THE TIME TO MAXIMUM HEIGHT IS t, THEN THE TIME OF FLIGHT IS 2t

MAXIMUM HEIGHT ATTAINED

$u = u\sin\theta \quad v = 0 \quad a = -g \quad H = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

$v^2 = u^2 + 2as$

$0 = (u\sin\theta)^2 - 2gH$

$2gH = (u\sin\theta)^2$

$H = \frac{(u\sin\theta)^2}{2g}$

HORIZONTAL MOTION (→)

INITIAL SPEED, $u = u\cos\theta$

ACCELERATION, $a = 0$

DISPLACEMENT = R

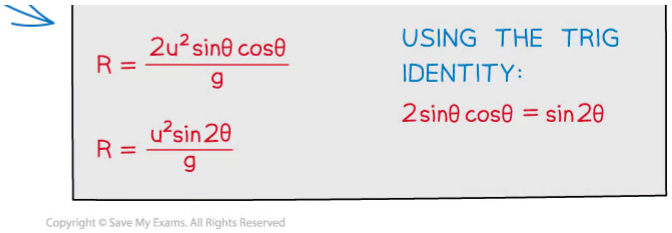
RANGE (R)

$u = u\cos\theta \quad t = \frac{2u\sin\theta}{g} \quad a = 0 \quad R = ?$

THE EQUATION THAT RELATES THESE QUANTITIES IS

DISTANCE = SPEED × TIME

$R = (u\cos\theta)t$



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How to find the time of flight, maximum height and range

Problems involving projectile motion

- There are two main considerations for solving problems involving two-dimensional motion of a projectile are:
  - **Constant velocity** in the **horizontal** direction
  - **Constant acceleration** in the **vertical** direction towards the Earth (due to gravity)
- The only force acting on the projectile, after it has been released, is the gravitational pull of the Earth on the object, or its **weight**
- Projectiles are often launched from an **angle** but are not necessarily always from the ground

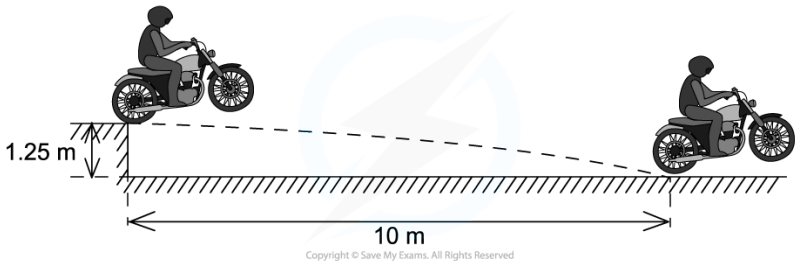
YOUR NOTES

YOUR NOTES

Worked Example

A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown.

What was the speed at take-off? (ignoring air resistance)



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IN THIS PROBLEM, WE NEED TO CONSIDER BOTH VERTICAL AND HORIZONTAL MOTION. LET'S CONSIDER THE VERTICAL MOTION FIRST. THE KNOWN VARIABLES ARE

$s = 1.25\ m$ $a = 9.81\ ms^{-2}$ $u = 0$ $t = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2s}{g}}$$

$$t = \sqrt{\frac{2 \times 1.25}{9.81}} = 0.5s$$

NEXT LET'S CONSIDER THE HORIZONTAL MOTION. THE KNOWN VARIABLES ARE

$s = 10\ m$ $a = 0$ $t = 0.5\ s$ $u = ?$

SINCE THE ACCELERATION IS ZERO, WE CAN USE

$$\text{VELOCITY} = \frac{\text{DISPLACEMENT}}{\text{TIME}}$$

$$v = \frac{10}{0.5} = 20\ ms^{-1}$$

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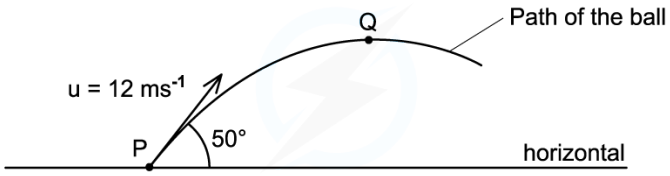
YOUR NOTES

Worked Example

A ball is thrown from a point P with an initial velocity u of 12 m s^{-1} at 50° to the horizontal.

What is the value of the maximum height at Q? (ignoring air resistance)



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IN THIS PROBLEM, WE ONLY NEED TO CONSIDER VERTICAL MOTION UP TO THE POINT Q. FIRST WE MUST LIST THE KNOWN VARIABLES

$u = 12\sin(50)$ $a = -9.81\ ms^{-2}$ $v = 0$ $H = ?$

THE EQUATION THAT LINKS THESE VARIABLES IS

$$v^2 = u^2 + 2as$$

$$2as = v^2 - u^2$$

$$s = \frac{(v^2 - u^2)}{2a}$$

$$H = \frac{0 - (12\sin 50)^2}{2 \times (-9.81)}$$

$$H = \frac{(12\sin 50)^2}{19.62} = 4.3\ m$$

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Exam Tip

Make sure you don't make these common mistakes:

- Mixing up positive and negative values for vectors
- Mixing up velocities and distances between horizontal and vertical motion
- Confusing the direction of sin θ and cos θ
- Not converting units (mm, cm, km etc.) to metres

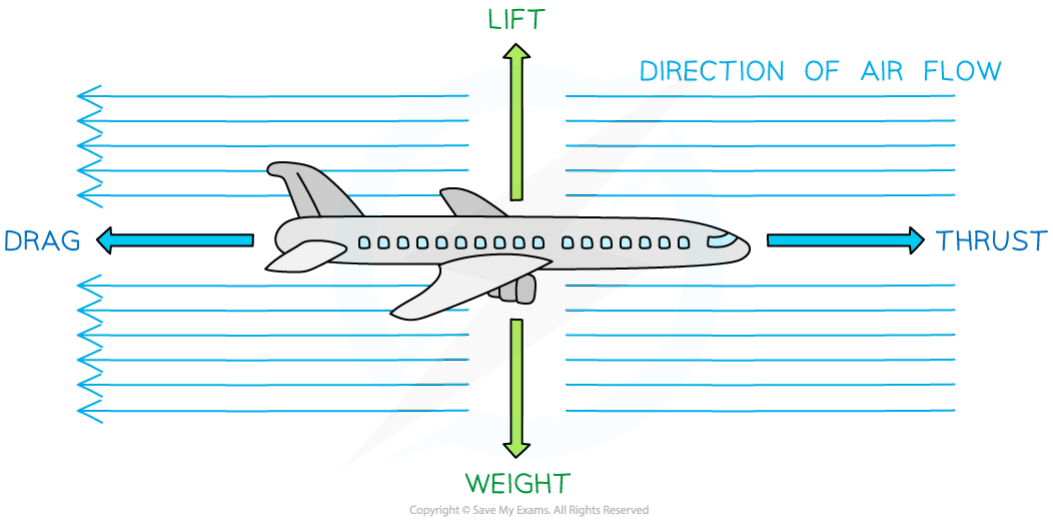
Further, it is worth noting that projectile motion is typically **symmetrical** when air resistance is **ignored** allowing for use of the peak to find the time of total flight or total horizontal distance by doubling the amount to get from the start point to the peak.

In these exam questions, unless specified, fluid resistance can be **ignored**

YOUR NOTES

Fluid Resistance

- Fluid resistance refers to the effects of gases and liquids on the motion of a body
- When an object moves through a fluid (a gas or a liquid), there are **resistive** forces for that movement
  - These forces are known as **drag** forces
- Examples of drag forces are **friction** and **air resistance**
- **Drag** forces:
  - Are always in the **opposite** direction to the motion of the object
  - Never speed an object up or start them moving
  - Slow down an object or keep them moving at a constant speed
  - Transfer energy from the kinetic store of the object to the thermal stores of the objects and the surroundings
- **Lift** is an **upwards** force on an object moving through a fluid. It is **perpendicular** to the fluid flow
  - For example, as an aeroplane moves through the air, the aeroplane pushes down on the air to change its direction
  - This causes an equal and opposite reaction as the air pushes upwards on the wings of the aeroplane (lift) due to Newton's Third Law



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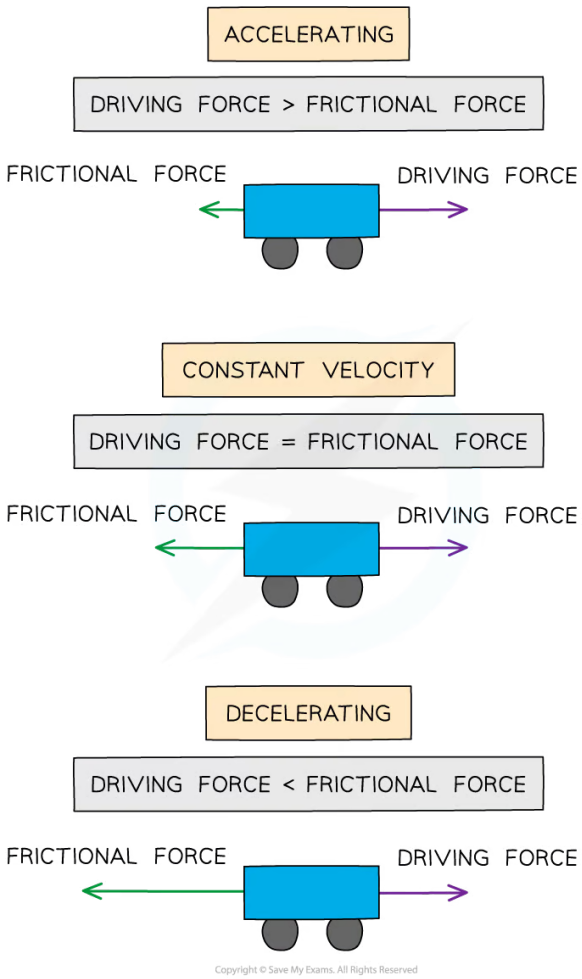
Drag forces are always in the opposite direction to the thrust (direction of motion). Lift is always in the opposite direction to the weight

- A key component of drag forces is that they increase with the **speed** of the object
- This is shown in the diagram below:

YOUR NOTES



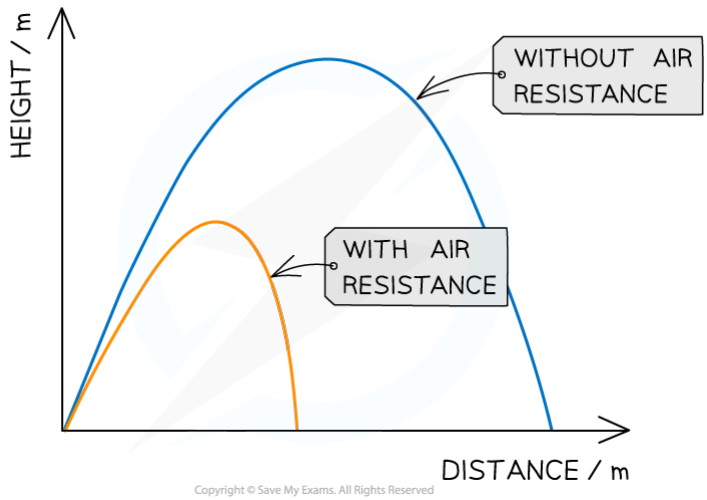
Frictional forces on a car increase with speed

Fluid Resistance in Projectile Motion

- In projectile motion, the factors that are affected by fluid resistance are:
  - Time of flight
  - Horizontal velocity
  - Horizontal acceleration
  - Range
  - Shape of trajectory

- **Air resistance** is the drag force which has the most significant effect on a projectile
- Air resistance decreases the **horizontal** component of the velocity of a projectile
  - This means both its **range** and **maximum height** will decrease compared to an identical situation with no air resistance (like a vacuum)

YOUR NOTES



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A projectile with air resistance travels a smaller distance and has a lower maximum height than one without air resistance

- When air resistance is applied, the path of the projectile no longer follows a parabola shape
  - Its path is now **steeper** on the way down than it is up
- The flight time will also **decrease** as the projectile is in the air for a **shorter** period of time
  - This is due to having a smaller range and lower maximum height
- In summary:

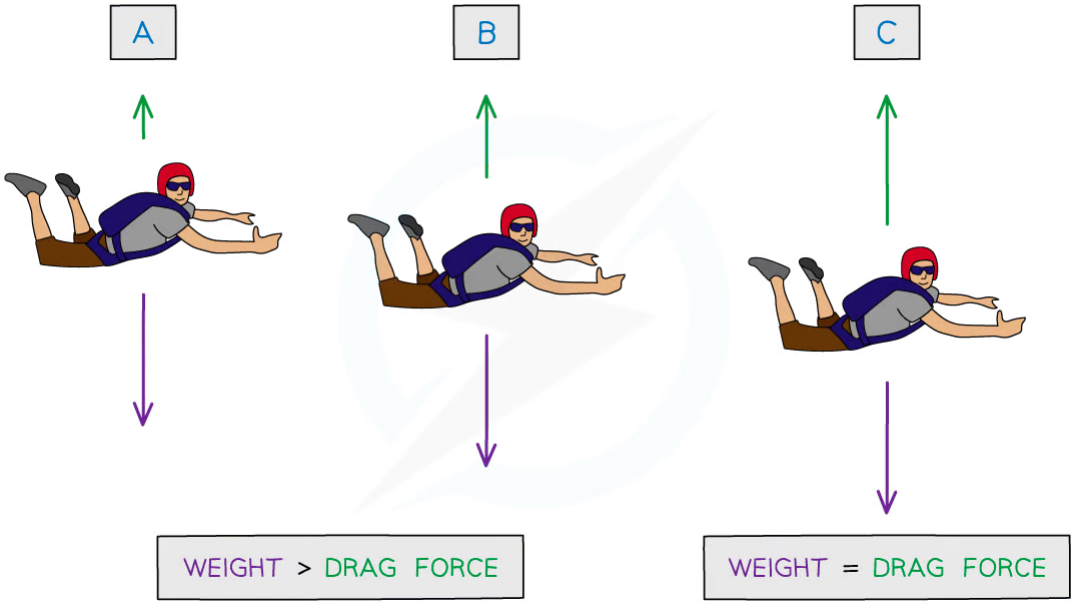
Air resistance affects	Effect of air resistance
time of flight	decreases
horizontal velocity	decreases
horizontal deceleration	increases
range	decreases
shape of trajectory	no longer a parabola

- The angle and launch speed of a projectile can be varied to **cover a longer range** or reach a **greater maximum height**, depending on the situation
  - For sports, such as the long jump or javelin, an optimum angle against air resistance is used to produce the greatest range (distance)
  - For gymnastics or ski jumper, the initial vertical velocity is made as large as possible to reach a greater maximum height and longer flight path

YOUR NOTES

Terminal Speed

- For a body in free fall, the only force acting is weight, and its acceleration *g* is only due to gravity
- The drag force from fluid resistance **increases** as the body **accelerates**
  - This increase in velocity means the drag force also **increases**
- Due to Newton's Second Law, this means the resultant force and therefore acceleration decreases (recall ***F* = *ma***)
- When the drag force is equal to the gravitational pull on the body, the body will no longer accelerate and will fall at a constant velocity
  - This velocity is called the **terminal velocity**
- Terminal velocity can occur for objects falling through a gas or a liquid



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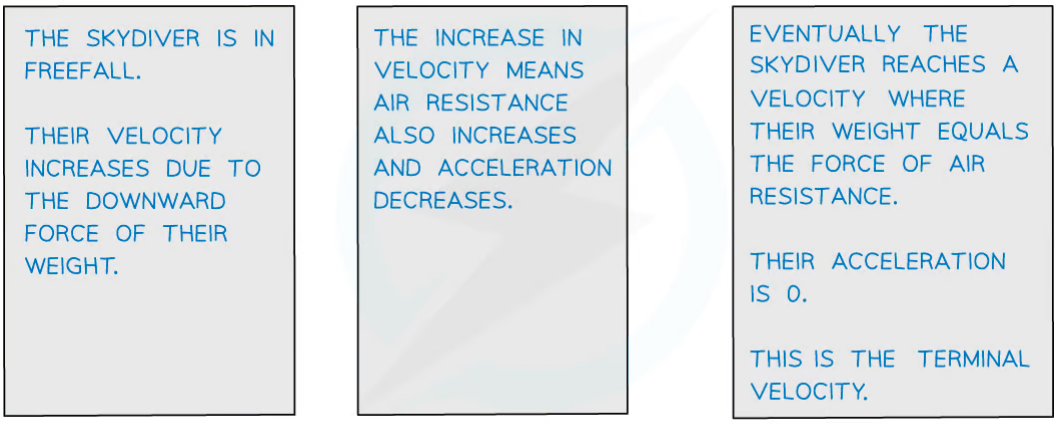

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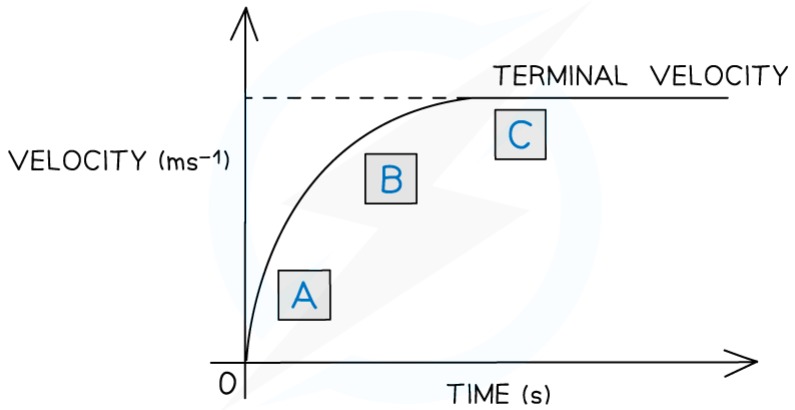

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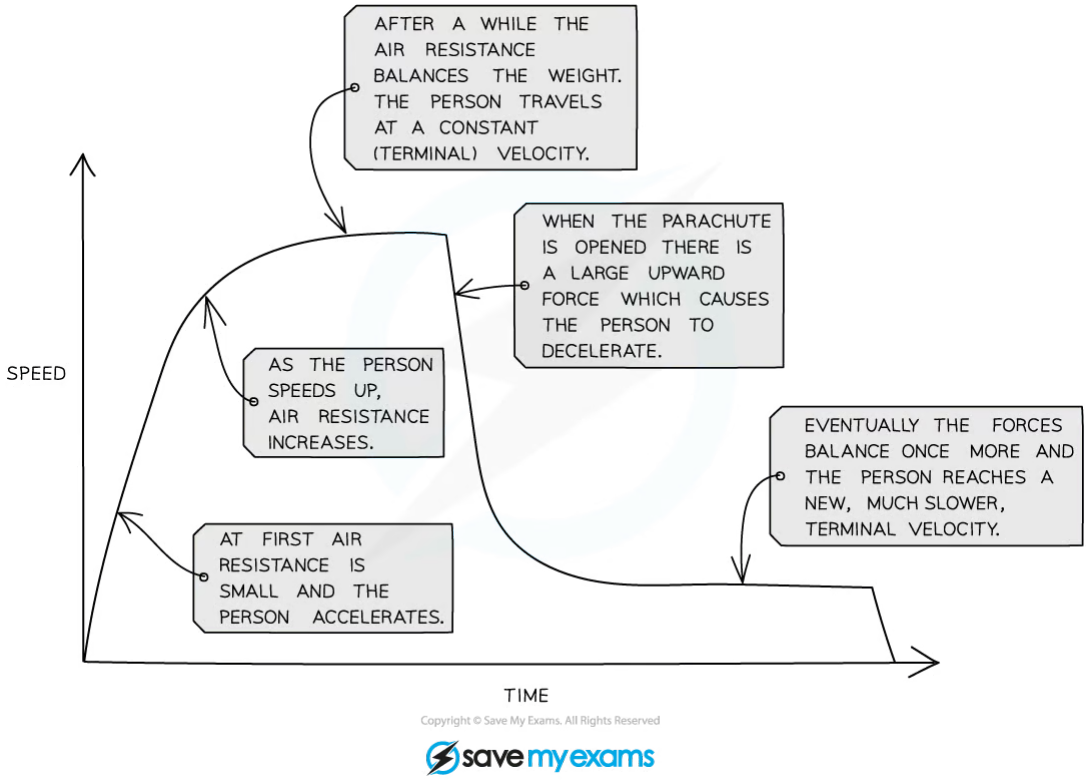
A skydiver in freefall reaching terminal velocity

- The graph shows how the velocity of the skydiver varies with time
- Since acceleration is equal to the **gradient** of a velocity-time graph, it can be seen that
  - The acceleration **decreases**
  - The acceleration eventually becomes **zero** when terminal velocity is reached
- After the skydiver deploys their parachute, they decelerate to a **lower terminal velocity** to reduce the impact on landing
- This is demonstrated by the graph below:

YOUR NOTES



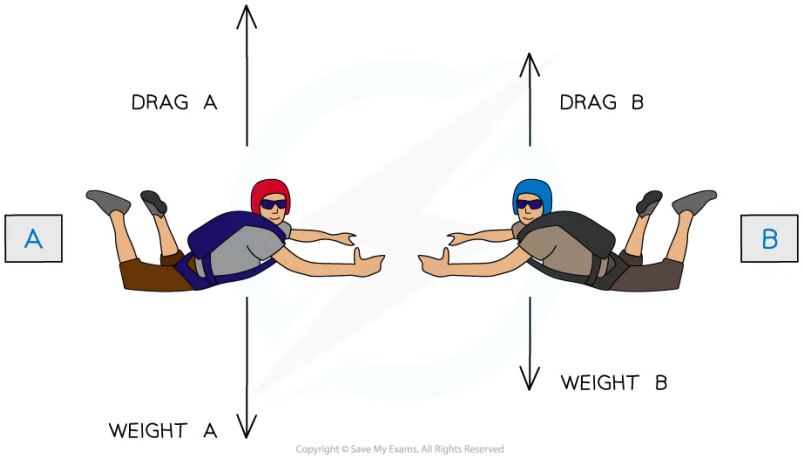
A graph showing the changes in speed of the skydiver throughout their entire journey in freefall

YOUR NOTES

Worked Example

Skydivers jump out of a plane at intervals of a few seconds.

Skydivers **A** and **B** want to join up as they fall.



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Skydiver **A** is heavier than Skydiver **B**. If the two skydivers want to reach terminal velocity together, who should jump first?

Answer:

- Skydiver **B** should jump first since he will take longer to reach terminal velocity
- This is because skydiver **A** has a higher mass, and hence, weight
- More weight means a greater speed, therefore, **A** will reach terminal velocity faster than **B**

Exam Tip

A common misconception is that skydivers move upwards when their parachutes are deployed - however, this is not the case, they are in fact **decelerating** to a lower terminal velocity.

If a question considers air resistance to be '**negligible**' this means in that question, air resistance is taken to be so small it will not make a difference to the motion of the body. You can take this to mean there are no drag forces acting on the body.

IB Physics HL

YOUR NOTES

Forces & Momentum

CONTENTS

Free-Body Diagrams
Newton's First Law
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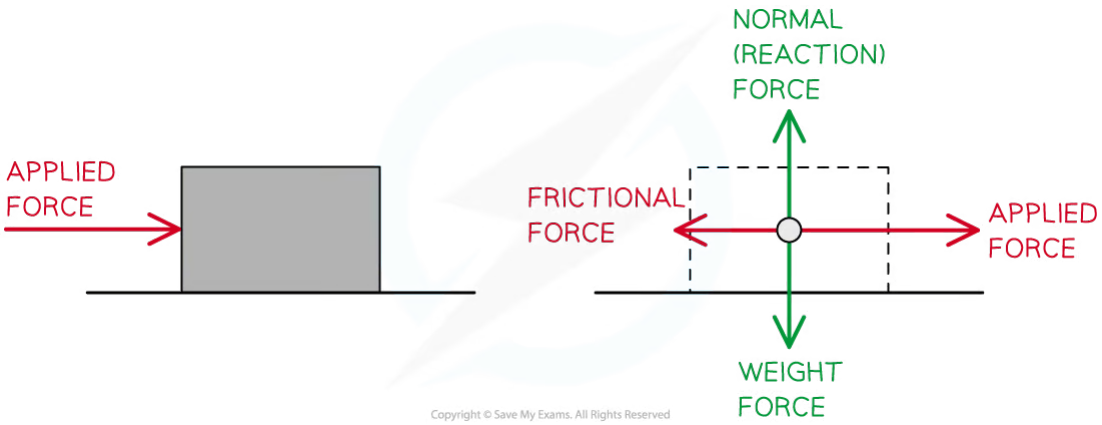
Angular Velocity
Centripetal Force
Centripetal Acceleration
Non-Uniform Circular Motion

YOUR NOTES

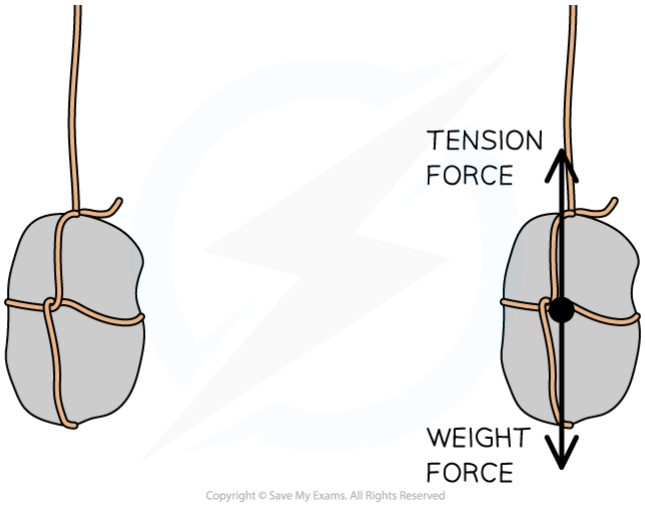
Free-Body Diagrams

- Forces are created from the interaction between bodies
  - For example, a push or pull
- In physics, during force interactions, it is common to represent situations as simply as possible without losing information
  - When considering force interactions, objects are represented as **point** particles
  - These point particles should be placed at the **centre of mass** of the object
- Forces are represented by **arrows** because forces are vectors
  - The length of the arrow gives the **magnitude** of the force, and its **direction** gives the force's direction
- The below example shows the forces acting on an object when pushed to the right over a rough surface



Point particle representation of the forces acting on a moving object

- The below example shows the forces acting on an object suspended from a stationary rope

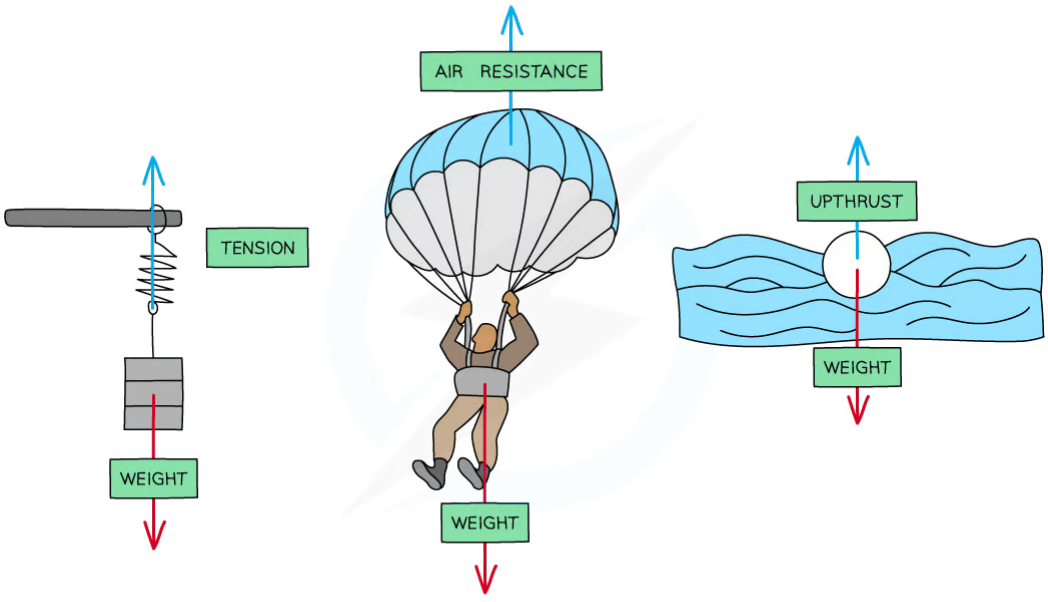


Forces on an object suspended from a stationary rope

YOUR NOTES

Free-body Diagrams

- As situations become more complex, there are often forces on multiple objects in different directions
- Forces acting on a body can be represented by a **free-body diagram**
- Each force is represented as a **vector** arrow, where each arrow:
  - Is scaled to the **magnitude** of the force it represents
  - Points in the **direction** that the force acts
  - Is **labelled** with the name of the force it represents or an appropriate symbol
- Free body diagrams can be used:
  - To identify which forces act in which plane
  - To resolve the net force in a particular direction
- The rules for drawing a free-body diagram are the following:
  - The diagram is for **one** object at a time
  - The body is represented by a **point** (the centre of mass) or a box or circle where all the forces act on
  - Only the **forces** acting on the body are drawn
  - The forces must be in the correct **direction** and a proportional **length**
  - All forces must be **clearly labelled**



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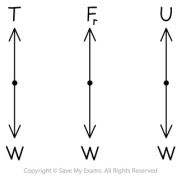

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Free-body diagrams for different situations

- The most common forces to apply are:
  - Weight (*W*) - always **towards** the **surface** of the planet
  - Tension (*T*) - always **away** from the mass
  - Normal Reaction Force (*N*) - **upwards** from a surface
  - Upthrust (*U*) - **upwards** if the mass is in a **fluid** (gas or liquid)
  - Frictional Forces (F_r) - in the **opposite** direction to the motion of the mass

YOUR NOTES

YOUR NOTES

Worked Example

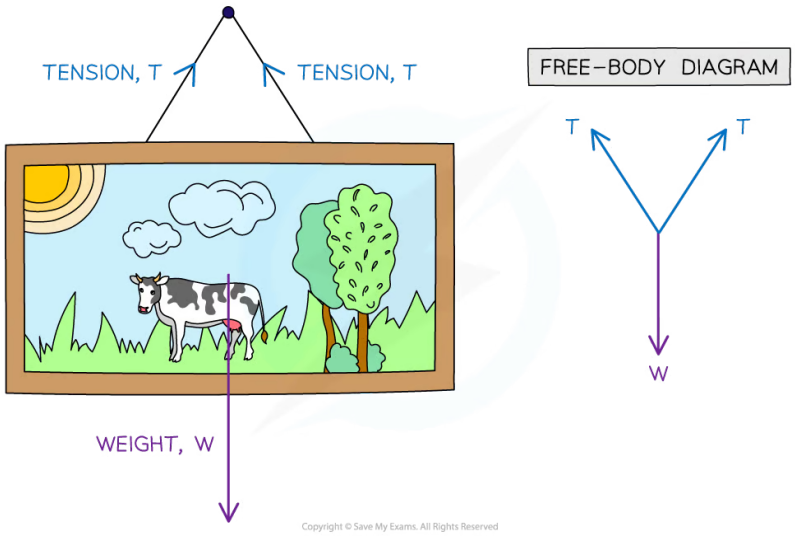
Draw free-body diagrams for the following scenarios:

(a)
A picture frame hanging from a nail.

(b)
A box sliding down a slope.

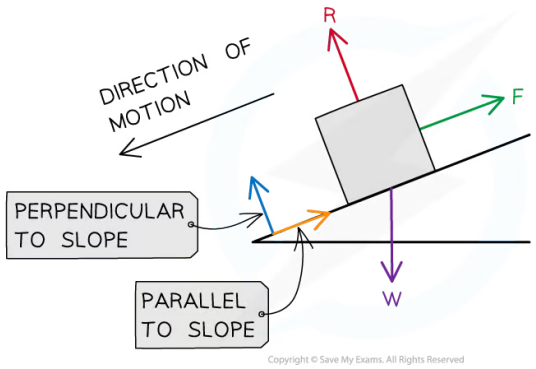
Answer:

(a) A picture frame hanging from a nail:



- The size of the arrows should be such that the 3 forces would make a closed triangle as they are **balanced**

(b) A box sliding down a slope:



- There are three forces acting on the box:
  - The **normal contact force**, *R*, acts perpendicular to the slope
  - **Friction**, *F*, acts parallel to the slope and in the opposite direction to the direction of motion

- **Weight**, *W*, acts down towards the Earth

YOUR NOTES

Worked Example

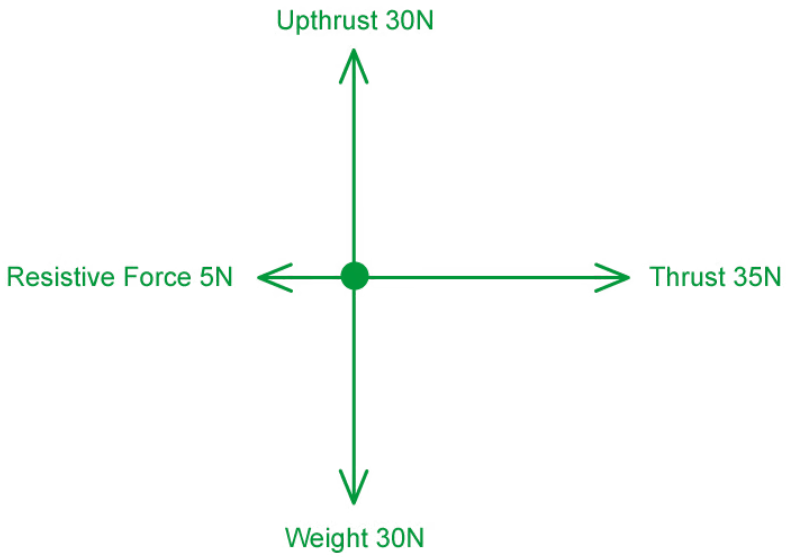
Draw a free-body diagram of a toy sailboat with a weight of 30 N floating in water that is being pulled to the right by a thrust of 35 N and has a total resistive force of 5 N.

Answer:

Step 1: Identify all of the forces acting upon the object in question, including any forces that may be implied

- **Weight** = 30 N downwards
- **Upthrust** from the water (as the object is floating) = 30 N upwards
- **Applied** force = 35 N to the right
- **Resistive** force = 5 N to the left

Step 2: Draw in all of the force vectors (arrows), making sure the arrows start at the object and are directed away



Exam Tip

When labelling force vectors, it is important to use **conventional** and **appropriate** naming or symbols such as:

- ***W*** or Weight force or ***mg***
- ***N*** or ***R*** for normal reaction force (depending on your local context either of these could be acceptable)

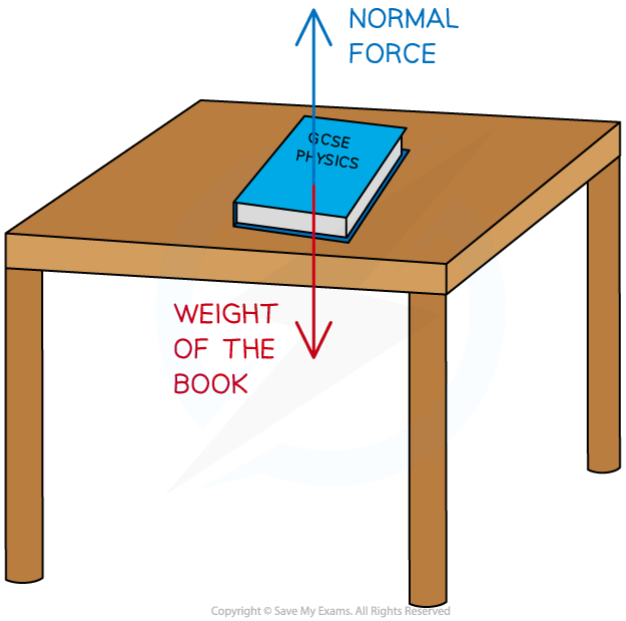
Using unexpected notation will **lose** you marks.

Make sure your arrows are roughly to scale with respect to the other forces in the image. In the second worked example, the 5 N force arrow needs to be considerably shorter than the 35 N thrust arrow. This shows clearly now that there is a resultant force to the right by the thrust.

Determining Resultant Forces

YOUR NOTES

- Free-body diagrams can be analysed to find the **resultant force** on a system
- A **resultant force** is a single force that describes all of the forces operating on a body
  - When many forces are applied to an object they can be **combined**
  - This produces one overall force which describes the **combined action** of all of the forces
- This single resultant force determines:
  - The **direction** in which the object will move as a result of all of the forces
  - The **magnitude** of the total force experienced by the object
- The resultant force is sometimes called the **net force**
- Forces can combine to produce
  - **Balanced** forces
  - **Unbalanced** forces
- **Balanced** forces mean that the forces have combined in such a way that they cancel each other out and no resultant force acts on the body
  - For example, the **weight** of a book on a desk is balanced by the **normal** force of the desk
  - As a result, **no resultant force** is experienced by the book; the forces acting on the book and the table are **equal** and **balanced**

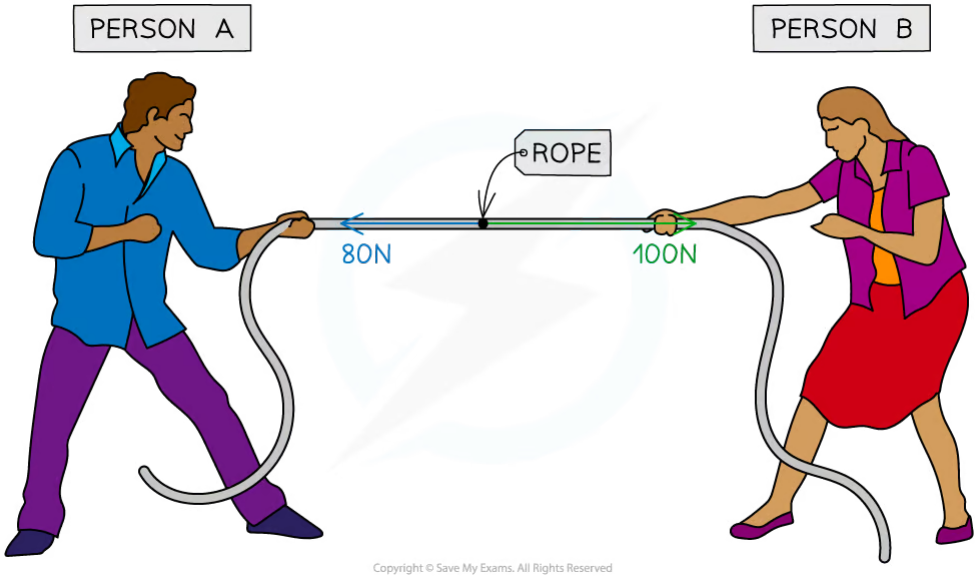


A book resting on a table is an example of balanced forces

- **Unbalanced** forces mean that the forces have combined in such a way that they do not cancel out completely and there is a **resultant force** on the object
  - For example, two people play a game of tug-of-war, working against each other on opposite sides of the rope

YOUR NOTES

- If person **A** pulls with 80 N to the left and person **B** pulls with 100 N to the right, these forces **do not cancel** each other out completely
- Since person **B** pulled with more force than person **A** the forces will be **unbalanced** and the rope will experience a **resultant force** of **(100 – 80 =) 20 N** to the **right**



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A tug-of-war is an example of when forces can become unbalanced

Resultant forces in one-dimension

- The resultant force in a one-dimensional situation i.e. when the forces are directed along the **same** plane, can be found by **combining vectors**
- Combining force vectors involves **adding** or **subtracting** all of the forces acting on the object
- This is easiest to visualise when they are drawn as a **free-body diagram**
  - Forces working in opposite directions are **subtracted** from each other
  - Forces working in the same direction are **added** together
- If the forces acting in opposite directions are equal in size, then there will be no resultant force – the forces are said to be **balanced**

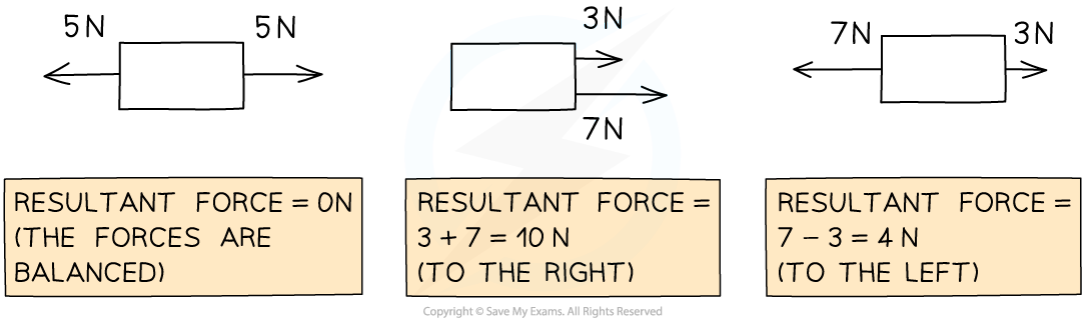

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Diagram showing the resultant forces on three different objects

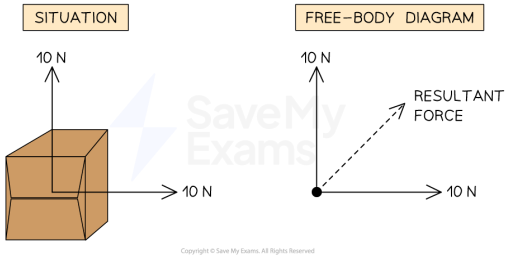
- Imagine the forces on the boxes as **two people** pushing and pulling on either side

YOUR NOTES

- In the first scenario, the two people are evenly matched - the box **doesn't move**
- In the second scenario, the two people are pushing on the same side of the box, it moves to the **right** with their combined strength
- In the third scenario, the two people are pushing against each other and are not evenly matched, so there is a **resultant force to the left**

Resultant forces in two-dimensions

- The resultant force in a two-dimensional situation i.e. when the forces are **not** on the same plane, can be found from **resolving vectors**
- Resolving force vectors involves using **Pythagoras** or **trigonometry** to determine the resultant of all of the forces acting on the object



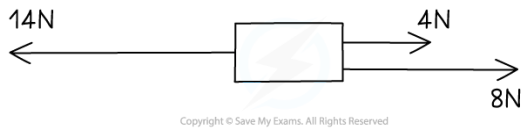
The resultant force is easier to visualise using a free-body diagram

- For example, the two 10 N forces acting on the cardboard box produce a resultant force of
  - $F_R = \sqrt{10^2 + 10^2} = 14\ \text{N}$
- More on these calculations can be found in Combining & Resolving Vectors

YOUR NOTES

Worked Example

Calculate the magnitude and direction of the resultant force on the object shown in the diagram below.



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Answer:

Step 1: Decide on the direction you will define as positive and negative

- Take the right as **positive** and the left as **negative**

Step 2: Add up all of the forces directed to the right

- The forces in the positive (right) direction are:

$$4\ N + 8\ N = 12\ N$$

Step 3: Add these to the forces on the left

- Combined with the forces in the negative (left) direction, the resultant force is:

$$12\ N - 14\ N = -2\ N$$

Step 4: Evaluate the direction of the resultant force

- Since the resultant force is negative, this is in the negative direction i.e. the **left**

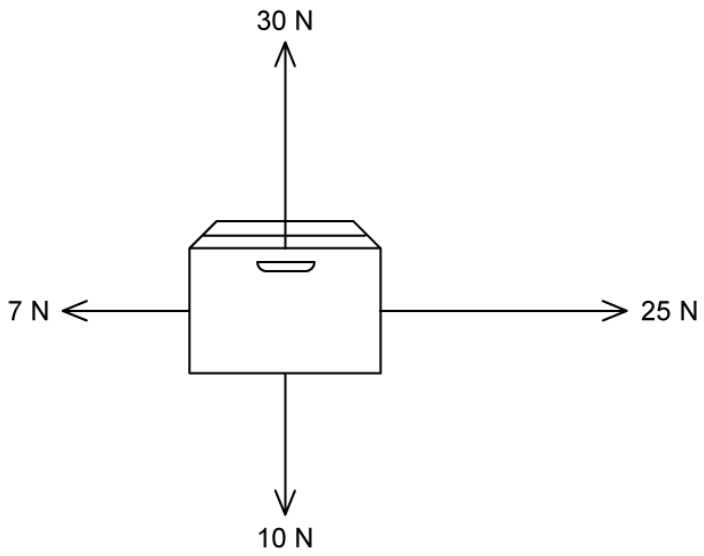
Step 5: State the magnitude and direction of the resultant force

- The resultant force is **2 N** to the **left**

YOUR NOTES

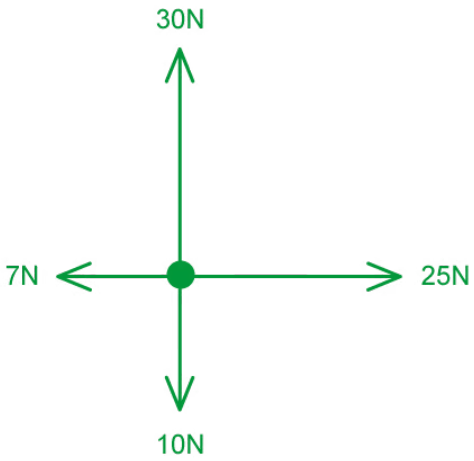
Worked Example

Calculate the magnitude and direction of the resultant force acting on the cardboard box shown in the diagram below.



Answer:

Step 1: Sketch the free-body diagram for the situation



Step 2: Determine the resultant horizontal force

- Taking the right as positive

$$25\ N - 7\ N = 18\ N \text{ (to the right)}$$

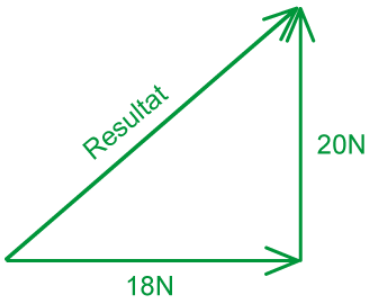
Step 3: Determine the resultant vertical force

- Take upwards as positive

$$30\ N - 10\ N = 20\ N \text{ (upwards)}$$

YOUR NOTES

Step 4: Calculate the resultant force



- Using Pythagoras' theorem

$$\sqrt{18^2 + 20^2} = 27\ \text{N}$$

Exam Tip

Take a look at the 'Tools' section of the course to learn how to combine and resolve vectors. You should be comfortable with these calculations for the whole of the forces topic.

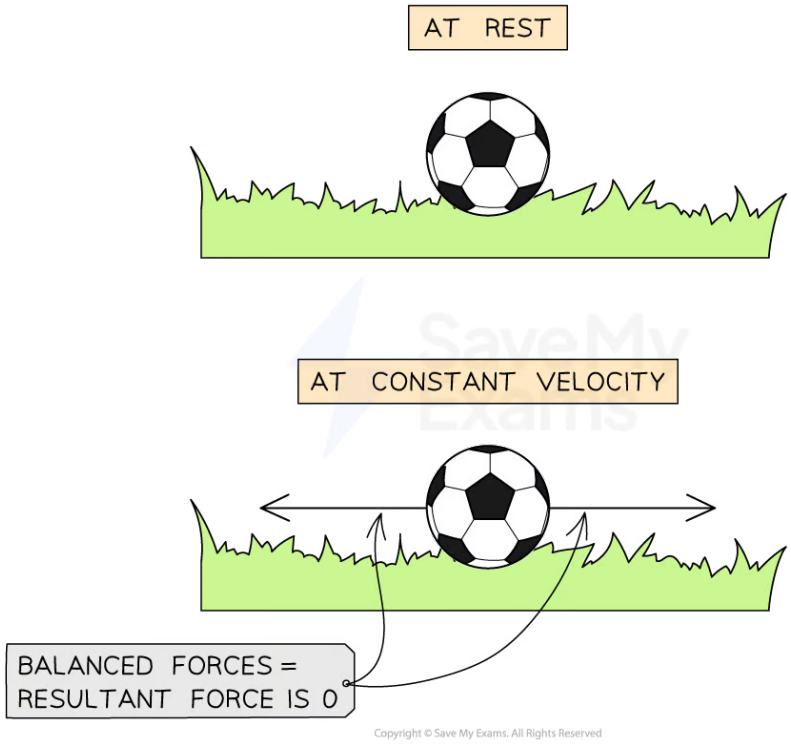
YOUR NOTES

Newton's First Law

- Newton's first law of motion states:

  A body will remain at rest or move with constant velocity unless acted on by a resultant force

- If the resultant force acting on an object is **zero**, it is said to be in **translational equilibrium**
- If the resultant force is zero (the forces on a body are balanced), the body must be either:
  - At **rest**
  - Moving at a **constant velocity**



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For both cases of the football being at rest or moving at a constant velocity, its resultant force is 0

- Since force is a vector, it is easier to split the forces into **horizontal** and **vertical** components
- If the forces are **balanced**:
  - The forces acting to the **left** = the forces acting to the **right**
  - The forces acting **upward** = the forces acting **downward**
- The resultant force is the single force obtained by combining **all** the forces acting on the body

YOUR NOTES

Worked Example

If there are no external forces acting on the car other than friction, and it is moving at a constant velocity, what is the value of the frictional force *F*?

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Answer:

Since the car is moving at a constant velocity, there is no resultant force. This means that the driving force and the frictional forces are balanced.

Therefore, *F* = 6 kN

Exam Tip

This law may sound counter-intuitive for an object that is moving at constant velocity. How can it be moving if the forces on it are balanced?

This is because a resultant force causes an **acceleration**. An object moving at constant velocity has no acceleration, so its forces must be balanced, which means the resultant force is zero. The drag forces are invisible to us, which makes this tricky to see.

YOUR NOTES

Newton's Second Law

- Newton's second law of motion states:

  The resultant force on an object is directly proportional to its acceleration

- This can also be written as:

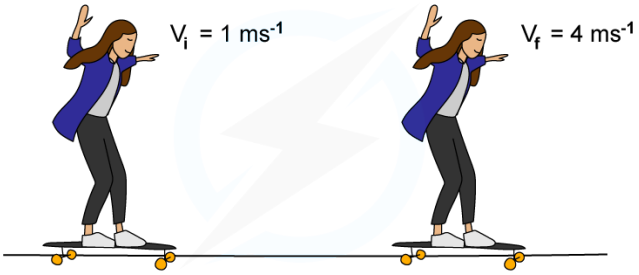
$$F = ma$$

- Where:
  - F = resultant force (N)
  - m = mass (kg)
  - a = acceleration (m s^{-2})
- This relationship means that objects will **accelerate** if there is a **resultant force** acting upon then
- An **unbalanced** force acting on a body **means** it experiences a **resultant force**
  - If the resultant force is along the direction of motion, it will speed up (**accelerate**) or slow down (**decelerate**) the body
  - If the resultant force is at an angle, it will change the direction of the body

YOUR NOTES

Worked Example

A girl is riding her skateboard down the road and increases her speed from 1 m s^{-1} to 4 m s^{-1} in 2.5 s. The force driving her forward is 72 N. Calculate the combined mass of the girl and the skateboard.



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STEP 1

NEWTON'S SECOND LAW STATES THE RESULTANT FORCE IS EQUAL TO THE RATE OF CHANGE IN MOMENTUM

$$F = \frac{\Delta p}{\Delta t}$$

STEP 2

FIND CHANGE IN MOMENTUM Δp

Δp = FINAL MOMENTUM − INITIAL MOMENTUM

$$\Delta p = mv_f - mv_i$$

STEP 3

SUBSTITUTE ALL VALUES INTO NEWTON'S SECOND LAW

$$72\ N = \frac{m(4-1)}{2.5}$$

MASS m IS CONSTANT SO CAN BE TAKEN OUT AS FACTOR

STEP 4

REARRANGE FOR MASS m

$$m = \frac{72 \times 2.5}{3} = 60\ kg$$

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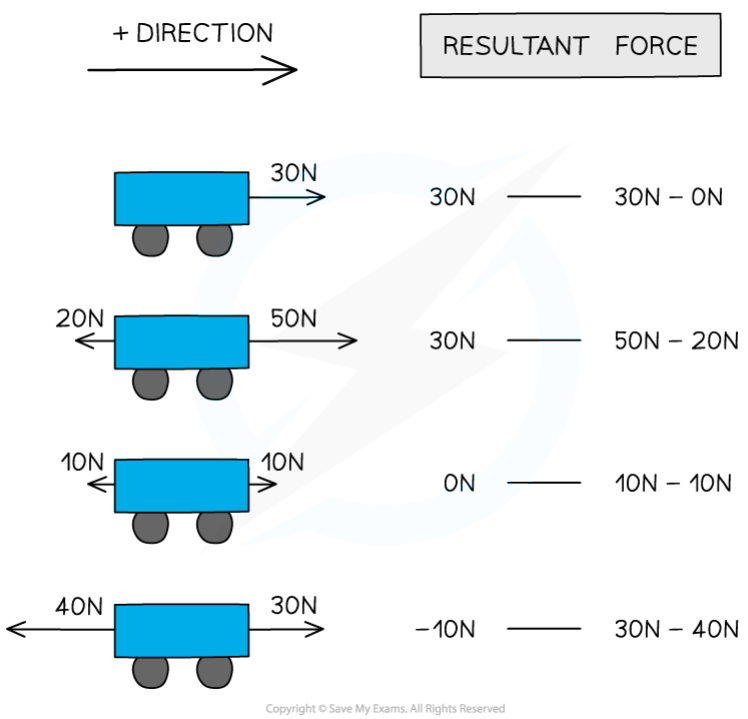
Resultant Force

- Since force is a vector, every force on a body has a magnitude and direction
  - The resultant force is, therefore, the **vector sum** of all the forces acting on the body
- The direction is given by either the positive or negative direction as shown in the examples below



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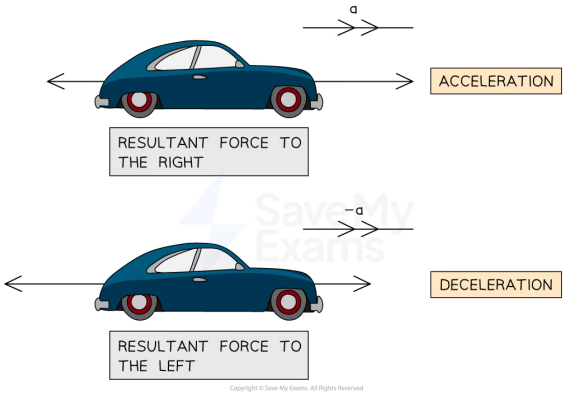
Resultant forces on a body can be positive or negative depending on their direction

- The resultant force could also be at an **angle**, in which case, the **magnitude** and **direction** of the resultant force can be determined using either:
  - **Calculation** (usually simple geometry, such as Pythagoras' Theorem or the sine and cosine rules)
  - **Scale drawing**
- This is covered further in Scale Diagrams

Acceleration

- Since acceleration is a **vector**, it can be either **positive** or **negative** depending on the direction of the resultant force
  - If the resultant force is in the **same** direction as the motion of an object, the acceleration is **positive**
  - If the resultant force is in the **opposite** direction to the motion of an object, the acceleration is **negative**
- An object may continue in the **same** direction however with a **resultant** force in the opposite direction to its motion
  - This means it will **slow down** (decelerate) and eventually come to a **stop**



- If there are no drag forces, or they're negligible, the acceleration is **independent** of the **mass** of an object
  - This has been shown in experiments by astronauts who have dropped a feather and a hammer on the Moon from the same height
  - Both the hammer and feather drop to the Moon's surface **at the same time**

YOUR NOTES

Worked Example

Three forces, 4 N, 8 N, and 24 N act on an object with a mass of 5 kg. Which acceleration is **not** possible with any combination of these three forces?

A. $1\,\text{m s}^{-2}$

B. $4\,\text{m s}^{-2}$

C. $7\,\text{m s}^{-2}$

D. $10\,\text{m s}^{-2}$

Answer:

Step 1: List the values given

- Three possible forces at any angle of choice: 4 N, 8 N, and 24 N
- Mass of object = 5 kg

Step 2: Consider the relevant equation

- Newton's second law relates force and acceleration:

$$F = m \times a$$

Step 3: Rearrange to make acceleration the focus

$$a = \frac{F}{m}$$

Step 4: Investigate the minimum possible acceleration

- The minimum acceleration would occur when the forces were acting **against** each other
- This is when just the 4 N force is acting on the body
- Now check the acceleration:

$$\frac{4}{5} = 0.8\ \text{m s}^{-2}$$

Step 4: Investigate the maximum possible acceleration

- The maximum acceleration would occur when **all** three forces are acting in the same direction
- This is a total force of

$$4 + 8 + 24 = 36\ \text{N}$$

- With acceleration:

$$\frac{36}{5} = 7.2\ \text{m s}^{-2}$$

Step 5: Consider this range and the options

- Since option **D** is higher than 7.2 m s^{-2}; it is not possible that these three forces can produce 10 m s^{-2} acceleration for this mass
- **Option D** is the **correct** answer, as it is the only one that is not possible

YOUR NOTES

YOUR NOTES

Worked Example

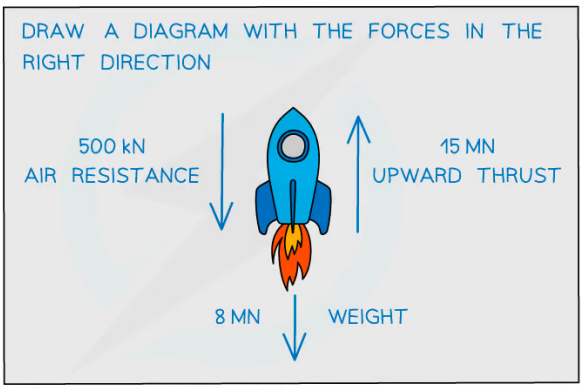
A rocket produces an upward thrust of 15 MN and has a weight of 8 MN.

(a)
When in flight, the force due to air resistance is 500 kN. Determine the resultant force on the rocket.

(b)
The mass of the rocket is 0.8×10^5 kg.

Calculate the magnitude and direction of the acceleration of the rocket.

A. STEP 1



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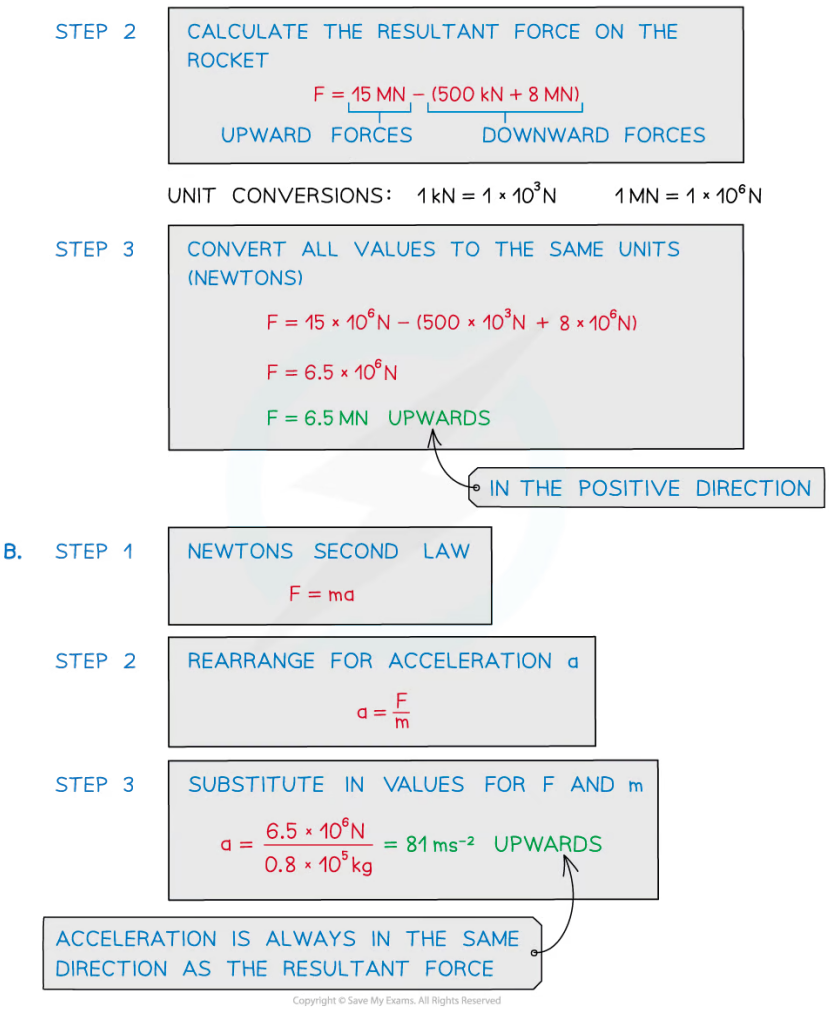

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Exam Tip

The direction you consider **positive** is **your choice**, as long as the signs of the numbers (positive or negative) are consistent throughout the question.

It is a general rule to consider the direction the object is initially travelling in as positive. Therefore all vectors in the direction of motion will be positive and opposing vectors, such as drag forces, will be negative.

YOUR NOTES

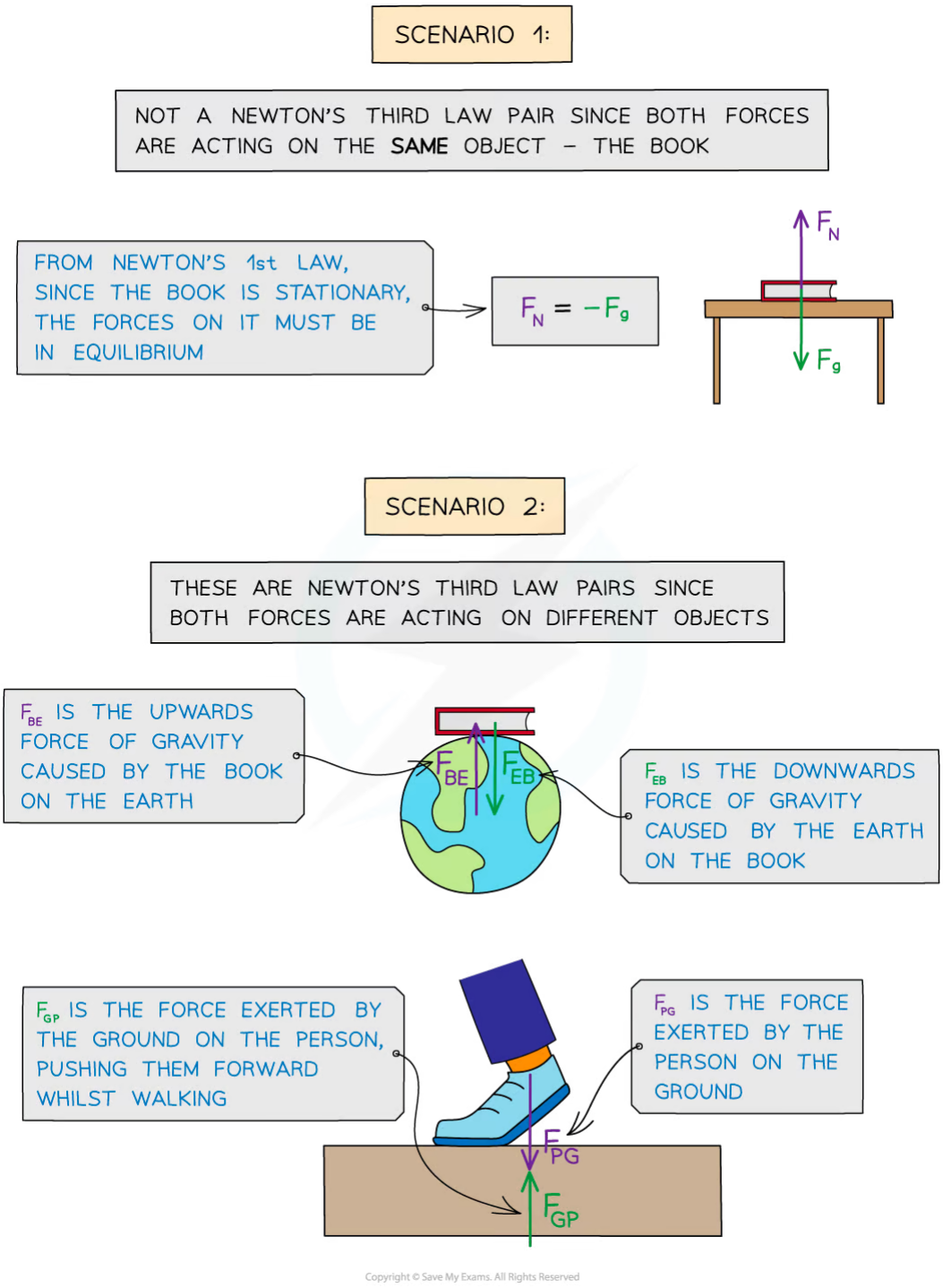
Newton's Third Law

- Newton's third law of motion states:

  Every action has an equal and opposite reaction

- Forces arise in **pairs**
  - If object **A** exerts a force on object **B**, then object **B** exerts an **equal** and **opposite** force on object **A**
- A Newton's third law force pair must be:
  - The same **type** of force – for example, if object **A** exerts a **gravitational force** on object **B**, then object **B** exerts an equal and opposite **gravitational force** on object **A**
  - The same **magnitude**
  - **Opposite** in **direction**
  - On **different objects** – this is why the weight and contact force are not a Newton's third law force pair, as they act on the same object

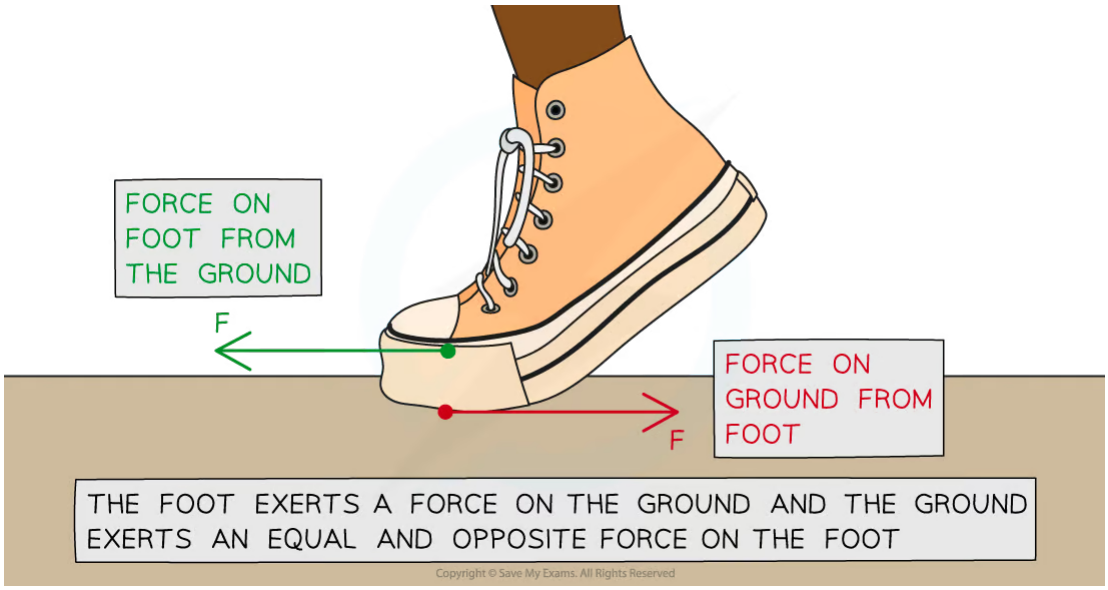
YOUR NOTES



Newton's third law force pairs

- Newton's third law explains the forces that enable someone to walk
- The image below shows an example of a pair of equal and opposite forces acting on two objects (the ground and a foot):

YOUR NOTES



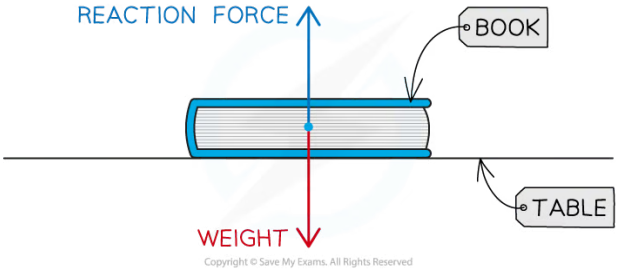
Newton's Third Law: The foot pushes the ground backwards, and the ground pushes the foot forwards

- One force is from the foot that pushes the ground backwards
- The other is an **equal and opposite** force from the ground that pushes the foot forwards

YOUR NOTES

Worked Example

A physics textbook is at rest on a dining room table. Eugene draws a free-body force diagram for the book and labels the forces acting on it.



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Eugene says the diagram is an example of Newton's third law of motion. William disagrees with Eugene and says the diagram is an example of Newton's first law of motion.

By referring to the free-body force diagram, state and explain who is correct.

Answer:

Step 1: State Newton's first law of motion

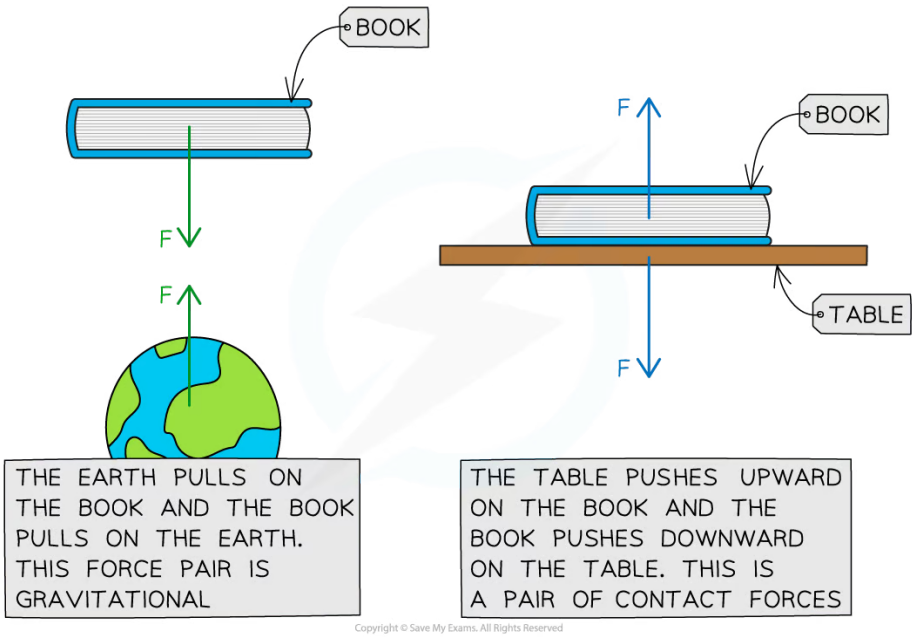
- Objects will remain at rest, or move with a constant velocity unless acted on by a resultant force

Step 2: State Newton's third law of motion

- Whenever two bodies interact, the forces they exert on each other are equal and opposite

Step 3: Check if the diagram satisfies the two conditions for identifying Newton's third law

- In each case, Newton's third law identifies pairs of equal and opposite forces, of the same **type**, acting on two **different** objects
- The diagram only involves **one object**
- Furthermore, the forces acting on the object are different types of force - one is a **contact force** (from the table) and the other is a **gravitational force** on the book (from the Earth) - its **weight**
- The image below shows how to apply Newton's third law correctly in this case, considering the pairs of forces acting:



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Step 4: Conclude which person is correct

- In this case, **William is** correct
- The free-body force diagram in the question is an example of **Newton's first law**
- The book is **at rest** because the two forces acting on it are **balanced** - i.e. there is no **resultant force**

Exam Tip

Just because you see two forces in opposite directions doesn't mean they are a Newton's third law force pairs! You must remember the specific criteria.

YOUR NOTES

Contact Forces

- A contact force is defined as:

  A force which acts between objects that are physically touching

- Examples of contact forces include:
  - Friction
  - Air resistance
  - Tension
  - Normal (reaction) force

Friction

- Friction is a force that opposes **motion**
- Occurs when objects rub against each other e.g. car wheels on the ground

Air resistance

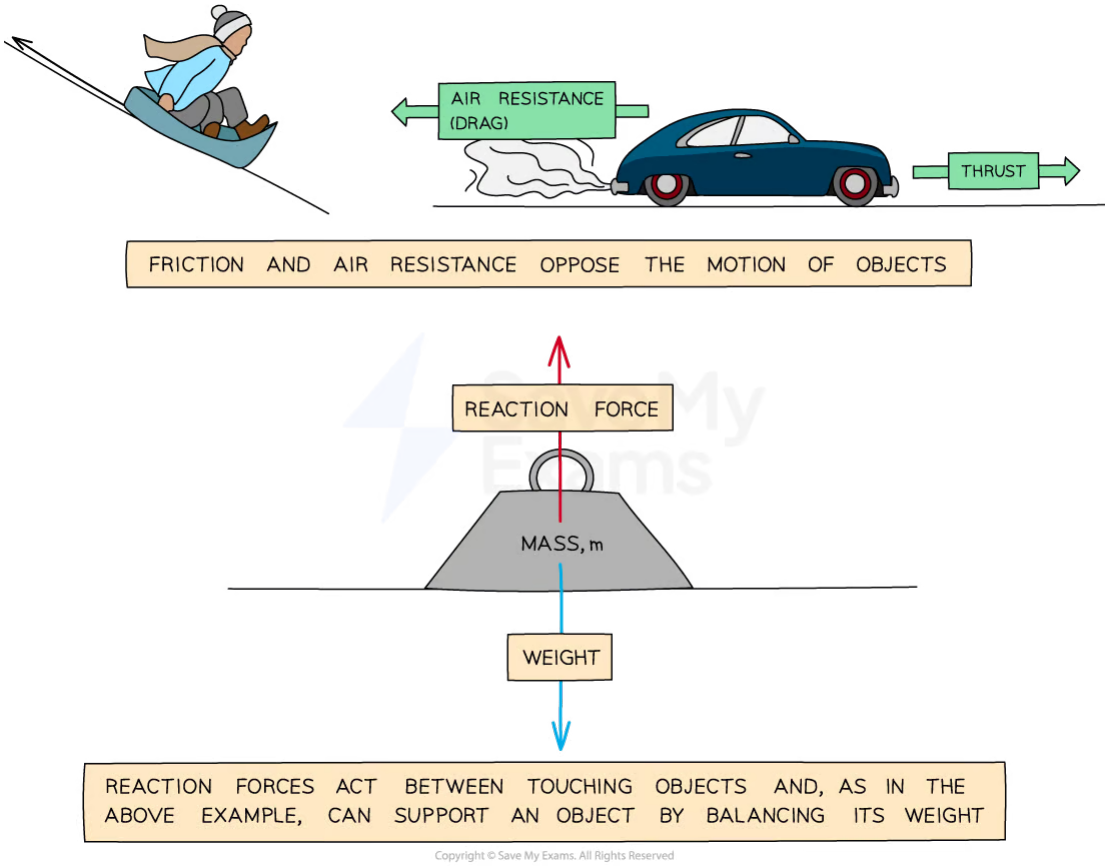
- Air resistance is a type of **friction**
- Occurs when an object moves through air e.g. an object in freefall

Tension

- Tension is a force that pulls two objects connected by a length of something, such as a string or rope
- Occurs when a **force** is applied to the length e.g. a spring extending

Normal / reaction force

- Reaction forces occur when an object is supported by a surface
- It is the component of the contact force acting **perpendicular** to the surface that counteracts the body e.g. a book on a table



Examples of contact forces

YOUR NOTES

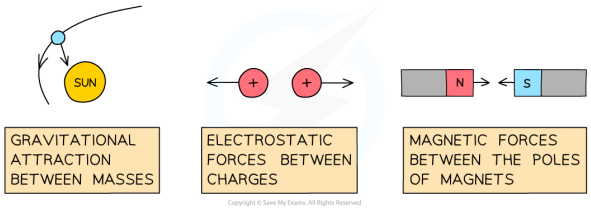
Non-Contact Forces

Non-Contact Forces

- A non-contact force is defined as:

  A force which acts at a distance, without any contact between bodies, due to the action of a field

- Examples of non-contact forces include:
  - Gravitational force, F_g
  - Electrostatic force, F_e
  - Magnetic force, F_m
- **Gravitational force:**
  - The **attractive** force experienced by two objects with mass in a gravitational field e.g the force between a planet and a comet
- **Electrostatic force:**
  - A force experienced by **charged** objects in an electric field which can be attractive or repulsive e.g. the attraction between a proton and an electron
- **Magnetic force:**
  - A force experienced between magnetic poles in a magnetic field that can be attractive or repulsive e.g. the attraction between the north and south poles of magnets



Examples of non-contact forces

Worked Example

Ben drags a sledge behind him as he climbs up a hill.

Describe the contact and non-contact forces involved in this scenario.

Answer:

Step 1: Identify the contact forces

- Ben pulls on a rope attached to the sledge producing **tension** in the rope
- The ground supports Ben and the sledge with a **reaction** force
- There is **friction** between the sledge and the ground which opposes the motion
- There is also **friction** between Ben's shoes and the ground allowing Ben to climb the hill without slipping down
- A very small force due to **air resistance** slows Ben's motion up the hill

Step 2: Identify the non-contact forces

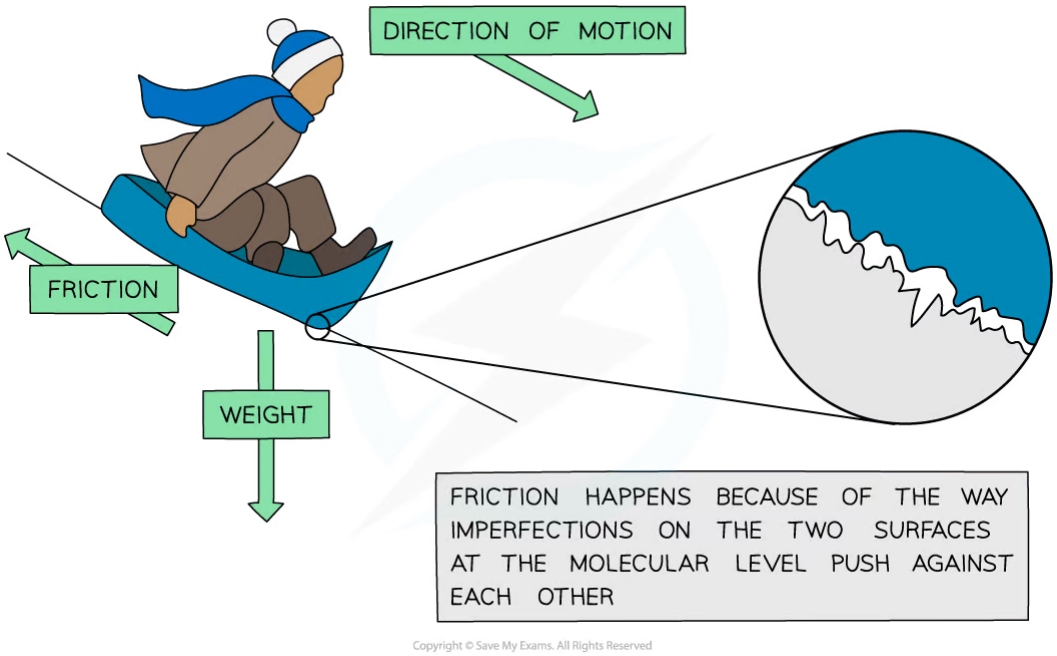
- The **gravitational attraction** between the Earth and Ben pulls Ben and the sledge downwards

YOUR NOTES

YOUR NOTES

Frictional Forces

- Friction is a force that works in **opposition** to the motion of an object
- It occurs between two **solid bodies** that are **in contact** with one another
  - The opposition of **friction slows** down the motion of the object
- When friction is present, energy is transferred in the form of **heat**
  - This raises the **temperature** (thermal energy) of the object and its surroundings
  - The work done against the frictional forces causes this rise in the temperature
- **Imperfections** at the interface between the object and the surface bump into and rub up against each other
  - Not only does this slow the object down but also causes an increase in **thermal energy**



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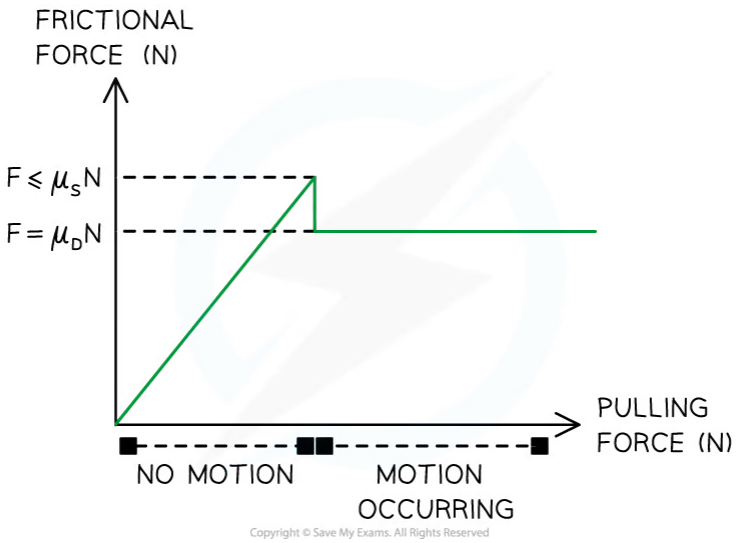
The interface between the ground and the sled is bumpy which is the source of the frictional force

Static & Dynamic Friction

- There are two kinds of friction to consider for IB DP Physics
  - **Static friction** occurs when a body is stationary on a surface
  - **Dynamic friction** occurs when a body is in motion on a surface, such as in the sled example above
- The surface frictional force always acts in a direction **parallel** to the plane of contact between a body and a surface

YOUR NOTES

- Both of these forms of friction depend on the **normal reaction force**, F_N of the object sitting upon the other
- Static friction will **match** any pushing force that acts against it until it can **no longer** hold the two objects stationary
  - Static friction **increases** in magnitude **until movement begins** and dynamic friction occurs
- For any given situation, **static friction** should reach a maximum value that is **larger** than that of **dynamic friction**
  - For a constant pushing force, **dynamic** friction will be a **constant**
- This is because there are more forces at work keeping an object stationary than there are forces working to resist an object once it is in motion



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The relationship between frictional forces and motion

- The equation for static friction is given by:

$$F_f \leq \mu_s F_N$$

- Where:
  - F_f = surface frictional force (N)
  - μ_S = coefficient of static friction
  - F_N = normal reaction force (N)
- The **coefficient** of static **friction** is a number between 0 and 1 but does not include those numbers
  - It is a ratio of the force of static friction and the normal force
  - The **larger** the coefficient of static friction, the **harder** it is to move those two objects past one another
- The equation for dynamic friction is given by:

$$F_f = \mu_D F_N$$

- Where:
  - F = dynamic frictional force (N)
  - μ_D = coefficient of dynamic friction
  - R = normal reaction force (N)

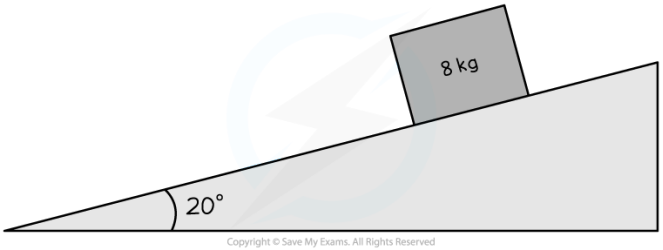
- The coefficient of dynamic friction has similar properties to that of static friction
- Yet, **dynamic friction** has a **definite force** value for a given situation
  - Whereas the force of **static friction** has an **increasing force** value

YOUR NOTES

YOUR NOTES

Worked Example

An 8.0 kg block sits on an incline of 20 degrees from the horizontal. It is stationary and does have a frictional force acting upon it.



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Determine the minimum possible value of the coefficient of static friction.

Answer:

Step 1: List the known quantities

- Mass of the block, $m = 8.0$ kg
- Angle between the slope and the horizontal, $\theta = 20°$

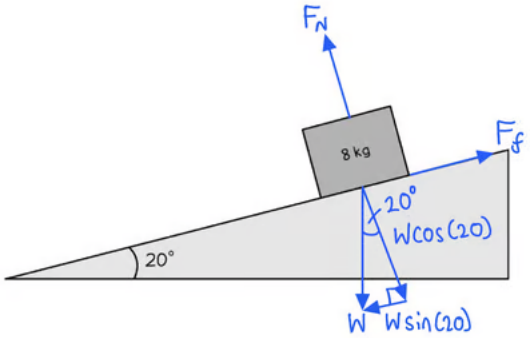
Step 2: Determine the weight of the block

- The weight will act directly downwards and comes from the interaction of mass and acceleration due to gravity

$$W = m \times g$$

$$8.0 \times 9.81 = 78.48 \text{ N } \textbf{downwards}$$

Step 3: Break the weight down into components based on the slope angle



- The component that is parallel to the slope and provides a force moving the block down the slope is equal to the frictional force up the slope:

$$F_f = W\sin(20)$$

$$F_f = 78.48 \times \sin(20) = 26.8 \text{ N}$$

YOUR NOTES

- The component that is perpendicular to the slope and the same magnitude as the normal reaction force can be found from:

$$F_N = 78.48 \times \cos(20) = 73.7 \text{ N}$$

Step 4: Use the equation of static friction to find the minimum value of the coefficient of static friction

- The equation for static friction is:

$$F_f \leq \mu_s F_N$$

- In this case, the F_f is the 26.8 N pushing the block down the slope
- The F_N is the normal reaction force which has the same magnitude as the perpendicular component of the weight force which is 73.7 N
- Therefore the value can be added and μ_S solved for:

- Rearrange for μ_S

$$\mu_s \geq \frac{26.8}{73.7}$$

$$\mu_s \geq 0.36$$

Step 5: State the final answer

- The coefficient for static friction must be at least **0.36 or greater** for this situation

YOUR NOTES

Hooke's Law

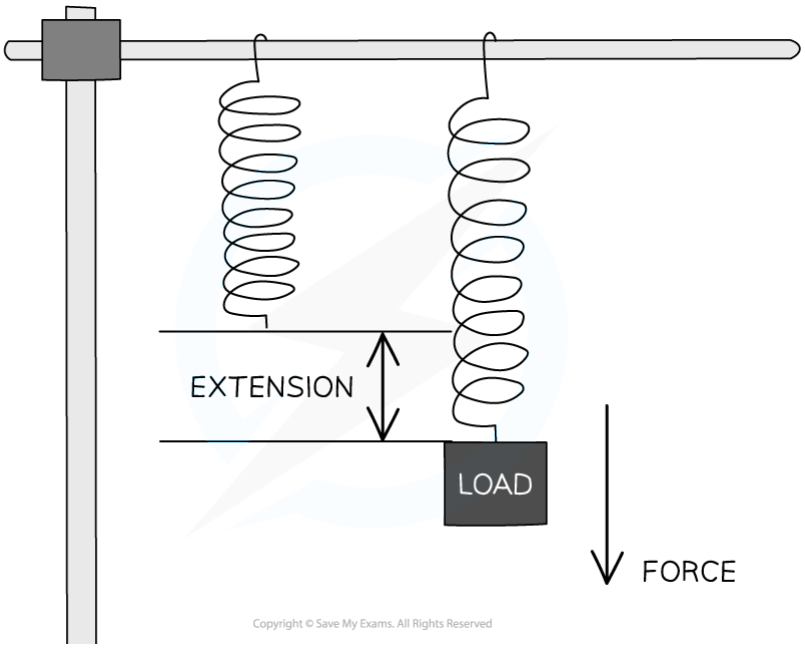
- When a force *F* is added to the bottom of a spring, it **stretches**
  - This phenomenon occurs for any material with elasticity, such as a wire or a bungee rope
- A material obeys Hooke's Law if:

  The extension of the material is directly proportional to the applied force (load) up to the limit of proportionality

- This linear relationship is represented by the Hooke's law equation:

$$F_{\mathrm{H}} = -kx$$

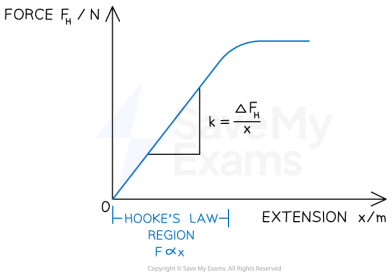
- Where:
  - F_{H} = elastic restoring force (N)
  - k = spring constant (N m^{-1})
  - x = extension (m)
- The spring constant, *k* is a property of the material being stretched and measures the **stiffness** of a material
  - The **larger** the spring constant, the **stiffer** the material
- Hooke's Law applies to both **extensions** and **compressions**:
  - The extension of an object is determined by how much it has **increased** in length
  - The compression of an object is determined by how much it has **decreased** in length
- The extension *x* is the difference between the **unstretched** and **stretched** length of a spring



Stretching a spring with a load produces a force that leads to an extension

Force-Extension Graphs

- The way a material responds to a given force can be shown on a force-extension graph
- Every material will have a unique force-extension graph depending on how brittle or ductile it is
- A material may obey Hooke's Law up to a point
  - This is shown on its force-extension graph by a **straight line through the origin**
- As more force is added, the graph may start to curve slightly



The Hooke's Law region of a force-extension graph is a straight line. The spring constant is the gradient of that region

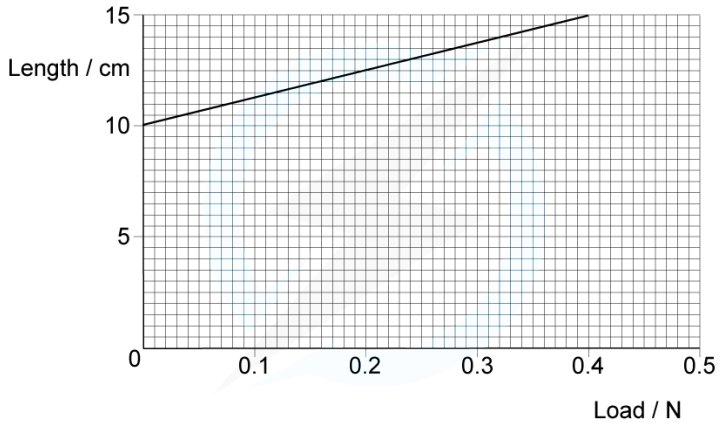
- The **gradient** of this graph is equal to the **spring constant *k***

YOUR NOTES

Worked Example

A spring was stretched with increasing load.

The graph of the results is shown below.



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Determine the spring constant.

STEP 1

REARRANGE FROM HOOKE'S LAW, THE SPRING CONSTANT IS

$$k = \frac{F}{\Delta L}$$

STEP 2

THE GRADIENT OF A FORCE–EXTENSION GRAPH IS THE SPRING CONSTANT

$$k = \frac{\Delta F}{\Delta L}$$

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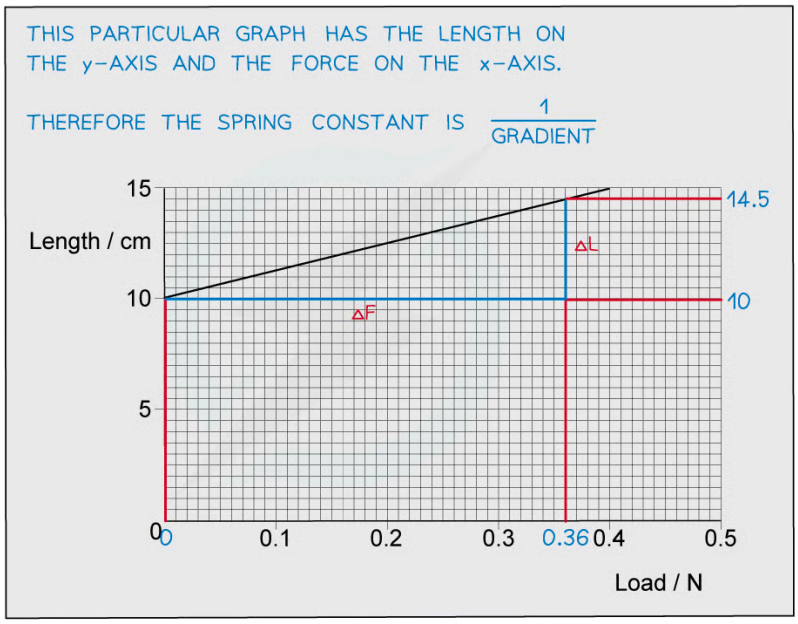
YOUR NOTES

STEP 3



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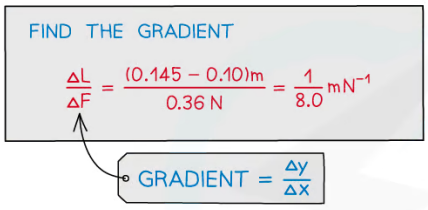
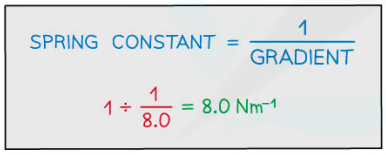
STEP 4



STEP 5



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Exam Tip

Always double check the axes before finding the spring constant as the gradient of a force-extension graph.

Exam questions often swap the force (or load) onto the x-axis and extension (or length) on the y-axis. In this case, the gradient is **not** the spring constant, it is $\frac{1}{k}$ instead.

Make sure that you put the **extension** of the object into the equation for *x* and not just the length.

YOUR NOTES

Stoke's Law

Viscous Drag

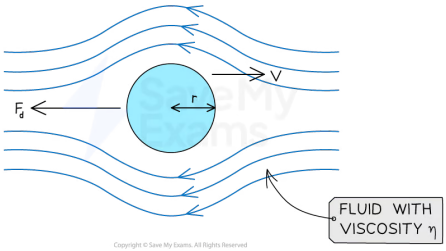
- Viscous drag is defined as:

  the frictional force between an object and a fluid which opposes the motion between the object and the fluid

- This drag force is often from air resistance
- Viscous drag is calculated using Stoke's Law:

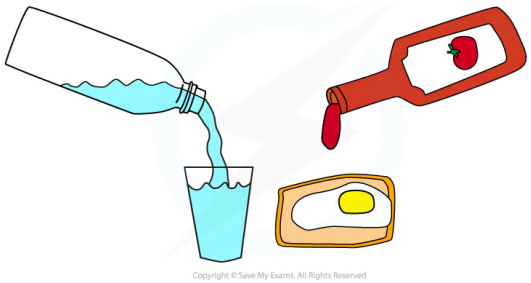
$$F_d = 6\pi\eta r v$$

- Where
  - F_d = viscous drag force (N)
  - η = fluid viscosity (N s m^{-2} **or** Pa s)
  - r = radius of the sphere (m)
  - v = velocity of the sphere through the fluid (ms^{-1})



A sphere travelling through air will experience a drag force that depends on its radius, velocity and the viscosity of the liquid

- The viscosity of a fluid can be thought of as its thickness, or how much it resists flowing
  - Fluids with **low viscosity** are **easy** to pour, while those with **high viscosity** are **difficult** to pour



Water has a lower viscosity than ketchup as it is easier to pour and flow

YOUR NOTES

- The **coefficient of viscosity** is a property of the fluid (at a given temperature) that indicates how much it will resist flow
  - **The rate of flow** of a fluid is inversely proportional to the coefficient of viscosity
- The size of the force depends on the:
  - Speed of the object
  - Size of the object
  - Shape of the object

Worked Example

A spherical stone of volume $2.7 \times 10^{-4}\ m^3$ falls through the air and experiences a drag force of 3 mN at a particular instant. Air has a viscosity of 1.81×10^{-5} Pa s. Calculate the speed of the stone at that instant.

Answer:

Step 1: List the known quantities

- Volume of stone, $V = 2.7 \times 10^{-4}\ m^3$
- Drag force, $F_d = 3\ mN = 3 \times 10^{-3}\ N$
- Viscosity of air, $\eta = 1.81 \times 10^{-5}$ Pa s

Step 2: Calculate the radius of the sphere, *r*

- The volume of a sphere is

$$V = \frac{4}{3}\pi r^3$$

- Therefore, the radius, *r* is:

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3 \times (2.7 \times 10^{-4})}{4\pi}} = 0.04\ m$$

Step 3: Rearrange the Stoke's law equation for the velocity, *v*

$$F_d = 6\pi\eta r v$$

$$v = \frac{F_d}{6\pi\eta r}$$

Step 4: Substitute in the known values

$$v = \frac{3 \times 10^{-3}}{6\pi \times (1.81 \times 10^{-5}) \times 0.04} = 220\ m\ s^{-1}$$

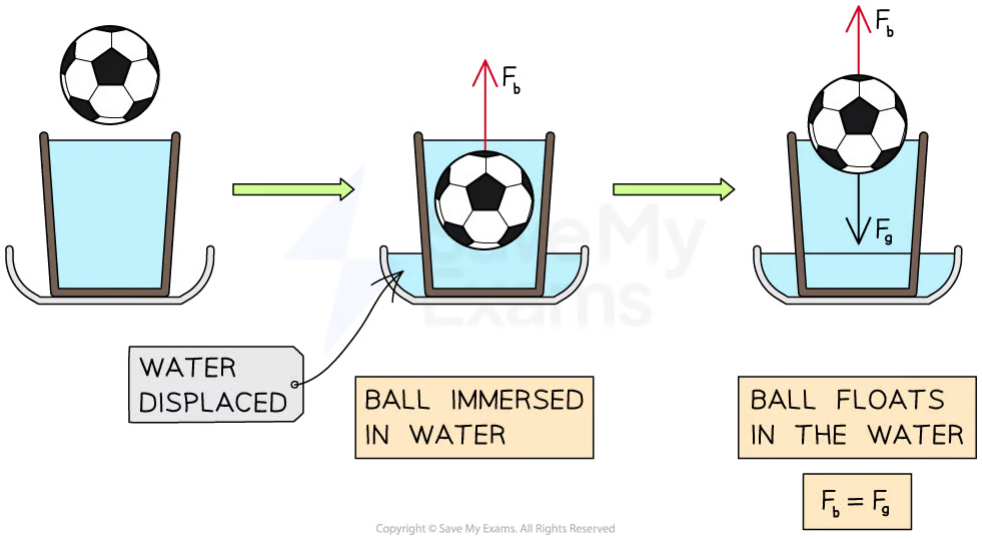
YOUR NOTES

Upthrust

- Buoyancy is experienced by a body partially or totally immersed in a fluid, such as a water
  - The **size** of the force that produces this is equal to the weight of the fluid displacement (sometimes referred to as Archimedes principle)
- This force, called upthrust, keeps boats afloat and allows balloons to rise through the air
- When a body travels through a fluid, it also experiences a **buoyancy force** (upthrust) due to the displacement of the fluid

$$F_b = \rho V g$$

- Where:
  - F_b = buoyancy force (N)
  - ρ = density of the fluid (kg m^{-3})
  - V_g = volume of the fluid displaced (m^3)
  - g = gravitational field strength (m s^{-2})
- If you were to take a hollow ball and submerge it into a bucket of water, you would feel some resistance
- Some water will flow out of the bucket as it is displaced by the ball
- The buoyancy force, F_b will act upwards on the ball to bring it to the surface
- The ball will remain stationary floating when its weight acting downwards, F_g equals the buoyancy force acting upwards, F_b



The ball floats when it is balanced by the buoyancy force and its weight

- Notice that

$$F_g = \rho V g = \frac{m}{V} V g = mg$$

which is the **weight** of the object submerged in the fluid

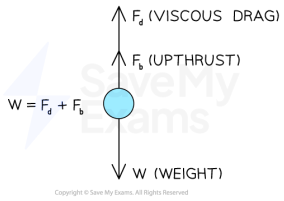
- Where:
  - m = mass of the ball (kg)
  - ρ = density of the ball (kg m^{-3})
  - V = volume of the ball (m^3)

Drag Force at Terminal Velocity

- Terminal velocity is useful when working with Stoke's Law since at terminal velocity the forces in each direction are **balanced**

$$W_s = F_d + F_b \text{ (equation 1)}$$

- Where:
  - W_s = weight of the sphere
  - F_d = the drag force (N)
  - F_b = the buoyancy force / upthrust (N)



At terminal velocity, the forces on the sphere are balanced

- The weight of the sphere is found using volume, density and gravitational field strength

$$W_s = \rho_s V_s g$$

$$W_s = \frac{4}{3}\pi r^3 \rho_s g \text{ (equation 2)}$$

- Where
  - V_s = volume of the sphere (m^3)
  - ρ_s = density of the sphere (kg m^{-3})
  - r = radius of the sphere (m)
  - g = gravitational field strength (N kg^{-1})
- Recall Stoke's Law

$$F_d = 6\pi\eta r v \text{ (equation 3)}$$

- Where v is the terminal velocity
- The buoyancy force equals the weight of the displaced fluid
  - The **volume** of displaced fluid is the **same** as the **volume** of the sphere
  - The **weight** of the fluid is found from **volume**, **density** and **gravitational**

YOUR NOTES

$$F_b = \frac{4}{3}\pi r^3 \rho_f g \text{ (equation 4)}$$

- Substitute equations 2, 3 and 4 into equation 1

$$\frac{4}{3}\pi r^3 \rho_s g = 6\pi\eta r v + \frac{4}{3}\pi r^3 \rho_f g$$

- Rearrange to make terminal velocity the subject of the equation

$$v = \frac{\frac{4}{3}\pi r^3 g(\rho_s - \rho_f)}{6\pi\eta r} = \frac{4\pi r^3 g(\rho_s - \rho_f)}{18\pi\eta r}$$

- Finally, cancel out *r* from the top and bottom to find an expression for **terminal velocity** in terms of the **radius of the sphere** and the **coefficient of viscosity**

$$v = \frac{2\pi r^2 g(\rho_s - \rho_f)}{9\pi\eta}$$

- This final equation shows that terminal velocity is:
  - **directly proportional** to the **square** of the **radius** of the sphere
  - **inversely proportional** to the **viscosity** of the fluid

YOUR NOTES

Worked Example

Icebergs typically float with a large volume of ice beneath the water. Ice has a density of 917 kg m^{-3} and a volume of V_i.

The density of seawater is 1020 kg m^{-3}.

What fraction of the iceberg is above the water?

A. 0.10 V_i **B.** 0.90 V_i **C.** 0.97 V_i **D.** 0.20 V_i

ANSWER: A

STEP 1

ACCORDING TO ARCHIMEDES' PRINCIPLE, THE UPTHRUST IS EQUAL TO THE WEIGHT OF THE SEAWATER DISPLACED BY THE ICEBERG

ICEBERG WEIGHT $W_i = m_i g$

BUOYANCY FORCE IS THE WEIGHT OF THE DISPLACED WATER $W_w = m_w g$

STEP 2

SINCE THE ICEBERG IS FLOATING, ITS WEIGHT IS EXACTLY EQUAL TO THE BUOYANCY FORCE

$$W_i = W_w$$

$$m_i g = m_w g$$

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YOUR NOTES

STEP 3

REARRANGE DENSITY EQUATION FOR MASS

$$m = \rho V$$

$$\rho_i V_i g = \rho_w V_w g$$

STEP 4

CANCELLING g SHOWS THE FRACTION OF ICE UNDERWATER IS GIVEN BY RATIO OF DENSITIES

$$\frac{V_i}{V_w} = \frac{\rho_w}{\rho_i}$$

STEP 5

REARRANGE FOR V_w

$$V_w = \frac{\rho_i V_i}{\rho_w}$$

$$V_w = \frac{917}{1020} V_i = 0.9 V_i$$

STEP 6

THEREFORE 90% OF THE ICEBERG'S VOLUME IS SUBMERGED UNDERWATER

THIS MEANS THAT $1 - 0.9 = 0.1\ V_i$ IS ABOVE WATER

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Exam Tip

Remember that ρ in the buoyancy force equation is the density of the **fluid** and **not** the object itself!

YOUR NOTES

Conservation of Linear Momentum

Linear Momentum

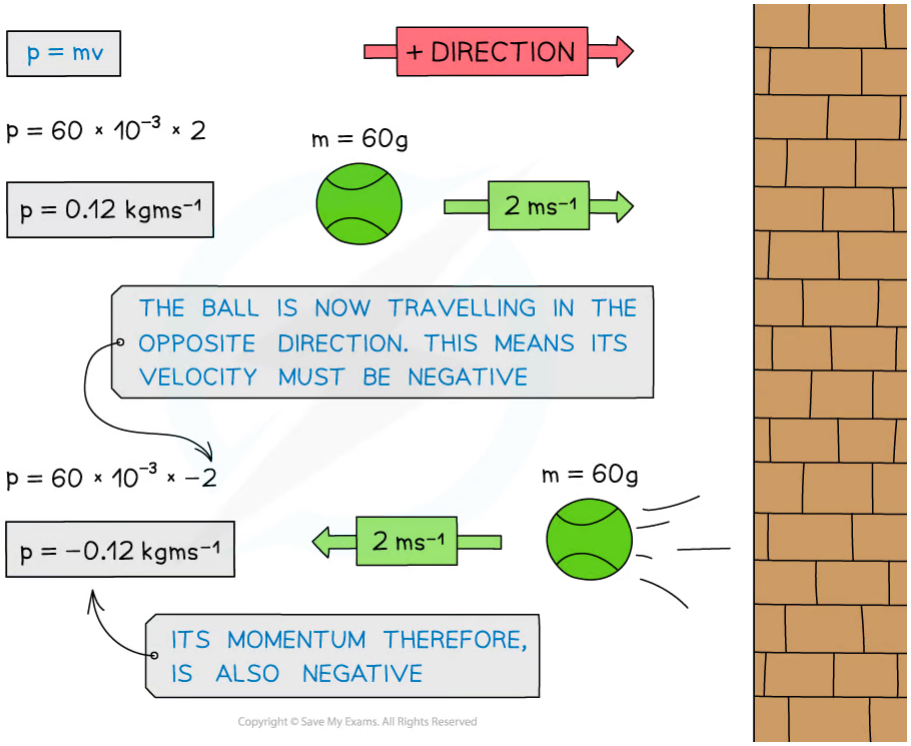
- The momentum of an object is defined as the product of **mass** and **velocity**

$$p = mv$$

- Where:
  - p = momentum (kg m s^{-1})
  - m = mass (kg)
  - v = velocity (m s^{-1})
- Momentum is a **vector** quantity - it has both a **magnitude** and a **direction**
- Therefore, it can have a negative or a positive value depending on its direction
  - If an object travelling to the **right** has **positive** momentum, then an object travelling to the **left** (in the opposite direction) has a **negative** momentum
  - The negative or positive directions are **defined by the observer** on a case-by-case basis

Direction of Momentum

- If a ball of mass 60 g travels at 2 m s^{-1}, it will have a momentum of 0.12 kg m s^{-1}
- If it then hits a wall and rebounds in the exact opposite direction at the same speed, it will have a momentum of –0.12 kg m s^{-1}



When the ball is travelling in the opposite direction, its velocity is negative. Since momentum = mass × velocity, its momentum is also negative

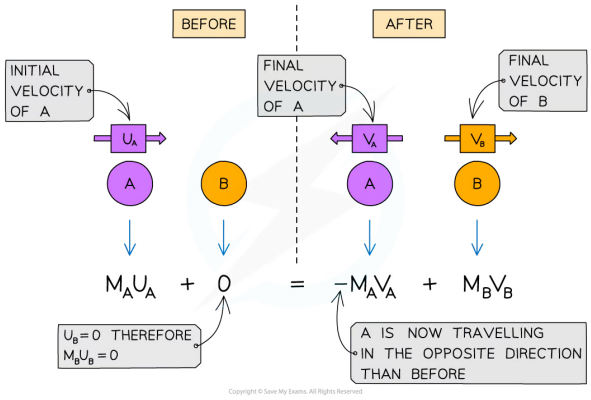
YOUR NOTES

Conservation of Linear Momentum

- The principle of conservation of linear momentum states that:

  The total linear momentum before a collision is equal to the total linear momentum after a collision unless the system is acted on by a resultant external force

- Linear momentum is the momentum of an object that only moves in **one dimension**
- Momentum is a **vector** quantity
  - This means oppositely-directed vectors can cancel each other out resulting in a net momentum of zero
  - If after a collision an object starts to move in the opposite direction to which it was initially travelling, its velocity will now be **negative**
- Momentum, just like energy, is **always conserved**
- If Ball A moves with a velocity of u_A and collides with Ball B which is stationary, then after the collision both balls travel in opposite directions (and the positive direction is to the right) it can be seen that:
  - The momentum before the collision is $m_A u_A + 0$
  - The momentum after the collision is $-m_A v_A + m_B v_B$
- Notice the minus sign is because mass A travels in the **opposite** direction to the initial travel (as the right is taken as positive, the left is negative)
- If an object is stationary, like B is before the collision, then it has a momentum of **0**

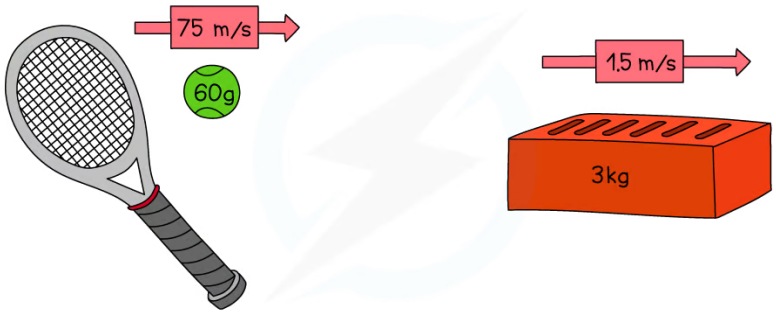


The conservation of momentum for two objects A and B colliding then moving apart

YOUR NOTES

Worked Example

Determine which object has the greatest momentum.



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Answer:

MOMENTUM = MASS × VELOCITY

MOMENTUM = 0.06 kg × 75 m/s
= 4.5 kg m/s

MOMENTUM = MASS × VELOCITY

MOMENTUM = 3 kg × 1.5 m/s
= 4.5 kg m/s

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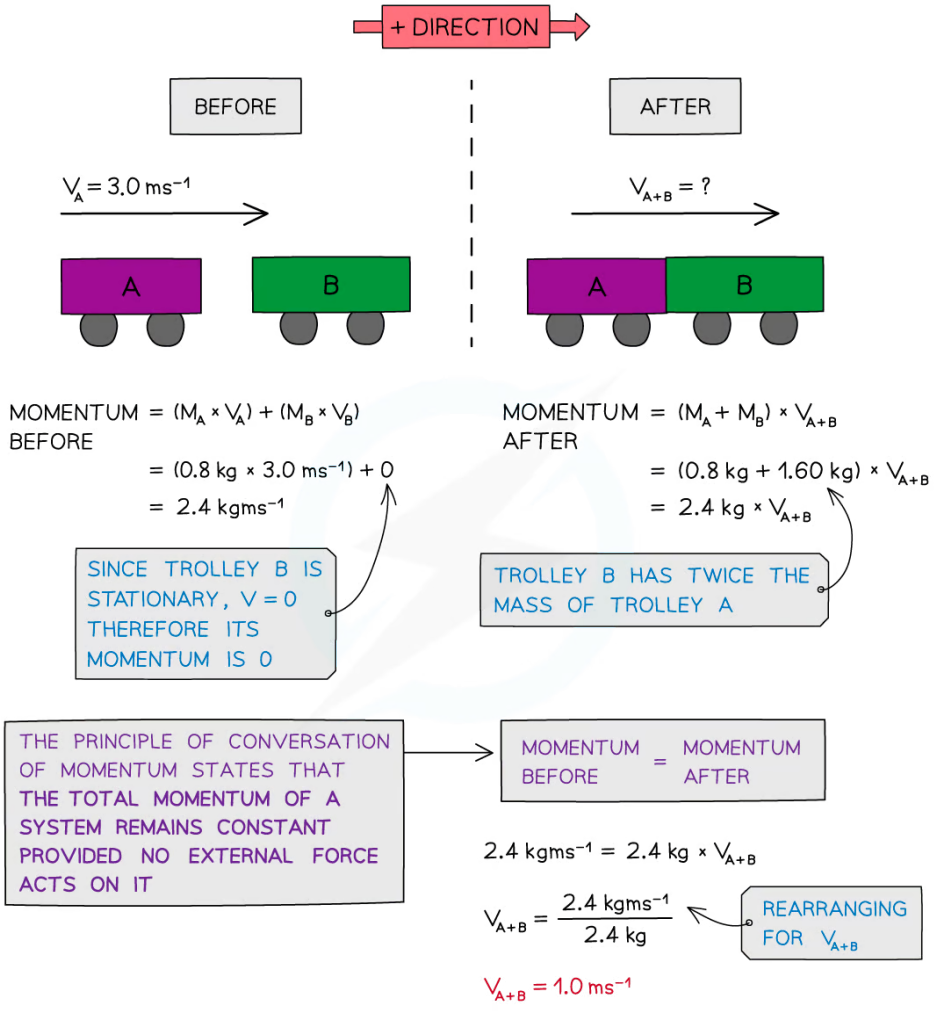
- Both the tennis ball and the brick have the **same momentum**
- Even though the brick is much **heavier** than the ball, the ball is travelling much **faster** than the brick
- This means that on impact, they would both exert a **similar force** (depending on the time it takes for each to come to rest)

YOUR NOTES

Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** whilst travelling at 3.0 m s^{-1}. Trolley **B** has twice the mass of trolley **A**. On impact, the trolleys stick together.

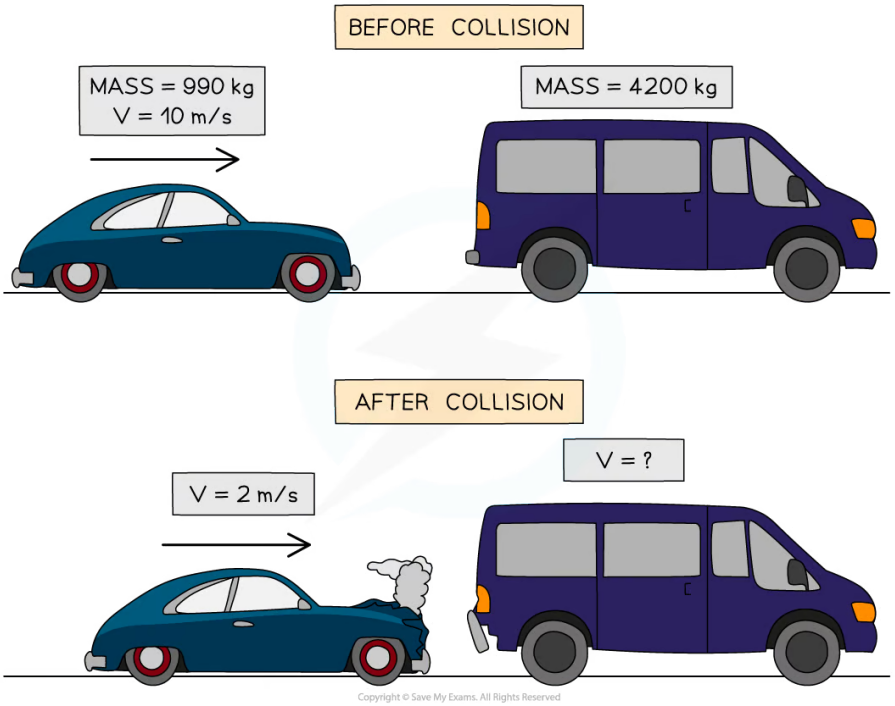
Using the conversation of momentum, calculate the common velocity of both trolleys after the collision.



YOUR NOTES

Worked Example

The diagram shows a car and a van which is initially at rest, just before and just after the car collides with the van.



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Use the idea of conservation of momentum to calculate the velocity of the van when it is pushed forward by the collision.

Answer:

Step 1: State the principle of conversation of momentum

- In a closed system, the total momentum before an event is equal to the total momentum after the event

Step 2: Calculate total momentum before the collision

$$p = mv$$

- Momentum of the car:

$$p = 990 \times 10 = 9900 \text{ kg m/s}$$

- Momentum of the van:

The van is at rest, therefore v = 0 m/s and p = 0 kg m/s

- Total momentum before:

$$p_{before} = 9900 + 0 = 9900 \text{ kg m/s}$$

Exam Tip

If it is not given in the question already, drawing a diagram of before and after helps keep track of all the masses and velocities (and directions) in the conversation of momentum questions. Even if one is given, label all the values that you have been given in the question to make sure you're substituting in the correct masses and velocities.

YOUR NOTES

YOUR NOTES

Impulse & Momentum

- When you are hit by an object, the effect of the force on you depends on whether it bounces off you or you catch it
- This is because the **change in momentum** of the ball is greater when it bounces off you
  - A greater **force** is also produced, which is why it hurts a lot more than if you just catch it
- The applied external impulse is also equal to the **change in momentum** of the system

$$J = \Delta p = mv - mu$$

- Where:
  - Δp = change in momentum (kg m s^{-1})
  - m = mass (kg)
  - v = final velocity (m s^{-1})
  - u = initial velocity (m s^{-1})
- It can also be defined in terms of the average resultant force, F:

$$J = F\Delta t$$

- Where:
  - J = impulse (N s)
  - F = average resultant force (N)
  - Δt = time of contact (s)

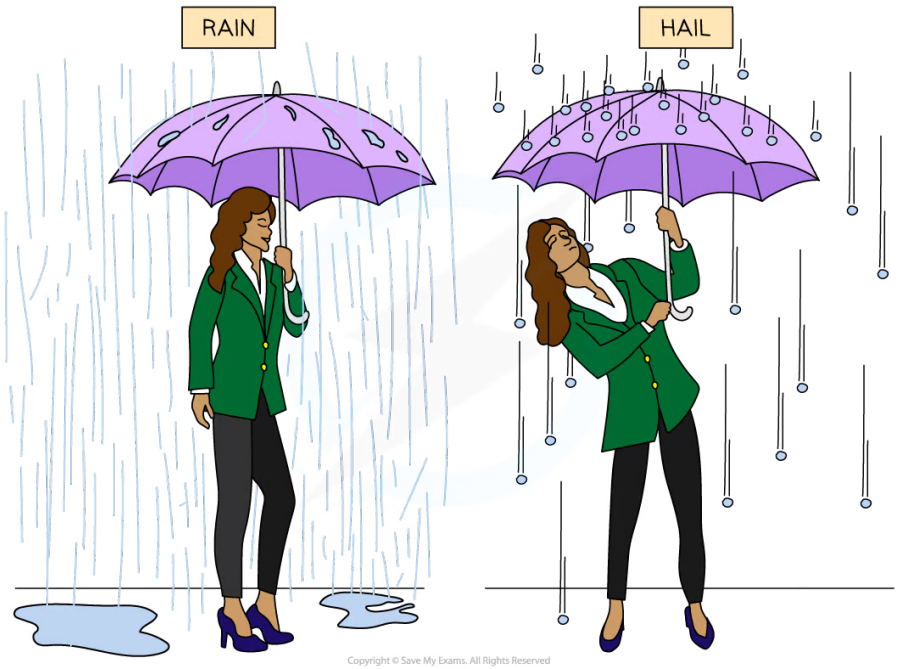
- This equation is only used when the force F is **constant**
  - Since the impulse is **proportional** to the force, it is also a **vector**
  - The impulse is in the **same direction** as the **force**

- The impulse quantifies the effect of a force acting over a time interval
  - This means a **small force acting over a long time** has the same effect as a **large force acting over a short time**

Impulse Examples

- An example of impulse in everyday life is the effect of rain on an umbrella, compared to hail (frozen water droplets)
  - When rain hits an umbrella, the water droplets tend to splatter and fall off it and there is only a very **small** change in momentum
  - However, hailstones have a **larger mass** and tend to bounce back off the umbrella, creating a **greater** change in momentum
  - Therefore, the impulse on an umbrella is **greater** in hail than in rain
  - This means that **more force** is required to hold an umbrella upright in hail compared to rain

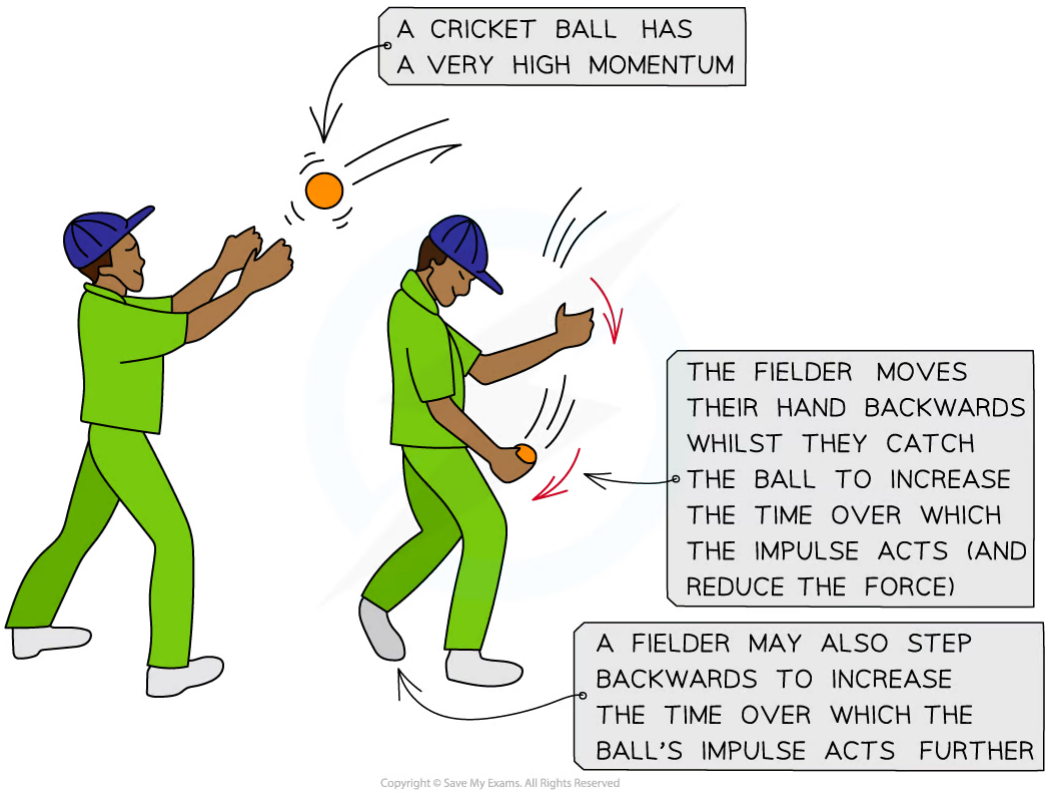
YOUR NOTES



Since hailstones bounce back off an umbrella, compared to water droplets from rain, there is a greater impulse on an umbrella in hail than in rain

- The concept of impulse is used to prevent injury
  - Increasing the time in which the change in momentum occurs, reduces the force
- For example, in cricket:
  - A cricket ball travels at very high speeds and therefore has a **high momentum**
  - When a fielder catches the ball, it exerts a force on their hands
  - Stopping a ball with high momentum abruptly will cause a large force on their hands
  - This is because a change in momentum (impulse) acts over a **short period of time** which creates a **large force** on the fielder's hands and could cause serious injury
  - A fielder moves their hands back when they catch the ball, which **increases the time** for its change in momentum to occur
  - This means there will be **less force** exerted on the fielder's hands and therefore less chance of injury

YOUR NOTES



A cricket fielder moves their hands backwards when catching a cricket ball to reduce the force it will exert on their hands

YOUR NOTES

Worked Example

A 58 g tennis ball moving horizontally to the left at a speed of 30 m s^{-1} is struck by a tennis racket which returns the ball to the right at 20 m s^{-1}.

(a)
Calculate the impulse delivered to the ball by the racket
(b)
State the direction of the impulse

Answer:

(a)

Step 1: **List the known quantities**

- Taking the initial direction of the ball as positive (the left)
  - Initial velocity, $u = 30$ m s^{-1}
  - Final velocity, $v = -20$ m s^{-1}
  - Mass, $m = 58$ g $= 58 \times 10^{-3}$ kg

Step 2: Write down the impulse equation

$$J = \Delta p = mv - mu = m(v - u)$$

Step 3: Substitute in the known values

$$J = (58 \times 10^{-3}) \times (-20 - 30) = -2.9 \text{ N s}$$

(b)

Step 1: State the direction of the impulse

- Since the impulse is negative, it must be in the opposite direction to which the tennis ball was initially travelling
- Therefore, (since the left is taken as positive) the direction of the impulse is to the **right**

Exam Tip

Remember that if an object changes direction, then this must be reflected by the change in the sign of the velocity (and impulse). This is the most common mistake made by students. Velocity, impulse, force and momentum are all **vectors!**

For example, if the left is taken as positive and therefore the right as negative, an impulse of 20 N s to the right is equal to –20 N s

YOUR NOTES

Force & Momentum

- The resultant force on a body is the **rate of change of momentum**
- The change in momentum is defined as:

$$\Delta p = p_f - p_i$$

- Where:
  - Δp = change in momentum (kg m s^{-1})
  - p_f = final momentum (kg m s^{-1})
  - p_i = initial momentum (kg m s^{-1})
- These can be expressed as follows:

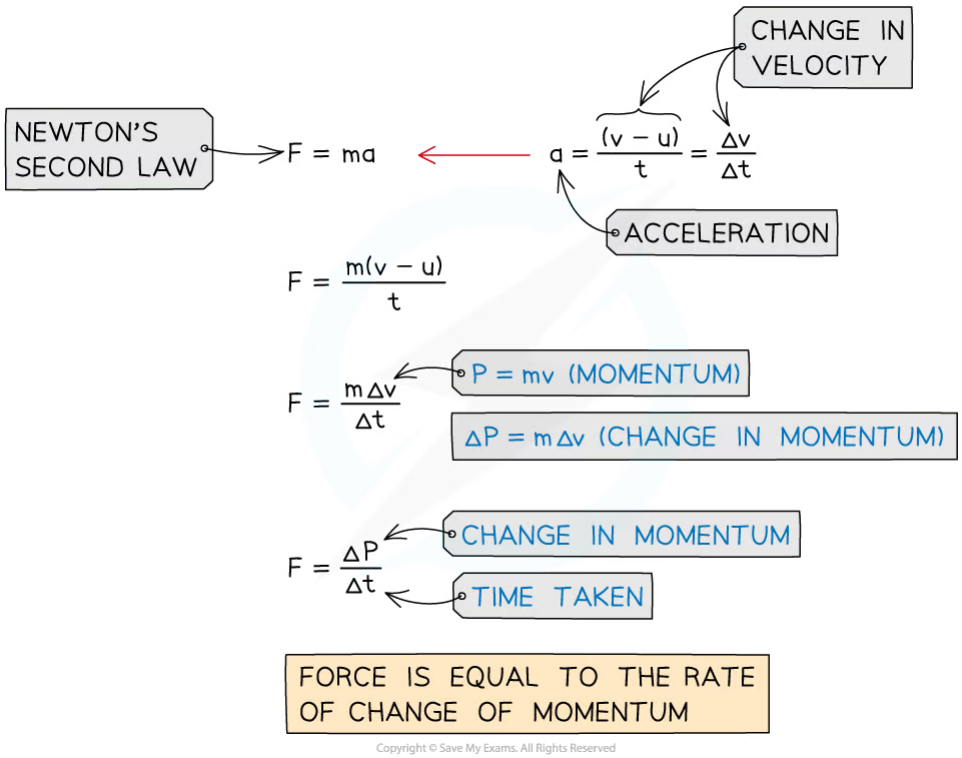
$$F = \frac{\Delta p}{\Delta t}$$

- Where:
  - F = resultant force (N)
  - Δt = change in time (s)
- This equation can be used in situations where the **mass** of the body is **not** constant
- It should be noted that the force in this situation is equivalent to Newton's second law:

$$F = ma$$

- This equation can **only** be used when the **mass** is **constant**
- The force and momentum equation can be derived from Newton's second law and the definition of acceleration

YOUR NOTES



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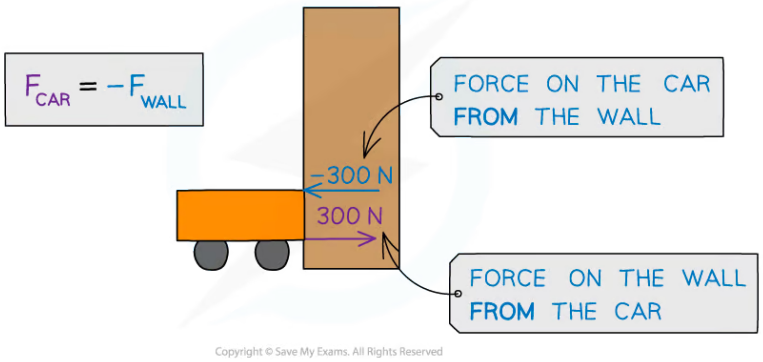
Direction of Forces

- Force and momentum are **vectors** so they can take either positive or negative values
- The force that is equal to the rate of change of momentum is still the **resultant force**
- A force on an object will be negative if it acts in the opposite direction to its initial velocity
  - This means that the force is **produced by** the object it has collided with



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The wall produces a force of -300N on the car and (due to Newton's Third Law) the car also produces a force of 300 N back onto the wall

YOUR NOTES

Worked Example

A car of mass 1500 kg hits a wall at an initial velocity of 15 m s^{-1}. It then rebounds off the wall at 5 m s^{-1}. The car is in contact with the wall for 3.0 seconds.

Calculate the average force experienced by the car.

STEP 1

FORCE IS EQUAL TO THE RATE OF CHANGE IN MOMENTUM

$$F = \frac{\Delta p}{\Delta t}$$

STEP 2

CHANGE IN MOMENTUM

Δp = FINAL MOMENTUM − INITIAL MOMENTUM

STEP 3

INITIAL MOMENTUM

INITIAL MOMENTUM = MASS × INITIAL VELOCITY

$$P_i = m \times v_i$$
$$= 1500\ kg \times 15\ ms^{-1}$$
$$P_i = 22500\ kgms^{-1}$$

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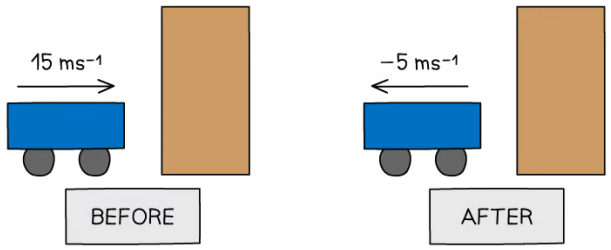

STEP 4

FINAL MOMENTUM

FINAL MOMENTUM = MASS × FINAL VELOCITY

$$P_f = m \times v_f$$
$$= 1500\ kg \times -5\ ms^{-1}$$
$$P_f = -7500\ kgms^{-1}$$

STEP 5

CALCULATE CHANGE IN MOMENTUM Δp

$$\Delta p = -7500 - 22500 = -30000\ kgms^{-1}$$

STEP 6

SUBSTITUTE THIS VALUE BACK INTO THE FORCE EQUATION

$$F = \frac{\Delta p}{\Delta t} = \frac{-30000}{3} = -10000N$$

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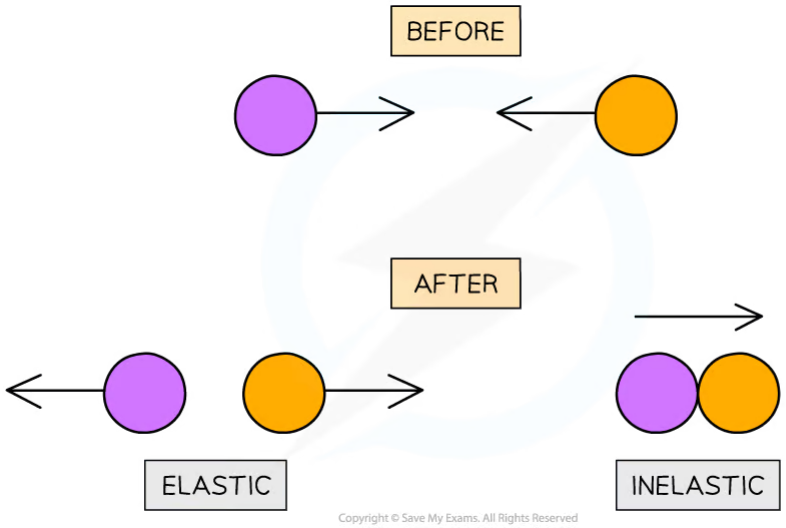
Exam Tip

The direction you consider positive is **your choice**, as long the signs of the numbers (positive or negative) are **consistent** with this throughout the question. In an exam question, carefully consider what produces the force(s) acting. Look out for words such as '**from**' or '**acting on**' to determine this. It can be really helpful to draw a force diagram to figure out what is going on.

YOUR NOTES

Collisions & Explosions in One-Dimension

- In both collisions and explosions, **momentum is always conserved**
  - However, **kinetic energy** might not always be
- A collision (or explosion) is either:
  - **Elastic** – if the kinetic energy **is** conserved
  - **Inelastic** – if the kinetic energy is **not** conserved
- Collisions are when objects strike against each other
  - **Elastic** collisions are commonly those where the objects colliding do not stick together and then they **move in opposite directions**
  - **Inelastic** collisions are commonly those where objects collide and **stick together** after the collision



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Elastic collisions are where two objects move in opposite directions. Inelastic collisions are where two objects stick together

- An explosion is commonly to do with **recoil**
  - For example, a gun recoiling after shooting a bullet or an unstable nucleus emitting an alpha particle and a daughter nucleus
- To find out whether a collision is elastic or inelastic, **compare the kinetic energy before and after the collision**
- The equation for kinetic energy is:

$$E_k = \frac{1}{2}mv^2$$

- Where:
  - E_k = kinetic energy (J)
  - m = mass (kg)

- v = velocity ($m\ s^{-1}$)

YOUR NOTES

Worked Example

Two similar spheres, each of mass m and velocity v are travelling towards each other. The spheres have a head-on elastic collision.

What is the total kinetic energy after the impact?

A. $\frac{1}{2}mv^2$ **B.** 0 **C.** mv^2 **D.** $2mv$

ANSWER: C

IN AN ELASTIC COLLISION, KINETIC ENERGY IS CONSERVED.

THIS MEANS KINETIC ENERGY BEFORE = KINETIC ENERGY AFTER.

KINETIC ENERGY BEFORE = $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$.

IN AN ELASTIC COLLISION, KINETIC ENERGY AFTER WILL ALSO EQUAL mv^2.

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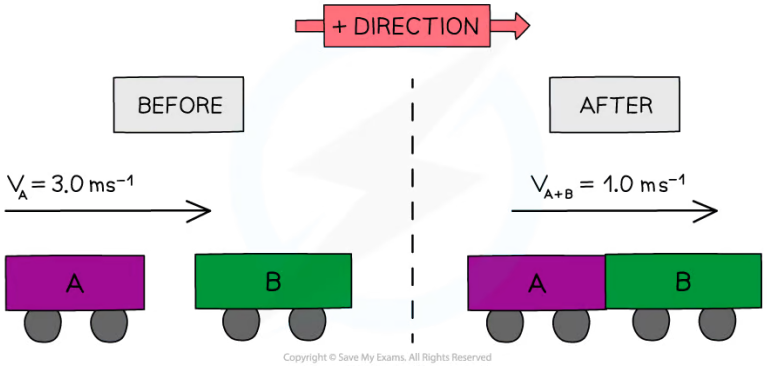
YOUR NOTES

Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** at speed 3.0 m s^{-1}. Trolley **B** has twice the mass of trolley **A**. The trolleys stick together and travel at a velocity of 1.0 m s^{-1}.

Determine whether this is an elastic or inelastic collision.



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KINETIC ENERGY = 1/2 mv²

KINETIC ENERGY BEFORE $= \frac{1}{2} \times M_A \times (V_A)^2 + \frac{1}{2} \times M_B \times (V_B^2)$

$= \frac{1}{2} \times 0.8 \times (3.0)^2 + 0$ ← $V_B = 0$

$= 3.6\ J$

KINETIC ENERGY AFTER $= \frac{1}{2} \times M_{A+B} \times (V_{A+B})^2$

$= \frac{1}{2} \times 2.4 \times (1.0)^2$

$= 1.2\ J$

THIS IS AN INELASTIC COLLISION SINCE KINETIC ENERGY IS **NOT** CONSERVED

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Exam Tip

If an object is **stationary** or at **rest**, its initial velocity is **0**, therefore, the momentum and kinetic energy are also equal to 0.

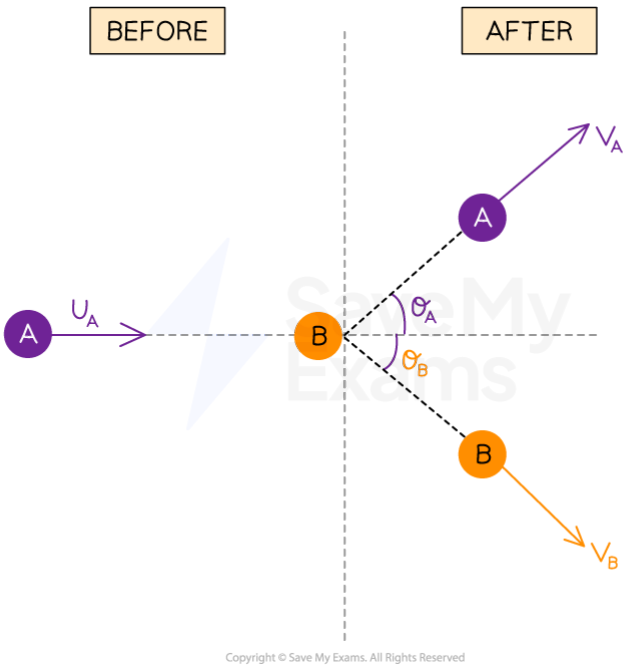
When a collision occurs in which two objects are stuck together, treat the final object as a **single** object with a **mass** equal to the **sum** of the masses of the two individual objects.

Despite velocity being a vector, kinetic energy is a **scalar** quantity and therefore will **never** include a minus sign - this is because in the kinetic energy formula, mass is scalar and the v^2 will always give a positive value whether it's a negative or positive velocity.

YOUR NOTES

Collisions & Explosions in Two-Dimensions

- We know that momentum is always **conserved**
- This doesn't just apply to the motion of colliding objects in one dimension (in one line), but this is true in **every** direction
- Since momentum is a **vector**, it can be split into its horizontal and vertical component
  - This is done by resolving vectors
- Consider again the two colliding balls A and B
- Before the collision, ball A is moving at speed u_A and hits stationary ball B
  - Ball A moves away at speed v_A and angle θ_A
  - Ball B moves away at speed v_B and angle θ_B



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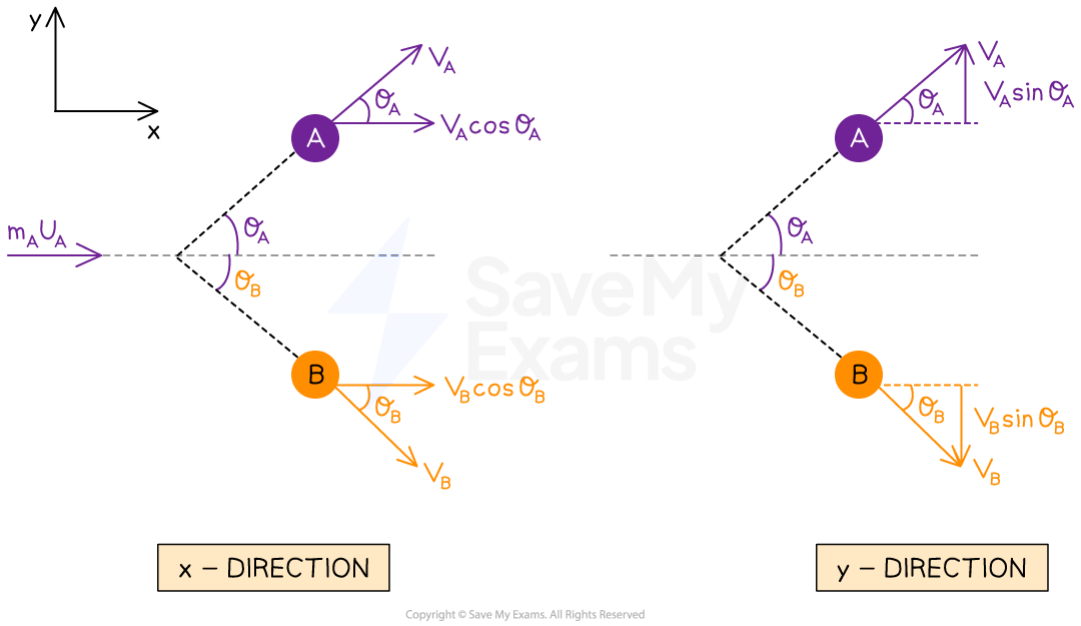
- This time, they move off in different directions, so we now need to consider their momentum in the *x* direction **and** separately, their momentum in the *y* direction
  - This is done by resolving the **velocity** vector of each ball after the collision



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- Applying the conservation of momentum along the ***x*** direction gives

$$m_A u_A + 0 = m_A v_A \cos\theta_A + m_B v_B \cos\theta_B$$

- Applying the conservation of momentum along the ***y*** direction gives

$$0 + 0 = m_A v_A \sin\theta_A - m_B v_B \sin\theta_B$$

- The minus sign now comes from B moving **downwards**, whilst positive *y* is considered **upwards**
- The momentum **before** in the *y* direction is **0** for both balls A and B because B is stationary and A is **only** travelling in the ***x*** direction, so u_A has no vertical component
- Since there are two equations involving sine and cosine, it is helpful to remember the trigonometric identity:

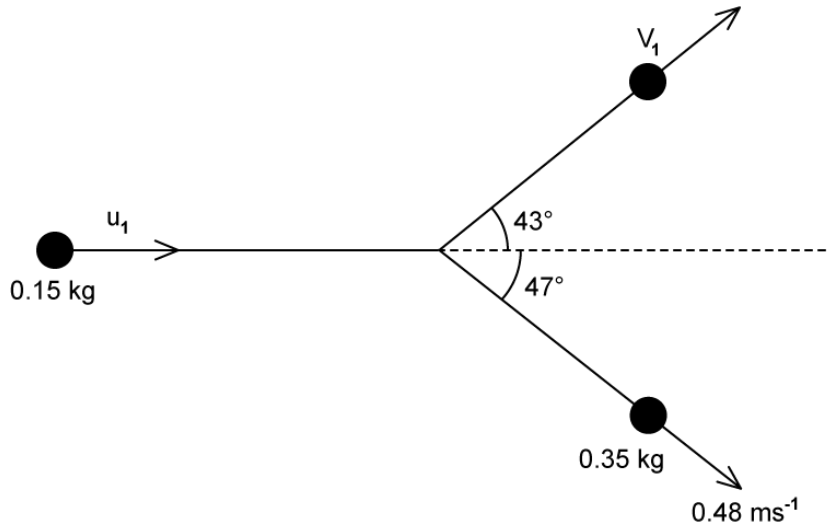
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

- When the collision is elastic, the conservation of linear momentum and energy indicates that $\theta_A + \theta_B = 90°$

YOUR NOTES

Worked Example

A snooker ball of mass 0.15 kg collides with a stationary snooker ball of mass 0.35 kg. After the collision, the second snooker ball moves away with a speed of 0.48 m s^{-1}. The paths of the balls make angles of 43° and 47° with the original direction of the first snooker ball.



Calculate the speed u_1 and v_1 of the first snooker ball before and after the collision.

Answer

Step 1: List the known quantities

- Mass of the first snooker ball, $m_1 = 0.15$ kg
- Mass of the second snooker ball, $m_2 = 0.35$ kg
- Velocity of second ball after, $v_2 = 0.48$ m s^{-1}
- Angle of the first ball, $\theta_1 = 43°$
- Angle of the first ball, $\theta_2 = 47°$

Step 2: State the equation for the conservation of momentum in the *y* (vertical) direction

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$$

Step 3: Calculate the speed of the first ball after the collision, v_1

- Use the conservation of momentum in the *y* direction to calculate the speed of the first snooker ball after the collision

$$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$$

$$v_1 = \frac{m_2 v_2 \sin\theta_2}{m_1 \sin\theta_1}$$

$$v_1 = \frac{0.35 \times 0.48 \times \sin(47)}{0.15 \times \sin(43)} = 1.2 \text{ m s}^{-1}$$

Step 3: State the equation for the conservation of momentum in the *x* (horizontal) direction

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

Step 4: Calculate the speed of the first ball before the collision, u_1

$$u_1 = \frac{m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2}{m_1}$$

$$u_1 = \frac{(0.15 \times 0.56 \times \cos(43)) + (0.35 \times 0.48 \times \cos(47))}{0.15} = 1.6 \text{ m s}^{-1}$$

YOUR NOTES

Exam Tip

Make sure you clearly label your diagram or write out the known quantities before you substitute values into the question. It's very easy to substitute in the incorrect velocity or mass. Use subscripts such as '1' '2' or 'A' 'B' depending on the question to help keep track of these.

Although you will get full marks either way, it may be easier in these equations to rearrange first and then substitute instead of the other way around, to keep track of the multiple masses and velocities.

Make sure your calculator is in degree mode if your angles are given in degrees!

If you use the fraction function to input your values, remember that you need to close the brackets on the trig functions or it will give you the wrong answer.

- Eg. $\frac{(0.35 \times 0.48 \times \sin(47))}{(0.15 \times \sin(43))} = 1.2\ \mathrm{m\,s^{-1}}$

And if you input the values into your calculator as numerator ÷ denominator, make sure you put brackets around the whole denominator.

- Eg. $Ans \div (0.15 \times \sin(43))$

The trig equation for $\tan\theta$ is also given on your data sheet under 'Mathematical equations', as well as that for resolving forces.

YOUR NOTES

Angular Velocity

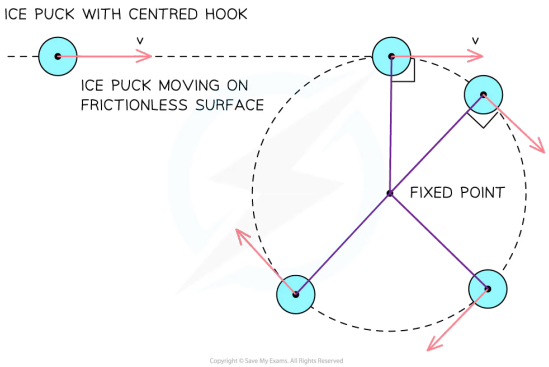
Motion in a Straight Line

- When an object moves in a straight line at a constant speed its motion can be described as follows:
  - The object moves at a constant velocity, *v*
  - Constant velocity means zero acceleration, *a*
  - Newton's First Law of motion says the object will continue to travel in a straight line at a constant speed unless acted on by another force
  - Newton's Second Law of motion says that for zero acceleration there is no net or resultant force
- For example, an ice hockey puck moving across a flat frictionless ice rink

An ice puck moving in a straight line

Motion in a Circle

- If one end of a string was attached to the puck, and the other attached to a fixed point, it would no longer travel in a straight line, it would begin to travel in a circle



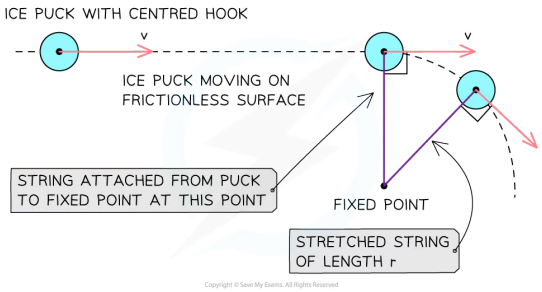
The red arrows represent the velocity vectors of the puck. If the string were cut, the puck would move off in the direction shown by the red vector, as predicted by Newton's first law.

- The motion of the puck can now be described as follows:
  - As the puck moves it stretches the string a little to a length *r*
  - The stretched string applies a force to the puck pulling it so that it moves in a circle of radius *r* around the fixed point
- The force acts at 90° to the velocity so there is no force component in the direction of velocity

YOUR NOTES

- As a result, the **magnitude** of the velocity is constant
- However, the **direction** of the velocity **changes**

- As it starts to move in a circle the tension of the string continues to pull the puck at 90° to the velocity
  - The speed does not change, hence, this is called **uniform circular motion**



The applied force (tension) from the string causes the puck to move with uniform circular motion

Time Period & Frequency

- If the circle has a radius *r*, then the distance through which the puck moves as it completes one rotation is equal to the circumference of the circle = 2π*r*
- The speed of the puck is therefore equal to:

$$speed = \frac{distance\ travelled}{time\ taken} = \frac{2\pi r}{T}$$

- Where:
  - *r* = the radius of the circle (m)
  - *T* = the time period (s)
- This is the same as the time period in waves and simple harmonic motion (SHM)
- The frequency, *f*, can be determined from the equation:

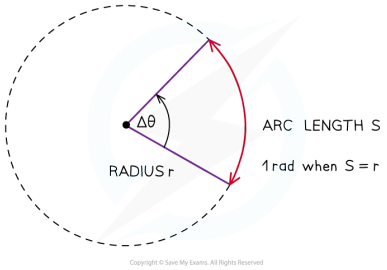
$$f = \frac{1}{T}$$

- Where:
  - *f* = frequency (Hz)
  - *T* = the time period (s)

Angles in Radians

- A **radian** (rad) is defined as:

> **The angle subtended at the centre of a circle by an arc equal in length to the radius of the circle**



When the angle is equal to one radian, the length of the arc (S) is equal to the radius (r) of the circle

- Radians are commonly written in terms of π
- The angle in radians for a complete circle (360°) is equal to:

$$\frac{circumference\ of\ circle}{radius} = \frac{2\pi r}{r} = 2\pi$$

- Use the following equation to convert from degrees to radians:

$$\theta° \times \frac{\pi}{180} = \theta \text{ rad}$$

Table of common degrees to radians conversions

Degrees (°)	Radians (rads)
360	2π
270	$\frac{3\pi}{2}$
180	π
90	$\frac{\pi}{2}$

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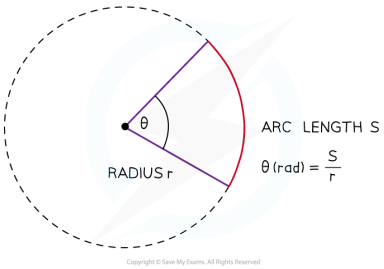
Angular Displacement

- In circular motion, it is more convenient to measure angular displacement in units of radians rather than units of degrees
- Angular displacement is defined as:

The change in angle, in radians, of a body as it rotates around a circle

YOUR NOTES

- Where:
  - $\Delta\theta$ = angular displacement, or angle of rotation (radians)
  - S = length of the arc, or the distance travelled around the circle (m)
  - r = radius of the circle (m)



An angle in radians, subtended at the centre of a circle, is the arc length divided by the radius of the circle

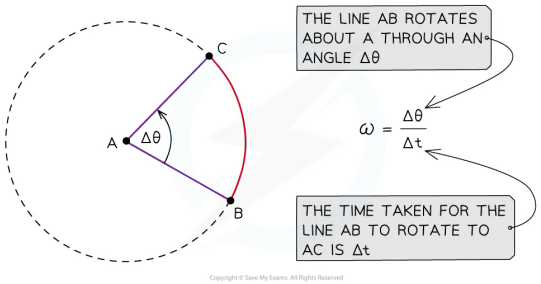
Angular Velocity

- Any object rotating with a uniform circular motion has a constant speed but constantly changing velocity
- Its velocity is changing so it is **accelerating**
  - But at the same time, it is moving at a constant speed
- The angular speed, ω, of a body in circular motion is defined as:

  The rate of change in angular displacement with respect to time

- Angular speed is a **scalar** quantity and is measured in rad s^{-1}

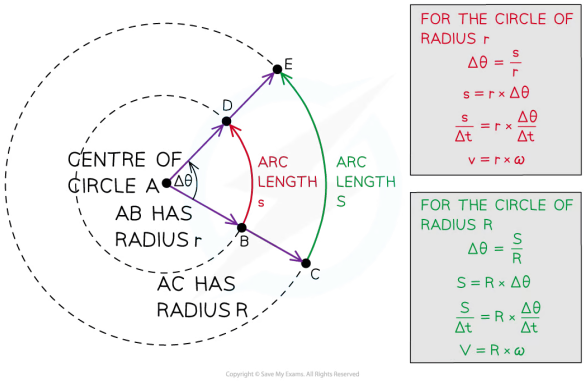
- The angular speed does not depend on the length of the line AB
- The line AB will sweep out an angle of 2π rad in a time T



The angular speed is ω is the rate at which the line AB rotates

Angular Velocity & Linear Speed

- Angular velocity is a **vector** quantity and is measured in rad s^{-1}
- Angular speed is the **magnitude** of the angular velocity
- Although the angular speed doesn't depend on the radius of the circle, the **linear** speed **does**



The angle Δθ is swept out in a time Δt, but the arc lengths s and S are different and so are the linear speeds

- The linear speed, *v*, is related to the angular speed, ω, by the equation:

$$v = r\omega$$

- Where:
  - *v* = linear speed (m s^{-1})
  - *r* = radius of circle (m)
  - ω = angular speed (rad s^{-1})
- Taking the angular displacement of a complete cycle as 2π, the angular speed ω can be calculated using the equation:

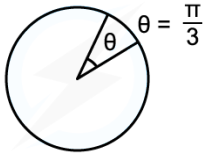
$$\omega = 2\pi f = \frac{2\pi}{T}$$

- Therefore, the linear velocity can also be written as:

$$v = \frac{2\pi r}{T}$$

Worked Example

Convert the following angular displacement into degrees:



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STEP 1 REARRANGE DEGREES TO RADIANS CONVERSION EQUATION

DEGREES → RADIANS $\theta^\circ \times \frac{\pi}{180} = \theta \text{ RAD}$

RADIANS → DEGREES $\theta \text{ RAD} \times \frac{180}{\pi} = \theta^\circ$

STEP 2 SUBSTITUTE VALUE

π's WILL CANCEL OUT

$$\frac{\pi}{3} \text{ RAD} \times \frac{180}{\pi} = \frac{180^\circ}{3} = 60^\circ$$

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YOUR NOTES

YOUR NOTES

Worked Example

A bird flies in a horizontal circle with an angular speed of 5.25 rad s^{-1} of radius 650 m.

Calculate:

(a)
The linear speed of the bird

(b)
The frequency of the bird flying in a complete circle

a) STEP 1

LINEAR SPEED EQUATION

$v = r\omega$

STEP 2

SUBSTITUTE IN VALUES

$V = 650 \times 5.25 = 3412.5 = 3410\ ms^{-1}$ (3 s.f.)

b) STEP 1

ANGULAR SPEED WITH FREQUENCY EQUATION

$\omega = 2\pi f$

STEP 2

REARRANGE FOR FREQUENCY

$f = \frac{\omega}{2\pi}$

STEP 3

SUBSTITUTE IN VALUES

$f = \frac{5.25}{2\pi} = 0.83556... = 0.836\ Hz$ (3 s.f.)

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Exam Tip

Remember the units of angular velocity as **rad** s^{-1}, so any angles used in calculations must be in **radians** and **not** degrees!

T is the time period which is the time taken for **one full** revolution.

YOUR NOTES

Centripetal Force

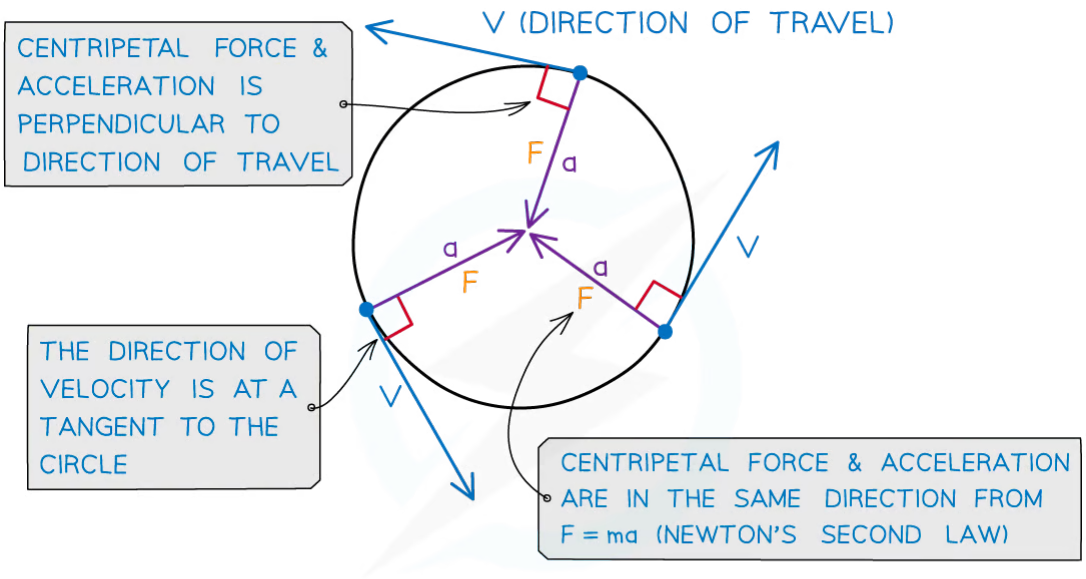
- Velocity and acceleration are both vector quantities
- An object in uniform circular motion is **continuously changing direction,** and therefore is **constantly changing velocity**
  - The object must therefore be **accelerating**
- This is called the **centripetal acceleration** and is **perpendicular** to the direction of the linear speed
  - Centripetal means it acts **towards the centre** of the circular path
- From Newton's second law, this must mean there is a resultant force acting upon it
  - This is known as the **centripetal force** and is what keeps the object moving in a circle
  - This means the object changes direction **even if** its magnitude of velocity remains constant
- The centripetal force (*F*) is defined as:

  The resultant force perpendicular to the velocity required to keep a body in a uniform circular motion which acts towards the centre of the circle

- The magnitude of the centripetal force *F* can be calculated using:

$$F = \frac{mv^2}{r} = mr\omega^2$$

- Where:
  - F = centripetal force (N)
  - v = linear speed (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)



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Centripetal force is always perpendicular to the linear velocity (i.e, the direction of travel)

- The direction of the centripetal force is:
  - In the **same** direction as the centripetal **acceleration** (towards the centre of the circle)
    - This is due to Newton's Second Law
  - **Perpendicular** to the linear **velocity**
- The centripetal force is **not** a separate force of its own
- It can be any type of force, depending on the situation, which keeps an object moving in a circular path
  - For example, tension, friction, gravitational, electrical or magnetic

YOUR NOTES

Examples of centripetal force

Situation	Centripetal force
Car travelling around a roundabout	Friction between car tyres and the road
Ball attached to a rope moving in a circle	Tension in the rope
Earth orbiting the Sun	Gravitational force

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- When solving circular motion problems involving one of these forces, the equation for centripetal force can be equated to the relevant force equation
- For example, for a mass orbiting a planet in a circular path, the **centripetal force** is provided by the **gravitational force**

- When an object travels in circular motion, there is **no work done**
  - This is because there is **no** change in kinetic energy

Horizontal Circular Motion

- An example of horizontal circular motion is a vehicle driving on a curved road
- The forces acting on the vehicle are:
  - The **friction** between the tyres and the road
  - The **weight** of the vehicle downwards

- In this case, the centripetal force required to make this turn is provided by the frictional force
  - This is because the force of friction acts towards the centre of the circular path

- Since the centripetal force is provided by the force of friction, the following equation can be written:

$$\frac{mv^2}{r} = \mu mg$$

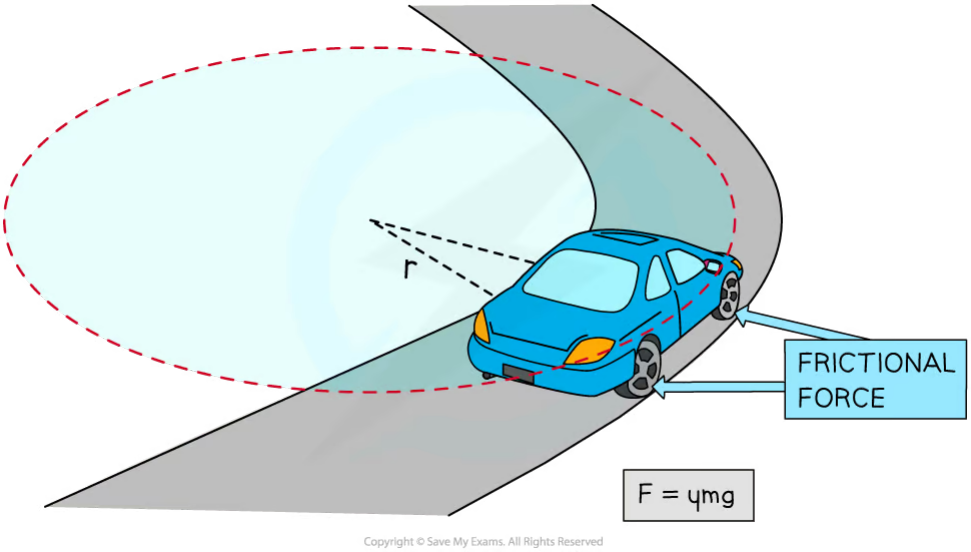
- Where:
  - m = mass of the vehicle (kg)
  - v = speed of the vehicle (m s^{-1})
  - r = radius of the circular path (m)
  - μ = static coefficient of friction
  - g = acceleration due to gravity (m s^{-2})

- Rearranging this equation for v gives:

$$v^2 = \mu g r$$

$$v_{max} = \sqrt{\mu g r}$$

- This expression gives the maximum speed at which the vehicle can travel around the curved road without skidding
  - If the speed exceeds this, then the vehicle is likely to skid
  - This is because the centripetal force required to keep the car in a circular path could not be provided by friction, as it would be too large



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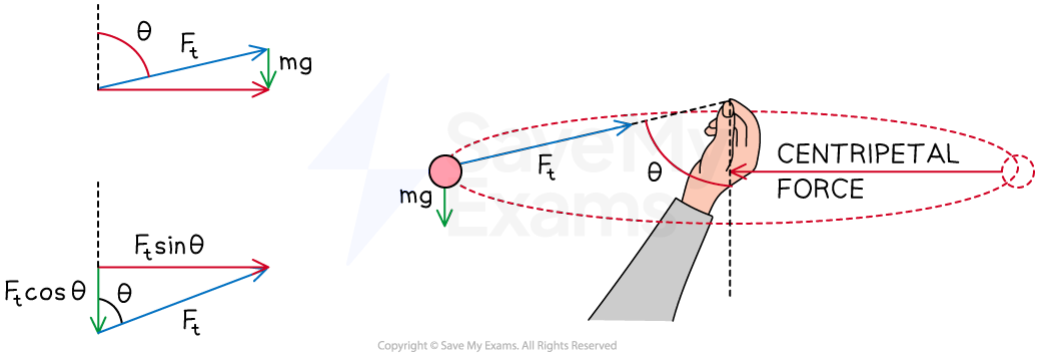
The frictional force provides the centripetal force

- Therefore, in order for a vehicle to avoid skidding on a curved road of radius *r*, its speed must satisfy the equation

$$v < \sqrt{\mu g r}$$

- In this case, the **tension** is the **centripetal force** as it acts towards the centre of the circle
- A mass attached to a string rotating around is another example of horizontal circular motion
- This time, the weight of the mass will be acting as well as the tension of the string



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A mass attached to a string will have its weight acting meaning the string is at an angle

YOUR NOTES

- The weight *mg* of the mass needs to be balanced by the **vertical** component of the tension

$$F_t \cos\theta = mg$$

- This means the string will always be at an **angle** and never perfectly horizontal
- The ball's linear velocity, *v* is still perpendicularly to the tension and its weight, *mg* points **downwards**
- All three forces are perpendicular to each other, so no other component contributes to the centripetal force, just the tension
- The centripetal force is still towards the centre of the circle, but now is just the **horizontal** component of the tension

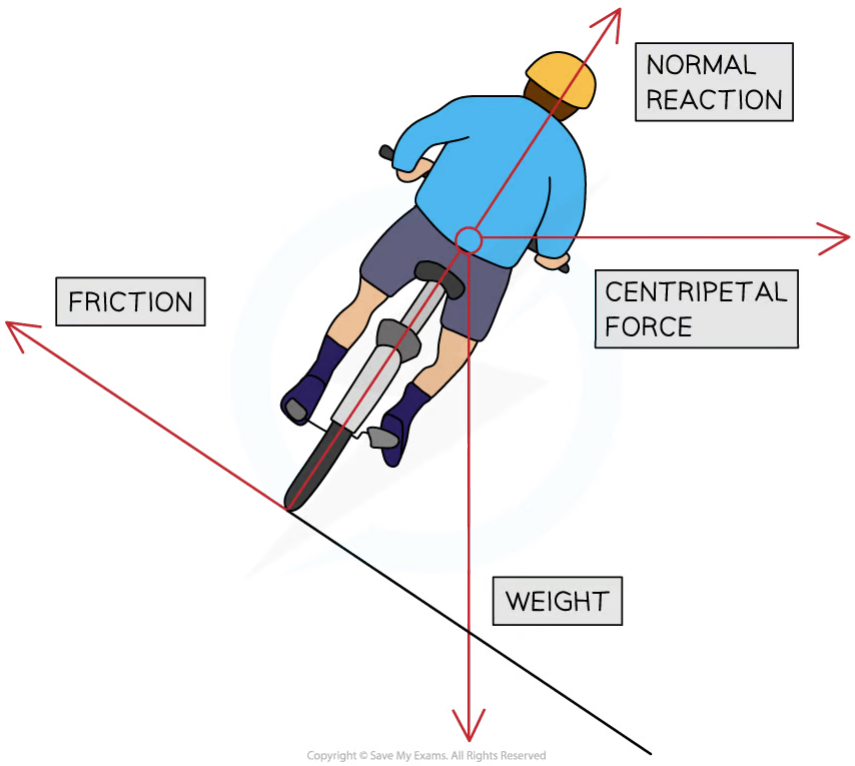
$$F_t \sin\theta = \frac{mv^2}{r}$$

- This is an important example of resolving vectors properly. The vertical component does not always have '$\sin\theta$', it depends on what θ is defined as

Banking

- A banked road, or track, is a curved surface where the outer edge is raised higher than the inner edge
  - The purpose of this is to make it safer for vehicles to travel on the curved road, or track, at a reasonable speed without skidding
- When a road is banked, the centripetal force no longer depends on the friction between the tyres and the road
- Instead, the centripetal force depends solely on the **horizontal component** of the **normal force**

YOUR NOTES



During banking, the horizontal component of the normal reaction force provides the centripetal force

YOUR NOTES

Worked Example

A 300 g ball is made to travel in a circle of radius 0.8 m on the end of a string. If the maximum force the ball can withstand before breaking is 60 N, what is the maximum speed of the ball?

Answer:

Step 1: List the known quantities

- Mass, $m = 300\ g = 300 \times 10^{-3}\ kg$
- Radius, $r = 0.8\ m$
- Resultant force, $F = 60\ N$

Step 2: Rearrange the centripetal force equation for *v*

$$F_{max} = \frac{mv^2_{max}}{r}$$

$$v_{max} = \sqrt{\frac{rF_{max}}{m}}$$

Step 3: Substitute in the values

$$v_{max} = \sqrt{\frac{0.8 \times 60}{300 \times 10^{-3}}} = 12.6\ m\ s^{-1}$$

Exam Tip

The linear speed, *v* is sometimes referred to as the 'tangential' speed.

The centripetal force equation is not given in your data book, but you are given in the equations for centripetal acceleration. You just need to multiply them by mass *m* since the centripetal force $F = ma$.

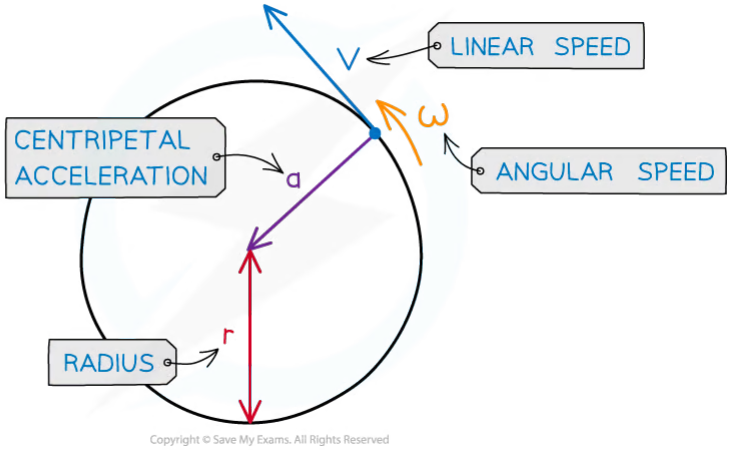
It is important you understand the foundations of circular motion, especially how to use the equations. This will heavily link with kepler's laws and magnetic fields.

YOUR NOTES

Calculating Centripetal Acceleration

- Centripetal acceleration is defined as:

> **The acceleration of an object towards the centre of a circle when an object is in motion (rotating) around a circle at a constant speed**



Centripetal acceleration is always directed toward the centre of the circle, and is perpendicular to the object's velocity

- It is directed towards the centre of the circle as it is in the **same** direction as the centripetal force
- It can be defined using the radius *r* and linear speed *v*:

$$a = \frac{v^2}{r}$$

- Where:
  - *a* = centripetal acceleration (m s^{-2})
  - *v* = linear speed (m s^{-1})
  - *r* = radius of the circular orbit (m)
- Using the equation relating angular speed ω and linear speed *v*:

$$v = r\omega$$

- Where:
  - ω = angular speed (rad s^{-1})
- These equations can be combined to give another form of the centripetal acceleration equation:

$$a = \omega^2 r$$

- Alternatively, since we know angular velocity is:

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- Where:
  - f = frequency (Hz)
  - T = time period (s)
- This means the centripetal acceleration can also be written as:

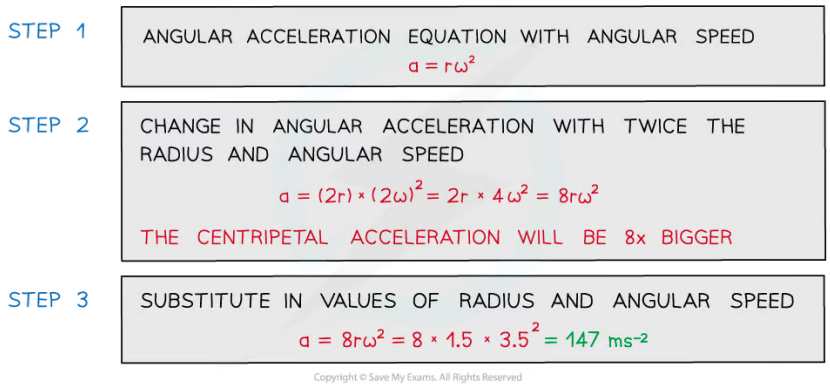
$$a = \left(\frac{2\pi}{T}\right)^2 r = \frac{4\pi^2 r}{T}$$

- This equation shows how the centripetal acceleration relates to the linear speed and the angular speed

Worked Example

A ball tied to a string is rotating in a horizontal circle with a radius of 1.5 m and an angular speed of 3.5 rad s^{-1}.

Calculate its centripetal acceleration if the radius was twice as large and angular speed was twice as fast.



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Exam Tip

The equations for centripetal acceleration are given on your data sheet in multiple forms. Which form you use depends on what you're given in the question i.e. v or ω

YOUR NOTES

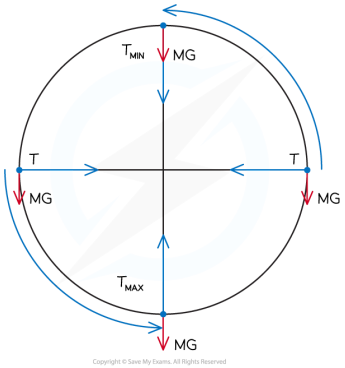
YOUR NOTES

Non-Uniform Circular Motion

- Some bodies are in **non-uniform** circular motion
- This happens when there is a **changing resultant force** such as in a **vertical** circle

- An example of vertical circular motion is swinging a ball on a string in a vertical circle
- The forces acting on the ball are:
  - The **tension** in the string
  - The **weight** of the ball downwards

- As the ball moves around the circle, the **direction** of the tension will change continuously
- The **magnitude** of the tension will also vary continuously, reaching a **maximum** value at the **bottom** and a **minimum** value at the **top**
  - This is because the direction of the weight of the ball never changes, so the resultant force will vary depending on the position of the ball in the circle



- At the bottom of the circle, the tension must overcome the weight, this can be written as:

$$T_{max} = \frac{mv^2}{r} + mg$$

- As a result, the acceleration, and hence, the **speed** of the ball will be **slower** at the top
- At the top of the circle, the tension and weight act in the same direction, this can be written as:

$$T_{min} = \frac{mv^2}{r} - mg$$

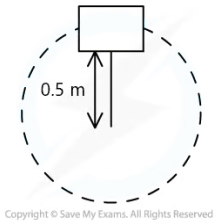
- As a result, the acceleration, and hence, the **speed** of the ball will be **faster** at the bottom

YOUR NOTES

Worked Example

A bucket of mass 8.0 kg is filled with water and is attached to a string of length 0.5 m.

What is the minimum speed the bucket must have at the top of the circle so no water spills out?



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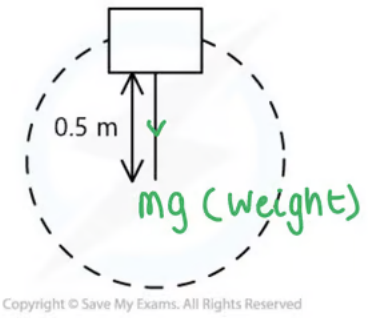
Answer:

Step 1: Draw the forces on the bucket at the top



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- Although tension is in the rope, at the very top, the tension is 0

Step 2: Calculate the centripetal force

- The weight of the bucket = *mg*
- This is equal to the centripetal force since it is directed towards the centre of the circle

$$mg = \frac{mv^2}{r}$$

Step 3: Rearrange for velocity *v*

- *m* cancels from both sides

$$v = \sqrt{gr}$$

Step 4: Substitute in values

$$v = \sqrt{9.81 \times 0.5} = 2.21 \text{ m s}^{-1}$$

YOUR NOTES

Work, Energy & Power

CONTENTS

Principle of Conservation of Energy
Sankey Diagrams
Work Done
Kinetic Energy
Gravitational Potential Energy
Elastic Potential Energy
Conservation of Mechanical Energy
Energy & Power
Efficiency Formula
Energy Density

YOUR NOTES

Principle of Conservation of Energy

- The Principle of conservation of energy states that:

  Energy cannot be created or destroyed, it can only be transferred from one form to another

- This means the total amount of energy in a closed system **remains constant**, although how much of **each form** there is **may change**

Types of Energy

FORM	WHAT IS IT?
KINETIC	THE ENERGY OF A MOVING OBJECT.
GRAVITATIONAL POTENTIAL	THE ENERGY SOMETHING GAINS WHEN YOU LIFT IT UP, AND WHICH IT LOSES WHEN IT FALLS.
ELASTIC	THE ENERGY OF A STRETCHED SPRING OR ELASTIC BAND.(SOMETIMES CALLED STRAIN ENERGY)
CHEMICAL	THE ENERGY CONTAINED IN A CHEMICAL SUBSTANCE.
NUCLEAR	THE ENERGY CONTAINED WITHIN THE NUCLEUS OF AN ATOM.
INTERNAL	THE ENERGY SOMETHING HAS DUE TO ITS TEMPERATURE (OR STATE). (SOMETIMES REFERRED TO AS THERMAL OR HEAT ENERGY)

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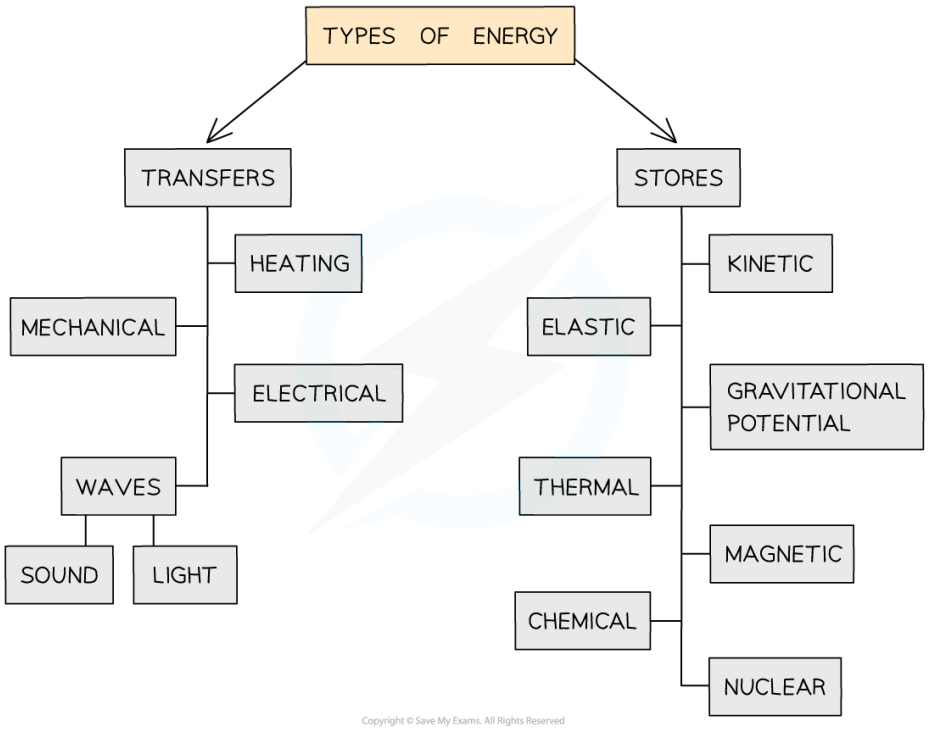
YOUR NOTES



Energy types can be separated into transfers or stores

- The differences between **energy stores** and **energy transfers** can be confusing, so it is important to learn their differences and the different types of energy involved in each

Energy Stores

- **Energy stores** keep or store energy within one part of a system
- Kinetic energy store: the amount of energy in a kinetic energy store depends on the **speed** of the object
  - A car with a high kinetic energy store within the engine can drive more quickly than a car with a smaller kinetic energy store
    - A racing car has a higher kinetic energy store than a tractor
- Gravitational potential energy store: the amount of energy stored in an object depending on its **height**
  - The higher an object, the more gravitational potential energy it has
    - When a diver stands on a board 5 m high he has more gravitational potential energy than when standing on a board 3 m high
- Magnetic energy store: magnets store magnetic energy until a magnetic material is present in its field
  - A piece of iron will move when it enters a magnetic field
  - The magnetic energy store is transferred to the kinetic energy of the iron
- Chemical energy store: energy is stored as chemical energy and a chemical reaction takes place to release and transfer it
  - Chemical energy is stored in our **body** for use when we think and move
  - Chemical energy is stored in a **battery** to be transferred to **electrical** energy

- Thermal energy store: all objects store **thermal** energy
  - An object that is **hotter** stores **more** thermal energy than an object that is **colder**
- Nuclear energy store: in a nuclear reactor, energy is stored as uranium-235 until it is bombarded by neutrons and a considerable amount of thermal energy is released

YOUR NOTES

Energy Transfers

- **Energy transfers:** give or transfer energy to different parts of a system
- They act as a **pathway** around an energy system
- **Electrical** transfer: when charge flows to produce an electric current
  - The current transfers the energy
- **Mechanical** transfer: this occurs when a force is applied to move an object
  - This could be pushing a book across a desk
  - It could also be sound waves passing through a material causing the particles to move
- **Heating** transfer: The internal energy of an object is determined by the temperature of the object
  - Energy is transferred from hotter to cooler areas
- **Waves** transfer: When sound travels through a material it causes the particles to vibrate
  - Energy is transferred from an object that is moving/vibrating to generate sound waves through other objects by the movement of particles
  - Light energy is transferred from the sun so we can see

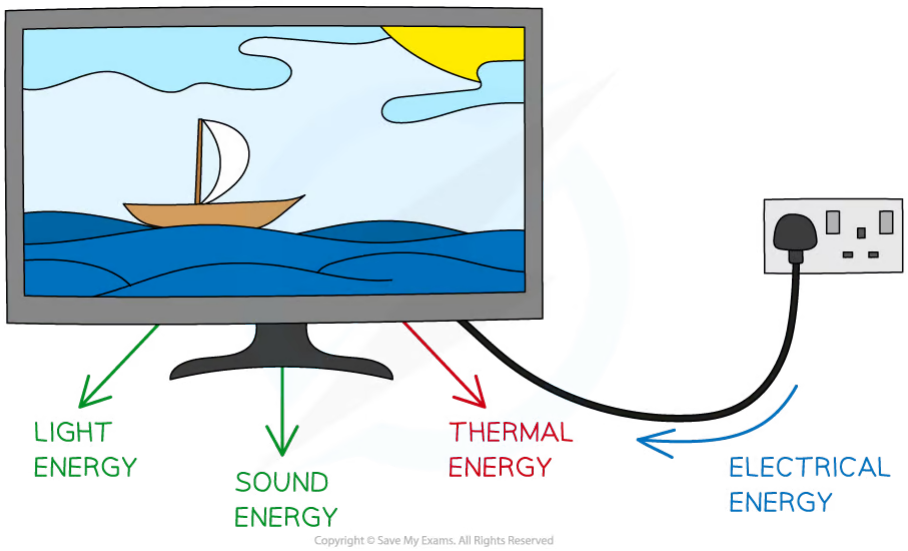
Energy Dissipation

- When energy is transferred from one form to another, not all the energy will end up in the desired form (or place)
- **Dissipation** is used to describe ways in which energy is **wasted**
- Any energy not transferred to useful energy stores is wasted because it is lost to the surroundings
- These are commonly in the form of **thermal (heat), light, or sound** energy
- What counts as **wasted energy** depends on the system
- For example, in a **television**:

$$\text{electrical energy} \rightarrow \text{light energy} + \text{sound energy} + \text{thermal energy}$$

- Light and sound energy are useful energy transfers whereas thermal energy (from the heating up of wires) is wasted

YOUR NOTES



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Useful and wasted energy conversions for a television

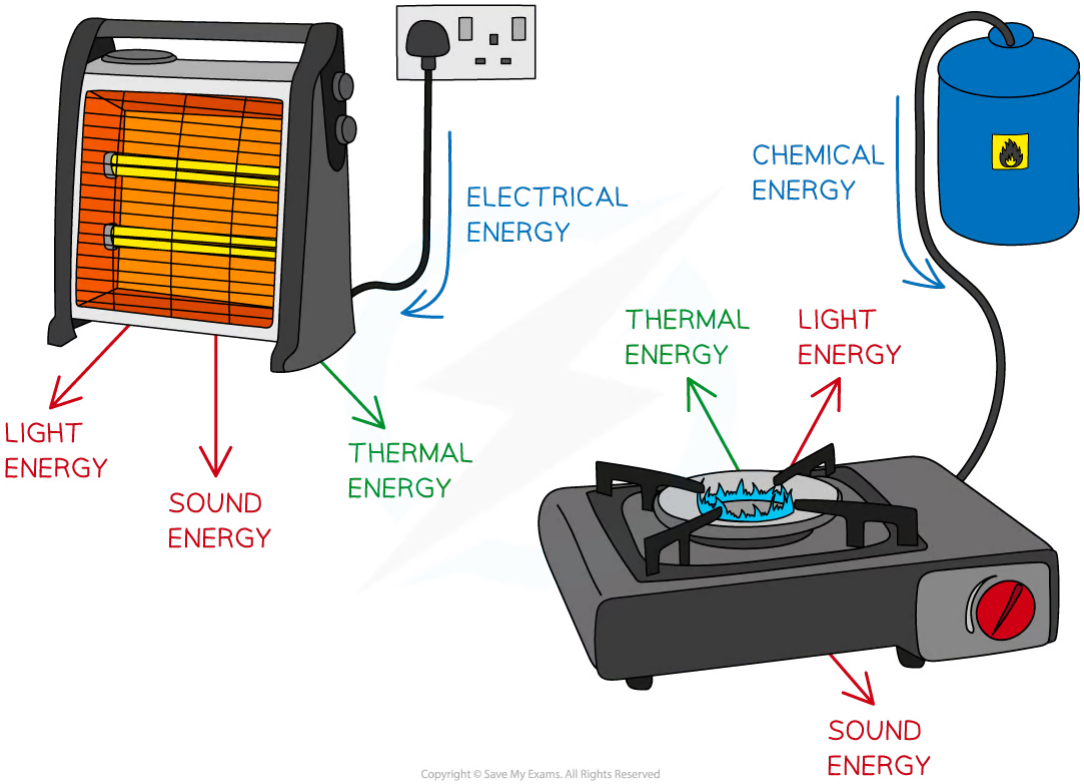
- The energy changes in an **electrical heater**:

electrical energy → thermal energy + sound energy + light energy

- In a **gas cooker**, the energy transfers are similar but the initial source of energy is different:

chemical energy → thermal energy + sound energy + light energy

- In both these cases, thermal energy is **useful**, whereas sound and light are not



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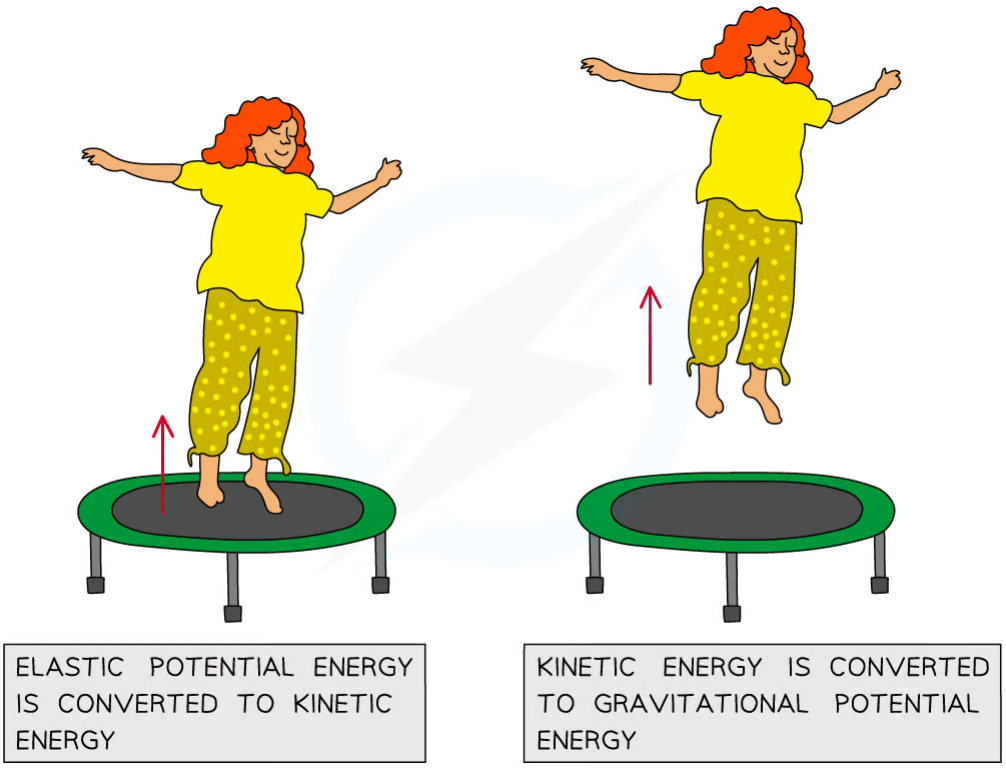
Useful and wasted energy conversions in an electric heater and gas cooker

YOUR NOTES

Applications of Energy Conservation

- In mechanical systems when energy is transferred between stores it is equivalent to the work done:
  - A falling object (in a vacuum, where no energy is not dissipated into the surroundings): **gravitational potential energy → kinetic energy**
  - Horizontal mass on a spring: **elastic potential energy → kinetic energy**
- We can also say energy is transferred between stores and transfers:
  - A battery connected to a bulb: **chemical energy → electrical energy → light energy** (if connected to a bulb)
  - A car: **chemical energy → mechanical energy → kinetic energy**



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Energy transfers whilst jumping on a trampoline

- There may also be work done against resistive forces such as **friction**
- For example, if an object travels up a rough inclined surface, then

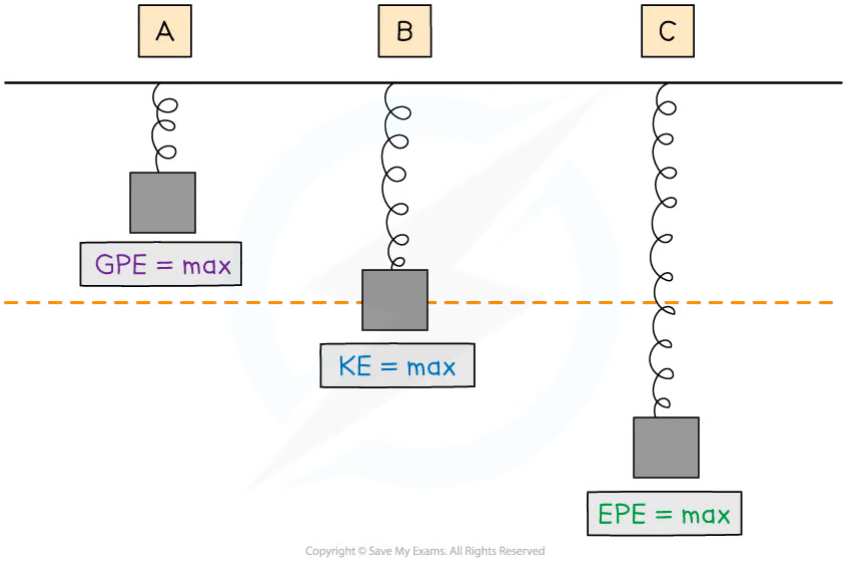
Loss in kinetic energy = Gain in gravitational potential energy + Work done against friction

Spring Energy Conservation

- When a vertical spring is extended and contracted, its energy is converted into other forms
- Although the total energy of the spring will remain constant, it will have changing amounts of:
  - **Elastic** potential energy (EPE)

- **Kinetic** energy (KE)
- **Gravitational** potential energy (GPE)

- When a vertical mass is hanging on a spring and it moves up and down, its energy will convert between the three in various amounts



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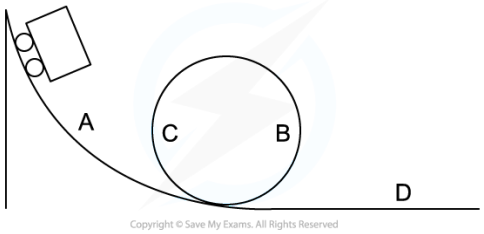
- At position **A:**
  - The spring has some EPE since it is slightly compressed
  - Its KE is 0 since it is stationary
  - Its **GPE is at a maximum** because the mass is at its highest point
- At position **B:**
  - The spring has some EPE since it is slightly stretched
  - Its **KE is at a maximum** as it passes through the equilibrium position at its maximum speed
  - It has some GPE since the mass is still above the ground
- At position **C:**
  - The spring has its maximum EPE because it is at its maximum extension
  - Its KE is 0 since it is stationary
  - Its **GPE is at a minimum** because it is at its lowest point above the Earth's surface
- For a horizontal mass on a spring system, there is no gravitational potential energy to consider. The spring only converts between kinetic and elastic potential energy

YOUR NOTES

Worked Example

The diagram shows a rollercoaster going down a track.

The rollercoaster takes the path A → B → C → D.



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Which statement is true about the energy changes that occur for the rollercoaster down this track?

A. KE - GPE - GPE - KE

B. KE - GPE - KE - GPE

C. GPE - KE - KE - GPE

D. GPE - KE - GPE - KE

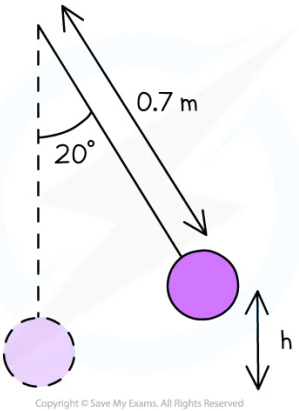
ANSWER: D

- **At point A:**
  - The rollercoaster is raised above the ground, therefore it has **GPE**
  - As it travels down the track, **GPE** is converted to **KE** and the roller coaster speeds up
- **At point B:**
  - **KE** is converted to **GPE** as the rollercoaster rises up the loop
- **At point C:**
  - This **GPE** is converted back into **KE** as the rollercoaster travels back down the loop
- **At point D:**
  - The flat terrain means the rollercoaster only has **KE**

YOUR NOTES

Worked Example

A simple pendulum has a mass of 640 g and a length of 0.7 m. It is pulled out to an angle of 20° from the vertical.The pendulum is released. Assuming negligible air resistance, calculate the maximum speed of the pendulum bob as it passes through the vertical position.



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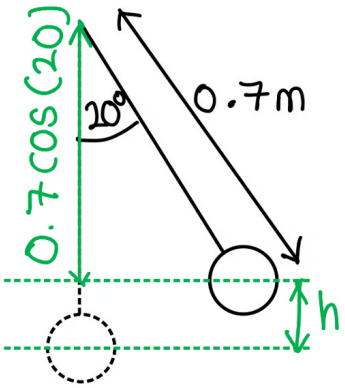
Step 1: Determine the energy transfers

When the pendulum bob is at its maximum height, it has gravitational potential energy.

As it is dropped to the vertical position, all this is converted into kinetic energy

Step 2: Calculate the height dropped



Height Δh dropped = 0.7 – 0.7cos(20)

Step 3: Calculate the loss in gravitational potential energy (GPE)

$$GPE = mg\Delta h = (640 \times 10^{-3}) \times 9.81 \times (0.7 - 0.7\cos(20))$$

$$GPE = 0.265 \text{ J}$$

Step 4: Determine the equation for the speed

The GPE = KE at the vertical position

$$KE = GPE = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2 \times GPE}{m}}$$

Step 5: Substitute in the values

$$v = \sqrt{\frac{2 \times 0.265}{640 \times 10^{-3}}} = 0.91 \text{ m s}^{-1}$$

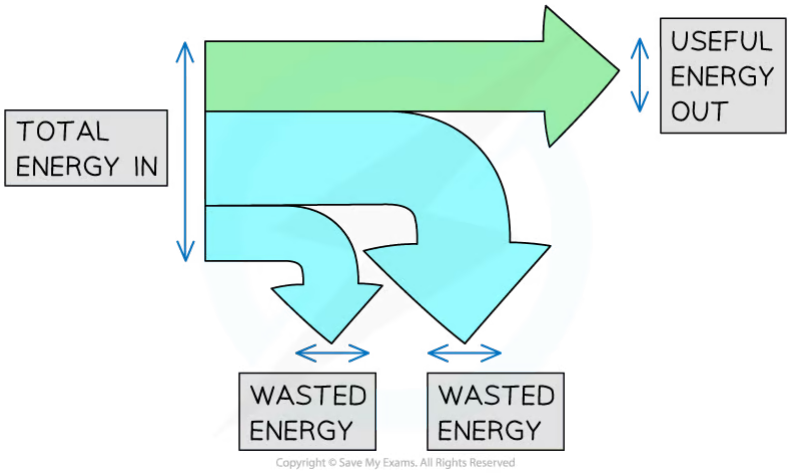
Exam Tip

Treat each scenario separately when it comes to energy conservation. Make sure you're familiar with the different energy stores and transfer that occur so you can apply to each and every scenario.

YOUR NOTES

Sankey Diagrams

- **Sankey diagrams** are used to represent energy transfers
- The arrow in a Sankey diagram represents the transfer of energy:
  - The end of the arrow pointing to the **right** represents the energy that ends up in the **desired** store (the **useful energy output**)
  - The end(s) that point(s) down represents the **wasted energy**



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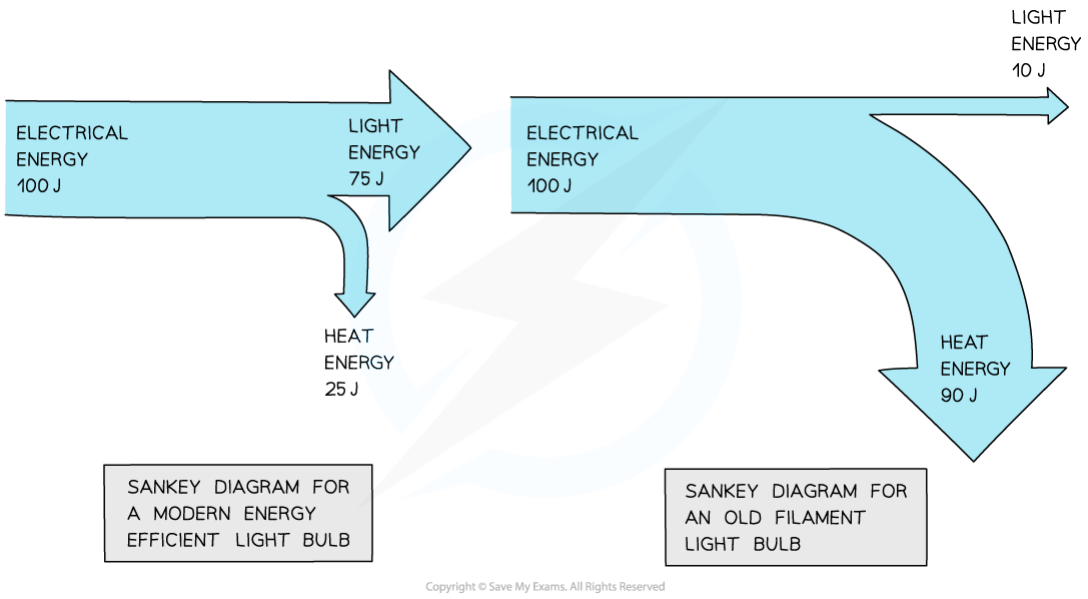
Total energy in, wasted energy and useful energy out shown on a Sankey diagram

- The width of each arrow is proportional to the amount of energy going to each store
- As a result of the conversation of energy:

Total energy in = Useful energy out + Wasted energy

- A Sankey diagram for a modern efficient light bulb will look very different from that for an old filament light bulb
- A more efficient light bulb has **less** wasted energy
  - This is shown by the smaller arrow downwards representing the heat energy



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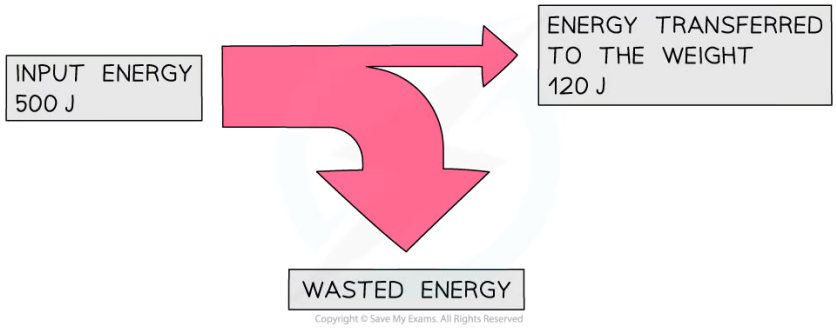
Sankey diagram for modern vs. old filament light bulb

YOUR NOTES

Worked Example

An electric motor is used to lift a weight. The diagram represents the energy transfers in the motor.



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How much energy is wasted?

Answer:

Step 1: State the conservation of energy

- Energy cannot be created or destroyed, it can only be moved from one store to another
- This means that:

Total energy in = Useful energy out + Wasted energy

Step 2: Rearrange the equation for the wasted energy

Wasted energy = Total energy in – Useful energy out

Step 3: Substitute the values from the diagram

500 – 120 = **380 J**

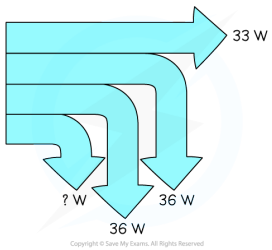
YOUR NOTES

Worked Example

A small electric car is driven by a 120 watt motor.

The useful power output of the motor is measured to be 33 W. While 36 W of power is wasted on friction losses and a further 36 W is wasted on traction losses.

Further power is lost by the electric car during operation. This situation is shown in the diagram below.



Determine the remaining power loss for the electric car when operating.

Answer:

Step 1: State the conservation of energy

- Energy cannot be created or destroyed, it can only be moved from one store to another
- This also applies to power
- This means that:

Total power in = Useful power out + Friction losses + Traction losses + Wasted power other losses

Step 2: Rearrange the equation for the wasted power

Wasted power other losses = Total power in – (Useful power out + Friction losses + Traction losses)

Step 3: Substitute the values from the diagram

120 – (33 + 36 + 36) = **15 W**

Exam Tip

- Drawing good Sankey diagrams takes practice.
- Start by planning your diagram:
  - How wide are you going to make the input arrow?
  - How wide will the 'useful energy out' arrow need to be?
  - How wide must the 'wasted energy' arrow be?
- Next, start drawing the diagram one step at a time:
  - Draw the left hand side of the arrow, along with the line going across the top
  - Next add the 'useful energy out' arrow, making sure it is the correct width
  - Now carefully mark the start and end of the wasted arrow – make sure your marks are the correct distance apart!
  - Finally join the markings together, finishing the 'wasted energy' arrow

YOUR NOTES

YOUR NOTES

Work Done

- Work is defined as

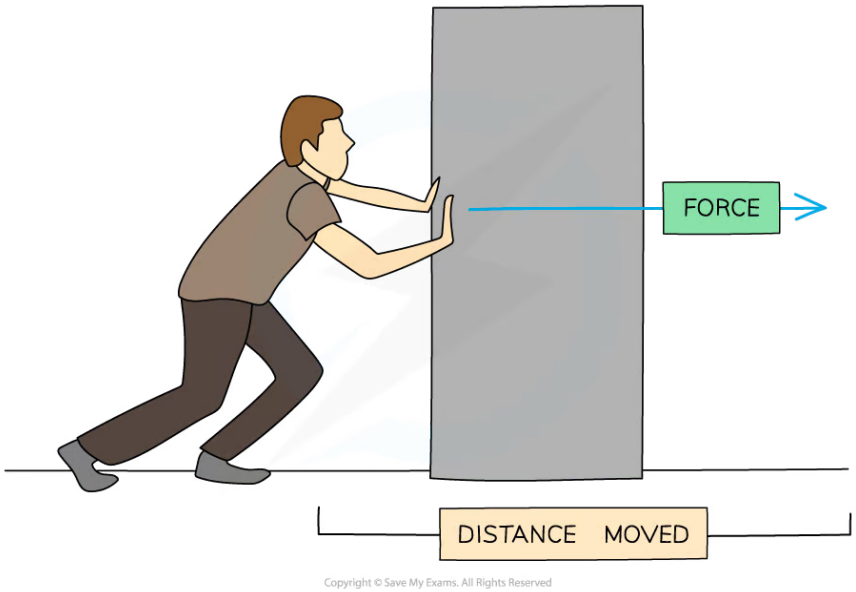
  The transfer of energy when an external force causes an object to move over a certain distance

- The work done by the **resultant force** on a system is equal to the change in the **energy** of the system
- If a constant force is applied in the line of an object's displacement (i.e. parallel to it), the work done can be calculated using the equation:

$$W = Fs$$

- Where:
  - W = work done (J)
  - F = average force applied (N)
  - s = displacement (m)

- In the diagram below, the man's pushing force on the block is doing work as it is transferring energy to the block



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Work is done when a force is used to move an object over a distance

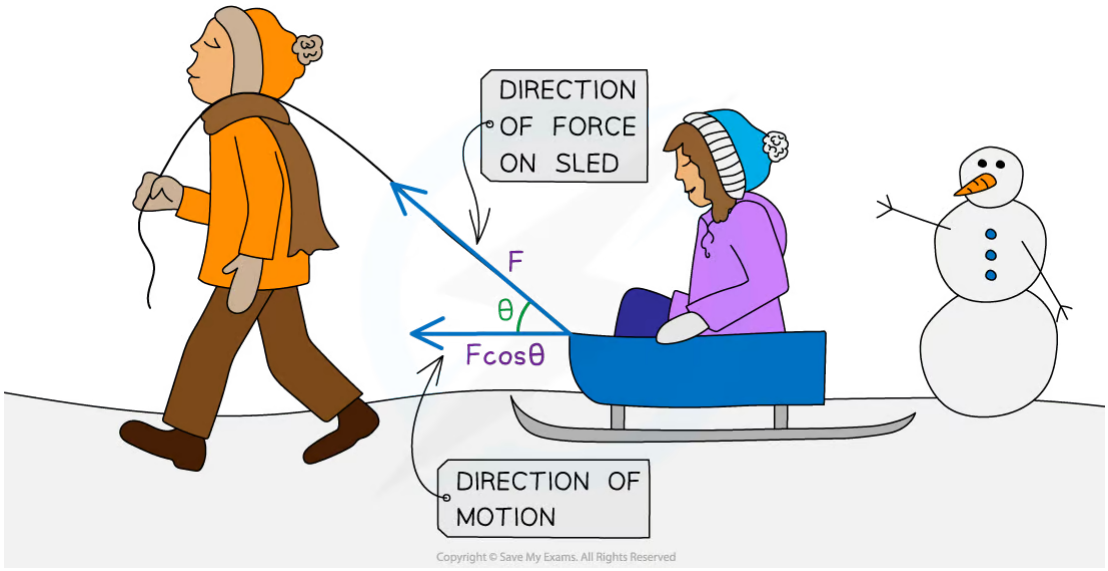
- When pushing a block, **work is done against friction** to give the box kinetic energy to move
- The kinetic energy is transferred to other forms of energy such as heat and sound
  - Usually, if a force acts **in** the direction that an object is moving then the object will **gain energy**
  - If the force acts in the **opposite** direction to the movement then the object will **lose energy**

YOUR NOTES

- When plotting a graph of average force applied against displacement, the **area** under the graph is equal to the **work done**
- Sometimes the direction of motion of an object is **not parallel** to the direction of the force
- If the force is at an **angle θ** to the object's displacement, the work done is calculated by:

$$W = Fs\cos\theta$$

- Where θ is the angle, in degrees, between the direction of the force and the motion
  - When θ is 0 (the force is in the direction of motion) then $\cos\theta = 1$ and $W = Fs$
- This may not always be cos θ, since this is just for **horizontal** motion
- For vertical motion, it would be sin θ

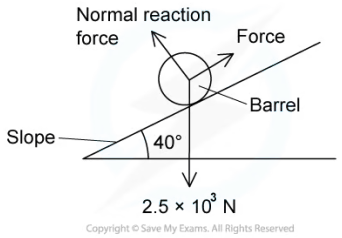


When the force is at an angle, only the component of the force in the direction of motion is considered for the work done

YOUR NOTES

Worked Example

The diagram shows a barrel of weight 2.5×10^3 N on a frictionless slope inclined at 40° to the horizontal.



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A force is applied to the barrel to move it up the slope at a constant speed. The force is parallel to the slope.

What is the work done in moving the barrel a distance of 6.0 m up the slope?

A. 7.2×10^3 J **B.** 2.5×10^4 J **C.** 1.1×10^4 J **D.** 9.6×10^3 J

ANSWER: D

STEP 1

WORK DONE EQUATION

$W = F \times d$

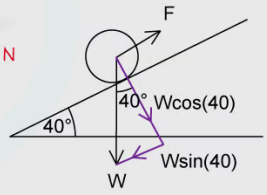
STEP 2

CALCULATE THE FORCE IN THE DIRECTION OF TRAVEL

THE FORCE NEEDED TO PUSH THE BARREL NEEDS TO OVERCOME THE COMPONENT OF THE BARREL'S WEIGHT. SINCE THE FORCE IS PARALLEL TO THE SLOPE, THE COMPONENT OF THE WEIGHT WE NEED IS THE ONE PARALLEL TO THE SLOPE.

$F = W\sin(40) = 2.5 \times 10^3 \times \sin(40) = 1607$ N

THIS IS THE FORCE IN THE SAME DIRECTION AS THE DISPLACEMENT



STEP 3

SUBSTITUTE F AND d INTO THE WORK DONE EQUATION

$W = 1607 \text{ N} \times 6.0 \text{ m} = 9.6 \times 10^3 \text{ J}$

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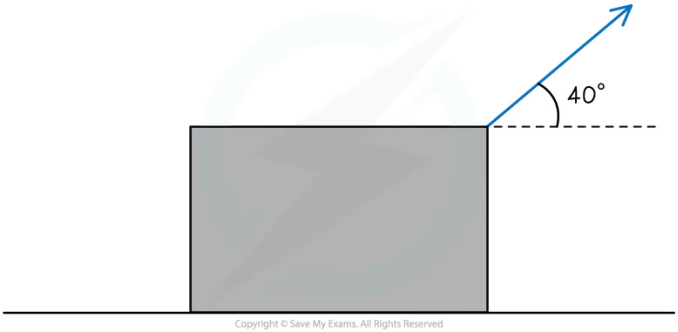
YOUR NOTES

Worked Example

An 80 kg person pulls a 15 kg box along using a rope which is at 40° from the horizontal as shown below. The person is pulling with a force of 40 N and moves the box 20 m horizontally from its starting position against a constant friction force of 5.0 N.

Calculate the work that has been done on the box in the direction of its motion.

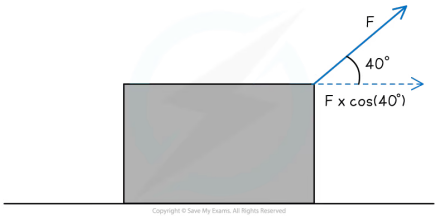


Answer

Step 1: List the known quantities

- The angle between the rope and the horizontal, $\theta = 40°$
- The pulling force (along rope) = 40 N
- Horizontal distance moved by box, s = 20 m
- Frictional force = 5.0 N

Step 2: Resolve the pulling force in the rope into its horizontal component

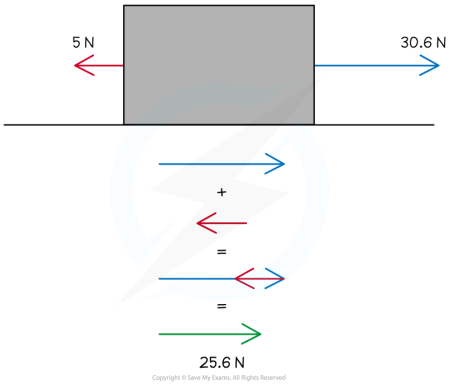


- The horizontal component of the pulling force is the only part of the pulling force aligned with the direction of work
- Hence, that is the component that is needed to continue solving this problem
- The horizontal component can be resolved from:

$$\cos(40) \times 40 = 30.6 \text{ N to the right}$$

Step 3: Find the net force for the motion

- The net force can be found from the interaction between the horizontal component of the pulling force and the friction force:



$30.6 - 5 = 25.6$ N to the right

Step 4: Calculate the work done

- Use the equation for work given the net force and distance moved in the horizontal

$$W = Fs\cos\theta$$

- The **cos θ** has already been accounted for so that the net force could be found when combined with friction
- Therefore:

$$W = Fs = 25.6 \times 20 = 512 \text{ J}$$

W = **512 J**

Exam Tip

Sometimes exam questions will include more values than you need to use in the solution - this is purposefully done to confuse you. For example, in the second worked example above, the question supplies the mass of the person and the box, however, these quantities are not needed for the calculation.

Always consider the horizontal and vertical components of the force. The component needed is the one that is **parallel to the displacement**. The equation with $\cos\theta$ is given on your data sheet, but you only need to use this if the force is applied at an angle to the displacement.

YOUR NOTES

Kinetic Energy

- Kinetic energy (E_k) is the energy an object has due to its **translational motion** (i.e. because it's moving)
  - The **faster** an object is moving, the **greater** its kinetic energy

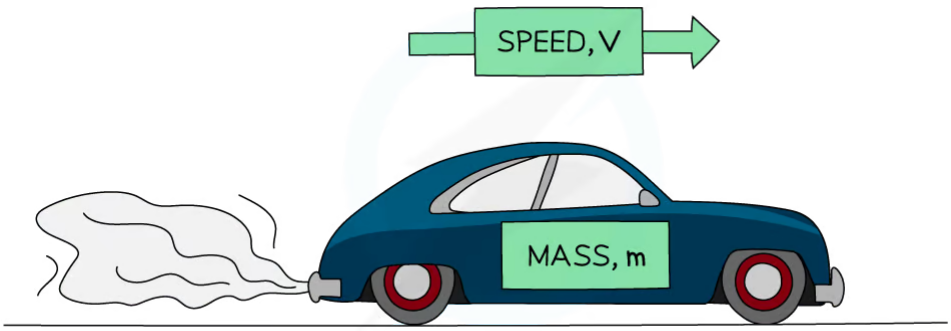
When an object is falling, it is **gaining** kinetic energy since it is **accelerating** under gravity

- This energy is transferred from the gravitational potential energy it is losing
- An object will **maintain** this kinetic energy unless its **speed** or **mass** changes

- Kinetic energy can be calculated using the following equation:

$$E_k = \frac{1}{2}mv^2$$

- Where:
  - E_k = kinetic energy (J)
  - m = mass (kg)
  - v = velocity (m s^{-1})



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Kinetic energy (KE): The energy an object has when it is moving

- Another quantity that also depends on mass m and velocity v is **momentum**
- Therefore, kinetic energy can be written in terms of momentum p, using the equation

$$E_k = \frac{p^2}{2m}$$

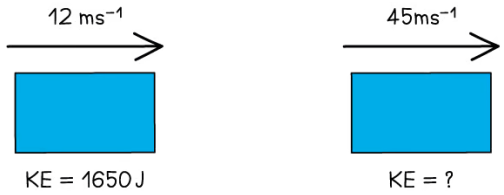
- Where:
  - p = momentum (kg m s^{-1})

- This form is very useful in particle physics, when comparing the momentum and kinetic energy of a particle

YOUR NOTES

Worked Example

A body travelling with a speed of 12 m s^{-1} has kinetic energy 1650 J. If the speed of the body is increased to 45 m s^{-1}, estimate what is its new kinetic energy.



STEP 1

EQUATION FOR KINETIC ENERGY

$KE = \frac{1}{2}mv^2$

STEP 2

MASS WILL NOT CHANGE, SO CAN BE CALCULATED FROM ITS INITIAL KINETIC ENERGY

REARRANGE FOR MASS m

$m = \frac{2 \times KE}{v^2} = \frac{2 \times 1650}{12^2} = 23\ kg$

STEP 3

SUBSTITUTE INTO KINETIC ENERGY EQUATION

USING VALUE OF MASS AND NEW VALUE OF VELOCITY

$KE = \frac{1}{2} \times 23 \times 45^2 = 23000\ J$ (2 s.f)

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Exam Tip

When using the kinetic energy equation, note that only the **speed** is **squared**, not the mass or the ½. If a question asks about the '**loss** of kinetic energy', remember **not** to include a negative sign since energy is a **scalar** quantity.

Both variations of the kinetic energy equation are given in your data booklet. You will most likely use $\frac{1}{2}mv^2$ in a mechanics question, and $\frac{p^2}{2m}$ in particle physics.

If you are not convinced these are in fact the same equation:

$$E_k = \frac{p^2}{2m} = \frac{(mv)^2}{2m} = \frac{m^2v^2}{2m} = \frac{mv^2}{2} = \frac{1}{2}mv^2$$

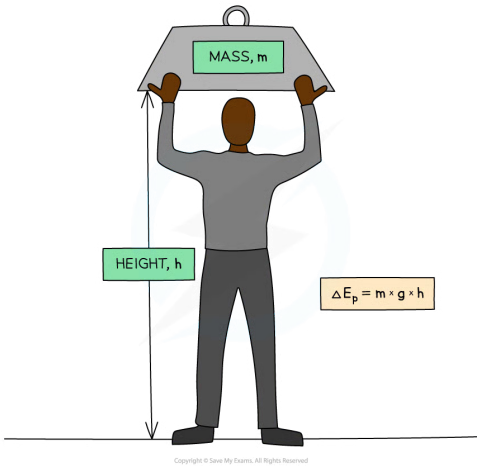
YOUR NOTES

Gravitational Potential Energy

- Gravitational potential energy E_p is the energy stored in a mass due to its position in a gravitational field
  - If a mass is **lifted** up, it will **gain** E_p (converted **from** other forms of energy)
  - If a mass **falls**, it will **lose** E_p (and be converted **to** other forms of energy)
- The equation for gravitational potential energy when **close** to the **surface** of the **Earth** is:

$$\Delta E_p = mg\Delta h$$

- Where:
  - ΔE_p = gravitational potential energy (J)
  - m = mass (kg)
  - g = gravitational field strength (9.81 N kg^{-1})
  - Δh = change in height (m)



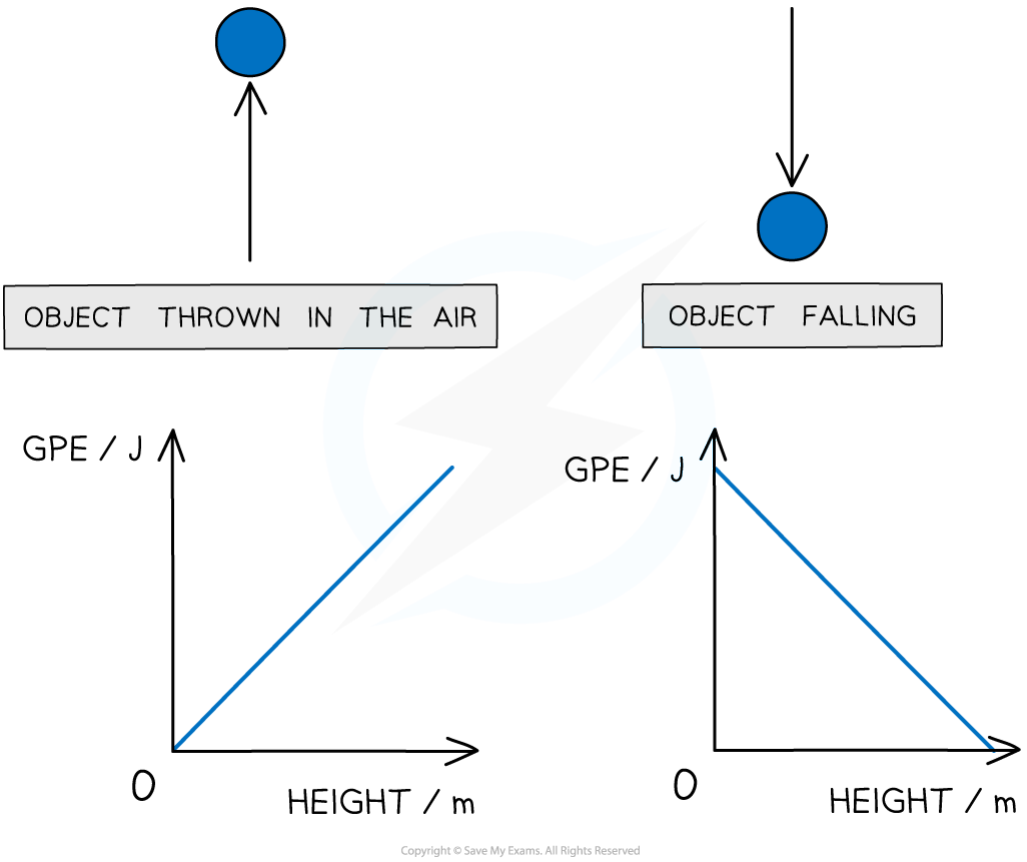
Gravitational potential energy (GPE): The energy an object has when lifted up

- The potential energy on the Earth's surface at ground level is taken to be equal to 0
- This equation is only relevant for energy changes in a **uniform gravitational field** (such as near the Earth's surface)
- A different potential energy is used in the gravitational fields topic, because the field is no longer uniform outside of the Earth's surface

GPE vs Height

- The two graphs below show how GPE changes with height for a ball being thrown up in the air and when falling down (**ignoring air resistance**)

YOUR NOTES



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Graphs showing the linear relationship between GPE and height

- Since the graphs are straight lines, GPE and height are said to have a **linear** relationship
  - These graphs would be identical for GPE against time instead of height

YOUR NOTES

Worked Example

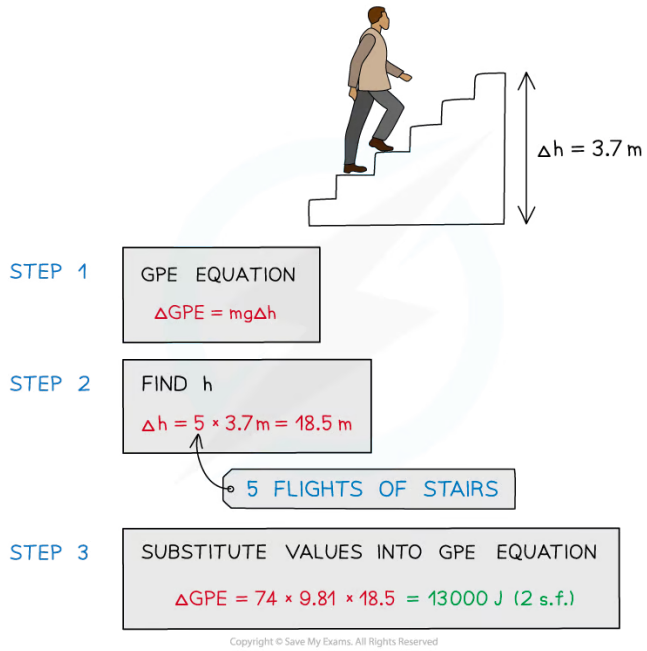
To get to his apartment a man has to climb five flights of stairs.

The height of each flight is 3.7 m and the man has a mass of 74 kg.

What is the approximate gain in the man's gravitational potential energy during the climb?

A. 13 000 J **B.** 2700 J **C.** 1500 J **D.** 12 500 J

ANSWER: A



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Elastic Potential Energy

- Elastic potential energy is defined as

  The energy stored within a material (e.g. in a spring) when it is stretched or compressed

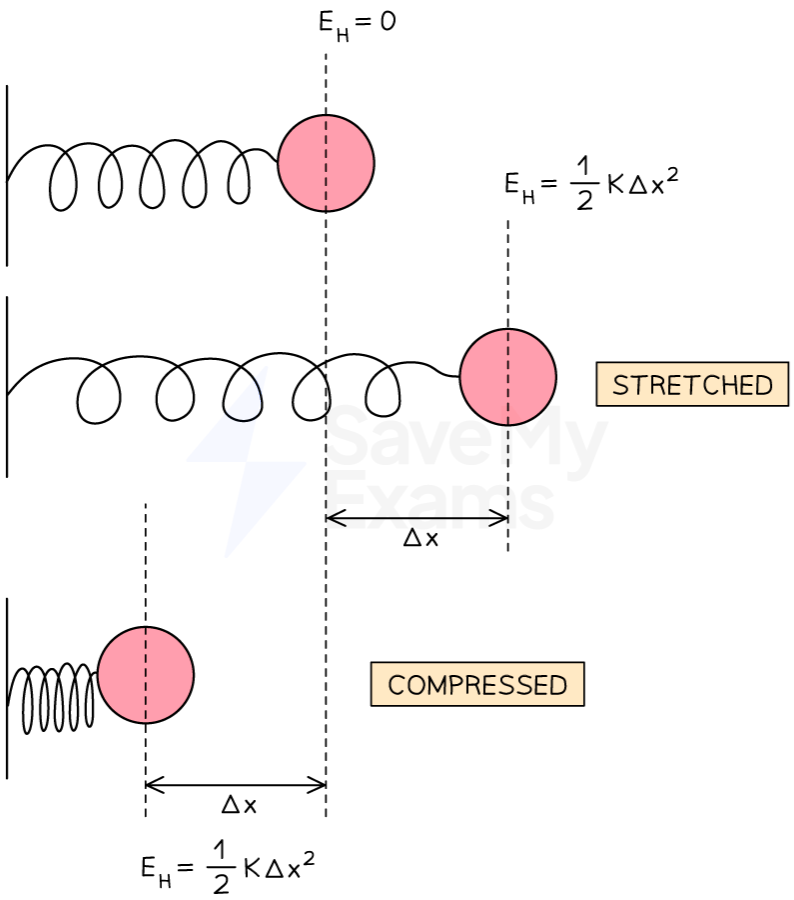
- Therefore, for a material obeying Hooke's Law, elastic potential energy is equal to:

$$E_H = \frac{1}{2} k \Delta x^2$$

- Where:
  - k = spring constant of the spring (N m^{-1})
  - Δx = extension of the spring (m)
- This can also be written as:

$$E_H = \frac{1}{2} F \Delta x$$

- Where:
  - F = restoring force (N)
- This force is the same restoring force as in Hooke's law: $F = k \Delta x$



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A spring that is stretched or compressed has elastic potential energy

- It is very dangerous if a wire under large stress suddenly breaks
- This is because the elastic potential energy of the strained wire is **converted** into kinetic energy

$$E_H = E_K$$

$$\frac{1}{2} k \Delta x^2 = \frac{1}{2} m v^2$$

$$v \propto \Delta x$$

- This equation shows
  - The greater the **extension** of a wire Δx the greater the **speed** v it will have when it breaks

YOUR NOTES

Worked Example

A car's shock absorbers make a ride more comfortable by using a spring that absorbs energy when the car goes over a bump. One of these springs, with a spring constant of 50 kN m^{-1} is fixed next to a wheel and compressed a distance of 10 cm.

Calculate the energy stored by the compressed spring.

Answer:

Step 1: List the known values

- Spring constant, $k = 50\ \text{kN m}^{-1} = 50 \times 10^3\ \text{N m}^{-1}$
- Compression, $x = 10\ \text{cm} = 10 \times 10^{-2}\ \text{m}$

Step 2: Substitute the values into the elastic potential energy equation

$$E_H = \frac{1}{2} \times (50 \times 10^3) \times (10 \times 10^{-2})^2 = 250\ \text{J}$$

YOUR NOTES

Conservation of Mechanical Energy

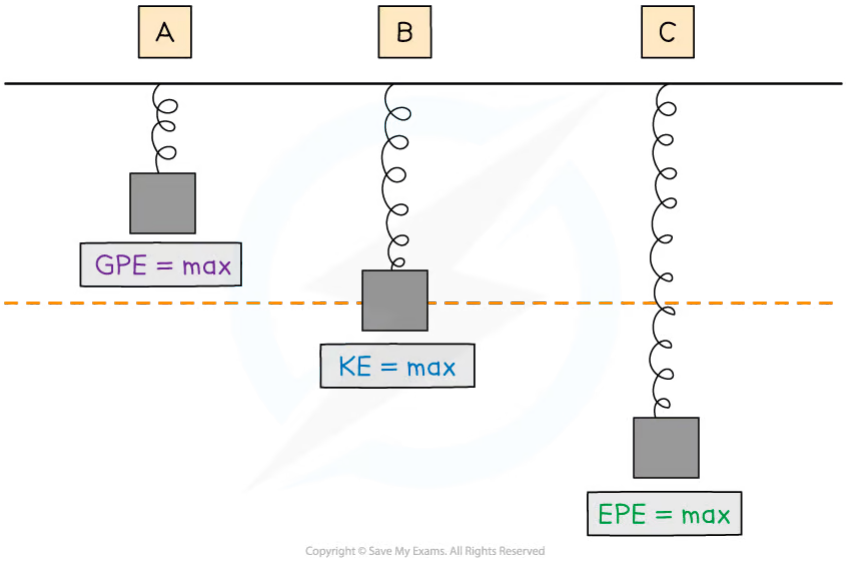
- Mechanical energy is the **sum** of kinetic energy, gravitational potential energy and elastic potential energy

$$\textbf{Mechanical energy} = E_k + \Delta E_p + E_H$$

- An example of a system that has mechanical energy is a spring and mass system
- The **change** in the total mechanical energy of a system should be interpreted in terms of the **work done** on the system by any non-conservative force.

- When a vertical spring is extended and contracted, its energy is converted into other forms
- Although the total energy of the spring will remain constant, it will have changing amounts of:
  - **Elastic** potential energy (E_H or EPE)
  - **Kinetic** energy (E_k or KPE)
  - **Gravitational** potential energy (E_p or GPE)

- When a vertical mass is hanging on a spring and it moves up and down, its energy will convert between the three in various amounts



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- At position **A:**
  - The spring has some EPE since it is slightly compressed
  - Its KE is 0 since it is stationary
  - Its **GPE is at a maximum** because the mass is at its highest point

- At position **B:**

- The spring has some EPE since it is slightly stretched
- Its **KE is at a maximum** as it passes through the equilibrium position at its maximum speed
- It has some GPE since the mass is still above the ground

- At position **C:**
  - The spring has its maximum EPE because it is at its maximum extension
  - Its KE is 0 since it is stationary
  - Its **GPE is at a minimum** because it is at its lowest point above the Earth's surface

- For a horizontal mass on a spring system, there is no gravitational potential energy to consider. The spring only converts between kinetic and elastic potential energy

YOUR NOTES

Conservation of Mechanical Energy

YOUR NOTES

- In the absence of frictional, resistive forces, the total **mechanical** energy of a system is **conserved**
  - This means the total kinetic, gravitational potential and elastic potential energy is the same throughout the motion of the system
- This is essential the conservation of energy
- There are many scenarios that involve the transfer of kinetic energy into gravitational potential, or vice versa
- Some examples are:
  - A swinging pendulum
  - Objects in freefall
  - Sports that involve falling, such as skiing and skydiving
- Using the principle of conservation of energy, and taking any drag forces as negligible:

Loss in gravitational potential energy = Gain in kinetic energy

- Another example is if a ball lands on a spring from a height
- In this case:

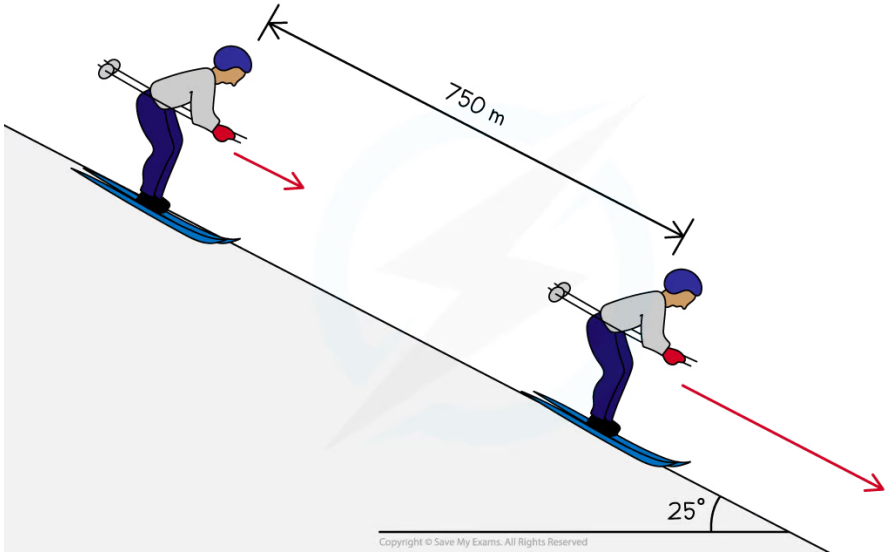
Loss in gravitational potential energy = Gain in elastic potential energy

- The change in energy is the **work done** on the system. The types of changes depend on the system

YOUR NOTES

Worked Example

The diagram below shows a skier on a slope descending 750 m at an angle of 25° to the horizontal.

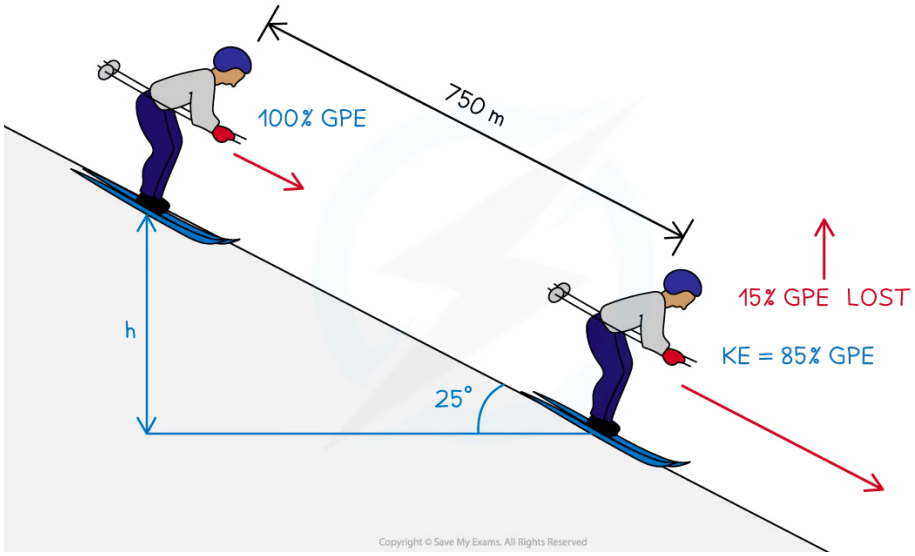


Calculate the final speed of the skier, assuming that he starts from rest and 15% of his initial gravitational potential energy is **not** transferred to kinetic energy.

Answer:

Step 1: Write down the known quantities

- Vertical height, $h = 750 \sin 25°$
- $E_k = 0.85\, E_p$



Step 2: Equate the equations for E_k and E_p

$$E_k = 0.85\, E_p$$

$$\tfrac{1}{2} mv^2 = 0.85 \times mgh$$

Step 3: Rearrange for final speed, *v*

$$\frac{1}{2} \cancel{m} v^2 = 0.85 \times \cancel{m} gh$$

$$v^2 = 0.85 \times 2gh$$

$$v = \sqrt{0.85 \times 2gh}$$

Step 4: Calculate the final speed, *v*

$$v = \sqrt{0.85 \times 2 \times 9.81 \times 750 \sin 25°} = 72.7$$

Final speed, $v = 73\ \text{m s}^{-1}$

Exam Tip

Exam questions often ask about the 'work done' in the process. This means how much energy is **transferred**. You must consider **all** the energy changes in the system and remember that mechanical energy is always conserved. This is essentially the conservation of energy.

YOUR NOTES

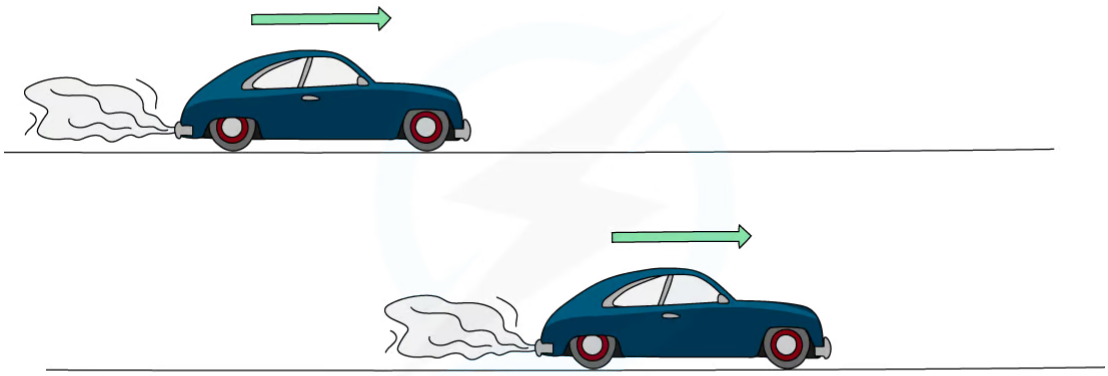
YOUR NOTES

Energy & Power

- The power of a mechanical process is the **rate at which energy is transferred**
- This energy transferred is the **work done**
- Therefore, power is:

The rate of work done (energy transfer)

- **Time** is an important consideration when it comes to **power**
- Two cars transfer the **same amount of energy**, or do the **same amount of work** to accelerate over a distance
- If one car has **more power**, it will transfer that energy, or do that work, in a **shorter amount of time**



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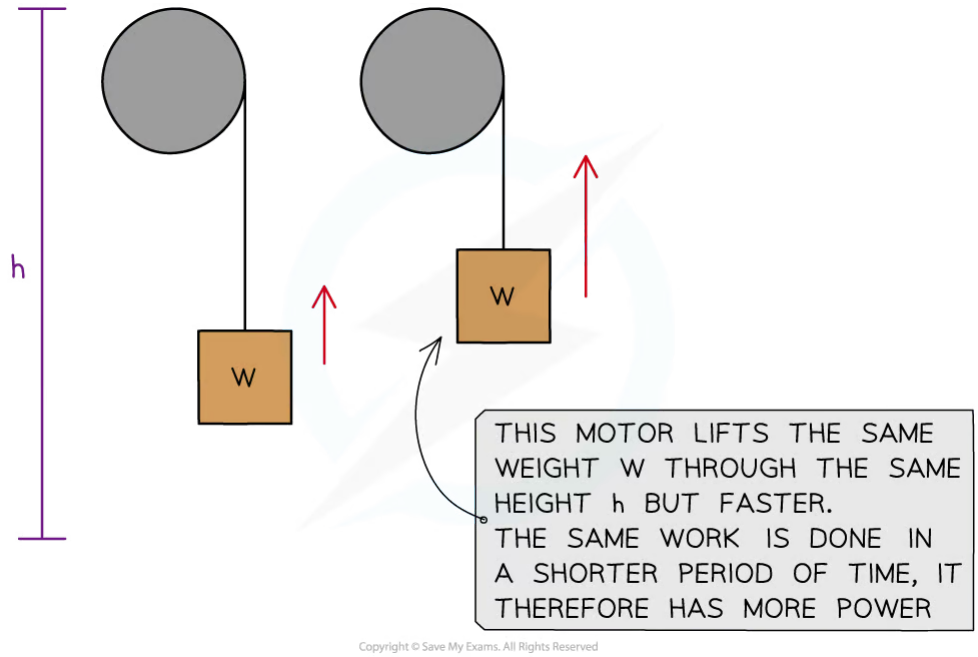
Two cars accelerate to the same final speed, but the one with the most power will reach that speed sooner

- Two electric motors:
  - lift the same weight
  - by the same height
  - but one motor lifts it **faster** than the other
- The motor that lifts the weight faster has more **power**



Two motors with different powers

- Power can be calculated using the equation:

$$P = \frac{\Delta W}{\Delta t} = Fv$$

- Where:
  - P = power (W)
  - ΔW = change in work done (J)
  - Δt = time interval (s)
  - F = force (N)
  - v = velocity (m s^{-1})
- The equation with F and v is only relevant where a **constant force** moves a body at **constant velocity**
  - Power is required in order to produce an acceleration
- The force must be applied in the **same** direction as the velocity

- Power is also used in electricity, with labels on lightbulbs which indicate their power, such as 60 W or 100 W
  - These indicate the amount of energy transferred by an electrical current rather than by a force doing work

The Watt

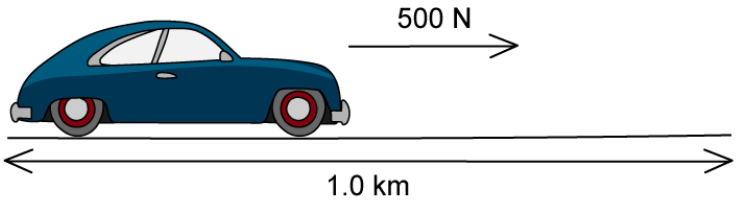
YOUR NOTES

- Power is measured in **watts (W)**
- The watt, *W*, is commonly used as the unit power (and radiant flux)
  - It is defined as $\mathbf{1\,W = 1\,J\,s^{-1}}$
- The SI unit for energy is $kg\,m^2\,s^{-3}$
- One watt is defined as:

A transfer of energy of 1 J in 1 s

Worked Example

A car engine exerts the following force for 1.0 km in 200 s.



Determine what is the average power developed by the engine.

STEP 1

EQUATION FOR POWER

$$\text{POWER} = \frac{\text{WORK DONE}}{\text{TIME}}$$

STEP 2

CALCULATE WORK DONE

$$W = F \times d$$
$$= 500\ N \times 1.0 \times 10^3\ m$$
$$= 5 \times 10^5\ J$$

STEP 3

SUBSTITUTE VALUES INTO POWER EQUATION

$$\text{POWER} = \frac{5 \times 10^5\ J}{200\ s} = 2500\ W = 2.5\ kW$$

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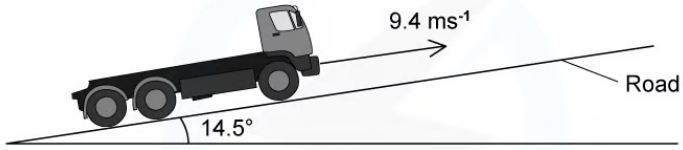
YOUR NOTES

Worked Example

A lorry moves up a road that is inclined at 14.5° to the horizontal.



The lorry has a mass of 3500 kg and is travelling at a constant speed of 9.4 m s^{-1}. The force due to air resistance is negligible. Calculate the useful power from the engine to move the lorry up the road.

STEP 1

EQUATION FOR POWER

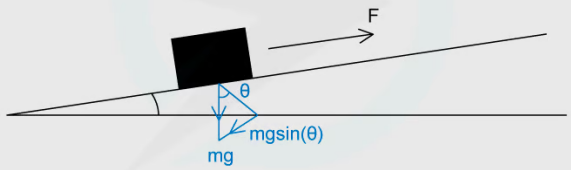
$P = F \times v$

STEP 2

CALCULATE THE FORCE

THE FORCE NEEDED TO MOVE THE LORRY UP THE ROAD IS THAT WHICH OVERCOMES THE COMPONENT OF ITS WEIGHT ACTING DOWN THE SLOPE

$F = mg \sin\theta = 3500 \times 9.81 \times \sin(14.5) = 8596.8$ N



STEP 3

SUBSTITUTE INTO POWER EQUATION

$P = 8\,596.8 \times 9.4 = 80\,809.9$ W $= 81\,000$ W $= 81$ kW (2.s.f.)

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Exam Tip

The force represented in exam questions will often be a **drag** force. Whilst this is in the **opposite** direction to its velocity, remember the force needed to calculate the **power** is equal to (or above) this drag force to **overcome** it therefore you equate it to that value.

YOUR NOTES

Efficiency Formula

- The efficiency of a system is a measure of how well energy is transferred in a system
- Efficiency is defined as:

> **The ratio of the useful power or energy transfer output from a system to its total power or energy transfer input**

- If a system has **high** efficiency, this means most of the energy transferred is **useful**
- If a system has **low** efficiency, this means most of the energy transferred is **wasted**
- Determining which type of energy is useful or wasted depends on the system
  - When electrical energy is converted to light in a lightbulb, the light energy is **useful** and the heat energy produced is **wasted**
  - When electrical energy is converted to heat for a heater, the heat energy is **useful** and the light and sound energy produced is **wasted**
- Efficiency is represented as a fraction, and can be calculated using the equation:

$$\eta = \frac{E\,(output)}{E\,(input)} = \frac{P\,(output)}{P\,(input)}$$

- Where:
  - η = efficiency (the greek letter "eta")
  - E = energy (J)
  - P = power (W)
- To turn this equation into a percentage, just × 100 %
- It can also be written in words as:

$$\eta = \frac{useful\ work\ out}{total\ work\ in} = \frac{total\ power\ out}{total\ power\ in}$$

- The energy can be of any form e.g. gravitational potential energy, kinetic energy

YOUR NOTES

Worked Example

An electric motor has an efficiency of 35 %. It lifts a 7.2 kg load through a height of 5 m in 3 s.

Calculate the power of the motor.

Answer:

Step 1: Write down the efficiency equation (as a percentage)

$$\eta = \frac{useful\ power\ out}{useful\ power\ in} \times 100\%$$

Step 2: Rearrange equation for the useful power in

$$useful\ power\ in = \frac{userful\ power\ out \times 100\%}{\eta}$$

Step 3: Calculate the power output

- The power output is equal to energy ÷ time
- The electric motor transferred electric energy into gravitational potential energy to lift the load

Gravitational potential energy = mgh = 7.2 × 9.81 × 5 = 353.16 J

$$\text{Power} = \frac{353.16}{3} = 117.72\ \text{W}$$

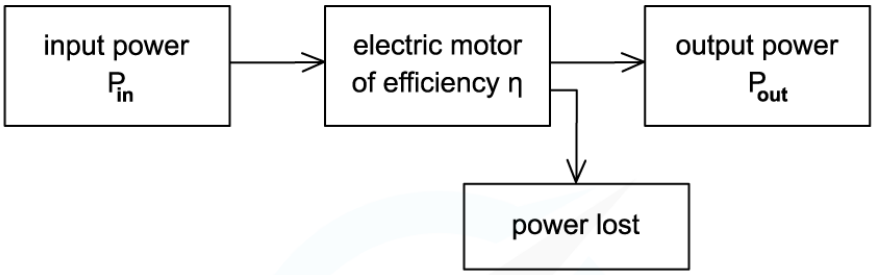
Step 4: Substitute values into power input equation

$$useful\ power\ in = \frac{117.72 \times 100}{35} = 336\ W$$

YOUR NOTES

Worked Example

An electric motor has an input power P_{in}, useful output power P_{out} and efficiency η.



What is the output power P_{out} of the motor?

A. ηP_{in} **B.** $\frac{-\eta P_{lost}}{\eta - 1}$ **C.** ηP_{lost} **D.** $-\eta P_{lost}(\eta - 1)$

ANSWER: B

EFFICIENCY = $\frac{\text{USEFUL POWER OUTPUT}}{\text{USEFUL POWER INTPUT}}$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{lost}}$$

MULTIPLY BY $P_{out} + P_{lost}$ ON BOTH SIDES $\eta(P_{out} + P_{lost}) = P_{out}$

EXPAND THE BRACKETS $\eta P_{out} + \eta P_{lost} = P_{out}$

$-P_{out}$ FROM BOTH SIDES $\eta P_{out} - P_{out} = -\eta P_{lost}$

TAKE POUT AS A FACTOR $P_{out}(\eta - 1) = -\eta P_{lost}$

DIVIDE BY $\eta - 1$ $P_{out} = \frac{-\eta P_{lost}}{\eta - 1}$

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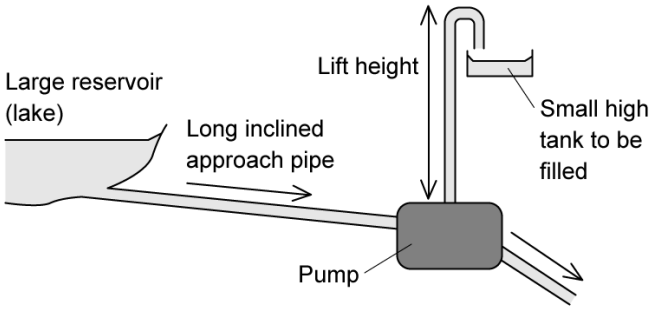
YOUR NOTES

Worked Example

The diagram shows a pump called a hydraulic ram.



In one such pump, the long approach pipe holds 700 kg of water. A valve shuts when the speed of this water reaches 3.5 m s^{-1} and the kinetic energy of this water is used to lift a small quantity of water by height of 12m.

The efficiency of the pump is 20%.

Which mass of water could be lifted 12 m?

A. 6.2 kg **B.** 4.6 kg **C.** 7.3 kg **D.** 0.24 kg

ANSWER: C

THE KINETIC ENERGY OF THE WATER IS CONVVERTED TO GRAVITATIONAL POTENTIAL ENERGY WHEN LIFTED BY 12m

$$KE = GPE$$

$$\frac{1}{2}mv^2 = mgh$$

SINCE EFFICIENCY IS 20% ONLY 20% OF THE KINETIC ENERGY WILL BE CONVERTED.

$$0.2 \times \frac{1}{2}mv^2 = mgh$$

$$0.2 \times \frac{1}{2} \times 700 \times (3.5)^2 = m \times 9.81 \times 12$$

$$857.5 = m \times 117.72$$

$$\frac{857.5}{117.72} = m$$

$$m = 7.3 \text{ kg (2 s.f.)}$$

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- The pump is what converts the water's **kinetic energy** into **gravitational potential energy**
- Since its efficiency is 20%, the kinetic energy can be multiplied by 0.2 since only 20% of the **kinetic energy** will be converted (**not** 20% of the gravitational

potential energy)

YOUR NOTES

Exam Tip

Efficiency can be in a ratio (between 0 and 1) or percentage format (between 0% and 100%).

If the question asks for efficiency as a ratio, give your answer as a **fraction** or **decimal**. If the answer is required as a percentage, remember to multiply the ratio by **100** to convert it: if the ratio = 0.25, percentage = 0.25 × 100 = 25 %.

Remember that efficiency has **no units**. It is a ratio with both the numerator and denominator with the same units.

YOUR NOTES

Energy Density

- A fuel is anything that can be burned to produce heat, which can be used for an engine to work
- The amount of energy that a fuel can provide for the amount of fuel used is an important consideration for the modern world
  - When this is compared by **volume** is known as **energy density**
- Energy density is a measure of the amount of **energy per unit volume** of a fuel
  - It is measured in J m^{-3}
- Different fuels contain different amounts of energy, which make them suitable for certain uses e.g. petrol for running vehicles
- Some examples are:

Energy Density of Fuels Table

Fuel	Energy density / MJ L^{-1}
coal	72
crude oil	37
diesel	36
sugar	26
vegetable oil	30
wood	3

- 1 L (litre) is 0.001 m^3
- This means that we can get **more** energy per unit volume of coal than we can wood
- This is why coal is burnt for fuel, whilst wood isn't - there isn't enough energy from it for it to be of use for practical purposes such as an engine or turning a turbine

IB Physics HL

YOUR NOTES

Rigid Body Mechanics

CONTENTS

Torque & Couples (HL)
Rotational Equilibrium (HL)
Angular Displacement, Velocity & Acceleration (HL)
Angular Acceleration Formula (HL)
Moment of Inertia (HL)
Newton's Second Law for Rotation (HL)
Angular Momentum (HL)
Angular Impulse (HL)
Rotational Kinetic Energy (HL)

YOUR NOTES

Torque & Couples

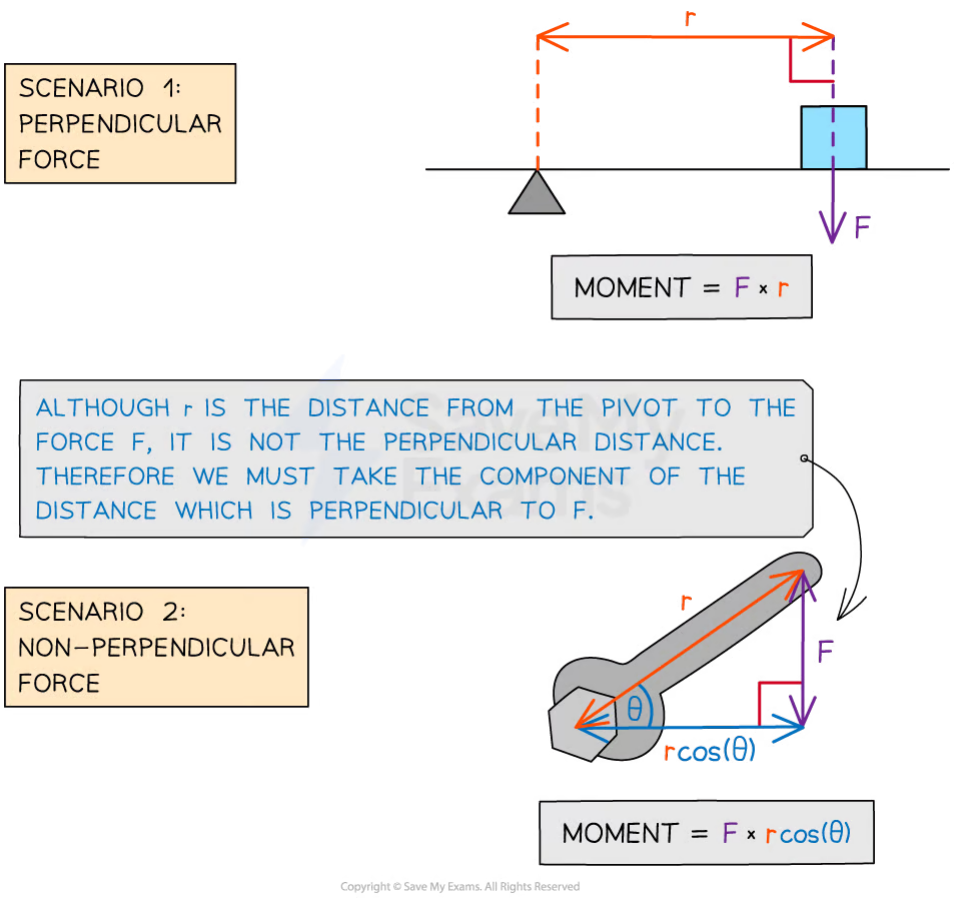
Moment of a Force

- A moment is the **turning effect** of a force around a particular point
- Moments occur when forces cause objects to **rotate** about some pivot
- The moment of a force is given by

Moment (**N m**) = Force (**N**) × perpendicular distance from the pivot (**m**)

- The SI unit for the moment is newton metres (**N m**)

Moments of perpendicular & non-perpendicular forces



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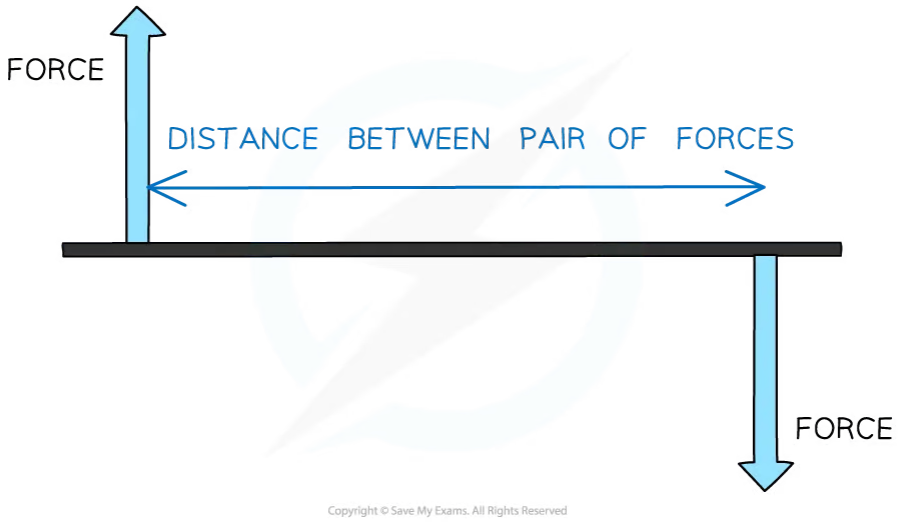
The force might not always be perpendicular to the distance. In this case, use trigonometry to resolve in the perpendicular direction

Couples

- A couple is a **pair** of equal and opposite coplanar forces that act to produce **rotation** only
- A couple consists of a pair of forces that are:
  - Equal in magnitude

- Opposite in direction
- Perpendicular to the distance between them

Force couple diagram



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A couple must consist of two equal and opposite forces separated by a perpendicular distance

- Unlike moments of a single force, the moment of a couple doesn't depend on a pivot
- The moment of a couple is equal to:

  Moment (**N m**) = Force (**N**) × Perpendicular distance between the lines of action of the forces (**m**)

- A couple does not produce a net linear force
  - However, it does produce a turning effect called a **torque**

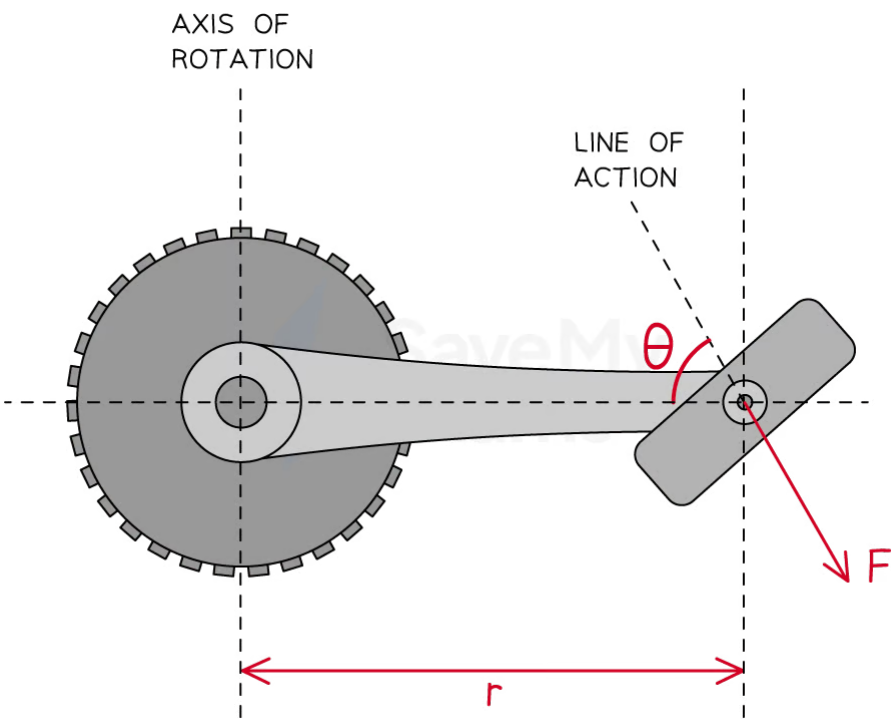
Torque

- The change in rotational motion due to a turning force is called **torque**
- The torque of a force *F* about an axis is given by

$$\tau = Fr \sin \theta$$

Torque of a non-perpendicular force

YOUR NOTES



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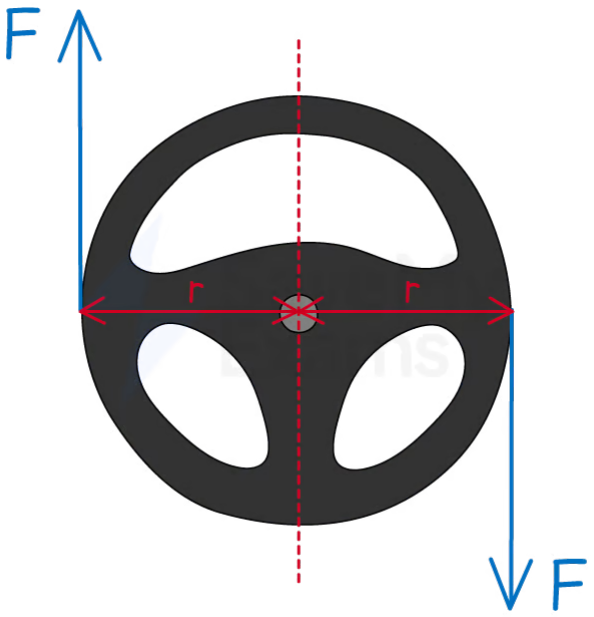
The torque applied by a cyclist on a bicycle pedal can be determined using the magnitude of the applied force and the perpendicular component of the distance between the line of action of the force and the axis of rotation

- For scenarios where the forces are perpendicular ($\theta = 90°$) to one another, the equation simplifies to

$$\tau = Fr$$

- Where:
  - τ = torque (N m)
  - F = applied force (N)
  - r = perpendicular distance between the axis of rotation and the line of action of the force (m)
  - θ = angle between the force and the axis of rotation (°)

Torque of a couple on a steering wheel



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The steering wheel is in rotational equilibrium since the resultant force and resultant torque are zero. This means it does not have linear or angular acceleration.

- When applied to a couple, torque can be described as

  The sum of the moments produced by each of the forces in the couple

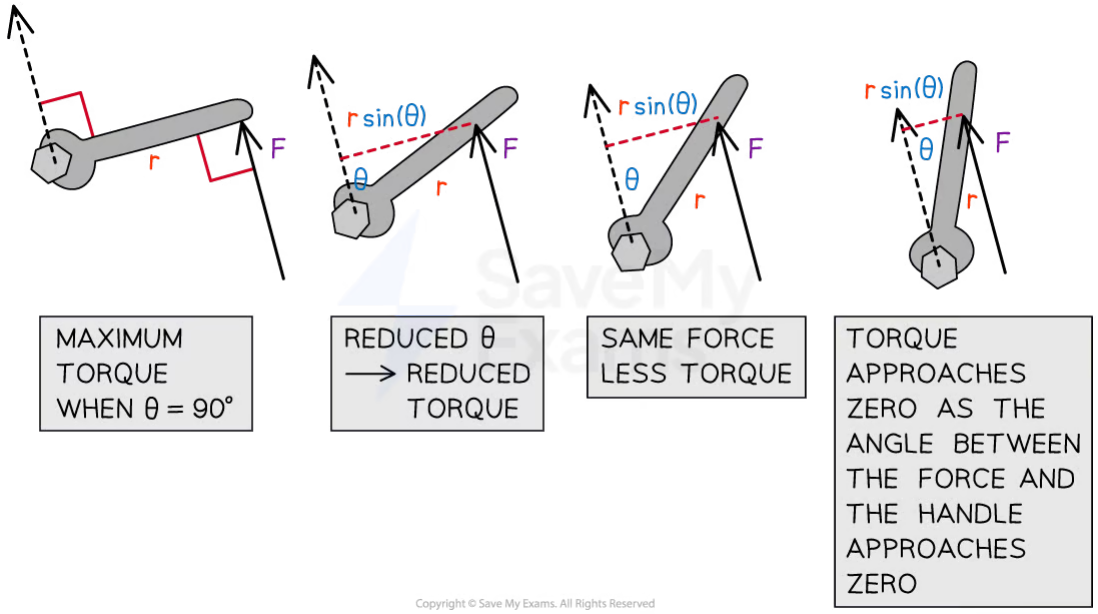
- For example, the torque provided by a couple on a steering wheel of radius *r* is

$$\tau = (F \times r\sin\theta) + (F \times r\sin\theta) = 2Fr\sin\theta$$

Since $\theta = 90°$, then $\tau = 2Fr$

- Therefore, the torque of a couple is equal to **double** the magnitude of the torque of the individual forces
- The forces are equal and act in opposite directions
  - Therefore, couples produce a resultant force of **zero**
- Due to Newton's Second law (**F** = **ma**), the steering wheel does **not** accelerate
  - In other words, when the force is applied, the steering wheel rotates with a **constant** angular speed but remains in the same location

The effect of angle on torque



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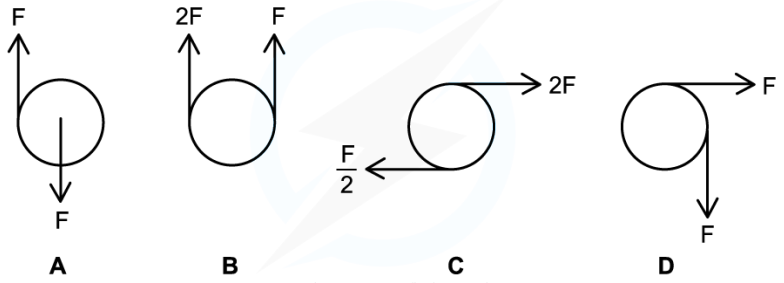
The force produces a maximum torque when applied perpendicular to the wrench. The same force is less effective when applied at non-perpendicular angles

Worked Example

Which pair of forces act as a couple on the circular object?



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Answer: A

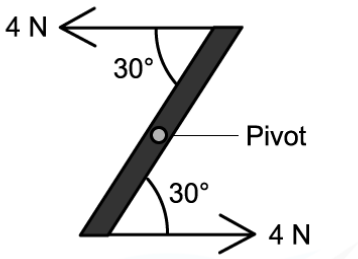
- In diagram **A**, the forces are:
  - Equal in size
  - In opposite directions
  - Perpendicular to the distance between them
- **B** is incorrect as the forces are in the same direction
- **C** is incorrect as the forces are different in size
- **D** is incorrect as the distance between the forces is not perpendicular

YOUR NOTES

Worked Example

A ruler of length 0.3 m is pivoted at its centre.

Two equal and opposite forces of magnitude 4 N are applied to the ends of the ruler, creating a couple as shown in the diagram.



Determine the magnitude of the torque of the couple on the ruler.

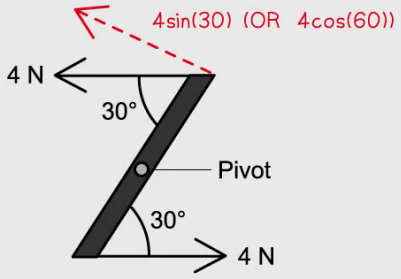
Answer:

STEP 1

TORQUE = FORCE × PERPENDICULAR DISTANCE

STEP 2

FIND THE COMPONENT OF THE FORCE THAT IS PERPENDICULAR TO THE DISTANCE



STEP 3

SUBSTITUTE VALUES INTO THE EQUATION

TORQUE = 4sin(30) N × 0.3 = 0.6 Nm

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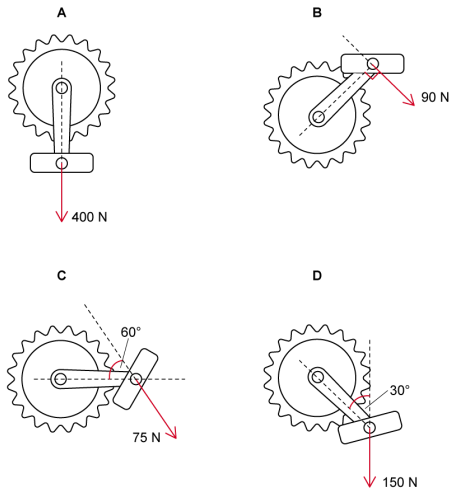
YOUR NOTES

Worked Example

The forces acting on a bicycle pedal at different positions during a ride are shown in the diagram below.

The distance from the pedal to the axis of rotation is 24 cm.



At which position is the magnitude of the torque the greatest?

Answer: B

- In position **A**: When $\theta = 180°$, $\sin\theta = 0$, so $\tau = 0$ i.e. the force exerts no torque on the pedal. When the pedal is at the bottom, no amount of pushing down will produce any torque on the pedal.
- In position **B**: When the angle θ is 90° and the force F is 90 N, the torque $\tau = 0.24 \times 90 \times \sin 90° =$ **21.6 N m**
- In position **C**: When the angle θ is 60° and the force F is 75 N, the torque $\tau = 0.24 \times 75 \times \sin 60° =$ **15.6 N m**
- In position **D**: When the angle θ is 30° and the force F is 150 N, the torque $\tau = 0.24 \times 150 \times \sin 30° =$ **18.0 N m**

YOUR NOTES

Exam Tip

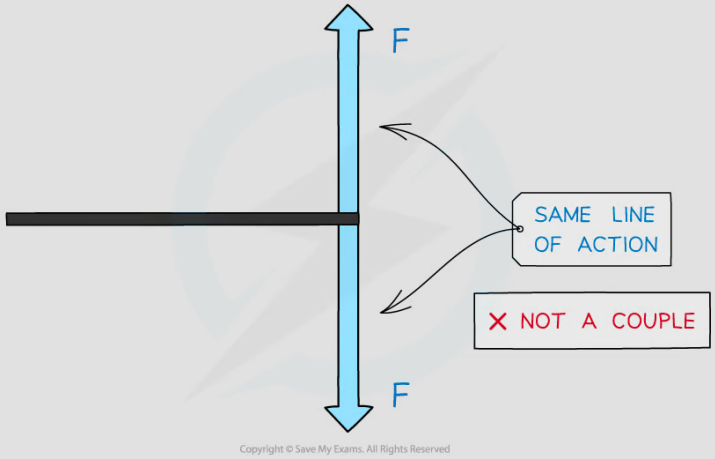
The terminology in this section can get confusing. For example, a moment is not a 'turning force' - the turning force is only part of the moment, the moment is the **effect** that the turning force has on the system when applied at a **distance** from a turning point, or pivot.

Also, torque is another term for 'moment', but in mechanics, torque can be used to describe a couple as a specific case of a moment.

Ultimately, when you carry out calculations, make sure you can identify

- The magnitude of the **applied force**
- The **perpendicular distance** between the force and the turning point (along the line of action)

Make sure you understand that the forces that make up a **couple** cannot share the same line of action (the line through the point at which the force is applied). An example of this is shown in the diagram below:



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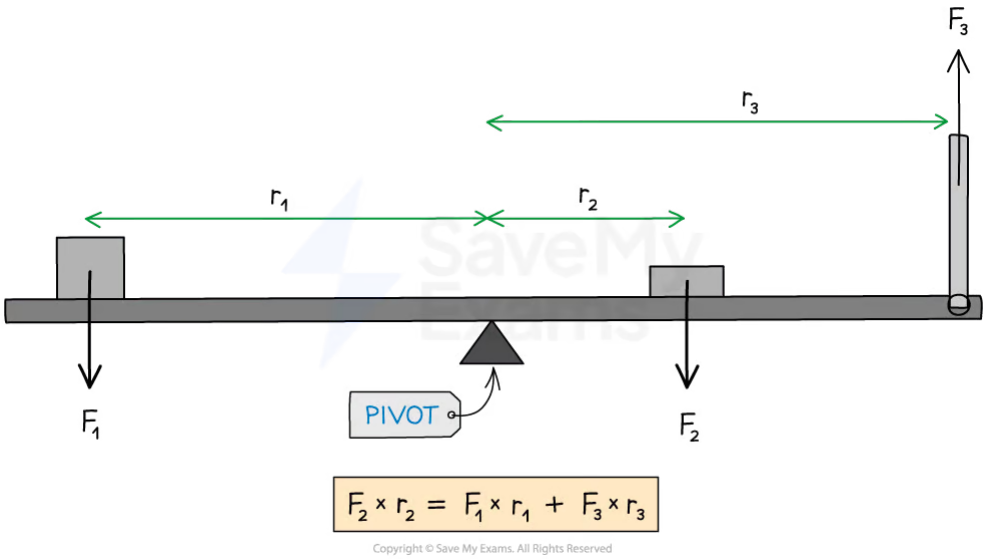
YOUR NOTES

Rotational Equilibrium

- A system is said to be in **rotational equilibrium** when
  - There is **no** resultant force acting
  - There is **no** resultant torque acting
- An object in rotational equilibrium will therefore remain **at rest**, or rotate with a **constant** angular velocity
  - This is analogous to Newton's First Law for translational equilibrium
- This means a body is in rotational equilibrium if

  The sum of the clockwise moments is equal to the sum of the anticlockwise moments
- This is also known as the principle of moments and can be applied to a range of scenarios, such as a balanced beam
  - A beam is an example of a rigid, extended body

A balanced beam in rotational equilibrium



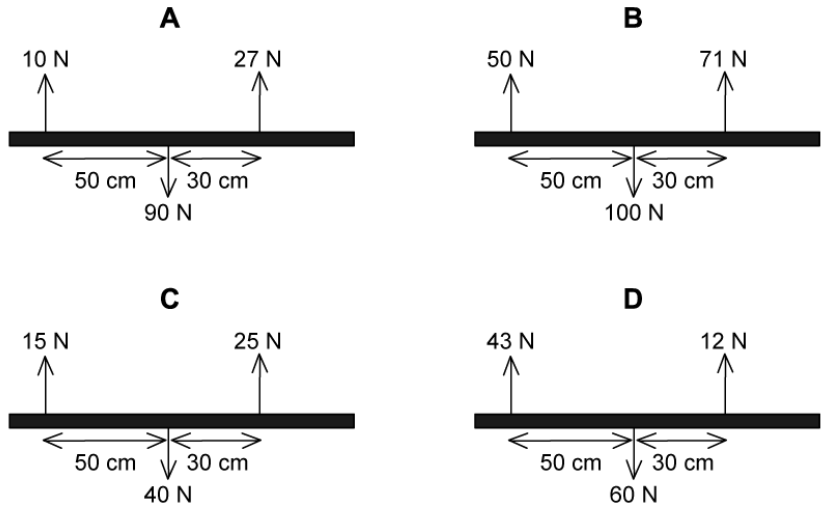
When the resultant force and resultant torque are both zero, the beam will be in rotational equilibrium

YOUR NOTES

Worked Example

Four beams of the same length each have three forces acting on them.

Which of the beams is in rotational equilibrium?



Answer: C

- A beam is in rotational equilibrium when there is **zero** resultant force and **zero** resultant torque acting on it
- In rotational equilibrium:

Total clockwise torque = Total anticlockwise torque

- Consider beam **C**, taking torques from the centre of the beam (where its weight acts) :

Torque, $\tau = Fr$ (as $\sin 90° = 1$)

Total clockwise torque = 15 × 50 = 750 N cm

Total anticlockwise torque = 25 × 30 = 750 N cm

- The total clockwise torque (750 N cm) = total anticlockwise torque (750 N cm), therefore, beam **C** is in rotational equilibrium

The other beams are **not** in rotational equilibrium because...

- Beam **A** has a resultant torque of 310 N cm anticlockwise
- Beam **B** has a resultant torque of 370 N cm clockwise
- Beam **D** has a resultant torque of 1790 N cm clockwise

YOUR NOTES

Exam Tip

When considering an object in rotational equilibrium, choosing certain points can simplify calculations of resultant torque. Remember you can choose **any** point, not just the axis of rotation.

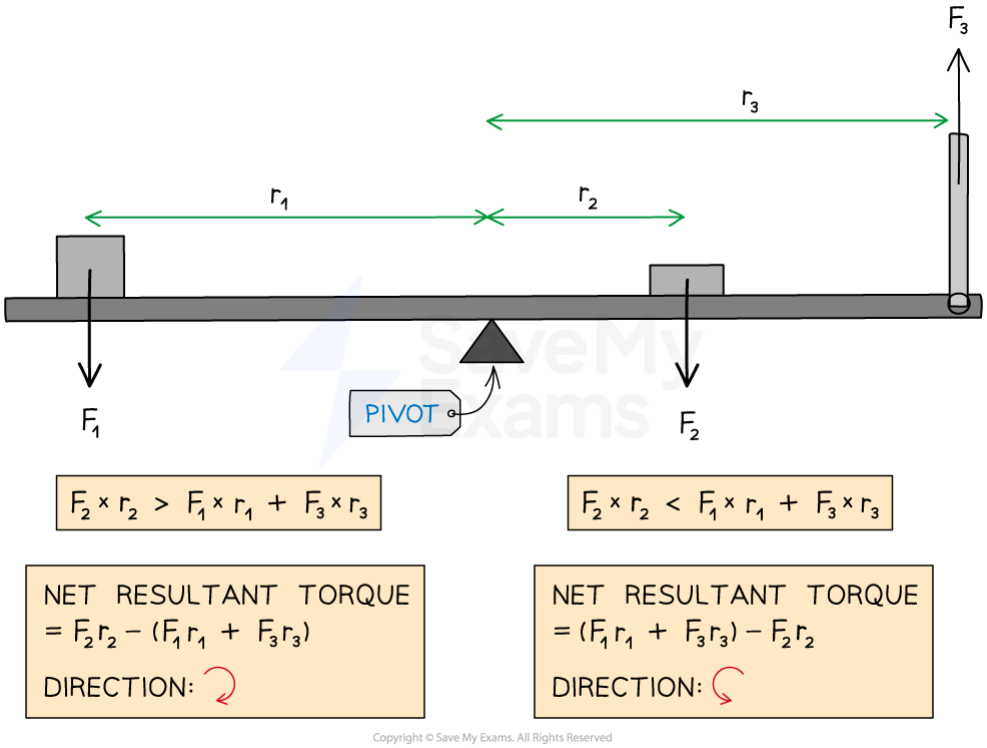
To simplify your calculation, choose a point where the torque of (most of) the forces are unknown, or when you need to determine where the resultant torque is **zero**. To do this, choose a point through which the lines of action of the forces pass

Unbalanced Torque

YOUR NOTES

- In the same way that a resultant force produces linear acceleration, a **resultant torque** produces **angular acceleration**
- The **direction** of the angular acceleration depends on the direction of the net resultant torque

Beam with an unbalanced torque



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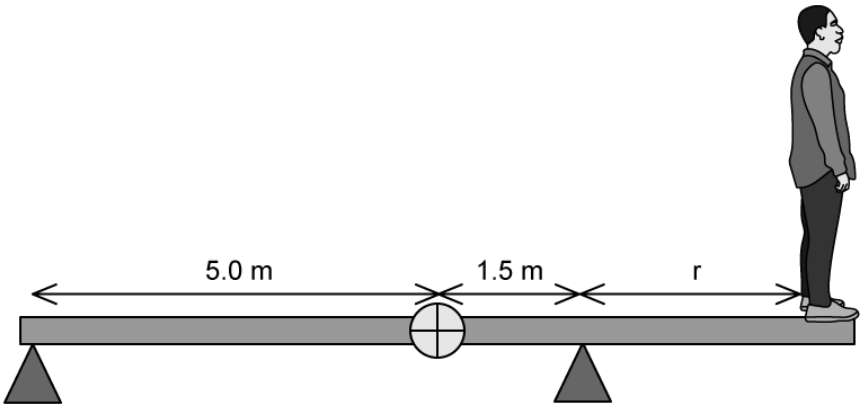
If there is a net resultant torque in the clockwise or anti-clockwise direction, the beam will also have an angular acceleration in that direction

YOUR NOTES

Worked Example

A uniform plank of mass 30 kg and length 10 m is supported at its left end and at a point 1.5 m from the centre.

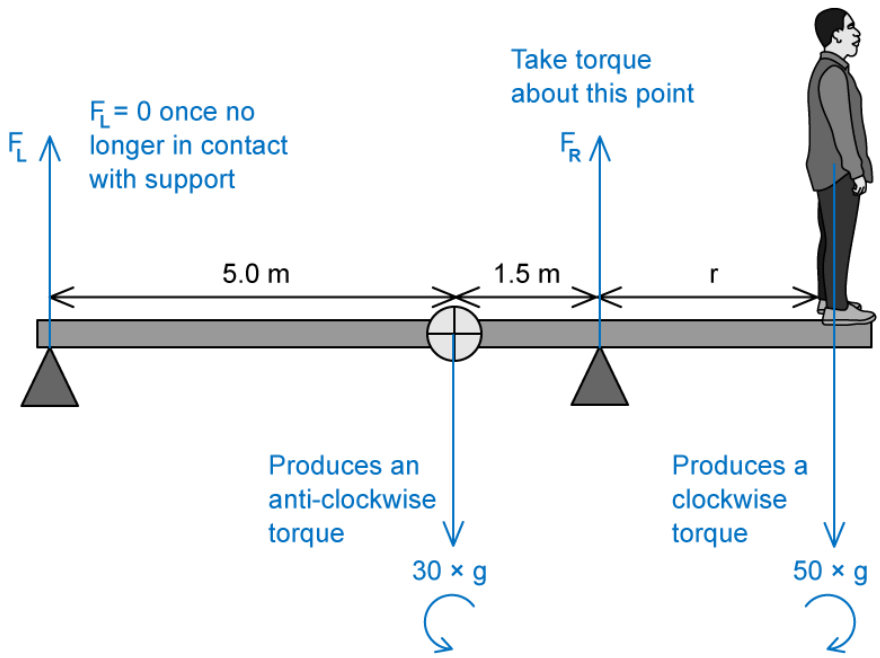


Calculate the maximum distance *r* to which a boy of mass 50 kg can walk without tipping the rod over.

Answer:

Step 1: Analyse the scenario and identify the forces

- Let the forces at each support be F_L (reaction force from the left support) and F_R (reaction force from the right support)
  - These are vertically upwards
- Just before the plank tips over, the system is in rotational equilibrium
- When the plank begins to tip over, the left support force F_L will become zero since the rod will no longer touch the support



Step 2: Take torques about the right support

- Torque = $Fr \sin\theta$ ($\theta = 90°$ for all)
- Clockwise torque = $50 \times g \times r$
- Anti-clockwise torque = $30 \times g \times 1.5$

Step 3: Equate the clockwise and anti-clockwise torques

$$30 \times g \times 1.5 = 50 \times g \times r$$

$$r = \frac{30\cancel{g} \times 1.5}{50\cancel{g}} = \frac{30 \times 1.5}{50}$$

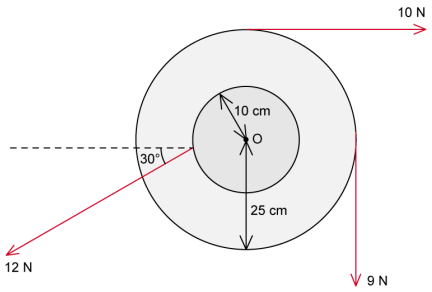
$$r = 0.90 \text{ m}$$

- Therefore, the plank will begin to tip once the boy is 0.90 m from the right support

YOUR NOTES

Worked Example

The diagram shows three forces acting on a wheel.



Determine the net resultant torque about the axis of rotation O. State whether the angular acceleration that is produced is clockwise or anticlockwise.

Answer:

Step 1: Recall the equation for torque

$$\tau = Fr\sin\theta$$

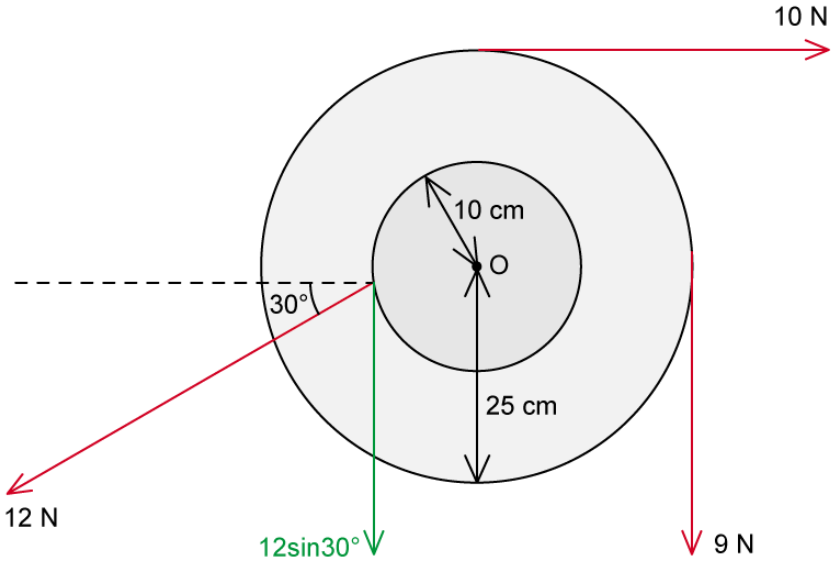
Step 2: Find the sum of the torques in the anti-clockwise direction

Torque of the 10 N force: $\tau = 10 \times 0.25 \times \sin 90° = 2.5\ \text{N m}$

Torque of the 9 N force: $\tau = 9 \times 0.25 \times \sin 90° = 2.25\ \text{N m}$

Total anti-clockwise torque = 2.5 + 2.25 = 4.75 N m

Step 3: Calculate the torque in the clockwise direction



Torque of the 12 N force: $\tau = 12 \times 0.1 \times \sin 30° = 0.6\ \text{N m}$

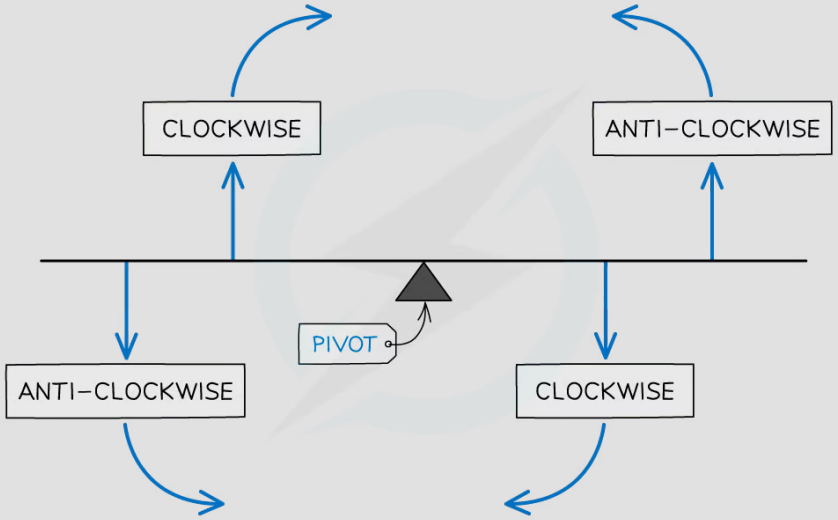
Step 4: Determine the net resultant torque

- Resultant torque = sum of anti-clockwise torques – sum of clockwise torques
- Resultant torque: $\sum \tau = 4.75 - 0.6 = 4.15$ N m, anticlockwise
- Direction of angular acceleration: **anticlockwise**

YOUR NOTES

Exam Tip

You should know that torque is a vector quantity, however, at this level, you will only need to consider whether it produces clockwise or anti-clockwise motion



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YOUR NOTES

Angular Displacement, Velocity & Acceleration

- A rigid rotating body can be described using the following properties:
  - Angular displacement
  - Angular velocity
  - Angular acceleration
- These properties can be inferred from the properties of objects moving in a straight line combined with the geometry of circles and arcs

Angular Displacement

- Angular displacement is defined as:

The change in angle through which a rigid body has rotated relative to a fixed point

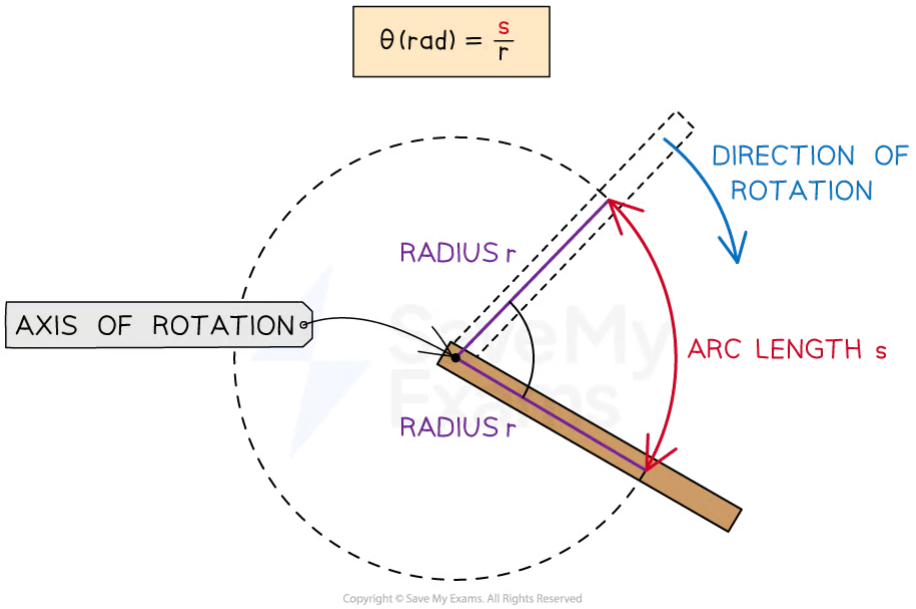
- Angular displacement is measured in radians

Angular displacement to linear displacement

- The linear displacement s at any point along a segment that is in rotation can be calculated using:

$$s = r\theta$$

- Where:
  - θ = angular displacement, or change in angle (radians)
  - s = length of the arc, or the linear distance travelled along a circular path (m)
  - r = radius of a circular path, or distance from the axis of rotation (m)



An angle in radians, subtended at the centre of a circle, is the arc length divided by the radius of the circle

YOUR NOTES

Angular Velocity

- The angular velocity ω of a rigid rotating body is defined as:

  The rate of change in angular displacement with respect to time

- Angular velocity is measured in rad s^{-1}
- This can be expressed as an equation:

$$\omega = \frac{\Delta \theta}{\Delta t}$$

- Where:
  - ω = angular velocity (rad s^{-1})
  - $\Delta\theta$ = angular displacement (rad)
  - Δt = change in time (s)

Angular velocity to linear velocity

- The linear speed v is related to the angular speed ω by the equation:

$$v = r\omega$$

- Where:
  - v = linear speed (m s^{-1})
  - r = distance from the axis of rotation (m)

- Taking the angular displacement of a complete cycle as 2π, angular velocity ω can also be expressed as:

$$\omega = \frac{v}{r} = 2\pi f = \frac{2\pi}{T}$$

- Rearranging gives the expression for linear speed:

$$v = 2\pi f r = \frac{2\pi r}{T}$$

- Where:
  - f = frequency of the rotation (Hz)
  - T = time period of the rotation (s)

Angular Acceleration

- Angular acceleration α is defined as

  The rate of change of angular velocity with time

- Angular acceleration is measured in rad s^{-2}
- This can be expressed as an equation:

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

- Where:
  - α = angular acceleration (rad s^{-2})
  - $\Delta \omega$ = change in angular velocity, or $\Delta \omega = \omega_f - \omega_i$ (rad s^{-1})
  - Δt = change in time (s)

Angular acceleration to linear acceleration

- Using the definition of angular velocity ω with the equation for angular acceleration α gives:

$$\Delta \omega = \frac{\Delta v}{r}$$

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\Delta v}{r \Delta t} = \frac{a}{r}$$

- Rearranging gives the expression for linear acceleration:

$$a = r\alpha$$

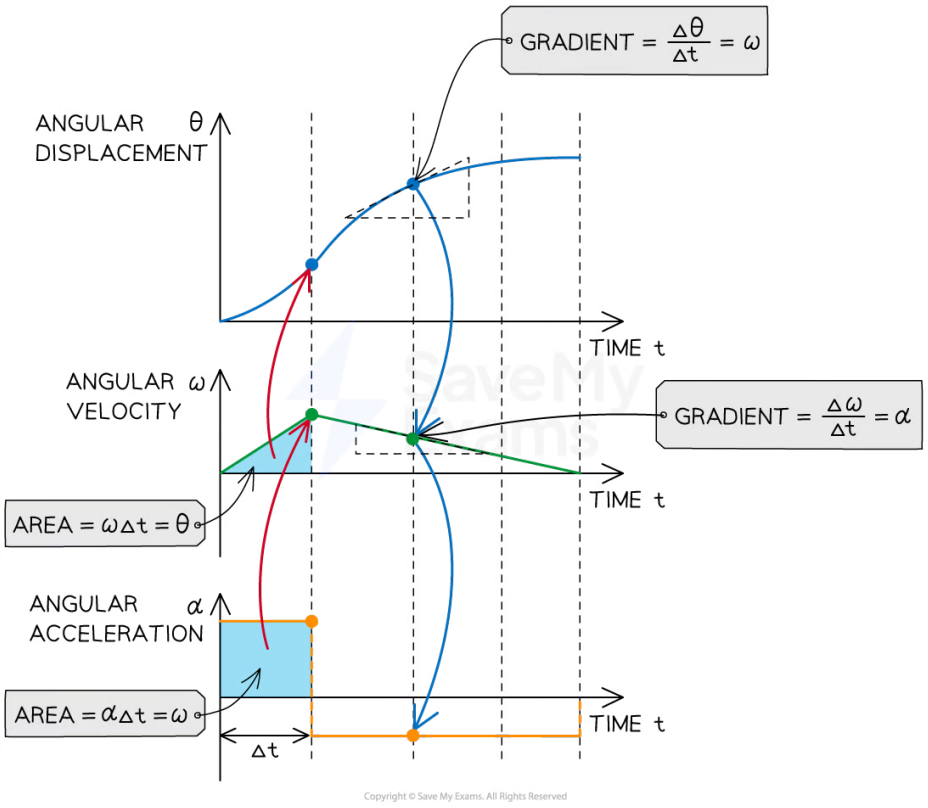
- Where:
  - a = linear acceleration (m s^{-2})
  - r = distance from the axis of rotation (m)
  - Δv = change in linear velocity, or $\Delta v = v - u$ (m s^{-1})

Graphs of Rotational Motion

- Graphs of rotational motion can be interpreted in the same way as linear motion graphs

YOUR NOTES

YOUR NOTES



Graphs of angular displacement, angular velocity and angular acceleration

- Angular displacement is equal to...
  - The **area under** the angular velocity-time graph
- Angular velocity is equal to...
  - The **gradient** of the angular displacement-time graph
  - The **area under** the angular acceleration-time graph
- Angular acceleration is equal to...
  - The **gradient** of the angular velocity-time graph

Summary of linear and angular variables

Variable	Linear	Angular
displacement	$s = r\theta$	$\theta = \frac{s}{r}$
velocity	$v = r\omega$	$\omega = \frac{v}{r}$
acceleration	$a = r\alpha$	$\alpha = \frac{a}{r}$

Exam Tip

While there are many similarities between the angular quantities used in this topic and the angular quantities used in the circular motion topic, make sure you are clear on the distinctions between the two, for example, **angular** acceleration and **centripetal** acceleration are **not** the same thing!

YOUR NOTES

YOUR NOTES

Angular Acceleration Formula

- The kinematic equations of motion for uniform linear acceleration can also be re-written for rotational motion
- The four kinematic equations for uniform **linear** acceleration are

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{(u + v)t}{2}$$

- This leads to the four kinematic equations for uniform **rotational** acceleration

$$\omega_i = \omega_f + \alpha t$$

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{(\omega_i + \omega_f)t}{2}$$

- The five linear variables have been swapped for the rotational equivalents, as shown in the table below

Variable	Linear	Rotational
displacement	s	θ
initial velocity	u	ω_i
final velocity	v	ω_f
acceleration	a	α
time	t	t

Worked Example

The turntable of a record player is spinning at an angular velocity of 45 RPM just before it is turned off. It then decelerates at a constant rate of 0.8 rad s^{-2}.

Determine the number of rotations the turntable completes before coming to a stop.

Answer:

Step 1: List the known quantities

- Initial angular velocity, ω_i = 45 RPM
- Final angular velocity, ω_f = 0
- Angular acceleration, α = 0.8 rad s^{-2}
- Angular displacement, $\Delta\theta$ = ?

Step 2: Convert the angular velocity from RPM to rad s^{-1}

- One revolution corresponds to 2π radians, and RPM = revolutions per minute, so

$$\omega = 2\pi f \text{ and } f = \frac{RPM}{60} \text{ (to convert to seconds)}$$

$$\omega_i = \frac{2\pi \times \text{RPM}}{60} = \frac{2\pi \times 45}{60} = \frac{3\pi}{2} \text{ rad s}^{-1}$$

Step 3: Select the most appropriate kinematic equation

- We know the values of ω_i, ω_f and α, and we are looking for angular displacement θ, so the best equation to use would be

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

Step 4: Rearrange and calculate the angular displacement $\Delta\theta$

$$0 = \omega_i^2 - 2\alpha\Delta\theta$$

$$\Delta\theta = \frac{\omega_i^2}{2\alpha} = \frac{\left(\frac{3\pi}{2}\right)^2}{2 \times 0.8}$$

Angular displacement, $\Delta\theta$ = 13.88 rad

Step 5: Determine the number of rotations in $\Delta\theta$

- There are 2π radians in 1 rotation
- Therefore, the number of rotations = $\frac{13.88}{2\pi}$ = 2.2
- This means the turntable spins 2.2 times before coming to a stop

YOUR NOTES

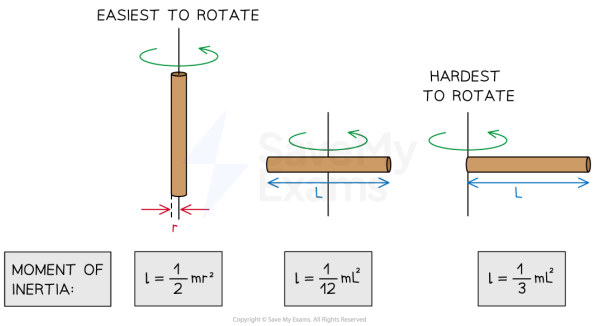
YOUR NOTES

Moment of Inertia

- In linear motion, the resistance to a change of motion, i.e. linear acceleration, is known as **inertia**
  - The larger the mass an object has, the greater its inertia
- In rotational motion, the distribution of mass around an axis must be considered, using **moments of inertia**
- The moment of inertia of a rigid, extended body is defined as:

  The resistance to a change of rotational motion, depending on the distribution of mass around a chosen axis of rotation

- Moment of inertia is measured in kg m^2
- The moment of inertia of a body corresponds to how 'easy' or 'hard' it is to rotate, and this is dependent on many factors, including
  - Its **shape**
  - Its **density**
  - Its **orientation** (relative to an axis of rotation)
- These factors allow an object's **distribution of mass** to be taken into account
  - It also means that the moment of inertia of a singular object can **change** depending on its orientation in relation to the chosen axis of rotation
- For example, the moment of inertia of a **thin rod** is different for each of the following orientations:
  - Rotation about its **vertical axis**
  - Rotation about its **centre of mass**
  - Rotation about **one end**



The moment of inertia of a body can change depending on its orientation relative to the axis of rotation

- These are just a few of the possible orientations of the axis of rotation for a thin rod
  - There is an **infinite** range of possible axes, and therefore moments of inertia
  - This also applies to nearly all rigid, extended objects that could be considered

Exam Tip

Make sure you are clear on the distinction between linear motion and rotational motion here. The implications of considering the distribution of masses in relation to an axis of rotation, as opposed to considering them as uniform, have important consequences when carrying out calculations

YOUR NOTES

YOUR NOTES

Calculating Moments of Inertia

- The moment of inertia I of a point mass is equal to

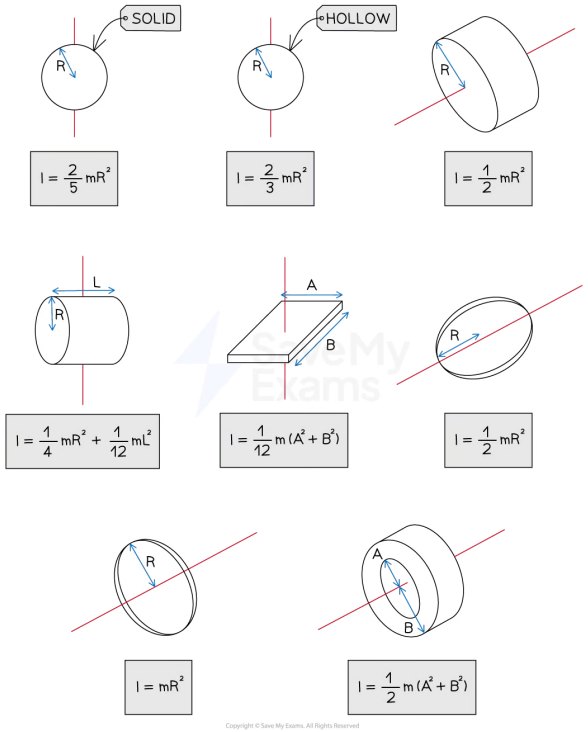
$$I = mr^2$$

- Where:
  - I = moment of inertia (kg m^2)
  - m = mass of the object (kg)
  - r = distance from its axis of rotation (m)

- A point mass is the simplest type of object to consider, the moment of inertia of a non-point mass can be calculated using

$$I = \sum mr^2$$

- This means that the sum of the moments of inertia of all the point masses in the system gives the total moment of inertia of the system
- Some moments of inertia of common shapes are shown below:

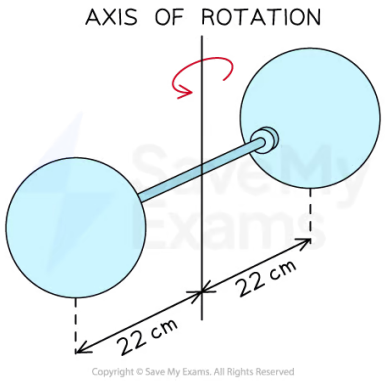


Moments of inertia of common shapes, where R represents radius and L represents length, as shown

Worked Example

Two solid spheres form a dumbbell when attached to each end of a thin rod. The dumbbell rotates with the centre of mass of each sphere at a distance of 22 cm from the axis of rotation, as shown in the diagram.

The thin rod has a mass of 20 g. Each sphere has a radius of 4 cm and a mass of 750 g.



Moment of inertia of a thin rod about its centre = $\frac{1}{12}mL^2$

Moment of inertia of a solid sphere = $\frac{2}{5}mr^2$

Determine

(a)
the overall moment of inertia of the dumbbell arrangement
(b)
the ratio of the moment of inertia of the thin rod to the overall moment of inertia of the dumbbell arrangement

Answer:

(a)

- The overall moment of inertia of the dumbbell is the sum of all the moments of inertia in the arrangement

$$I = \sum mr^2 = 2 \times \left(\frac{2}{5} m_{sphere} r^2\right) + \frac{1}{12} m_{rod} L^2$$

- Where:
  - Mass of a sphere, m_{sphere} = 750 g = 0.75 kg
  - Distance from axis to each sphere, r = 22 cm = 0.22 m
  - Mass of the rod, m_{rod} = 20 g = 0.02 kg
  - Length of the rod, L = 2 × (22 – 4) = 36 cm = 0.36 m

$$I = 2 \times \left(\frac{2}{5} \times 0.75 \times 0.22^2 \right) + \left(\frac{1}{12} \times 0.02 \times 0.36^2 \right)$$

Moment of inertia of the dumbbell: $I = 0.029 \text{ kg m}^2$

(b)

- The moment of inertia of the thin rod is

$$I_{rod} = \frac{1}{12} \times 0.02 \times 0.36^2 = 2.16 \times 10^{-4} \text{ kg m}^2$$

- Therefore, the ratio I_{rod} / I is

$$\frac{I_{rod}}{I} = \frac{2.16 \times 10^{-4}}{0.029} = 0.0071$$

- This means the rod contributes about 0.7% of the overall moment of inertia of the dumbbell

Exam Tip

You will never be expected to memorise the moments of inertia of different shapes, they will **always** be given in an exam question where required

YOUR NOTES

Newton's Second Law for Rotation

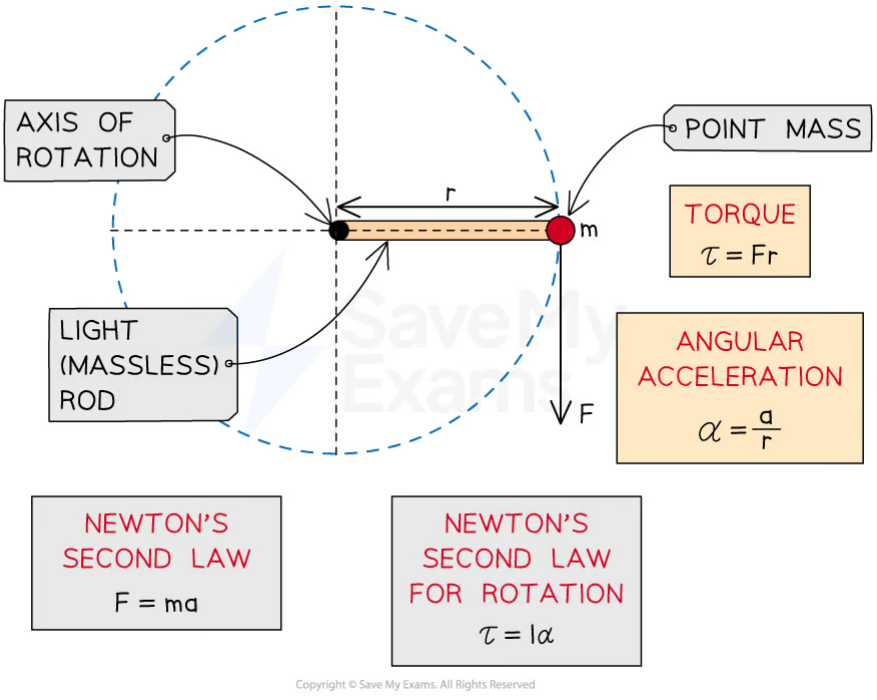
- In linear motion, the **force** required to give an object a certain **acceleration** depends on its **mass**

$$F = ma$$

- This is Newton's Second Law of **linear motion**, where:
  - F = force (N)
  - m = mass (kg)
  - a = linear acceleration (m s^{-2})
- In rotational motion, the **torque** required to give a rotating object a certain **angular acceleration** depends on its **moment of inertia**

$$\tau = I\alpha$$

- This is Newton's Second Law of **rotational motion**, where:
  - τ = torque (N m)
  - I = moment of inertia (kg m^2)
  - α = angular acceleration (rad s^{-2})



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Newton's second law for rotating bodies is equivalent to Newton's second law for linear motion

- This equation comes from the fact that **torque** is the rotational equivalent of **force**:

Force: $F = ma$

YOUR NOTES

Torque: $\tau = Fr$

- Where:
  - r = distance from the axis of rotation (m)
- Combining these equations gives:

$$\tau = r(ma)$$

- The **moment of inertia** of a rotating body can be thought of as analogous to **mass**
  - The inertia of a mass describes its ability to resist changes to **linear** motion, which is referring to **linear acceleration**
  - Similarly, the moment of inertia of a mass describes its ability to resist changes to **rotational** motion, which is referring to **angular acceleration**

Angular acceleration: $\alpha = \frac{a}{r}$

Moment of inertia (point mass): $I = mr^2$

- Using these equations with the equations for force and torque leads to:

$$\tau = r(mr\alpha)$$

$$\tau = (mr^2)\alpha$$

$$\tau = I\alpha$$

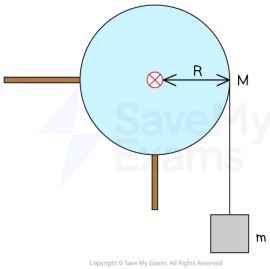
Comparison of linear and rotational variables in Newton's Second Law

Linear variable	Rotational variable
Force, F	Torque, τ
Mass, m	Moment of inertia, I
Acceleration, a	Angular acceleration, α
Newton's Second Law, $F \propto a$	Newton's Second Law, $\tau \propto \alpha$
$F = ma$	$\tau = I\alpha$

Worked Example

A block of mass *m* is attached to a string that is wrapped around a cylindrical pulley of mass *M* and radius *R*, as shown in the diagram.

The moment of inertia of the cylindrical pulley about its axis is $\frac{1}{2}MR^2$.

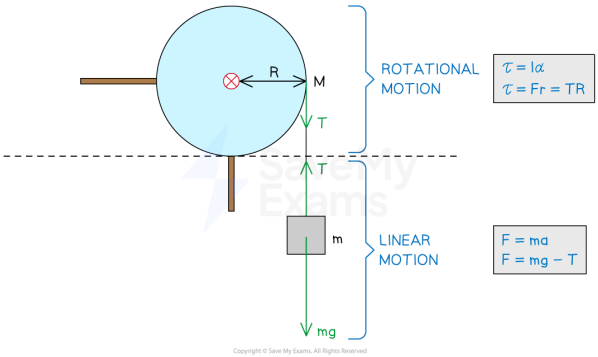


When the block is released, the pulley begins to turn as the block falls.

Write an expression for the acceleration of the block.

Answer:

Step 1: Identify the forces acting on the block



Step 2: Apply Newton's second law to the motion of the block

$$F = ma$$

$$mg - T = ma \quad \textbf{eq. (1)}$$

Step 3: Apply Newton's second law to the rotation of the pulley

$$\tau = I\alpha$$

$$TR = I\alpha$$

Step 4: Write the equation for the pulley in terms of acceleration a

- The angular acceleration α of the pulley is:

$$\alpha = \frac{a}{R}$$

- Substitute this into the previous equation:

$$TR = I\frac{a}{R}$$

- Substitute in the expression for the moment of inertia and simplify:

Moment of inertia of the cylinder: $I = \frac{1}{2}MR^2$

$$TR = \left(\frac{1}{2}MR^2\right)\frac{a}{R}$$

$$T = \left(\frac{1}{2}MR^2\right)\frac{a}{R^2} = \frac{1}{2}Ma$$

$$T = \frac{1}{2}Ma \text{ eq. (2)}$$

Step 5: Substitute eq. (2) into eq. (1) and rearrange for acceleration a

$$mg - \frac{1}{2}Ma = ma$$

$$mg = ma + \frac{1}{2}Ma = a\left(m + \frac{M}{2}\right)$$

Acceleration of the block: $a = \dfrac{mg}{m + \dfrac{M}{2}}$

YOUR NOTES

Angular Momentum

- Angular momentum is the **rotational equivalent** of linear momentum, which is defined by mass × velocity, or

$$p = mv$$

- Therefore, angular momentum L is defined by

$$L = I\omega$$

- Where:
  - L = angular momentum (kg m^2 rad s^{-1})
  - I = moment of inertia (kg m^2)
  - ω = angular velocity (rad s^{-1})

Angular Momentum of a Point Mass

- The moment of inertia of a rotating point mass *m* which is a distance *r* from an axis of rotation is equal to

$$I = mr^2$$

- The angular velocity of the point mass is given by

$$\omega = \frac{v}{r}$$

- Therefore, the angular momentum of the point mass is equal to

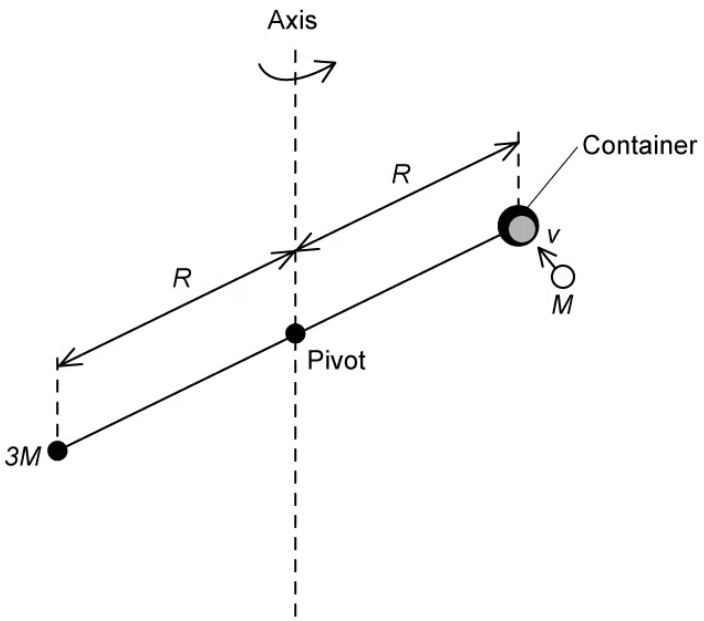
$$L = I\omega = (mr^2) \times \frac{v}{r} = mvr$$

YOUR NOTES

Worked Example

A horizontal rigid bar is pivoted at its centre so that it is free to rotate. A point particle of mass $3M$ is attached at one end of the bar and a container is attached at the other end, both are at a distance of R from the central pivot.

A point particle of mass M moves with velocity v at right angles to the rod as shown in the diagram.



The particle collides with the container and stays within it as the system starts to rotate about the vertical axis with angular velocity ω.

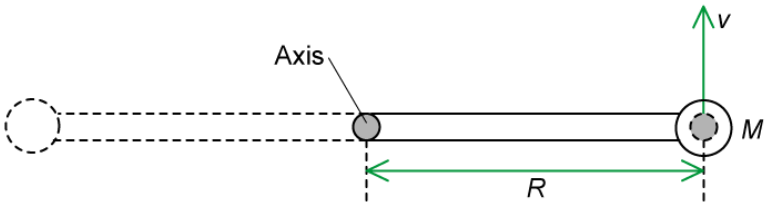
The mass of the rod and the container are negligible.

Write an expression for the angular momentum of the system about the vertical axis:

(a)
just before the collision, in terms of M, v and R
(b)
just after the collision, in terms of M, R and ω.

Answer:

(a) Just before the collision:



- Angular momentum is equal to:

$$L = I\omega$$

- The moment of inertia of a point particle is

$$I = mr^2$$

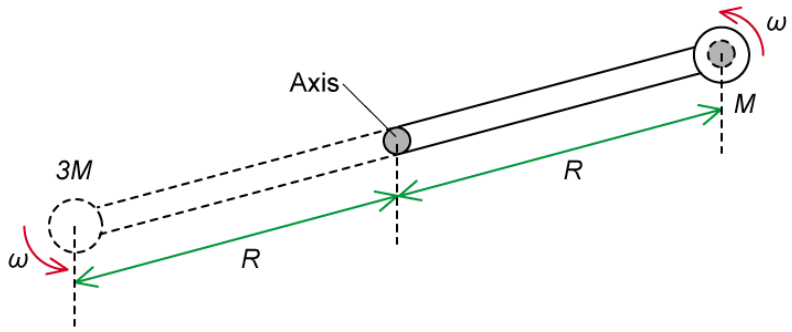
- Linear velocity is related to angular velocity by

$$v = \omega r$$

- The rod, container and 3*M* mass are all stationary before the collision, so we only need to consider the angular momentum of the point particle
- Where:
  - Mass of the particle, $m = M$
  - Distance of the particle from the axis, $r = R$
  - Angular velocity of the particle, $\omega = \frac{v}{R}$
- Therefore, the angular momentum of the system before the collision is:

$$L = (MR^2) \times \frac{v}{R} = MvR$$

(b) After the collision:



- The whole system rotates with an angular velocity of ω
- We are considering the rod and the container as massless, so we only need to consider the angular momentum of the two masses *M* and 3*M*
- Therefore, the angular momentum of the system after the collision is:

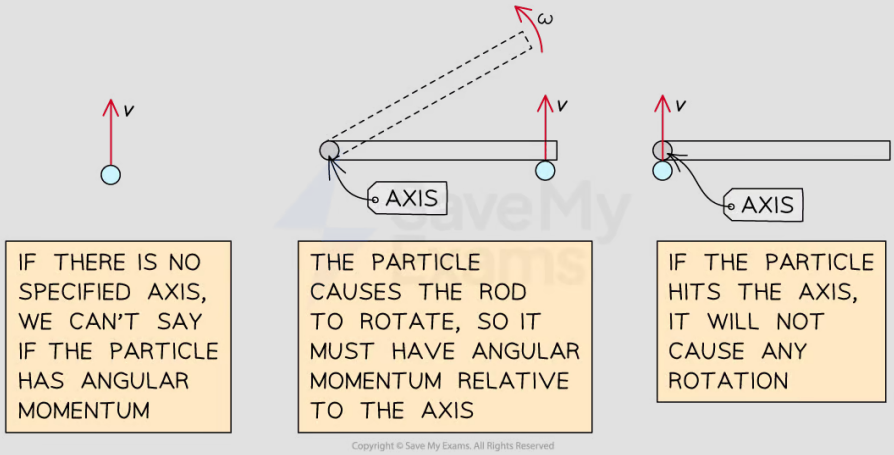
$$L = (MR^2)\omega + (3MR^2)\omega$$

$$L = 4MR^2\omega$$

YOUR NOTES

Exam Tip

You should know that objects travelling in straight lines **can** have angular momentum - just make sure you understand that it all depends on the position of the object in relation to the axis of rotation being considered



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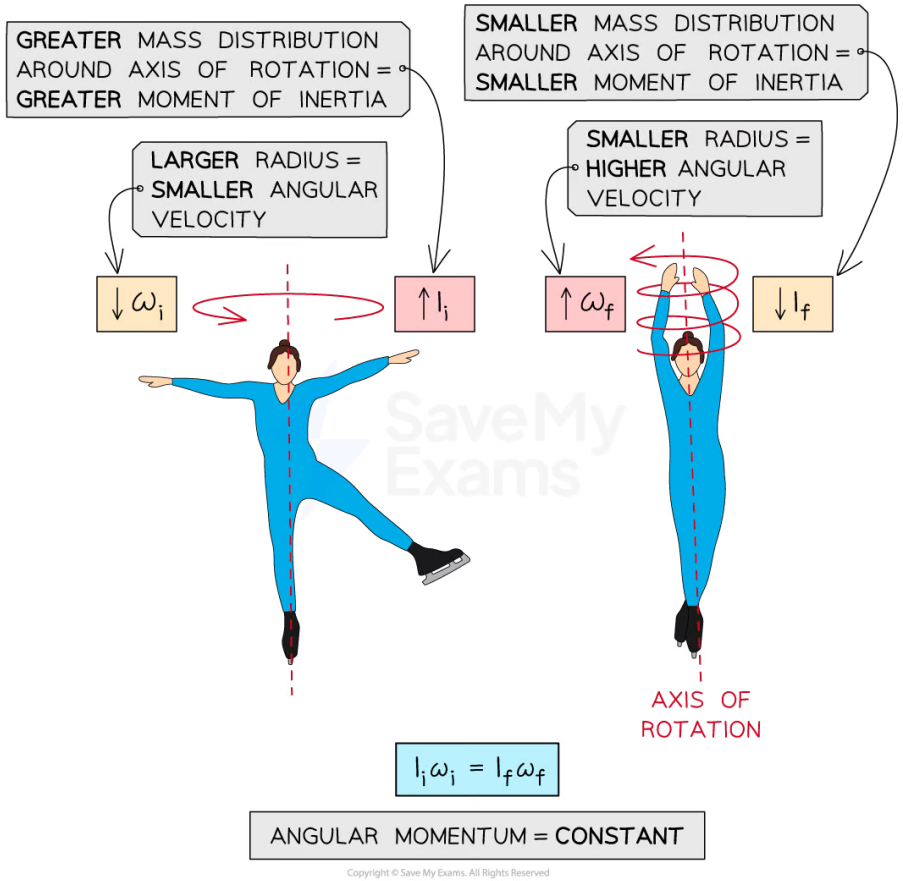
Conservation of Angular Momentum

YOUR NOTES

- As with linear momentum, angular momentum is **always conserved**
- The principle of conservation of angular momentum states:

> **The angular momentum of a system always remains constant, unless a net torque is acting on the system**

- This conservation law has many real-world applications, for example
  - A person on a **spinning chair** spins faster while their arms and legs are contracted and slower while extended
  - Objects in **elliptical orbits** travel faster nearer the object they orbit and slower when further away
  - **Ice skaters** can change their rotation velocity by extending or contracting their arms
  - **Tornados** spin faster as their radius decreases



Ice skaters can change their moment of inertia by extending or contracting their arms and legs. Due to conservation of angular momentum, this allows them to spin faster or slower

- Problems involving a **change in angular momentum** can be solved using the equation:

$$I_i \omega_i = I_f \omega_f$$

- Where:

- I_i = initial moment of inertia (kg m^2)
- ω_i = initial angular velocity (rad s^{-1})
- I_f = final moment of inertia (kg m^2)
- ω_f = final angular velocity (rad s^{-1})

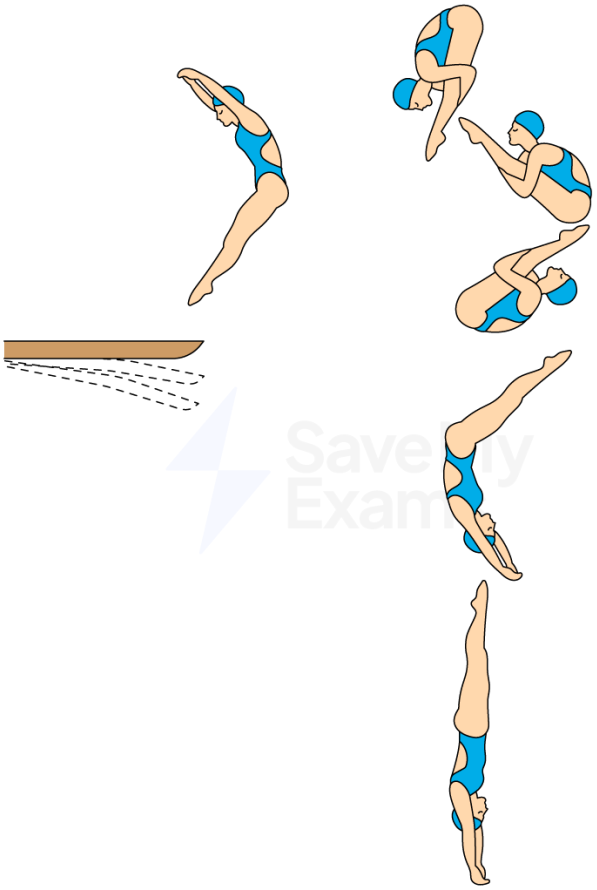
YOUR NOTES

YOUR NOTES

Worked Example

The diagram shows the different positions of a diver between jumping off a springboard and entering the water.

During their fall, the diver pulls their arms and legs into a tight tuck position while in the air and straightens them before entering the water.



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Which row correctly describes the changes to the diver's moment of inertia and angular velocity as they bring their limbs closer to their body?

	moment of inertia	angular velocity
A.	increases	increases
B.	decreases	increases
C.	increases	decreases
D.	decreases	decreases

Answer: B

- After the diver leaves the springboard, there is no longer a resultant torque acting on them
  - This means their angular momentum remains **constant** throughout the dive
- Due to the conservation of angular momentum:

$$I_i \omega_i = I_f \omega_f = constant$$

- When the diver tucks their arms and legs in closer to their body, they **decrease** their moment of inertia
  - This eliminates options **A** & **C**
- Therefore, to conserve angular momentum, when the diver's moment of inertia decreases, their angular velocity must **increase**

YOUR NOTES

Worked Example

A spherical star of mass *M* and radius *R* rotates about its axis. The star explodes, ejecting mass in space radially and symmetrically. The remaining star is left with a mass of $\frac{1}{10}M$ and a radius of $\frac{1}{50}R$.

Calculate the ratio of the star's final angular velocity to its initial angular velocity.

The moment of inertia of a sphere is $\frac{2}{5}MR^2$

Answer:

- Before the star explodes:
  - Initial moment of inertia, $I_i = \frac{2}{5}MR^2$
  - Initial angular velocity = ω_i
- After the star explodes:
  - Final moment of inertia, $I_f = \frac{2}{5}\left(\frac{1}{10}M\right)\left(\frac{1}{50}R\right)^2$
  - Final angular velocity = ω_f
- From the conservation of angular momentum:

$$I_i\omega_i = I_f\omega_f$$

$$\left(\frac{2}{5}MR^2\right)\omega_i = \left(\frac{2}{5}\times\frac{1}{10}M\times\left(\frac{1}{50}\right)^2R^2\right)\omega_f$$

$$\left(\frac{2}{5}\cancel{MR^2}\right)\omega_i = \left(\frac{2}{5}\cancel{MR^2}\right)\frac{1}{25\,000}\omega_f$$

- Therefore, the ratio of the star's final angular velocity to its initial angular velocity is:

$$\frac{\omega_f}{\omega_i} = 25\,000$$

YOUR NOTES

Angular Impulse

- In linear motion, the **resultant force** on a body can be defined as the rate of change of linear momentum:

$$F = \frac{\Delta p}{\Delta t}$$

- This leads to the definition of linear impulse:

> **An average resultant force F acting for a time Δt produces a change in linear momentum Δp**

$$\Delta p = F\Delta t = \Delta(mv)$$

- Similarly, the **resultant torque** on a body can be defined as the rate of change of angular momentum:

$$\tau = \frac{\Delta L}{\Delta t}$$

- Where:
  - τ = resultant torque on a body (N m)
  - ΔL = change in angular momentum (kg m^2 s^{-1})
  - Δt = time interval (s)

- This leads to the definition of angular impulse:

> **An average resultant torque τ acting for a time Δt produces a change in angular momentum ΔL**

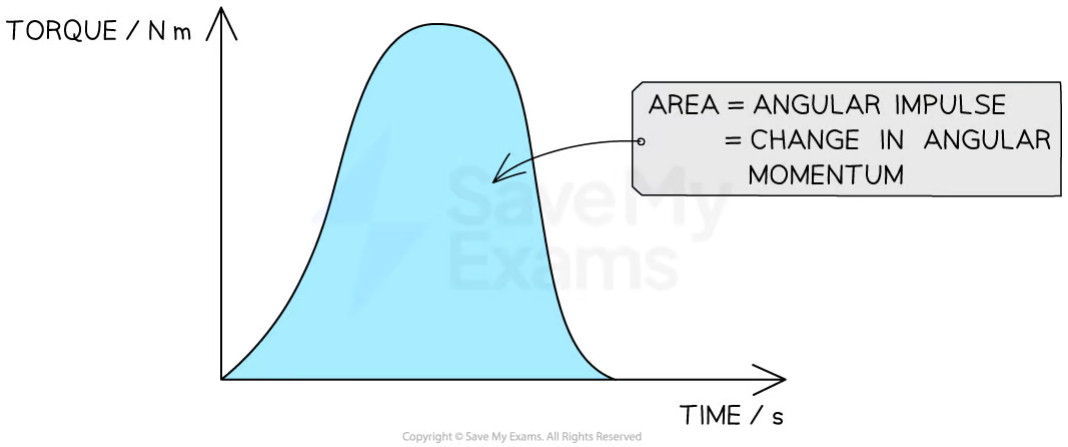
$$\Delta L = \tau\Delta t = \Delta(I\omega)$$

- Angular impulse is measured in kg m^2 s^{-1}, or N m s
- This equation requires the use of a **constant** resultant torque
  - If the resultant torque changes, then an **average** of the values must be used

- Angular impulse describes the effect of a torque acting over a time interval
  - This means a **small torque** acting over a **long time** has the same effect as a **large torque** acting over a **short time**

Angular Impulse on a Torque-Time Graph

- The area under a torque-time graph is equal to the angular impulse or the change in angular momentum



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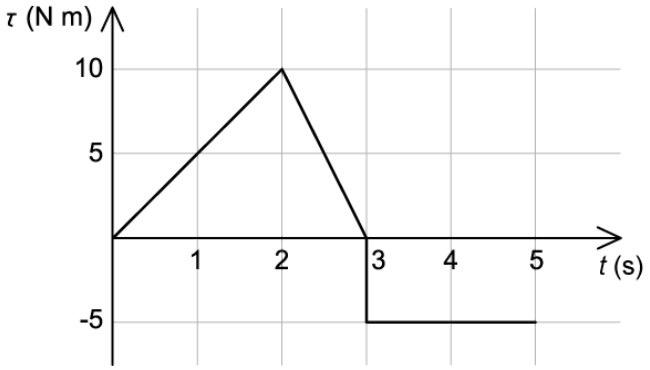
When the torque is not constant, the angular impulse is the area under a torque–time graph

Worked Example

The graph shows the variation of time t with the net torque τ on an object which has a moment of inertia of 5.0 kg m^2.



At t = 0, the object rotates with an angular velocity of 2.0 rad s^{-1} clockwise.

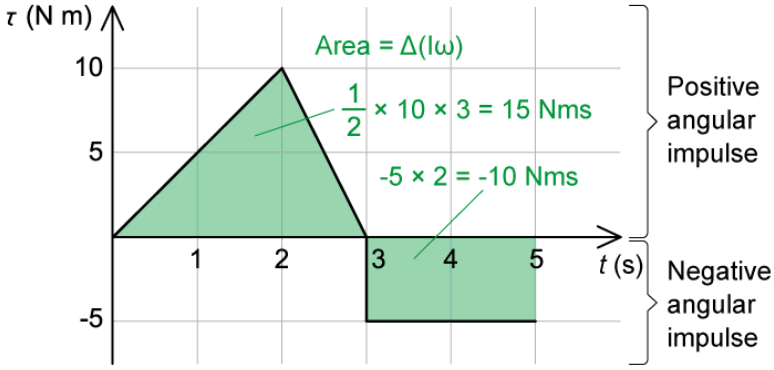
Determine the magnitude and direction of rotation of the angular velocity at t = 5 s.

In this question, take anticlockwise as the positive direction.

Answer:

- The area under a torque-time graph is equal to angular impulse, or the change in angular momentum

$$\Delta L = \tau \times \Delta t$$



- The area under the positive curve (triangle) = $\frac{1}{2} \times 10 \times 3 = 15 \text{ N m s}$
- The area under the negative curve (rectangle) = $-5 \times 2 = -10 \text{ N m s}$
- Therefore, the overall change in angular momentum is

$$\Delta L = 15 - 10 = 5 \text{ N m s}$$

- The change in angular momentum is equal to

$$\Delta L = \Delta(I\omega) = I(\omega_f - \omega_i)$$

- Where
  - Moment of inertia, $I = 5.0$ kg m^2
  - Initial angular velocity, $\omega_i = -2.0$ rad s^{-1} (clockwise is the negative direction)
- Therefore, when t = 5 s, the angular velocity is

$$5 \times \left(\omega_f - (-2)\right) = 5$$

$\omega_f = -1.0$ rad s^{-1} in the clockwise direction

YOUR NOTES

YOUR NOTES

Rotational Kinetic Energy

- A body moving with **linear velocity** has an associated linear kinetic energy given by

$$E_k = \frac{1}{2}mv^2$$

$$E_k = \frac{p^2}{2m}$$

- Similarly, a rotating body with **angular velocity** has an associated **rotational kinetic energy** given by

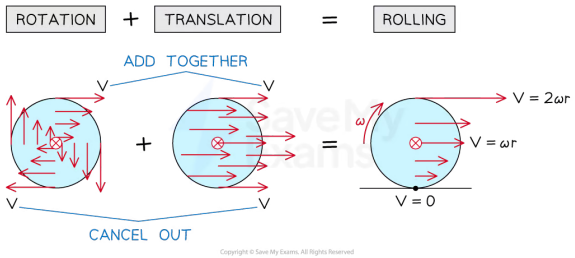
$$E_k = \frac{1}{2}I\omega^2$$

$$E_k = \frac{L^2}{2I}$$

- Where:
  - E_k = rotational kinetic energy (J)
  - I = moment of inertia (kg m^2)
  - ω = angular velocity (rad s^{-1})
  - L = angular momentum (kg m^2 s^{-1})

Rolling without slipping

- Circular objects, such as wheels, are made to move with **both** linear and rotational motion
  - For example, the wheels of a car, or bicycle **rotate** causing it to move **forward**
- Rolling motion without slipping is a combination of **rotating** and **sliding** (translational) motion
- When a disc **rotates**:
  - Each point on the disc has a **different** linear velocity depending on its distance from the centre $(v \propto r)$
  - The linear velocity is the **same** at all points on the **circumference**
- When a disc **slips**, or **slides**:
  - There is **not enough friction** present to allow the object to roll
  - Each point on the object has the **same** linear velocity
  - The angular velocity is **zero**
- So, when a disc **rolls without slipping**:
  - There is enough **friction** present to initiate rotational motion allowing the object to roll
  - The point in contact with the surface has a **velocity of zero**
  - The centre of mass has a velocity of $v = \omega r$
  - The top point has a velocity of $2v$ or $2\omega r$



Rolling motion is a combination of rotational and translational motion. The resultant velocity at the bottom is zero and the resultant velocity at the top is 2v

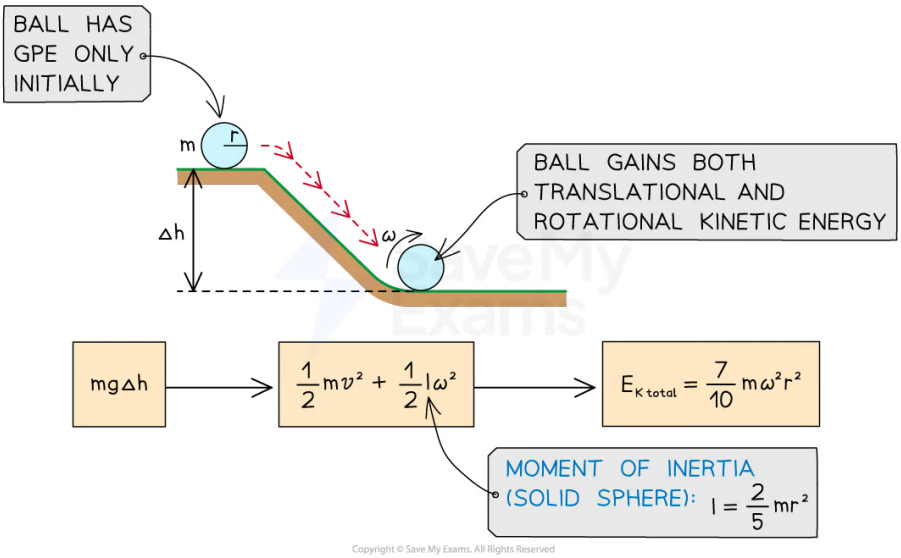
Rolling down a slope

- Another common scenario involving rotational and translational motion is an object (usually a ball or a disc) **rolling down a slope**
- At the **top** of the slope, a stationary object will have gravitational potential energy equal to

$$E_p = mg\Delta h$$

- As the object **rolls** down the slope, the gravitational potential energy will be transferred to **both** translational (linear) and rotational kinetic energy
- At the **bottom** of the slope, the **total kinetic energy** of the object will be equal to

$$E_{K\ total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$



- The linear or angular velocity can then be determined by
  - Equating E_p and $E_{K\ total}$
  - Using the equation for the moment of inertia of the object
  - Using the relationship between linear and angular velocity $v = \omega r$
- For example, for a ball (a solid sphere) of mass *m* and radius *r*, its moment of inertia is

$$I = \frac{2}{5}mr^2$$

YOUR NOTES

- Equating the equations for E_p and $E_{K\ total}$ and simplifying gives

$$mg\Delta h = \frac{1}{2}m(\omega r)^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2$$

$$mg\Delta h = \frac{1}{2}m\omega^2 r^2 + \frac{1}{5}m\omega^2 r^2$$

$$mg\Delta h = \frac{7}{10}m\omega^2 r^2$$

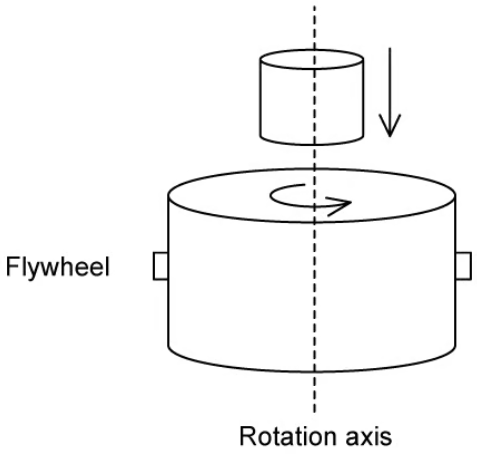
YOUR NOTES

Worked Example

A flywheel of mass *M* and radius *R* rotates at a constant angular velocity ω about an axis through its centre. The rotational kinetic energy of the flywheel is E_K.

The moment of inertia of the flywheel is $\frac{1}{2}MR^2$.

A second flywheel of mass $\frac{1}{2}M$ and radius $\frac{1}{2}R$ is placed on top of the first flywheel. The new angular velocity of the combined flywheels is $\frac{2}{3}\omega$.



What is the new rotational kinetic energy of the combined flywheels?

A. $\frac{E_K}{2}$ **B.** $\frac{E_K}{4}$ **C.** $\frac{E_K}{8}$ **D.** $\frac{E_K}{24}$

Answer: A

- The kinetic energy of the first flywheel is

$$E_K = \frac{1}{2}I\omega^2 = \frac{1}{2}\times\left(\frac{1}{2}MR^2\right)\times\omega^2$$

$$E_K = \frac{1}{4}MR^2\omega^2$$

- The combined flywheels have a total moment of inertia of

$$I_{new} = I_1 + I_2$$

$$I_{new} = \frac{1}{2}MR^2 + \frac{1}{2}\left(\frac{1}{2}M\right)\left(\frac{1}{2}R\right)^2$$

$$I_{new} = \frac{9}{16}MR^2$$

- The kinetic energy of the combined flywheels is

$$E_{K\,new} = \frac{1}{2}I_{new}\omega_{new}{}^2 = \frac{1}{2}\times\left(\frac{9}{16}MR^2\right)\times\left(\frac{2}{3}\omega\right)^2$$

$$E_{K\,new} = \frac{1}{2}\times\left(\frac{1}{4}MR^2\omega^2\right) = \frac{1}{2}E_K$$

YOUR NOTES

YOUR NOTES

Galilean & Special Relativity

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YOUR NOTES

Reference Frames

- The term **relative** is used often in Physics to make it clear which point of view we are referring to
  - For example, the velocity of a car **relative** to someone stationary is different to the velocity measured by another car travelling alongside the initial car at the same speed
- A **reference frame**, or a **frame of reference**, refers to the **position** of an object, it is defined as:

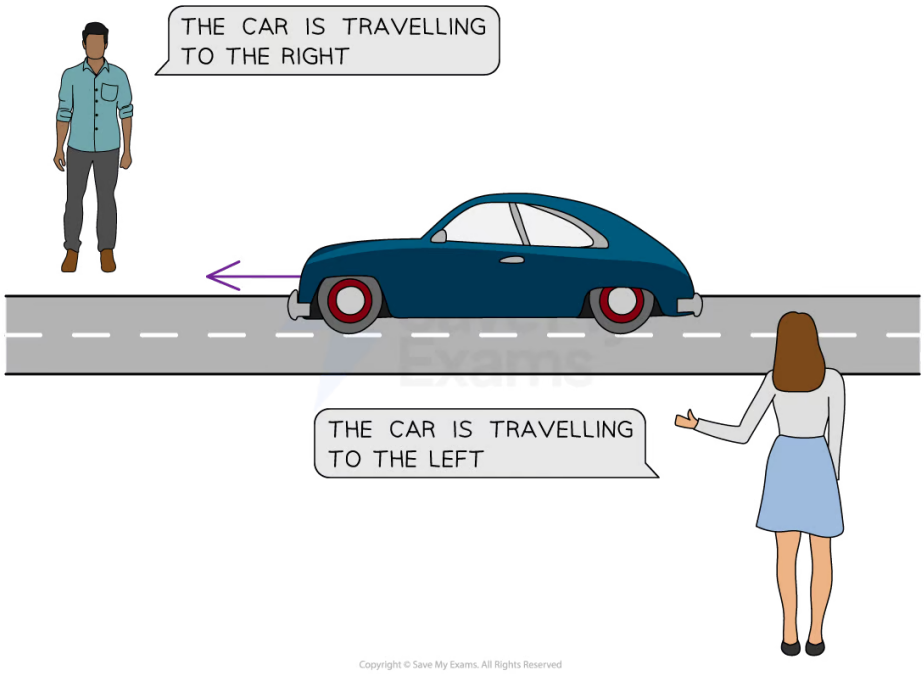
  A set of coordinates to record the position and time and events

- For example, you currently sitting on your chair at your desk is your current reference frame
  - You feel as if you are stationary, despite the fact the Earth is revolving on its axis and orbiting the sun
- It is the point of view where an object, at a specific co-ordinate, is **at rest**

Examples of Reference Frames

- An everyday example is the **direction** of an object from your point of view in comparison to someone else
- In this example, a car is driving down a road and two people are standing on opposite sides of that road
- Despite the car moving in one direction, each person will view its direction **relative to them** differently
  - The person on one side of the road would say the car is moving to the right, and the person on the other side of the road would say the car is moving to the left
  - Both are correct, but they are viewing the car's motion from **different points of reference**

Diagram showing different points of reference for a moving car



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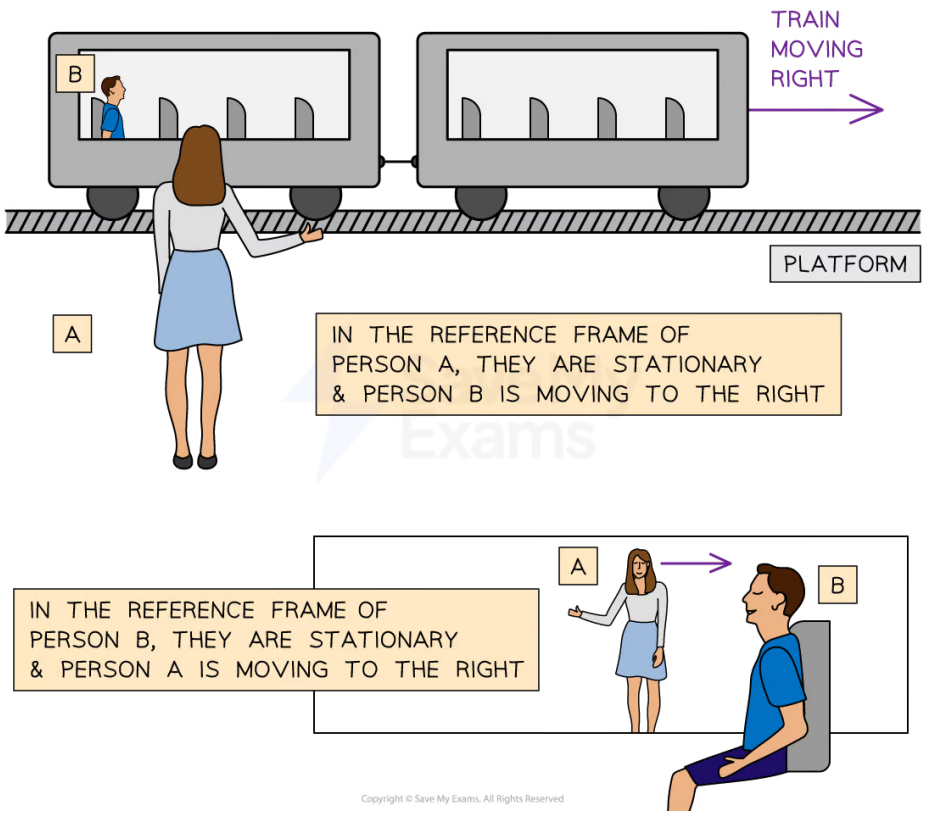
Each person has a different frame of reference, so they interpret the direction of the car differently relative to themselves

- Another common example is of a train pulling out of a station, where Person A is on the platform and Person B is on the train
- As the train begins to move, Person A, on the platform, views Person B, on the train, moving to the right
  - Therefore, according to Person A, they, themselves, are stationary and Person B is moving to the right
- Things look a little different from Person B's perspective
- As the train begins to move (to the left from Person B's perspective), Person B, on the train, views Person A, on the platform, moving to the right
  - Therefore, according to Person B, they, themselves, are stationary (as they cannot feel the train moving to the left) and Person A is moving to the right

Diagram demonstrating different reference frames for a train leaving a station



Person A and B are both stationary in their own reference frames and see the other as moving

- Therefore, frames of reference are used to specify the relationship between a moving and stationary object

Inertial Frames of Reference

- An **inertial** reference frame is

  A reference frame that is non-accelerating

- Therefore, all inertial reference frames are moving at **constant velocity** with **respect** to each other
- There is no such thing as an absolute reference frame in our Universe
  - In other words, there is no place in the Universe that is completely stationary
  - Everything is always moving relative to everything else

YOUR NOTES

Worked Example

A student is cycling to school with their friend who is also cycling exactly in line. As they cycle past a bus stop, they wave to their aunt who is stationary at the bus stop as she waits for her bus.

The student's aunt estimates the speed of the students to be 5 m s^{-1}.

At what speed would the friend measure the student to be travelling?

A 5 m s^{-1}

B -5 m s^{-1}

C 0 m s^{-1}

D 2 m s^{-1}

Answer:

The correct answer is **C** because:

- We must think about the friend's reference frame for this question, in which they are stationary (according to them)
- Since the friend is cycling **in line** with the student, this means they measure the student to be travelling at **0 m s^{-1}** relative to them

Exam Tip

In exam questions, look out for terms such as 'for the reference frame of...', 'in the reference frame of...' or 'relative to ...' to know which reference frame is being referred to. You can think of it as 'What do they see from their **point of view?'.** This becomes important when you learn about Galilean relativity and Lorentz transformations.

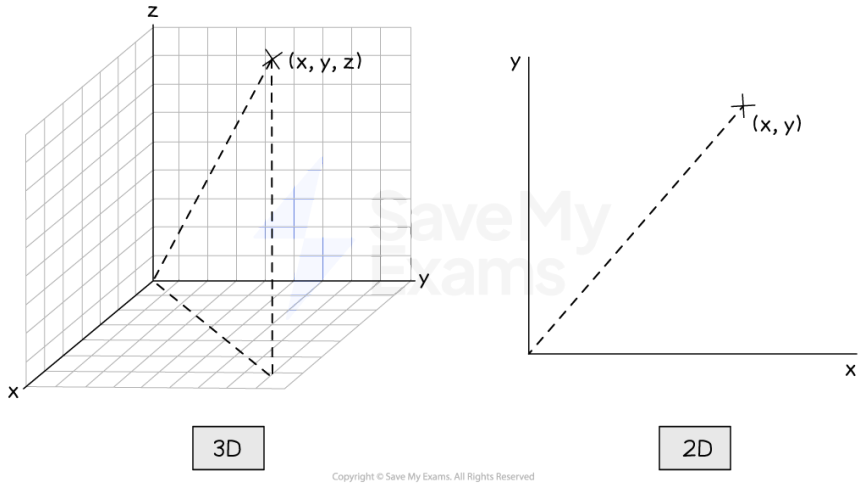
You will not come across non-inertial reference frames (i.e. ones where a frame is accelerating) in your exam.

YOUR NOTES

Newton's Postulates of Time & Space

- We use **inertial** reference frames because Newton's laws of motion are the **same** in all of them
  - This is known as **Galilean Relativity**
- For example, an object in an inertial reference frame will continue moving in a straight line with constant velocity unless acted upon by a force
  - This is in accordance with Newton's first law of motion
- This means that the **same** laws of Physics apply, regardless of one's frame of motion relative to another, as long as they are moving in a **straight line** at a **constant velocity**
- For an object moving with constant velocity in one reference frame, it will still have a constant (but **different**) velocity in another reference frame
- The Cartesian coordinate system is generally used for reference frames

Cartesian co-ordinates in 3D and 2D diagram



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Cartesian coordinates are used to represent a point in space

- Although there are an infinite number of inertial frames of reference in the Universe, there are ways to move between them

Exam Tip

Remember, anything that is moving in a curved line is **accelerating**. Therefore, you will not come across reference frames of something moving in a circle or an arc, but only straight lines with no acceleration.

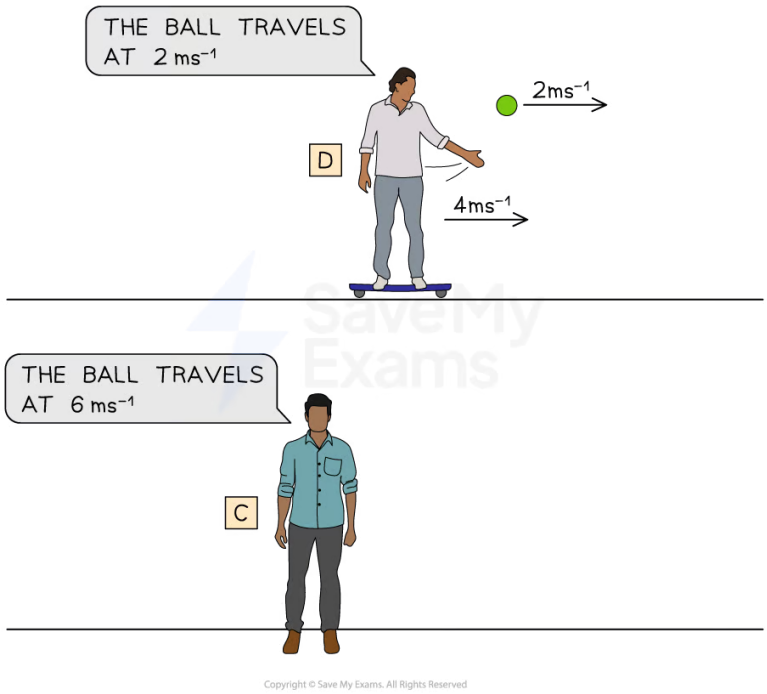
Cartesian co-ordinates are technically used to refer to a point in 3D (x, y, z), although, your exam questions will focus on movement in 2D (x, y) as this is easier to draw diagrams for.

YOUR NOTES

Galilean Transformation Equations

- **Galilean transformation equations** are used to **convert** between coordinates of space and time in one frame of reference to another for an event
  - This is because the **velocity**, **position** and **time** of an event appear differently from different reference frames
- For example, Person D is on a skateboard travelling at 4 m s^{-1} when they throw a ball in a straight line at a constant velocity of 2 m s^{-1}. Person C is a stationary observer of the event.
  - In Person D's reference frame, the ball is travelling at 2 m s^{-1}
  - In Person C's reference frame, the ball is travelling at 4 + 2 = 6 m s^{-1}
- So what is the speed of the ball? Well, it depends!

Diagram showing the difference in velocity of an object in two reference frames



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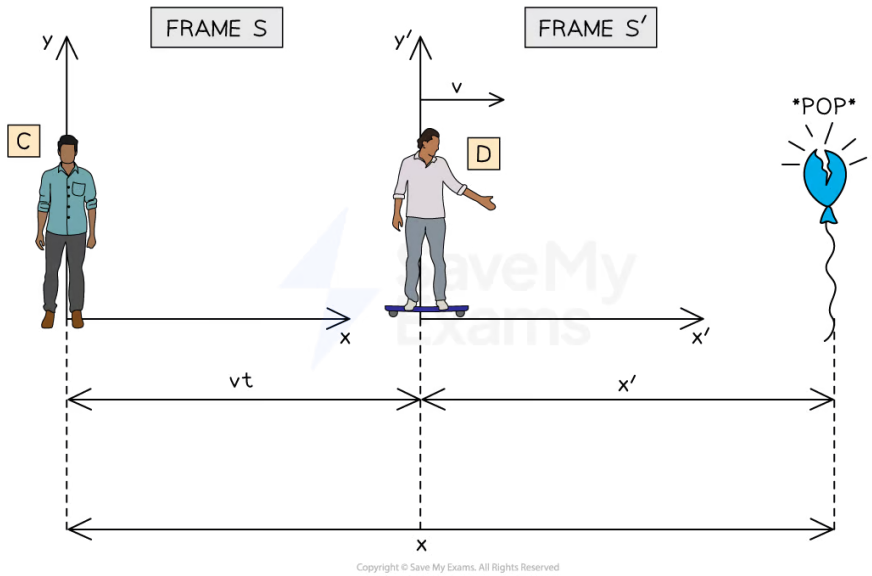
Person C measures the ball to be travelling faster than when measured by Person D

- Mathematically, the position and time of an event in one reference frame, S, is represented by the coordinates (x, y, t)
- The position and time of an event in a **second** reference frame, moving **relative** to the first one, S', is represented by the co-ordinates (x', y', t')
  - Remember from the train example in Reference Frames, both Person A and Person B view the other as moving because they are viewing the event from two different frames of reference
- Now consider another example:
- Person C is stationary whilst Person D is moving away from Person C at velocity v

YOUR NOTES

- Both Person C and Person D witness a balloon pop at some distance away
  - Person D, in reference frame S', measures the balloon pop at a distance ***x'*** away
  - Person C, in reference frame S, measures the balloon pop at a distance ***x = x' + vt*** away

Diagram showing a stationary and moving reference frame



Person C and D measure the distance at which a balloon pops differently

- Now let's see the event from Person D's frame of reference (S')
- From Person D's point of view, they are stationary and it is Person C that is travelling **away** from them at speed *v*
  - Since this is the opposite direction to the *v* observed by Person C, Person C's velocity from Person D's perspective is **–*v***
  - Therefore, Person D measures the balloon to pop at a distance ***x' = x – vt*** away
- Remember the *vt* comes from distance (*x*) = speed (*v*) × time (*t*).
- In both frames of reference, the time the balloon pops are still the same, hence ***t = t'***

- In summary:

Table of the Galilean relativity equations for a stationary and moving reference frame

Stationary reference frame	Moving reference frame	Transformation
x	x'	$x' = x - vt$ $x = x' + vt$
y	y'	$y = y'$

z	z'	$z = z'$
t	t'	$t = t'$

- Co-ordinates y and z are also the same in both reference frames because the relative motion is only in the ***x*** direction

YOUR NOTES

YOUR NOTES

Worked Example

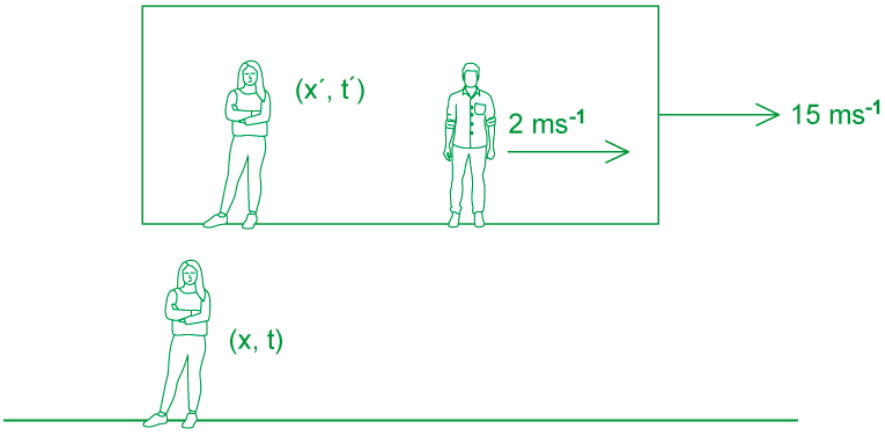
A train travels through a station at a constant velocity of 15 m s^{-1}. One observer is sitting inside the train and another sits on the platform. As they pass each other, they start their stopwatches and watch a child on the train run at a constant speed in the same direction as the motion of the train. The observer on the train measures the speed of the child to be 2 m s^{-1}.

(a)
According to the observer on the train, how far has the child moved after 10 s?
(b)
According to the observer on the platform, how far has the child moved after 10 s?

Answer:

Draw a quick sketch of the situation

- Label the stationary (x, t) and moving (x', t') reference frames



(a) Calculate the distance the child travels according to the observer on the train

- The observer measures the child to be travelling away at 2 m s^{-1}
- Therefore, using the equation

$$distance\ (x) = speed \times time$$

- The distance, x' is:

$$x' = 2 \times 10 = 20 \text{ m}$$

(b) Calculate the distance the child travels according to the observer on the platform

- This observer is stationary in their reference frame, so we use the equation

$$x = x' + vt$$

$$x = 20 + (15 \times 10) = 170 \text{ m}$$

- v is the speed of the **whole** reference frame

YOUR NOTES

Exam Tip

v is still a **velocity** and hence is a **vector**. Therefore, the **direction** matters i.e. a velocity in the opposite direction is $-v$.

Notice you only need to remember $x = x' + vt$

- $x' = x - vt$ is the same equation but rearranged for x' as the subject instead

You must be very careful about **which observer** you are referring to in any exam question.

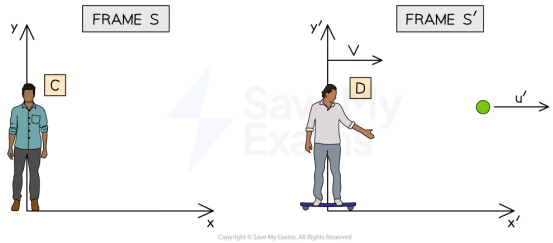
- In part (a) of the worked example, the child is running away from the observer on the train, because the observer on the train assumes that they are stationary
- However, the observer on the platform witnesses the child running **and** the train moving too, so adds both of these velocities to get the total velocity of the child to be much higher

YOUR NOTES

Velocity Addition Equation

- Galilean transformations can also be used to transform **velocities**
  - This is known as **velocity addition**
- Velocity addition is used when there are **multiple velocities** in the scenario
- Let's go back to the example of Person D on a skateboard throwing a ball directly in **front** of them in a straight line
- In this example:
  - *u* is the speed of the ball measured in frame S (by Person C)
  - *u'* is the speed of the ball measured in frame S' (by Person D)
  - *v* is the speed of Person D

Diagram showing velocity addition for two objects moving in the same direction

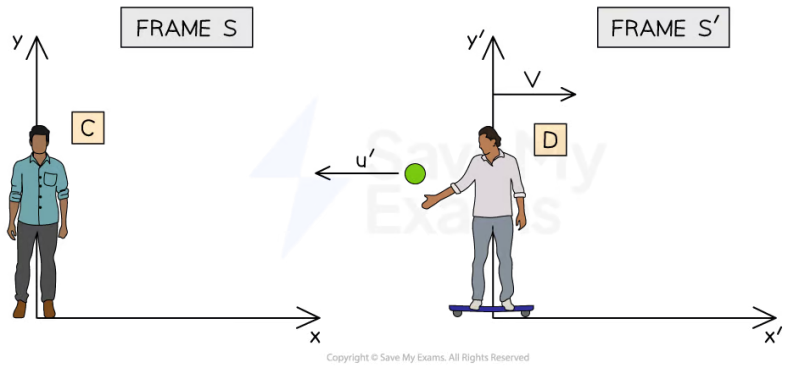


In Person C's reference frame, the ball is travelling at a speed of u' + v

- Therefore:
  - Person D (frame S') measures the velocity of the ball to be ***u'***
  - Whilst Person C (frame S) measures the velocity of the ball to be ***u = v + u'***
- Hence, the velocity of the ball from Person D's reference frame **in terms of** speeds *v* and *u* is
  - ***u' = u – v***

- Velocities are vectors, so their direction must be taken into account
- Let's say Person D now throws the ball directly **behind** them in a straight line at constant velocity
  - *u*, *u'* and *v* still refer to the same objects
- This time, since *u'* is in the **opposite** direction to *v*, it is now ***–u'***
- Therefore:
  - Person D (frame S') measures the velocity of the ball to be ***–u'***
  - Whilst Person C (frame S) measures the velocity of the ball to be ***u = v – u'***

Diagram showing velocity addition for two objects moving in opposite directions



In Person C's reference frame, the ball is now travelling at a speed of v – u'

- In summary:

Table of the velocity addition equations for a stationary and moving reference frame

Stationary reference frame	Moving reference frame	Transformation
u	u'	$u = u' + v$ $u' = u - v$

Worked Example

Two travellers X and Y walk past each other at an airport. Traveller X walks at 1.5 m s^{-1} whilst Traveller Y walks at 2.1 m s^{-1}.

Calculate the velocity of:

(a)
Traveller Y relative to Traveller X
(b)
Traveller X relative to Traveller Y

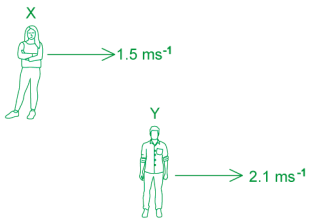
Traveller X now walks onto a travelator walkway in the airport at the same speed. The travelator moves at 0.3 m s^{-1}. Traveller Y decides to take a seat facing the travelator.

Calculate the velocity of:

(c)
Traveller X relative to Traveller Y when they are walking in the same direction as the travelator
(d)
Traveller X relative to Traveller Y when they are walking in the opposite direction to the travelator (just for fun)

Answer:

(a) The velocity of Traveller Y relative to Traveller X is:



- X is the reference frame at rest
- Y is the reference frame that is moving
- Therefore, from Traveller X's perspective, Traveller Y is moving at:

$$2.1 - 1.5 = 0.6 \text{ m s}^{-1}$$

- From Traveller X's frame of reference, Traveller Y is moving at a speed of 0.6 m s^{-1}
- Traveller Y is moving faster than Traveller X by 0.6 m s^{-1}

(b) The velocity of Traveller X relative to Traveller Y is:

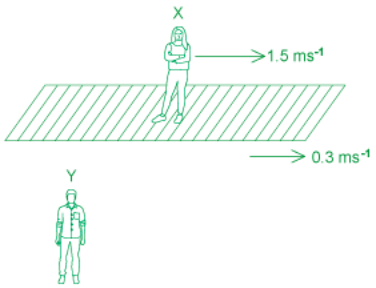
- Y is the reference frame at rest
- X is the reference frame that is moving

- Therefore, from Traveller Y's perspective, Traveller X is moving at:

$$1.5 - 2.1 = -0.6 \text{ m s}^{-1}$$

- From Traveller Y's frame of reference, Traveller X is moving at a speed of −0.6 m s^{-1}
- Traveller X is moving slower than Traveller Y by 0.6 m s^{-1}

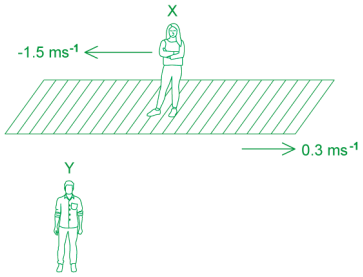
(c) The velocity of Traveller X relative to Traveller Y when walking in the same direction as the travelator



- Y is the reference frame at rest
- X is the reference frame that is moving **with** the travelator
  - u = velocity that Y measures X to be
  - u' = velocity of the travelator
  - v = velocity of X
- Therefore, they are moving at:

$$u = u' + v = 0.3 + 1.5 = 1.8 \text{ m s}^{-1}$$

(d) The velocity of Traveller X relative to Traveller Y when walking in the opposite direction to the travelator



- Y is the reference frame at rest
- X is the reference frame that is moving **against** the travelator
  - u = velocity that Y measures X to be
  - u' = velocity of the travelator
  - $-v$ = velocity of X
- Therefore, they are moving at:

$$u = u' - v = 0.3 - 1.5 = -1.2 \text{ m s}^{-1}$$

YOUR NOTES

Exam Tip

Always watch out for the **direction** of objects in velocity addition, don't just plug in numbers into the equation!

It helps to draw a quick sketch of the scenario in your exam and label the velocities. It doesn't matter which direction you take as positive, as long as you are consistent throughout your question.

Again, $u = v + u'$ can be rearranged for u' to give $u' = u - v$ i.e. the speed of the object in the other reference frame.

YOUR NOTES

The Postulates of Special Relativity

- Galilean relativity states that Newton's laws of motion are the same in all inertial reference frames
- Newton then treated space and time as fixed and absolute
  - This means the time interval between two events in one frame (x, y) is the same as the time interval between another frame (x', y')
- However, this is **not** what happens when we are close to the speed of light
  - Space and time become **relative**, meaning, the length of an object or a time interval **depends** on the frame of reference
- **Velocity addition** works with speeds much **lower** than the **speed of light (c)**
- It doesn't work for objects travelling closer to the speed of light
  - According to Galilean relativity, if a rocket ship travels at 0.7c and releases a probe directly in front of it at 0.5c, a stationary observer would view this at 0.7c + 0.5c = 1.2c
  - **However**, we know that **nothing** can travel faster than the speed of light, so this is **not** possible
- Einstein's two **postulates of special relativity** are:

First Postulate

The laws of physics are the same in all inertial frames of reference

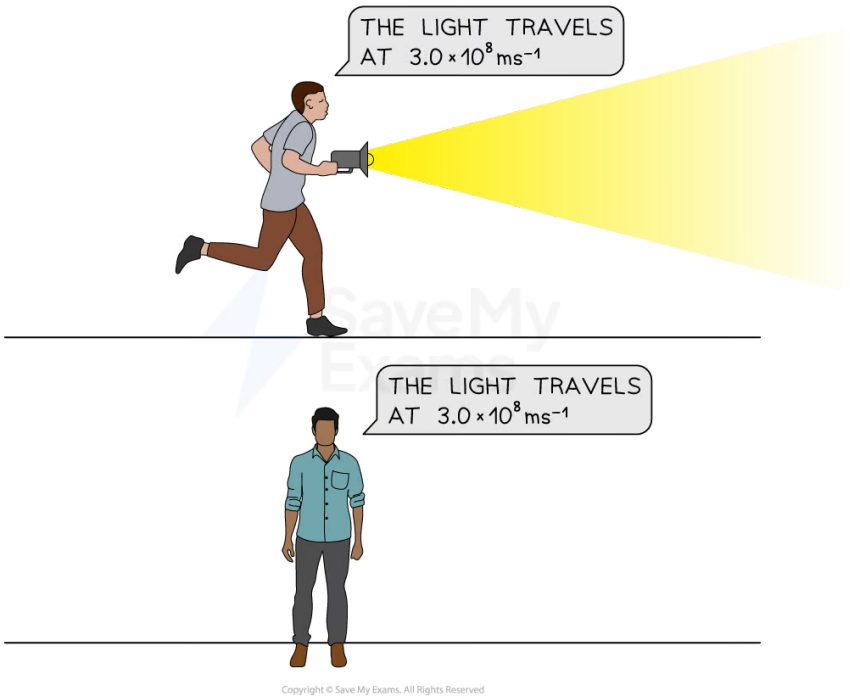
- In our own reference frame, we are always **stationary**
- This means in practice, we should **not** be able to tell whether we are moving or not
  - Someone conducting a physics experiment on a moving train versus on a stationary platform should produce the **exact same** results

Second Postulate

The speed of light, c, in a vacuum, is the same in all inertial frames of reference

- Two different observers will always measure the speed of light to be the same value, c in their reference frame
  - It makes no difference whether they are travelling or not. If it did, you would know whether you are moving, which counteracts the first postulate
- For example, a runner holding a flashlight in front of them will measure the speed of the light as **c**
  - However, someone stationary observing the runner will **also** see the speed of light as c and **not** c + the velocity of the runner
- This only works for the **speed of light**, not any other speed

Diagram demonstrating Einstein's postulate of special relativity



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Both a moving and stationary observer would measure the same speed of light from the torch. They are happy to finally agree on something

Exam Tip

You must remember these two postulates, as they play an important part conceptually and mathematically in further equations in special relativity.

YOUR NOTES

Lorentz Transformation Equations

- To relate measurements from one reference frame to another in Galilean relativity, we used Galilean transformations
- However, Galilean transformations can no longer be used to describe distances, times and speeds for objects travelling close to the speed of light
- Einstein's postulates of special relativity lead to the **Lorentz transformation** equations for the coordinates of an event in two inertial reference frames

The Lorentz Factor

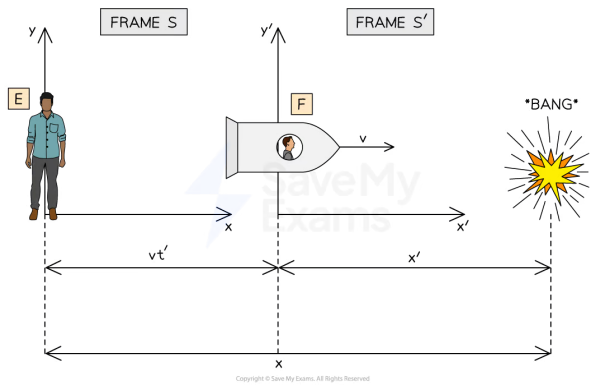
- Lorentz transformations are a **correction** of the Galilean transformations for speeds close to the speed of light, by multiplying by a scaling factor called the **Lorentz factor**, γ

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- As v is always greater than c (the speed of light), this means γ is always **greater than 1**
- This is especially important for time dilation and length contraction

Lorentz Transformation Equations

- Again we have the reference frame S measuring with co-ordinates (x, y, t), and S' with co-ordinates (x', y', t')
- Person F is moving **away** from Person E in their rocket ship at speed v which is close to the speed of light c
  - Person E is a stationary observer on Earth
- Both Person E and Person F witness a loud bang some distance away
  - Person F measures the loud bang to be a distance $\boldsymbol{x'}$ away
  - Person E measures the loud bang to be a distance $\boldsymbol{x = \gamma(x' + vt')}$ away



People E measures the loud bang to be at a different distance to person F

- However, motion is relative

YOUR NOTES

- According to Person F, they are stationary and Person E is moving away from them at a velocity v in the **opposite** direction
- So from Person F's point of view:
  - Person E's velocity is $-v$
  - Therefore, Person F measures the bang to happen at a distance $x' = \gamma(x - vt)$ away
- But, the time the bang happens is **not** the same in both frames of reference! $t \neq t'$
- These are the exact same equations as the Galilean transformation equations, just with the **added** Lorentz factor
- In summary, the Lorentz equations from frame S → S' are:

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

- Where:
  - (x, y, z, t) = the co-ordinates measured from one reference frame
  - (x', y', z', t') = the co-ordinates measured from another reference frame **moving at speed v relative to it**

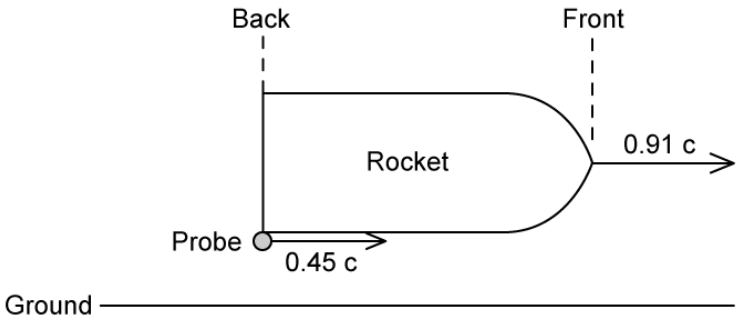
Galilean vs. Lorentz Transformations

Lorentz ($v \approx c$)	Galilean ($v << c$)
$x' = \gamma(x - vt)$	$x' = x - vt$
$x = \gamma(x' + vt')$	$x = x' + vt$
$t' = \gamma\left(t - \frac{vx}{c^2}\right)$	$t' = t$
$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$	$t = t'$
$y = y'$	$y = y'$
$z = z'$	$z = z'$

- Notice that **time** is different between reference frames t and t' for objects travelling close to the speed of light, whilst in Galilean transformations, time was absolute (it doesn't change) between reference frames

Worked Example

A rocket of proper length 150 m moves to the right with speed 0.91c relative to the ground.



A probe is released from the back of the rocket at speed 0.45c relative to the rocket.

Determine the time it takes the probe to reach the front of the rocket according to an observer

(a)
At rest in the rocket.

(b)
At rest on the ground.

Answer:

(a)

Step 1: List the known quantities

- Length of the rocket, $l = 150$ m
- Speed of the probe, $v' = 0.40c$

Step 2: Analyse the situation

- In the reference frame of an observer at rest in the rocket, they are **stationary**
- Therefore, the probe travels at a constant speed 0.45c across the full length of the rocket of 150 m

$$t' = \frac{l}{v'} = \frac{150}{0.45c} = \frac{150}{0.45 \times (3 \times 10^8)}$$

$$t' = 1.11 \times 10^{-6}\ \text{s}$$

- t' and v' are used because they are the times and velocity of the **moving** object **in the reference frame** of the observer at **rest**

(b)

Step 1: List the known quantities

- Length of the rocket, $l = 150$ m
- Speed of the rocket, $v = 0.91c$
- Time taken for the probe to reach the front of the rocket, $t' = 1.11 \times 10^{-6}$ s

Step 2: Analyse the situation

- In reference to an observer at rest on the ground, they will see the probe taking **longer** to reach the front of the ship
- Since object in question, the probe, is **moving** in both reference frames, we need to use a Lorentz transformation

Step 3: Calculate the gamma factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.91c)^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1 - 0.91^2}} = 2.412$$

Step 3: Substitute values into the Lorentz transformation

- Since the observer is at **rest**, the Lorentz equation for time t must be used

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$t = 2.412\left((1.11 \times 10^{-6}) + \frac{(0.91c)(150)}{c^2}\right) = 2.412\left((1.11 \times 10^{-6}) + \frac{(0.91)(150)}{3 \times 10^8}\right)$$

$$t = 3.8 \times 10^{-6} \text{ s}$$

YOUR NOTES

Exam Tip

Always check that your value of γ is greater than 1.

You will often be given a speed v **in terms of c** e.g. $v = 0.90c$ etc. When you put this value into the gamma factor, this is **squared**. Therefore, you do not need to put in 3.0×10^8 at all into your calculator, as the c^2 will cancel.

E.g. $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1 - \dfrac{(0.90c)^2}{c^2}}} = \dfrac{1}{\sqrt{1 - \dfrac{(0.90)^2(c)^2}{c^2}}}$

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{(0.90)^2(c)^2}{c^2}}} = \frac{1}{\sqrt{1 - (0.90)^2}}$$

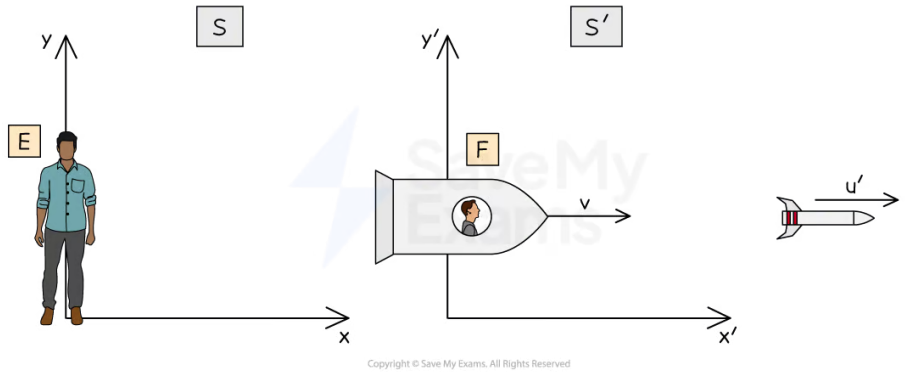
The equations for x', t' and γ are given in your data booklet, but you must remember the sign change if you want to calculate x or t (from the rest frame) instead!

Some textbooks may go further into this for your understanding, you will **not** be expected to derive these equations in your exam. You will only be assessed on how to use them.

YOUR NOTES

Velocity Addition Transformations

- Similar to velocity addition to Galilean transformations, the Lorentz transformation equations lead to relativistic velocity addition equations
- These are again used when there are **multiple** velocities in the scenario but now some are **close to the speed of light**
- Let's go back to the example of Person F in the rocket ship. They now release a missile in front of them
- In this example:
  - *u* is the speed of the missile measured in frame S (by Person E)
  - *u'* is the speed of the missile measured in frame S' (by Person F)
  - *v* is the speed of frame S' (Person F)



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Person F releases a missile in front of them. Both observers will view the missile travelling at different speeds

- In **Galilean** velocity addition, when *v* << c, these were:
  - The speed of the missile as measured by Person E: $u = u' + v$
  - Or, $u' = u - v$
- If *v* and *u'* are close to the speed of light, we have to use **Lorentz** velocity addition transformations instead
- These equations are:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

- Where:
  - *u* = the velocity of an object measured from the **stationary** reference frame
  - *u'* = the velocity of an object measured from a **moving** reference frame

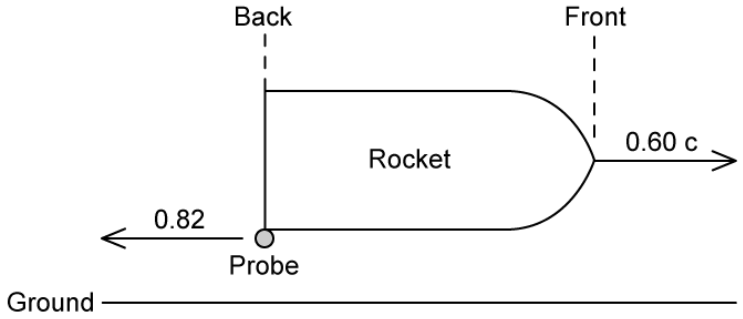
- v = the velocity of the moving reference frame
- c = the speed of light

YOUR NOTES

YOUR NOTES

Worked Example

A rocket moves to the right with speed 0.60c relative to the ground.



A probe is released from the back of the rocket at speed 0.82c relative to the rocket.

Calculate the speed of the probe relative to the ground.

Answer:

Step 1: List the known quantities

- Speed of the rocket, $v = 0.60c$
- Speed of the probe relative to the rocket, $u' = 0.82c$

Step 2: Analyse the situation

- We have multiple velocities in this scenario in terms of c, so we need to use the **Lorentz** velocity addition equations
- The probe is travelling in the **opposite** direction to the rocket, so its velocity is –0.82c
- We want the speed relative to the ground, which is a reference frame **at rest**, so this is u

Step 3: Substitute values into the equation

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

$$u = \frac{-0.82c + 0.60c}{1 + \frac{(-0.82c)(0.60c)}{c^2}} = \frac{(-0.82 + 0.60)c}{1 + (-0.82)(0.60)}$$

$$u = -0.43c$$

Exam Tip

Be very careful which reference frame you are asked to calculate the velocity from, as this determines whether you find u or u'. Notice the equations are very similar, except one is with – and the other +. However, the signs will match on the numerator and denominator.

The equation for u' is given in your data booklet.

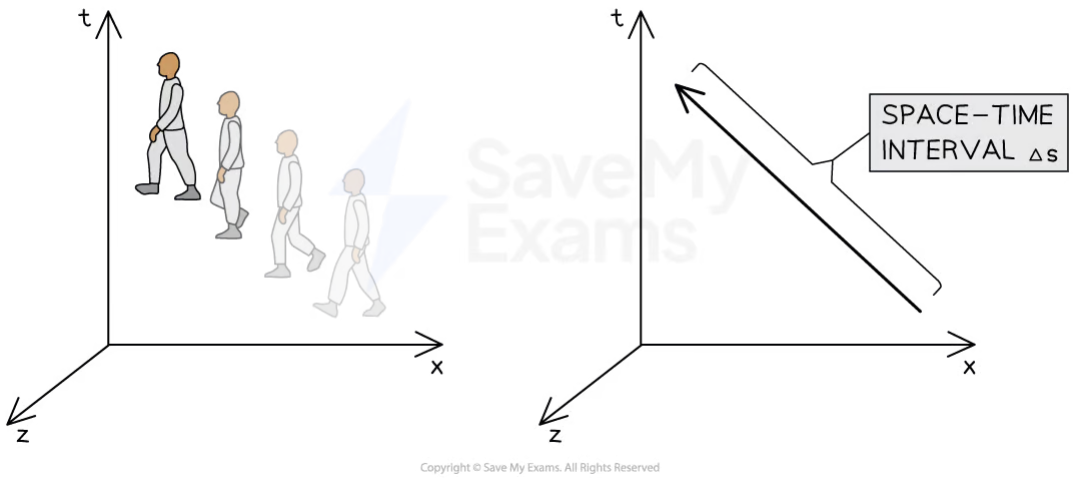
Anytime you see the word 'relativistic' in physics such as 'relativistic speeds' it just means 'close to the speed of light'. Physics gets a bit weird at this point!

It is fine, and often encouraged, to give your final answers for relativistic velocities in terms of c. In the denominator of the velocity addition equations, the c^2 will cancel out if two velocities u and v are given in terms of c.

YOUR NOTES

Space-Time Interval

- Einstein discovered that time and distance **changes** when moving from one inertial reference frame to another when travelling at speeds close to the speed of light
  - In other words, these reference frames are **not absolute**
- However, some quantities **are** the same in all inertial frames. These are called **invariant**
- These are:
  - Proper time, t_0
  - Proper length, L_0
  - Space-time interval, Δs
- These are a product of Einstein's second postulate
- In Galilean relativity:
  - A space and interval is the **same** in all reference frames, i.e. $\Delta t = \Delta t'$ and $\Delta x = \Delta x'$
- In special relativity:
  - These are replaced with a **space-time** interval, as space and time are connected together as 4 coordinates (x,y,z,t) for an event
  - Motion can be represented as spanning both space **and** time using this coordinate system
- The diagram below shows a person moving in both space x and z **and** in time t
  - They can also move in the y direction, but 4 dimensions are not possible to draw accurately here (in 3-dimensional space)



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Motion in space-time. The length of the arrow for the space-time interval is the same for all inertial reference frames

- An interval in space-time is an **invariant** quantity in all inertial reference frames and is defined as:

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

YOUR NOTES

- Where:
  - Δt = time interval / separation (s)
  - c = the speed of light
  - Δx = spacial separation (m)
  - Δs = space-time interval (m)
- This means that in two inertial reference frames, although Δt and Δx will be different in both frames, Δs will be the **same**
- These will be used in space-time diagrams

Worked Example

An inertial reference frame S' moves relative to S with a speed close to the speed of light. When clocks in both frames show zero the origins of the two frames coincide.

An event P has coordinates x = 2 m and ct = 0 in frame S, and x = 2.3 m in frame S'. Show that the time coordinate of event P in frame S' is −1.1 m.

Answer:

Step 1: List the known quantities:

- Spacial separation in frame S, $\Delta x = 2$ m
- Time separation in frame S, $c\Delta t = 0$
- Spacial separation in frame S', $\Delta x' = 2.3$ m

Step 2: Calculate the space-time interval in frame S

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$(\Delta s)^2 = (0) - (2)^2 = -4$$

Step 3: Substitute values into the space-time interval for S'

- Δs is the same (invariant) in both reference frames

$$(\Delta s)^2 = (c\Delta t')^2 - (\Delta x')^2$$

$$(c\Delta t')^2 = (\Delta s)^2 + (\Delta x')^2$$

$$(c\Delta t')^2 = -4 + (2.3)^2 = 1.29$$

$$c\Delta t' = \sqrt{1.29} = \pm 1.14$$

Exam Tip

The units still work out on both sides of the equation. Remember, $c\Delta t$ is a **speed** × **time** which is a **distance** in metres, so is Δx so Δs is in metres.

Whether ct' in the worked example is + or − will come in later with space-time diagrams.

Proper Time & Length

YOUR NOTES

- In special relativity, we have found that distances and times are **relative**
- This means the length of an object and the time interval of an event **changes** when observed in frames that are **moving relative** to that **object**
  - More on this is explored in length contraction and time dilation
- We need to define the difference mathematically between the time and lengths measured between each frame, to know which one is being referred to

What is Proper Time and Proper Length?

- **Proper time interval**, Δt_0 is defined as:

  The time interval measured in a reference frame where the object is at rest (relative to the observer)

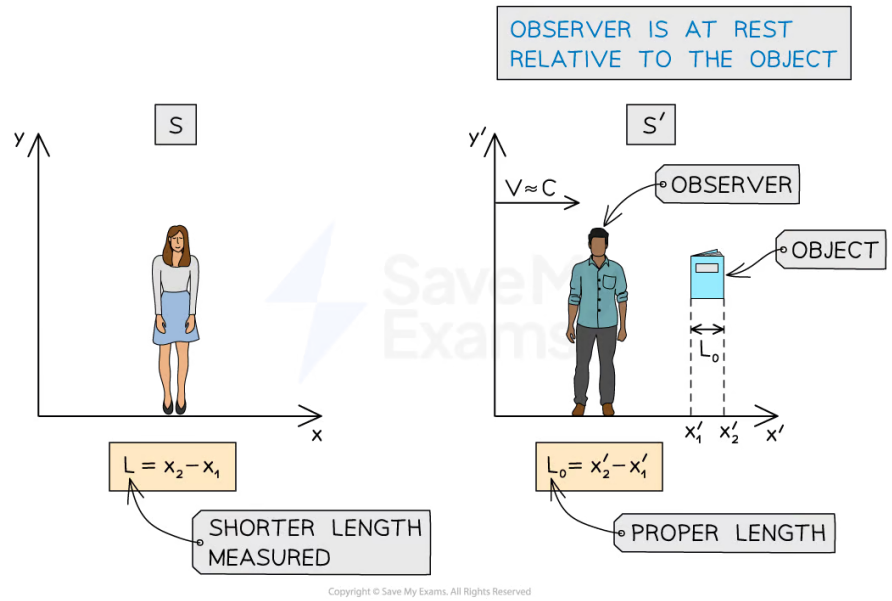
- **Proper length**, L_0 is defined as:

  The length measured in a reference frame where the object is at rest (relative to the observer)

- These can be measured either in the moving frame S' **or** a rest frame S
  - This depends on the reference frame you are calculating the length and time from

- For example, if a person in moving frame S' (e.g. on a train) measures the length of a book, they are **at rest relative** to the book
  - They measure the distance between points x_2' and x_1'
  - This is the proper length, L_0 although they are technically moving (but they don't know this - otherwise it would go against Einstein's first postulate)

- However, for an observer in frame S, at rest (e.g. on a platform
  - They will measure the distance between points x_2 and x_1
  - This a shortened length, L



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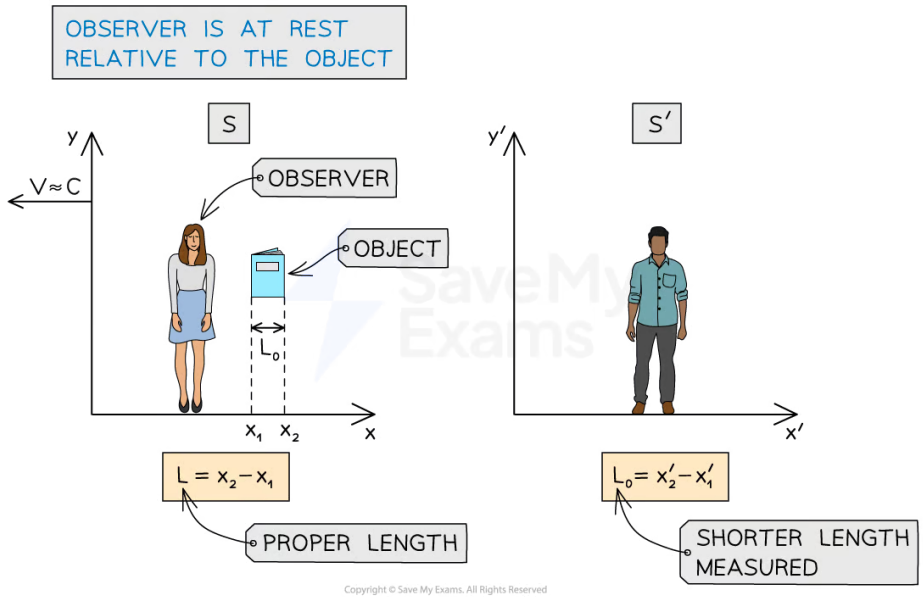
The observer in S' is measuring the proper length, because they are at rest relative to the object

- For example, if a person in the stationary frame S (e.g. on the platform) measures the length of a book, **they** are now **at rest relative** to the book
  - They measure the distance between points x_2 and x_1
  - This is the proper length, L_0
- However, an observer in frame S', which is moving (e.g. on a train) sees **themselves** at rest but instead sees frame S as moving (remember, motion is **relative**)
  - They will measure the distance between points x_2' and x_1'
  - This a shortened length, L



The observer in S is measuring the proper length because they are at rest relative to the object

- The same rules apply for the proper time, t_0, except that the time measured from a frame **moving** relative to an event will be measured **longer**

Exam Tip

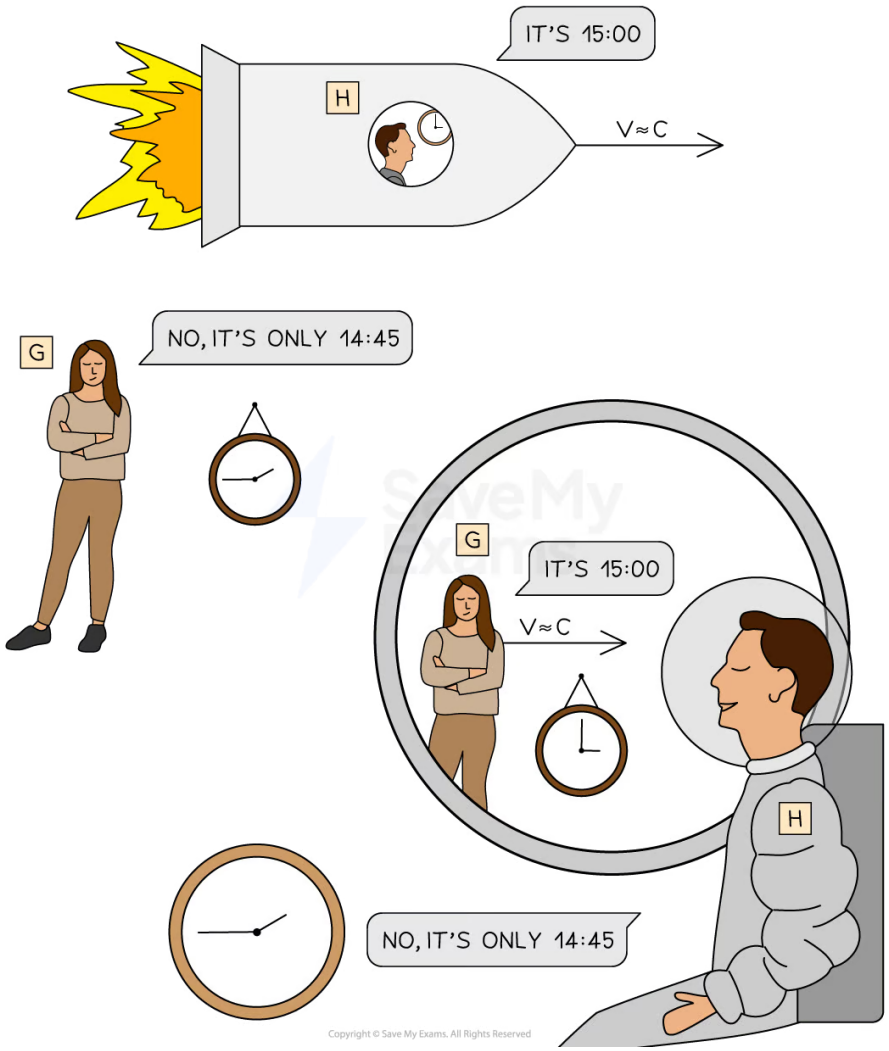
Do not misinterpret this as the time or length measured in the stationary reference frame! This is not the case. It could be the time and length measured in the **moving** frame too, because it is a length and time measured by an observer **at rest relative to the object**

Again, this only works for objects moving close to the speed of light. You will never encounter this in everyday life!

YOUR NOTES

Time Dilation

- When objects travel close to the speed of light, to an observer **moving relative** to that object, it looks as if the object has **slowed down**
  - This is best demonstrated by **clocks**
- Observer H is in a rocket moving close to the speed of light
  - They will see their clock ticking at a regular pace, say, it is reading 15:00
- Observer G at rest on Earth, with remarkable eyesight, will measure the clock as ticking **slower**
  - They will observe that time has **slowed down** in the spaceship from **their** reference frame i.e. they may see the time as 14:45 instead of 15:00
- However, the **same** occurs the **other way around**
- For observer H on the rocket, it is observer G that is **moving relative** to them
  - Therefore, observer H will measure observer G's clock as ticking **slower** i.e. they see time **slow down** on Earth from **their** reference frame

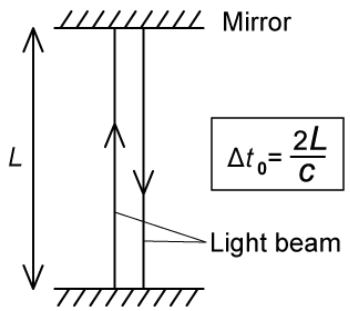


A stationary observer in their own reference frame views clocks as running slower in the moving reference frame. We're back to disagreeing

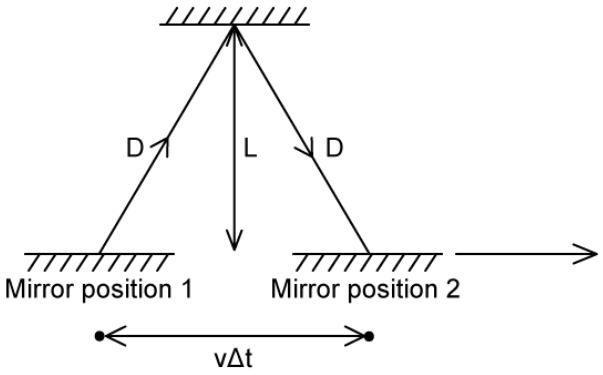
Time Dilation Equation

- Consider a light clock. This consists of two mirrors facing each other with a beam of light travelling up and down between them
- An observer is at **rest relative** to the clock on a train platform and watches the light reflect between the mirrors
- The distance between the mirrors is *L* and the light travels at a speed c
- Therefore, the time interval for the light to travel from the top mirror back down to the bottom is:

$$\Delta t_0 = \frac{2L}{c}$$



- Another light clock is on a moving train, **relative** to the initial observer on the platform, travelling at constant velocity *v*
- The stationary observer on the platform sees the light clock on the train and watches the reflection of the rays between the mirrors
- It appears that the light rays travel to the right at an angle to the direction of motion



- The observer on the train platform sees the mirror at:
  - Position 1, when the light leaves the bottom mirror
  - Position 2, when the light returns to it
- The length of the light path is no longer *L* but a longer path, *D*
- The distance travelled by the light ray is now 2D, and the time observed between the reflections is now

$$\Delta t = \frac{2D}{c}$$

- The apparent distance horizontally travelled by the mirror is $v\Delta t$ where *v* is the speed of the train
- Notice that this is part of a right-angled triangle, so using Pythagoras' theorem we can see that:

$$D^2 = L^2 + \left(\frac{v\,\Delta t}{2}\right)^2$$

- Where: $D = \frac{c\Delta t}{2}$

YOUR NOTES

- We want to find Δt, the time taken for the light ray to travel up and down in the reference frame of the stationary observer on the train platform, who is moving **relative** to the light clock on the train
  - Remember, although it is the train that is moving, in the reference frame of an observer on the train it is the observer on the **platform** that is moving

$$\Delta t = \frac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Remember the gamma factor $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is from the Lorentz transformation equations
- Therefore:

$$\Delta t = \gamma \Delta t_0$$

- Where:
  - Δt = the time interval measured from an observer **moving relative** to the time interval being measured (s)
  - Δt_0 = the **proper** time interval (s)
- As $\gamma > 1$, this means that the $\Delta t > \Delta t_0$
  - In other words, a clock observed from a reference frame moving **relative** to it will be measured to tick slower than a clock that is at rest in its frame of reference
- The observer on the **platform** will view the **train** clock as moving **slower**
- The observer on the **train** will view the **platform** as moving **slower**
- You may be wondering why it's the time that slows down for the light beam, and not the light beam just speeding up to hit each mirror at the same frequency
  - This is due to Einstein's second postulate
  - Both Observers G and H **must** measure the speed of light to be c, so it doesn't slow down or speed up according to either reference frame
- It is important to note that the time has been measured at the same **position**
  - In other words, the time interval is the position at which the light leaves the first mirror and at which it returns to the second mirror in the reference frame of the mirror

YOUR NOTES

Worked Example

Alex's spacecraft is on a journey to a star travelling at 0.7 c. Emma is on a space station on Earth at rest. According to Emma, the distance from the base station to the star is 14.2 ly.

Show that Alex measures the time taken for her to travel from the base station to the star to be about 14.5 years.

Answer:

Step 1: List the known quantities:

- Distance of space station according to Emma = 14.2 ly
- Speed of Alex's spacecraft, $v = 0.7c$

Step 2: Analyse the situation

- We are trying to find the time that Alex measures for **her** travel i.e. the time she would measure on her own clock in the spaceship which **she is stationary relative to**
- This is the proper time, Δt_0

Step 3: Calculate the time taken according to Emma, Δt

- 1 ly (light year) is the **distance**, *s*, light (at speed c) travels in a year
- Therefore it takes the light 14.2 years (time) to travel the distance at speed c

$$s = speed \times time = 14.2c \text{ m}$$

- Therefore, the time taken according to Emma is:

$$\Delta t = \frac{s}{v} = \frac{14.2c}{0.7c} = \frac{14.2}{0.7} = 20.29 \text{ years}$$

Step 4: Substitute values into the time dilation equation

$$\Delta t = \gamma \Delta t_0 \quad \Rightarrow \quad \Delta t_0 = \frac{\Delta t}{\gamma}$$

$$\Delta t_0 = \frac{20.29}{\frac{1}{\sqrt{1 - \frac{(0.7c)^2}{c^2}}}} = \frac{20.29}{\frac{1}{\sqrt{1 - (0.7)^2}}} = 14.5 \text{ years}$$

Step 5: Check whether your answer makes sense

- Since Emma (who is stationary) is viewing Alex's clock (which is moving) she would measure a **longer** time for Alex to reach the star than Alex will
- As Emma records 20.29 years, but Alex only records 14.5 years, this time makes sense

YOUR NOTES

Exam Tip

A nice way to remember this is 'moving clocks run slower'. The caveat is what is considered 'moving' **depends on the reference frame.**

You will **not** be expected to remember this derivation, but it's helpful to know where all the factors have come from. The time dilation equation is given on your data sheet.

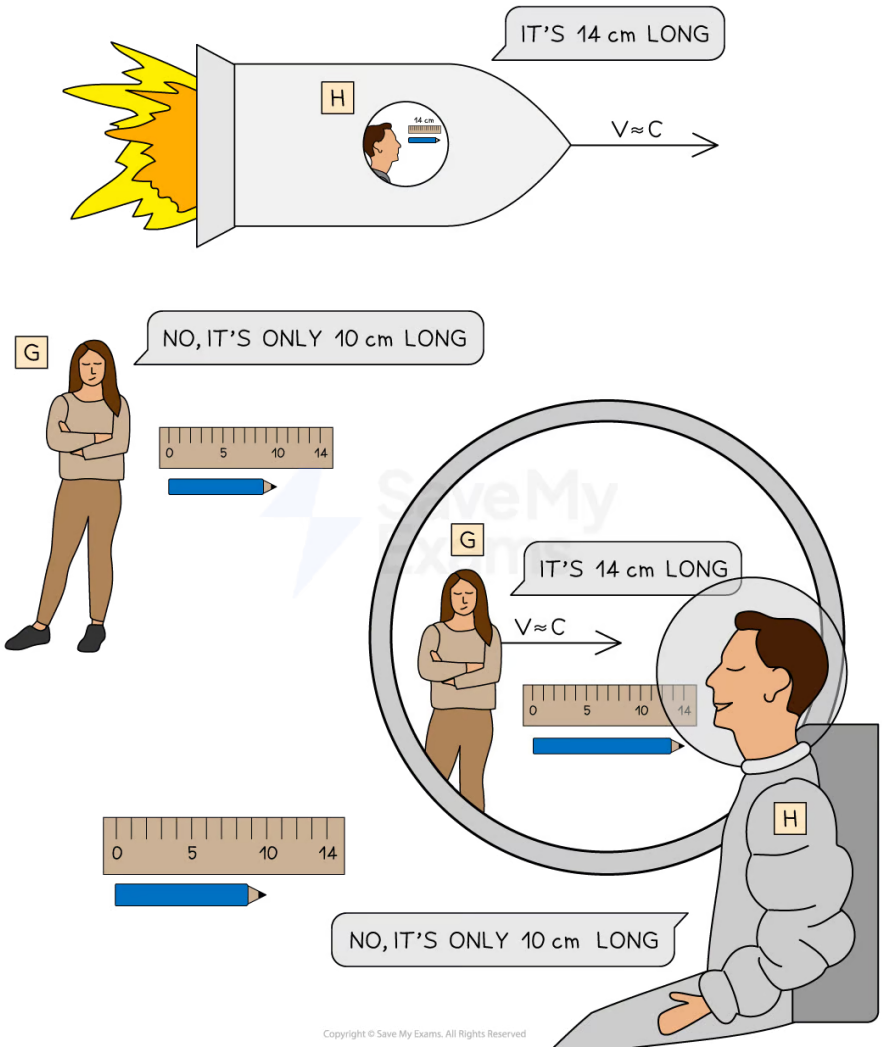
The notion of 'proper time' is incredibly important here, as it depends on the reference frame the time interval is being measured from.

The maths for the derivation is only using $speed = \frac{distance}{time}$ and Pythagoras' theorem.

YOUR NOTES

Length Contraction

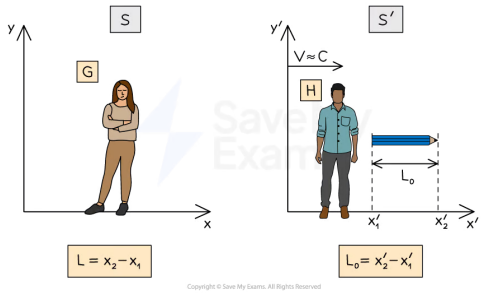
- When objects travel close to the speed of light, to an observer **moving relative** to that object, it looks as if the object has **become shorter**
  - This is best demonstrated using **rulers**
- Observer H, in their rocket moving close to the speed of light, measures the length of their pencil to be 14 cm
- Observer G, at rest on Earth, would measure (with remarkable eyesight) the length of the pencil to be **shorter**
  - They will see lengths **contracted** in the spaceship from **their** reference frame, e.g. the length may appear to be 10 cm instead of 14 cm
- However, the **same** occurs the **other way around**
- For observer H on the rocket, it is observer G that is **moving relative** to them
  - Therefore, observer H would measure the length of observer G's pencil as **shorter** i.e observer H, on the rocket, sees lengths **contracted** on Earth from **their** reference frame



A stationary observer in their own reference frame views lengths as shorter in the moving reference frame

Length Contraction Equation

- The length of an object is the difference in the position of its ends
- Consider the observers G and H measuring the length of a pencil, which is stationary in reference frame S' (for observer H)



YOUR NOTES

Observer G measures the length of the pencil to be different to observer H

- As observer H is in the moving frame S', they measure the length of the ruler as:

$$\Delta x' = x_2' - x_1' = L_0$$

- This is the **proper length, L_0** as the pencil is **not** moving relative to observer H
  - Both the pencil and observer H are, however, moving relative to observer G
- Observer G needs to measure the length of the pencil by measuring the position of its ends at the same time (just like observer H did)
- They measure the length of the ruler to be:

$$\Delta x = x_2 - x_1 = L$$

- This is the **observed length, L** as the pencil **is** moving relative to observer G
  - **Lorentz** transformations tell us how the x and x' are related
- We want to find Δx, the length measured in the reference frame of the stationary observer on Earth (G), who is moving **relative** to the observer on the rocket (H)
- Transforming these distances gives:

$$x_1' = \gamma(x_1 - vt)$$

$$x_2' = \gamma(x_2 - vt)$$

- These are then substituted into the equation for the proper length, L_0:

$$x_2' - x_1' = \gamma(x_2 - vt) - \gamma(x_1 - vt) = \gamma(x_2 - x_1)$$

$$L_0 = \gamma L$$

- Therefore:

$$L = \frac{L_0}{\gamma}$$

- Where:
  - L = the length measured by an observer **moving relative** to the length being measured (m)
  - L_0 = the **proper** length (m)
- As $\gamma > 1$, this means that the $L < L_0$
  - In other words, lengths measured from a reference frame moving **relative** to the object will be measured as shorter than the lengths measured at rest from within their frame of reference
- Similar to time dilation, length contraction is also due to Einstein's second postulate
  - Both observers G and H **must** measure the speed of light to be c

- Since the time for observer H will run slower, according to observer G (i.e. t increases), then for c to stay the same, the length of the object, L must **decrease**

- It is important to note that the length has been measured at the same **time**
  - This length is the difference between the ends of the pencil, with both ends measured at the same time

- The ruler used in both reference frames is **stationary** in their **own** reference frame
  - Otherwise, observer G would see the ruler on observer H's rocket contracting as well and wouldn't measure any difference in length

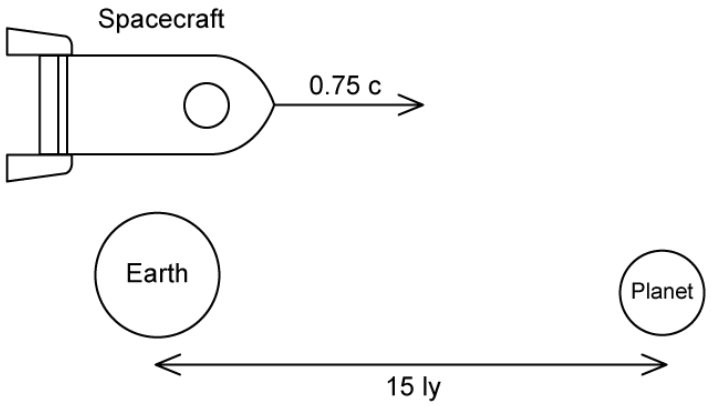
YOUR NOTES

YOUR NOTES

Worked Example

A spacecraft leaves Earth and moves towards a planet.

The spacecraft moves at a speed of 0.75c relative to the Earth. The planet is a distance of 15 ly away according to the observer on Earth.



The spacecraft passes a space station that is at rest relative to the Earth. The proper length of the space station is 482 m.

Calculate the length of the space station according to the observer in the spacecraft.

Answer:

Step 1: List the known quantities

- Speed of the spacecraft, $v = 0.75c$
- Proper length of the space station, $L_0 = 482$ m

Step 2: Analyse the situation

- We are trying to find the length of the space station in the reference frame of the observer in the **spacecraft**
- In this observer's reference frame, it is the **space station** that is moving away from them at 0.75c
- Therefore, we a measuring a length in the moving reference frame (relative to the spacecraft) - this is the length, ***L***

Step 3: Substitute values into the length contraction equation

$$L = \frac{L_0}{\gamma} = \frac{L_0}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$L = \frac{482}{\frac{1}{\sqrt{1 - \frac{(0.75c)^2}{c^2}}}} = \frac{482}{\frac{1}{\sqrt{1 - (0.75)^2}}} = 319 \text{ m}$$

Step 4: Check whether your answer makes sense

- As the observer in the spacecraft is stationary, the length of the space station they measure should be **shorter** than the proper length
- As the length recorded from the spacecraft is 319 years, and the proper length is 482 m, this length makes sense

Exam Tip

You will **not** be expected to remember this derivation, but it's helpful to know where all the factors have come from. The time dilation equation is given on your data sheet.

The notion of 'proper length' is incredibly important here, as it depends on the reference frame the length is being measured from.

You will find in some exam questions you can use time dilation **or** length contraction, you will receive marks for either way.

YOUR NOTES

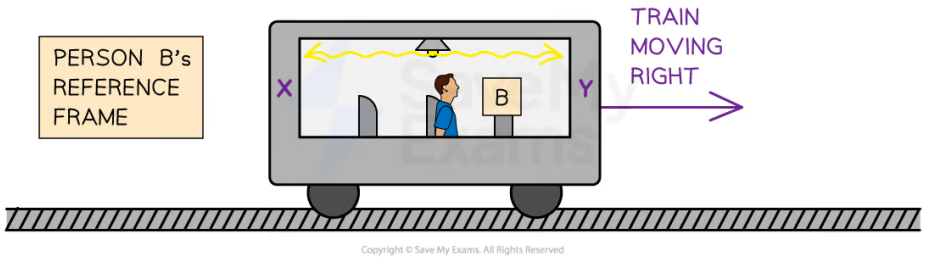
Simultaneity in Special Relativity

- The term 'simultaneous' means to occur at the same **time**
- The relativity of simultaneity states that

Whether two spatially simultaneous events happen at the same time is not absolute, but depends on the observer's reference frame

- This means that in one reference frame, two events that occur at **different** points in space seem to happen at the **same** time, whilst in another reference frame **moving** relative to the first the events seem to happen one after another
  - This was not the case in **Galilean** relativity, where simultaneity was **absolute**

- This is best shown in the following example
- Person B is in a train carriage moving to the right at constant velocity
- They switch on a lamp above them and they observe that the light from the lamp reaches the two ends of the carriage, points X and Y, at the same time



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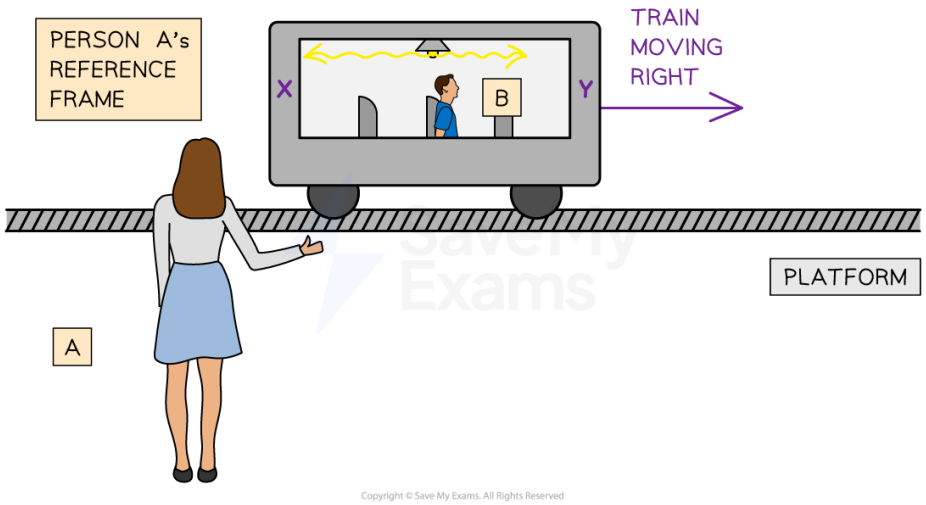
Person B sees the light from the lamp reach point X and Y at the same time

- Meanwhile, Person A is stationary on a platform and observes the train travel past
- Person A will still see the light from the lamp move to both ends of the carriage at the same speed (c)
  - This is in line with Einstein's second postulate
- However, Person A will see the light reach point X **before** it reaches point Y
  - This is because the whole carriage is moving to the right (relative to Person A), so the left side of the carriage is moving **towards** the light ray and the right side of the carriage is moving **away** from the light ray
  - This means that Person A will see the light ray reach point Y slightly later



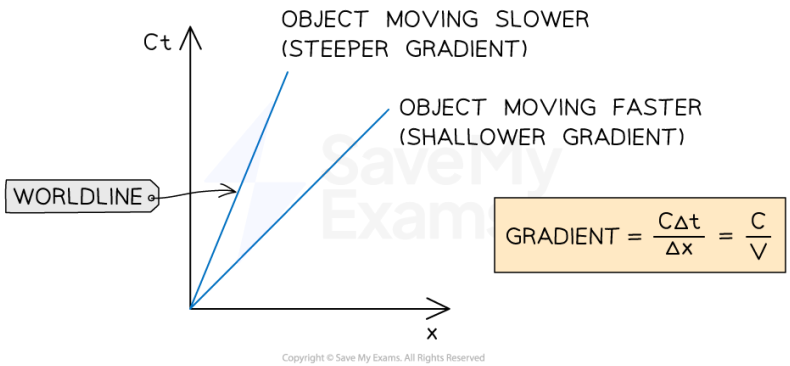
Person A sees the light from the lamp reach point X before point Y

- This image is exaggerated to show the point; the difference between the times will be very very small and is dependent upon the speed of the train carriage
- Simultaneity can be visualised using **space-time** diagrams

YOUR NOTES

Space-Time Diagrams

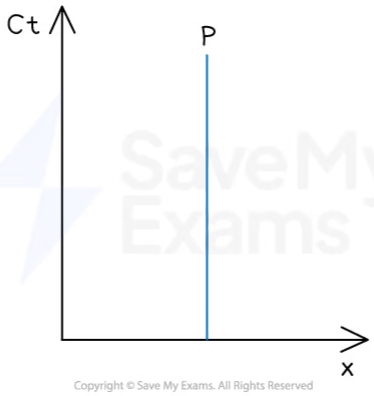
- **Spacetime** (or Minkowski) **diagrams** represent an object's motion in spacetime
- They help to visualise
  - Time dilation
  - Length contraction
  - Simultaneity
- Since 4D (*x*, *y*, *z*, *t*) diagrams cannot be drawn on a 2D page, we collapse 3D space (*x*, *y*, *z*) into 1 spacial dimension and keep time as its own dimension
- This gives a **spacetime diagram**
  - Lines drawn on a spacetime diagram are called **world lines**
- Instead of the usual distance-time graphs, we plot
  - The horiztonal axis as ***x***
  - The vertical axis as **ct**



Objects moving slower are represented by a steeper gradient on a spacetime gradient

- Note that both axes have dimensions of **length**
  - This makes it easy to compare values on one axis and another
- This means the worldlines have a gradient of
  - $\frac{c\Delta t}{\Delta x} = \frac{c}{v}$ where the velocity is $v = \frac{\Delta x}{\Delta t}$
- This means:
  - The **steeper** the gradient, the **slower** the object is moving
  - The **shallower** the gradient, the **faster** the object is moving
- *ct* was first seen when the spacetime interval was introduced, and c is chosen deliberately so our diagram is oriented around the speed of light
  - *ct* is a sort of 'distance in time'

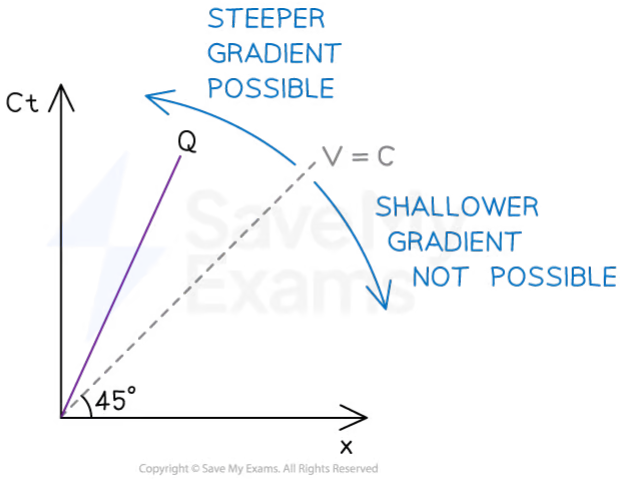
Worldline at Rest



A worldline for an object at rest

- P is an object **at rest**
  - It has an infinite slope meaning $v = 0$
- It is only moving in time, not in space

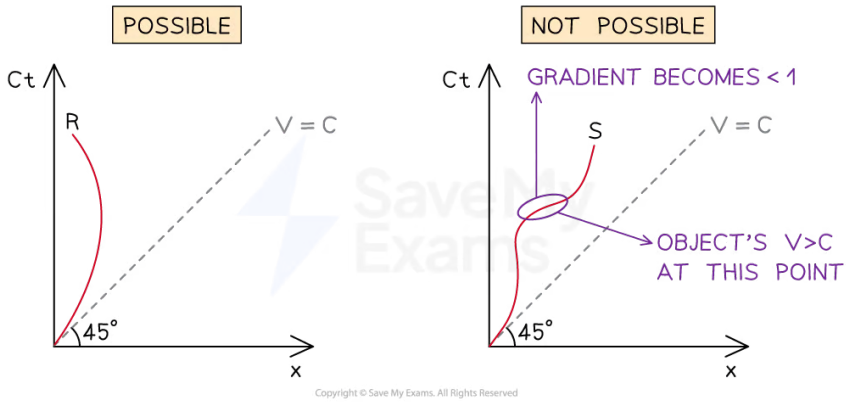
Worldline at Constant Velocity



A worldline at constant velocity has to have a gradient steeper than 1

- Q is an object at **constant velocity**
- The object moves some distance over some time
- For an object moving at the speed of light, c (e.g. a massless photon), its worldline has a gradient of $\frac{c}{v} = \frac{c}{c} = 1$ (a 45° angle)
- Since nothing can travel faster than the speed of light (i.e. v cannot be greater than c), then objects can only have a gradient **greater than 1**
  - Therefore, a gradient of **1** is the **lowest** possible gradient
- Q's motion does not need to start at the origin. As long as it has a gradient of less than 1, it can start anywhere on the x-axis

Worldline accelerating

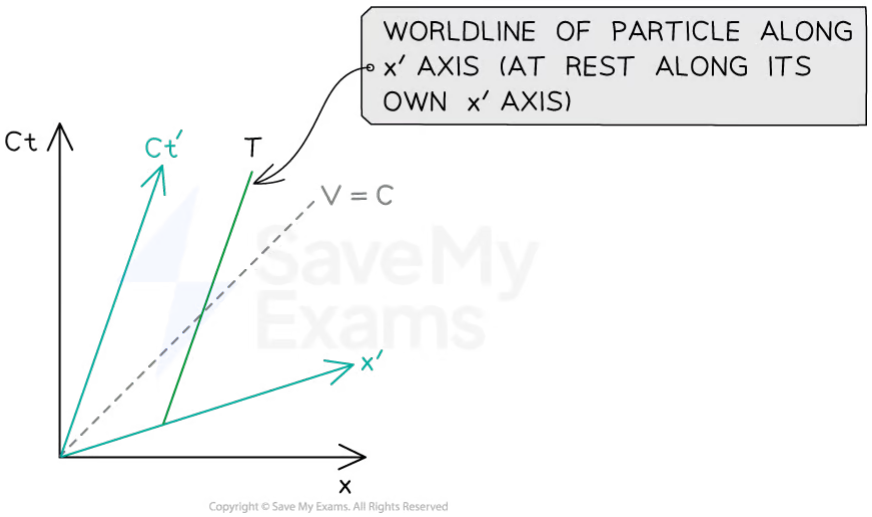


All points on a worldline representing an object accelerating must have a gradient steeper than 1

- R is an object with a **varying velocity** that represents **possible motion**
  - It has a small gradient (larger velocity), which increases (decreasing velocity)
  - Its gradient never gets less than 1
- S is an object with a **varying velocity** that represents **impossible motion**
  - At one point, it has a gradient of less than 1 implying a velocity greater than c
  - Even though it does not physically cross the $v = c$ gradient line, it is still **not** possible because a portion of the line has a gradient of less than one

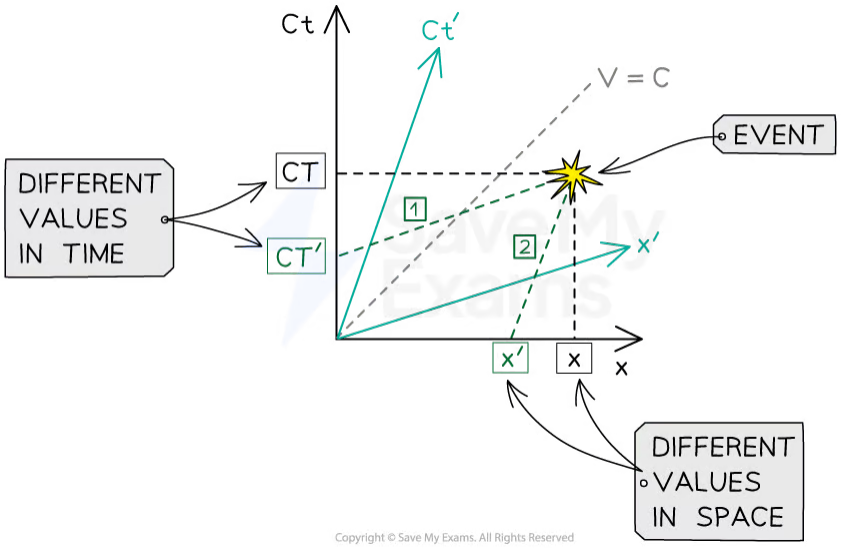
Multiple Reference Frames

- Every **point** on a spacetime diagram represents an **event**, for example, an object moving
- More than one inertial reference frame can be represented on a spacetime diagram
  - ct and x represent the co-ordinate axes for an observer in frame S
  - ct' and x' represent the co-ordinate axes for an observer in frame S' (moving at speed v with respect to frame S)
- Therefore, we can combine two separate spacetime diagrams for different inertial reference frames moving at constant speed relative to each other
  - The axes for the ct' and x' are at an angle
- The worldline T shows the equivalent worldline of P, but now in the S' reference frame
  - This represents an object at rest in its own co-ordinate system



Worldline of a particle at rest in the reference frame S'

- The axes are tilted because the ct' reference frame is travelling at speed *v* relative to the *ct* reference frame
  - The *x'* axis must also be tilted in order for the speed of light (the dashed line in the middle) to be the **same** in **both** reference frames
- The **scales** on the time axes ***ct*** and ***ct'*** and on the space axes ***x*** and ***x'*** of two inertial reference frames **moving** relative to one another are **not** the same and are defined by lines of constant spacetime interval
- If an event occurs (such as a flash of light), both reference frames will measure a different time and different position with respect to each other
  - This can be seen on a spacetime diagram



An event is shown in reference frames S and S' with differing values of distance and time

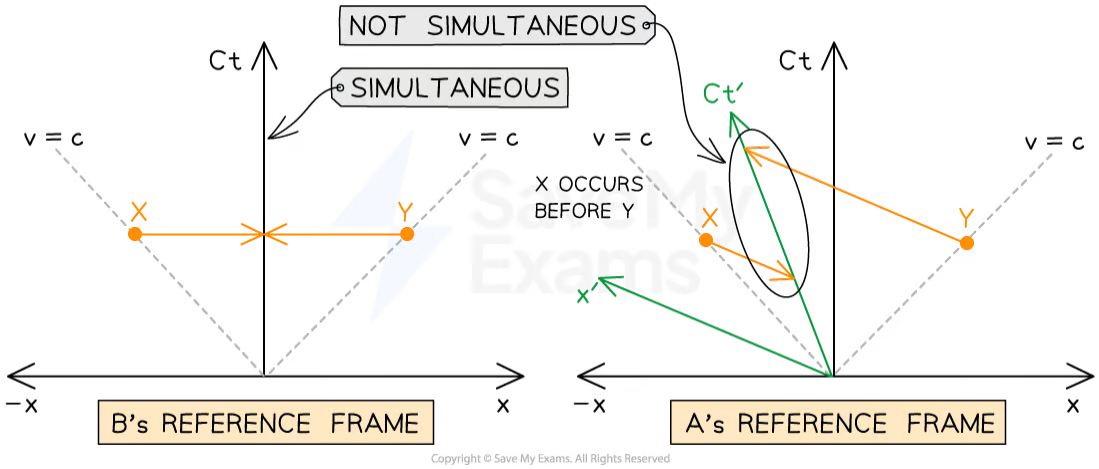
- The event in the S reference frame occurs at (X, cT)
- The event in the S' reference frame occurs at (X', cT')

YOUR NOTES

- The co-ordinates in the S' reference frame are determined by lines 1 and 2
  - Line 1 is a line **parallel** to the x' axis
  - Line 2 is a line **parallel** to the ct' axis
- The clocks in both frames show zero at the origins where two frames collide i.e. both observers start their clocks at the same time to measure any time intervals

Simultaneity

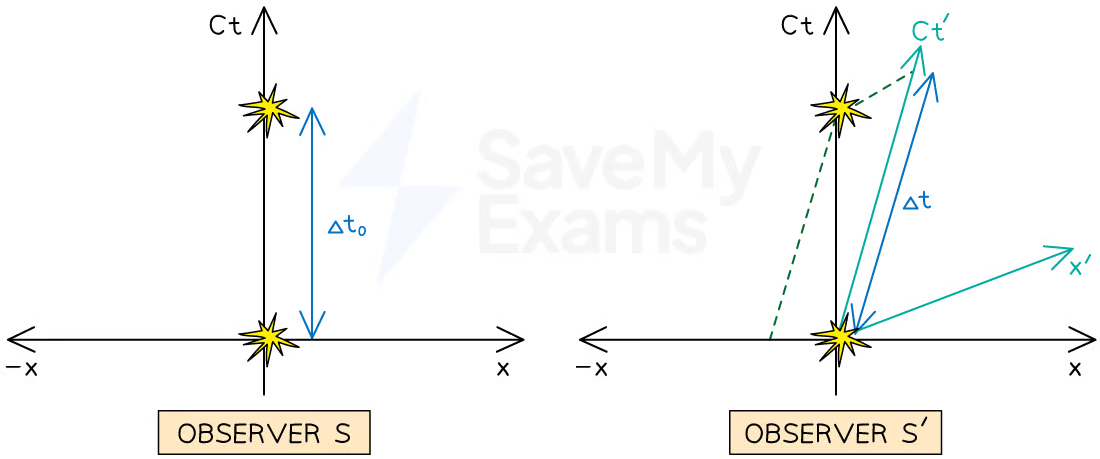
- We can now see that simultaneous events in one frame are not simultaneous in another moving inertial reference frame
- Let's go back to Observers A and B in Simultaneity in Special Relativity
  - We can see that Observer B sees the light reach points X and Y at the same time, whilst Observer A (in the ct'–x' co-ordinate system) sees the light from the lamp reach point X before point Y on a spacetime diagram



Simultaneity is not possible for two reference frames moving relative to each other

Time Dilation

- Consider two flashes of light at $x = 0$ in the S reference frame that occur one after another
- When these flashes are observed in the S' frame, we can see the time between the flashes is **longer**
  - The time between them has **increased (dilated)**



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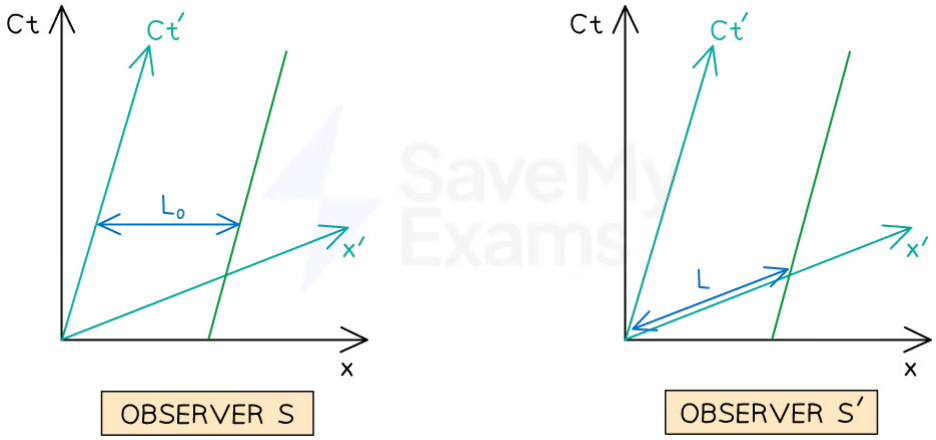
Spacetime diagrams representing time dilation

- Another difference is that in the S' reference frame, the first flash now occurs on the -*x* axis
  - This just means it takes place to the **left** of the observer

Length Contraction

- Consider a rod measured in the S reference frame where the rod moving relative to S
- The rod has the same speed as it does in the S' reference frame (which is also moving relative to S)



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Spacetime diagrams representing length contraction

- The length of the rod is measured by measuring each side at the same time
  - The observer in frame S will measure a length *L*
- When the observer in frame S' measures the rod, they see the rod as stationary (as it is moving at the same speed as the observer)
  - The observer in frame S will measure a length *L*'
- *L* is **shorter** than *L*', which means that the length has been **shortened** (**contracted**) when measured by Observer S, who is **moving** relative to the rod

- Although it is the rod that is moving, remember, it is at rest **in its own reference frame** and Observer S is moving relative to it

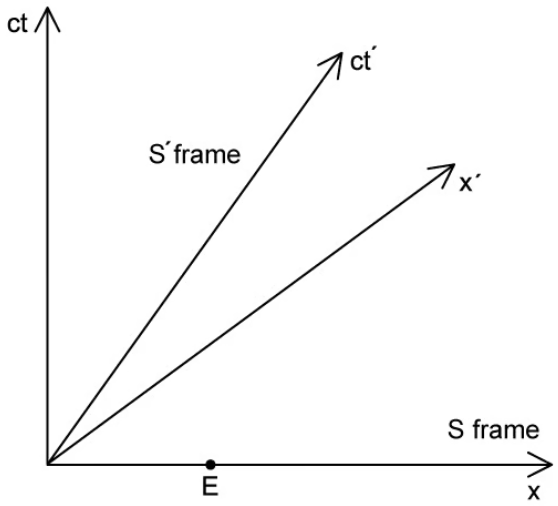
- This occurs from the fact that measurements that are simultaneous in one reference frame are not simultaneous in another

YOUR NOTES

YOUR NOTES

Worked Example

The spacetime diagram shows the axes of an inertial reference frame S and the axes of a second inertial reference frame S' that moves relative to S with speed 0.6432c. When clocks in both frames show zero the origins of the two frames coincide.



Event E has co-ordinates x = 1.5 m and ct = 0 in frame S.

(a)
Label, on the diagram,

(i) the space co-ordinate of event E in the S' frame. Label this event with the letter Q.

(ii) the event that has co-ordinates x' = 1.5 m and ct' = 0. Label this event with the letter R.

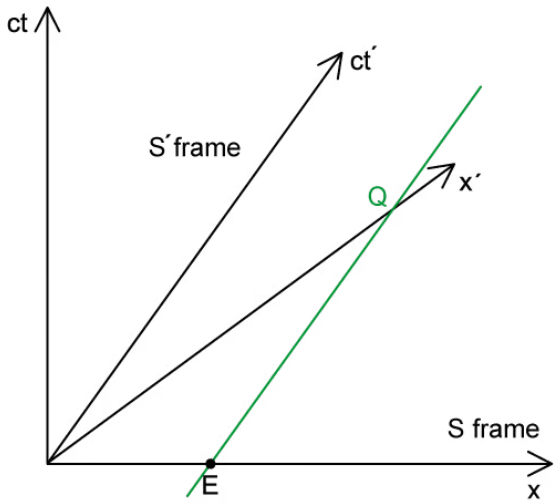
(b)
A rod at rest in frame S has a proper length of 1.5 m. At t = 0, the left-hand end of the rod is at x = 0 and the right-hand end is at x = 1.5 m.

Using the spacetime diagram, outline without calculation, why observers in frame S' measure the length of the rod to be less than 1.5 m.

Answer:

(a)

(i) Draw a line parallel to the ct' axis



(ii)

Step 1: List the known quantities

- Speed of the spacecraft, $v = 0.6432c$
- Position of event in frame S, $x = 1.5$ m

Step 1: Calculate the *x*' co-ordinate of point Q

- To convert between a position (or time) from one co-ordinate system and another, we can use **Lorentz** transformations

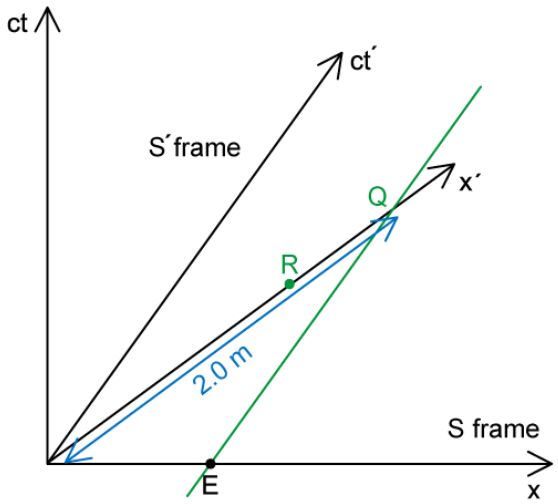
$$x' = \gamma(x - vt)$$

$$x' = \frac{1}{\sqrt{1 - \frac{(0.6432c)^2}{c^2}}}(1.5 - 0) = 1.959 = 2.0 \text{ m}$$

- Since there are no other objects involved, speed $v = 0$

Step 2: Label this point on the axes as R

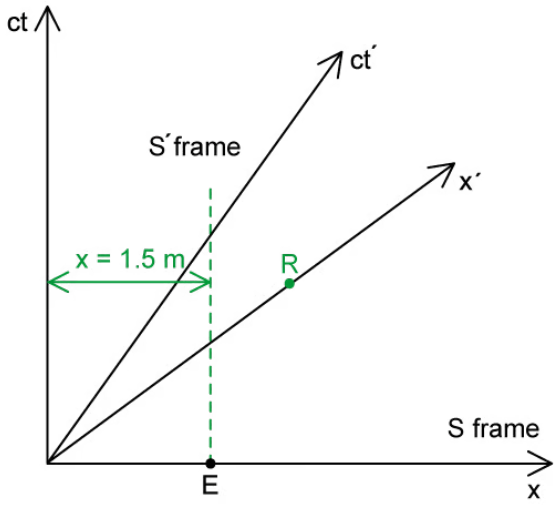
- The co-ordinates are $x' = 1.5$ m and $ct' = 0$
- Point R (at 1.5 m) is roughly $\frac{2}{3}$ of the distance of Q (at 2.0 m)



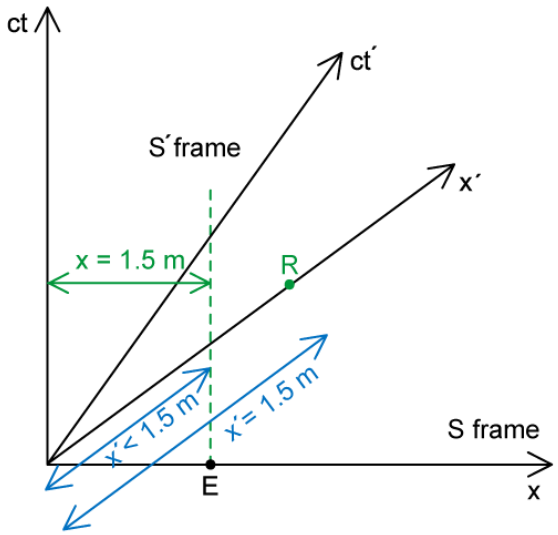
(b)

Step 1: Outline why observers in frame S' measure the length of the rod to be less than 1.5 m

- The ends of the rod must be recorded at the same time in frame S'
- This is shown on the spacetime diagram:



- The right-hand side of the rod intersects the *x*' axis at a co-ordinate that is less than 1.5 m



Exam Tip

This all might sound counter-intuitive because we're used to thinking of position versus time with distance-time graphs, rather than time versus position. Remember, now the gradient is $\frac{1}{velocity}$ instead of equating to the velocity.

The important thing about worldlines is not their value but their **gradient**. Where they start doesn't matter, whether at the origin or along the *x* axis, their gradients cannot be less than 1.

Make sure you never write c', as there is no such thing. c is the same in all reference frames.

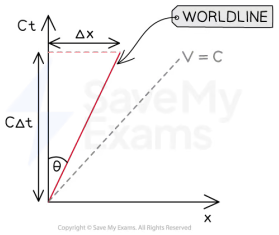
Notice that reading from the *ct*' and *x*' co-ordinate axis is actually no different reading from *ct* and *x*, it's just that they're slanted so it looks a bit different, but the principles are still the same.

Exam questions will generally have the units of *ct* and *x* in **light years (ly)**, so make sure you're comfortable with this definition.

Velocity on a Space-Time Diagram

YOUR NOTES

- The worldline for a moving particle on a spacetime diagram using the *x*-*ct* axis is a diagonal line



- The velocity of the particle can be calculated by the angle of the moving particle's worldline with the *ct* axis
- When we are using the *x*-*ct* axis, we can see that:

$$\tan\theta = \frac{opposite}{adjacent} = \frac{\Delta x}{c\Delta t}$$

- From mechanics, we know that velocity is the rate of change of displacement:

$$\frac{\Delta x}{\Delta t} = v$$

- Therefore:

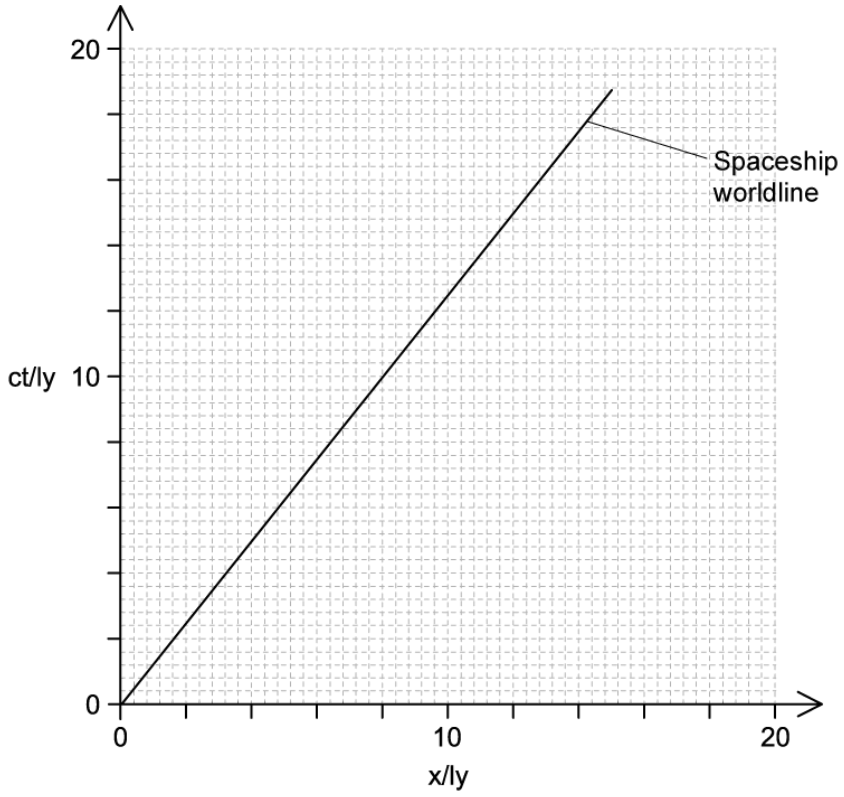
$$\tan\theta = \frac{v}{c}$$

- Where:
  - θ = angle between the world line and the *ct* axis (°)
  - v = velocity of the object (m s^{-1})
  - c = speed of light

YOUR NOTES

Worked Example

A spaceship travels away from Earth in the direction of a nearby planet. A spacetime diagram for the Earth's reference frame shows the worldline of the spaceship. Assume the clock on the Earth, the clock on the planet, and the clock on the spaceship were all synchronized when ct = 0.



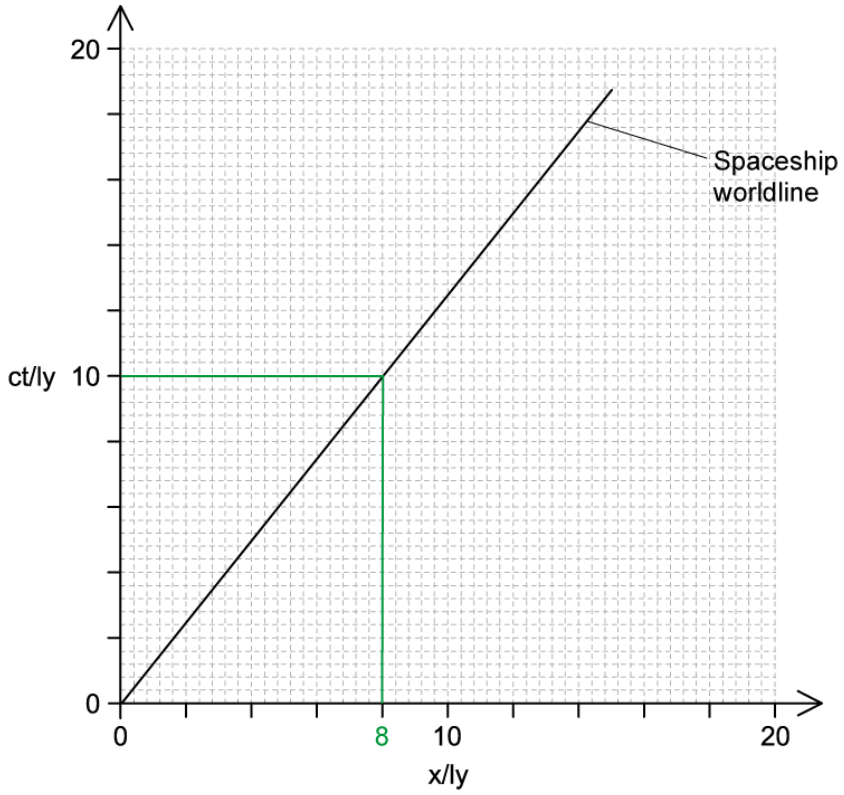
Show, using the spacetime diagram, that the speed of the spaceship relative to the Earth is 0.80c.

Answer:

Method 1: Using the gradient

Step 1: Choose a co-ordinate pair

- Choose any corresponding value of ct and x e.g. $ct = 10$, $x = 8$



Step 2: Calculate the gradient

- The gradient of a spacetime graph is $\frac{c}{v}$

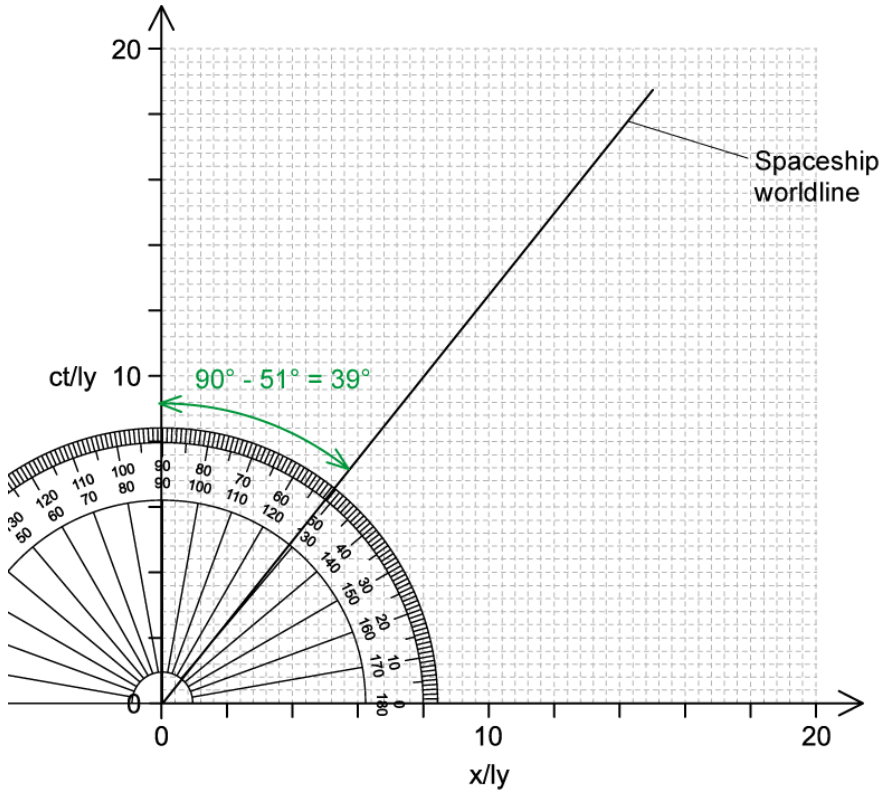
$$gradient = \frac{c}{v} = \frac{10}{8}$$

Step 3: Calculate the velocity, *v*

$$v = \frac{8}{10}c = 0.8\ c$$

Method 2: Measure the angle

Step 1: Measure angle θ using a protractor



- $\theta = 39°$

Step 2: Substitute into the velocity equation

$$\tan\theta = \tan(39) = 0.8 = \frac{v}{c}$$

$$v = 0.8\ c$$

Exam Tip

Any discussion of world lines of moving particles will only be limited to **constant** velocity in your exam.

Make sure you have your **protractor** with you in your exam. An exam question could ask you to measure the angle θ yourself from the diagram on your exam paper.

YOUR NOTES

Muon Lifetime Experiment

- Muon decay experiments provide **experimental evidence** for time dilation and length contraction
- **Muons** are unstable, subatomic particles that are around 200 times heavier than an electron and are produced in the upper atmosphere as a result of pion decays produced by cosmic rays
- Muons travel at 0.98c and have a half-life of 1.6 μs (or mean lifetime of 2.2 μs)
  - The distance they travel in one half-life is around 470 m
- A considerable number of muons can be detected on the Earth's surface, which is about 10 km from the distance they are created
  - Therefore, according to Newtonian Physics, very few muons are expected to reach the surface as this is about 21 half-lives!
- The detection of the muons is a product of time dilation (or length contraction, depending on the viewpoint of the observer)

Muon Decay From Time Dilation

- According to the muon's reference frame, time is dilated so its half-life is **longer**
- We can see this from the time dilation equation

$$\Delta t = \gamma \Delta t_0$$

- Where:
  - The gamma factor, $\gamma = \dfrac{1}{\sqrt{1 - (0.98)^2}} = 5$
  - Δt = the half-life measured by an observer on Earth
  - Δt_0 = the proper time for the half-life measured in the muon's inertial frame
- Therefore, in the reference frame of an observer on Earth, the muons have a lifetime of

$$\Delta t = 5 \times 1.6 = 8 \ \mu s$$

- The time to travel 10 km at 0.98c is 33 μs or 4.1 half-lives, so a significant number of muons remain undecayed at the surface

Muon Decay From Length Contraction

- According to the muon's reference frame, length is contracted so the distance the need to travel is **shorter**
- We can see this from the length contraction equation

$$L = \frac{L_0}{\gamma}$$

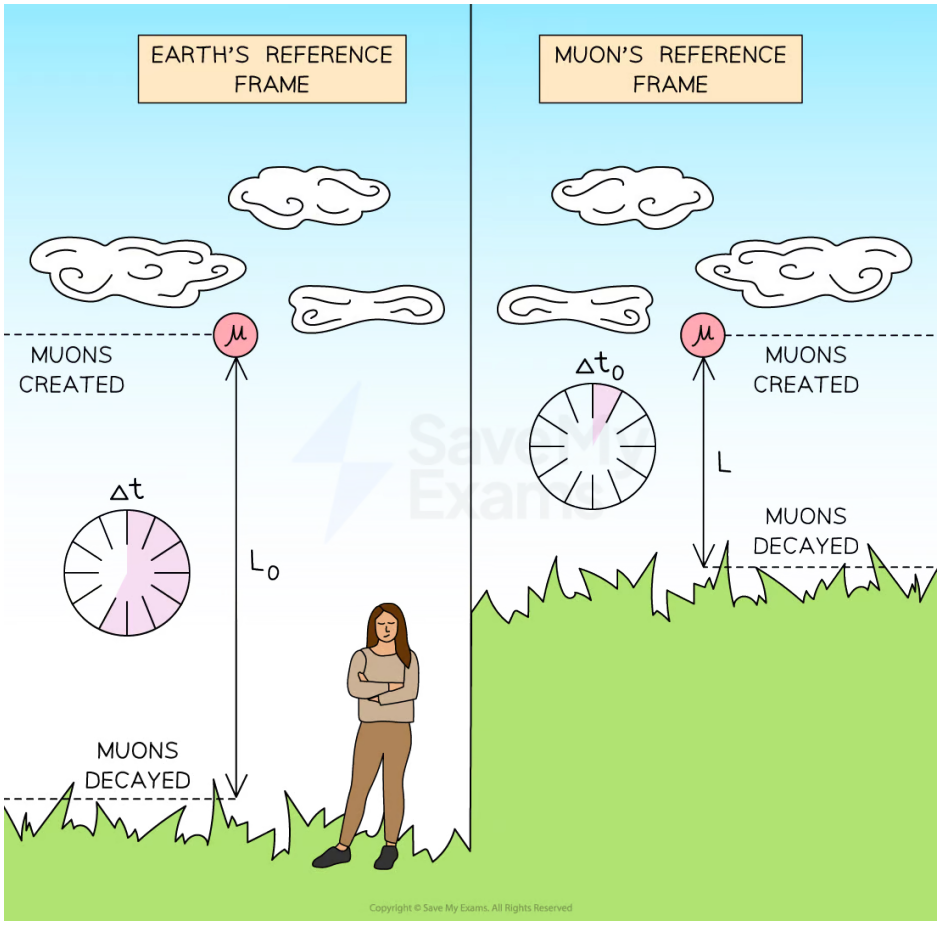
- Where:
  - L_0 = the proper length for the distance measured in the muon's inertial frame
  - L = the distance measured by an observer on Earth

YOUR NOTES

- Therefore, in the reference frame of the muons, they only have to travel a distance:

$$L = \frac{10\,000}{5} = 2000 \text{ m}$$

- To travel this distance takes a time of $\frac{2000}{0.98c}$ = 6.8 μs which is about 4.3 half-lives again, so a significant number of muons remain undecayed at the surface



Muon decay from the Earth's and a muon's reference frame

Worked Example

Muons are created at a height of 4250 m above the Earth's surface. The muons move vertically downward at a speed of 0.980c relative to the Earth's surface. The gamma factor for this speed is 5.00. The half-life of a muon in its rest frame is 1.6 μs.

(a)
Estimate the fraction of the original number of muons that will reach the Earth's surface before decaying, from the Earth's frame of reference, according to:

(i)
Newtonian mechanics

(ii)
Special relativity

(b)
Demonstrate how an observer moving with the same velocity as the muons, accounts for the answer to (a)(ii).

Answer:

(a) (i)

Step 1: List the known quantities

- Height of muon creation above Earth's surface, $h = 4250$ m
- Speed of muons, $v = 0.980c$
- Lifetime of muon, $t = 1.6\ \mu s = 1.6 \times 10^{-6}$ s

Step 2: Calculate the time to travel for the muon

$$time = \frac{distance}{speed} = \frac{h}{v}$$

$$t = \frac{4250}{0.98 \times (3.0 \times 10^8)} = 1.45 \times 10^{-5}\ \text{s}$$

Step 3: Calculate the number of half-lives

$$\frac{1.45 \times 10^{-5}}{1.6 \times 10^{-6}} = 9 \text{ half-lives}$$

Step 4: Calculate the fraction of the original muons that arrive

$$\frac{1}{2^9} \times 100\ \% = 0.2\ \%$$

(a) (ii)

Step 1: List the known quantities

- Time for the muon to travel, $\Delta t_0 = 1.45 \times 10^{-5}$ s

Step 2: Calculate the time travelled in the muons rest frame

$$\Delta t = \gamma \Delta t_0$$

$$\Delta t = 5 \times (1.6 \times 10^{-6}) = 8 \times 10^{-6}$$

Step 3: Calculate the number of half-lives

$$\frac{1.45 \times 10^{-5}}{8 \times 10^{-6}} = 1.8 \text{ half-lives}$$

Step 4: Calculate the fraction of the original muons that arrive

$$\frac{1}{2^{1.8}} \times 100\ \% = 29\ \%$$

(b)

Step 1: Analyse the situation

- An observer moving with the same velocity as the muons will measure the **distance** to the surface to be **shorter** by a factor of $\left(\frac{9}{1.8}\right) = 5$ **OR** length contraction occurs

Step 2: Calculate the distance travelled in the muon's rest frame

$$L = \frac{L_0}{\gamma}$$

$$L = \frac{4250}{5} = 850 \text{ m}$$

Step 3: Calculate the time to travel

$$\text{time taken} = \frac{distance}{speed} = \frac{850}{0.98 \times (3 \times 10^8)} = 2.9 \times 10^{-6}$$

Step 4: Calculate the number of half-lives

$$\frac{2.9 \times 10^{-6}}{1.6 \times 10^{-6}} = 1.8 \text{ half-lives (same as (b))}$$

YOUR NOTES

Exam Tip

Remember that it is the observer on **Earth** that viewed the muons' lifetime or half-life as **longer** (time dilation), whilst it is the **muons'** reference frame that views the distance needed to travel as **shorter** (length contraction).

Always do a sense check with your answer, you must always end up with a longer time or shorter distance for the muons to be observed on the Earth's surface.

Any exam questions on this topic will only use the following equations:

- Time dilation
- Length contraction
- $speed = \frac{distance}{time}$

Calculating half-lives through $\frac{1}{2^{number\ of\ half-lives}}$ is a common way to calculate the number of muons remaining:

- After 1 half-life, $\frac{1}{2}$ the original muons remain
- After 2 half-lives, $\frac{1}{4}$ or $\frac{1}{2^2}$ of the original muons remain
- After 3 half-lives, $\frac{1}{8}$ or $\frac{1}{2^3}$ of the original muons remain, and so on

IB Physics HL

YOUR NOTES

Thermal Energy Transfers

CONTENTS

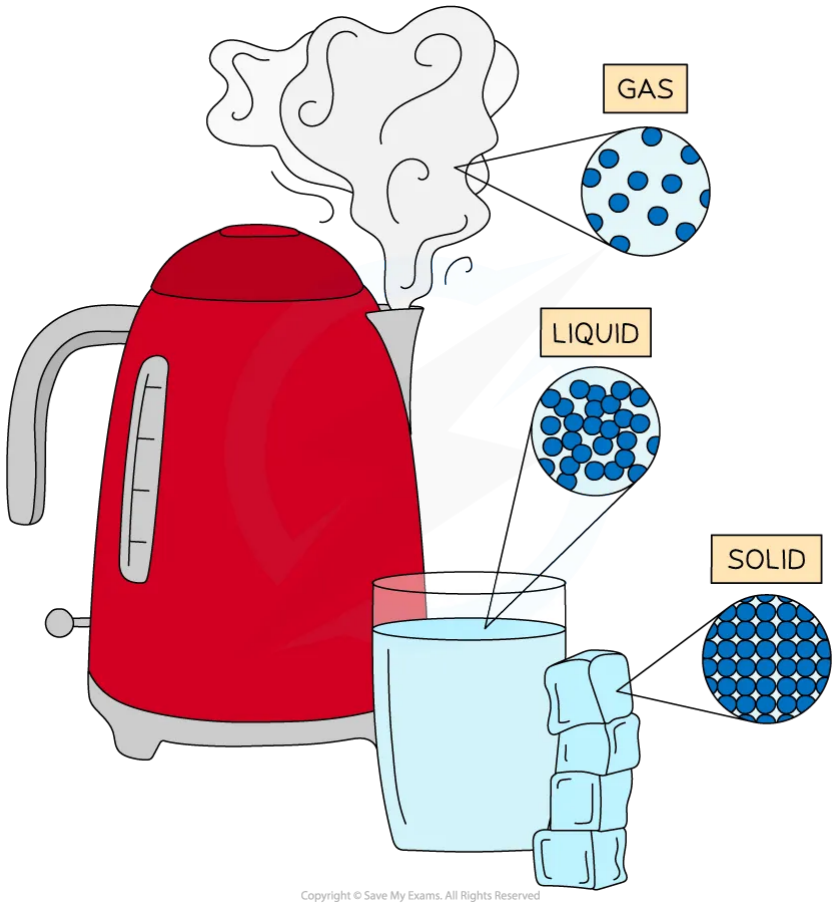
Solids, Liquids & Gases
Density
Temperature Scales
Temperature & Kinetic Energy
Internal Energy
Thermal Equilibrium
Changes of State
Specific Heat Capacity
Specific Latent Heat
Thermal Conduction
Thermal Convection
Thermal Radiation
Apparent Brightness & Luminosity
Stefan-Boltzmann Law
Wien's Displacement Law

YOUR NOTES

Solids, Liquids & Gases

- The three states of matter are **solid**, **liquid** and **gas**
- The **kinetic theory of matter** is a model that attempts to explain the properties of the three states of matter
  - In this model, particles are assumed to be small spheres

Water as a solid, liquid, and a gas



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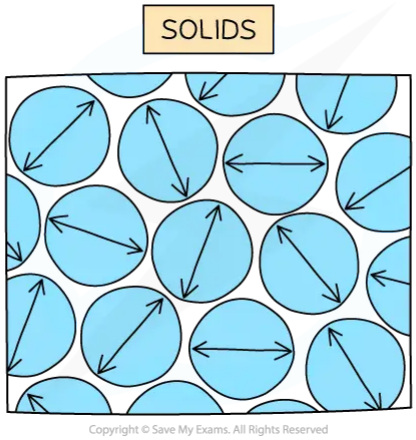
Water has three states of matter; solid ice, liquid water and gaseous steam. The difference between each state is the arrangement of the particles

Solids

- Particles in a solid...
  - Are **closely packed**
  - Are arranged in a fixed pattern (**lattice structure**)
  - Can only **vibrate** about their fixed positions
  - Have **low energies** compared to particles in liquids and gases
  - Therefore, do **not** have enough energy to **overcome** the **intermolecular forces** of attraction holding them together

YOUR NOTES

Molecular arrangement in a solid



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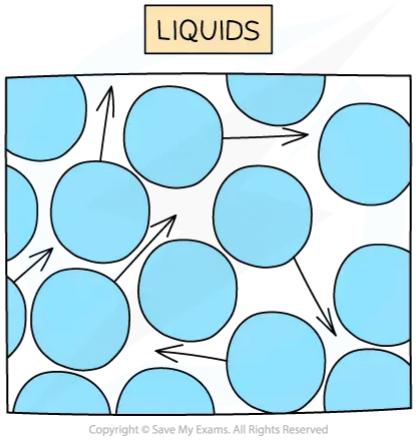
In a solid, particles are arranged in a fixed pattern, with no spaces between them, and are only able to vibrate about their fixed positions

- As a result of the arrangement and behaviour of their particles, solids...
  - Have a fixed shape (although some solids can be deformed when forces are applied)
  - Have a fixed volume
  - Are very difficult to compress
  - Have higher densities than liquids and gases

Liquids

- Particles in a liquid...
  - Are **closely packed**
  - Are **randomly** arranged (i.e. there is no fixed pattern)
  - Can **flow** past each other
  - Have **higher energies** than particles in solids, but lower energies than gas particles
  - Therefore, have enough energy to **partially overcome** the **intermolecular forces** of attraction holding them together

Molecular arrangement in a liquid



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YOUR NOTES

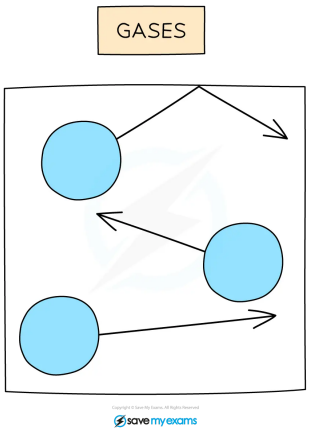
In a liquid, particles are arranged randomly and are able to flow past one another

- As a result of the arrangement and behaviour of their particles, liquids...
  - Do not have a fixed shape and take the shape of the container they are held in
  - Have a fixed volume
  - Are difficult to compress
  - Have lower densities than solids, but higher densities than gases

Gases

- Particles in a gas...
  - Are **far apart** (the average distance between the particles is ~10 times greater than the distance between the particles in solids and liquids)
  - Are **randomly** arranged
  - Move around in **all directions** at a variety of speeds, occasionally **colliding** with each other and with the walls of the container they are in
  - Are negligible in size compared to the volume occupied by the gas
  - Have **higher energies** than particles in solids and liquids
  - Therefore, have enough energy to overcome the **intermolecular forces** of attraction holding them together

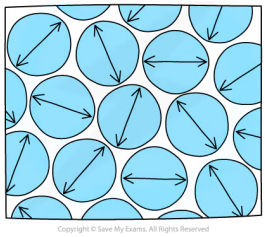
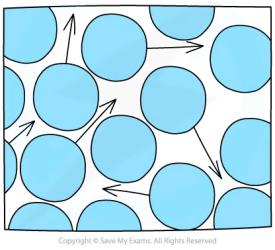
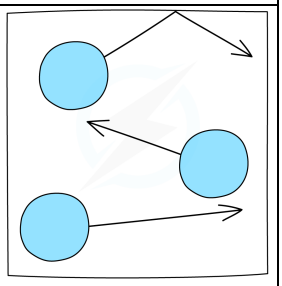
Molecular arrangement in a gas



In a gas, particles can move around freely in all directions (shown by the arrows)

- As a result of the arrangement and behaviour of their particles, gases...
  - Do not have a fixed shape and take the shape of the container they are held in
  - Do not have a fixed volume and expand to completely fill the available volume
  - Can be compressed
  - Have the lowest densities (~1000 times smaller than the densities of solids and liquids)

State of Matter	Solid	Liquid	Gas
Particle arrangement	Fixed pattern (lattice structure)	Random	Random

Space between particles	No space	Some space	Large space
Particle movement	Vibrates around a fixed position	Flows past each other	Moves around at different speeds
Particle energy	Low	Medium	High
Substance shape	Fixed	Not fixed	Not fixed
Substance volume	Fixed	Fixed	Not fixed
Substance density	High	Medium	Low
2D diagram of particle arrangement			

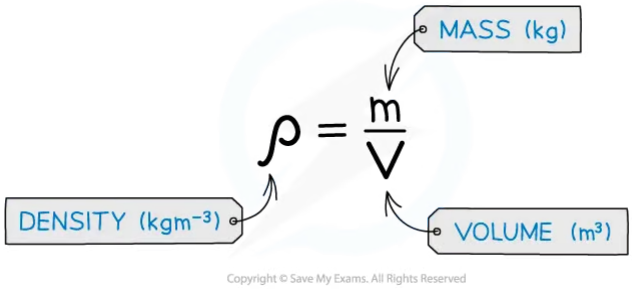
YOUR NOTES

YOUR NOTES

Density

- Density is the **mass per unit volume** of an object
  - Objects made from low-density materials typically have a lower mass
  - For example, a balloon is less dense than a small bar of lead despite occupying a larger volume
- The units of density depend on the units used for mass and volume:
  - If the mass is measured in *g* and volume in **cm^3**, then the density will be in **g / cm^3**
  - If the mass is measured in *kg* and volume in **m^3**, then the density will be in **kg / m^3**

How to calculate density



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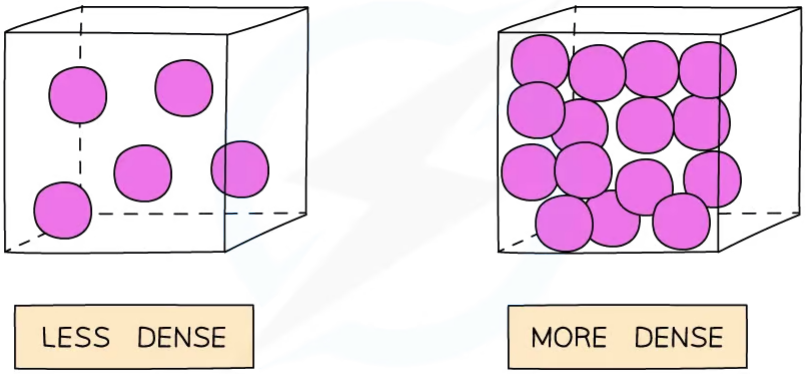

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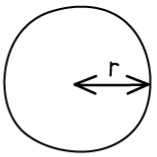
Gases are less dense than solids due to their molecular arrangements

- The volume of an object may not always be given directly but can be calculated using the appropriate formula

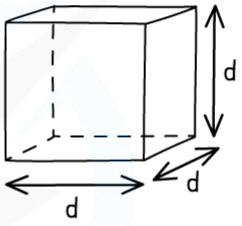
Volumes of common 3D shapes

YOUR NOTES

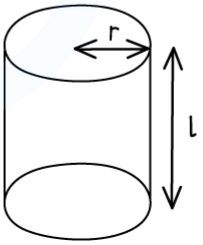
SPHERE: $\frac{4}{3}\pi r^3$



CUBE: d^3



CYLINDER: $\pi r^2 \times l$



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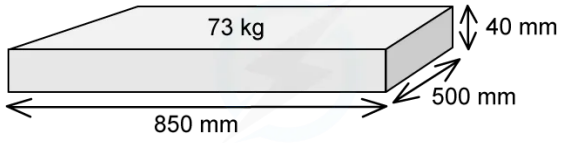
YOUR NOTES

Worked Example

A paving slab has a mass of 73 kg and dimensions 40 mm x 500 mm x 850 mm.

Calculate the density, in kg m^{-3} of the material from which the paving slab is made.



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Answer:

STEP 1

EQUATION FOR DENSITY

$$\rho = \frac{M}{V}$$

STEP 2

CALCULATE THE VOLUME

$V = 40\ mm \times 500\ mm \times 850\ mm = 1.7 \times 10^{7}\ mm^3$

STEP 3

CONVERT mm^3 TO m^3

$1\ mm = 0.001\ m = 1 \times 10^{-3}\ m$

$1\ mm^3 = (0.001)^3\ m^3 = (1 \times 10^{-3})^3\ m^3 = 1 \times 10^{-9}\ m^3$

$V = 1.7 \times 10^{7} \times 1 \times 10^{-9} = 1.7 \times 10^{-2}\ m$

STEP 4

SUBSTITUTE MASS AND VOLUME INTO DENSITY EQUATION

$$\rho = \frac{73\ kg}{1.7 \times 10^{-2}\ m^3} = 4300\ kgm^{-3}\ (2\ s.f.)$$

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YOUR NOTES

Worked Example

Liquids are about 1000 times denser than gases. Let d be the diameter of a molecule.

Estimate the average intermolecular distance in a gas. Give your answer in terms of d.

Answer:

Step 1: Recall the equation for density

$$\rho = \frac{m}{v}$$

Step 2: Write down the relationship between the density of a gas ρ_{gas} and the density of a liquid ρ_{liquid}

$$\rho_{liquid} = 1000 \times \rho_{gas}$$

Step 3: Substitute into the density equation to show the relationship between the masses and volumes of a liquid a gas

$$\frac{m_{liquid}}{v_{liquid}} = 1000 \times \left(\frac{m_{gas}}{v_{gas}} \right)$$

Step 4: Since the mass stays the same, the relationship between the densities translates into a relationship between volumes as mass cancels out

$$\frac{\cancel{m_{liquid}}}{v_{liquid}} = 1000 \times \left(\frac{\cancel{m_{gas}}}{v_{gas}} \right)$$

$$v_{gas} = 1000 \times v_{liquid}$$

Step 4: Relate the volume to the average distance between the molecules, x

- The average distance x between the molecules is related to the cube root of the volume

$$x = \sqrt[3]{1000\, v_{liquid}} = 10 \times d$$

YOUR NOTES

Exam Tip

When converting a **larger** unit to a **smaller** one, you **multiply (×)**

- E.g. 125 m = 125 × 100 = 12 500 cm

When you convert a **smaller** unit to a **larger** one, you **divide (÷)**

- E.g. 5 g = 5 / 1000 = 0.005 or 5×10^{-3} kg

When dealing with squared or cubic conversions, cube or square the conversion factor too

- E.g. $1\ \text{mm}^3 = \frac{1}{1000^3} = 1 \times 10^{-9}\ \text{m}^3$
- E.g. $1\ \text{cm}^3 = \frac{1}{100^3} = 1 \times 10^{-6}\ \text{m}^3$

YOUR NOTES

Temperature Scales

- On the thermodynamic (Kelvin) temperature scale, **absolute zero** refers to the lowest possible temperature
  - This is equal to 0 K or –273 °C
- It is not possible to have a temperature lower than 0 K
  - This means a temperature in Kelvin will **never** be a negative value
- Absolute zero is defined as:

The temperature at which the molecules in a substance have zero kinetic energy

- This means for a system at 0 K, it is not possible to remove any more energy from it
- Even in space, the temperature is roughly 2.7 K, just above absolute zero

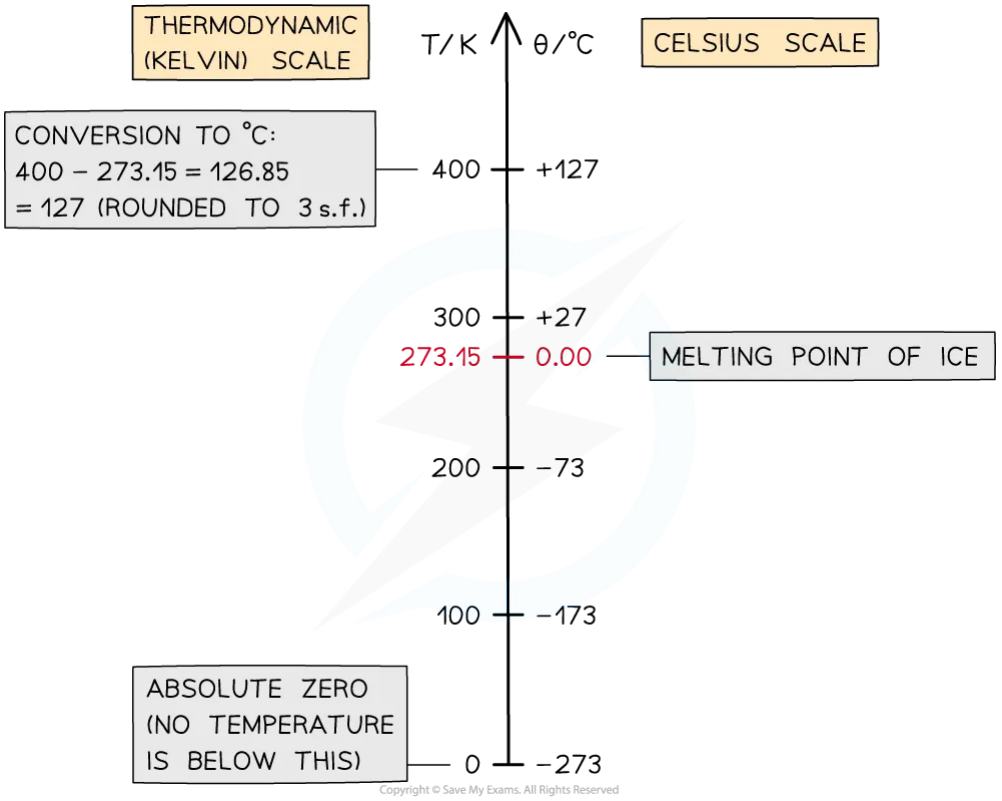
How to use the Kelvin Scale

- To convert between temperatures θ in the Celsius scale, and T in the Kelvin scale, use the following conversion:

$$\theta / °C = T / K - 273$$

$$T / K = \theta / °C + 273$$

Conversion chart between temperature scales in Kelvin and Celsius



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- The divisions on both scales are equal, which means

A change in a temperature of 1 K is equal to a change in temperature of 1 °C

Worked Example

In many ideal gas problems, room temperature is considered to be 300 K.

What is this temperature in degrees Celsius?

Answer:

Step 1: Recall the Kelvin to Celsius conversion

$$\theta / °C = T / K - 273$$

Step 2: Substitute in the value of 300 K

$$300\ K - 273 = 27\ °C$$

Exam Tip

If you forget in the exam whether it's + or –273, check the data booklet!

$$\text{Temperature (K)} = \text{temperature (°C)} + 273$$

This helpful reminder is there under 'unit conversions'

YOUR NOTES

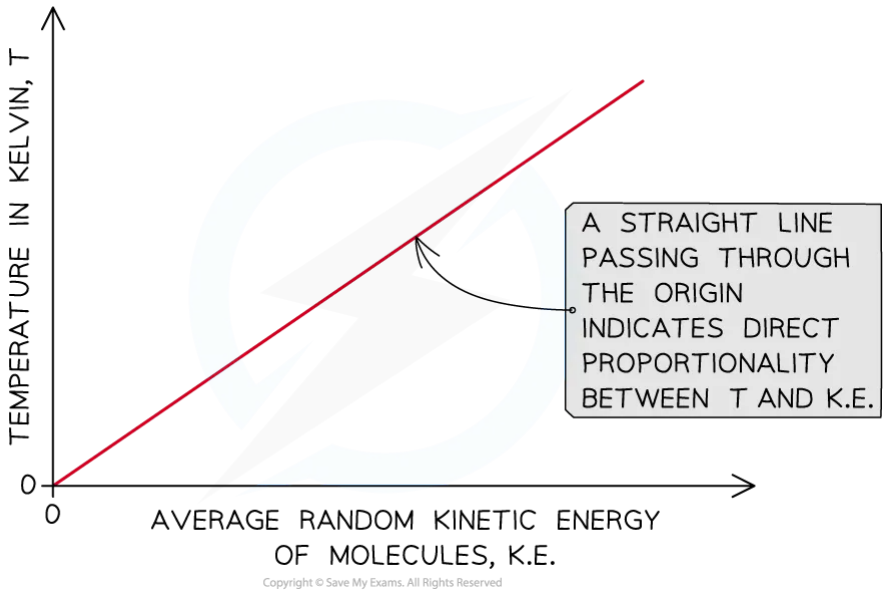
Temperature & Kinetic Energy

- Particles in gases usually have a range of speeds
- The average random kinetic energy of the particles E_k can be calculated using the equation

$$E_k = \frac{3}{2} k_B T$$

- Where:
  - E_k = average random kinetic energy of the particles in joules (J)
  - $k_B = 1.38 \times 10^{-23}$ J K^{-1} (Boltzmann's constant)
  - T = absolute temperature in kelvin (K)
- This tells us that the absolute temperature of a body is **directly proportional** to the average kinetic energy of the molecules within the body

Relationship between absolute temperature and average random kinetic energy of molecules



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YOUR NOTES

Worked Example

The surface temperature of the Sun is 5800 K and contains mainly hydrogen atoms.

Calculate the average speed of the hydrogen atoms, in km s^{-1}, near the surface of the Sun.

Answer:

Step 1: List the known quantities

- Temperature, $T = 5800$ K
- Mass of a hydrogen atom = mass of a proton, $m_p = 1.673 \times 10^{-27}$ kg
- Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J K^{-1}

Step 2: Equate the equations relating kinetic energy with temperature and speed

- Average kinetic energy of a molecule: $E_k = \frac{3}{2} k_B T$
- Kinetic energy: $E_k = \frac{1}{2} mv^2$

$$\frac{3}{2} k_B T = \frac{1}{2} m_p v^2$$

Step 3: Rearrange for average speed and calculate

$$\frac{3k_B T}{m_p} = v^2$$

$$v = \sqrt{\frac{3k_B T}{m_p}} = \sqrt{\frac{3 \times (1.38 \times 10^{-23}) \times 5800}{1.673 \times 10^{-27}}}$$

Average speed: $v = 11\,980$ m $s^{-1} = 12$ km s^{-1}

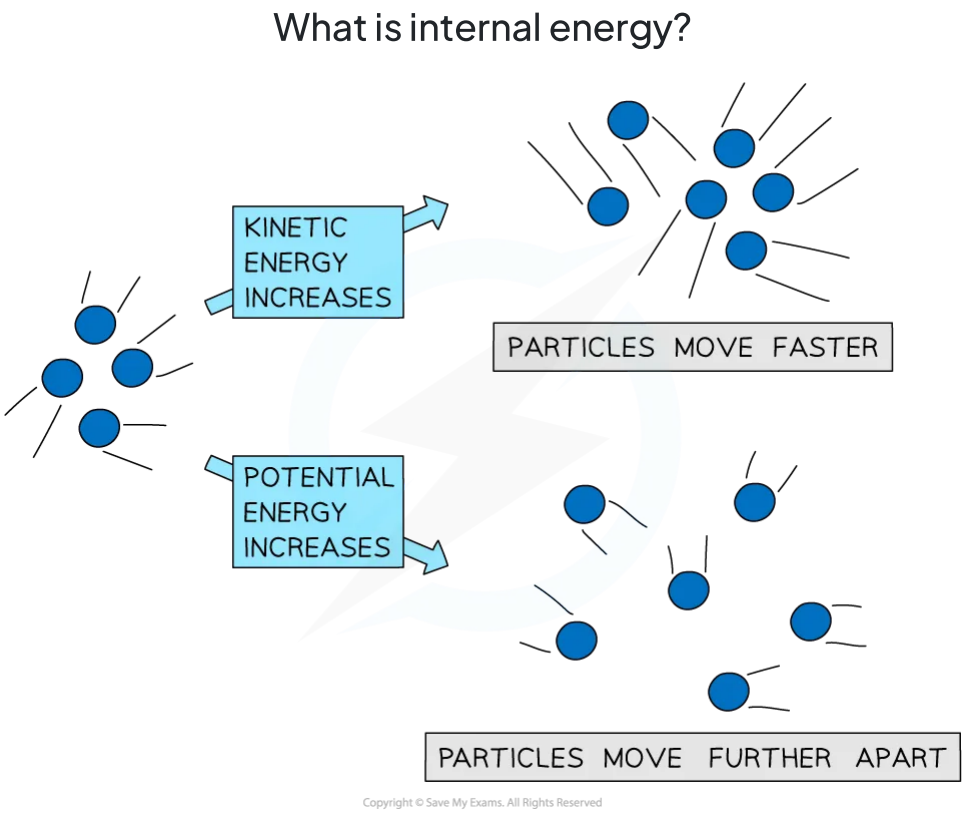
YOUR NOTES

Internal Energy

- When a substance gains or loses thermal energy, its **internal energy** increases or decreases
- The internal energy of a substance is defined as:

  The sum of the total kinetic energy and the total intermolecular potential energy of the particles within the substance

- As thermal energy is transferred to a substance, two things can happen:
  - An increase in the average kinetic energy of the molecules within the substance - i.e. the molecules vibrate and move at higher speeds
  - An increase in the potential energy of the molecules within the substance - i.e. the particles get further away from each other or move closer to each other



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Kinetic energy and potential energy are the two energy stores that make up internal energy

- Temperature is a measure of the average kinetic energy of the molecules
  - Therefore, only an **increase** in the average kinetic energy of the molecules will result in an **increase in temperature** of the substance
  - Due to thermal expansion, when the temperature of a substance increases, the **potential energy** of the molecules also increases

- When **only** the **potential energy** of the molecules changes, the **temperature** of the substance **does not change**
  - This is the case for all **state changes** (e.g. melting, boiling)

Exam Tip

Remember that a change in internal energy does not necessarily correspond to a change in temperature

- A change in the average kinetic energy of the molecules corresponds to a change in temperature
- A change in the average potential energy of the molecules does not affect the temperature

YOUR NOTES

YOUR NOTES

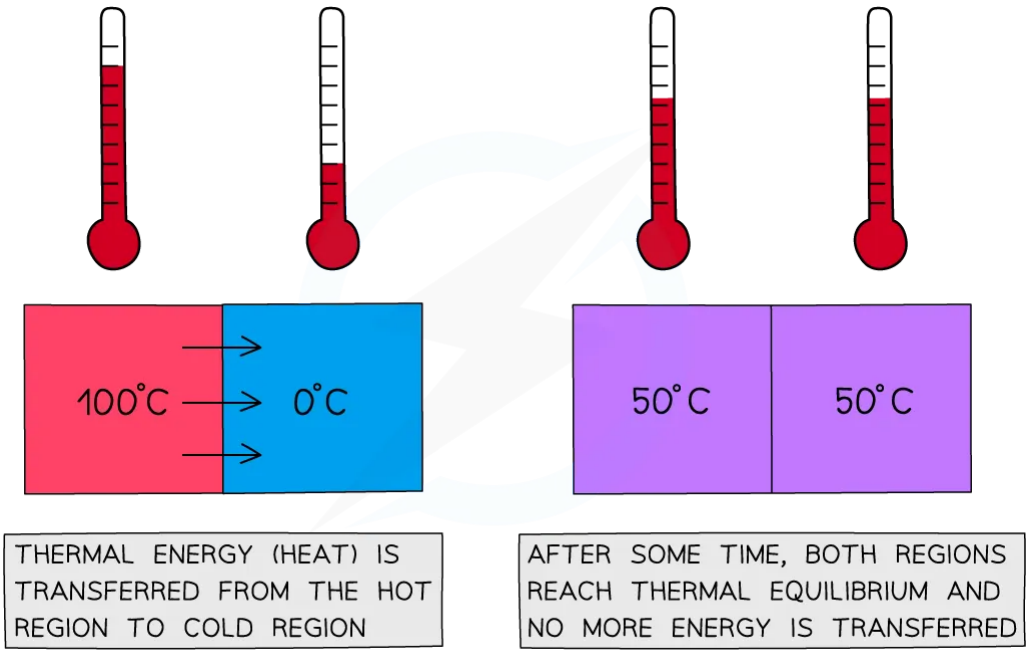
Thermal Equilibrium

- Thermal energy is **always** transferred from a hotter region to a lower region
  - Eventually, once the regions reach the same temperature, no more thermal energy is transferred
- Thermal equilibrium is defined as:

  When two substances in contact with each other no longer exchange any heat energy and both reach an equal temperature

- There is no longer thermal energy transfer between the regions

Thermal equilibrium and the direction of energy flow



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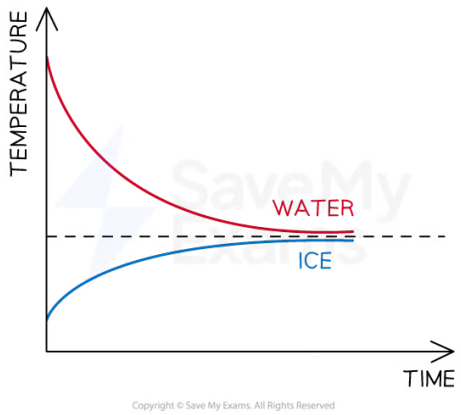
Two regions of different temperatures will eventually reach thermal equilibrium

- The two regions need to be in **thermal contact** for this to occur
  - The **hotter** region will cool down and the **cooler** region will heat up until they reach the **same** temperature

Graph showing water and ice reaching thermal equilibrium



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The final temperature of the water depends on the initial temperature difference between the water and ice

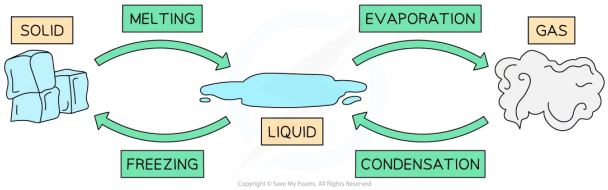
- An example of this is ice in room temperature water
  - The ice cubes heat up as thermal energy is transferred **from** the water
  - Therefore, the water cools down as thermal energy is transferred away from the water **to** the ice

YOUR NOTES

Changes of State

- A **change of state,** or **phase** change, happens whenever matter changes from one phase (solid, liquid or gas) into another
  - During a phase change, thermal energy is transferred to or from a substance
- This energy transfer does **not change the temperature** of the substance undergoing the phase change, meaning
  - The thermal energy provided (or removed) does not affect the kinetic energy of the molecules within the substance
  - Only the potential energy (i.e. the spacing between the atoms or molecules) is affected
- The four main phase changes are:
  - **Melting** – i.e. when a substance changes from solid to liquid as it absorbs thermal energy
  - **Freezing** – i.e. when a substance changes from liquid to solid as it releases thermal energy
  - **Vaporisation** (or **boiling**) – i.e. when a substance changes from liquid to gas as it absorbs thermal energy
  - **Condensation** – i.e. when a substance changes from gas to liquid as it releases thermal energy

Phase changes of water



- Each substance has its own melting (or freezing) and boiling points
  - For example, the freezing point of water is 0 °C and its boiling point is 100 °C
- Possible phase changes of water include:
  - Solid ice **melting** into liquid water at 0 °C
  - Liquid water **boiling** and changing into gaseous water vapour at 100 °C
- Both these changes happen when **thermal energy** is absorbed
  - If thermal energy is released from water vapour at 100 °C, it condenses back into water
  - If water continues to release thermal energy, it cools down until it reaches 0 °C and **freezes** into ice
- Melting and freezing happen at the **melting / freezing point** of a substance
- Vaporisation and condensation happen at the **boiling point** of a substance

Specific Heat Capacity

- The amount of thermal energy needed to change the temperature of an object depends on:
  - The **change in temperature** required ΔT, i.e. the larger the change in temperature the more energy is needed
  - The **mass** of the object m, i.e. the greater the mass the more energy is needed
  - The **specific heat capacity** c of the given substance, i.e. the higher the specific heat capacity the more energy is needed

Substance	Specific Heat Capacity ($J\,kg^{-1}\,K^{-1}$)
Water	4200
Ice	2200
Aluminium	900
Copper	390
Gold	130

- The equation for the thermal energy transferred, Q, is then given by

$$Q = mc\Delta T$$

- Where:
  - m = mass of the substance (kg)
  - ΔT = change in temperature (K or °C)
  - c = specific heat capacity of the substance ($J\,kg^{-1}\,K^{-1}$)
- The specific heat capacity of a substance is defined as:

  The amount of energy required to change the temperature of 1 kg of a substance by 1 K (or 1°C)

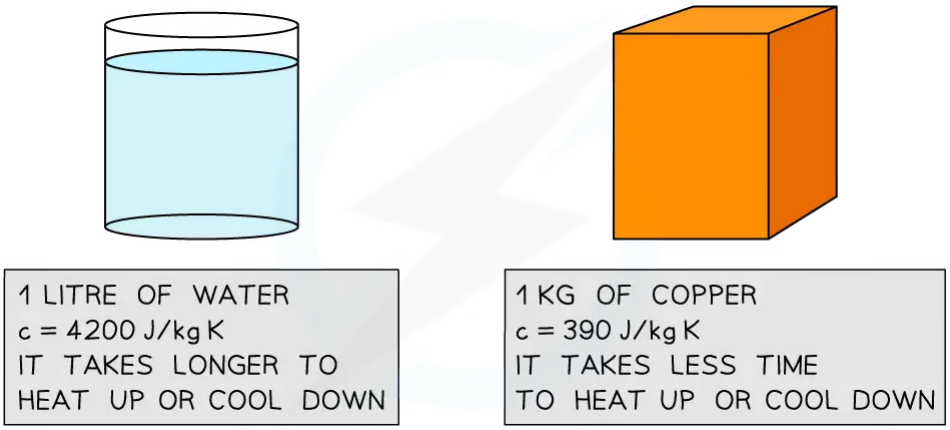
- This definition can be explained when the above equation is rearranged for c:

$$c = \frac{Q}{m\Delta T}$$

- This means the **higher** the specific heat capacity of a substance, the **longer** it takes for the substance to warm up or cool down
  - Note that the specific heat capacity is measured in $J\,kg^{-1}\,K^{-1}$

Specific heat capacity of water and copper

YOUR NOTES



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The high specific heat capacity of water means it heats up and cools down much slower than metals, such as copper

YOUR NOTES

Worked Example

A piece of copper of mass 50 g is heated until it reaches a temperature of 120 °C. The copper is removed from the heat and immediately placed into 250 mL of water at 25 °C.

The temperature of the water and copper is measured until they reach thermal equilibrium.

Determine the final temperature of the copper and water, in degrees Celsius (°C), assuming no heat is lost to the surroundings.

- The specific heat capacity of water is 4200 J kg^{-1} K^{-1}
- The specific heat capacity of copper is 390 J kg^{-1} K^{-1}

Answer:

Step 1: Write down the known quantities

- Mass of copper, m_c = 50 g = 0.05 kg
- Mass of water, m_w = 250 ml = 0.25 kg (since 1 litre = 1 kg)
- Initial temperature of copper, T_c = 120 °C
- Initial temperature of water, T_w = 25°C
- Specific heat capacity of water, c_w = 4200 J kg^{-1} K^{-1}
- Specific heat capacity of copper, c_c = 390 J kg^{-1} K^{-1}

Step 2: Write down the equation for thermal energy

$$Q = mc\Delta T$$

Step 3: Equate the equations for the energy transferred from the copper to the water

- The copper is at a higher initial temperature than the water, hence thermal energy will be transferred from the copper to the water
- The energy lost by the copper = the energy gained by the water

$$-Q_c = Q_w$$

$$-m_c c_c \Delta T_c = m_w c_w \Delta T_w$$

Step 4: Determine the final temperature T_f of the copper and water

- Since the water and copper reach thermal equilibrium, their final temperature T_f will be **the same**

$$-m_c c_c (T_f - 120) = m_w c_w (T_f - 25)$$

$$-0.05 \times 390 \times (T_f - 120) = 0.25 \times 4200 \times (T_f - 25)$$

$$2340 - 19.5T_f = 1050T_f - 26\,250$$

$$1069.5\ T_f = 28\,590$$

$$T_f = \frac{28\,590}{1069.5} = 26.7\ °C$$

Exam Tip

You should notice that changes in temperature ΔT can usually be written in degrees Celsius (although this is not the SI base unit for temperature) and do not need to be converted into kelvin (K). This is because differences in absolute temperatures always correspond to differences in Celsius temperature.

If the question asks to determine the initial or final temperature of a substance, make sure you always check the unit of measure (°C or K) in which you are required to give your final answer.

YOUR NOTES

YOUR NOTES

Specific Latent Heat

- During a **phase change** (i.e. a change of state) thermal energy is transferred to a substance or removed from it
  - During a phase change, the **temperature** of the substance **does not change**
- In this case, the thermal energy is calculated as follows:

$$Q = mL$$

- Where:
  - Q = heat energy transferred (J)
  - m = mass of the substance in kilograms (kg)
  - L = specific latent heat of the substance (J kg^{-1})
- The specific latent heat of a substance is defined as:

> **The amount of energy required to change the state of 1 kg of a substance without changing its temperature**

- This definition can be explained when the above equation is rearranged for L:

$$L = \frac{Q}{m}$$

- This means that the higher the specific latent heat of a substance, the greater the energy needed to change its state
  - Note that the specific latent heat is measured in J kg^{-1}
- The amount of energy required to melt (or solidify) a substance is **not** the same as the amount of energy required to evaporate (or condense) the same substance
- Hence, there are two types of specific heat:
  - **Specific latent heat of fusion, L_f**
  - **Specific latent heat of vaporisation, L_v**

Specific Latent Heat of Fusion

- Specific latent heat of **fusion** is defined as:

> **The energy released when 1 kg of liquid freezes to become solid at constant temperature**
> OR
> **The energy absorbed when 1 kg of solid melts to become liquid at constant temperature**

- This is because fusion applies to the following phase changes:
  - Solid to liquid
  - Liquid to solid

Specific Latent Heat of Vaporisation

YOUR NOTES

- Specific latent heat of **vaporisation** is defined as:

  The energy released when 1 kg of gas condenses to become liquid at constant temperature
  OR
  The energy absorbed when 1 kg of liquid evaporates to become gas at constant temperature

- This is because vaporisation applies to the following phase changes:
  - Liquid to gas
  - Gas to liquid

What is the difference between the latent heat of vaporisation and fusion?

- For a given substance, the value of the specific latent heat of vaporisation is always **higher** than the value of the specific latent heat of fusion
  - In other words, $L_v > L_f$
- This means more energy is required to **evaporate** (or condense) a substance than is needed to **melt** it (or solidify it)
- During melting (**fusion**):
  - The intermolecular forces of attraction only need to be partially overcome to turn from a solid to a liquid
- During evaporation (**vaporisation**):
  - The intermolecular forces of attraction need to be completely overcome to turn from liquid to gas
  - This requires a lot more energy

Substance	Specific Latent Heat of Fusion ($J\ kg^{-1}$)	Specific Latent Heat of Vaporisation ($J\ kg^{-1}$)
Water	4.0×10^5	1.1×10^7
Aluminium	3.3×10^5	2.3×10^6
Copper	2.1×10^5	4.7×10^6
Gold	6.3×10^4	1.7×10^6

YOUR NOTES

Worked Example

Determine the energy needed to melt 200 g of ice at 0°C.

- The specific latent heat of fusion of water is 3.3×10^5 J kg^{-1}
- The specific latent heat of vaporisation of water is 2.3×10^6 J kg^{-1}

Answer:

Step 1: Determine whether to use latent heat of fusion or vaporisation

- We need to use the specific latent heat of **fusion** because the phase change occurring is from **solid** to **liquid**

Step 2: List the known quantities

- Mass of the ice, $m = 200$ g $= 0.2$ kg
- Specific latent heat of fusion of water, $L_f = 3.3 \times 10^5$ J kg^{-1}

Step 3: Write down the equation for the latent heat of fusion

$$Q = mL_f$$

Step 4: Substitute numbers into the equation

$$Q = 0.2 \times (3.3 \times 10^5)$$

$$Q = 66\,000 = 66 \text{ kJ}$$

YOUR NOTES

Worked Example

Energy is supplied to a heater at a rate of 2500 W.

Determine the time taken, in minutes, to boil 500 ml of water at 100°C. Ignore energy losses.

- The specific latent heat of fusion of water is 3.3×10^5 J kg^{-1}
- The specific latent heat of vaporisation of water is 2.3×10^6 J kg^{-1}

Answer:

Step 1: Determine whether to use latent heat of fusion or vaporisation

- We need to use the specific latent heat of **vaporisation** because the phase change occurring is from **liquid** to **gas**

Step 2: Write down the known quantities

- Power, $P = 2500$ W
- Mass, $m = 500$ ml $= 0.5$ kg (since1 litre = 1 kg)
- Specific latent heat of vaporisation of water, $L_V = 2.3 \times 10^6$ J kg^{-1}

Step 3: Recall the equations for power and latent heat of

- Power: $P = \frac{E}{t} \quad \Rightarrow \quad E = Pt$
- Thermal energy: $Q = mL_v$

Step 4: Equate the two expressions for energy

$$Pt = mL_v$$

Step 5: Rearrange for the time *t* and substitute in the values

$$t = \frac{mL_v}{P}$$

$$t = \frac{0.5 \times (2.3 \times 10^6)}{2500}$$

$t = 460$ s $= 7.7$ min

Heating & Cooling Curves

YOUR NOTES

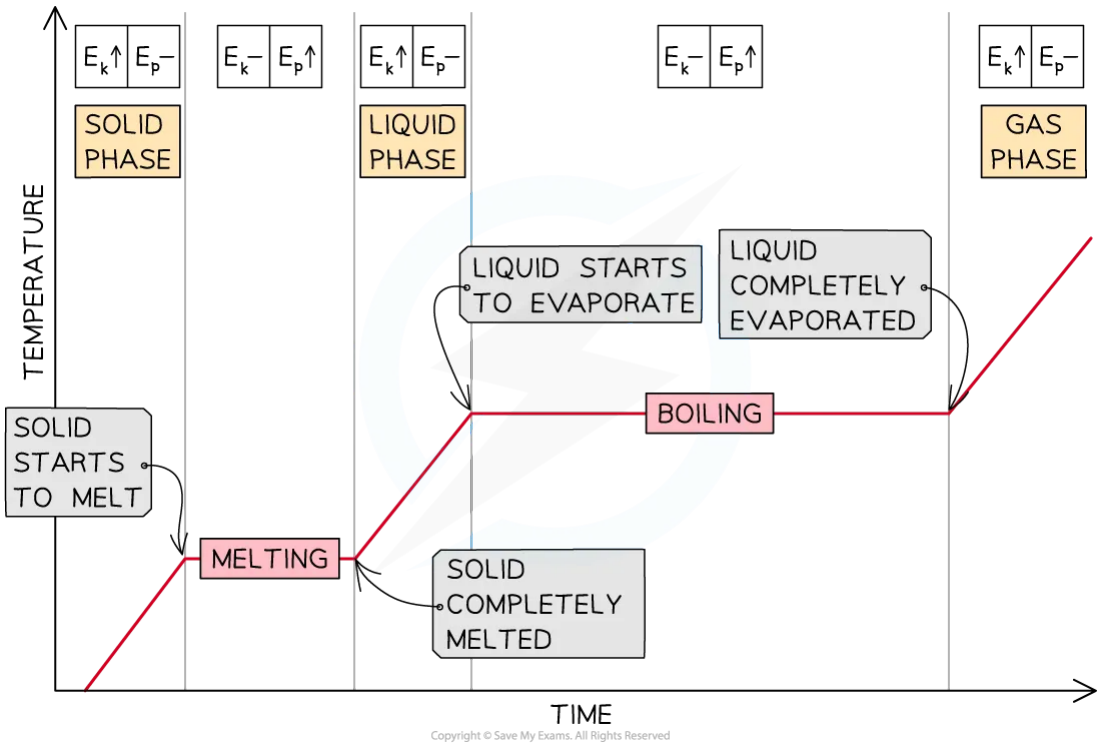
- A **heating** or **cooling curve** shows how the temperature of a substance changes with time
- The two main features of these curves are
  - The flat sections
  - The non-flat sections
- The flat sections show...
  - No **change in temperature** over time
  - The substance is undergoing a **phase change**
  - The thermal energy supplied to or removed from the substance only affects the **potential energy of the particles**
- The non-flat sections show...
  - **Changes in temperature** over time
  - The substance is **heating up** or **cooling down**
  - That the thermal energy supplied to or removed from the substance changes the **average kinetic energy of the particles**, hence resulting in an overall change in the temperature of the substance

Heating Curves

- When energy is supplied to a **solid**, its temperature will increase until it reaches its **melting point**
  - Once at the melting point, the temperature remains **constant** until the substance has **melted** completely
- If energy continues to be supplied, the temperature of the **liquid** will increase until it reaches its **boiling point**
  - Once at the boiling point, the temperature remains **constant** until the substance has **vaporised** completely

Heating Curve of a Substance

YOUR NOTES



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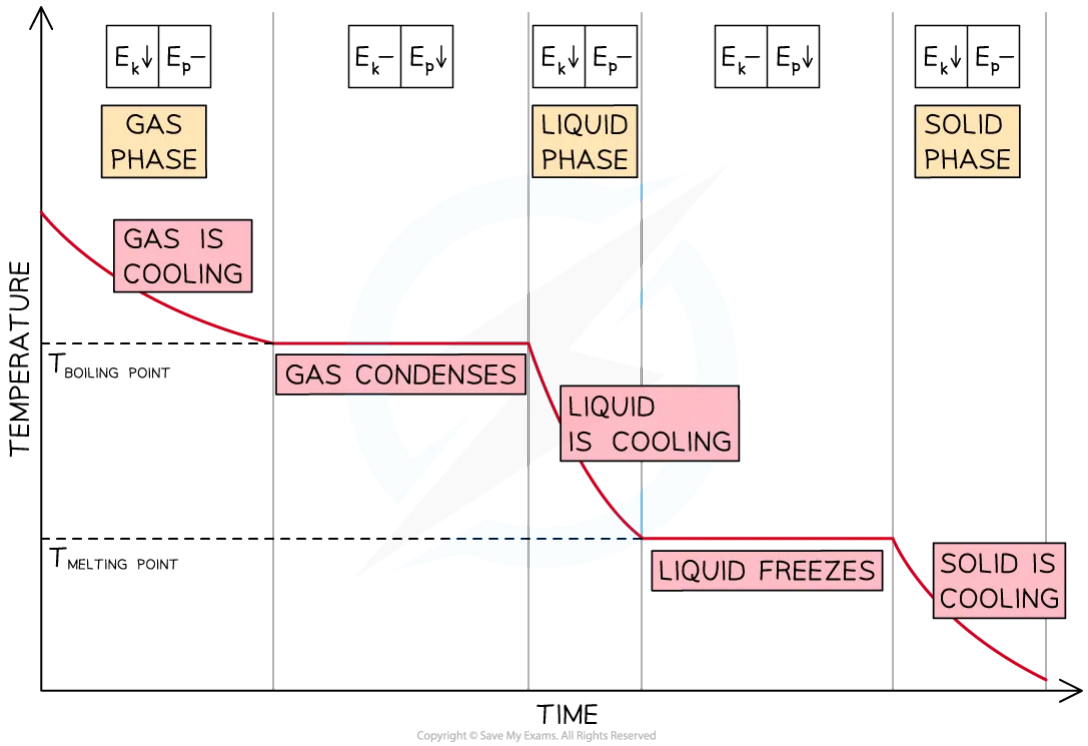
Cooling Curves

- When energy is removed from a **gas**, its temperature will decrease until it reaches its **boiling point**
  - Once at the boiling point, the temperature remains **constant** until the substance has **condensed** completely
- If energy continues to be removed, the temperature of the **liquid** will decrease until it reaches its **freezing point**
  - Once at the freezing point, the temperature remains **constant** until the substance has **frozen** completely

Cooling Curve of a Substance

YOUR NOTES



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- Heating or cooling curves can also display how the temperature of a substance changes with energy
  - In the following worked example, **energy** (in J) is plotted on the **x-axis** instead of time

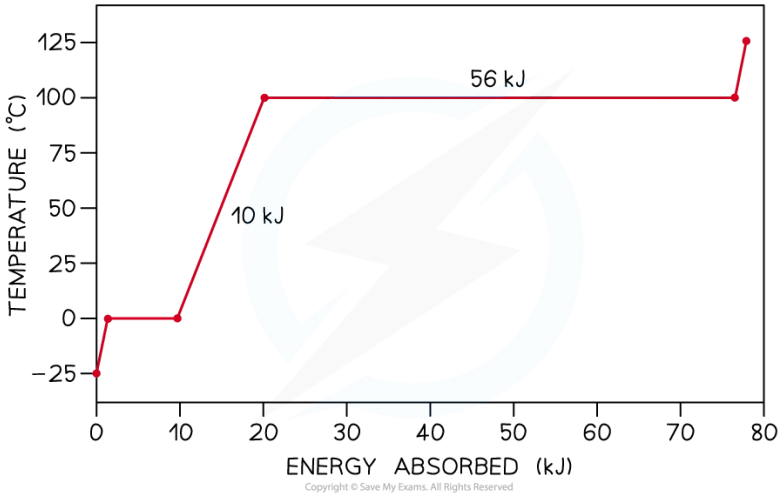
YOUR NOTES

Worked Example

The graph below is the heating curve for a 24 g cube of ice being heated at a constant rate.

Using the graph, calculate

(a)
The specific heat capacity of water in its liquid phase
(b)
The specific latent heat of vaporisation of water



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Answer:

Write down the known quantities:

- Mass of the ice, m = 24 g = 0.024 kg
- Melting point of water = 0°C
- Boiling point of water = 100°C
- Change in temperature of liquid water, ΔT = 100°C

(a)

Step 1: Identify the liquid phase on the graph and read off the amount of energy absorbed

- The non-flat region between 0°C and 100°C represents water in its liquid phase being heated up
- Energy absorbed by liquid water = 10 kJ = 10 000 J

Step 2: Write down the equation for specific heat capacity and rearrange for c

$$Q = mc\Delta T \quad \Rightarrow \quad c = \frac{Q}{m\Delta T}$$

YOUR NOTES

Step 3: Substitute in the values to calculate the specific heat capacity of water c

$$c = \frac{10\,000}{0.024 \times 100}$$

$c = 4167 = 4200\ J\,kg^{-1}\,°C^{-1}$

(b)

Step 1: Identify the phase change of vaporisation on the graph and read off the amount of energy absorbed

- The second flat section at 100°C indicates the phase change from water into steam (vaporisation)
- Energy absorbed during vaporisation = 56 kJ = 56 000 J

Step 2: Write down the equation for specific latent heat of vaporisation and rearrange for L_v

$$Q = mL_v \quad \Rightarrow \quad L_v = \frac{Q}{m}$$

Step 3: Substitute in the values to calculate specific latent heat of vaporisation of water L_v

$$L_v = \frac{56\,000}{0.024}$$

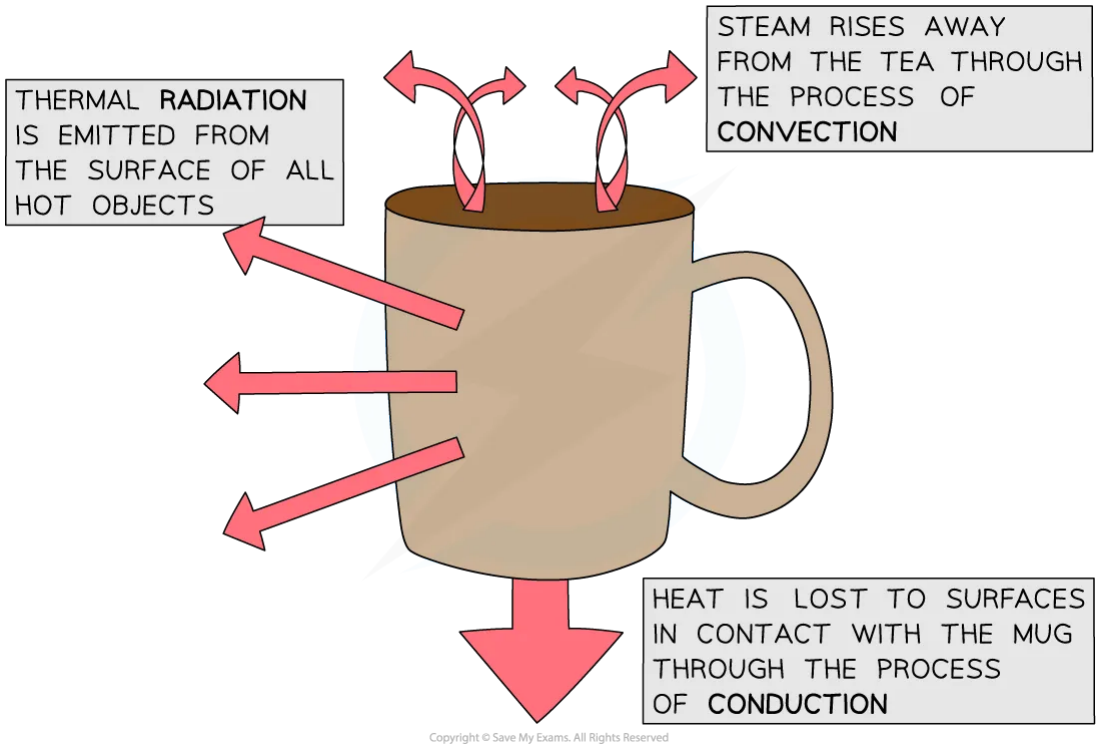
$L_v = 2.3 \times 10^6\ J\,kg^{-1}$

YOUR NOTES

Thermal Conduction

- Thermal energy can be transferred from a hotter area to a cooler area through one of the following mechanisms:
  - Conduction
  - Convection
  - Radiation

Thermal conduction, convection and radiation in a mug of tea



- Objects will **always** lose heat until they are in thermal equilibrium with their surroundings
  - For example, a mug of hot tea will cool down until it reaches room temperature

Conduction

- Conduction is the main method of thermal energy transfer in **solids**
- Conduction occurs when:

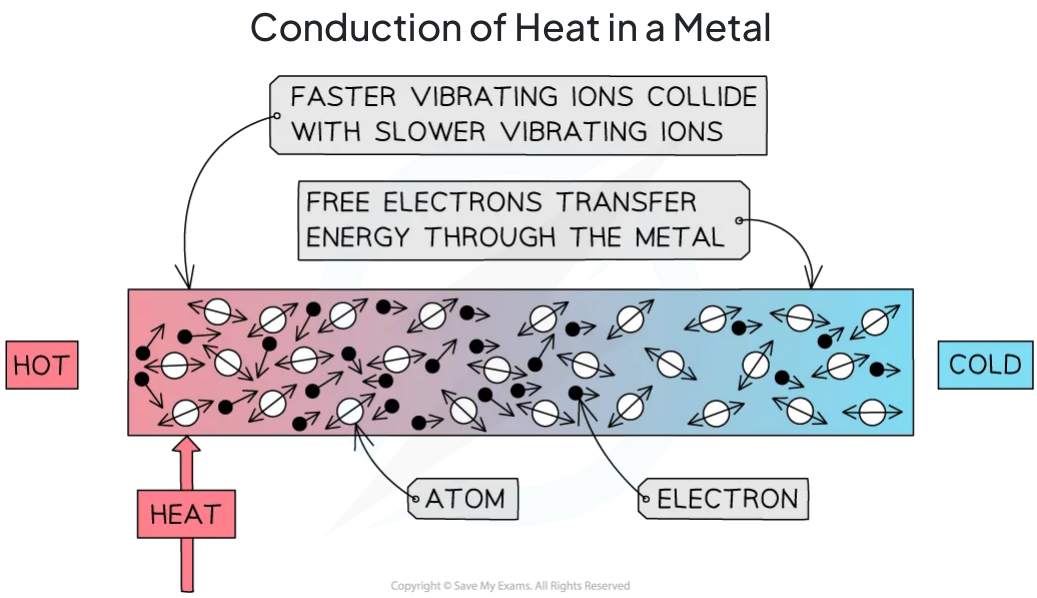
  Two solids of different temperatures come in contact with one another, thermal energy is transferred from the hotter object to the cooler object

- Metals are the best thermal **conductors**
  - This is because they have a high number of **free electrons**
- Non-metals, such as plastic or glass, are poor at conducting heat
  - Poor conductors of heat tend to also be poor conductors of electricity

YOUR NOTES

- This suggests a link between the mechanisms behind both types of conduction

- Liquids and gases are even poorer thermal conductors
  - This is because the atoms are **further apart**



During conduction, the atoms in a solid vibrate and collide with each other

- Conduction can occur through two mechanisms:
  - Atomic vibrations
  - Free electron collisions

- When a substance is heated, the atoms, or ions, start to move around, or **vibrate**, more
  - The atoms at the hotter end of the solid will vibrate more than the atoms at the cooler end
  - As they do so, they **bump into each other**, transferring energy from atom to atom
  - These collisions transfer internal energy until **thermal equilibrium** is achieved throughout the substance
  - This occurs in **all solids**, metals and non-metals alike

- Metals are especially good at conducting heat due to their high number of **delocalised electrons**
  - These can collide with the atoms, increasing the rate of transfer of **vibrations** through the material
  - This allows metals to achieve thermal equilibrium **faster** than non-metals

Worked Example

Determine which of the following metals is likely to be the best thermal conductor, and which is likely to be the worst.

Metal	Density / g cm^{-3}	Relative atomic mass
Copper	8.96	63.55
Steel	7.85	55.85
Aluminium	2.71	26.98

Assume that each metal contributes one free electron per atom.

Answer:

Step 1: Use dimensional analysis to determine the equation for the number of free electrons

- Units for number of free electrons per cubic centimetre, $[n] = cm^{-3}$
- Units for density, $[\rho] = g\ cm^{-3}$
- Units for Avogadro's number, $[N_A] = mol^{-1}$
- Units for relative atomic mass, $[A] = g\ mol^{-1}$

$$[n]^a = [\rho]^b\,[N_A]^c\,[A]^d$$

$$(cm^{-3})^a = (g\ cm^{-3})^b\,(mol^{-1})^c\,(g\ mol^{-1})^d$$

- The only unit present on both sides is cm^{-3}, therefore:

$$a = b = 1$$

- No other units are present on both sides, so:

$$c + d = 0$$

$$b + d = 0$$

$$\therefore d = -1, c = 1$$

Step 2: Write out the equation for the number of free electrons per cubic centimetre

$$[n]^1 = [\rho]^1\,[N_A]^1\,[A]^{-1}$$

$$n = \frac{\rho N_A}{A}$$

Step 3: Calculate the number of free electrons in each metal

- Avogadro constant, $N_A = 6.02 \times 10^{23}\ mol^{-1}$ (this is given in the data booklet)

YOUR NOTES

Copper: $n = \frac{8.96 \times (6.02 \times 10^{23})}{63.55} = 8.49 \times 10^{22}\ \text{cm}^{-3}$

Steel: $n = \frac{7.85 \times (6.02 \times 10^{23})}{55.85} = 8.46 \times 10^{22}\ \text{cm}^{-3}$

Aluminium: $n = \frac{2.71 \times (6.02 \times 10^{23})}{26.98} = 6.05 \times 10^{22}\ \text{cm}^{-3}$

Step 4: Rank the metals from best thermal conductor to worst

- Best thermal conductor = **copper** (highest number of free electrons)
- Worst thermal conductor = **aluminium** (lowest number of free electrons)

Exam Tip

If a question mentions thermal energy transfers and **metals**, the answer will likely be about conduction!

YOUR NOTES

Thermal Conductivity

- The conductivity of a material can be quantified by its **thermal conductivity**
- Thermal conductivity is defined as

> **The ability of a substance to transfer heat via conduction**

- It is denoted by the symbol *k* and has units of $W\,m^{-1}\,K^{-1}$
- The thermal conductivities of some common materials are shown in the table below

Substance	Thermal conductivity / $W\,m^{-1}\,K^{-1}$
air	0.024
rubber	0.13
water	0.6
ice	1.6
iron	80
copper	400
silver	429
diamond	1600

- Excellent thermal **conductors**...
  - Have **high** values of thermal conductivity
  - Transfer thermal energy at a **fast** rate
  - (Usually) contain a **large** number of delocalised electrons (diamond being the obvious exception)
- Poor thermal conductors (**insulators**)...
  - Have **low** values of thermal conductivity
  - Transfer thermal energy at a **slow** rate
  - Contain **few** delocalised electrons

Temperature Gradient Formula

- When there is a temperature **difference** between two points, thermal energy will flow from the region of **higher** temperature to the region of **lower** temperature
  - This is known as a **temperature gradient**
- The **rate** of the **heat transfer** via conduction is given by

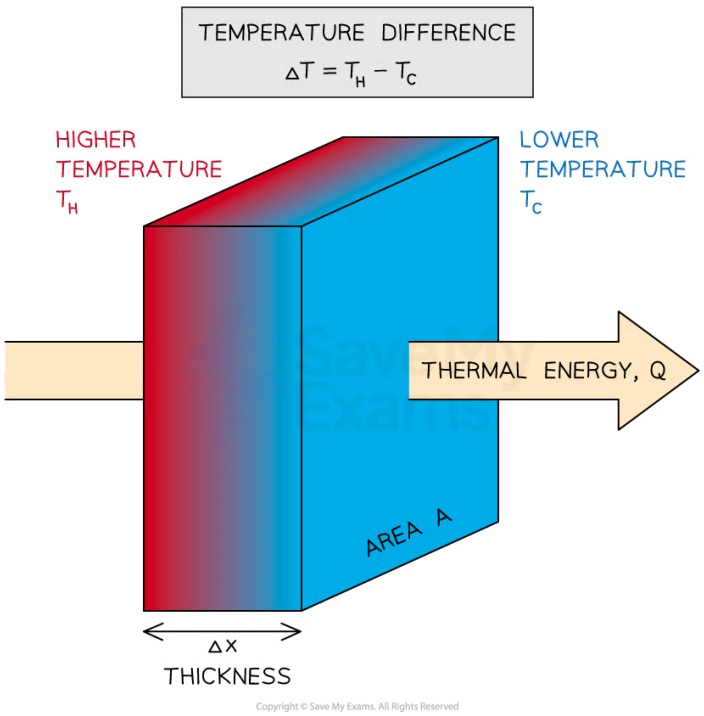
$$\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{\Delta x}$$

- Where

YOUR NOTES

- $\frac{\Delta Q}{\Delta t}$ = flow of thermal energy per second (W)
- k = thermal conductivity of the material (W m^{-1} K^{-1})
- A = cross-sectional area (m^2)
- ΔT = temperature difference (K or °C)
- Δx = thickness of the material (m)

Conduction of thermal energy through a solid

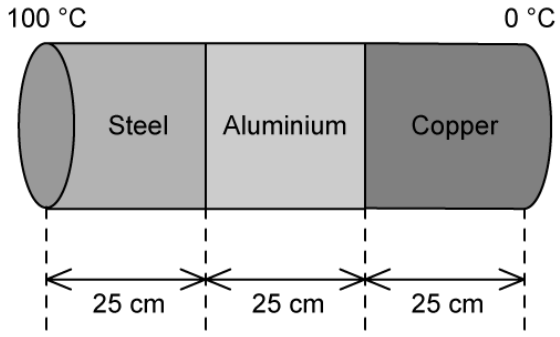


Thermal energy flows down a temperature gradient. The rate of energy transfer depends on the properties of the material

YOUR NOTES

Worked Example

A composite rod is made of three rods; steel, aluminium and copper. Each rod has the same length and cross-section, as shown in the diagram.



The steel end is held at 100°C and the copper end is held at 0°C.

Determine the temperatures at the steel-aluminium junction and the aluminium-copper junction.

Assume that the rods are perfectly insulated from the surroundings.

- Thermal conductivity of steel = 60 W m^{-1} K^{-1}
- Thermal conductivity of aluminium = 240 W m^{-1} K^{-1}
- Thermal conductivity of copper = 400 W m^{-1} K^{-1}

Answer:

Step 1: Analyse the scenario and set up an equation

- As the rods have identical dimensions, the amount of heat flowing through each rod must be the same
- Therefore, rate of energy transfer in steel = rate of energy transfer in aluminium

$$\frac{\Delta Q_s}{\Delta t} = \frac{\Delta Q_a}{\Delta t} = \frac{\Delta Q_c}{\Delta t}$$

$$k_s A \frac{\Delta T_s}{\Delta x} = k_a A \frac{\Delta T_a}{\Delta x} = k_c A \frac{\Delta T_c}{\Delta x}$$

$$k_s \Delta T_s = k_a \Delta T_a = k_c \Delta T_c$$

Step 2: Form two simultaneous equations and substitute in the values of Δ*T* and *k*

- Temperature difference in steel: $\Delta T_s = (100 - T_1)$
- Temperature difference in aluminium: $\Delta T_a = (T_1 - T_2)$
- Temperature difference in copper: $\Delta T_c = (T_2 - 0)$

$$k_s(100 - T_1) = k_a(T_1 - T_2) \quad \Rightarrow \quad 60(100 - T_1) = 240(T_1 - T_2) \text{ eq. (1)}$$

$$k_a(T_1 - T_2) = k_c(T_2 - 0) \quad \Rightarrow \quad 240(T_1 - T_2) = 400(T_2 - 0) \text{ eq. (2)}$$

Step 3: Expand and simplify eq. (1)

$$6000 - 60T_1 = 240T_1 - 240T_2$$

$$300T_1 - 240T_2 = 6000$$

$$5T_1 - 4T_2 = 100$$

Step 4: Expand and simplify eq. (2)

$$240T_1 - 240T_2 = 400T_2$$

$$240T_1 = 640T_2$$

$$T_2 = \frac{3}{8}T_1$$

Step 5: Determine the temperature at the steel-aluminium junction T_1

$$5T_1 - 4\left(\frac{3}{8}T_1\right) = 100$$

$$\frac{7}{2}T_1 = 100$$

$$T_1 = \frac{200}{7} = 28.6 = 29\ °C$$

Step 6: Determine the temperature at the aluminium-copper junction T_2

$$T_2 = \frac{3}{8} \times \frac{200}{7} = 10.7 = 11\ °C$$

YOUR NOTES

YOUR NOTES

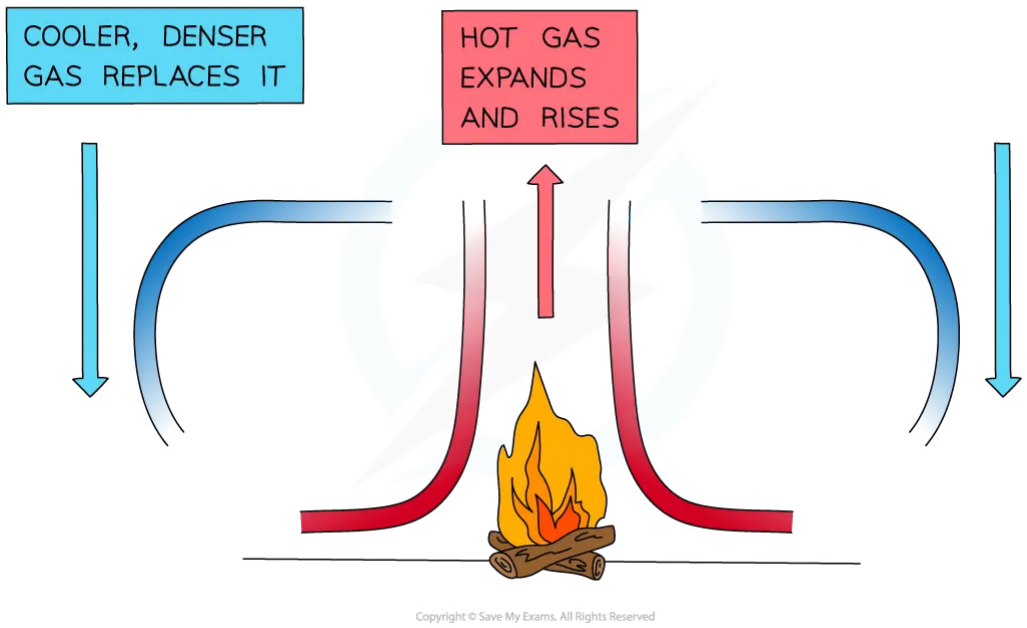
Thermal Convection

- Convection occurs when:

  A fluid is heated causing the movement of groups of atoms or molecules due to variations in density

- Convection is the main way that heat travels through **liquids** and **gases**
  - Convection **cannot occur in solids**
- When a fluid (a liquid or a gas) is heated from below:
  - The heated molecules gain kinetic energy and push each other apart, increasing the space between particles, thus making the fluid **expand**
  - This makes the hot part of the fluid **less dense** than the surrounding fluid
  - The **hot fluid rises**, and the cooler (surrounding) fluid moves in to take its place
  - Eventually, the hot fluid cools, contracts and sinks back down again
  - The resulting motion is called a **convection current**

How is thermal energy transferred during convection?



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The thermal energy from a fire forms a convection current in the air around it. The hot air rises and the cool air sinks.

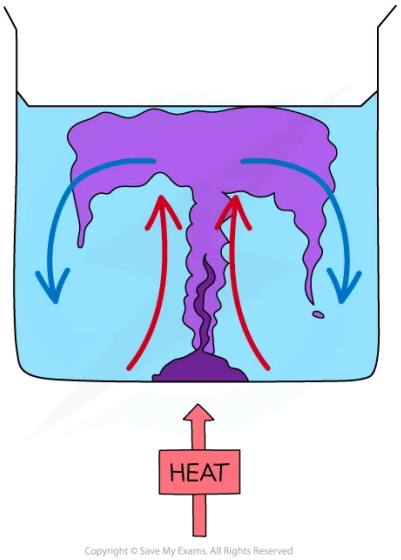
YOUR NOTES

Worked Example

Discuss **one** example, in nature or in the lab, in which convection takes place.

Answer:

Step 1: Draw a diagram to illustrate the convection currents



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Step 2: Describe the situation

- One method of observing a convection current is by heating a beaker of water containing potassium permanganate crystals
- Heat is initially transferred through the glass wall of the beaker by conduction
- The water in the region of the Bunsen flame is heated and expands, becomes **less dense** and **rises**
  - This causes the dissolved purple crystal to flow up with the water
- Meanwhile, when the water at the top of the beaker cools, it becomes **denser** again and falls
- The process continues which leads to a **convection current** where heat is transferred through the liquid
  - The dissolved purple crystal follows this current which is what is observed during this experiment

Other examples of convection:

- Atmospheric convection / winds / sea breezes
- Thunderheads (a cloud that appears before a thunderstorm)
- Convection currents in the Earth's mantle (which can lead to continental drift)
- Ocean currents
- Solar conventions / sunspots / flares

Exam Tip

If a question refers to thermal energy transfers in a fluid (a **liquid** or **gas**) that isn't trapped, then make sure your answer mentions that convection currents will likely form!

YOUR NOTES

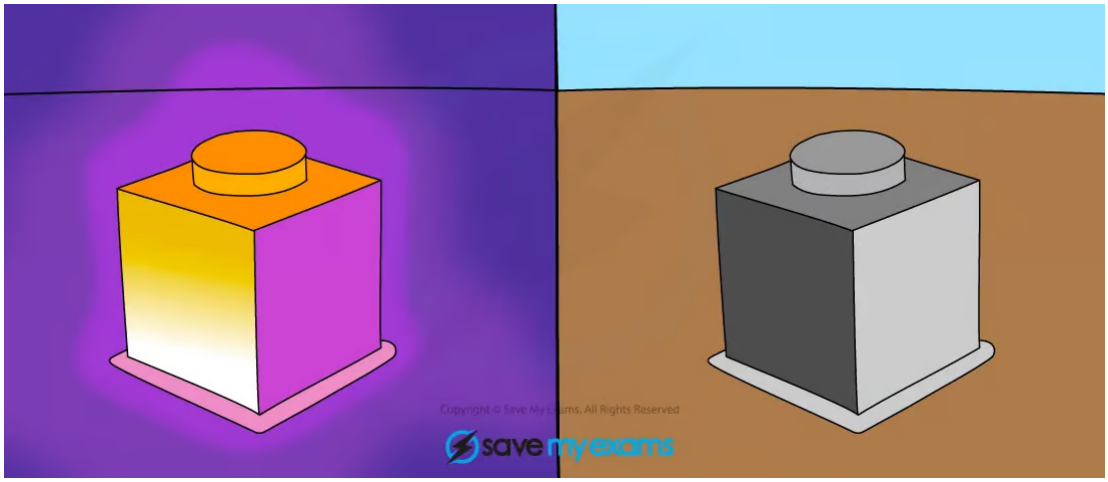
YOUR NOTES

Thermal Radiation

- All bodies (objects), no matter what temperature, emit a spectrum of **thermal radiation** in the form of electromagnetic waves
  - These electromagnetic waves usually lie in the **infrared** region of the spectrum
- Thermal radiation is defined as:

  Heat transfer by means of electromagnetic radiation normally in the infrared region
- The **hotter** the object, the **more** infrared radiation it radiates in a given time
  - This is because atoms and molecules above absolute zero are in constant motion
  - Electric charges within the atoms in a material vibrate causing **electromagnetic radiation** to be emitted
  - Therefore, the higher the **temperature**, the greater the **thermal motion** of the atoms and the greater the **rate** of emission of **radiation**
- Thermal radiation is the **only** method of thermal energy transfer that does not require **matter** in order to **move or propagate**
  - Therefore, thermal radiation is the only way heat can travel through a **vacuum**

Thermal radiation demonstration on a Leslie cube



An image of a hot object, known as a Leslie cube, taken in both Infrared and visible light. The black surface emits more thermal radiation (infrared) than the shiny surface

- The amount of thermal radiation emitted by an object depends on a number of factors:
  - The **surface colour** of the object (black = more radiation)
  - The **texture** of the surface (shiny surfaces = more radiation)
  - The **surface area** of the object (greater surface area = more area for radiation to be emitted from)

YOUR NOTES

Worked Example

A hot meteorite hits the surface of the Moon.

Identify and discuss the principal means by which the meteorite can dissipate thermal energy.

Answer:

Step 1: Identify the types of thermal energy transfer

- An object can lose energy through conduction, convection or radiation
- In this case, the hot meteorite will only be able to lose energy via **conduction** and **radiation**

Step 2: Explain these choices

- The meteorite can lose heat energy through conduction because it is in contact with the surface of the Moon
- The Moon does not have an atmosphere, so convection is **not** possible
- Infrared photons emitted by the meteorite are able to travel through a vacuum, so heat loss via radiation **is** possible

Exam Tip

If a question refers to the colour of something (**black, white or shiny**) then the answer will likely be about thermal radiation!

If a question involves a **vacuum** (empty space) remember that **radiation** is the **only** mechanism in which heat can be transferred, as conduction and convection require particles to transfer heat.

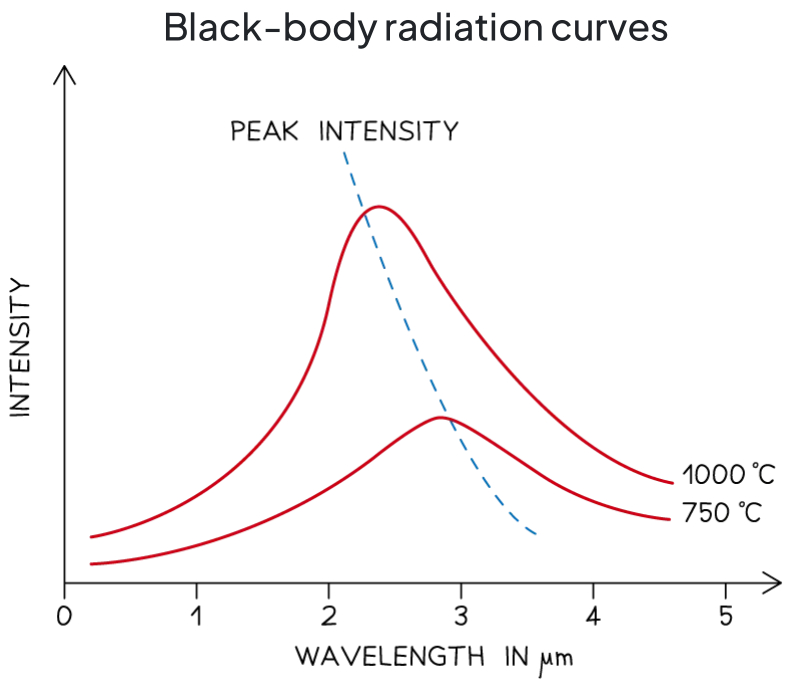
YOUR NOTES

Black-Body Radiation

- Black-body radiation is the name given to the **thermal radiation** emitted by all bodies
  - Black-body radiation can be emitted in the form of infrared light, but also visible light or other wavelengths, depending on the temperature
- A perfect black body is defined as:

  An object that absorbs all of the radiation incident on it and does not reflect or transmit any

- Since a **good absorber** is also a **good emitter**, a perfect black body would be the best possible emitter too
- As a result, an object which completely absorbs all radiation will be black
  - This is because the colour black is what is seen when **all** colours from the visible light spectrum are absorbed
- The **intensity** and **wavelength** distribution of any emitted waves depends on the **temperature** of the body
- This can be represented on a black-body radiation curve of **intensity** against **wavelength**
  - As the temperature increases, the peak of the curve moves
  - This moves to a **lower** wavelength and a **higher** intensity



Black body spectrum for objects of different temperatures

- From the electromagnetic spectrum, waves with a **smaller** wavelength have **higher** energy (e.g. UV rays, X-rays)
  - The **hotter** the object, the **greater** the amount of infrared radiation it radiates in a given time

- This increases the thermal energy emitted and therefore the **wavelength** of the emitted radiation **decreases**
    - At room temperature, objects emit thermal radiation in the infrared region of the spectrum
    - At around 1000°C, an object will emit a significant amount of **red visible light**
    - At around 6000°C, an object will mainly emit **white** or **blue visible light** (and some ultraviolet)
    - At even higher temperatures, objects will emit **ultraviolet** or even **X-rays**

YOUR NOTES

YOUR NOTES

Apparent Brightness & Luminosity

- The apparent brightness *b* of a star is defined as:

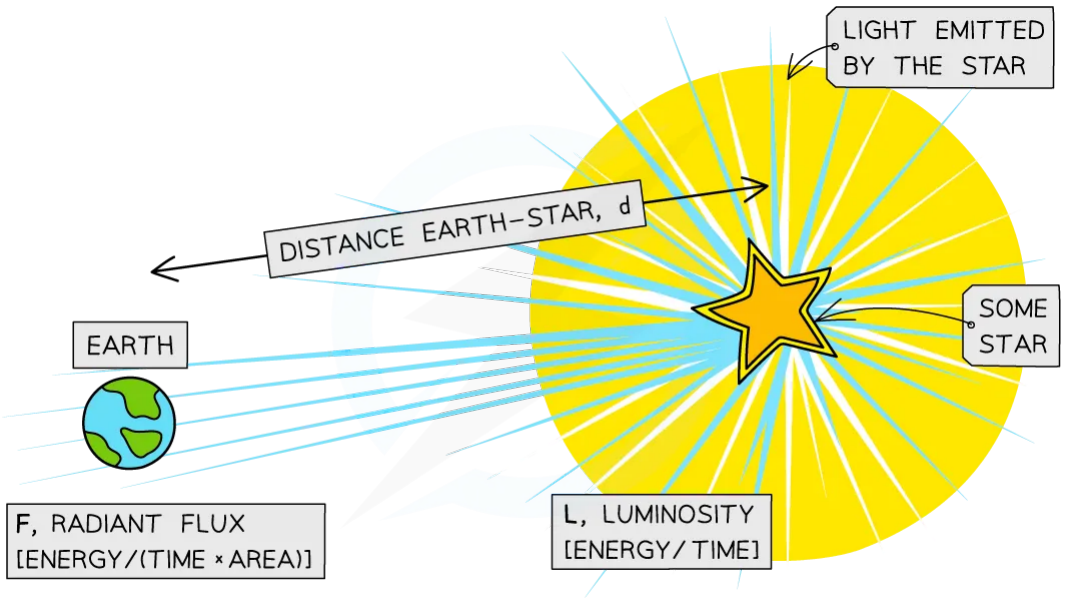
  The intensity of radiation received on Earth from a star

- Apparent brightness is measured in **watts per metre squared (W m^{-2})**
- The apparent brightness of a star depends on two main factors:
  - How much light the star emits
  - How far away the star is (more distant stars are usually fainter than nearby stars)
- How much light the star emits is given by the luminosity *L* of the star, which is defined as:

  The total power output of radiation emitted by a star

- Luminosity is measured in units of **watts (W)**

What is the difference between apparent brightness and luminosity?



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The luminosity is the total power output of the star, whereas the apparent brightness is what is measured on Earth

- Knowing the luminosity and apparent brightness of a star is useful because it allows us to determine how far away it is from the Earth
- This is because
  - Luminosity tells us how bright the star is at its **surface**
  - Apparent brightness tells us how bright the star is as observed from the **Earth**

- Therefore, by the time the radiation from the distant star reaches the Earth, it will have spread out over a **very large area**
    - This means the intensity of the radiation detected on Earth will only be a **fraction** of the value of the star's luminosity

YOUR NOTES

Inverse Square Law of Radiation

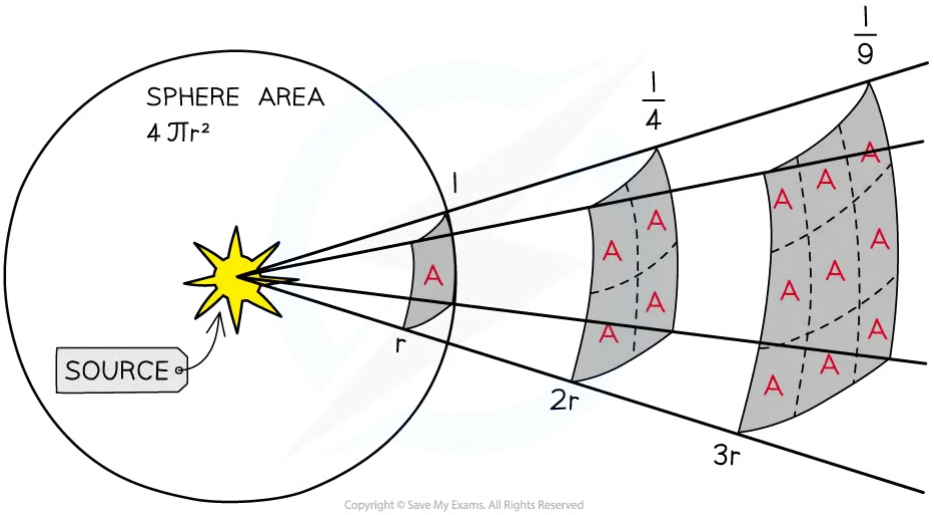
YOUR NOTES

- Light sources which are **farther** away appear **fainter** because the light it emits is **spread out** over a greater area
- The moment the light leaves the surface of the star, it begins to spread out uniformly through a **spherical** shell
  - The surface area of a sphere is equal to $\mathbf{4\pi r^2}$
- The radius r of this sphere is equal to the distance d between the star and the Earth
  - By the time the radiation reaches the Earth, it has been spread over an area of $\mathbf{4\pi d^2}$
- The **inverse square law of radiation** can be calculated using:

$$b = \frac{L}{4\pi d^2}$$

- Where:
  - b = apparent brightness, or observed intensity on Earth ($W\,m^{-2}$)
  - L = luminosity of the source (W)
  - d = distance between the star and the Earth (m)
- This equation assumes:
  - The power from the star radiates **uniformly** through space
  - No radiation is **absorbed** between the star and the Earth
- This equation tells us:
  - For a given star, the luminosity is **constant**
  - The intensity of the emitted light follows an inverse square law
  - For stars with the **same luminosity**, the star with the **greater** apparent brightness is **closer** to the Earth

Inverse square law of radiation



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When the light is twice as far away, it has spread over four times the area, hence the intensity is four times smaller

YOUR NOTES

Worked Example

A star has a known luminosity of 9.7×10^{27} W. Observations of the star show that the apparent brightness of light received on Earth from the star is 114 nW m^{-2}.

Determine the distance of the star from Earth.

Answer:

Step 1: Write down the known quantities

- Luminosity, $L = 9.7 \times 10^{27}$ W
- Apparent brightness, $b = 114$ nW $m^{-2} = 114 \times 10^{-9}$ W m^{-2}

Step 2: Write down the inverse square law of radiation and rearrange for distance d

$$b = \frac{L}{4\pi d^2}$$

$$d = \sqrt{\frac{L}{4\pi b}}$$

Step 3: Substitute in the values and calculate the distance d

$$d = \sqrt{\frac{9.7 \times 10^{27}}{4\pi \times (114 \times 10^{-9})}}$$

distance, $d = 8.2 \times 10^{16}$ m

Stefan-Boltzmann Law

- The total power *P* radiated by a perfect black body depends on **two** factors:
  - It's absolute temperature
  - It's surface area
- The relationship between these is known as **Stefan's Law** or the **Stefan-Boltzmann Law**, which states:

> **The total energy emitted by a black body per unit area per second is proportional to the fourth power of the absolute temperature of the body**

- The Stefan-Boltzmann Law can be calculated using:

$$P = \sigma A T^4$$

- Where:
  - *P* = total power emitted across all wavelengths (W)
  - σ = the Stefan-Boltzmann constant
  - *A* = surface area of the body (m)
  - *T* = absolute temperature of the body (K)
- The Stefan-Boltzmann law is often used to calculate the luminosity of celestial objects, such as stars
- The surface area of a star (or other spherical object) is equal to $\boldsymbol{A = 4\pi r^2}$
  - Where *r* = radius of the star
- The Stefan-Boltzmann equation then becomes:

$$L = 4\pi r^2 \sigma T^4$$

- Where:
  - *L* = luminosity of the star (W)
  - *r* = radius of the star (m)
  - σ = the Stefan-Boltzmann constant
  - *T* = surface temperature of the star (K)

YOUR NOTES

Worked Example

The surface temperature of Proxima Centauri, the nearest star to Earth, is 3000 K and its luminosity is 6.506×10^{23} W.

Calculate the radius of Proxima Centauri in solar radii and show your working clearly.

Solar radius $R_\odot = 6.96 \times 10^8$ m

Answer:

Step 1: List the known quantities:

- Surface temperature, $T = 3000$ K
- Luminosity, $L = 6.506 \times 10^{23}$ W
- Stefan's constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
- Radius of the Sun, $R_\odot = 6.96 \times 10^8$ m

Step 2: Write down the Stefan–Boltzmann equation and rearrange for radius r

$$L = 4\pi R^2 \sigma T^4$$

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

Step 3: Substitute the values into the equation

$$R = \sqrt{\frac{6.506 \times 10^{23}}{4\pi \times (5.67 \times 10^{-8}) \times 3000^4}}$$

Radius of Proxima Centauri: $R = 1.061 \times 10^8$ m

Step 4: Find the ratio of the radii of Proxima Centauri and the Sun

$$\frac{R}{R_\odot} = \frac{1.061 \times 10^8}{6.96 \times 10^8} = 0.152\ R_\odot$$

- Proxima Centauri has a radius which is about 0.152 times smaller than the Sun

YOUR NOTES

Wien's Displacement Law

- Wien's displacement law relates the observed wavelength of light from an object to its surface temperature, it states:

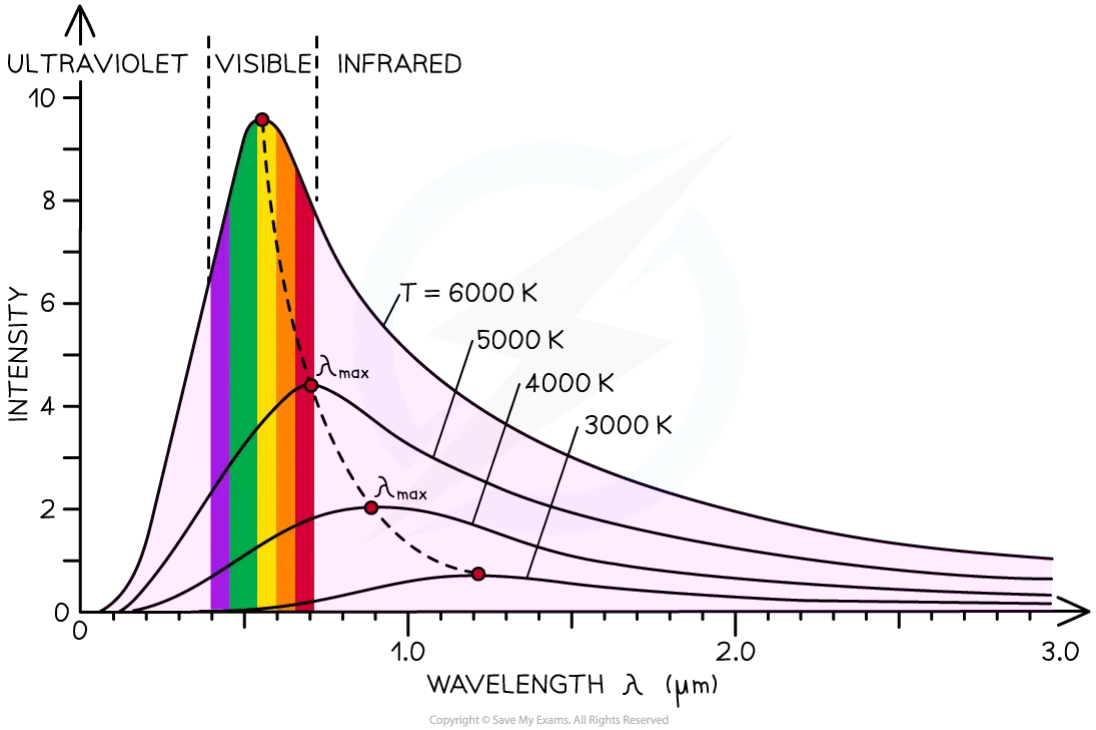
> **The black body radiation curve for different temperatures peaks at a wavelength that is inversely proportional to the temperature**

- This relation can be written as:

$$\lambda_{max} \propto \frac{1}{T}$$

- Where:
  - λ_{max} = the maximum wavelength emitted by an object at the peak intensity (m)
  - T = the surface temperature of an object (K)

Wien's Displacement Law Graph



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The intensity-wavelength graph shows how thermodynamic temperature links to the peak wavelength for four different bodies

- Wien's Law equation is given by:

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K}$$

- This equation shows that the higher the temperature of a body...

- The shorter the wavelength at the peak intensity, so **hotter** objects tend to be **white or blue,** and **cooler** objects tend to be **red or yellow**
- The greater the intensity of the radiation at each wavelength

YOUR NOTES

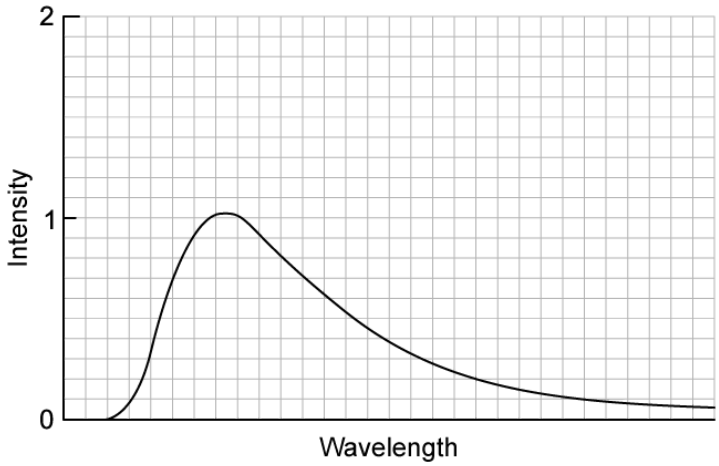
Table to compare surface temperature and star colour

Star Colour	Temperature / K
blue	>33 000
blue-white	10 000 – 30 000
white	7500 – 10 000
yellow-white	6000 – 7500
yellow	5000 – 6000
orange	3500 – 5000
red	<3500

YOUR NOTES

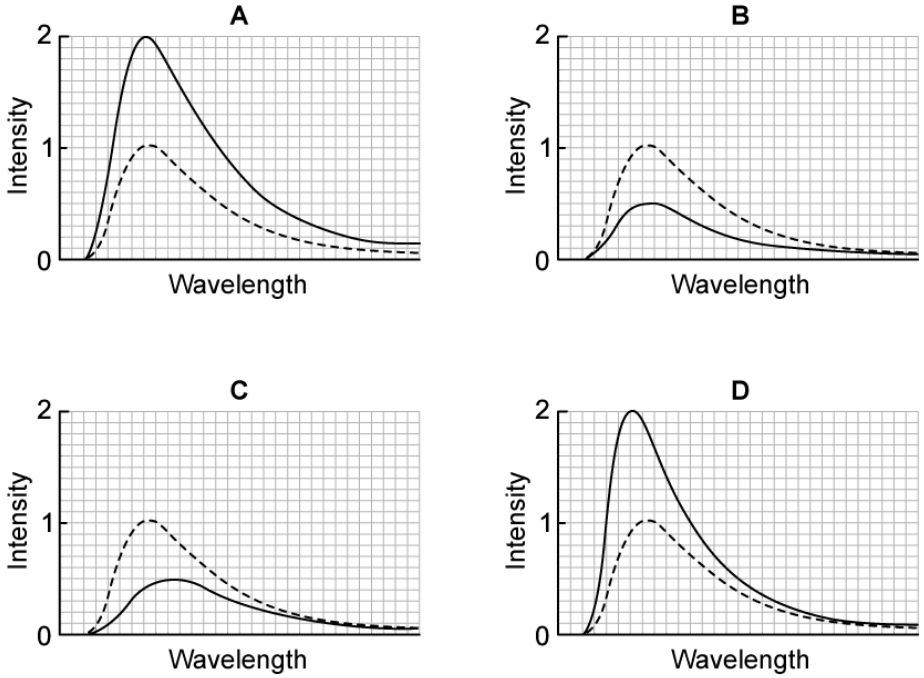
Worked Example

The black-body radiation curve of an object at 900 K is shown in the diagram below.



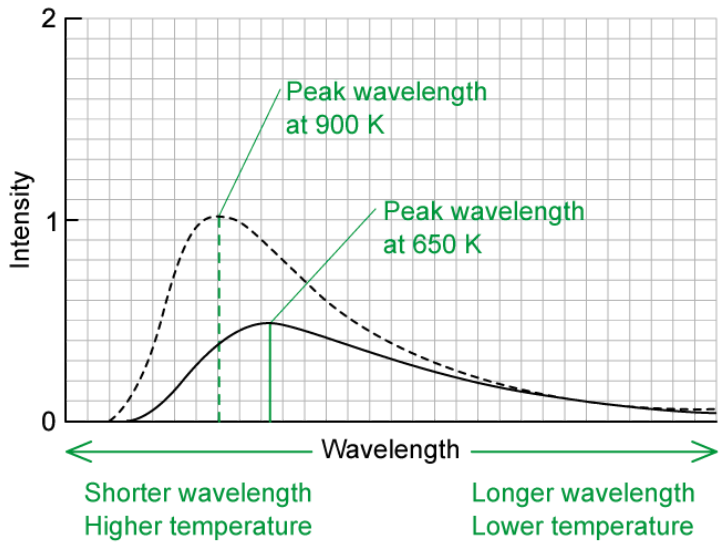
Which of the following shows the black-body radiation curve of an object at 650 K?

The dashed line represents the curve of the object at 900 K.



Answer: C

- From Wien's displacement law: $\lambda_{max} \propto \frac{1}{T}$
- Therefore, a curve with a **longer peak wavelength** will correspond to a **lower temperature**


2
1
0
Intensity
Peak wavelength
at 900 K
Peak wavelength
at 650 K
Wavelength
Shorter wavelength
Higher temperature
Longer wavelength
Lower temperature

YOUR NOTES

Worked Example

The spectrum of the star Rigel in the constellation of Orion peaks at a wavelength of 263 nm, while the spectrum of the star Betelgeuse peaks at a wavelength of 828 nm.

Determine which of these two stars, Betelgeuse or Rigel, is cooler.

Answer:

Step 1: List the known quantities

- Maximum emission wavelength of Rigel = 263 nm = 263×10^{-9} m
- Maximum emission wavelength of Betelgeuse = 828×10^{-9} m

Step 2: Write down Wien's displacement law and rearrange for temperature T

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K}$$

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}}$$

Step 3: Calculate the surface temperature of each star

$$\text{Rigel: } T = \frac{2.9 \times 10^{-3}}{263 \times 10^{-9}} = 11\,027 = 11\,000 \text{ K}$$

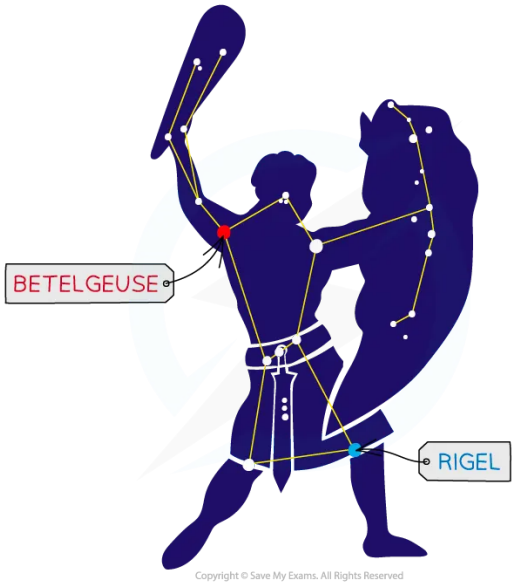
$$\text{Betelgeuse: } T = \frac{2.9 \times 10^{-3}}{828 \times 10^{-9}} = 3502 = 3500 \text{ K}$$

Step 4: Write a concluding sentence

- Betelgeuse has a surface temperature of 3500 K, therefore, it is much cooler than Rigel

Wien's law and stars in the Orion constellation

YOUR NOTES



The Orion Constellation; cooler stars, such as Betelgeuse, appear red or yellow, while hotter stars, such as Rigel, appear white or blue

Exam Tip

Note that the temperature used in Wien's Law is in **Kelvin** (K). Remember to convert from ^{o}C if the temperature is given in degrees in the question before using the Wien's Law equation.

IB Physics HL

YOUR NOTES

Greenhouse Effect

CONTENTS

Albedo & Emissivity
The Solar Constant
Greenhouse Gases
The Greenhouse Effect
Energy Balance Problems

Emissivity

- Stars are good approximations to a black body, whereas planets are not
  - This can be quantified using the emissivity
- Emissivity, e, is defined as

> **The power radiated by a surface divided by the power radiated from a black body of the same surface area and temperature**

- It can be calculated using the equation

$$e = \frac{\text{power radiated by an object}}{\text{power emitted by a black body}}$$

- Calculations of the emissivity assume that the black body:
  - Is at the same temperature as the object
  - Has the same dimensions as the object
- For a perfect black body, emissivity is equal to 1
- When using the Stefan-Boltzmann law for an object which is not a black body, the equation becomes:

$$P = e\sigma AT^4$$

- Where:
  - P = total power emitted by the object (W)
  - e = emissivity of the object
  - σ = the Stefan-Boltzmann constant
  - A = total surface area of the object black body (m^2)
  - T = absolute temperature of the body (K)

Exam Tip

You will be expected to remember that a perfect black body has an emissivity of 1 - this information is **not** included in the data booklet!

Albedo

YOUR NOTES

- Albedo, *a*, is defined as

  The proportion of light that is reflected by a given surface

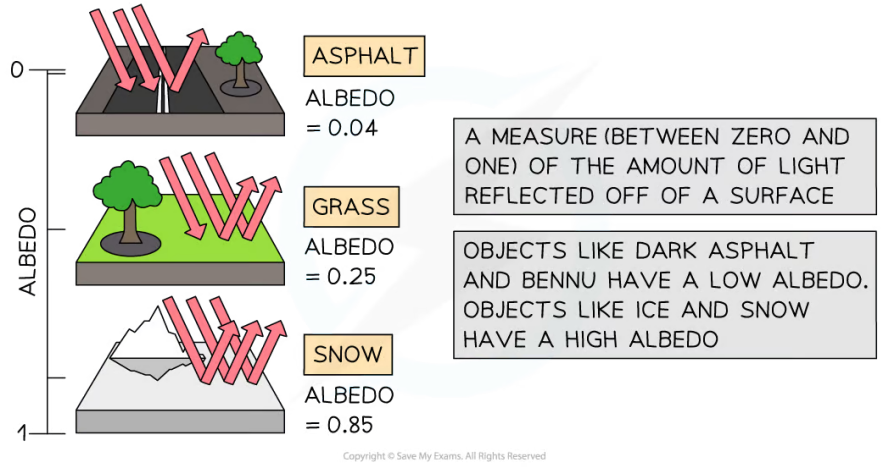
- It can be calculated using the equation

$$a = \frac{\text{total scattered power}}{\text{total incident power}}$$

- More specifically, the albedo of a **planet** is defined as

  The ratio between the total scattered, or reflected, radiation and the total incident radiation of that planet

- Earth's albedo is generally taken to be 0.3, which means 30% of the Sun's rays that reach the ground are reflected, or scattered, back into the atmosphere
- Earth's albedo varies daily and depends on:
  - **Cloud formations** and **season** – the thicker the cloud cover, the higher the degree of reflection
  - **Latitude**
  - **Terrain** – different materials reflect light to different degrees
- It is useful to know the albedo of common materials:
  - Fresh asphalt = 0.04
  - Bare soil = 0.17
  - Green grass = 0.25
  - Desert sand = 0.40
  - New concrete = 0.55
  - Ocean ice = 0.50 - 0.70
  - Fresh snow = 0.85
- Albedo has no units because it is a **ratio** (or fraction) of power



YOUR NOTES

Worked Example

The average albedo of fresh snow is 0.85

Calculate the ratio $\frac{\text{energy absorbed by fresh snow}}{\text{energy reflected by fresh snow}}$

Answer:

Step 1: Define albedo

- Albedo = the proportion of radiation that is reflected
- Therefore, the energy reflected by fresh snow = 0.85

Step 2: Identify the proportion of radiation that is absorbed

- If 85% of the radiation is reflected, we can assume that 15% is absorbed
- Therefore, the energy absorbed by fresh snow = 1 – 0.85 = 0.15

Step 3: Calculate the ratio

$$\frac{\text{energy absorbed by fresh snow}}{\text{energy reflected by fresh snow}} = \frac{0.15}{0.85} = 0.18$$

YOUR NOTES

The Solar Constant

- Since life on Earth is entirely dependent on the Sun's energy, it is useful to quantify how much of its energy reaches the top of the atmosphere
  - This is known as the solar constant, *S*

- The solar constant is defined as:

  The amount of solar radiation across all wavelengths that is incident in one second on one square metre at the mean distance of the Earth from the Sun

- The value of the solar constant varies year-round because:
  - The Earth is in an **elliptical orbit** around the Sun, meaning at certain times of year the Earth is closer to the Sun, and at other times of year it is further away
  - The **Sun's output varies** by about 0.1% during its 11-year sunspot cycle
- Calculations of the solar constant assume that:
  - This radiation is incident on a **plane perpendicular** to the Earth's surface
  - The Earth is at its **mean distance** from the Sun
- Different planets have different solar constants
  - Venus' solar constant is higher than Earth's because it is closer to the Sun

Incoming Radiative Power

- The surface area of a planet, with radius *r*, equals the surface area of a sphere, $4\pi r^2$
- A planet's radiative intensity covers a cross-sectional area of πr^2
- So the mean value of the radiative power or intensity is:

$$S \times \left(\frac{\pi r^2}{4 \pi r^2} \right) = \frac{S}{4}$$

YOUR NOTES

Worked Example

The Sun emits 4×10^{26} J in one second. The mean distance of the Earth from the Sun is 1.5×10^{11} m.

Using this data, calculate the solar constant.

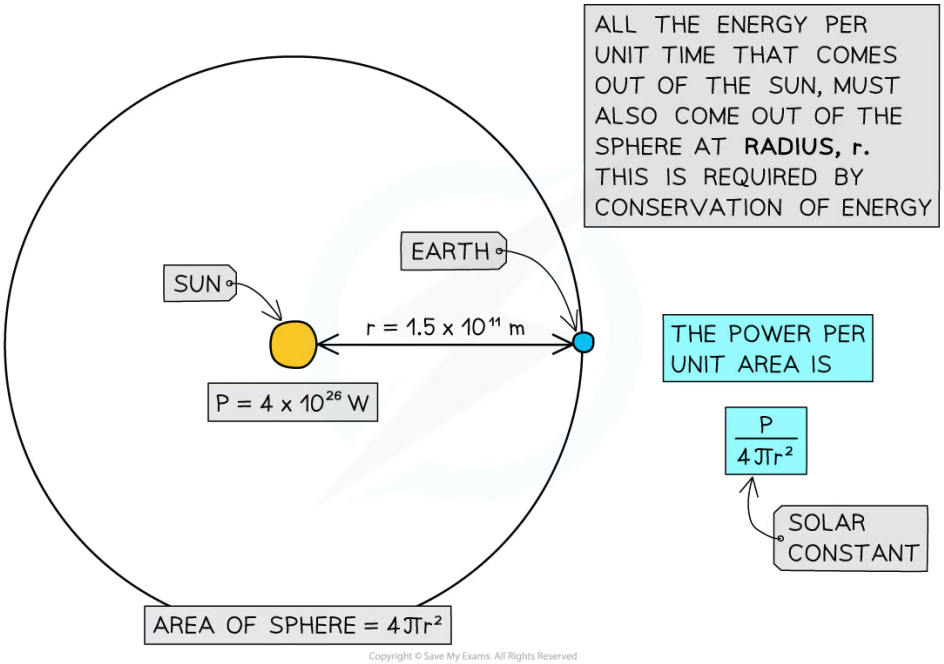
Answer:

Step 1: List the known quantities

- Power output of Sun, $P = 4 \times 10^{26}$ W
- Distance between the Earth and Sun, $r = 1.5 \times 10^{11}$ m

Step 2: Model the scenario using geometry

- As light leaves the surface of the Sun, it begins to spread out uniformly through a spherical shell
- The surface area of a sphere = $4\pi r^2$
- The radius r of this sphere is equal to the distance between the Sun and the Earth



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Step 3: Write an equation to calculate the solar constant

$$\text{Solar constant} = \frac{P}{4\pi r^2}$$

Step 4: Calculate the solar constant

$$\text{Solar constant} = \frac{4 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2} = 1415 \text{ W m}^{-2}$$

Solar constant = **1.4 kW m^{-2}** (2 s.f)

YOUR NOTES

The Main Greenhouse Gases

- The main greenhouse gases have both natural and human-generated origins
- In order of decreasing contributions, these are:
  - **Water vapour (H_2O)** - evaporation from the oceans / seas and plants
  - **Carbon dioxide (CO_2)** - volcanic eruptions, wildfires and respiration
  - **Methane (CH_4)** - emission from oceans and soils as part of decomposition, termites also emit methane
  - **Nitrous oxide (N_2O)** - soils and oceans
- When radiation from the Sun hits the Earth, it is radiated back from the Earth's surface as long-wave radiation
- A greenhouse gas is a gas that absorbs this re-radiated radiation, trapping it in the Earth's atmosphere so that it is not lost to space
  - Greenhouse gases in the atmosphere have a similar effect to the glass in a greenhouse, hence the term greenhouse gas
- There are many greenhouse gases, and those that contribute most to the greenhouse effect are:
  - Carbon dioxide (CO_2)
  - Water vapour (H_2O)
- These have the most significant impact on the greenhouse effect
- There are other greenhouse gases which have a lesser effect, such as:
  - Ozone (O_2 and O_3)
  - Methane
  - Nitrous oxides

Exam Tip

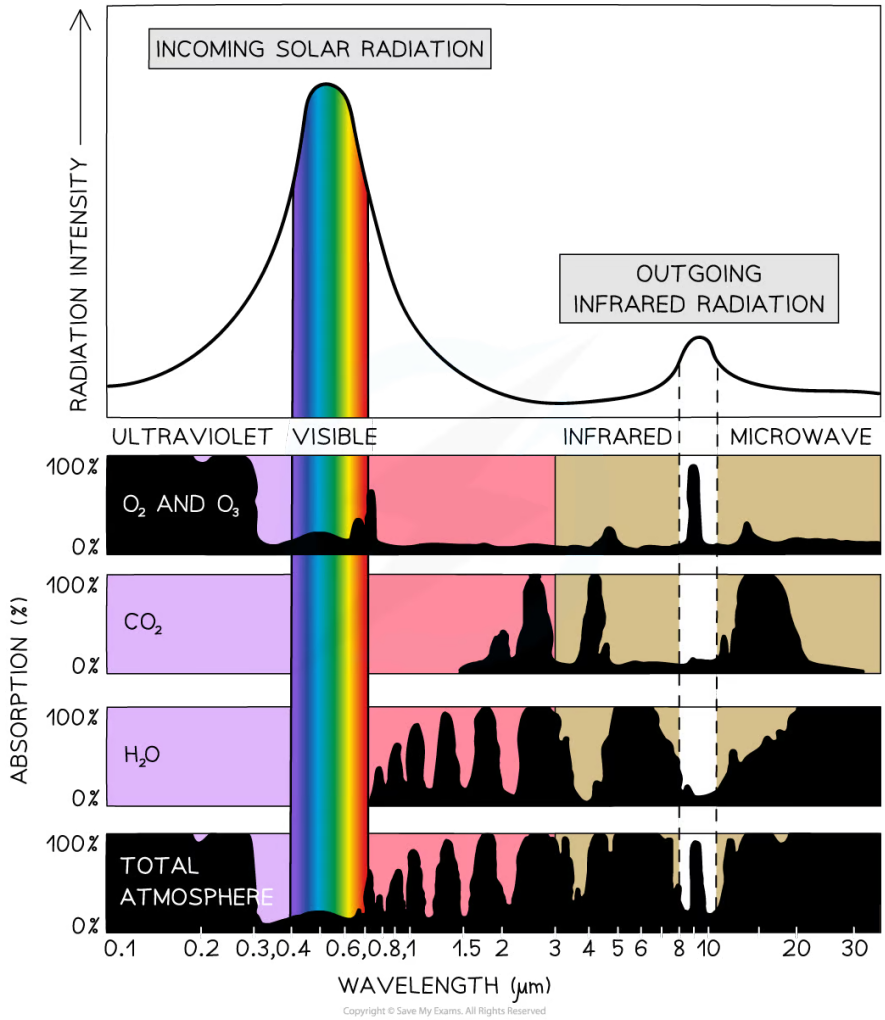
You may have heard of a separate environmental concern, described as the 'hole in the ozone layer'; this is **not** something that you need to know about. Ozone is an atmospheric gas that absorbs harmful UV radiation before it reaches earth, but any concerns about ozone depletion have nothing to do with the greenhouse effect. The problem of ozone depletion is one that has improved significantly due to measures taken to reduce certain types of emissions; humans can get it right sometimes!

You do not need to know the specific sources of each type of greenhouse gas – all you need to know is that each greenhouse gas has both natural and man-made origins

Greenhouse Gases & Infrared Radiation

YOUR NOTES

- While only around 25% of the (mostly short wavelength) solar radiation is absorbed by the **atmosphere** on its way to Earth, around 80% of the (long wavelength) re-emitted radiation from Earth is **absorbed** on its way back into the atmosphere
  - For example, incoming UV radiation is absorbed by **ozone**
  - Re-emitted **infrared** radiation is absorbed by the main greenhouse gases
- This absorbed radiation keeps Earth at a **habitable** temperature
  - However, if there is an imbalance in the chemical composition of the atmosphere, this can lead to fluctuations in the Earth's mean surface temperature
- The relative significance of a greenhouse gas depends on its concentration in the Earth's atmosphere and how much the gas can absorb specific wavelengths of radiation



The degree of absorption for a particular molecule varies depending on the wavelength of the radiation. The dark parts show the percentage of radiation that is absorbed by each type of greenhouse gas.

YOUR NOTES

Ozone (O_3)

- Ozone absorbs close to 100% of the Sun's incoming ultraviolet rays
- It also strongly absorbs the wavelengths of the outgoing infrared radiation leaving the Earth's atmosphere, between 9 μm and 10 μm
- However, it is not a significant contributor to the greenhouse effect as it is found in much smaller concentrations in the atmosphere

Carbon dioxide (CO_2)

- Carbon dioxide is a good absorber of infrared radiation with wavelengths between 1.5 - 30 μm
- In particular, it strongly absorbs radiation with a wavelength of 15 μm
- The increasing concentration of carbon dioxide in the atmosphere makes it one of the most significant contributors to the greenhouse effect

Water vapour (H_2O)

- Water vapour is the best absorber of infrared radiation with wavelengths between 0.8 - 35 μm
- The concentration of water vapour in the atmosphere increases as the air becomes warmer

Total atmosphere

- Overall, most of the ultraviolet, infrared and microwave radiation is absorbed by the atmosphere
- The atmosphere is mostly transparent to incoming visible radiation, which means that the gases in the atmosphere do not absorb or emit much visible radiation

YOUR NOTES

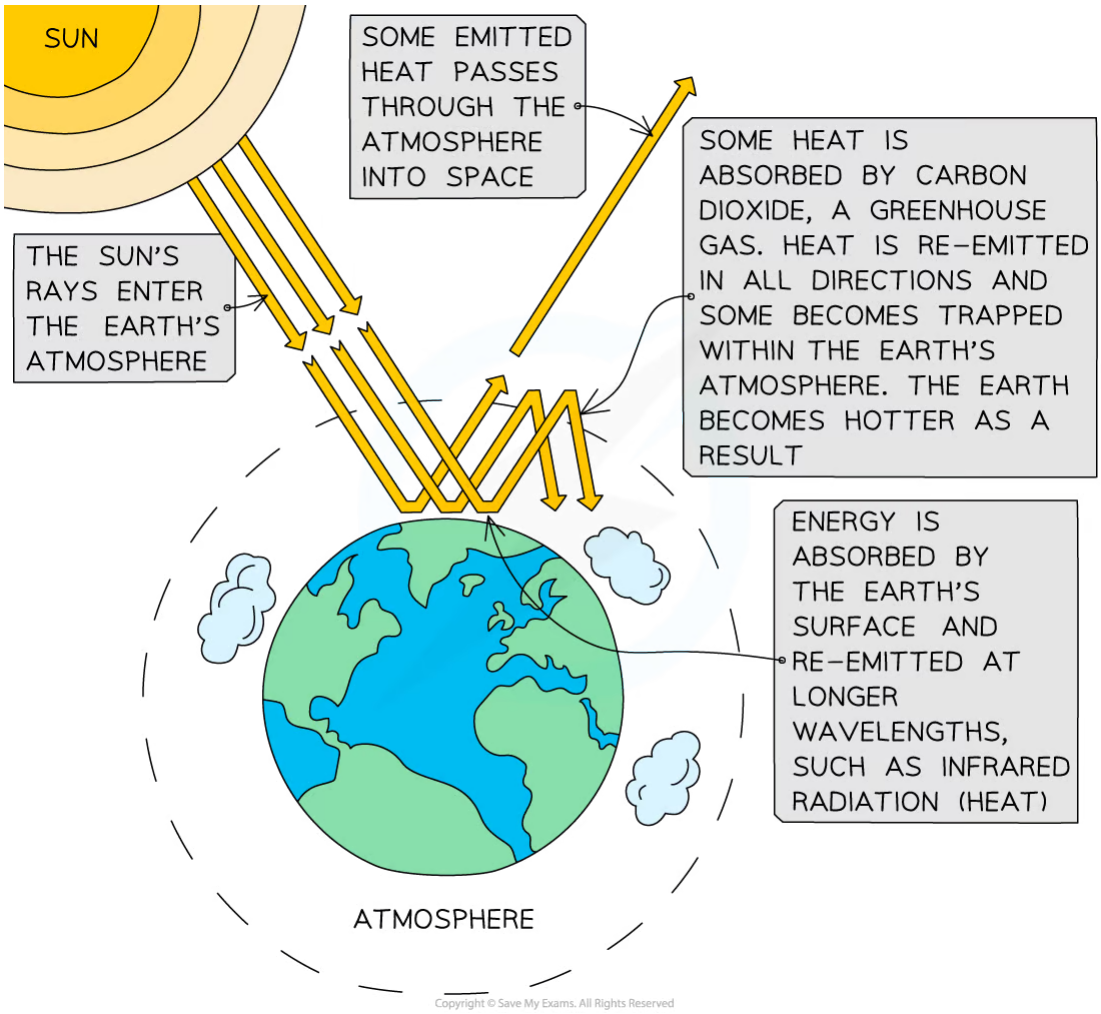
The Greenhouse Effect

- While only around 25% of the (primarily **short wavelength**) solar radiation is absorbed by the atmosphere on its way to Earth, around 80% of the (**long wavelength**) re-emitted radiation from Earth is absorbed on its way back into the atmosphere
  - For example, incoming UV radiation is absorbed by ozone
  - Re-emitted infrared radiation is absorbed by greenhouse gases
- This absorbed radiation keeps Earth at a habitable temperature
  - However, if there is an imbalance in the chemical composition of the atmosphere, this can lead to fluctuations in the Earth's mean surface temperature

Resonance Model of Global Warming

- Incoming radiation from the Sun predominantly takes the form of ultraviolet and visible radiation
- Visible light is not absorbed by the atmosphere, instead, it is absorbed by the Earth's surface
- At night, the **Earth re-radiates** this radiation as infrared
- Some of this radiation is **absorbed** by the **Earth's atmosphere** and some of the radiation is reflected back into space
- The greenhouse gases present in the atmosphere absorb infrared radiation and reflect it back towards the Earth's surface
  - The higher the concentration of greenhouse gases present, the more infrared radiation there is remaining in the Earth's surface-atmosphere system
- Therefore, heat energy becomes trapped inside Earth's atmosphere and accumulates
  - This leads to the greenhouse effect and an increase in average mean temperatures on Earth

YOUR NOTES

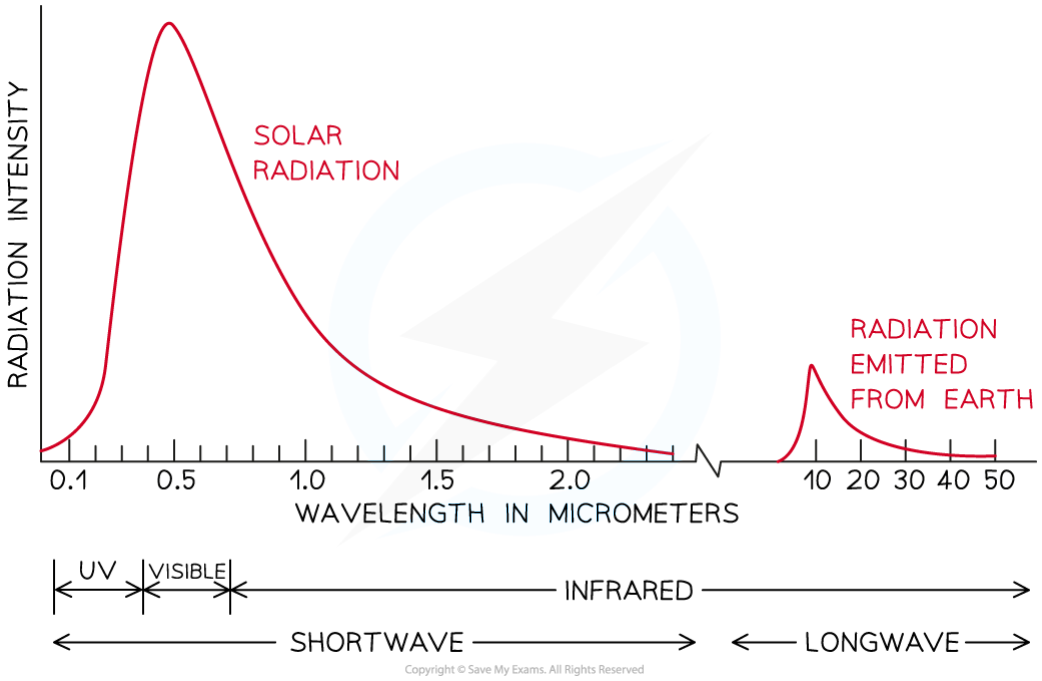


Greenhouse gases absorb the long-wave radiation emitted by Earth, warming the atmosphere

Molecular Energy Level Model

- The greenhouse effect occurs due to the particular molecular structure of greenhouse gases
  - High-frequency UV light is energetic and able to break bonds within molecules
  - Infrared light, on the other hand, causes atoms to vibrate
- The greenhouse gases have a **natural frequency** that falls in the infrared region
  - This means when they absorb infrared light, they begin to resonate, causing the molecules to heat up
  - They absorb the infrared radiation and subsequently emit it back towards the Earth's surface

YOUR NOTES



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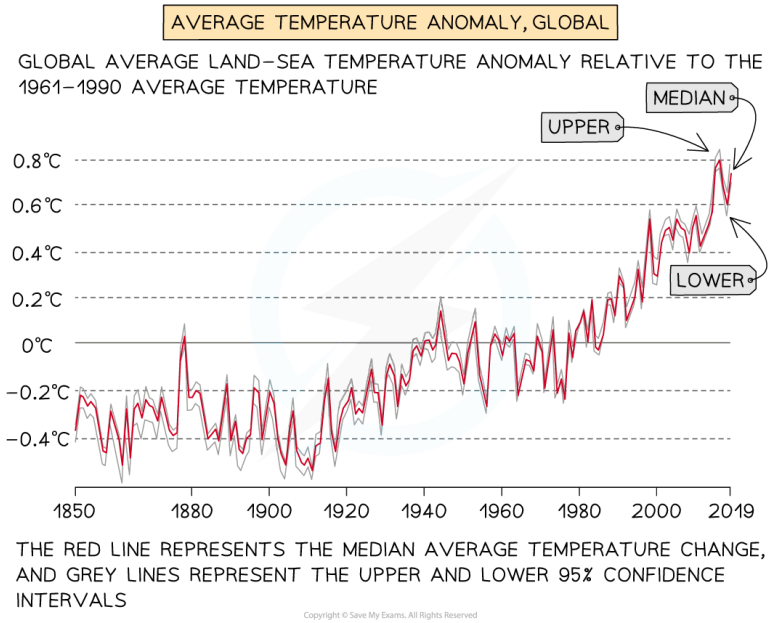
Solar radiation is primarily short-wave, while the radiation that is re-emitted by earth is long-wave radiation

The Enhanced Greenhouse Effect

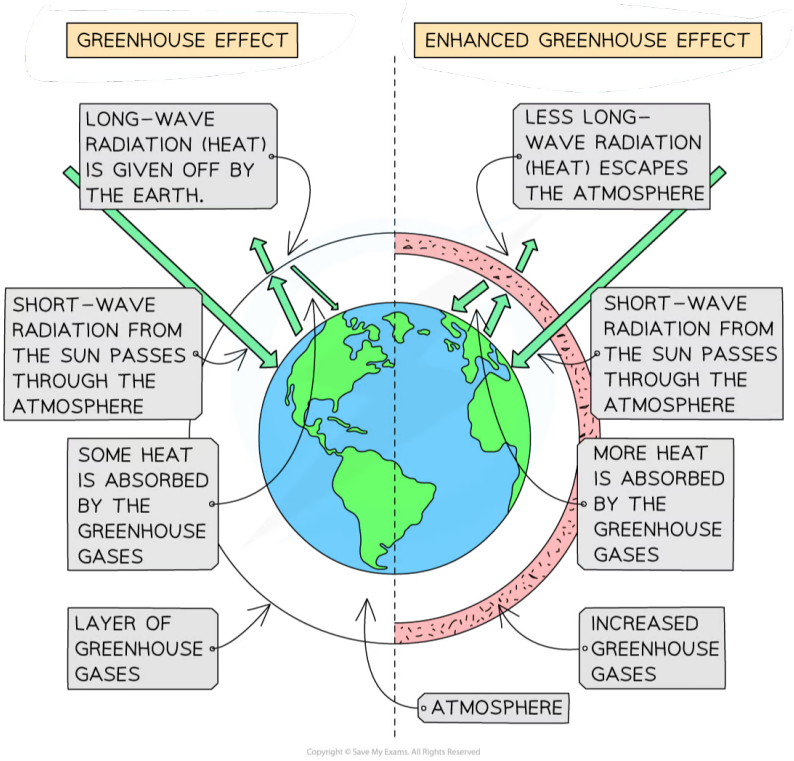
YOUR NOTES

- Human activity is increasing the number of greenhouse gases in the atmosphere:
  - **Carbon dioxide (CO_2)** levels in the atmosphere have increased by more than 100 parts per million (ppm) to 420ppm in 2020
- Increased amounts of greenhouse gases have led to the **enhanced greenhouse effect:**
  - Less long-wave radiation (heat) can escape the atmosphere
  - Average global temperatures have increased over 1°C since pre-industrial times



The average global temperature has risen remarkably in the past 100 years

YOUR NOTES



The greenhouse effect arises due to natural causes, but the enhanced greenhouse effect arises due to human activity

Human Sources of Greenhouse Gases

Greenhouse Gas	Sources from human activity
Carbon Dioxide (CO_2)	• Burning of fossil fuels - power stations, vehicles • Burning of wood • Deforestation - trees utilise CO_2 in photosynthesis. The fewer trees there are the less CO_2 is removed from the atmosphere
Methane (CH_4)	• Decay of organic matter - manure, waste in landfill, crops
Nitrous Oxide (N_2O)	• Artificial fertilisers • Burning fossil fuels

Worked Example

Which of the following is the result of the enhanced greenhouse effect?

A. increasing global average temperature due to natural causes

B. decreasing global average temperature due to human activity

C. increasing global average temperature due to human activity

D. decreasing global average temperature due to natural causes

Answer: C

- The enhanced greenhouse effect causes the average global temperature to **increase** and is the result of **human activity**

Exam Tip

Remember not all scientists agree about the causes of climate change. There are a few scientists who argue that global warming is the result of the Earth's natural climate pattern and not the result of human activities.

YOUR NOTES

YOUR NOTES

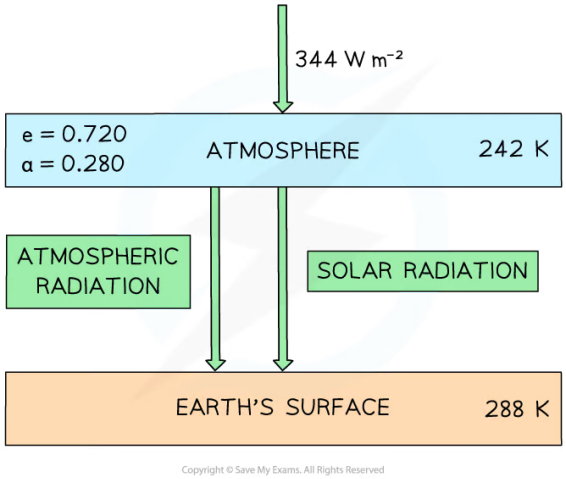
Energy Balance Problems

- It is useful to consider Earth's energy balance in terms of how much incoming energy from the Sun is used and how much is returned to space
- If incoming and outgoing energy are in balance, the Earth's temperature will remain constant
- This can be used to create models which can help climate scientists predict temperature fluctuations based on current and increased concentrations of greenhouse gases
    - At it's simplest, the model involves a one-layer atmosphere above the Earth's surface

YOUR NOTES

Worked Example

The diagram below shows a simple energy balance climate model in which the atmosphere and the Earth's surface are treated as two bodies.



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The Earth's surface receives both solar radiation and radiation emitted from the atmosphere.

At current atmospheric greenhouse gas concentrations, the temperature of Earth's atmosphere is set to increase by 6 K.

Data for this model:

- Current mean temperature of the Earth's atmosphere = 242 K
- Current mean temperature of the Earth's surface = 288 K
- Solar power per unit area at top of the atmosphere = 344 W m^{-2}
- Emissivity of the atmosphere, e = 0.720
- Albedo of the atmosphere, a = 0.280

Use this data to estimate the increase in temperature of the Earth's surface.

Answer:

Step 1: List the known quantities

- Solar power above atmosphere, P_a = 344 W m^{-2}
- Emissivity of the atmosphere, e = 0.720
- Emissivity of the surface, e = 1
- New temperature of Earth's atmosphere, T_a = 242 + 6 = 248 K
- Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
- Power absorbed at the Earth's surface = P_s
- New temperature of Earth's surface = T_s

Step 2: Calculate the solar power absorbed at the Earth's surface

YOUR NOTES

- This can be calculated using the emissivity and the solar power above the atmosphere

$$P_s = e \times P_a$$

$$P_s = 0.720 \times 344 = 247.68 = 248\ \text{W m}^{-2}$$

Step 3: Write the equation for the power emitted by a body

$$P = e\sigma T^4$$

Step 4: Calculate the new power radiated by the atmosphere

$$P = 0.720 \times (5.67 \times 10^{-8}) \times 248^4 = 154.43 = 154\ \text{W m}^{-2}$$

Step 5: Calculate the new power absorbed by the Earth's surface

- The power absorbed by the Earth's surface is a sum of the solar radiation that reaches the surface plus the power radiated by the atmosphere

$$\text{New power, } P_s = 248 + 154 = 402\ \text{W m}^{-2}$$

Step 6: Calculate the new temperature of the Earth's surface

- The Earth's surface can be assumed to be a black body, hence e = 1

$$P_s = \sigma T_s^4$$

$$400 = (5.67 \times 10^{-8}) \times T_s^4$$

$$T_s = \sqrt[4]{\frac{400}{5.67 \times 10^{-8}}} = 290\ \text{K}$$

Step 7: Determine the increase in temperature

$$\Delta T = 290 - 288 = 2\ \text{K}$$

Exam Tip

In simplified climate models, you can generally assume the Earth's surface and the atmosphere:

- Act as black bodies - this means the emissivity of the surface will be equal to 1!
- Remain at a constant temperature

IB Physics HL

YOUR NOTES

Gas Laws

CONTENTS

Gas Pressure
Amount of Substance
Gas Laws
Ideal Gas Equation
Kinetic Theory of Gases
Derivation of the Kinetic Theory of Gases Equation
Average Kinetic Energy of a Molecule

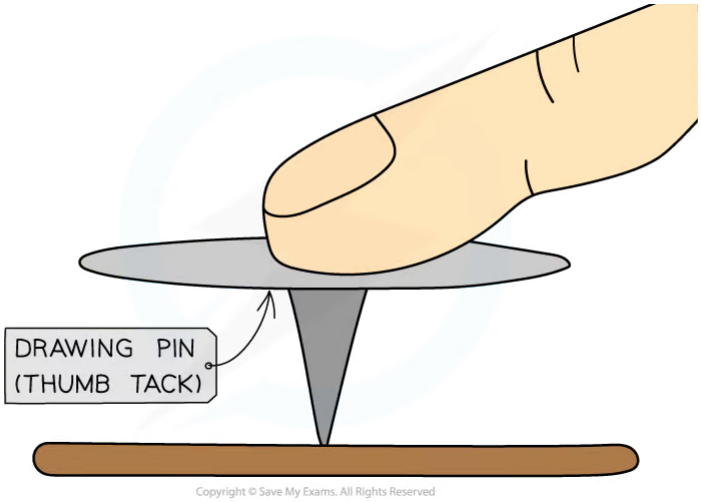
YOUR NOTES

Calculating Gas Pressure

- **Pressure** is defined as

  The concentration of a force or the force per unit area

- For example, when a drawing pin is pushed downwards:
  - It is pushed into the surface, rather than up towards the finger
  - This is because the sharp point is more **concentrated** (a small area) creating a **larger** pressure



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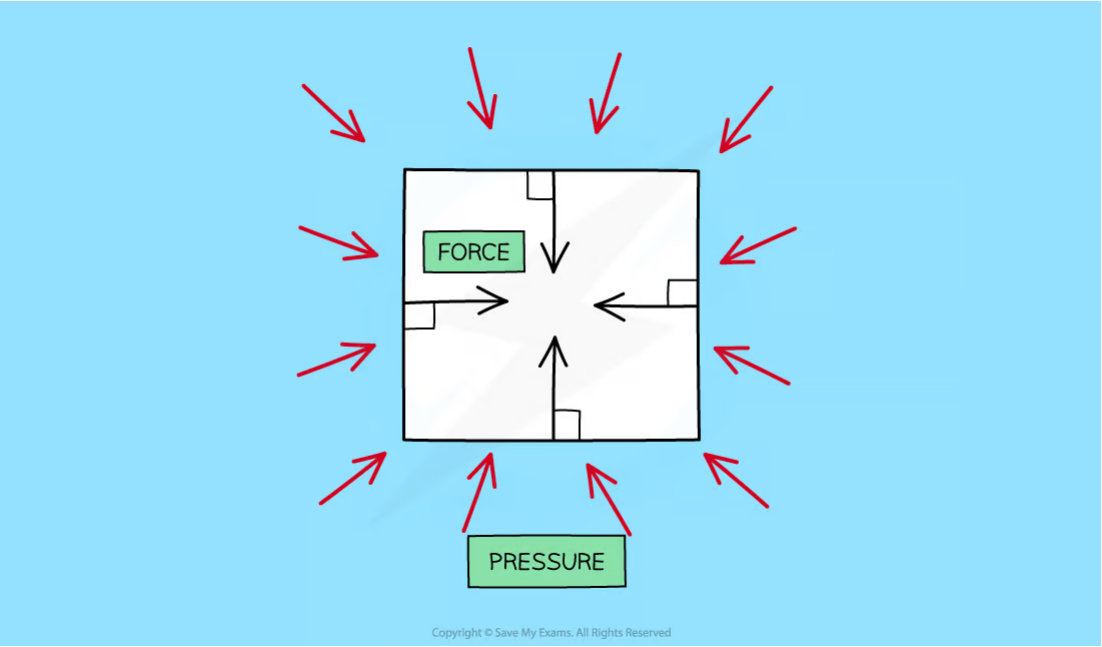
When you push a drawing pin, it goes into the surface (rather than your finger)

- When an object is immersed in a liquid, the liquid will exert pressure, squeezing the object
  - The pressure exerted on objects in fluids creates **forces** against surfaces
  - These forces act at **90 degrees** (at right angles) to the surface

YOUR NOTES



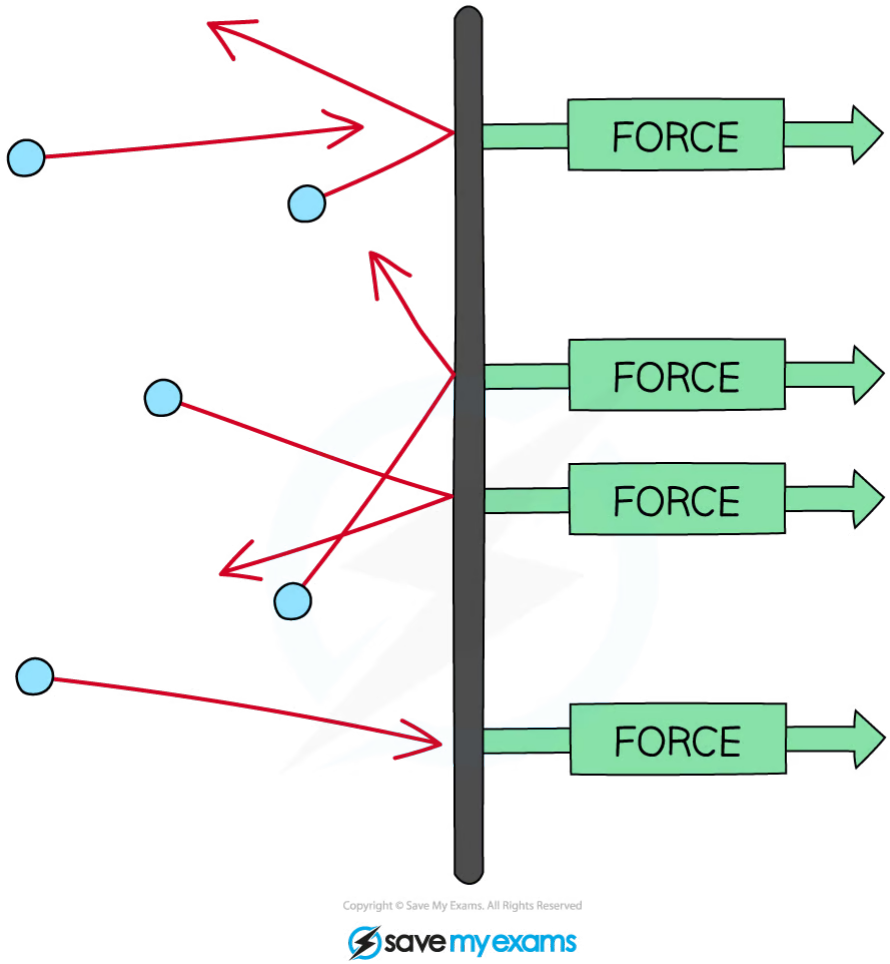
The pressure of a fluid on an object creates a force normal (at right angles) to the surface

- The equation for pressure is:

$$P = \frac{F}{A}$$

- Where:
  - P = pressure (Pa)
  - F = force (N)
  - A = cross-sectional area (m^2)
- Pressure is measured in **Pascals (Pa)**
- This equation is only relevant when gas molecules exert a force **perpendicular** to the surface

YOUR NOTES

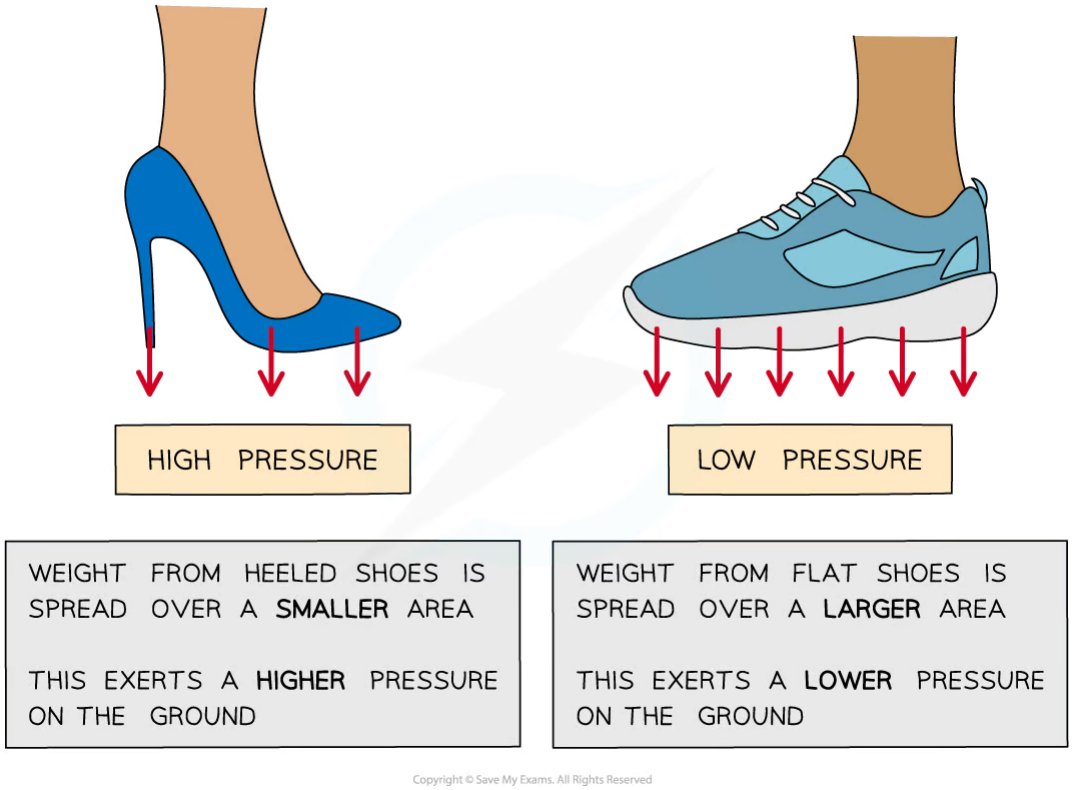


Gas molecules bouncing off the walls of a container

- It is possible for someone to experience this force by closing their mouth and forcing air into their cheeks
- The strain on the cheeks is due to the force of the gas particles pushing at right angles to the cheeks

- This equation means:
  - If a force is spread over a **large** area it will result in a **small** pressure
  - If it is spread over a **small** area it will result in a **large** pressure

YOUR NOTES



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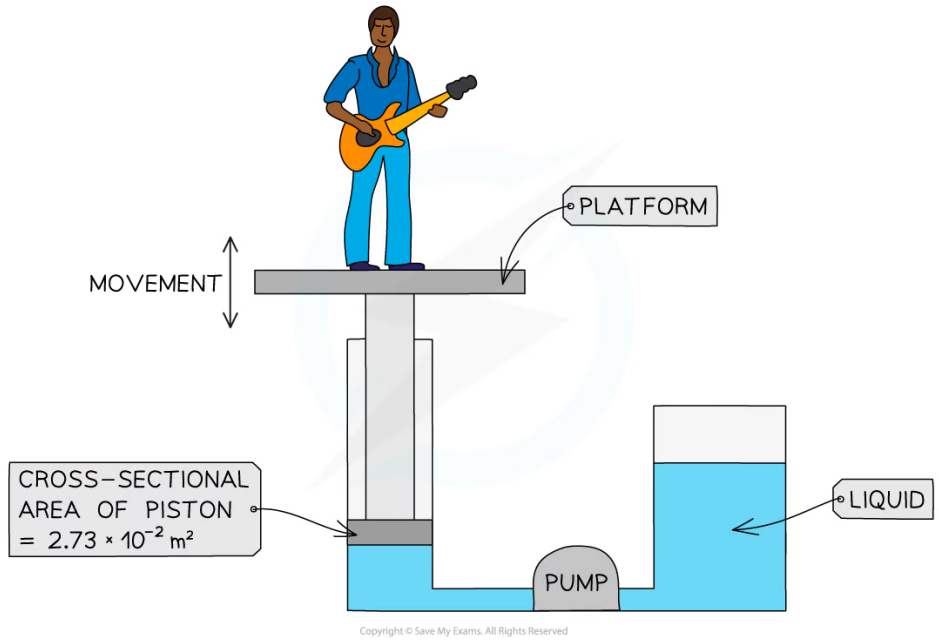
High heels produce a higher pressure on the ground because of their smaller area, compared to flat shoes

YOUR NOTES

Worked Example

The diagram below shows the parts of the lifting machine used to move a platform of cross-sectional area $2.73 \times 10^{-2}\ m^2$ up and down.



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The pump creates pressure in the liquid of 5.28×10^5 Pa to move the platform upwards.

Calculate the force that the liquid applies to the piston.

Answer:

Step 1: List the known quantities

- Cross-sectional area, $A = 2.73 \times 10^{-2}\ m^2$
- Pressure, $P = 5.28 \times 10^5$ Pa

Step 2: Write down the relevant equation

$$p = \frac{F}{A}$$

Step 3: Rearrange for the force, *F*

$$F = p \times A$$

Step 4: Substitute the values into the equation

$$F = (5.28 \times 10^5) \times (2.73 \times 10^{-2}) = 14\,414.4$$

Step 5: Round to the appropriate number of significant figures and quote the correct unit

$F = 14\,400\ \text{N} = 14.4\ \text{kN}\ (3\ \text{s.f})$

YOUR NOTES

Exam Tip

Make sure A is always the **cross-sectional** area of the surface that the force is being applied upon, especially if there are multiple areas given in the question.

Amount of Substance

- The **mole** is one of the seven SI base units
  - It is used to measure the **amount of substance**, not a mass
- One mole is defined as follows:

> **The amount of substance that contains as many elementary entities as the number of atoms in 12 g of carbon-12**

- This amount of substance is exactly $6.02214076 \times 10^{23}$ elementary entities (i.e. particles, atoms, molecules)
  - At IB level, this number can be rounded to 6.02×10^{23}
- One mole of gas contains a number of particles (atoms or molecules) equal to the **Avogadro Constant**
- For example, 1 mole of sodium (Na) contains 6.02×10^{23} **atoms** of sodium
- The number of atoms can be determined if the number of moles is known by multiplying by N_A.
  - For example: 2.0 mol of nitrogen contains: $2.0 \times N_A = 2.0 \times 6.02 \times 10^{23} =$ **1.20×10^{24}** atoms
- The number of moles, n of a substance can be calculated using the equation

$$n = \frac{N}{N_A}$$

- Where:
  - N = number of gas particles
  - n = number of moles of gas (mol)
  - $N_A = 6.02 \times 10^{23}$ mol^{-1} (Avogadro constant)

Molar Mass

- One mole of any element is equal to the **relative atomic mass** of that element in grams
  - E.g. Helium has an atomic mass of 4 - this means 1 mole of helium has a mass of 4 g
- If the substance is a compound, add up the relative atomic masses, for example, water (H_2O) is made up of
  - 2 hydrogen atoms (each with atomic mass of 1) and 1 oxygen atom (atomic mass of 16)
  - So, 1 mole of water would have a mass of $(2 \times 1) + 16 = 18$ g
- The **molar mass,** m of a substance is defined as the mass m of the substance divided by the amount (in moles) of that substance
- The molar mass is calculated as follows:

$$m_r = \frac{m}{n}$$

YOUR NOTES

- Where:
  - m_r = molar mass in g mol^{-1}
  - m = mass in grams (g)
  - n = number of moles (mol)

Worked Example

120 moles of nitrogen gas are in a container.

Calculate the number of nitrogen gas molecules in the container.

Answer:

Step 1: List the known quantities

- Number of moles, $n = 120$

Step 2: Rearrange the number of moles equation for the number of molecules, *N*

$$N = nN_A$$

Step 3: Substitute in the values

$$N = (6.02 \times 10^{23}) \times 120 = 7.2 \times 10^{25} \text{ molecules}$$

Worked Example

A container is filled with 2.6×10^{20} molecules of argon gas which has a mass number of 40.

Calculate the total mass of argon in the container.

Answer:

Step 1: List the known quantities

- Number of argon gas molecules, $N = 2.6 \times 10^{20}$
- Molar mass of argon, $m_r = 40$ g mol^{-1} (same as the atomic mass)

Step 2: Calculate the number of moles of argon in the container

$$n = \frac{N}{N_A} = \frac{2.6 \times 10^{20}}{6.02 \times 10^{23}} = 4.3 \times 10^{-4} \text{ moles}$$

Step 3: Calculate the total mass of the argon in the container

$$m = m_r \times n = 40 \times (4.3 \times 10^{-4}) = 0.0172\, g$$

Exam Tip

The value of the Avogadro constant is given on your data sheet, so you don't need to learn this value. However, it does help make calculations quicker if you do!

Take care with the units for molar mass, this is **g mol^{-1}**, so multiplying by the number of moles gives a mass in grams and **not** kilograms.

Gas Laws

- An **ideal gas** is one which obeys the relation:

$$pV \propto T$$

- Where:
  - p = pressure of the gas (Pa)
  - V = volume of the gas (m^3)
  - T = thermodynamic temperature (K)
- Turning this into an equation gives:

$$\frac{PV}{T} = \text{constant}$$

- This is derived from the empirical gas laws for constant pressure, constant temperature and constant volume

Empirical Gas Laws

- The ideal gas laws are the experimental relationships between pressure *P*, volume *V* and the temperature *T* of an ideal gas
  - **Boyle's Law** (constant temperature)
  - **Charles's Law** (constant pressure)
  - **Gay-Lussac's Law** (constant volume)
- The mass and the number of molecules of the gas are assumed to be **constant** for all experiments

Boyle's Law

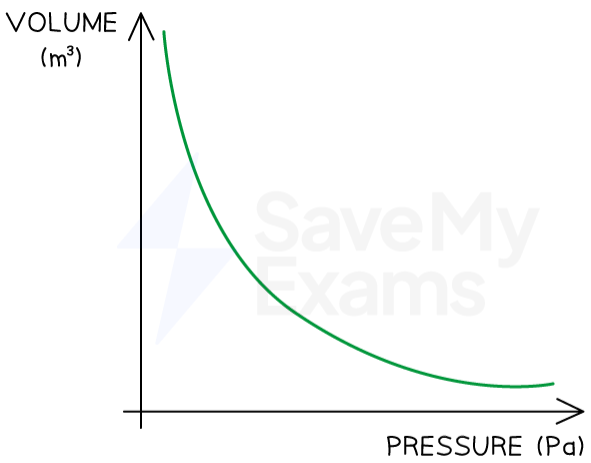
- If the temperature *T* of an ideal gas is constant, then **Boyle's Law** is given by:

$$P \propto \frac{1}{V}$$

- This means the pressure is **inversely proportional** to the volume of a gas
- The relationship between the pressure and volume for a fixed mass of gas at constant temperature can also be written as:

$$P_1 V_1 = P_2 V_2$$

- Where:
  - P_1 = initial pressure (Pa)
  - P_2 = final pressure (Pa)
  - V_1 = initial volume (m^3)
  - V_2 = final volume (m^3)
- The variation of volume and pressure at a constant temperature is shown below:



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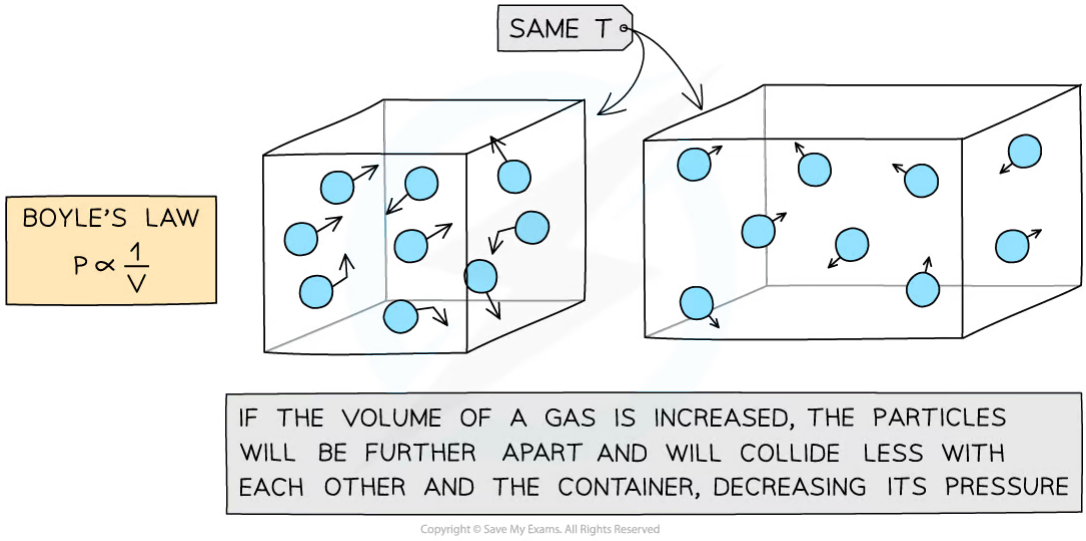
Boyle's Law graph showing pressure is inversely proportional to volume when temperature is constant

- If the temperature increases, the graph is further from the origin and vice versa



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Boyle's law: gas pressure is inversely proportional to the volume it occupies

Charles's Law

- If the pressure *P* of an ideal gas is constant, then **Charles's law** is given by:

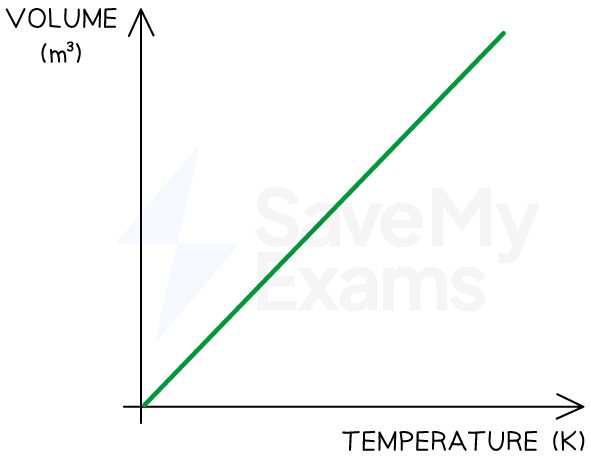
$$V \propto T$$

- This means the volume is **proportional** to the temperature of a gas
- The relationship between the volume and thermodynamic temperature for a fixed mass of gas at constant pressure can also be written as:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

YOUR NOTES

- Where:
  - V_1 = initial volume (m^3)
  - V_2 = final volume (m^3)
  - T_1 = initial temperature (K)
  - T_2 = final temperature (K)
- The variation of volume and temperature at a constant pressure is shown below:



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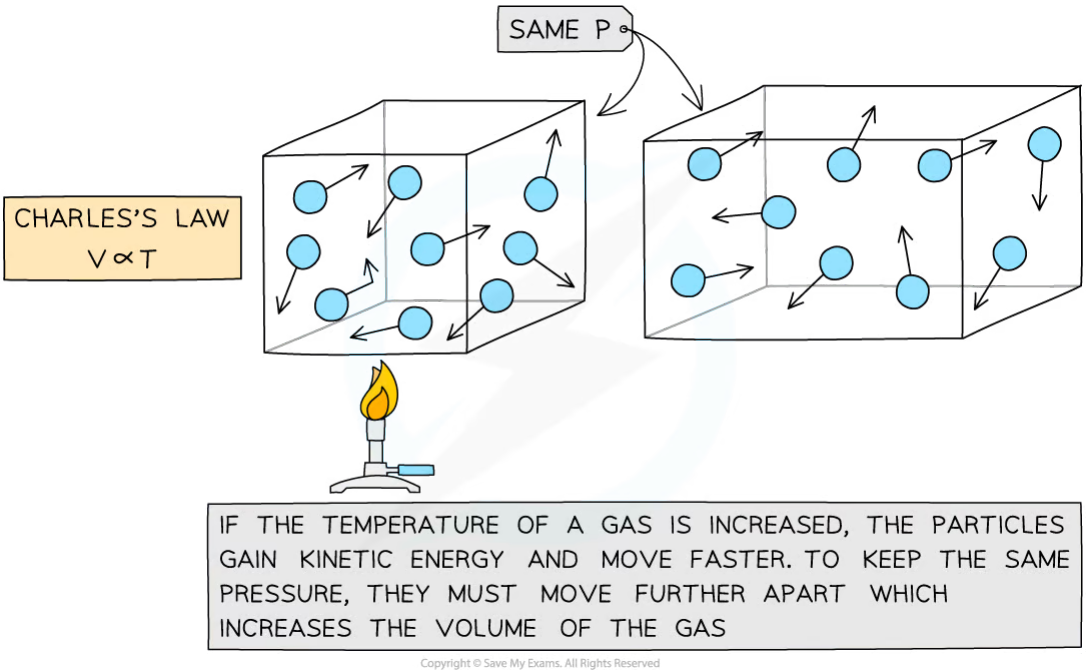
Charles's Law graph showing temperature (in K) is directly proportional to the volume when pressure is constant



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Charles's law: the temperature of a gas is proportional to the volume it occupies

Gay-Lussac's (Pressure) Law

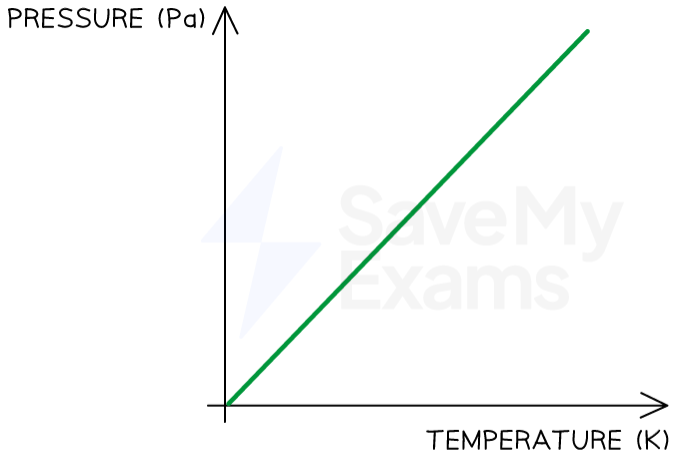
- If the volume *V* of an ideal gas is constant, the **Gay-Lussac's** or **Pressure law** is given by:

$$P \propto T$$

- This means the pressure is **proportional** to the temperature
- The relationship between the pressure and thermodynamic temperature for a fixed mass of gas at constant volume can also be written as:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

- Where:
  - P_1 = initial pressure (Pa)
  - P_2 = final pressure (Pa)
  - T_1 = initial temperature (K)
  - T_2 = final temperature (K)
- The variation of pressure and temperature at a constant volume is shown below:



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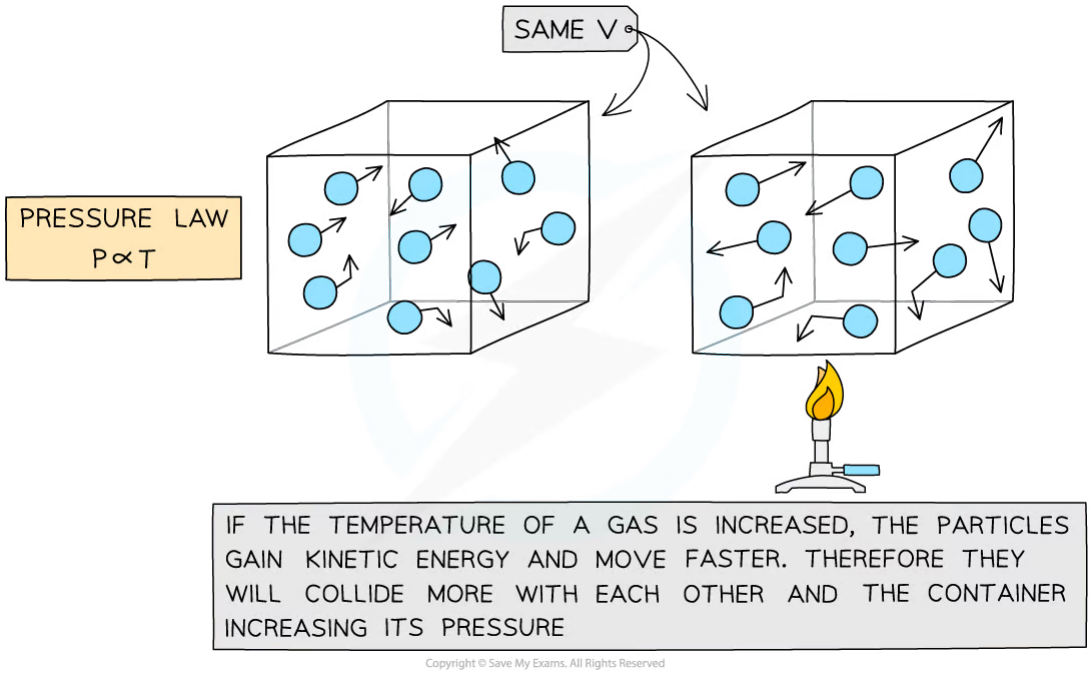
Gay-Lussac's Law graph showing temperature (in K) is directly proportional to pressure when volume is constant

YOUR NOTES



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Pressure law: the temperature of a gas is proportional to the gas pressure

YOUR NOTES

Worked Example

An ideal gas is in a container of volume 4.5×10^{-3} m^3.The gas is at a temperature of 30 °C and a pressure of 6.2×10^5 Pa.

Calculate the pressure of the ideal gas in the same container when it is heated to 40 °C.

Answer:

Step 1: List the known quantities

- Volume, $V = 4.5 \times 10^{-3}$ m^3
- Inital temperature, $T_1 = 30$ °C
- Pressure, $P = 6.2 \times 10^5$ Pa
- Final temperature, $T_2 = 40$ °C

Step 2: State the ideal gas equation

$$pV \propto T$$

$$pV = kT$$

- Where k = the constant of proportionality

Step 3: Rearrange for the constant of proportionality

$$k = \frac{pV}{T}$$

Step 4: Convert temperature T into Kelvin

$$\theta \text{ °C} + 273.15 = T \text{ K}$$

$$30 \text{ °C} + 273.15 = 303.15 \text{ K}$$

Step 5: Substitute in known value into the constant of the proportionality equation

$$k = \frac{6.2 \times 10^5 \times 4.5 \times 10^{-3}}{303.15} = 9.203...$$

Step 6: Rearrange the ideal gas relation equation for pressure

$$p = \frac{kT}{V}$$

Step 7: Substitute in new values

$$k = 9.203$$

$$T = 40 \text{ °C} = 40 + 273.15 = 313.15 \text{ K}$$

$$p = \frac{(9.203...) \times 313.15}{4.5 \times 10^{-3}} = 640.45... \times 10^3 \text{ Pa} = \mathbf{640 \text{ kPa}}$$

Exam Tip

Make sure to convert the temperature into **kelvin** before substituting into any gas law equation.

To know which gas laws to use, look at what values you've already been provided in the question. If it helps, label these as V_1, T_1 etc. or look out for what quantity has been kept **constant**.

In the worked example, make sure not to get mixed up with this constant k with the Boltzmann constant k_B. The constant depends on which ideal gas equation you use. It will be either nR or k_BT.

YOUR NOTES

Ideal Gas Equation

- The empirical gas laws can be combined with Avogadro's law to give a single constant, known as the **ideal gas constant**, ***R***

	Boyle's Law	Charle's Law	Pressure Law	Avogadro's Law
relationship	$PV = constant$	$V \propto T$	$P \propto T$	$V \propto n$
constants	T, n	P, n	V, n	P, T

- An ideal gas is defined as:

  A gas which obeys the ideal gas equation at all pressures, volumes and temperatures

- Combining the four equations leads to the equation of state of an ideal gas:

$$PV = nRT$$

- Where:
  - P = pressure (Pa)
  - V = volume (m^3)
  - n = number of moles (mol)
  - R = 8.31 $J K^{-1} mol^{-1}$ (ideal gas constant)
  - T = temperature (K)
- The ideal gas constant R is the macroscopic equivalent of the Boltzmann constant k_B
  - The Boltzmann constant is associated with the thermal energy of **microscopic** particles
- The Boltzmann constant is defined as the ratio of the ideal gas constant R and Avogadro's constant N_A:

$$k_B = \frac{R}{N_A}$$

- Where $N_A = \frac{N}{n}$
- This gives another form of the equation of state of an ideal gas:

$$PV = Nk_BT$$

- Where:
  - N = number of molecules
  - k_B = 1.38 × 10^{-23} $J K^{-1}$ (Boltzmann constant)

YOUR NOTES

Worked Example

A gas has a temperature of −55 °C and a pressure of 0.5 MPa. It occupies a volume of 0.02 m^3.

Calculate the number of gas particles.

Answer:

Step 1: Write down the known quantities

- Temperature, T = −55 °C = 218 K
- Pressure, p = 0.5 MPa = 0.5 × 10^6 Pa
- Volume, V = 0.02 m^3

Step 2: Write down the equation of state of ideal gases

$$PV = nRT$$

Step 3: Rearrange the above equation to calculate the number of moles *n*

$$n = \frac{PV}{RT}$$

Step 4: Substitute numbers into the equation

- From the data booklet, R = 8.31 $J K^{-1} mol^{-1}$

$$n = \frac{(0.5 \times 10^6) \times 0.02}{8.31 \times 218} = 5.5 \text{ moles}$$

Step 5: Calculate the number of particles *N*

$$n = \frac{N}{N_A} \Rightarrow N = nN_A$$

$$N = 5.5 \times (6.02 \times 10^{23}) = 3.3 \times 10^{24}$$

Exam Tip

The values for the gas constant, Avogadro constant and Boltzmann constant are all given in your data booklet.

Always make sure that temperature T is in **kelvin** for this topic. You must convert this from °C if not using the following conversion

$$\theta / °C = T / K - 273.15$$
$$T / K = \theta / °C + 273.15$$

YOUR NOTES

Kinetic Theory of Gases

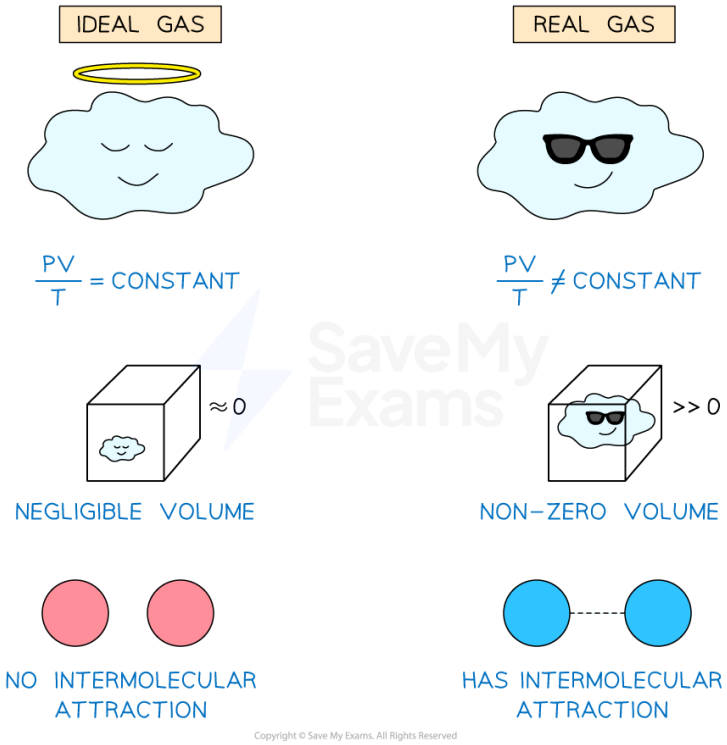
- Ideal gases are described in terms of the **kinetic theory**
  - This is a modelled system used to approximate the behaviour of real gases
- A gas consists of atoms or molecules moving around **randomly** at **high speeds**
- The kinetic theory of gases models the thermodynamic behaviour of gases by linking:
  - The **microscopic properties** of particles e.g. mass and speed
  - The **macroscopic properties** of particles e.g. pressure and volume
- Kinetic theory and ideal gases are based on a set of the following **assumptions**:

1. A gas consists of **many** identical molecules in a container. They all have the same mass
2. The volume of the molecules is negligible **compared to** the volume of the container. This means they can be considered **point** particles
3. The molecules are in continuous **random** motion at **high speeds**
4. The molecules obey Newton's laws of motion
5. The molecules collide elastically with other and the walls of the container
6. There are no intermolecular forces between the molecules **except during** collisions
7. The time of a collision between molecules is negligible compared to the time between collisions
8. External forces (e.g. gravity) are ignored
9. The number of molecules of gas in a container is **very large.** Therefore the **average** behaviour (eg. speed) is usually considered
10. Each particle exerts a force on the wall of the container with which it collides. This means the average of the forces produced by all gas particles results in a **uniform gas pressure**

Real & Ideal Gases

YOUR NOTES

- **Real gases** will not always obey the **ideal gas** equation
- This is because the assumptions of the kinetic theory of gases may not always be valid
  - For example, the assumption that there are no intermolecular forces between gas molecules is not particularly realistic
- However, under certain conditions, they can be **considered** ideal gases
- An ideal gas is a good approximation of a real gas when:
  - The gas pressure is **low**
  - The gas density is **low**
  - Temperature is sufficiently **higher** than the boiling point of the substance
- At very **high pressures**, **densities** and **low temperatures**, real gases do not obey the kinetic theory model
- This is because at high pressures and densities:
  - Molecules are closer together
  - Therefore, the volume of the molecules cannot be considered negligible
  - There will be attractive forces between the molecules
- At low temperatures:
  - A gas can change into a liquid, so the substance will no longer behave like a gas
- No gas is exactly ideal, but air at normal room temperature and pressure, based on experiments, comes pretty close



Real vs. ideal gases

Derivation of the Kinetic Theory of Gases Equation

- Gas pressure arises due to the collisions of the gas particles with the walls of the container that holds it
  - When a gas particle collides with a wall in the container, it exerts a **force** on the wall
  - The random motion of a large number of molecules exerting a force on the walls creates an overall **pressure**
  - This is because pressure = force per unit area
- When a gas particle hits a wall of the container, it undergoes a **change in momentum** due to the force exerted by the wall on the particle

$$\text{initial momentum} = mv$$

$$\text{final momentum} = -mv$$

- Therefore, the change in momentum Δp is:

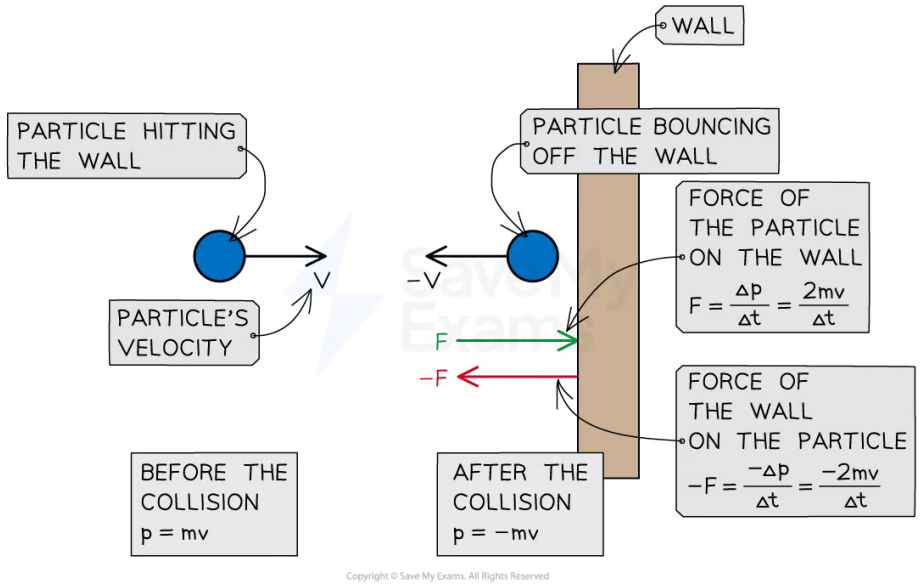
$$\Delta p = \text{final momentum} - \text{initial momentum}$$

$$\Delta p = -mv - (mv) = -2mv$$

- Force is equal to the rate of change of momentum (Newton's Second Law), so the force exerted by the wall on a molecule is:

$$F = \frac{\Delta p}{\Delta t} = -\frac{2mv}{\Delta t}$$

- According to Newton's Third Law, the particle exerts an **equal** and **opposite** force on the wall $\left(F = \frac{2mv}{\Delta t}\right)$



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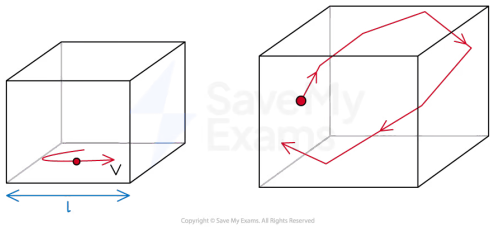
A particle hitting a wall of the container in which the gas is held experiences a force from the wall and a change in momentum. The particle exerts an equal and opposite force on the wall

YOUR NOTES

Derivation

- Picture a single molecule in a cube-shaped box with sides of equal length l
- The molecule has a mass *m* and moves with speed c, parallel to one side of the box
- It collides at regular intervals with the ends of the box, exerting a force and contributing to the pressure of the gas
- By calculating the pressure this one molecule exerts on one end of the box, the total pressure produced by all the molecules can be deduced



A single molecule in a box collides with the walls and exerts a pressure

1. Find the change in momentum as a single molecule hits a wall perpendicularly

- One assumption of the kinetic theory is that molecules **rebound elastically**
- This means there is no kinetic energy lost in the collision
- If they rebound in the opposite direction to their initial velocity, their final velocity is -c
- The change in momentum is therefore:

$$\Delta p = -mv - (+mv) = -mv - mv = -2mv$$

2. Calculate the number of collisions per second by the molecule on a wall

- The time between collisions from one wall to the other, and then back again, over a distance of $2l$ with speed *v* is:

$$\text{time between collisions} = \frac{distance}{speed} = \frac{2l}{v}$$

3. Find the change in momentum per second

- The force the molecule exerts on one wall, *F* is found using Newton's second law of motion:

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv}{\frac{2l}{v}} = \frac{mv^2}{l}$$

- The change in momentum is $+2mv$ since the force on the molecule from the wall is in the **opposite** direction to its change in momentum

4. Calculate the total pressure from *N* molecules

- The area of one wall is l^2
- The pressure, *P* can be written as:

$$P = \frac{F}{A} = \frac{\frac{mv^2}{l}}{l^2} = \frac{mv^2}{l^3}$$

- This is the pressure **exerted from one molecule**
- To account for the large number of *N* molecules, the pressure can now be written as:

$$P = \frac{Nmv^2}{l^3}$$

- Each molecule has a different velocity and they all contribute to the pressure
- The pressure is now defined as:

$$P = \frac{Nmv^2}{l^3}$$

5. Consider the effect of the molecule moving in 3D space

- The pressure equation still assumes all the molecules are travelling in the same direction and colliding with the same pair of opposite faces of the cube
- In reality, all molecules will be moving in three dimensions equally
- Splitting the velocity into its components c_x, c_y and c_z to denote the amount in the x, y and z directions, c^2 can be defined using pythagoras' theorem in 3D:

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

- Since there is nothing special about any particular direction, it can be determined that:

$$\overline{v}_x^2 = \overline{v}_y^2 = \overline{v}_z^2$$

- Therefore, *v* can be defined as:

$$\overline{v}_x^2 = \frac{1}{3}\overline{v}^2$$

6. Re-write the pressure equation

- The box is a cube with a side length l
- Therefore, volume of the cube $V = l^3$

$$P = \frac{Nmv^2}{V}$$

- Substituting the new values for $\overline{v}^2$ and l^3 back into the pressure equation obtains the final equation:

$$PV = \frac{1}{3}Nm\overline{v}^2$$

- $\overline{v}^2$ is often just written as v^2

$$PV = \frac{1}{3}Nmv^2$$

- This is known as the **Kinetic Theory of Gases equation**
- This can also be written using the density ρ of the gas:

$$\rho = \frac{mass}{volume} = \frac{Nm}{V}$$

- Rearranging the pressure equation for p and substituting the density ρ:

$$P = \frac{1}{3}\rho v^2$$

- Where:
  - P = pressure of the gas (Pa)
  - ρ = density of the gas (kg m^{-3})
  - v = mean square speed (m s^{-1})

YOUR NOTES

YOUR NOTES

Worked Example

An ideal gas has a density of 4.5 kg m^{-3} at a pressure of 9.3 x 10^5 Pa and a temperature of 504 K.

Determine the mean square speed of the gas molecules at 504 K.

Answer:

Step 1: List the known quantities

- Density of the gas, $\rho = 4.5$ kg m^{-3}
- Gas pressure, $P = 9.3 \times 10^5$ Pa
- Temperature, $T = 504$ K

Step 2: Rearrange the pressure equation for the mean square speed

$$P = \frac{1}{3}\rho v^2$$

$$v = \sqrt{\frac{3P}{\rho}}$$

Step 3: Substitute in the values

$$v = \sqrt{3 \times \frac{(9.3 \times 10)^5}{4.5}} = 787.4 = 790 \text{ m s}^{-1}$$

Exam Tip

Although you will not be asked to recall the derivation, you need to understand the physics that underlines it and how to use the equation in the final result.

The 'mean square' speed just means an **average** speed, as we're assuming that all the molecules travel at the same speed.

Average Kinetic Energy of a Molecule

- We think of an ideal gas as molecules that collide elastically in random motion
- When the molecules collide, momentum and energy are conserved and we assume **no** forces act between them
- This means no **work** needs to be done to change the position of the molecules, so they have no **potential energy**
  - Therefore, the internal energy U of an ideal, monatomic gas is equal to its **average kinetic energy** if the molecules are far enough apart
- The average kinetic energy, E_k for **one** molecule is equal to:

$$E_k = \frac{1}{2}mv^2 = \frac{3}{2}k_B T$$

- Where:
  - E_k = average kinetic energy of one molecule (J)
  - v = mean square speed of one molecule (m s^{-1})
  - m = mass of one molecule (kg)
  - k_B = Boltzmann constant
  - T = temperature of the gas (K)
- Since the **total** internal energy is the **total** kinetic energy, the internal energy of a gas U is defined as:

$$U = \frac{3}{2}Nk_B T$$

- Where:
  - U = internal energy (J)
  - N = number of molecules
- The relation to the amount of substance is:

$$U = \frac{3}{2}nRT$$

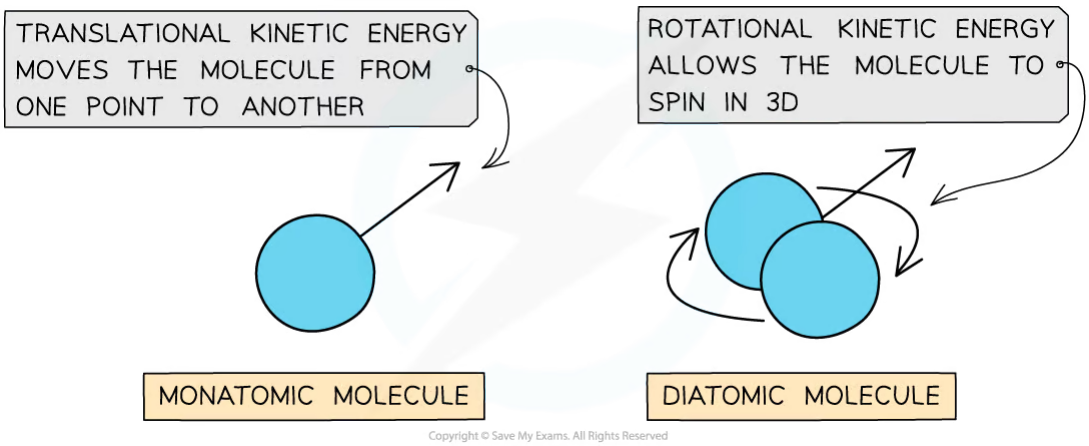
- Where:
  - n = number of moles
  - R = ideal gas constant
- When heat is transferred to a fixed volume of gas, the internal energy **increases** and hence, so does the temperature, since the equations show that

$$U \propto T$$

- This is only relevant for **monatomic** gases. Examples include:
  - Helium
  - Neon
  - Argon

YOUR NOTES

- A monatomic (one atom) molecule only has translational energy, whilst a diatomic (two-atom) molecule has both **translational** and **rotational energy**



A diatomic molecule has both rotational and translational kinetic energy

YOUR NOTES

Worked Example

600 J of thermal energy is transferred to 3 g of helium gas kept at a constant volume in a cylinder.

Helium has a mass number of 4.

Calculate the temperature of the gas.

Answer:

Step 1: List the known quantities

- Thermal (internal) energy, $U = 600$ J
- Molar mass of helium, $m_r = 4$ g mol^{-1} (from its mass number)
- Mass of helium gas, $m = 3$ g

Step 2: State the relevant equation

$$U = \frac{3}{2}nRT$$

Step 3: Calculate the number of moles of the gas

$$m_r = \frac{m}{n} \quad \Rightarrow \quad n = \frac{m}{m_r}$$

$$n = \frac{3}{4} = 0.75 \text{ moles}$$

Step 4: Rearrange the internal energy for the temperature, *T*

$$T = \frac{2U}{3nR}$$

Step 5: Substitute in the values

$$T = \frac{2 \times 600}{3 \times 0.75 \times 8.31} = 64 \text{ K}$$

Exam Tip

Calculations involving moles, molar mass and number of molecules are common in this topic. You should be confident with converting between these quantities for all calculations in thermal physics.

Remember the combination '*nRT*' and 'Nk_BT' is also in the **ideal gas equation**. This shows how all the equations link together.

If you are using the equation $E_k = \frac{3}{2}k_BT$, remember this is only for **one** molecule, so you will need to multiply it by the total number of molecules for a whole gas.

YOUR NOTES

IB Physics HL

YOUR NOTES

Thermodynamics

CONTENTS

Thermodynamic Systems (HL)

First Law of Thermodynamics (HL)

Entropy (HL)

Calculating Changes in Entropy (HL)

Second Law of Thermodynamics (HL)

Thermodynamic Processes (HL)

Heat Engines (HL)

The Carnot Cycle (HL)

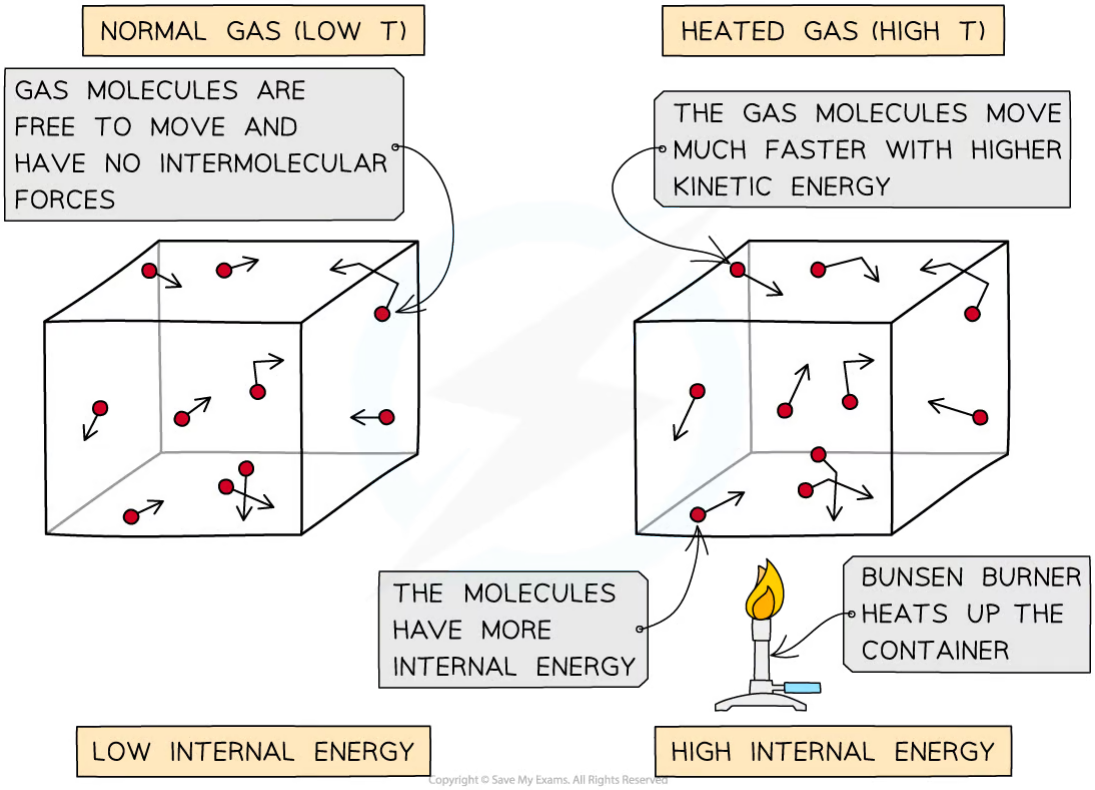
YOUR NOTES

Change in Internal Energy

- The change in the internal energy ΔU of an object is intrinsically related to a change in its temperature ΔT

$$\Delta U \propto \Delta T$$

- When a container containing gas molecules is heated up, the molecules begin to **move around faster,** increasing their kinetic energy
  - In a solid, where the molecules are tightly packed, molecules begin to **vibrate** more as they are heated
  - Molecules in liquids and solids have **both** kinetic and potential energy because they are close together and bound by intermolecular forces
- However, the molecules in an ideal gas are assumed to have **no intermolecular forces**
  - This means they do **not** possess potential energy, **only** kinetic energy



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As the container is heated up, the gas molecules move faster with higher kinetic energy and therefore higher internal energy

- The (change in) internal energy of an ideal gas is equal to:

$$\Delta U = \frac{3}{2} k_B \Delta T$$

YOUR NOTES

- Where
  - ΔU = change in internal energy (J)
  - k_B = Boltzmann constant
  - ΔT = change in temperature (K)
- Another form of this equation related to the translational kinetic energy of the **particles** is

$$\Delta U = \frac{3}{2} nR\Delta T$$

- Where:
  - n = number of moles of gas (mol)
  - R = molar gas constant

Worked Example

A student suggests that, when an ideal gas is heated from 50°C to 150°C, the internal energy of the gas is trebled.

State and explain whether the student's suggestion is correct.

Answer:

- The change in internal energy of an ideal gas is directly proportional to its change in temperature

$$\Delta U \propto \Delta T$$

- The temperature change is the **thermodynamic** temperature i.e. Kelvin
- The temperature change in degrees (from 50°C to 150°C) increases by three times
- The temperature change in Kelvin is:

$$50°C + 273.15 = 323.15\ K$$

$$150°C + 273.15 = 423.15\ K$$

$$\frac{423.15}{323.15} = 1.3$$

- The temperature change, in Kelvin, does **not** increase by three times, therefore, neither does the internal energy
- **Hence, the student is incorrect**

YOUR NOTES

Worked Example

An ideal gas expands at constant pressure. The following data are available:

amount of gas = 126 mol

initial temperature of gas = –23.0°C

final temperature of gas = +27.0°C

Determine the change in internal energy of the gas during this expansion.

Answer:

- The change in internal energy of a gas is equal to

$$\Delta U = \frac{3}{2} nR\Delta T$$

- Where
  - Amount of gas, n = 126 mol
  - Gas constant, R = 8.31 J mol^{-1} K^{-1}
  - Change in temperature, ΔT = 27 – (–23) = 50°C

$$\Delta U = \frac{3}{2} \times 126 \times 8.31 \times 50$$

$$\Delta U = 7.85 \times 10^4 \text{ J}$$

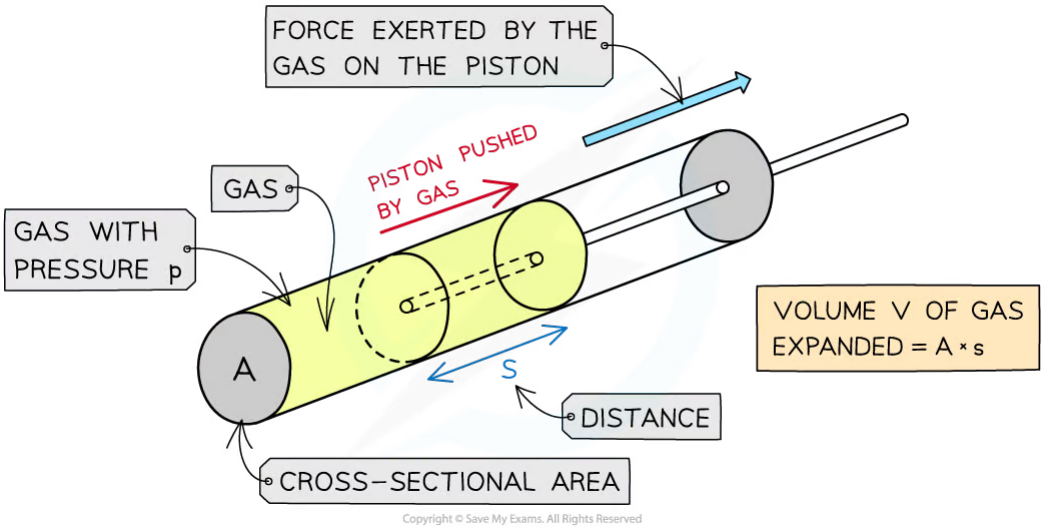
Exam Tip

If an exam question about an ideal gas asks for the **total internal energy**, remember that this is equal to the **total kinetic energy** since an ideal gas has **zero potential energy**

Work Done by a Gas

YOUR NOTES

- When a gas **expands**, it **does work** on its surroundings by exerting pressure on the walls of the container it's in
- For a gas inside a piston, the force exerted by the gas pushes the piston outwards
  - As a result, work is done **on** the piston **by** the gas
- Alternatively, if an external force is applied to the piston, the gas will be **compressed**
  - In this case, work is done **on** the gas **by** the piston



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The expansion of the gas does work on the piston by exerting a force over a distance, s

- Assuming the volume of gas is kept at **constant** pressure, this means the force *F* exerted by the gas on the piston is equal to:

$$p = \frac{F}{A} \quad \Rightarrow \quad F = pA$$

- Where *A* = cross-sectional area of the cylinder (m^2)
- The definition of work done is:

$$W = Fs$$

- Where *s* = displacement in the direction of force (m)
- The displacement *s* of the gas multiplied by the cross-sectional area *A* is equal to the increase in volume ΔV of the gas:

$$W = pAs$$

- This gives the equation for the work done when the volume of a gas changes at constant pressure:

$$W = p\Delta V$$

- Where:

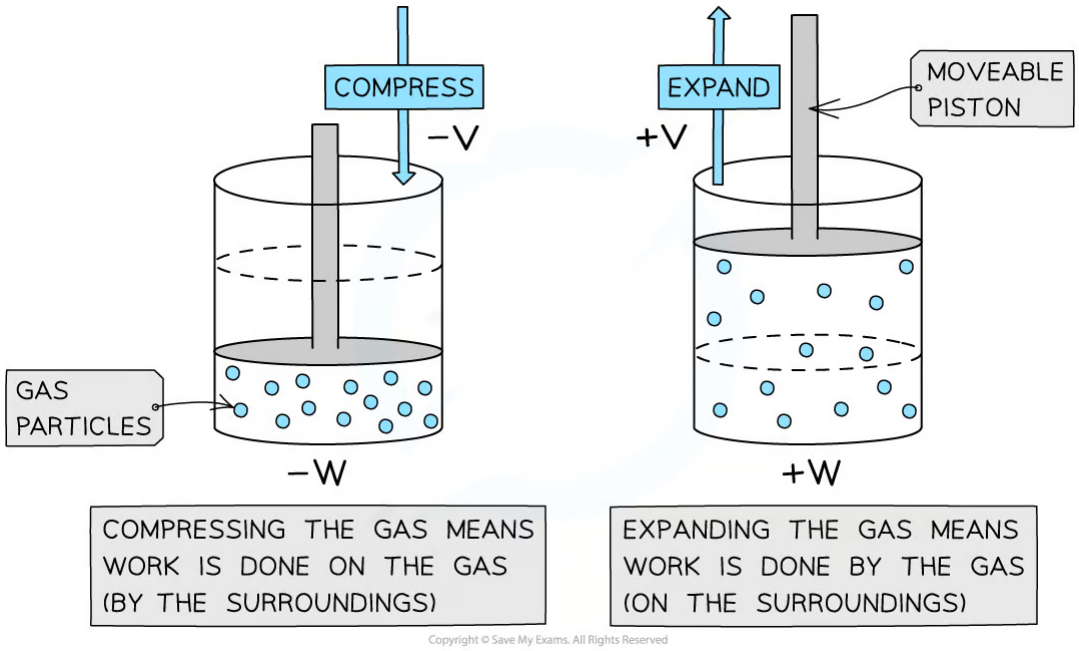
YOUR NOTES

- W = work done (J)
- p = pressure of the gas (Pa)
- ΔV = change in the volume of the gas (m^3)

- This equation assumes that the surrounding pressure does not change as the gas expands
  - This is true if the gas is expanding against the pressure of the atmosphere, which changes very slowly

p-V diagrams

- Pressure-volume (p-V) diagrams are often used to represent changes in the **state of a gas** in thermodynamic processes



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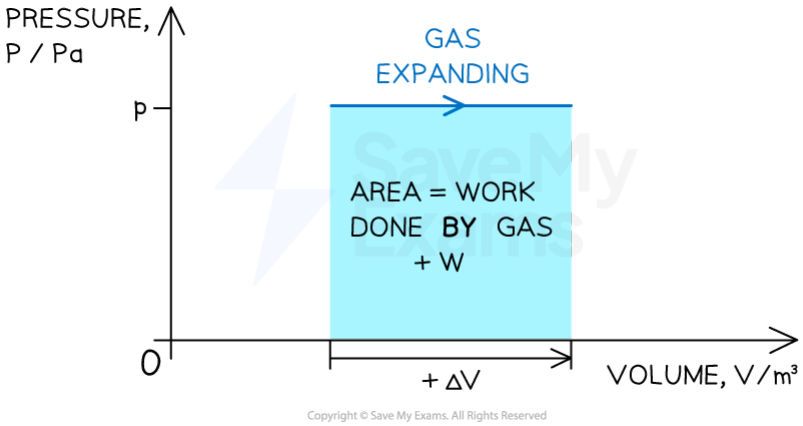
Positive or negative work done depends on whether the gas is compressed or expanded

- When a gas **expands** (at constant pressure) ***work done is positive***
  - Volume increases $+\Delta V$
  - Work is done **by** the gas $+W$



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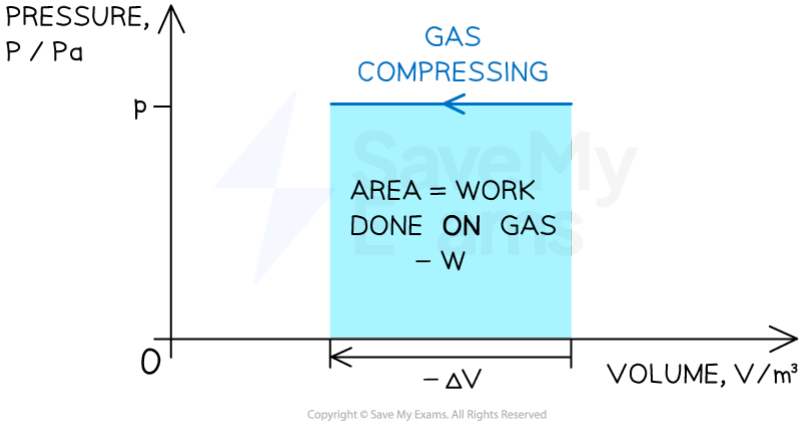
- When a gas is **compressed** (at constant pressure) ***work done W is negative***
  - Volume decreases $-\Delta V$
  - Work is done **on** the gas $-W$



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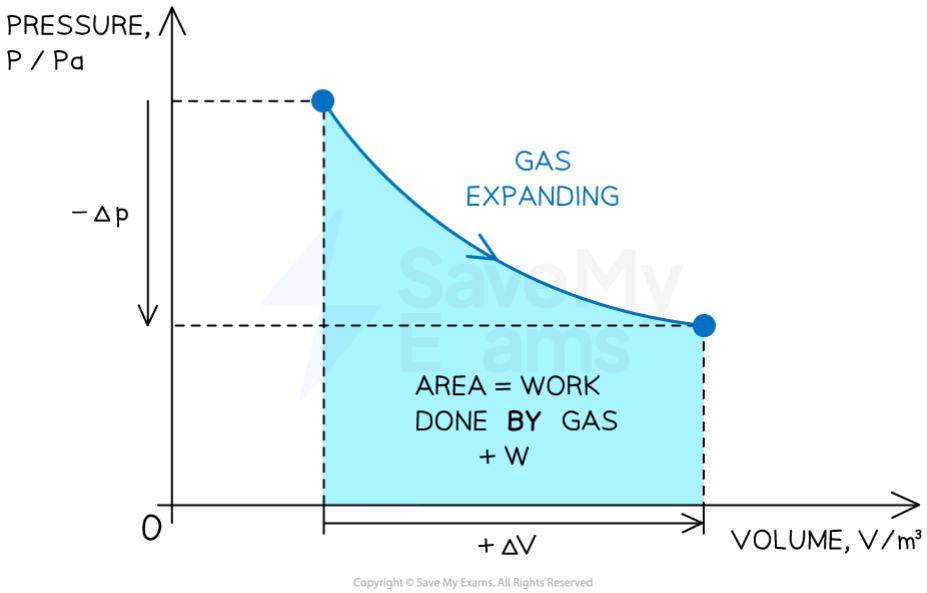
- When both the volume and pressure of gas **changes**

The work done can be determined from the area under a p-V diagram



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Worked Example

When a balloon is inflated, its rubber walls push against the air around it.

Calculate the work done when the balloon is blown up from 0.015 m^3 to 0.030 m^3.

Atmospheric pressure = 1.0×10^5 Pa.

Answer:

- The work done by a gas is equal to

$$W = p\Delta V$$

- Where the change in volume is

ΔV = final volume – initial volume = 0.030 – 0.015 = 0.015 m^3

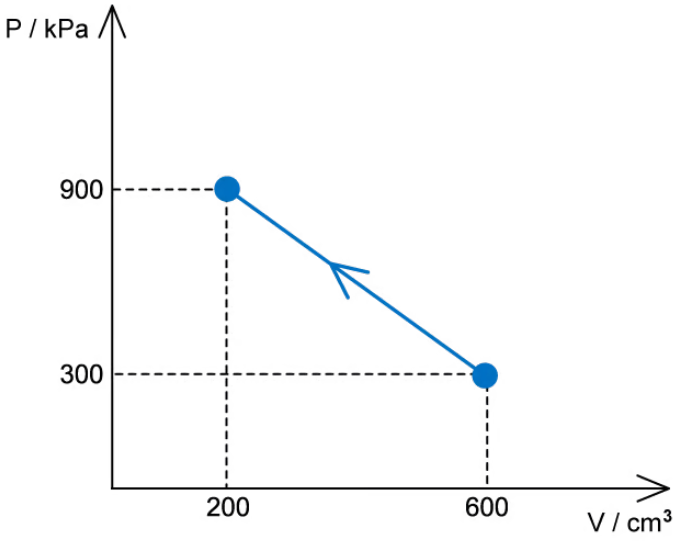
- Therefore, work done is

$W = (1.0 \times 10^5) \times 0.015 = 1500$ J

YOUR NOTES

Worked Example

An ideal gas is compressed, as shown on the graph below.



(a)
For this change, state and explain whether work is done on the gas or by the gas
(b)
Determine the value of the work done and state whether it is positive or negative

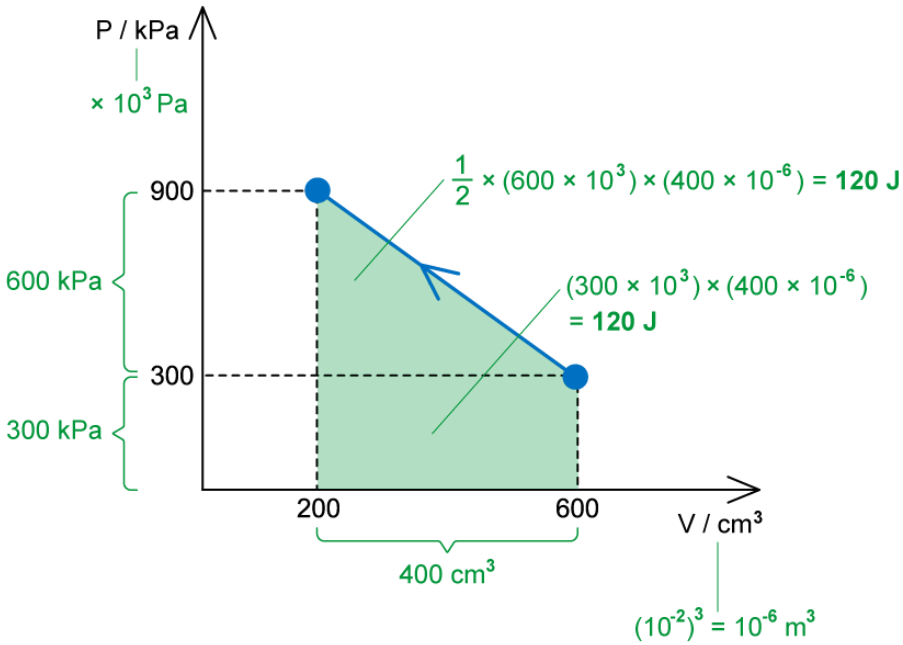
Answer:

(a)

- The volume decreases, therefore, work is done **by the gas**

(b)

- The work done is equal to the area under the p-V diagram



$$W = \frac{1}{2}(600 \times 10^3)(400 \times 10^{-6}) + (300 \times 10^3)(400 \times 10^{-6})$$

$W = -240$ J

- When volume decreases, the work done is **negative**

Exam Tip

The pressure *p* in the work done by a gas equation is not the pressure of the gas but the pressure of the surroundings. This is because when a gas expands, it does work **on** the surroundings.

YOUR NOTES

The First Law of Thermodynamics

- The first law of thermodynamics is based on the principle of conservation of energy
- When energy is put into a gas by heating it or doing work on it, its internal energy must increase:

energy supplied by heating = change in internal energy + work done on the system

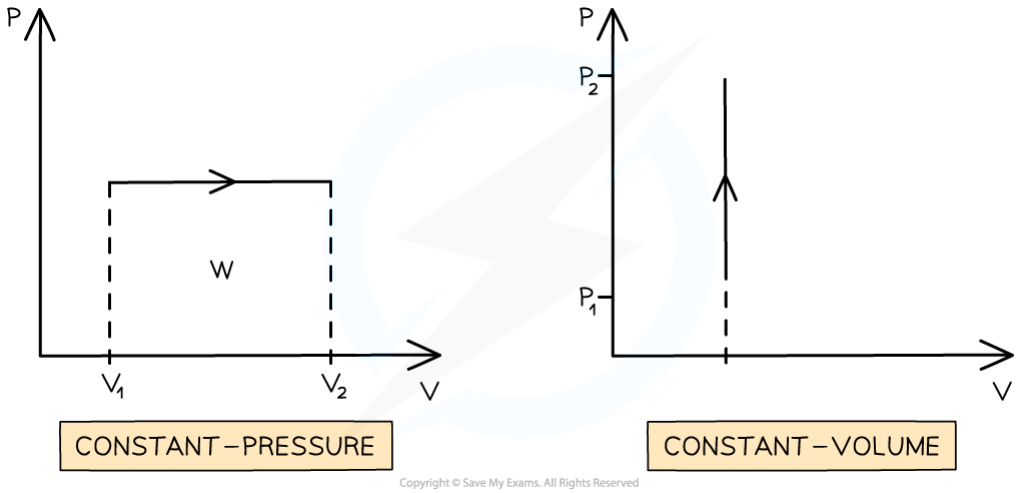
- The first law of thermodynamics is therefore defined as:

$$Q = \Delta U + W$$

- Where:
  - Q = energy supplied to the system by heating (J)
  - ΔU = change in internal energy (J)
  - W = work done by the system (J)
- The first law of thermodynamics applies to **all** situations, not just to gases
  - There is an important sign convention used for this equation
- A **positive** value for internal energy (+ΔU) means:
  - The internal energy ΔU **increases**
  - Heat Q is **added** to the system (+Q)
  - Work W is done **on** the system (−W)
- A **negative** value for internal energy (−ΔU) means:
  - The internal energy ΔU **decreases**
  - Heat Q is **taken away** from the system (−Q)
  - Work W is done **by** the system (+W)

Graphs of Constant Pressure & Volume

- Graphs of pressure p against volume V can provide information about the work done and internal energy of the gas
  - The work done is represented by the area under the line
- A constant pressure process is represented as a **horizontal line**
  - If the volume is increasing (expansion), work is done **by** the gas (on the surroundings) and internal energy decreases ($\Delta U = q - W$)
  - If the arrow is reversed and the volume is decreasing (compression), work is done **on** the gas and internal energy increases ($\Delta U = q + W$)
  - The volume of the gas is made smaller, so more collisions between the molecules of the gas and the walls of the container occur. This creates a higher pressure.
- A constant volume process is represented as a **vertical line**
  - In a process with constant volume, the area under the curve is **zero**
  - Therefore, no work is done when the volume stays the same



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Work is only done when the volume of a gas changes

YOUR NOTES

Worked Example

The volume occupied by 1.00 mol of a liquid at 50°C is 2.4×10^{-5} m^3. When the liquid is vaporised at an atmospheric pressure of 1.03×10^5 Pa, the vapour occupies a volume of 5.9×10^{-2} m^3.

The latent heat to vaporise 1.00 mol of this liquid at 50°C at atmospheric pressure is 3.48×10^4 J.

For this change of state, determine the increase in internal energy ΔU of the system.

Answer:

Step 1: List the known quantities

- Thermal energy, $Q = 3.48 \times 10^4$ J
- Atmospheric pressure, $p = 1.03 \times 10^5$ Pa
- Initial volume = 2.4×10^{-5} m^3
- Final volume = 5.9×10^{-2} m^3

Step 2: Calculate the work done *W*

- The work done by a gas at constant pressure is

$$W = p\Delta V$$

- Where the change in volume is:

ΔV = final volume – initial volume = $(5.9 \times 10^{-2}) - (2.4 \times 10^{-5}) = 0.059$ m^3

- Since the volume of the gas decreases, the work done is negative

$$W = (1.03 \times 10^5) \times 0.059 = 6077 = 6.08 \times 10^3 \text{ J}$$

$$W = -6.08 \times 10^3 \text{ J}$$

Step 3: Substitute the values into the equation for the first law of thermodynamics

- From the first law of thermodynamics:

$$\Delta U = Q + W$$

$$\Delta U = (3.48 \times 10^4) + (-6.08 \times 10^3) = 28\,720$$

- Increase in internal energy: $\Delta U = 28\,700$ J (3 s.f.)

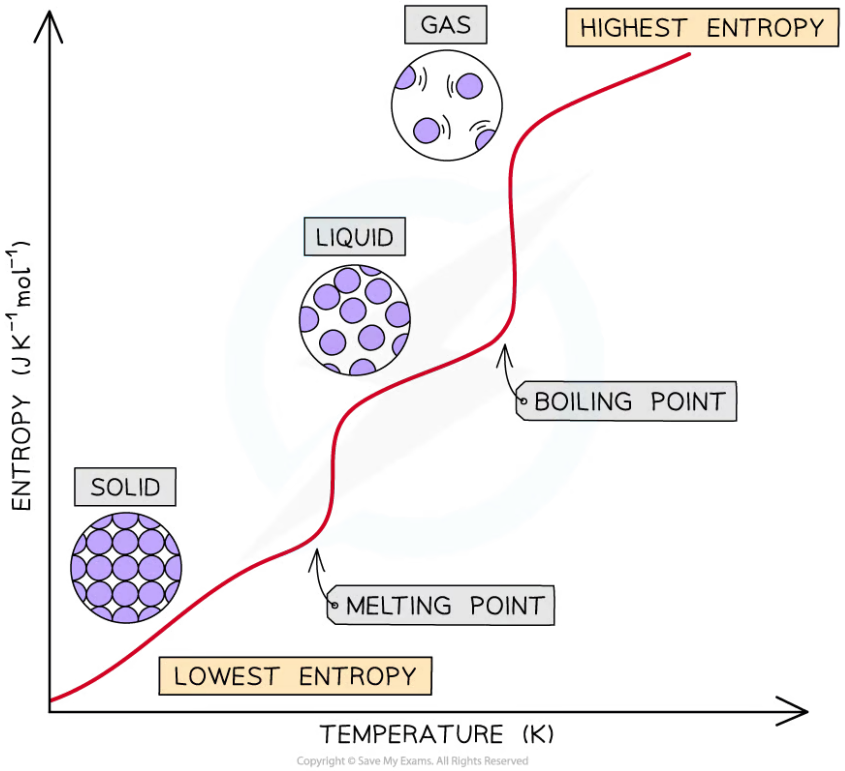
YOUR NOTES

Entropy

- The entropy S of a given system is a measure of the number of possible arrangements of the particles and their energies
  - In other words, it is a measure of how **disordered** a system is
  - When a system becomes more disordered, its entropy will **increase**
- The order of entropy for the different states of matter from **most** disordered to **least** is:

gas > liquid > solid

- The entropy of a substance changes during a **change in state**
- Entropy **increases** when a substance **melts** (solid → liquid) or **boils** (liquid → gas)
  - Increasing the temperature of a substance causes the particles to **vibrate** more
  - The particles in a gas can now freely move around and are far apart from each other
  - The entropy increases as the particles become more disordered
- Similarly, entropy **decreases** when a substance **condenses** (gas → liquid) or **freezes** (liquid → solid)
  - The particles are brought together and become arranged more regularly
  - The particles become less able to move as they become more ordered
  - There are fewer ways of arranging the energy, hence the entropy decreases



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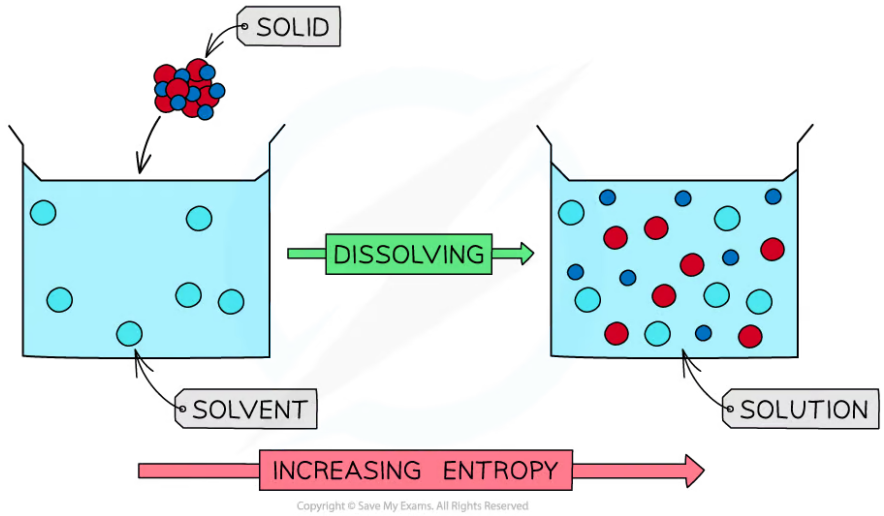
YOUR NOTES

The entropy of a substance increases when the temperature is raised as particles become more disordered

- Entropy can also increase when
  - A solid **dissolves** in a solvent
  - A gas **diffuses** in a container
- In both cases, entropy **increases** because:
  - The particles become more spread out
  - There is an increase in the number of ways of arranging the energy



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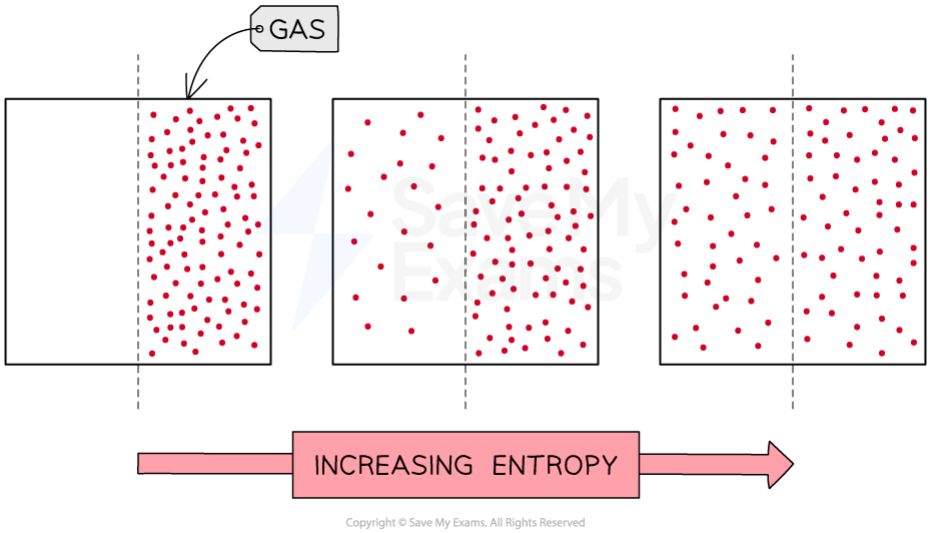
When a solid is dissolved in a solvent to form a dilute solution, the entropy increases as the particles become more disordered



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YOUR NOTES

Worked Example

A freezer door is opened while switched on and placed in a sealed room.

The entropy of the room

A. equals zero

B. increases

C. decreases

D. does not change

Answer: B

- A freezer is a heat pump, so thermal energy is transferred from inside the freezer and released at the back of the freezer
- While it runs with the door open, the internal energy of the contents of the freezer decreases
- The entropy of the contents of the freezer **decreases** because they are colder
- But the entropy of the room **increases** because it is hotter

Real Isolated Systems

- In thermodynamics, the distinction between **reversible** and **irreversible** processes at the macroscopic level is very important
- A reversible process is defined as:

  A process where there is no overall change in entropy as the system and its surroundings are returned to their original states

- Whereas, an irreversible process is defined as:

  A process which results in an increase in entropy as the system and its surroundings cannot return to their original states

- Processes in **real isolated systems** are almost always irreversible and consequently, the entropy of a real isolated system always increases

Non-Isolated Systems

YOUR NOTES

- While the entropy of an isolated system must always increase, the entropy of a non-isolated system can decrease
- An isolated system is defined as

  A system in which neither matter nor energy can be transferred in or out

- Whereas in a non-isolated system, matter and energy can be transferred in or out
  - In other words, a non-isolated system can be thought of as one which is part of a larger isolated system
- This means that the entropy of a non-isolated system can decrease locally, but this is compensated by an equal, or greater increase in the entropy of the surroundings

Calculating Changes in Entropy

- At a constant temperature *T*, the change in entropy on a **label macroscopic** level can be calculated using the equation

$$\Delta S = \frac{\Delta Q}{T}$$

- Where:
  - ΔS = change in entropy ($J\,K^{-1}$)
  - ΔQ = heat given to or removed from the system (J)
  - T = temperature of the system (K)
- When heat is given to a system (ΔQ = positive), **entropy increases**

$$\Delta S = +\frac{\Delta Q}{T}$$

$$\Delta Q > 0$$

$$\Delta S > 0$$

- When heat is removed from a system (ΔQ = negative), **entropy decreases**

$$\Delta S = -\frac{\Delta Q}{T}$$

$$\Delta Q < 0$$

$$\Delta S < 0$$

- For a reversible process that returns the system to its original state, **entropy is constant**

$$\Delta Q = 0$$

$$\Delta S = 0$$

Entropy & Microstates

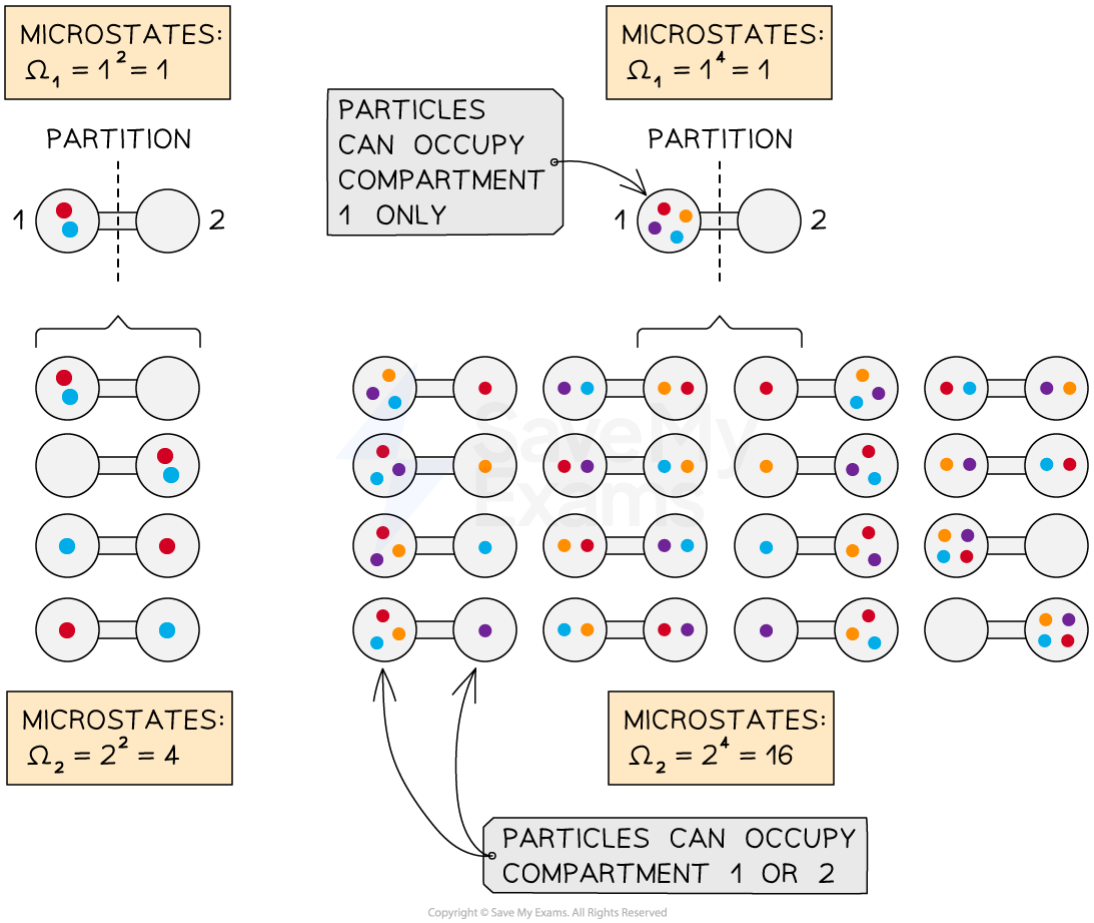
- The entropy of a system, on a **microscopic** level, can be calculated using the equation

$$S = k_B \ln \Omega$$

- Where:
  - S = entropy of a system of microscopic particles ($J\,K^{-1}$)
  - k_B = the Boltzmann constant
  - Ω = the number of possible microstates of the system
- Similarly, the change in entropy when the number of microstates increases from Ω_1 to Ω_2 is given by

$$\Delta S = k_B \ln \frac{\Omega_2}{\Omega_1}$$

- A **microstate** describes the number of states or possible arrangements of the particles in the system
  - A state can be defined by any microscopic or macroscopic property that is known about the system e.g. positions or velocities of molecules, energy, volume etc.
- An example that helps illustrate this is a two-compartment container which holds *N* distinguishable particles (i.e. each particle can be identified individually)
- Initially, all *N* particles are sealed in one of two compartments



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The number of possible microstates describes the number of different possible arrangements of particles in a system

- When the particles are confined to one compartment, we know the location of all the particles
  - Therefore, the number of microstates (possible arrangements) in the initial volume is $\Omega_1 = 1^N = 1$
  - It is **always** equal to 1, for example, when *N* = 2 or *N* = 4: $\Omega_1 = 1^2 = 1^4 = 1$

YOUR NOTES

- Once the partition is removed, the particles can spread out and occupy either one of the two compartments
  - The number of microstates (possible arrangements) in the final volume is $\Omega_2 = 2^N$
  - For example, when ***N* = 2**, the particles can be arranged 2^2 = **4 different ways**
  - Or, when ***N* = 4**, the particles can be arranged 2^4 = **16 different ways**
- The change in the entropy is therefore:

$$\Delta S = k_B \ln\left(\frac{\Omega_2}{\Omega_1}\right) = k_B \ln\left(\frac{2^N}{1^N}\right)$$

$$\Delta S = k_B \ln(2^N)$$

- It follows that the number of possible microstates can be equated to macroscopic properties of the gas, such as its **volume** increasing from *V* to 2*V*
- As the gas expands, the space it can occupy **doubles**, hence it gains an amount of entropy equal to:

$$\Delta S = Nk_B \ln(2V) - Nk_B \ln(V)$$

$$\Delta S = Nk_B \ln\left(\frac{2V}{V}\right) = Nk_B \ln(2)$$

- This gives the same result as above:

$$\Delta S = k_B \ln(2^N)$$

Exam Tip

Entropy is an incredibly important topic in physics and underpins many fundamental ideas from quantum mechanics to the determination of the Schwarzchild radius of a black hole, so don't worry if you feel a bit lost at first as it is quite a challenging concept to get your head around initially!

You might find it useful to think of microstates as a way of quantifying the **certainty** of information we have about the system

For example:

- A solid has **lower entropy** than a gas because we can be more certain about the location of the atoms in the solid
- A gas at a higher temperature (or pressure or volume) has a **higher entropy** than a similar gas at a lower temperature (or pressure or volume) because we become less certain about the location of the atoms by further increasing the possible locations they could occupy

Second Law of Thermodynamics

- The second law of thermodynamics states

  In every process, the total entropy of an isolated system always increases

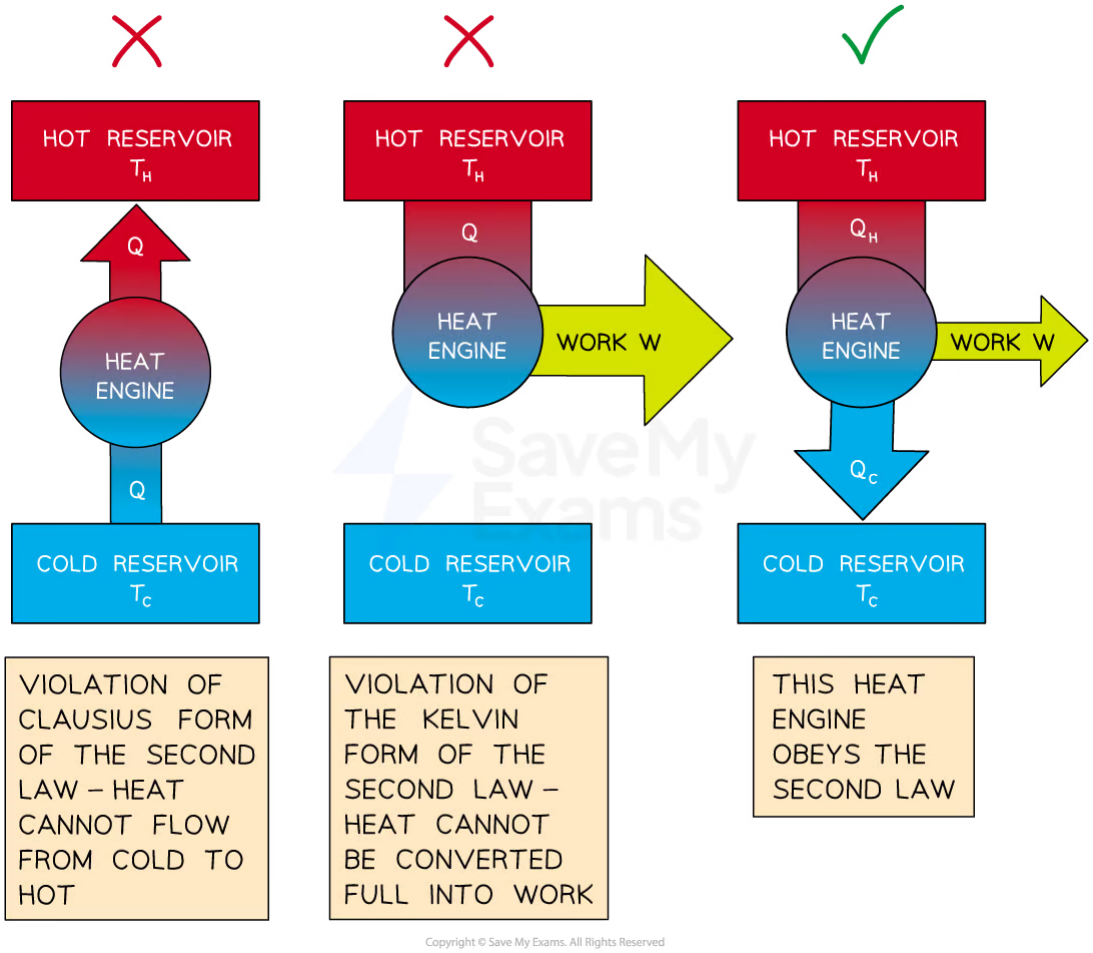
- In the case of a non-isolated system, it can be generalised to say that the entropy of the Universe must increase
- There are two alternative forms of this law which have slightly more practical applications
- The **Clausius form** of the second law states:

  Thermal energy cannot spontaneously transfer from a region of lower temperature to a region of higher temperature

- The use of the word 'spontaneously' is the key here, as heat pumps are an example of a way that heat can be transferred from a colder region to a hotter one by **doing work**
- This leads to the **Kelvin form** of the second law, which states

  When extracting energy from a heat reservoir, it is impossible to convert it all into work

- The different forms of the second law have significant implications for real-world applications, such as **heat engines**



Two of these engines cannot exist as they violate the Clausius and Kelvin forms of the second law of thermodynamics

Thermodynamic Processes

- The four main thermodynamic processes are
  - Isovolumetric $(W = 0)$
  - Isobaric $(\Delta p = 0)$
  - Isothermal $(\Delta T = 0)$
  - Adiabatic $(\Delta Q = 0)$

Constant pressure (isobaric)

- An isobaric process is defined as:

A process in which no change in pressure occurs

- This occurs when gases are allowed to expand or contract freely during a change in temperature
- When there is a change in volume ΔV at a constant pressure p, work done W is equal to

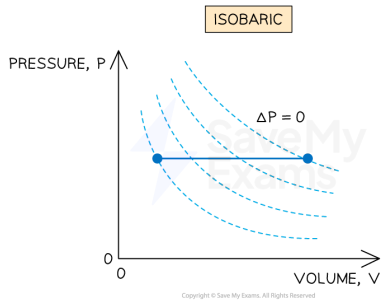
$$W = p\Delta V$$

- From the first law of thermodynamics:

$$Q = \Delta U + W$$

$$Q = \Delta U \pm p\Delta V$$

- The ± sign reflects whether work has been done on or by the gas as a result of the change in volume



Representing an isobaric process on a p-V diagram

Constant volume (isovolumetric)

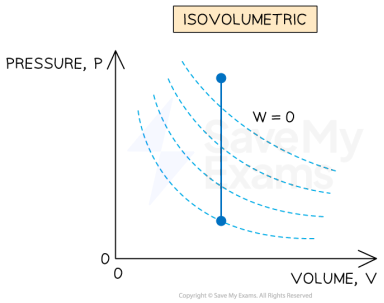
- An isovolumetric process is defined as:

A process where no change in volume occurs and the system does no work

- If there is no change in volume, then there is no work done on or by the gas, so $W = 0$
- Therefore, from the first law of thermodynamics:

$$Q = \Delta U + W = \Delta U + 0$$

$$Q = \Delta U$$



Representing an isovolumetric process on a p-V diagram

Constant temperature (isothermal)

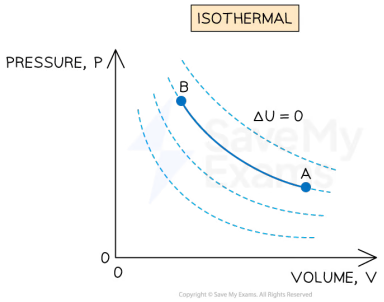
- An isothermal process is defined as:

  A process in which no change in temperature occurs

- If the temperature does not change, then the internal energy of the gas will not change, so $\Delta U = 0$
- Therefore, from the first law of thermodynamics:

$$Q = \Delta U + W = 0 + W$$

$$Q = W$$



Representing an isothermal process on a p-V diagram

Constant thermal energy (adiabatic)

- An adiabatic process is defined as:

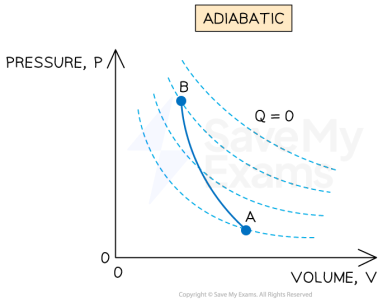
  A process where no heat is transferred into or out of the system

- If there is no heat entering or leaving the system then $Q = 0$
- Therefore, from the first law of thermodynamics:

$$Q = \Delta U + W = 0$$

$$W = -\Delta U$$

- This means that all the work done is at the expense of the system's internal energy
- Hence, an adiabatic process will usually be accompanied by a change in temperature



Representing an adiabatic process on a p-V diagram

Entropy in Thermodynamic Processes

- At a constant temperature *T*, the change in entropy is related to heat by

$$\Delta S = \frac{\Delta Q}{T}$$

- When heat is gained by a system $(\Delta Q > 0)$, entropy increases $(\Delta S > 0)$
- When heat is lost from a system $(\Delta Q < 0)$, entropy decreases $(\Delta S < 0)$
- For a reversible process $(\Delta Q = 0)$ that returns the system to its original state $(\Delta S = 0)$

Process		Heat gained or lost, ΔQ	Change in entropy, ΔS
Isothermal	Expansion	$\Delta Q > 0$ Heat gained = work done by gas	$\Delta S > 0$ Increases
	Compression	$\Delta Q < 0$ Heat lost = work done on gas	$\Delta S < 0$ Decreases
Isobaric	Expansion	$\Delta Q > 0$ Heat gained = increase in internal energy + work done by gas	$\Delta S > 0$ Increases
	Compression	$\Delta Q < 0$ Heat lost = decrease in internal energy + work done on gas	$\Delta S < 0$ Decreases
Isovolumetric	Pressure rise	$\Delta Q > 0$ Heat gained due to temperature rise	$\Delta S > 0$ Increases

	Pressure drop	$\Delta Q < 0$ Heat lost due to temperature drop	$\Delta S < 0$ Decreases
Adiabatic	Expansion	$\Delta Q = 0$ Pressure & temperature decrease with no heat gained or lost	$\Delta S = 0$ No change
	Compression	$\Delta Q = 0$ Pressure & temperature increase with no heat gained or lost	$\Delta S = 0$ No change

YOUR NOTES

YOUR NOTES

Worked Example

A quantity of energy Q is supplied to three ideal gases, **X**, **Y** and **Z**.

Gas **X** absorbs Q isothermally, gas **Y** isovolumetrically and gas **Z** isobarically.

Complete the table by inserting the words 'positive', 'zero' or 'negative' for the work done *W*, the change in internal energy ΔU and the temperature change ΔT for each gas.

	W	ΔU	ΔT
X			
Y			
Z			

Answer:

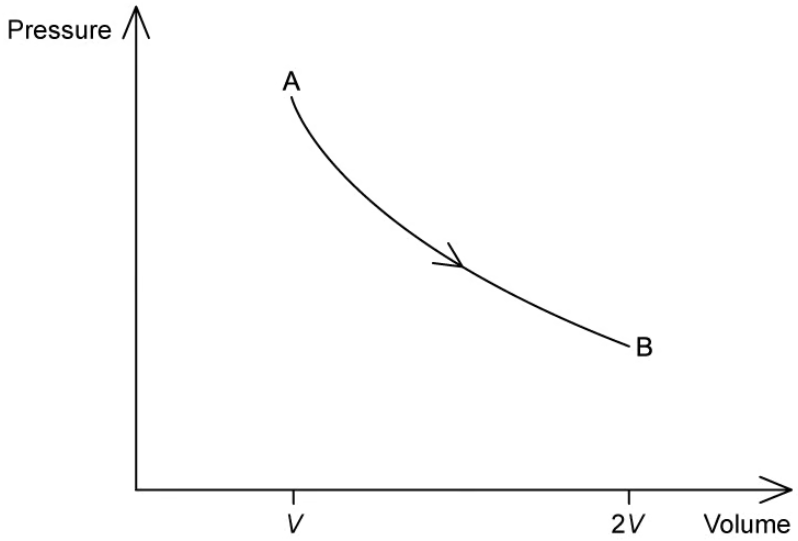
- **X**: Isothermal = constant temperature, no change in internal energy
  - Temperature: $\Delta T = 0$
  - Internal energy: $\Delta T \propto \Delta U$, so, $\Delta U = 0$
  - Work done: $Q = \Delta U + W \quad \Rightarrow \quad Q = +W$
- **Y**: Isovolumetric = constant volume, no work done
  - Work done: $W \propto \Delta V$, so, $W = 0$
  - Internal energy: $Q = \Delta U + W \quad \Rightarrow \quad Q = +\Delta U$
  - Temperature: $\Delta T \propto \Delta U$, so, $\Delta T > 0$
- **Z**: Isobaric = constant pressure
  - Work done: $\Delta p = 0$, so $W = p\Delta V$, so $W > 0$
  - Internal energy: $Q = \Delta U + W$, so $\Delta U > 0$
  - Temperature: $\Delta T \propto \Delta U$, so $\Delta T > 0$

	W	ΔU	ΔT
X	positive	0	0
Y	0	positive	positive
Z	positive	positive	positive

YOUR NOTES

Worked Example

A heat engine operates on the cycle shown in the pressure-volume diagram. One step in the cycle consists of an isothermal expansion of an ideal gas from state A of volume *V* to state B of volume 2*V*.



(a)

On the graph, complete the cycle ABCA by drawing curves to show

- an isovolumetric change from state B to state C
- an adiabatic compression from state C to state A

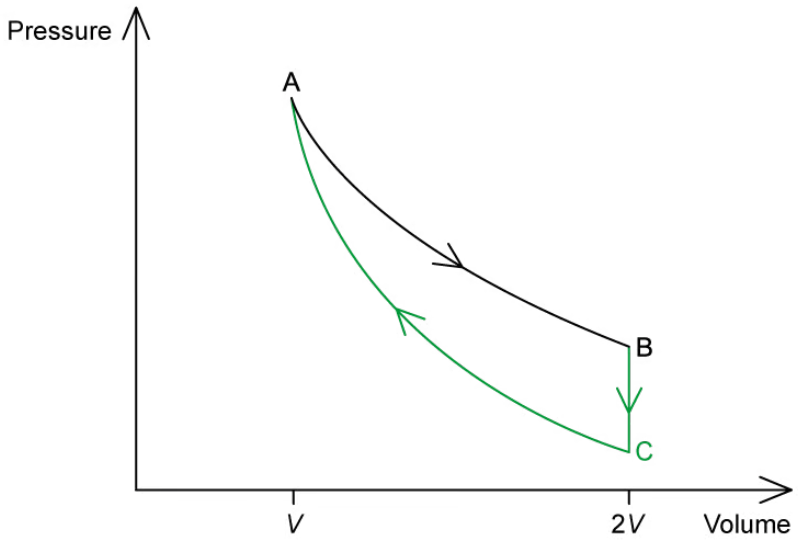
(b)

State and explain at which point in the cycle ABCA the entropy of the gas is the largest.

Answer:

(a)

- **Isovolumetric** = constant volume, no work done
- Next step is a compression (where pressure increases), so this step should involve a **pressure drop**
  - Hence, B to C: line drawn vertically down
- **Adiabatic** = no heat supplied or removed, **compression** = work is done on the gas, volume decreases
  - Hence, C to A: line curves up to meet A



(b)

- Entropy and heat (at a constant T) are related by

$$\Delta S = \frac{\Delta Q}{T}$$

- From state A to state B:
  - In an isothermal expansion, entropy **increases**
  - Because T = constant but the volume increases so work is done by gas, $\Delta Q > 0$ so $\Delta S > 0$
- From state B to state C:
  - In an isovolumetric change where pressure decreases, entropy **decreases**
  - Because temperature decreases, so energy has been removed, $\Delta Q < 0$ so $\Delta S < 0$
- From state C to state A:
  - In an adiabatic compression, entropy is **constant**
  - Because it is an adiabatic process, $\Delta Q = 0$ so $\Delta S = 0$
- Therefore, entropy is **greatest** at B

Adiabatic Processes

YOUR NOTES

- Adiabatic processes in monatomic ideal gases can be modelled by the equation

$$pV^{\frac{5}{3}} = constant$$

- Where:
  - p = pressure of the gas (Pa)
  - V = volume occupied by the gas (m^3)
- This equation can be used for calculating changes in pressure, volume and temperature for monatomic ideal gases

$$p_1 V_1^{\frac{5}{3}} = p_2 V_2^{\frac{5}{3}}$$

- Where:
  - p_1 = initial pressure (Pa)
  - p_2 = final pressure (Pa)
  - V_1 = initial volume (m^3)
  - V_2 = final volume (m^3)

YOUR NOTES

Worked Example

An ideal monatomic gas expands adiabatically from a state with pressure 7.5×10^5 Pa and volume 1.8×10^{-3} m^3 to a state of volume 4.2×10^{-3} m^3.

Calculate the new pressure of the gas.

Answer:

- For an ideal monatomic gas undergoing an adiabatic change:

$$pV^{\frac{5}{3}} = C$$

$$p_1 V_1^{\frac{5}{3}} = p_2 V_2^{\frac{5}{3}}$$

- Where:
  - Initial pressure, $p_1 = 7.5 \times 10^5$ Pa
  - Final pressure = p_2
  - Initial volume, $V_1 = 1.8 \times 10^{-3}$ m^3
  - Final volume, $V_2 = 4.2 \times 10^{-3}$ m^3

$$p_2 = p_1 \left(\frac{V_1}{V_2} \right)^{\frac{5}{3}}$$

$$p_2 = (7.5 \times 10^5) \times \left(\frac{1.8 \times 10^{-3}}{4.2 \times 10^{-3}} \right)^{\frac{5}{3}}$$

New pressure: $p_2 = 1.8 \times 10^5$ Pa

YOUR NOTES

Worked Example

An ideal monatomic gas is compressed adiabatically from a state with volume 3.1×10^{-3} m^3 and temperature 590 K to a state of volume 2.1×10^{-3} m^3.

Calculate the new temperature of the gas.

Answer:

- For an ideal monatomic gas undergoing an adiabatic change:

$$pV^{\frac{5}{3}} = C$$

$$p_1 V_1^{\frac{5}{3}} = p_2 V_2^{\frac{5}{3}}$$

- From the ideal gas law:

$$pV = nRT \quad \Rightarrow \quad p = \frac{nRT}{V}$$

$$\left(\frac{nRT_1}{V_1}\right) V_1^{\frac{5}{3}} = \left(\frac{nRT_2}{V_2}\right) V_2^{\frac{5}{3}}$$

$$T_1 V_1^{\frac{2}{3}} = T_2 V_2^{\frac{2}{3}}$$

- Where:
  - Initial temperature, $T_1 = 590$ K
  - Final temperature = T_2
  - Initial volume, $V_1 = 3.1 \times 10^{-3}$ m^3
  - Final volume, $V_2 = 2.1 \times 10^{-3}$ m^3

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\frac{2}{3}}$$

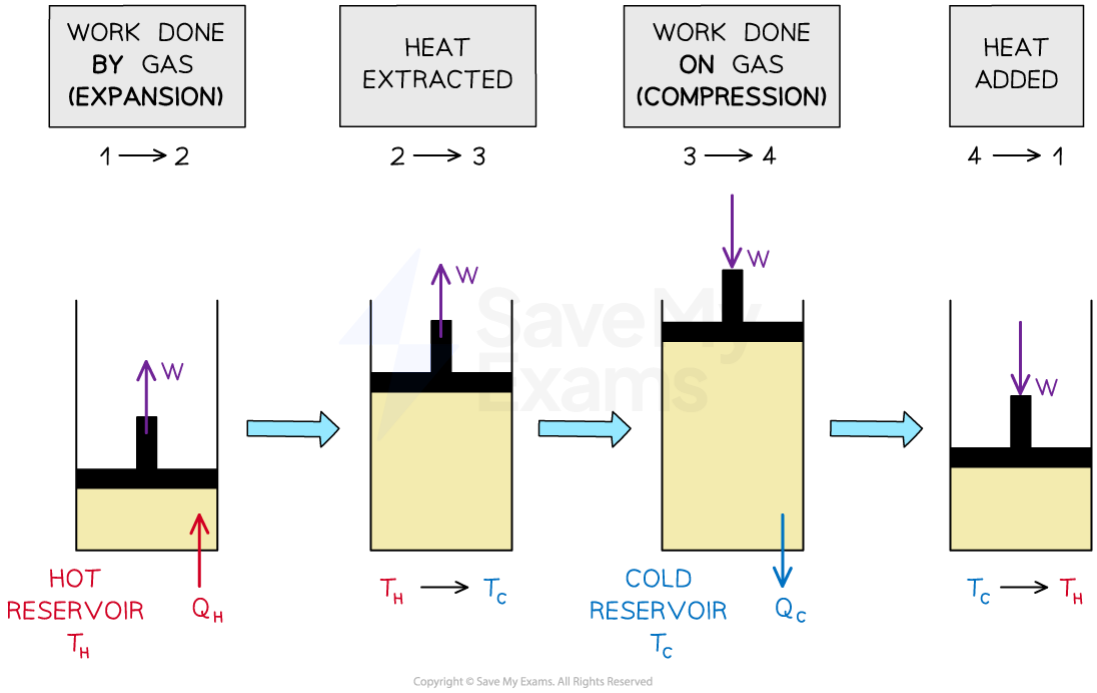
$$T_2 = 590 \times \left(\frac{3.1 \times 10^{-3}}{2.1 \times 10^{-3}}\right)^{\frac{2}{3}}$$

New temperature: $T_2 = 765$ K

YOUR NOTES

Heat Engines

- A heat engine is a device that converts **thermal energy** into **mechanical work**
- Heat engines operate through a series of thermodynamic processes which form a **closed** cycle
  - A closed cycle is one in which the system returns to its initial state
- A simple heat engine consists of a gas in a cylinder with a piston



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A simple heat engine converts thermal energy into mechanical work

- The steps in the operation of a cyclic heat engine process are:

1. Extract heat from a hot reservoir

- A hot reservoir (a source of thermal energy) at a high temperature T_H transfers heat Q_H into the engine

2. Use some of the extracted heat to perform work

- The gas does mechanical work as it expands which pushes the piston out

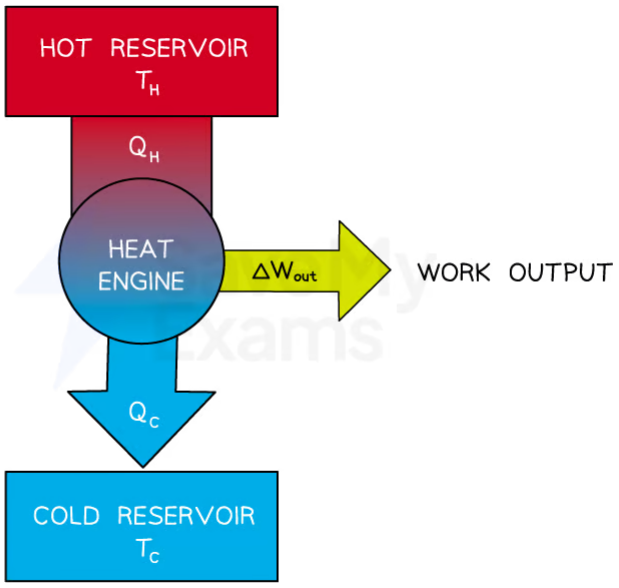
3. Release excess heat into a cold reservoir

- The gas is allowed to cool at constant volume, meanwhile, heat Q_C is released to the surroundings

- Some of the energy transferred into the engine is released into a cold reservoir (a sink for excess heat) at a lower temperature T_C

4. Repeat cycle

- Once the heat has been extracted, the piston is pushed down to compress the gas back to its original state
- The process can then be repeated as many times as needed, continuously converting heat into mechanical work



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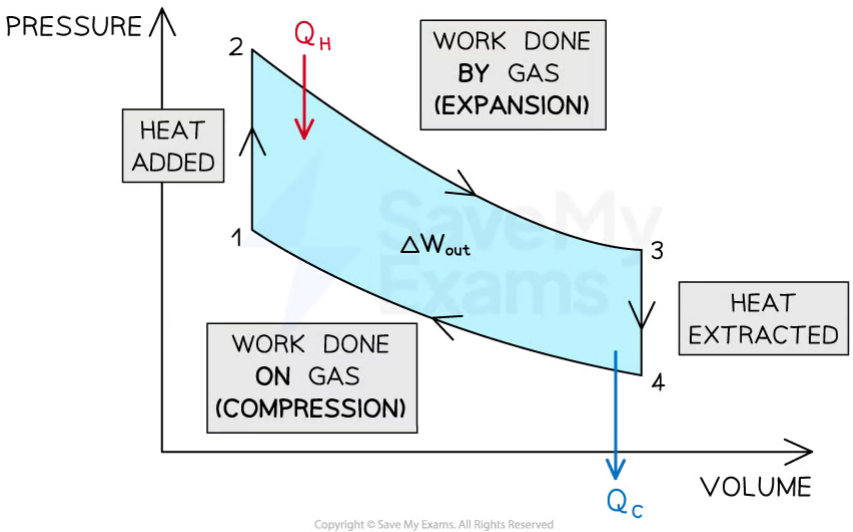
- For a cyclic heat engine process, the p-V diagram will form a closed loop
- The **area** inside the loop is equal to the **net work done** during one cycle



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- The net work done by the engine is:

$$\Delta W_{out} = Q_H - Q_C$$

- Where:
  - ΔW_{out} = useful work output of the heat engine (J)
  - Q_H = heat transferred from hot reservoir to engine (J)
  - Q_C = heat transferred from engine to cold reservoir (J)

YOUR NOTES

Efficiency of Heat Engines

YOUR NOTES

- The goal of a heat engine is to transfer thermal energy into useful mechanical work as efficiently as possible
- The thermodynamic efficiency of a heat engine can be calculated using

$$\text{efficiency} = \frac{useful\ work\ output}{total\ energy\ input}$$

$$\eta = \frac{W_{out}}{Q_H} = \frac{(Q_H - Q_C)}{Q_H}$$

$$\eta = 1 - \frac{Q_C}{Q_H}$$

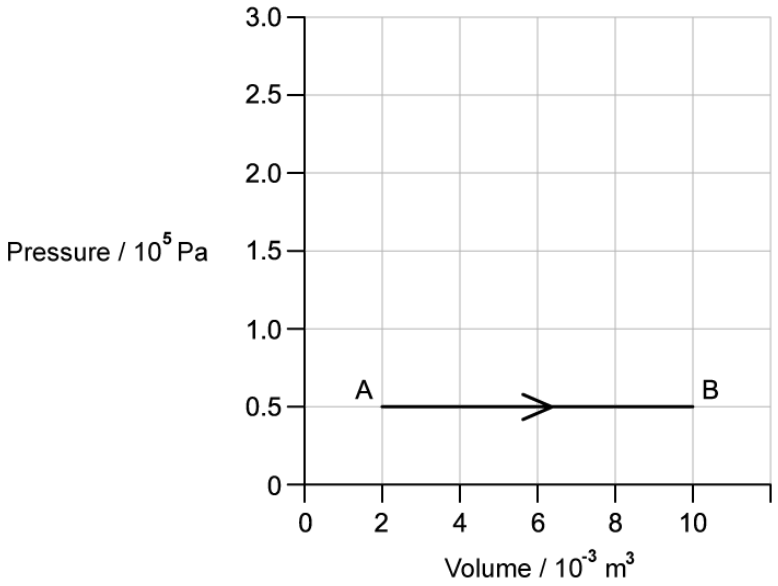
- Where:
  - η = efficiency of a heat engine
  - W_{out} = useful work output (J)
  - Q_H = total energy input from the hot reservoir (J)
  - Q_C = energy lost to the cold reservoir (J)

YOUR NOTES

Worked Example

The pressure-volume (*pV*) diagram shows part of a cycle ABCA of a heat engine.

The cycle consists of a change AB, followed by an adiabatic compression BC and then the gas cools at a constant volume to its original state.



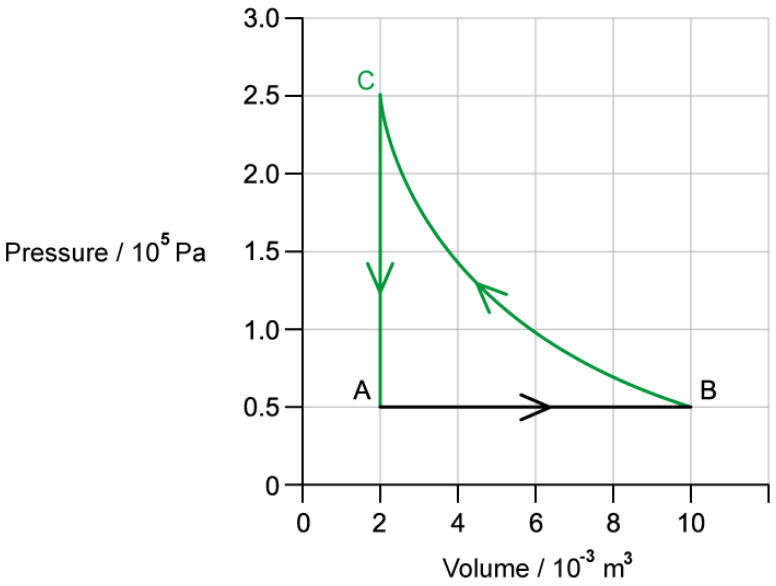
At C the pressure of the gas is 2.5×10^5 Pa. The total thermal energy transferred to the gas during the cycle is 1250 J.

(a)
Sketch, on the *pV* diagram, the complete cycle ABCA.
(b)
State the names of processes AB and CA.
(c)
Estimate the net useful work done by the gas during the cycle.
(d)
Determine the efficiency of the heat engine.

Answer:

(a)

- From B to C: adiabatic compression, so
  - The pressure increases to 2.5×10^5 Pa
  - The volume decreases to 2×10^{-3} m^3 (same as A)
- From C to A: the gas cools at a constant volume
  - The pressure decreases to 0.5×10^5 Pa
  - The volume stays at 2×10^{-3} m^3 (same as A)

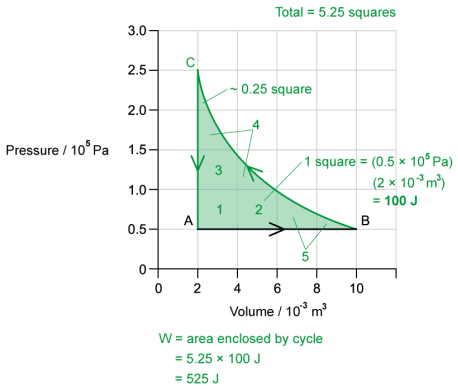


(b)

- Process AB = **isobaric expansion** (constant pressure, volume increasing)
- Process CA = **isovolumetric** temperature drop (constant volume, pressure/temperature decreasing)

(c)

- The net useful work done by the gas during the cycle is given by the area enclosed by the loop



- Work done by the gas $\approx 5.25 \times (0.5 \times 10^5) \times (2 \times 10^{-3}) = 525$ J

(d)

- The efficiency of a heat engine is given by

$$\eta = \frac{W_{out}}{Q_H}$$

- Where

- Useful work output, W_{out} = 525 J
- Total energy input, Q_H = 1250 J

$$\eta = \frac{525}{1250} = 0.42$$

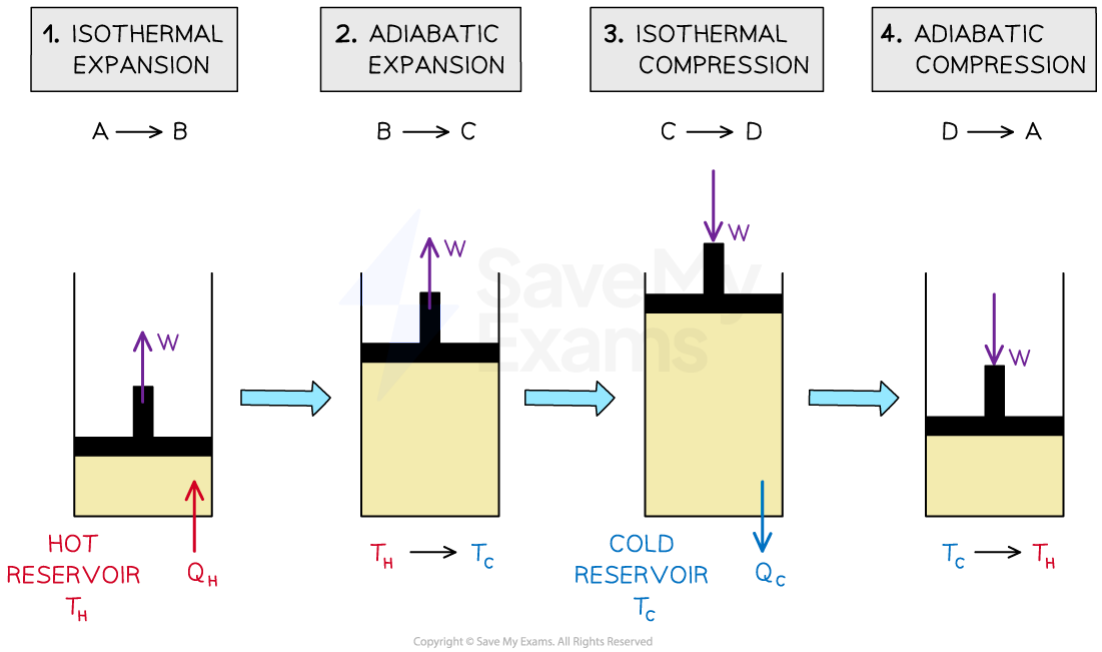
- Therefore, the heat engine has an efficiency of about 42%

YOUR NOTES

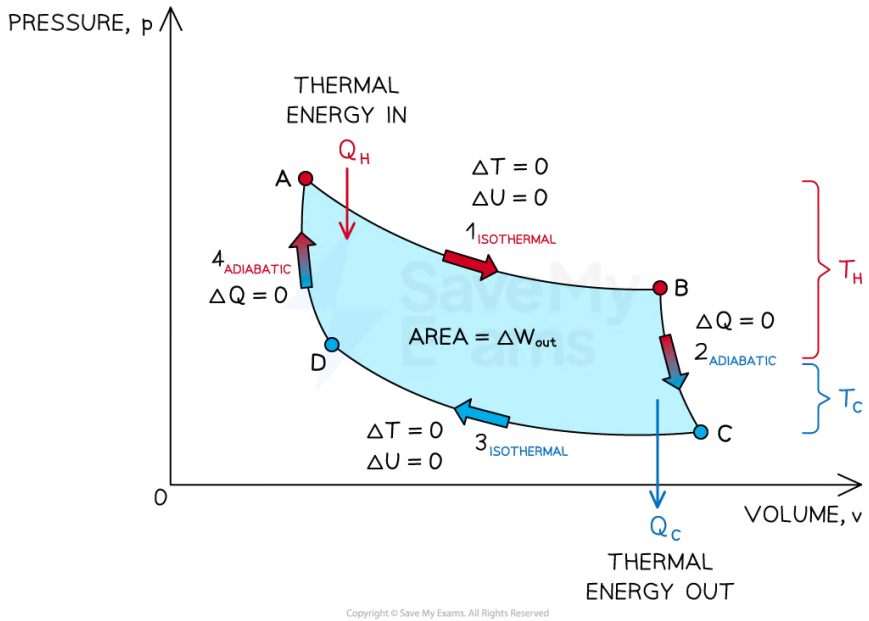
YOUR NOTES

The Carnot Cycle

- A thermodynamic system that runs at its greatest possible efficiency follows a cycle called the **Carnot cycle**



The four stage Carnot cycle of a gas in a piston



p-V diagram for the Carnot cycle. The enclosed area equals the work done after one cycle

- The Carnot cycle is an idealised and reversible process

YOUR NOTES

- It consists of four stages:

1. Isothermal expansion

  - The gas absorbs heat Q_H from a hot reservoir at temperature T_H
  - Work is done by the gas as it expands i.e. volume increases, temperature is constant $(\Delta T = 0)$
  - Work done by the gas = heat gained Q_H

2. Adiabatic expansion

  - The gas continues to expand
  - The gas does work on the surroundings as its volume increases and pressure decreases
  - The gas cools down from T_H to T_C, but no thermal energy is transferred $(\Delta Q = 0)$

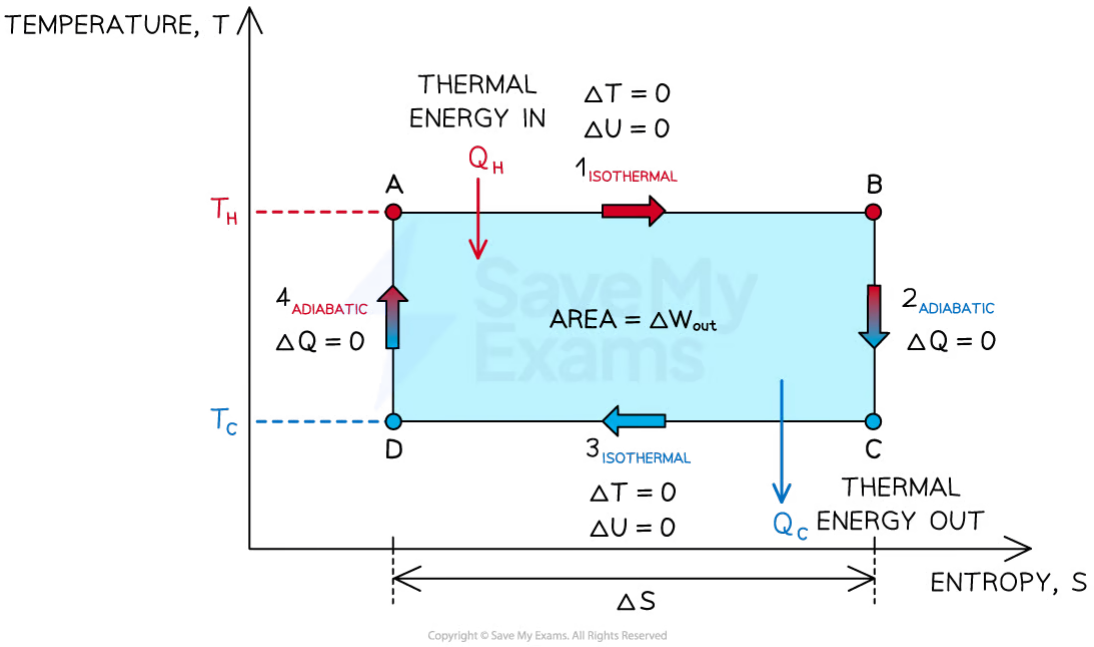
3. Isothermal compression

  - The gas is compressed and transfers heat Q_C to a cold reservoir at temperature T_C
  - Work is done on the gas as it is compressed i.e. volume decreases, temperature is constant $(\Delta T = 0)$
  - Work done on the gas = heat lost Q_C

4. Adiabatic compression

  - The gas continues to be compressed
  - Work is done on the gas as its volume decreases and pressure increases
  - The gas heats up from T_C to T_H, but no thermal energy is transferred $(\Delta Q = 0)$

- At the end of the fourth stage, the gas has returned to its original state and the cycle can be repeated as many times as needed



The variation of temperature and entropy throughout the Carnot cycle

- As the efficiency of a thermodynamic system increases, the **difference** between the temperatures of the hot and cold reservoirs increases
- The maximum theoretical efficiency of a heat engine using the Carnot cycle is:

$$\eta_C = 1 - \frac{T_C}{T_H}$$

- Where:
  - η_C = maximum theoretical efficiency (Carnot cycle only)
  - T_C = temperature in the cold reservoir (K)
  - T_H = temperature in the hot reservoir (K)

YOUR NOTES

Worked Example

In an idealised heat engine, the hot and cold reservoirs are held at temperatures of T_H and T_C respectively.

Using the equation for the change in entropy

$$\Delta S = \frac{\Delta Q}{T}$$

Show that the maximum theoretical efficiency of a heat engine is given by

$$\eta_C = 1 - \frac{T_C}{T_H}$$

Answer:

Step 1: Determine the change in entropy during isothermal expansion

- In isothermal expansion (AB): the gas absorbs heat Q_H from a hot reservoir at temperature T_H
- Therefore, the increase in entropy is:

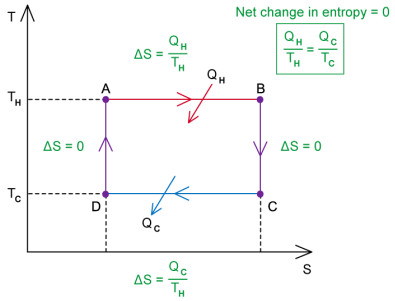
$$\Delta S_{AB} = \frac{Q_H}{T_H}$$

Step 2: Determine the change in entropy during isothermal compression

- In isothermal compression (CD): the gas transfers heat Q_C to a cold reservoir at temperature T_C
- Therefore, the decrease in entropy is:

$$\Delta S_{CD} = -\frac{Q_C}{T_C}$$

Step 3: Consider the net change in entropy over the cycle



- During adiabatic expansion (BC) and compression (DA), entropy does not change as there is no thermal energy gained or lost

$$\Delta S_{BC} = \Delta S_{DA} = 0$$

- We know that the overall entropy of the system does not change in a cyclic process, therefore

$$\Delta S = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CD} + \Delta S_{DA}$$

$$\Delta S = \frac{Q_H}{T_H} + 0 + \left(-\frac{Q_C}{T_C}\right) + 0 = 0$$

$$\frac{Q_H}{T_H} = \frac{Q_C}{T_C} \quad \Rightarrow \quad \frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

Step 4: Substitute the expression into the equation for the efficiency of a heat engine

- The efficiency of a heat engine is given by:

$$\eta = \frac{useful\ work}{input\ energy} = \frac{W}{Q_H}$$

- Where useful work is $W = Q_H - Q_C$

$$\eta = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

- Combining with the expression derived above gives:

$$\eta_C = 1 - \frac{T_C}{T_H}$$

YOUR NOTES

Worked Example

An engineer designs a heat engine that has an inlet temperature of 500 K and an outlet temperature of 300 K. The engineer claims that 100 kJ of thermal energy flows out of the hot reservoir and 25 kJ of thermal energy flows into the cold reservoir.

Determine, with reference to the second law of thermodynamics, whether this engine is thermodynamically possible.

Answer:

Step 1: Determine the efficiency of the proposed engine

- The efficiency of this engine would be

$$\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

- Where:
  - Heat transferred in, Q_H = 100 kJ
  - Heat transferred out, Q_C = 25 kJ

$$\text{Efficiency} = 1 - \frac{25}{100} = 0.75 = 75\%$$

Step 2: Determine the maximum theoretical (Carnot) efficiency of the proposed engine

- A Carnot engine operating between the same temperatures would have an efficiency of

$$\eta_C = 1 - \frac{T_C}{T_H}$$

- Where:
  - Inlet temperature, T_H = 500 K
  - Outlet temperature, T_C = 300 K

$$\text{Carnot efficiency: } \eta_C = 1 - \frac{300}{500} = 0.4 = 40\%$$

Step 3: Discuss the proposed engine in relation to the second law

- The Clausius form of the second law states: it is impossible for heat to flow from a cooler body to a hotter body without performing work
- This law sets an upper limit on the maximum possible efficiency of the transfer of thermal energy to mechanical energy in a heat engine

- The maximum possible efficiency of the proposed engine is 40%, but the engineer is proposing an efficiency of 75% i.e. an efficiency greater than the Carnot efficiency
- This violates the Clausius form of the second law, hence **the proposed engine is impossible**

YOUR NOTES

IB Physics HL

YOUR NOTES

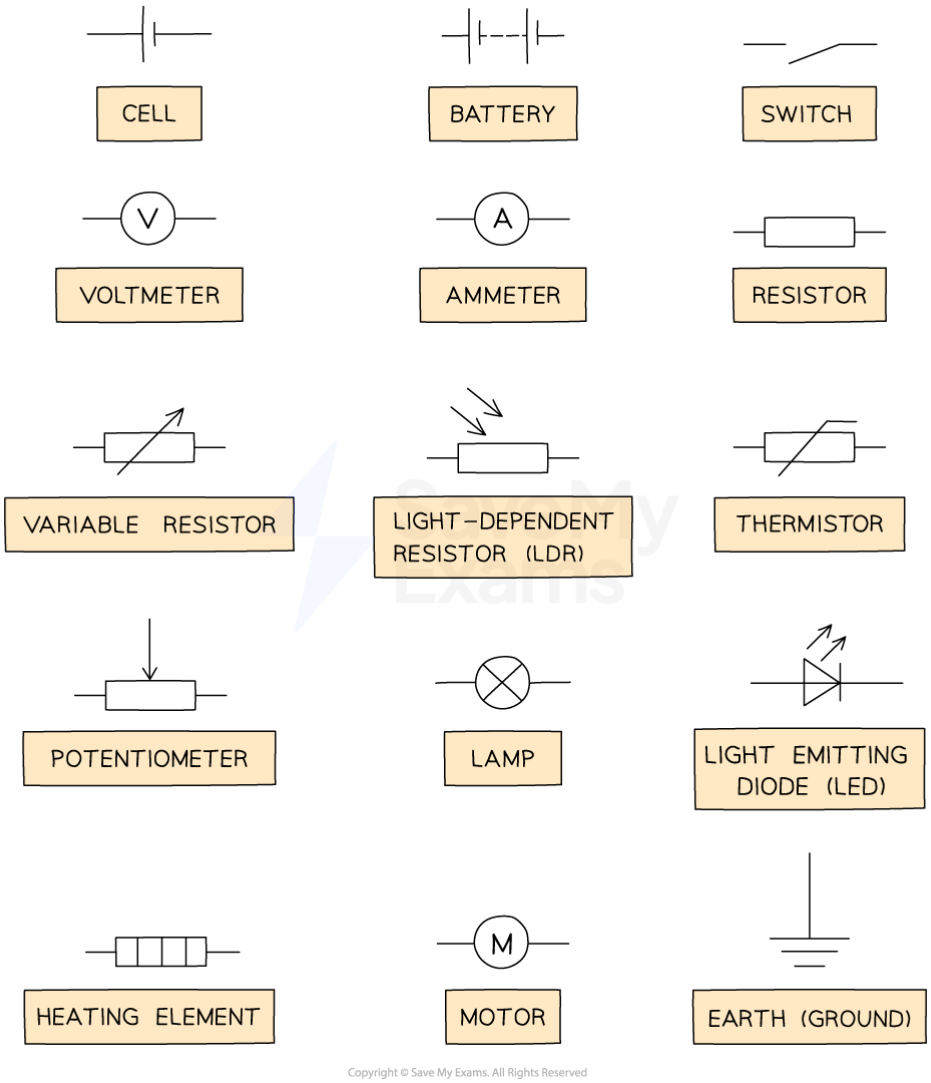
Current & Circuits

CONTENTS

Circuit Diagrams
Electric Current
Electric Potential Difference
Electrical Conductors & Insulators
Electric Resistance
Electrical Resistivity
I-V Characteristics
Series & Parallel Circuits
Electrical Power
Sources of Electrical Energy
Electromotive Force & Internal Resistance
Variable Resistance

Circuit Diagrams

- Circuit symbols are used to represent electrical components in circuit diagrams
- These symbols are universally recognised by all scientists in any language
- The following symbols are commonly used in circuit diagrams



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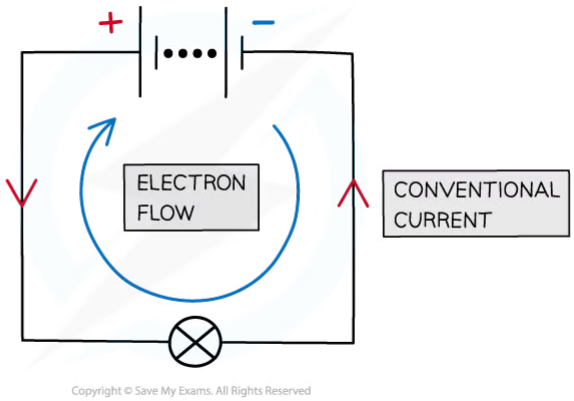
Circuit symbols

Cells & Batteries

- A device which converts energy from a chemical energy store to electrical store (one battery)
  - The direction of the current flow is from the **positive** (longer side) to the **negative** (shorter side) terminal
  - This is the opposite direction to the electron flow

YOUR NOTES



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- A **battery** is simply a power source made up of multiple cells

Switch

- A device which turns the circuit on (closed), or off (open)
- The switch allows or prevents the flow of current

Voltmeters & Ammeters

- A **voltmeter** is a device which measures the potential difference between two points in a circuit
- An **ammeter** is a device which measures the current flowing in a circuit

Fixed resistor

- A device which increases resistance to limit the flow of current
- As electrons flow through a resistor, they transform energy from its electric potential energy store into other stores (e.g. thermal energy)

Variable resistor

- A resistor with a slider that can be used to **change** its **resistance**
- As the resistance of the variable resistor increases, the current in the circuit decreases and vice versa

Light-dependent resistor (LDR):

- A resistor whose resistance depends on the light intensity
- As light intensity increases, the resistance of an LDR decreases and vice versa

Thermistor:

- A resistor whose resistance depends on its temperature
- As temperature increases, the resistance of a thermistor decreases and vice versa

Potentiometer

- A resistor with a sliding contact to form an adjustable voltage divider

Lamp

- A lighter emitting component consisting of heating a filament inside a glass cover

YOUR NOTES

Light-emitting diode (LED)

- A device that emits light when a current passes through it
- A diode only allows current to flow in one direction only

Heating element

- An element that converts energy from an electrical store into a thermal store through the process of resistance

Motor

- An element that converts energy from an electrical store into a mechanical store

Earth (ground)

- The point in the circuit which is grounded i.e. connected to the Earth
- A connection to the Earth allows an instantaneous discharge to occur if an appliance malfunctions
- Electrons are transferred directly to the Earth through a low-resistance wire

Drawing Circuit Diagrams

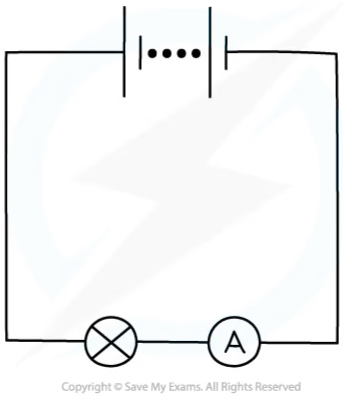
- Circuit **diagrams** represent the **arrangement** of components in a circuit
  - This is important, as some components need to be in a certain position in relation to the others to work
- Being able to draw and interpret circuit diagrams using circuit symbols is an essential skill in the electricity topic

A circuit diagram must include:

- An **energy source**
  - This is a source of **potential difference** so a current can flow
  - This can be a cell, battery, or a power supply
- A **closed path** or a **complete circuit**
  - Electrons need to flow in a complete loop for a current to flow
  - A circuit can be open and closed using a switch
- **Electrical components** (using the correct circuit symbol)
  - These could act as sensors that respond to the environment (LDR, thermistor)
  - Or, measure a value (ammeter, voltmeter)
  - Or, transfer electrical energy to other forms of energy (LED, lamp)

Measuring Current

- Electric current is measured using an **ammeter**
- Ammeters should always be connected in **series** within a circuit



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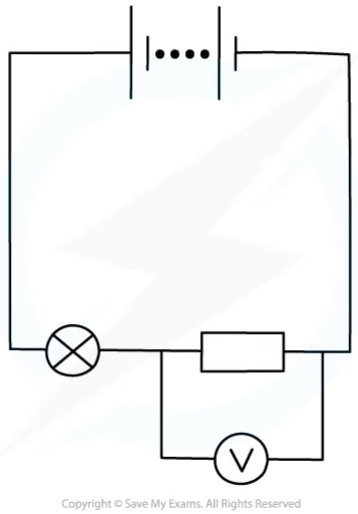
To measure the current flowing through a light bulb, an ammeter must be connected in series with it

- An **ideal** ammeter should have zero resistance
  - This way, it will not take any energy from the electrons flowing through it
  - Otherwise, it would alter the value of the current it is trying to measure

Measuring Potential Difference

- Potential difference (or voltage) is measured using a **voltmeter**
- A voltmeter is always set up **in parallel** to the component being measured
- This means that the voltmeter must be connected across the terminals of the component



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To measure the potential difference of a fixed resistor, a voltmeter must be connected in parallel to it

- A measurement of potential difference represents the difference between the values of electric potential on either side of the component
  - This is because 1 Volt (V) is equivalent to 1 Joule per Coulomb ($J\,C^{-1}$)
- Therefore, the key rules to remember are:

- An ammeter is always connected in **series**
- A voltmeter is always connected in **parallel** to the component the voltage is being measured
- The direction of current flow is always from the **positive** to the **negative** terminal of the power supply

YOUR NOTES

Exam Tip

The standard circuit symbols are given in the data booklet, so you do not need to memorise them. However, you must be able to identify them and draw them correctly.

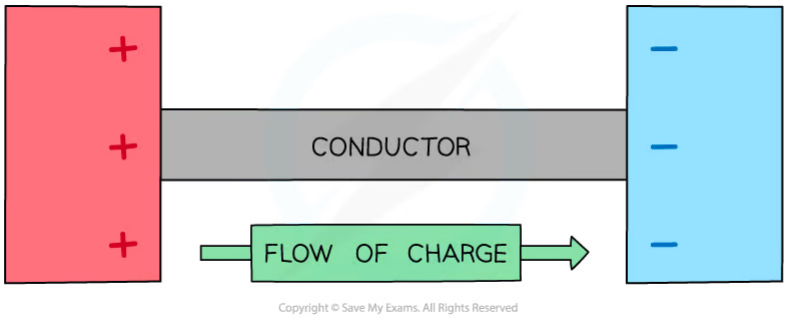
When asked to draw a circuit diagram, make sure to draw the wires as straight lines with a straight edge or a ruler and make it as neat as possible, especially the circuit symbols.

If the diagram is too small or there is ambiguity as to what a symbol represents, the examiner may not award you full marks!

YOUR NOTES

Electric Current

- Electric current is **the rate of flow of charge** carriers and is measured in units of **amperes (A)** or **amps**
- Charge can be either **positive** or **negative** and is measured in units **coulombs (C)**
- When two oppositely charged conductors are connected together (by a length of wire)
  - The charge will flow between the two conductors, giving rise to a current
  - The greater the flow of charge, the greater the electric current



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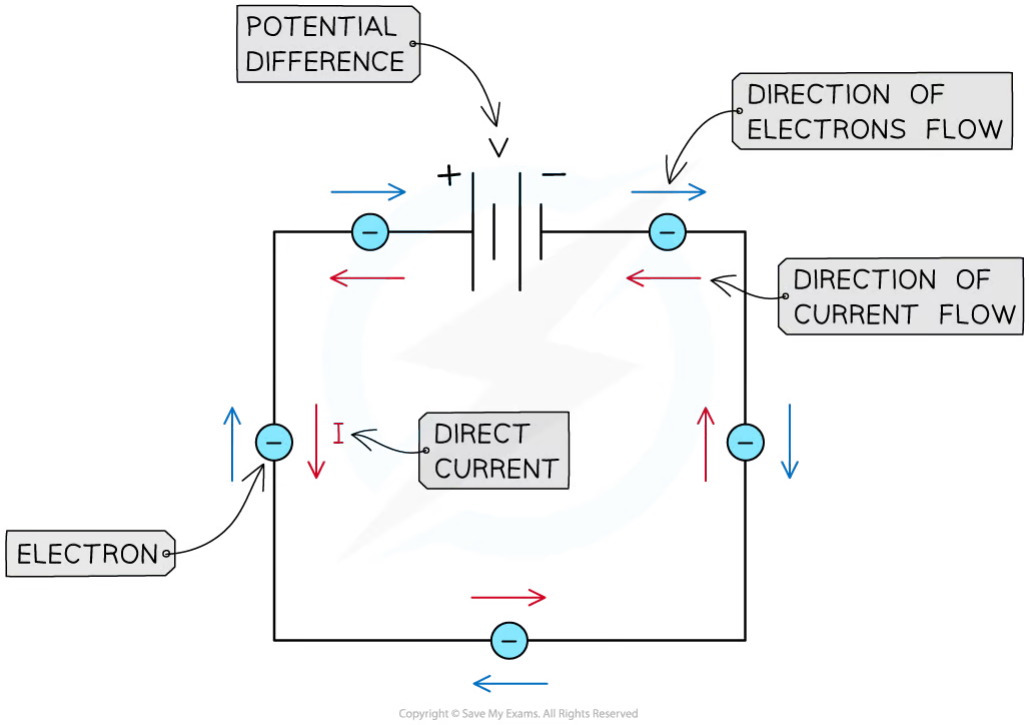
Charge can flow between two conductors. The direction of conventional current in metal is from positive to negative

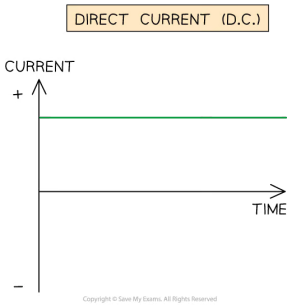
- In electrical wires, the current is a flow of **electrons**
- Electrons are **negatively** charged so they flow **away** from the negative terminal of a cell **towards** the positive terminal
- **Conventional current** is defined as the flow of positive charge from the **positive** terminal of a cell to the **negative** terminal
  - This is the **opposite** of the direction of electron flow, as the conventional current was described before the electric current was really understood



Direct current flows from the positive to the negative terminal of the battery in a circuit. Electrons flow in the opposite direction

- The potential difference in the circuit causes the current to flow
- Current is measured using an **ammeter**

- **Direct current** (**dc**) flows through the circuit in **one direction**
  - The direction of conventional current is from the **positive** terminal to the **negative** one
  - This is **opposite** to the **electrons** flow
- Direct current is produced when from cells and batteries



Direct current is current that only flows in one direction and has one value

YOUR NOTES

- The equation for current is:

$$I = \frac{\Delta q}{\Delta t}$$

- Where:
  - I = current (A)
  - Δq = change in charge (C)
  - Δt = time interval (s)

Worked Example

When will 8 mA of current pass through an electrical circuit?

A. When 1 J of energy is used by 1 C of charge

B. When a charge of 4 C passes in 500 s

C. When a charge of 8 C passes in 100 s

D. When a charge of 1 C passes in 8 s

Answer: B

- The equation relating current, charge and time is:

$$I = \frac{\Delta q}{\Delta t}$$

- Consider option **B**:

$$I = \frac{4}{500} = 8 \times 10^{-3} = 8\ \text{mA}$$

- Therefore, the correct answer is **B**
- **A** is incorrect as it does not contain a value for charge or time, so it can be ruled out
- **C** is incorrect as:

$$I = \frac{8}{100} = 80 \times 10^{-3} = 80\ \text{mA}$$

- **D** is incorrect as:

$$I = \frac{1}{8} = 125 \times 10^{-3} = 125\ \text{mA}$$

Exam Tip

Remember that conventional current flow is always from the **positive** to the **negative** terminal. This is important in understanding circuits and whether components such as an LED, which only works if the diode points in the direction of conventional current flow.

Current can be positive or negative. It is still a scalar quantity, but the sign just indicates its **direction**. Direct current (dc) will only ever be in one direction.

YOUR NOTES

Electric Potential Difference

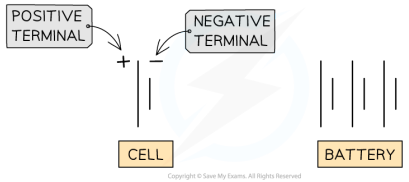
- Potential difference (p.d.) is a measure of the electrical **potential energy** transferred by an electron as it moves between two points in a conductor
- The definition of potential difference, also known as voltage, is:

> **The work done per unit charge on moving a positive charge between two points along the path of the current**

- Potential difference is measured in volts (V) and is calculated as follows:

$$V = \frac{W}{q}$$

- Where:
  - V = potential difference (V)
  - W = work done (J)
  - q = charge (C)
- From the above equation, one volt is equal to one joule per unit coulomb
  - $1\,V = 1\,J\,C^{-1}$
- The potential difference in a circuit is provided by **cells** or **batteries**
  - Each cell has a **positive terminal** (high potential location) and a **negative terminal** (low potential location)
  - A battery is a collection of cells arranged positive terminal to negative terminal



A cell and a battery made of three cells

- When both terminals of a cell, or battery, are connected to a loop of conducting wire, a **circuit** is formed
- The battery is the **source** of the potential difference needed for the electrons to flow
- Electrons gain electrical potential energy as they move through the battery and then through the wire
  - A little amount of their energy is transferred to the metal atoms in the wire
  - The flow of electrons is from the negative to the positive terminal

The Electronvolt

- The energy values associated to electrons and other microscopic particles are very small when expressed in SI units

YOUR NOTES

- For this reason, it is often more convenient to use another unit for energy - the **electronvolt** (**eV**)
- The electronvolt is defined as follows:

> **The amount of energy needed to move an electron through a potential difference of one volt**

- Electronvolts are measured in Joules (J), where e = elementary charge (C)

Worked Example

Determine the value of 1 eV in joules (J).

Answer:

Step 1: Recall the definition of electronvolt

- One electronvolt is the work *W* associated with an electron of charge e moving through a potential difference *V* = 1V

$$W = qV = eV$$

- Where e is the charge of an electron = 1.6×10^{-19} C

Step 2: Substitute this and the value of the voltage into the above equation for *W*

$$W = (1.6 \times 10^{-19}\ \text{C}) \times 1\ \text{V}$$

$$W = 1.6 \times 10^{-19}\ \text{J}$$

- One electronvolt is equal to 1.6×10^{-19} joules

Exam Tip

The value for the charge of an electron is given on your data sheet.

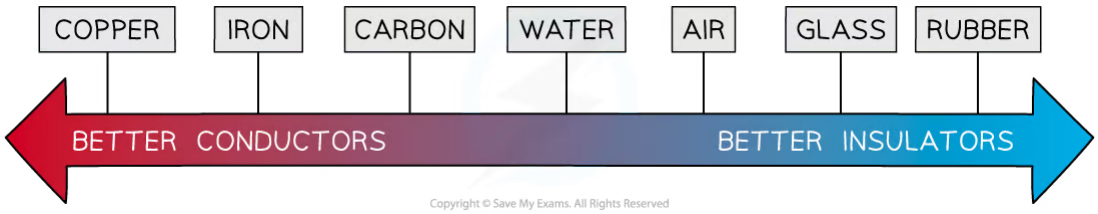
The units of electronvolt (eV) will appear in future topics such as a in Spectra & Atomic Energy Levels

YOUR NOTES

Electrical Conductors & Insulators

Conductors

- A **conductor** is a material that allows **charge** (usually electrons) to flow through it easily
- Examples of conductors are:
  - Silver
  - Copper
  - Aluminium
  - Steel
- Conductors tend to be **metals**



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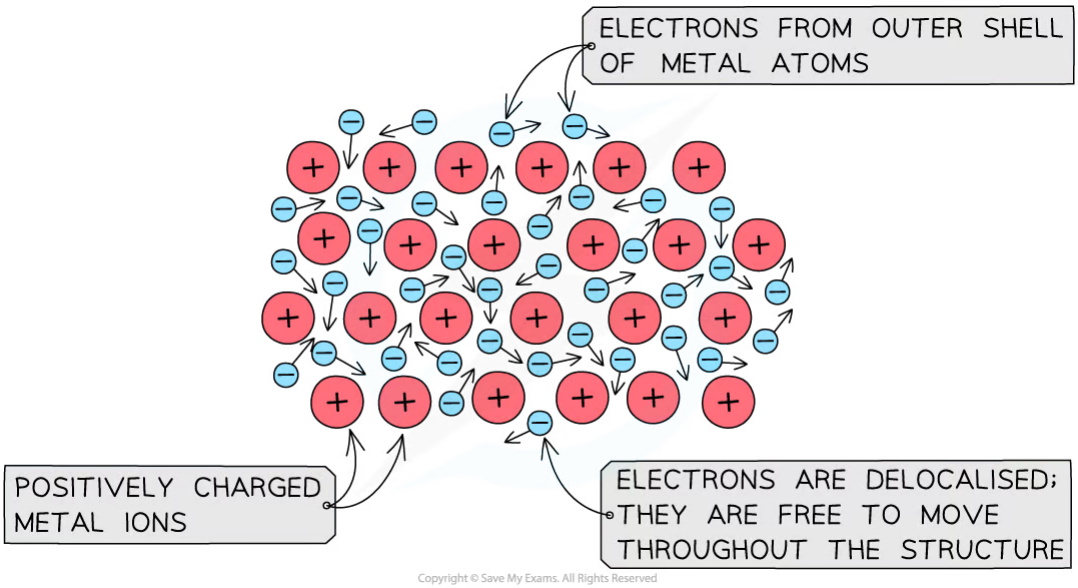
Different materials have different properties of conductivity

- On the atomic scale, conductors are made up of positively charged metal ions within a sea of delocalised electrons
- Metals are **excellent conductors** of electricity because:
  - Current is the rate of flow of electrons
  - So, the more easily electrons are able to flow in a material, the **better** it is at conducting electricity



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The lattice structure of a conductor with positive metal ions and delocalised electrons

Insulators

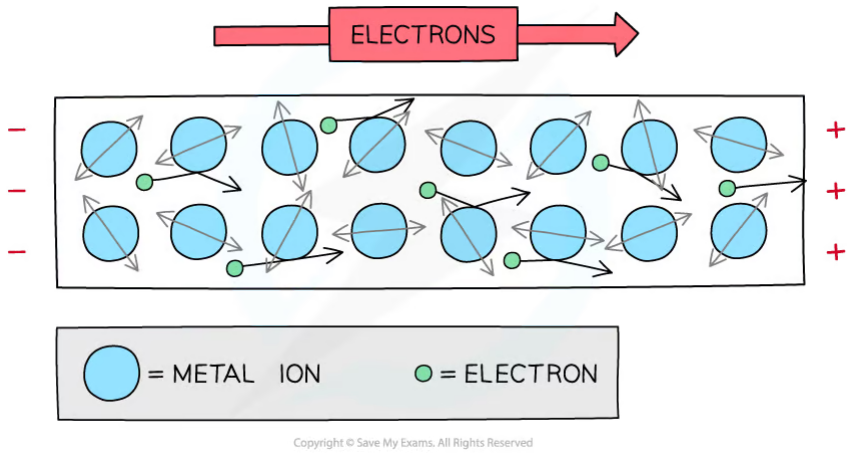
YOUR NOTES

- An **insulator** is a material that has **no free charges,** hence does **not** allow the flow of charge through them very easily
- Examples of insulators are:
  - Rubber
  - Plastic
  - Glass
  - Wood
- Some non-metals, such as wood, allow some charge to pass through them
- Despite not being very good at producing an electrical current, insulators are able to conduct **static electricity**
  - This occurs when an insulator builds up charge on its surface
  - When a charged insulator comes into contact with a conductor, the charge can be transferred

YOUR NOTES

Electric Resistance

- As electrons move through a conductor within a circuit (or any other component), they transfer some of their electrical **potential energy** to the **positive ions** of the metal



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Free electrons collide with metal ions which resist their flow

- This transfer of energy results in an increase in the kinetic energy of the atoms in the lattice
  - This raises the overall internal energy of the metal
- The macroscopic result of this transfer is the **heating up** of the wire which causes **resistance**
- Some metals heat up more than others
  - The greater the **heating effect**, the higher the **resistance**
  - Copper has a **low** electrical resistance, making it an ideal material to make wires from
- All electrical components have resistance to different degrees, including the wires and batteries
- Voltmeters and ammeters are said to be **ideal** when
  - An ideal voltmeter has **infinite** resistance, such that **no** current passes through it
  - An ideal ammeter has **zero** resistance, such that **all** the current passes through it

Exam Tip

Unless otherwise stated, voltmeters and ammeters will always be **ideal** in exam questions. This simply means you do not need to consider the resistance of an ammeter or voltmeter as part of the total resistance of the circuit

When non-ideal meters are used, the resistance will always be **constant**

Calculating Resistance

YOUR NOTES

- The resistance ***R*** of a component is defined as:

  The ratio of the potential difference across the component to the current flowing through it

- It is calculated as follows:

$$R = \frac{V}{I}$$

- Where:
  - V = potential difference (V)
  - I = electric current (A)
  - R = resistance (Ω)
- The units for resistance is **ohms** represented by the greek letter 'omega', Ω
- The **higher** the resistance of a component, the **lower** the current flowing through it and vice versa
- In terms of SI base units: $1\,\Omega = 1\,\text{kg}\,\text{m}^2\,\text{s}^{-3}\,\text{A}^{-2}$

YOUR NOTES

Worked Example

A charge of 5.0 C passes through a resistor at a constant rate in 30 s. The potential difference across the resistor is 2.0 V.

Calculate the resistance R of the resistor.

Answer:

Step 1: Write down the known quantities

- Charge, $\Delta q = 5.0$ C
- Time, $\Delta t = 30$ s
- Potential difference, $V = 2.0$ V

Step 2: Write down the equation for the resistance *R*

$$R = \frac{V}{I}$$

Step 3: Calculate the current *I* from the charge and time

$$I = \frac{\Delta q}{\Delta t}$$

Step 4: Substitute the numbers into the above equation

$$I = \frac{5.0}{30} = 0.17 \text{ A}$$

Step 5: Substitute this value of the current into the equation for the resistance given in Step 2

$$R = \frac{2.0}{0.17} = 12 \Omega$$

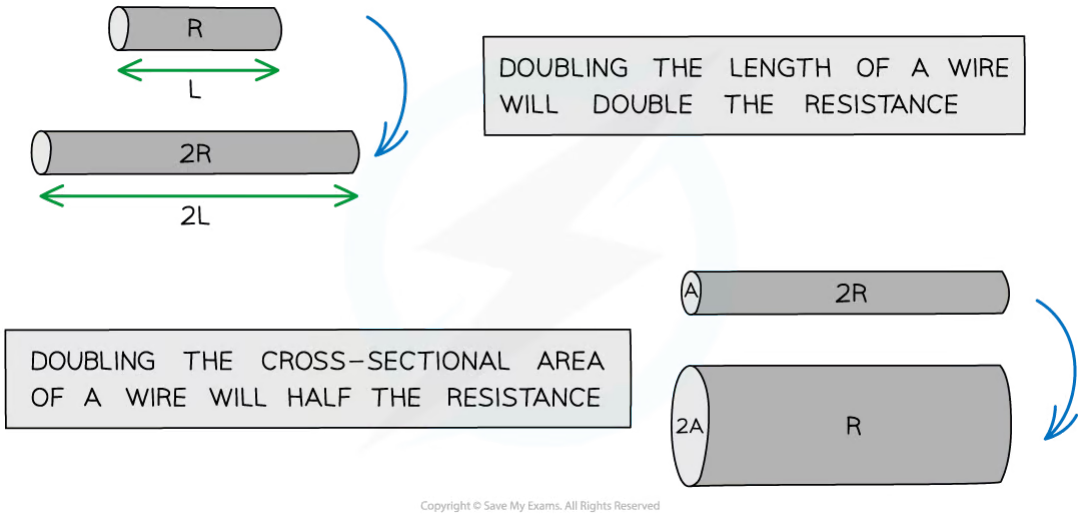
Exam Tip

It is common for current to be given in mA or potential difference in kV. For resistance to be in Ω, you must convert mA into A and kV into V!

YOUR NOTES

Electrical Resistivity

- The **resistance** of a sample depends on:
  - The material it is made of
  - The length of the sample
  - The cross-sectional area of the sample
- The resistance of a conductor (e.g. a wire) is:
  - **Directly proportional** to its **length**
  - **Inversely proportional** to its **cross-sectional area**



How the dimensions of a wire (i.e. length and width) affect its resistance

- The cross-sectional area of a **wire** is always modelled as a **circle**
  - Therefore, the cross-sectional area is πr^2, where r is the radius of the wire

Resistivity

- This leads to the definition of a new quantity, called **resistivity**
- Resistivity is a property describing the extent to which a material **opposes** the flow of electric current through it
- It is defined as follows:

> **The resistivity of a material is equal to the resistance per unit length of a material with unit cross-sectional area**

- The equation for the resistivity is:

$$\rho = \frac{RA}{L}$$

- Where:
  - ρ = resistivity in ohm-metres (Ω m)

YOUR NOTES

- R = resistance in ohms (Ω)
- A = cross-sectional area of material in square metres (m^2)
- L = length of material in meres (m)

- Resistivity is measured in ohm-metres (Ω m)
- Resistivity is the **property** of a material

Resistivity of Materials Table

	Material	Resistivity ρ / Ω m
Metals	Copper	1.7×10^{-8}
	Gold	2.4×10^{-8}
	Aluminium	2.6×10^{-8}
Insulators	Glass	10^{12}
	Sulfur	10^{15}

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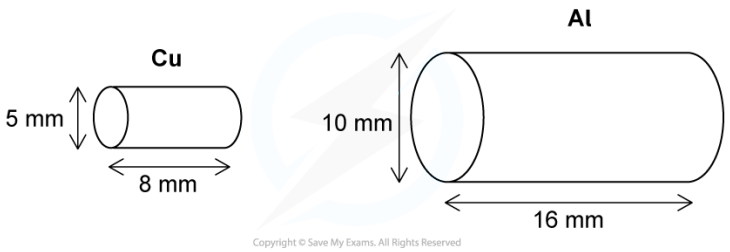
- Conductors, such as metals, have **low** values of resistivity
  - This makes metals, such as copper and aluminium, ideal for making wires as they have low values of resistance, which is why they are excellent conductors
- Whereas insulators have such **high** values of resistivity that virtually no current will flow through them
  - This is why insulating materials, such as plastic and rubber, are ideal for housing electrical wires, as they keep current flowing within the circuit and prevent users from receiving dangerous electric shocks

YOUR NOTES

Worked Example

Two conducting cylinders are made from copper and aluminium respectively. The dimensions of the cylinders are shown in the diagram below.



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- Resistivity of copper = $1.7 \times 10^{-8}\ \Omega\,m$
- Resistivity of aluminium = $2.6 \times 10^{-8}\ \Omega\,m$

Determine which cylinder is a better conductor.

Answer:

STEP 1

THE BETTER CONDUCTOR WILL HAVE LOWER RESISTANCE

STEP 2

RESISTANCE IS CALCULATED FROM

$$R = \frac{\rho L}{A}$$

STEP 3

RESISTANCE OF THE COPPER CYLINDER

THE CROSS-SECTIONAL AREA OF A CYLINDER IS A CIRCLE

mm → m

$$A = \pi \times r^2 = \pi \times \left(\frac{d}{2}\right)^2 = \pi \times \left(\frac{5 \times 10^{-3}}{2}\right)^2 = 2.0 \times 10^{-5}\ m^2$$

$$R = \frac{1.7 \times 10^{-8} \times 8 \times 10^{-3}}{2.0 \times 10^{-5}\ m^2} = 6.8 \times 10^{-6}\ \Omega$$

SUBSTITUTE VALUES INTO THE EQUATION

STEP 4

RESISTANCE OF THE ALUMINIUM CYLINDER

$$A = \pi \times r^2 = \pi \times \left(\frac{d}{2}\right)^2 = \pi \times \left(\frac{10 \times 10^{-3}}{2}\right)^2 = 7.9 \times 10^{-5}\ m^2$$

$$R = \frac{2.6 \times 10^{-8} \times 16 \times 10^{-3}}{7.9 \times 10^{-5}\ m^2} = 5.3 \times 10^{-6}\ \Omega$$

STEP 5

RESISTANCE OF ALUMINIUM CYLINDER < RESISTANCE OF COPPER CYLINDER

THE ALUMINIUM CYLINDER IS THE BETTER CONDUCTOR

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YOUR NOTES

Exam Tip

You won't need to memorise the value of the resistivity of any material, these will be given in the exam question. The equation for resistivity is also given in the data booklet.

The area of a circle equation is given on your data booklet as $A = \pi r^2$. In terms of diameter, this is

$$A = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$$

Remember, if the cross-sectional area of the shape is a circle (e.g. in a wire), it is proportional to the diameter or radius **squared**. This means if the diameter (or radius) doubles, the area quadruples causing the resistance to drop by a **quarter.**

YOUR NOTES

Ohm's Law

- Ohm's law states that:

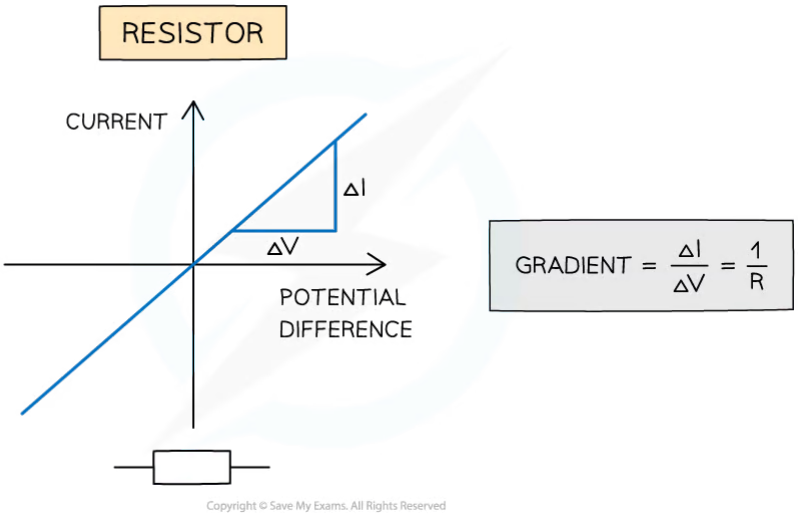
  For a component at a constant temperature, the current through it is proportional to the potential difference across it

- It is defined by the equation:

$$V = IR$$

- Where:
  - V = potential difference (V)
  - I = current (A)
  - R = resistance (Ω)

- An electrical component obeys Ohm's law if its graph of current against potential difference is a **straight line** through the origin
  - A **fixed resistor** obeys Ohm's law i.e. it is an **ohmic** component
  - A **filament lamp** does **not** obey Ohm's law i.e. it is a **non-ohmic** component



The current-voltage graph for a fixed resistor is a straight line through the origin. The fixed resistor is an ohmic component

- The resistance of an ohmic component can be calculated from the **gradient** of a current-voltage graph, since resistance is equal to

$$R = \frac{V}{I}$$

- If **current** I is on the ***y***-axis and **potential difference** V is on the ***x***-axis, then $R = \frac{1}{gradient}$

- If **potential difference** V is on the ***y***-axis and **current** I is on the ***x***-axis, then $R = gradient$
- Any metal conductor at a **constant temperature** can be considered an **ohmic** device
  - This is likely to be a fixed resistance
- Non-ohmic devices include:
  - Lamps
  - LEDs
  - Thermistors

YOUR NOTES

YOUR NOTES

Worked Example

The current flowing through a component varies with the potential difference *V* across it as shown.

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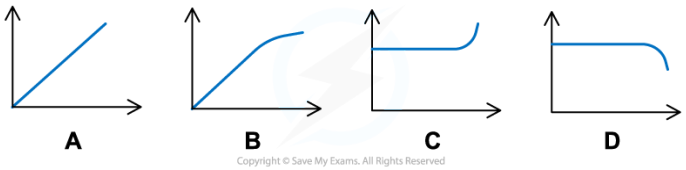
Which graph best represents how the resistance *R* varies with *V*?



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Answer: D

Step 1: Write down the equation for the resistance *R*

$$R = \frac{V}{I}$$

Step 2: Link the resistance to the gradient of the graph

$$gradient = \frac{I}{V} \quad \Rightarrow \quad R = \frac{1}{gradient}$$

Step 3: Identify the gradient of different sections of the graph and use it to deduce what happens to the resistance

- The first straight section of the graph has a constant gradient
  - So the resistance remains constant
- The second section is curved and the steepness of the line increases, so the gradient increases
  - So the resistance decreases

Step 4: Identify the correct graph out of the four proposed

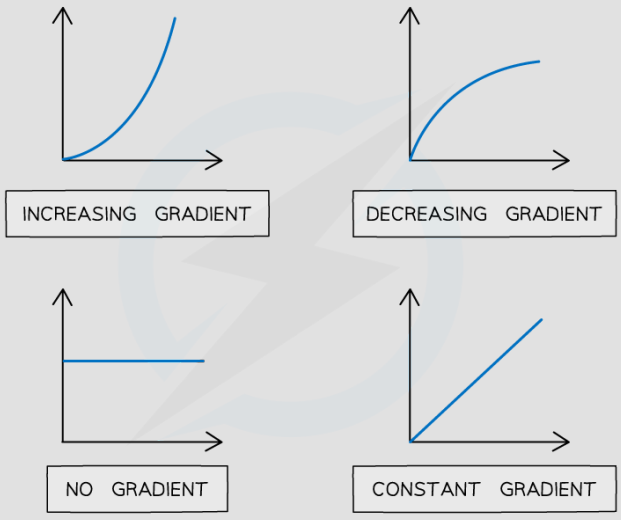
- Constant resistance is indicated by a straight horizontal line
  - So either **C** or **D** are correct
- Decreasing resistance is indicated by a line curving downwards
  - So only **D** is correct

YOUR NOTES

Exam Tip

When solving problems about Ohm's law you will often deal with graphs. You need to be confident in identifying and calculating their gradients.

- In maths, the gradient is the **slope** of the graph (i.e. $\frac{rise}{run}$)
- The graphs below show a summary of how the slope of the graph represents the gradient



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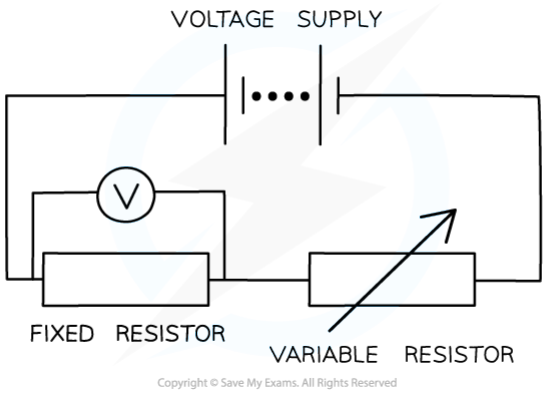
Although the Ohm's law equation is not given on your data sheet, you can see it is just rearranging $R = \frac{V}{I}$ to make V the subject.

I-V Characteristics

YOUR NOTES

- The Ohmic and non-Ohmic behaviour of an electrical conductor can be deduced by looking at its *I-V* (current-voltage) characteristics
  - This is usually plotted as a graph showing the variation of current against voltage
- The relation between potential difference across an electrical component (e.g. a fixed resistor) and current can be investigated through a circuit such as the one below



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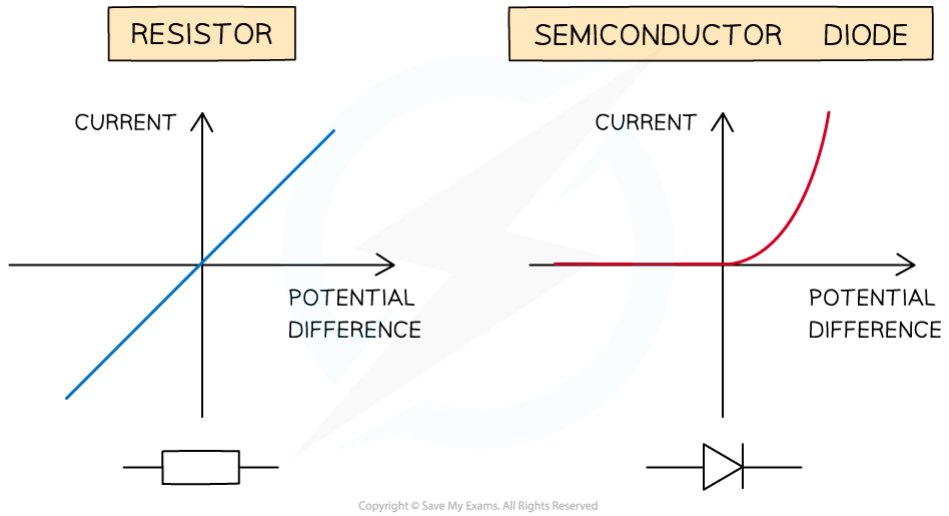
Circuit for plotting graphs of current against voltage. The component being investigated here is a fixed resistor

- By adjusting the resistance on the variable resistor:
  - The **current** in the circuit will **change**
  - For each value of the current I, the **potential difference** V can be **recorded**
- A graph of current against potential difference can then be plotted

I-V characteristics of common conductors

- Common ohmic conductors include
  - Wires (at constant temperature)
  - Resistors
- Common non-ohmic conductors include
  - Semiconductor diodes e.g. LEDs
  - Filament lamp
  - Thermistors & light-dependent resistors (LDRs)



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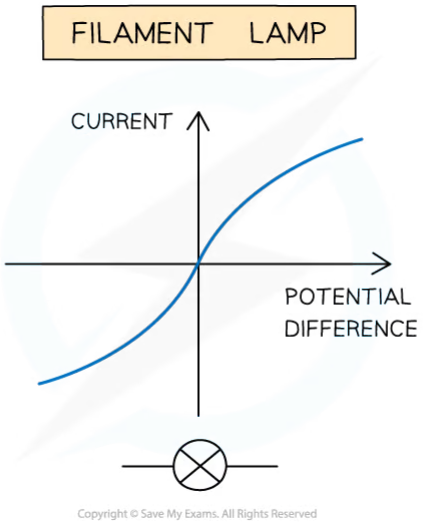
I–V characteristics for an ohmic conductor (e.g. resistor) and semiconductor diode (e.g. LED)

Resistor

- A resistor is an example of an Ohmic resistor
  - This means the current is **directly proportional** to the potential difference
  - Its *I-V* graph is a **straight line** through the origin

Semiconductor Diode

- When the current is in the direction of the arrowhead symbol, the diode is said to be **forward-biased**
  - There is a sharp increase in current
  - This is shown on the right side of the graph
- When the diode is switched around, it does not conduct and it is said to be **reverse biased**
  - The current through the diode is zero
  - This is shown on the left side of the graph
- The diode is a **non-ohmic** component
  - Its *I-V* graph is not a straight line through the origin
- A specific type of diode is an **LED**
  - The *I-V* characteristic graphs looks the exact same for this



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I–V characteristics for a filament lamp

Filament Lamp

- For **very small voltages**, the filament lamp behaves as an **ohmic** component
  - The middle section of the graph (around zero voltage) is straight and passes through the origin
- As voltage increases:
  - More current flows through the filament lamp and the **temperature** of the filament in the lamp **increases**
  - The **higher** the **temperature** of the filament, the **higher** its **resistance**
  - Since resistance opposes current, the **current flows** through the filament **at a slower rate**
  - This is shown by the curved section of the graph
- For slightly **higher voltages**, the filament lamp is **non-ohmic**
  - The *I-V* graph is a curve with decreasing gradient

Exam Tip

Make sure you're confident in drawing the *I–V* characteristics for different components, as you may be asked to sketch these from memory or to identify those given in exam questions

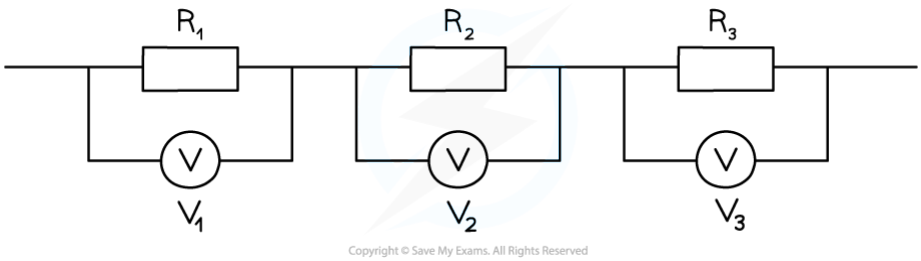
Series & Parallel Circuits

Resistors in Series

- In a **series** circuit:
  - The **current** is the **same** at any point
  - The **potential difference** is **split** across all components depending on their resistance
- When two or more components are connected in series:

  The combined resistance of the components is equal to the sum of individual resistances

- Three resistors connected in series will have a **total resistance** of $R = R_1 + R_2 + R_3$



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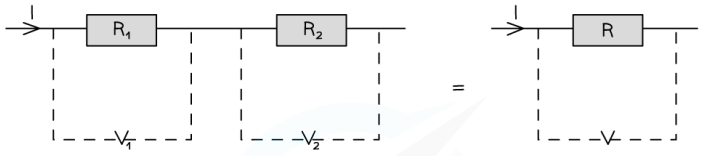
Resistors in series

- The equation for the total resistance of **resistors in series** is derived using the above rules for current and potential difference:

YOUR NOTES

CONSIDER TWO RESISTORS R_1 AND R_2 CONNECTED IN SERIES
A SINGLE RESISTOR R IS EQUIVALENT TO THEM

FROM THE ELECTRIC CURRENT RULE: THE CURRENT I THROUGH EACH RESISTOR IS THE SAME SINCE THEY ARE CONNECTED IN SERIES (NO JUNCTIONS)



FROM THE ELECTRICAL VOLTAGES RULE: THE TOTAL p.d OF BOTH RESISTORS IN A CLOSED CIRCUIT LOOP MUST EQUAL THE SUM OF THE p.ds (THE p.d ACROSS THE SINGLE RESISTOR)

$$V = V_1 + V_2$$

FROM OHM'S LAW, POTENTIAL DIFFERENCE IS GIVEN BY THE PRODUCT OF CURRENT AND RESISTANCE

$$IR = IR_1 + IR_2$$

SINCE CURRENT I IS THE SAME FOR ALL RESISTORS, DIVIDING BY I

$$R = R_1 + R_2$$

THIS EQUATION CAN BE EXTENDED SO THE EQUIVALENT RESISTOR R OF SEVERAL RESISTORS CONNECTED IN SERIES IS GIVEN BY

$$R = R_1 + R_2 + R_3 + R_4 ...$$

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- The equation for the combined resistance of resistors in series is therefore:

COMBINED RESISTANCE IN SERIES	$R = R_1 + R_2 + R_3 ...$

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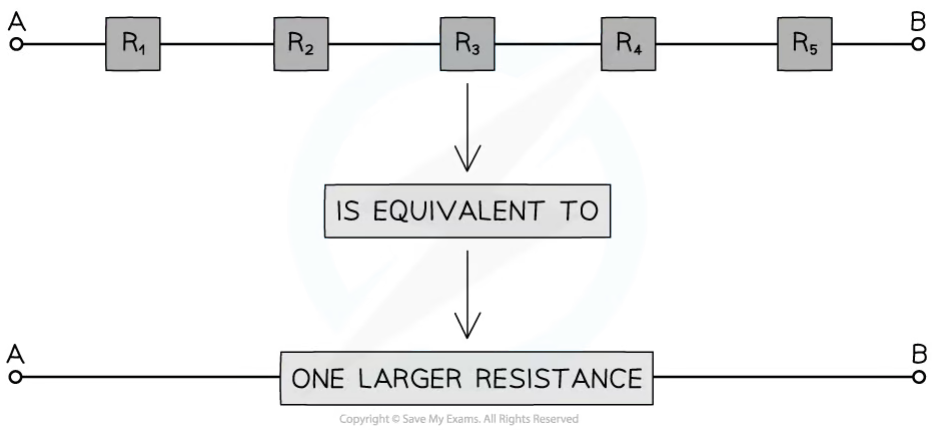
- This means the **total resistance increases**
  - The combined resistance is **more** than the resistance of any of the individual components
  - For example, if two resistors of equal resistance are connected in series, then the combined resistance will **double**



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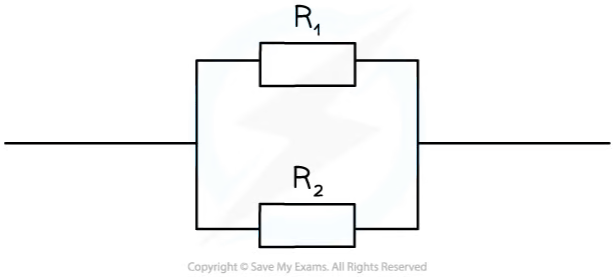
Connecting more resistors in series increases the overall resistance

Resistors in Parallel

- In a **parallel** circuit:
  - The total **current** is equal to the **sum** of the currents in each parallel **branch** of the circuit
  - The **potential difference** is the same across each loop
- In a parallel circuit, the combined resistance of the components requires the use of **reciprocals**

  The reciprocal of the combined resistance of two or more resistors is the sum of the reciprocals of the individual resistances

- Two resistors connected in parallel will have a **total resistance** of $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$



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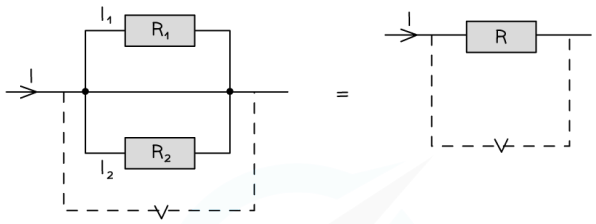
Resistors in parallel

- The equation for the total resistance of **resistors in parallel** is derived using the above rules for current and potential difference:

YOUR NOTES

CONSIDER TWO RESISTORS R_1 AND R_2 CONNECTED IN PARALLEL
A SINGLE RESISTOR R IS EQUIVALENT TO THEM

FROM THE ELECTRICAL CURRENT RULE: THE CURRENT THROUGH EACH RESISTOR WILL BE DIFFERENT DUE TO THE CURRENT SPLITTING AT THE JUNCTION
THE CURRENT THROUGH THE EQUIVALENT RESISTOR R WILL BE THE TOTAL CURRENT I



$$I = I_1 + I_2$$

FROM THE ELECTRICAL VOLTAGES RULE: THE p.d ACROSS EACH RESISTOR IN DIFFERENT BRANCHES IS THE SAME
THE RESISTOR R WILL HAVE THAT SAME p.d ACROSS IT TOO

$$V = V_1 = V_2$$

REARRANGING OHM'S LAW V = IR FOR CURRENT I

$$I = \frac{V}{R}$$

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

SINCE POTENTIAL DEFFERENCE V IS THE SAME FOR ALL RESISTORS, DIVIDING BY V

$$\frac{I}{R} = \frac{I}{R_1} + \frac{I}{R_2}$$

THIS EQUATION CAN BE EXTENDED SO THE EQUIVALENT RESISTOR R OF SEVERAL RESISTORS CONNECTED IN PARALLEL IS GIVEN BY

$$\frac{I}{R} = \frac{I}{R_1} + \frac{I}{R_2} + \frac{I}{R_3} \dots$$

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- The equation for the combined resistance of resistors in parallel is therefore:

COMBINED RESISTANCE IN PARALLEL	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots$

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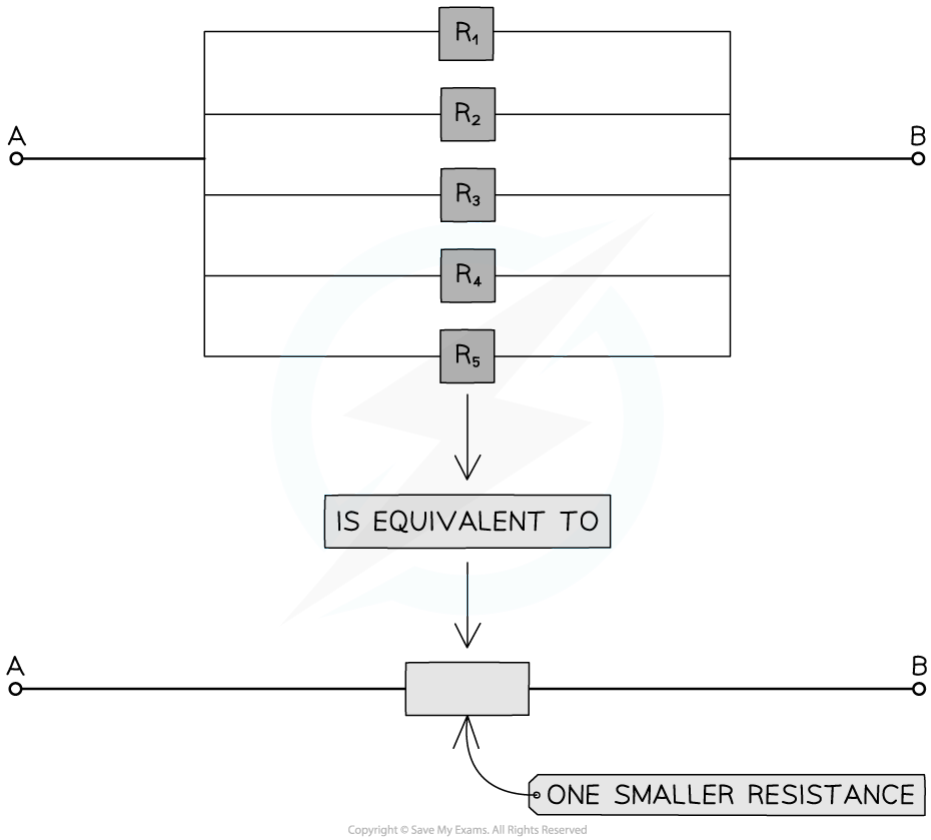
- This means the **total resistance decreases**
  - The combined resistance is **less** than the resistance of any of the individual components
  - For example, if two resistors of equal resistance are connected in parallel, then the combined resistance will **halve**



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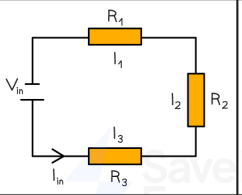
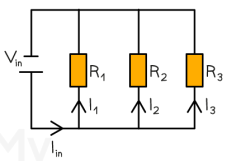
Connecting more resistors in parallel decreases the overall resistance

Summary of the rules in series and parallel circuits

- The table below summarises the rules for calculating current, potential difference and resistance within series and parallel circuits

Series and Parallel Circuits Summary

	Series	Parallel
Circuit		
Voltage	$V_{in} = V_1 + V_2 + V_3$	$V_{in} = V_1 = V_2 = V_3$
Current	$I_{in} = I_1 = I_2 = I_3$	$I_{in} = I_1 + I_2 + I_3$
Resistance	$R_S = R_1 + R_2 + R_3$	$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

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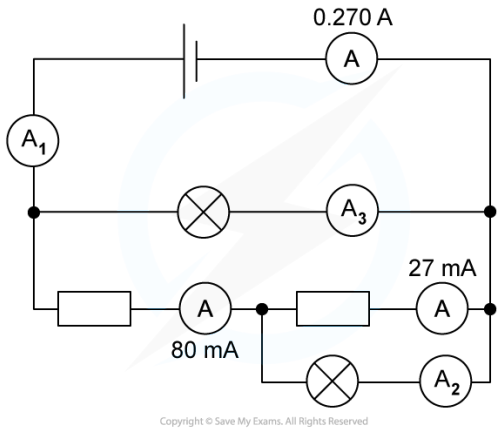
YOUR NOTES

Worked Example

For the circuit below, state the readings of ammeters A_1, A_2 and A_3.



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Answer:

Step 1: Recall the rules for current in series & parallel

- In a **series** circuit: The current is the same at any point $I_{in} = I_1 = I_2 = I_3$
- In a **parallel** circuit: The total current is equal to the sum of the currents in each parallel branch of the circuit $I_{in} = I_1 + I_2 + I_3$

Step 2: Determine the current in ammeter A_1

- Ammeter A_1 is in series with ammeter A
- Therefore, the current in A_1 = the current in A

$$A_1 = 0.270 \text{ A}$$

Step 3: Determine the current in ammeter A_3

- The current from ammeter A_1 enters splits into two branches
- One branch has a current of 80 mA = 0.080 A
- The current in the other branch is the reading of ammeter A_3

$$0.270 = 0.080 + A_3$$

$$A_3 = (0.270 - 0.080) = 0.190 \text{ A}$$

Step 4: Determine the current in ammeter A_2

- The current of 0.080 A splits into two branches
- One branch has a current of 27 mA = 0.027 A
- The current in the other branch is the reading of ammeter A_2

$$0.080 = 0.027 + A_2$$

$$A_2 = (0.080 - 0.027) \text{ A} = 0.053 \text{ A}$$

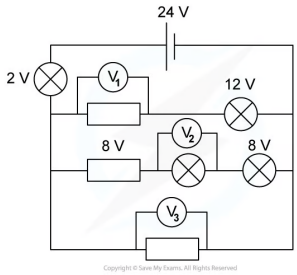
- The current readings of the three ammeters are:
  - $A_1 = 0.270\ A = 270\ mA$
  - $A_2 = 0.053\ A = 53\ mA$
  - $A_3 = 0.19\ A = 190\ mA$

YOUR NOTES

Worked Example

For the circuit below, state the readings of the voltmeters V_1, V_2 and V_3.

All the lamps and resistors have the same resistance.



Answer:

Step 1: Recall the rules for p.d. in series & parallel

- In a **series** circuit: The potential difference is split across all components (equally if the resistance is equal) $V_{in} = V_1 + V_2 + V_3$
- In a **parallel** circuit: The potential difference is the same across each loop $V_{in} = V_1 = V_2 = V_3$

Step 2: Determine the p.d. in voltmeter V_1

- The potential difference provided by the cell is the total input p.d., so, each loop must sum to 24 V
- The first loop consists of the 2 V lightbulb, the 12 V lightbulb and resistor V_1

$$2 + 12 + V_1 = 24 \text{ V}$$

$$V_1 = 24 - 2 - 12 = 10 \text{ V}$$

Step 3: Determine the p.d. in voltmeter V_2

- The second loop consists of the 2 V lightbulb, the 8 V resistor, lightbulb V_2, and the 8 V lightbulb

$$2 + 8 + V_2 + 8 = 24 \text{ V}$$

$$V_2 = 24 - 2 - 8 - 8 = 6 \text{ V}$$

Step 3: Determine the p.d. in voltmeter V_3

- The third loop consists of the 2 V lightbulb and resistor V_3

$$2 + V_3 = 24 \text{ V}$$

$$V_3 = 24 - 2 = 22 \text{ V}$$

- The potential difference readings of the three voltmeters are:
  - $V_1 = 10$ V

- $V_2 = 6\ V$
- $V_3 = 22\ V$

YOUR NOTES

Exam Tip

The equations for current, potential difference and resistance in series and parallel circuits is given in your data booklet. However, it is much quicker if you remember them!

YOUR NOTES

Electrical Power

- When an electrical current does work against electrical resistance:
  - Electrical energy is **dissipated as thermal energy** in the surroundings
  - The heat that is produced will dissipate via thermal conduction, convection and radiation
- The amount of heat produced depends on two factors:
  - **Current:** The greater the current, the more heat that is produced
  - **Resistance:** The higher the resistance, the greater the amount of heat produced (for a given current)
- Note that reducing the resistance can cause the current to increase
  - This could actually **increase** the amount of heat produced
- In mechanics, power *P* is defined as the **rate of doing work**
  - The potential difference is the **work done per unit charge**
  - Current is the **rate of flow of charge**
- Therefore, the electrical power is defined as the **rate of change of work done**:

$$P = \frac{E}{t} = \frac{W}{t}$$

- Where:
  - *P* = power (W)
  - *E* = energy transferred (J)
  - *W* = work done (J)
  - *t* = time (s)
- The work done is the energy transferred so the power is the **energy transferred per second** in an electrical component
- The power dissipated (produced) by an electrical device can also be written as

$$P = IV$$

- Where:
  - *I* = current (A)
  - *V* = potential difference (V)
- Using Ohm's Law ***V = IR*** to rearrange for either *V* or *I* and substituting into the power equation, means power can be written in terms of resistance *R*

$$P = I^2R = \frac{V^2}{R}$$

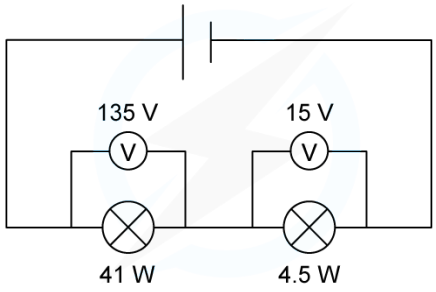
- Where *R* = resistance (Ω)

- This means that, for a given resistor, **doubling** the current (or voltage) will yield an electrical power **four** times greater

YOUR NOTES

Worked Example

Two lamps are connected in series to a 150 V power supply.



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Which statement most accurately describes what happens?

A. Both lamps light normally

B. The 15 V lamp blows

C. Only the 41 W lamp lights

D. Both lamps light at less than their normal brightness

ANSWER: A

STEP 1	CALCULATE CURRENT NEEDED FOR BOTH LAMPS TO OPERATE $P = IV$
STEP 2	REARRANGE FOR I $I = \frac{P}{V}$
STEP 3	FOR THE 41 W LAMP: $I = \frac{41\ W}{135\ V} = 0.3\ A$ FOR THE 4.5 W LAMP: $I = \frac{4.5\ W}{15\ V} = 0.3\ A$
STEP 4	FOR BOTH TO OPERATE AT THEIR NORMAL BRIGHTNESS, A CURRENT OF 0.3 A IS REQUIRED. SINCE THE LAMPS ARE CONNECTED IN SERIES, THE SAME CURRENT WOULD FLOW THROUGH BOTH.
STEP 5	THE LAMPS WILL LIGHT AT THEIR NORMAL BRIGHTNESS – OPTION A

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Exam Tip

You can use the mnemonic "Twinkle Twinkle Little Star, Power equals *I* squared *R*" to remember whether to multiply or divide by resistance in the power equations, although they are all given in your data booklet.

The most appropriate equation to use will depend on whether the value of current or voltage has been given in the question

YOUR NOTES

YOUR NOTES

Sources of Electrical Energy

- An electric cell stores chemical energy that can be transferred to electrical energy
- Electric cells, batteries and other sources of electrical energy come in many forms, such as
  - Chemical cells
  - Solar cells
  - Mains electricity
  - Wind generators

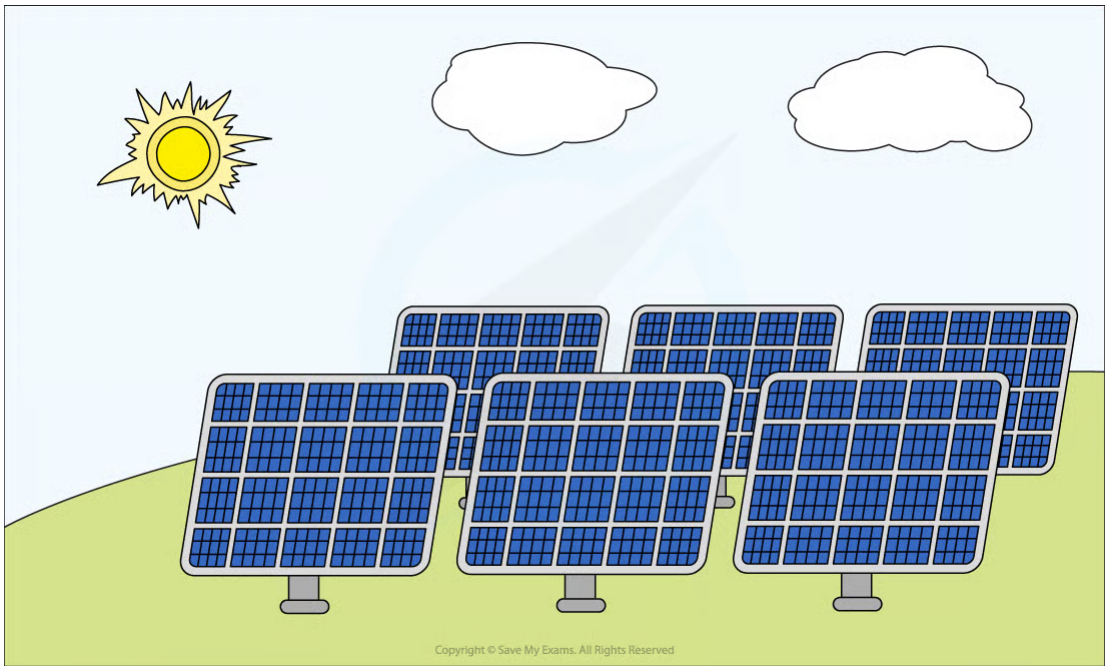
Chemical Cells

- Batteries, or chemical cells, utilise chemical reactions to provide a potential difference
  - They can be rechargeable or non-rechargeable
- Non-rechargeable batteries are by definition, only able to be **used once** as the chemicals within them are used up
  - Examples include alkaline, or AA batteries, common in many small devices
- Rechargeable cells can be **used many times** as they are attached to a charger and the **chemical reaction** is **reversed** allowing the cells to store energy for use once again
- Examples include:
  - Lithium-ion batteries used in laptops and other mobile devices
  - Lead-acid batteries such as those used in cars and other motor vehicles

Solar Cells

- Photovoltaic cells in solar panels convert **electromagnetic radiation** (photons) from the Sun into **electrical** energy
- When photons from the Sun are incident on the **solar heating panels**, the **light** energy is transferred into **thermal** energy
- The solar photons are absorbed by **electrons** on the surface of the photovoltaic cells, giving them enough energy to move and be **released** from the surface
- These electrons transfer the **thermal** energy into **electrical** energy which is then transferred to the external circuit

YOUR NOTES



Solar panels use solar cells to transfer energy from electromagnetic radiation into electrical energy

Advantages & Disadvantages

- Each energy source has its advantages and disadvantages, some of these are shown in the following table

Battery type	Advantages	Disadvantages
Single-use battery (e.g. alkaline AA battery)	high energy density convenient source of energy used in everyday appliances portable source of electrical energy potential to join many in series to increase p.d. low cost	non-rechargeable/limited power supply and will need replacing high internal resistance disposal issues/contributes to pollution made from non-renewable materials

Mobile phone battery (e.g. lithium-ion battery)	very high energy density high electrical efficiency convenient source of energy used in everyday appliances rechargeable, long lifetime fast charging time portable source of electrical energy low internal resistance	capacity of the cell degrades over time internal resistance increases over time expensive
Car battery (e.g. lead-acid battery)	low cost rechargeable able to deliver very high currents in a short time	low energy density - very heavy compared to power output internal resistance increases as battery degrades limited number of full discharge cycles disposal issues/contributes to pollution, uses toxic and corrosive materials made from non-renewable materials
Solar cell	unlimited supply of energy clean to produce the electricity freely available everywhere cheap maintenance no fuel is required for energy	variable output, highly dependent on weather conditions impacted by poor weather limited efficiency only available during the day requires large investment upfront requires large areas

YOUR NOTES

Wind generator	zero fuel costs, freely available no chemical pollution always sustainable and will never run out high set-up cost but becomes economical	inconsistent output, highly dependent on weather conditions requires favourable local conditions to be placed in windy locations noise/visual pollution
Mains electricity (fossil fuel generator)	extensive infrastructure in place high energy density of fuel reliable/available energy at any time well-known and developed technology	produces greenhouse gases unsustainable (non-renewable) produces pollution

Exam Tip

You do not need to know the inner workings of all the cells. However, you must be able to compare the advantages and disadvantages of using each cell as an energy source.

YOUR NOTES

YOUR NOTES

Electromotive Force

- When charge passes through a power supply such as a battery, it **gains** electrical energy
- The **electromotive force (e.m.f.)** is defined as

> **The amount of chemical energy converted to electrical energy per coulomb of charge (C) when charge passes through a power supply**

- Cells and batteries provide a **source** of e.m.f.
- E.m.f. is measured in **Volts** (V) and can be calculated using:

$$\text{E.M.F.} = \frac{\text{ENERGY TRANSFORMED FROM OTHER FORMS TO ELECTRICAL}}{\text{CHARGE}}$$

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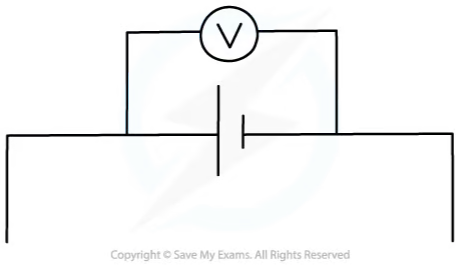
- Emf is also the potential difference across the cell when **no** current is flowing
- The emf of a cell can be measured by connecting a high-resistance voltmeter around the terminals of the cell in an open circuit



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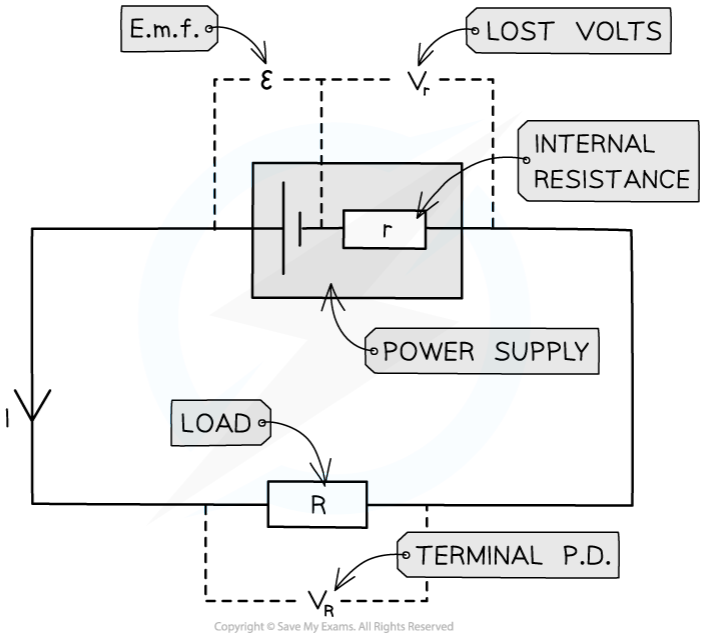
Emf is measured using a voltmeter connected in parallel with the cell

Internal Resistance

YOUR NOTES

- All power supplies have some resistance between their terminals
  - This is called **internal resistance** (*r*)
- This internal resistance causes the charge circulating to dissipate some electrical energy from the power supply itself
  - This is why the cell becomes warm after a period of time
- The internal resistance therefore causes **loss of voltage** or energy loss in a power supply
- A cell can be thought of as a source of e.m.f with an internal resistance connected in series. This is shown in the circuit diagram below:



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Circuit showing the e.m.f and internal resistance of a power supply

- Where:
  - Resistor *R* is the 'load resistor'
  - *r* is the internal resistance
  - ε is the e.m.f
  - V_r is the **lost volts**
  - V_R is the p.d across the load resistor, which is the same as the terminal p.d
- V_r is called the 'lost volts' as its the potential difference 'lost' due to the internal resistance in the cell
- The e.m.f is the **sum** of these **potential differences,** giving the equation below:

$$\varepsilon = I(R + r)$$

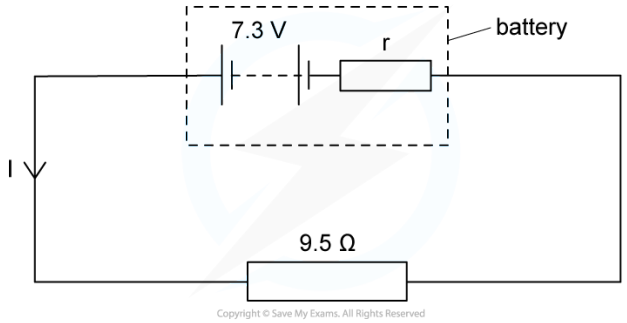
- Where:
  - ε = electromotive force (emf) (V)
  - *I* = current (A)

- R = resistance available to the rest of the circuit (Ω)
- r = internal resistance (Ω)

- Emf is, therefore, the total, or maximum, voltage available to the circuit

Worked Example

A battery of e.m.f 7.3 V and internal resistance r of 0.3 Ω is connected in series with a resistor of resistance 9.5 Ω.



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Determine:

(a) the current in the circuit

(b) the lost volts from the battery

Answer:

a.)

STEP 1 USING THE e.m.f EQUATION TO DETERMINE THE CURRENT I

$E = I(R + r)$

STEP 2 REARRANGE FOR I

$I = \frac{E}{(R + r)}$

STEP 3 SUBSTITUTE IN THE VALUES

$I = \frac{7.3}{(9.5 + 0.3)} = 0.745... = 0.7\ A$ (2 s.f.)

b.)

STEP 1 THE LOST VOLTS IS THE VOLTAGE LOST DUE TO INTERNAL RESISTANCE

LOST VOLTS = $I \times r$

STEP 2 SUBSTITUTE IN THE VALUES

LOST VOLTS = $0.7 \times 0.3 = 0.21 = 0.2$ (2 s.f.)

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Exam Tip

Make sure you substitute the correct resistance into the emf equation. *R* is the resistance available **to the rest of the circuit** and can vary depending on what components are in the circuit (e.g. adding or taking away resistors). The internal resistance *r* is a property of the **cell** and cannot change.

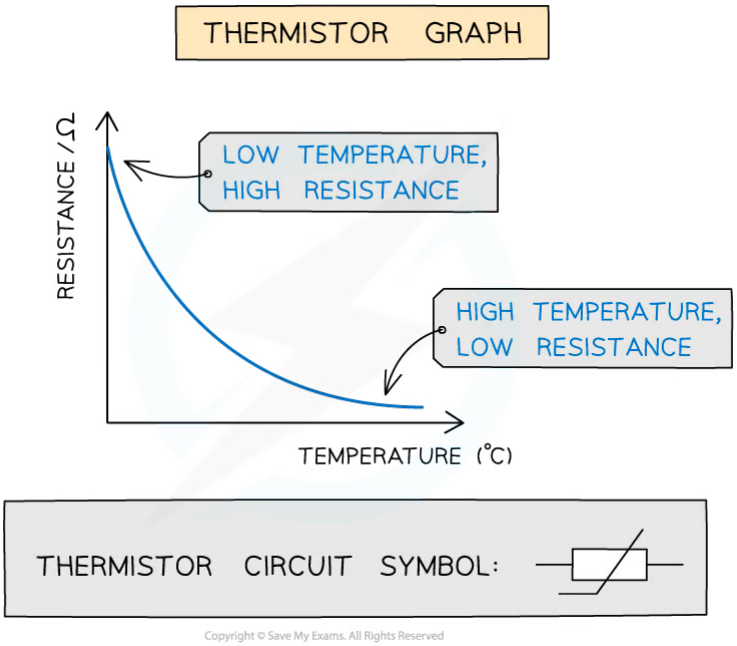
YOUR NOTES

YOUR NOTES

Variable Resistance

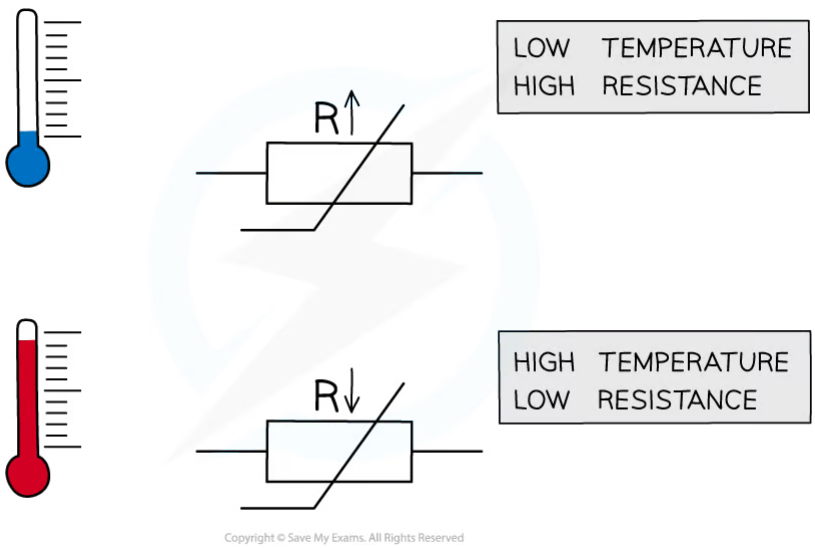
Thermistors

- A thermistor is a non-ohmic conductor and sensory resistor whose resistance varies with temperature
- Most thermistors are negative temperature coefficient (ntc) components.
  - This means that if the temperature **increases,** the resistance of the thermistor **decreases** (and vice versa)
- The temperature-resistance graph for a thermistor is shown below



Graph of temperature against resistance for a thermistor

- Thermistors are temperature sensors and are used in circuits in ovens, fire alarms and digital thermometers
  - As the thermistor gets **hotter**, its resistance **decreases**
  - As the thermistor gets **cooler**, its resistance **increases**



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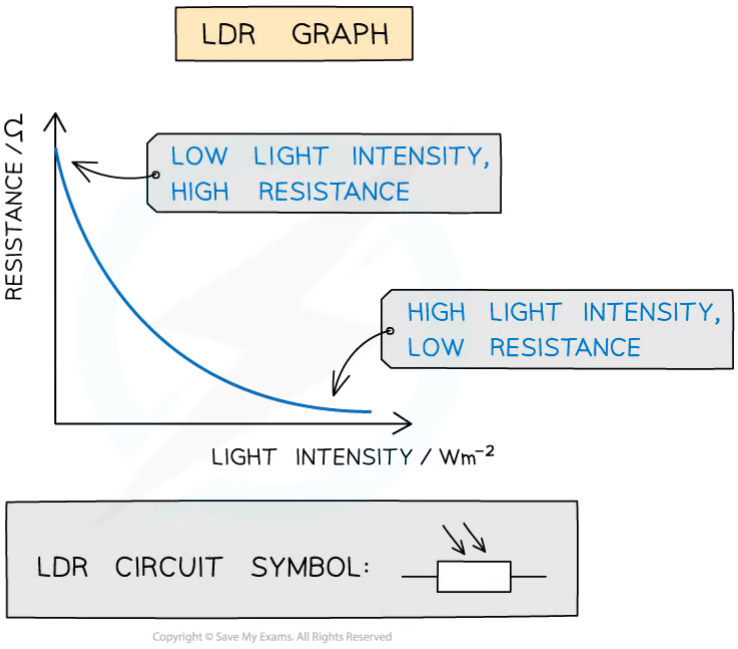
The resistance through a thermistor is dependent on the temperature of it

Light-dependent resistors (LDR)

- A light-dependent resistor (LDR) is a non-ohmic conductor and sensory resistor
- Its resistance automatically changes depending on the light energy falling onto it (illumination)
- This is shown by the following graph:



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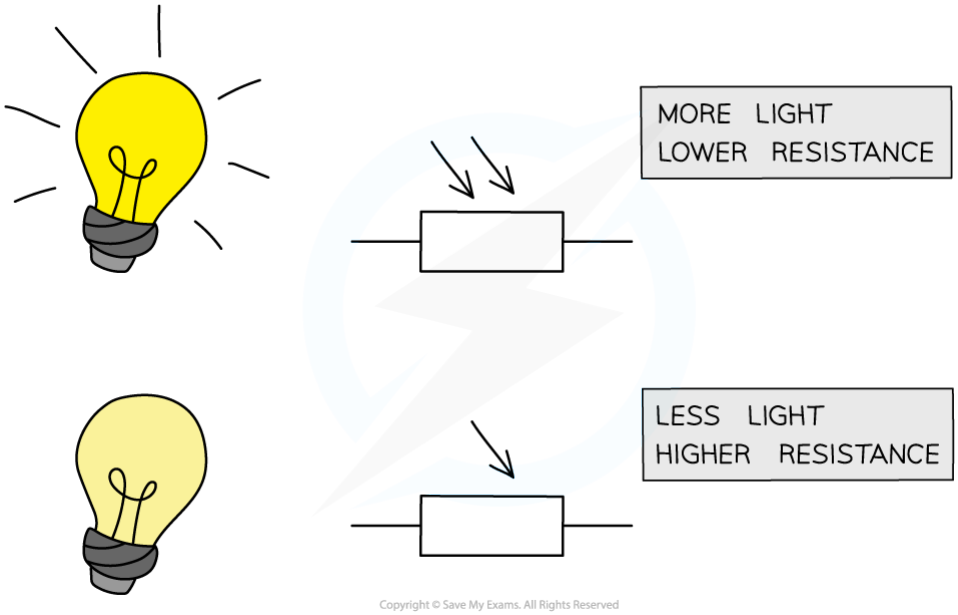
Graph of light intensity and resistance for an LDR

- As the **light intensity increases**, the **resistance** of an LDR **decreases**



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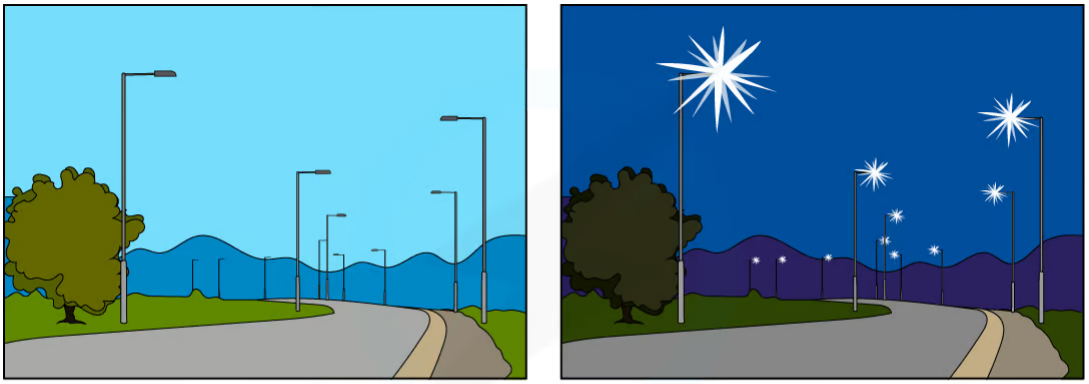
Resistance of an LDR depends on the light intensity falling on it

- LDRs can be used as light sensors, so, they are useful in circuits which automatically switch on lights when it gets dark, for example, street lighting and garden lights
  - In the dark, its resistance is very large (millions of ohms)
  - In bright light, its resistance is small (tens of ohms)



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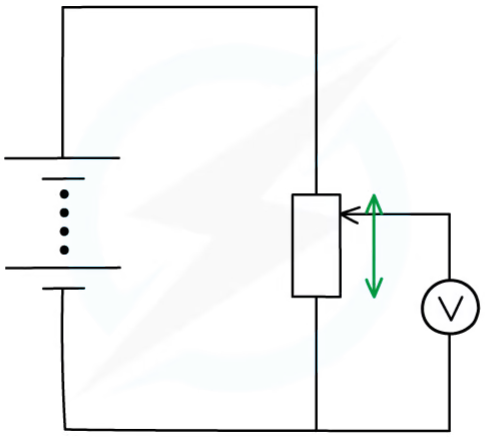
LDRs are used for automatic street lights

Potentiometer

- A potentiometer is similar to a variable resistor connected as a potential divider to give a continuously variable output voltage
- It can be used as a means of comparing potential differences in different parts of the circuit

YOUR NOTES

- It is recognised on a circuit diagram with a resistor fitted with a sliding contact
- The sliding contact has the effect of separating the potentiometer into two parts (an upper part and a lower part), both of which have different resistances



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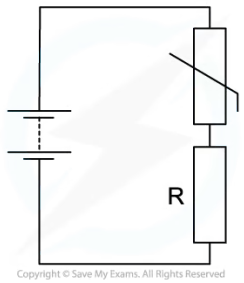
Moving the slider (the arrow in the diagram) changes the resistance (and hence potential di erence) of the upper and lower parts of the potentiometer

- If the slider in the above diagram is moved upwards, the resistance of the lower part will increase and so the potential difference across it will also increase
- Therefore, the variable resistor obtains a maximum or minimum value for the output voltage
- If the resistance is 3 Ω:
  - Maximum voltage is when the resistance is 3 Ω
  - Minimum voltage is when the resistance is 0 Ω

YOUR NOTES

Worked Example

A thermistor is connected in series with a resistor *R* and a battery.



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The resistance of the thermistor is equal to the resistance of *R* at room temperature. When the temperature of the thermistor decreases, which statement is correct?

A. The p.d across the thermistor increases

B. The current in *R* increases

C. The current through the thermistor decreases

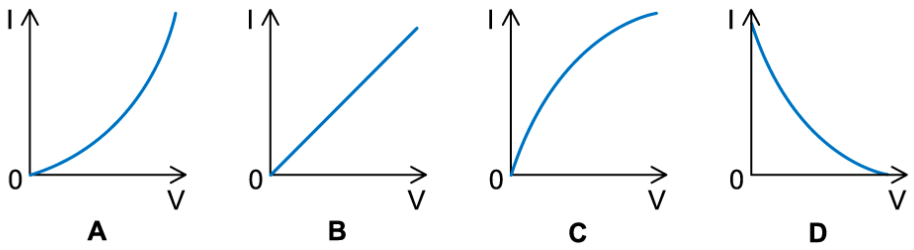
D. The p.d across *R* increases

ANSWER: A

- The resistance of the thermistor increases as the temperature decreases
- Since the thermistor and resistor *R* are connected in series, the current *I* in both of them is the same
- Ohm's law states that ***V = IR***
- Since the resistance of the thermistor increases, and *I* is the same, the potential difference *V* across it increases
- Therefore, statement **A** is correct

Worked Example

Which graph best represents the way in which the current I through an LDR depends upon the potential difference V across it?



ANSWER: B

- As the potential difference across the LDR increases, the light intensity increases causing its resistance to decrease
- Ohm's law states that ***V = IR***
- The resistance is equal to V/I or 1/R = I/V = gradient of the graph
- Since *R* decreases, the value of 1/R increases, so the gradient must increase
- Therefore, *I* increases with the p.d with an increasing gradient

YOUR NOTES

IB Physics HL

YOUR NOTES

Simple Harmonic Motion

CONTENTS

Describing Oscillations

Simple Harmonic Motion (SHM)

Time Period of a Mass–Spring System

Time Period of a Simple Pendulum

Energy Changes in Simple Harmonic Motion (SHM)

Calculating Energy Changes in SHM (HL)

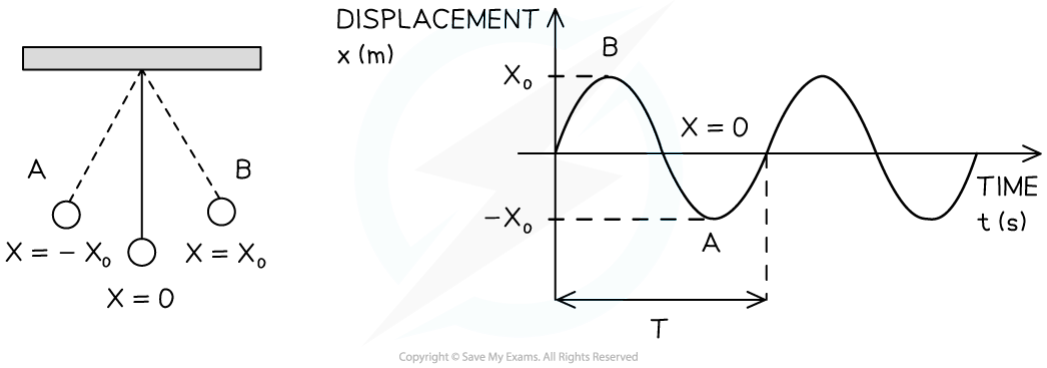
Phase Angles in Simple Harmonic Motion (SHM) (HL)

Equations for Simple Harmonic Motion (SHM) (HL)

Properties of Oscillations

- An **oscillation** is defined as follows:

  The repetitive variation with time *t* of the displacement *x* of an object about the equilibrium position (*x* = 0)



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A pendulum oscillates between A and B. On a displacement-time graph, the oscillating motion of the pendulum is represented by a wave, with an amplitude equal to x_0

- **Equilibrium position (*x* = 0)** is the position when there is no resultant force acting on an object

- **Displacement (*x*)** of a wave is the distance of a point on the wave from its equilibrium position
  - It is a vector quantity; it can be positive or negative and it is measured in metres (m)

- **Period (*T*)** or time period, is the **time interval** for one complete repetition and it is measured in seconds (s)
  - If the oscillations have a **constant period**, they are said to be **isochronous**

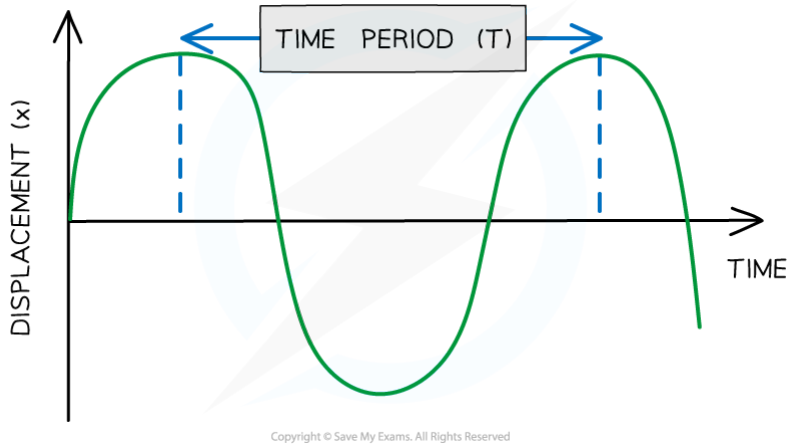

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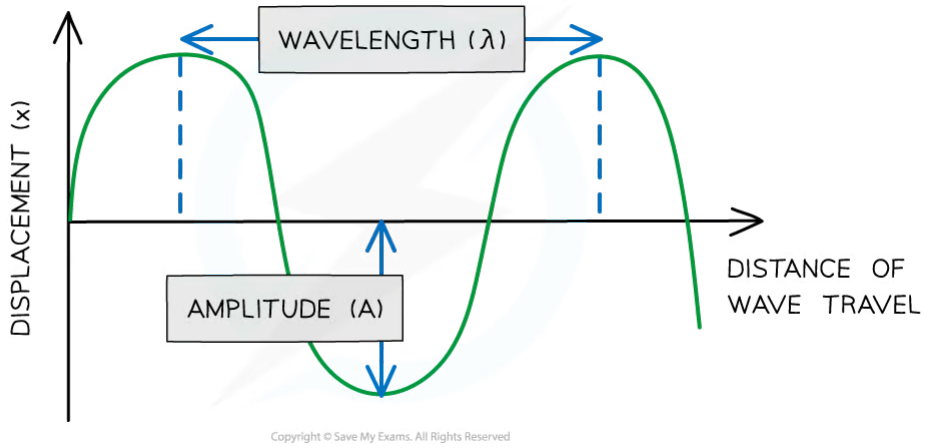
Diagram showing the time period of a wave

- **Amplitude (x_0)** is the **maximum value of the displacement** on either side of the equilibrium position is known as the **amplitude** of the oscillation
  - Amplitude is measured in metres (m)

- **Wavelength (λ)** is the **length of one complete oscillation** measured from the same point on two consecutive waves
  - Wavelength is measured in metres (m)



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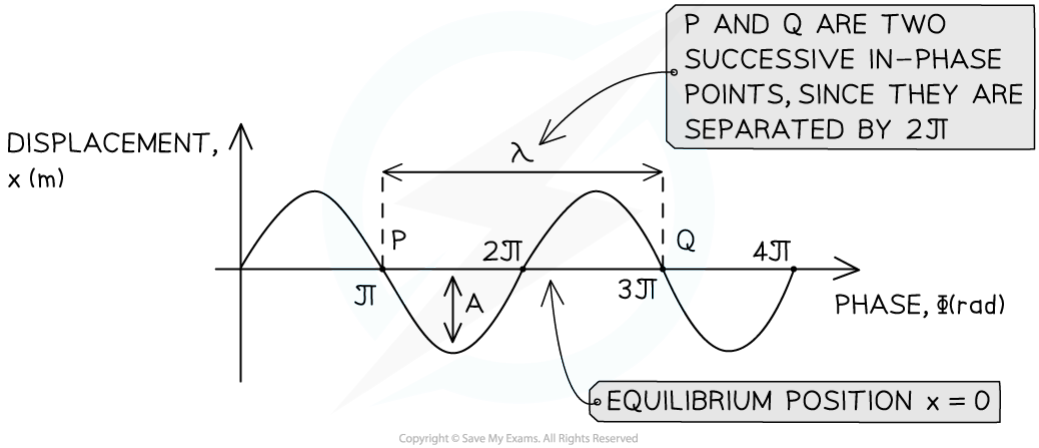
- **Frequency** (***f***) is the **number of oscillations per second** and it is measured in hertz (Hz)
  - Hz have the SI units of per second s^{-1}

Phase Difference

- Phase is a useful way to think about waves
- The phase of a wave can be considered in terms of:
  - Wavelength

- Degrees
- Radians

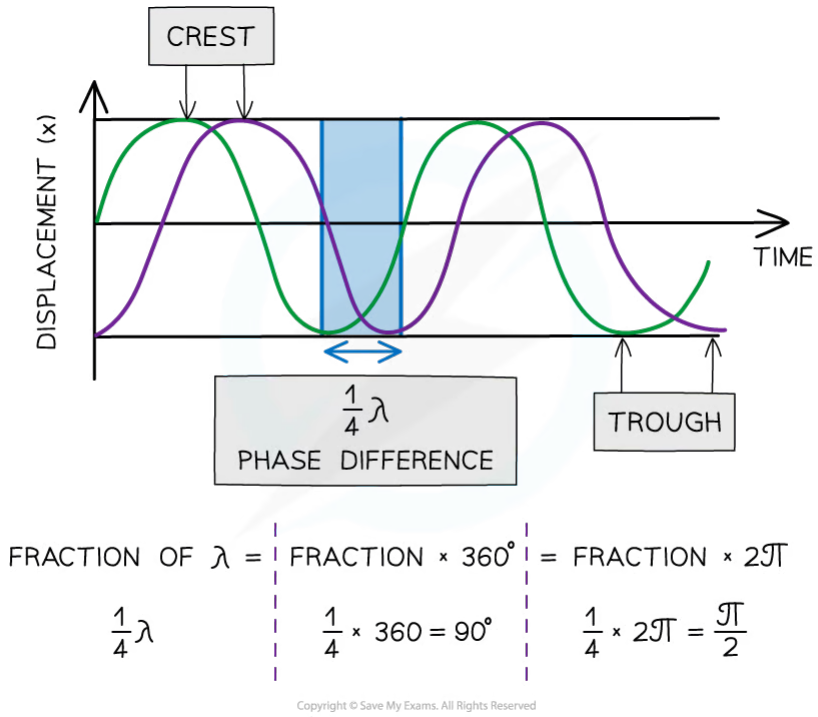
- One complete oscillation = 1 wavelength = 360° = 2π radians



Wavelength λ and amplitude A of a travelling wave

- The **phase difference** between two waves is a measure of **how much a point or a wave is in front or behind another**
- This can be found from the relative position of the crests or troughs of two different waves of the same frequency
  - When the crests of each wave, or the troughs of each wave are aligned, the waves are **in phase**
  - When the crest of one wave aligns with the trough of another, they are in **antiphase**

- The diagram below shows the green wave **leads** the purple wave by ¼ λ



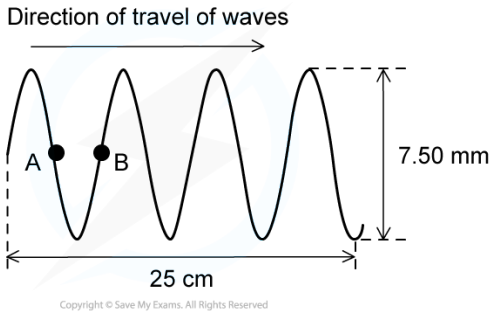
Two waves ¼ λ out of phase

- In contrast, the purple wave is said to **lag** behind the green wave by ¼ λ
- Phase difference is measured in **fractions of a wavelength, degrees** or **radians**
- The phase difference can be calculated from two different points on the same wave or the same point on two different waves
- The phase difference between two points can be described as:
  - **In phase** is **360°** or 2π radians
  - **In anti-phase** is **180°** or π radians

YOUR NOTES

Worked Example

Plane waves on the surface of water at a particular instant are represented by the diagram below.



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Determine:

(a) The amplitude

(b) The wavelength

(c) The phase difference between points **A** and **B**

Answer:

YOUR NOTES

A. THE AMPLITUDE

MAXIMUM DISPLACEMENT FROM THE EQUILIBRIUM POSITION

$7.50\ mm \div 2 = 3.75\ mm$

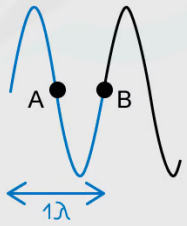
B. THE WAVELENGTH

DISTANCE BETWEEN POINTS ON SUCCESSIVE OSCILLATIONS OF THE WAVE THAT ARE IN PHASE

FROM DIAGRAM: $25cm = 3\frac{3}{4}$ WAVELENGTHS

$1\lambda = 25\ cm \div 3\frac{3}{4} = 6.67\ cm$

C. THE PHASE DIFFERENCE BETWEEN POINTS A AND B



POINTS A AND B HAVE $\frac{1}{2}\lambda$ DIFFERENCE = $\frac{1}{2} \times 360^\circ = 180^\circ$

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Exam Tip

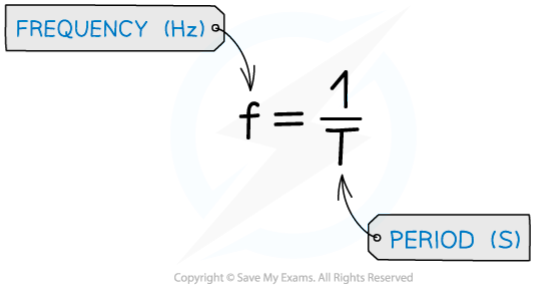
When labelling the wavelength and time period on a diagram:

- Make sure that your arrows go from the **very top** of a wave to the very top of the next one
- If your arrow is too short, you will lose marks
- The same goes for labelling amplitude, don't draw an arrow from the bottom to the top of the wave, this will lose you marks too.

Calculating Time Period of an Oscillation

YOUR NOTES

- The frequency and the period of an oscillation are related by the equation



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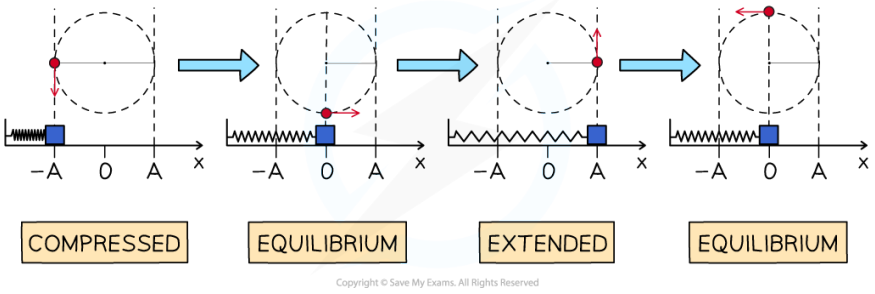
- **Angular frequency, ω,** in oscillations is equivalent to **angular velocity** in circular motion
- It can is defined as:

The rate of change of angular displacement with respect to time

- This can be written as an equation:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- Where:
  - ω = angular frequency (rad s^{-1})
  - 2π = circumference of a circle
  - *T* = time period (s)
  - *f* = frequency of oscillation (Hz)



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YOUR NOTES

Worked Example

A child on a swing performs 0.2 oscillations per second.

Calculate the period of the oscillation.

Answer:

Step 1: Write down the known quantities

- Frequency, $f = 0.2$ Hz

Step 2: Write down the relationship between the period *T* and the frequency *f*

$$T = \frac{1}{f}$$

Step 3: Substitute the value of the frequency into the above equation and calculate the period

$$T = \frac{1}{0.2} = 5.0 \text{ s}$$

YOUR NOTES

Worked Example

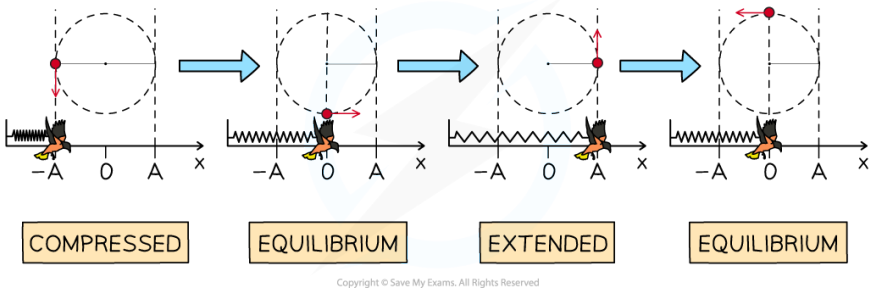
A cuckoo in a cuckoo clock emerges from a fully compressed position to a fully extended position in 1.5 seconds.

Calculate the angular frequency of the cuckoo as it emerges from the clock.

Answer:

Step 1: Consider the motion of the cuckoo

- The cuckoo goes from being fully compressed to fully extended which means that it travels for an **angular displacement** of half a circle and not a full circle
- So, instead of 2π the distance will be π



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Step 2: Substitute into the equation for angular velocity and time period

$$\omega = \frac{2\pi}{T} = \frac{\pi}{1.5} = 2.09 \text{ rad s}^{-1}$$

Step 3: State the final answer

- The angular frequency of the cuckoo as it emerges from the clock is 2.1 rad s^{-1} (2 s.f.)

Conditions for Simple Harmonic Motion

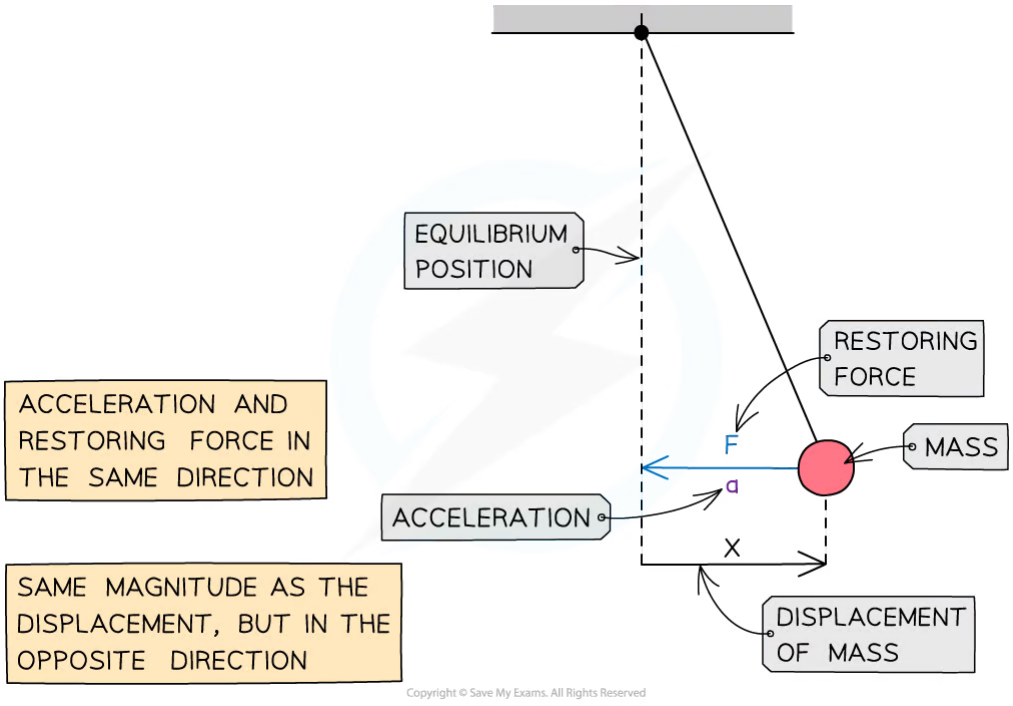
- **Simple harmonic motion (SHM)** is a specific type of oscillation
- An oscillation is said to be SHM when:

> **The acceleration of a body is proportional to its displacement but acts in the opposite direction**

- Acceleration *a* and displacement *x* can be represented by the defining equation of SHM:

$$a \propto -x$$

- The two **conditions** required for an object to be in simple harmonic motion are therefore:
  - The acceleration is **proportional** to the displacement
  - The acceleration is in the **opposite direction** to the displacement
- The **negative sign** shows that the acceleration will **always** be towards the centre of the oscillation



Force, acceleration and displacement of a pendulum in SHM

Worked Example

Explain why a person jumping on a trampoline is **not** an example of simple harmonic motion.

Answer:

Step 1: Recall the conditions for simple harmonic motion

- The conditions required for SHM:
  - The restoring force/acceleration is **proportional** to the displacement
  - The restoring force/acceleration is in the **opposite direction** to the displacement

Step 2: Consider the forces in the scenario given

- When the person is not in contact with the trampoline, the restoring force is equal to their weight, which is constant
- The value of their weight does not change, even if they jump higher (increase displacement)

Step 3: Write a concluding sentence

- The restoring force on the person is not proportional to their distance from the equilibrium position, therefore, this scenario does not fulfil the conditions for SHM

YOUR NOTES

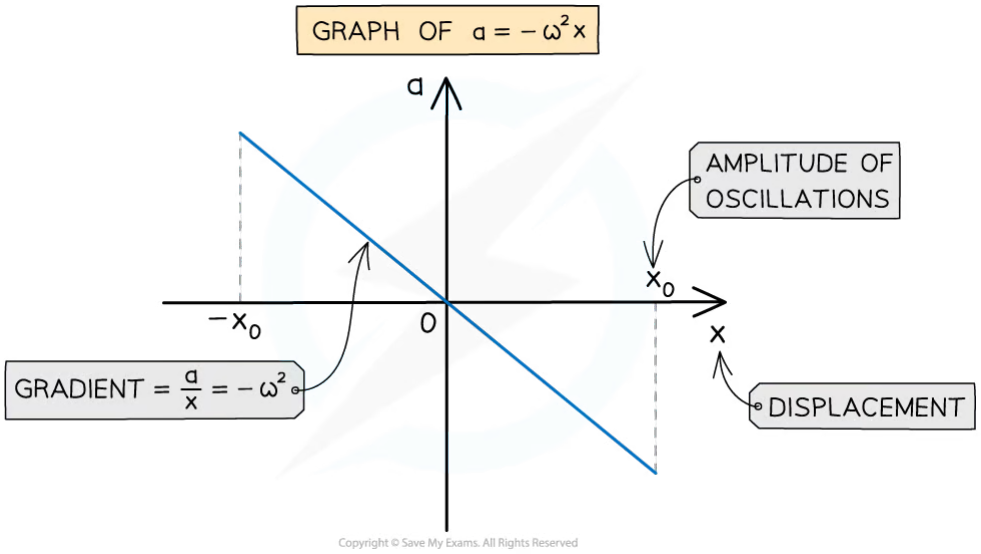
The Defining Equation of Simple Harmonic Motion

YOUR NOTES

- The acceleration of an oscillator can be calculated using the **defining equation of SHM**:

$$a = -\omega^2 x$$

- Where:
  - a = acceleration of the oscillator (m s^{-2})
  - x = displacement of the oscillator from its equilibrium position (m)
- The equation shows that:
  - The acceleration reaches its maximum value when the displacement is at a maximum i.e. $x = x_0$
  - The minus sign shows that when the object is displacement to the **right**, the direction of the acceleration is to the **left**
- An object in SHM will have a restoring force acting on it to return it to its equilibrium position
- This restoring force will be directly proportional, but in the **opposite direction**, to the displacement of the object from the equilibrium position
  - **Note:** the restoring force and acceleration act in the **same direction**



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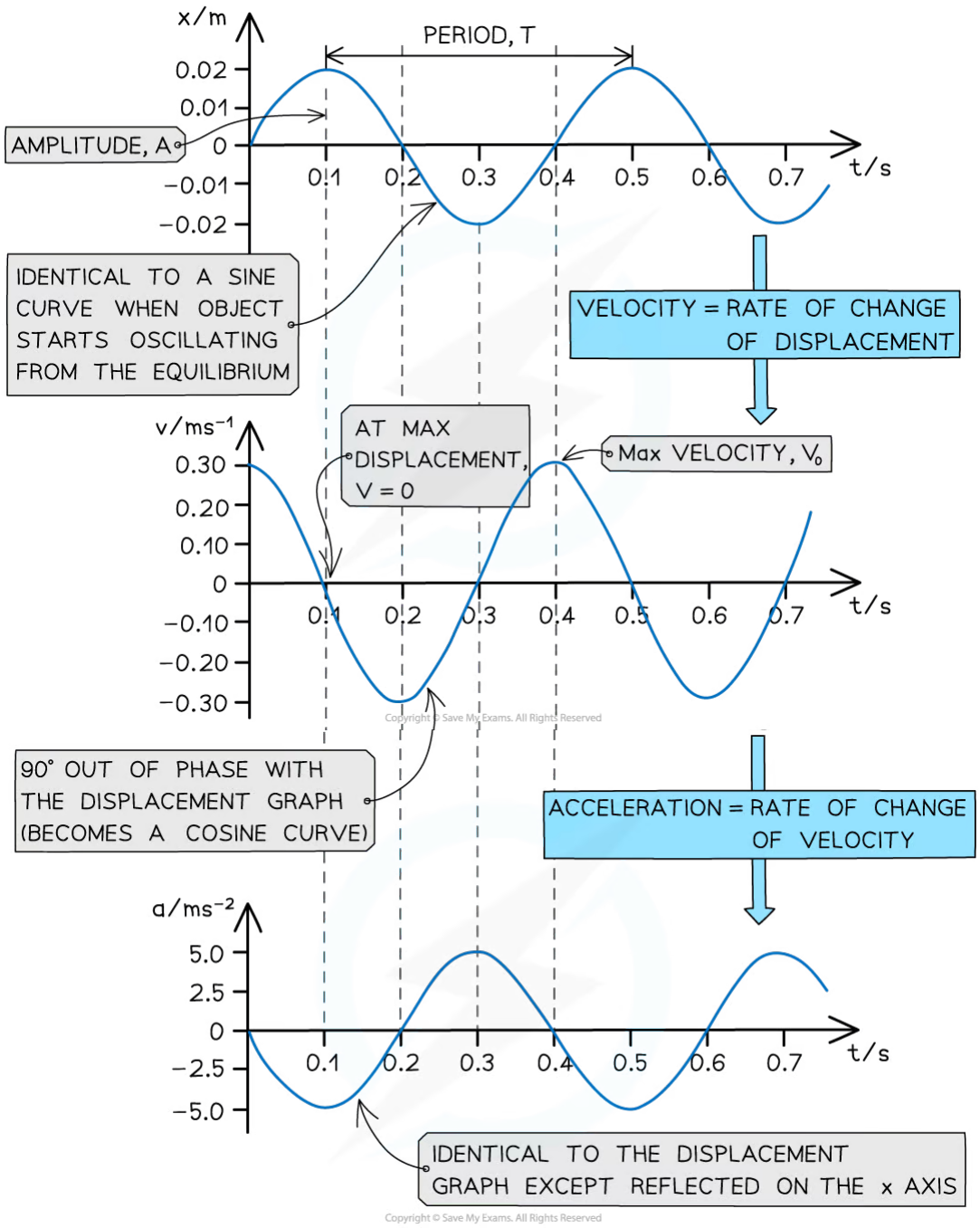
The acceleration of an object in SHM is directly proportional to the negative displacement

Graphs of Simple Harmonic Motion

- The displacement, velocity and acceleration of an object in simple harmonic motion can be represented by graphs against time
- All undamped SHM graphs are represented by **periodic functions**
  - This means they can all be described by sine and cosine curves



The displacement, velocity and acceleration graphs in SHM are all 90° out of phase with each other

Key features of the displacement-time SHM graph:

- The amplitude of oscillations *A* can be found from the maximum value of *x*
- The time period of oscillations *T* can be found from reading the time taken for one full cycle
- The graph might not always start at 0

- If the oscillation starts at the positive or negative amplitude, the displacement will be at its maximum

YOUR NOTES

Key features of the velocity-time SHM graph:

- It is 90° out of phase with the displacement-time graph
- Velocity is equal to the rate of change of displacement
- So, the velocity of an oscillator at any time can be determined from the **gradient of the displacement-time graph:**

$$v = \frac{\Delta x}{\Delta t}$$

- An oscillator moves the fastest at its equilibrium position
- Therefore, the velocity is at its **maximum** when the **displacement is zero**

Key features of the acceleration-time SHM graph:

- The acceleration graph is a reflection of the displacement graph on the x-axis
- This means when a mass has positive displacement (to the right) the acceleration is in the opposite direction (to the left) and vice versa
- It is 90° out of phase with the velocity-time graph
- Acceleration is equal to the rate of change of velocity
- So, the acceleration of an oscillator at any time can be determined from the **gradient of the velocity-time graph:**

$$a = \frac{\Delta v}{\Delta t}$$

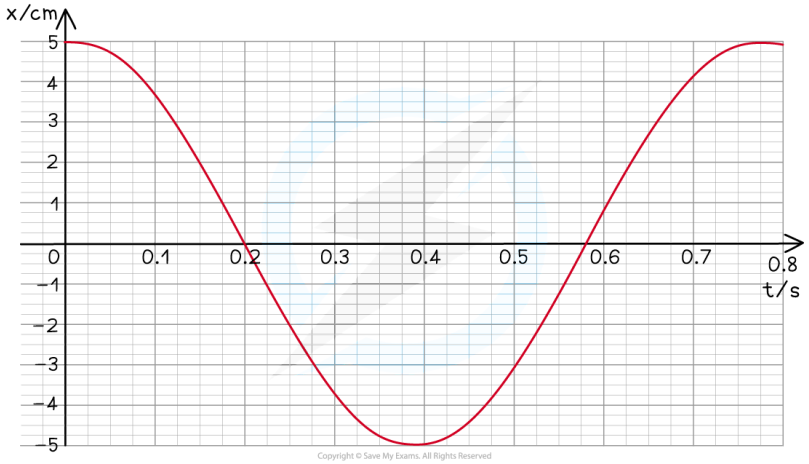
- The maximum value of the acceleration is when the oscillator is at its **maximum displacement**

YOUR NOTES

Worked Example

A swing is pulled 5 cm and then released.

The variation of the horizontal displacement *x* of the swing with time t is shown on the graph below.



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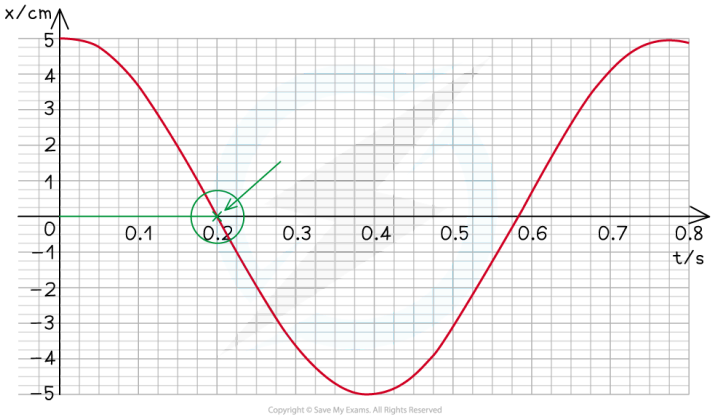
The swing exhibits simple harmonic motion.

Use data from the graph to determine at what time the velocity of the swing is first at its maximum.

Answer:

- The velocity is at its maximum when the displacement x = 0



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- From the graph, the first time the pendulum reaches maximum displacement is 0.2 s

Exam Tip

The defining equation of SHM shows acceleration, as a positive value, and displacement, –x as a negative one. This reminds us that acceleration and displacement are **vector** quantities and are always in the opposite direction to each other in SHM.

Since displacement is a vector quantity, remember to keep the minus sign in your solutions if they are negative. Getting the marks will depend on keeping your positive and negative numbers distinct from each other! Also remember that your calculator must be in **radians** mode when using the cosine and sine functions. This is because the angular frequency ω is calculated in rad s^{-1}, **not** degrees.

Time Period of a Mass–Spring System

- When an object is moving in SHM a **restoring force** F is always trying to return the object back to its equilibrium position
- The force is proportional to the displacement x from that equilibrium position

$$F = -kx$$

- Where:
  - F = the restoring force (N)
  - x = the displacement of the object from the equilibrium position (m)
  - k = spring constant (N m^{-1})
- From Newton's Second Law:

$$F = ma$$

- Where:
  - m = mass of the object (kg)
  - a = acceleration (m s^{-2})
- Combining these equations and rearranging for the acceleration gives

$$ma = -kx \quad \Rightarrow \quad a = -\frac{k}{m}x$$

- The defining equation of SHM is given by:

$$a = -\omega^2 x$$

- Where ω = angular frequency (rad s^{-1})
- Combining these expressions for acceleration gives:

$$-\omega^2 x = -\frac{k}{m}x$$

- This can be simplified to give an expression for ω:

$$\omega^2 = \frac{k}{m} \quad \Rightarrow \quad \omega = \sqrt{\frac{k}{m}}$$

- The equation relating angular frequency, ω, and time period, T, is:

$$\omega = \frac{2\pi}{T}$$

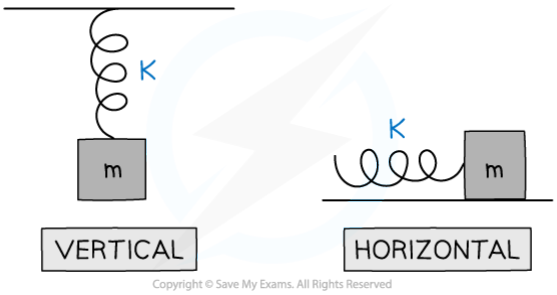
- Finally, combining these expressions leads to an equation for the **time period of a mass-spring system**:

YOUR NOTES

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- This equation applies for both a **horizontal** or **vertical** mass-spring system



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A mass-spring system can be either vertical or horizontal. The time period equation applies to both.

- The equation shows that:
  - The **higher** the spring constant *k*, the **stiffer** the spring and hence, the **shorter** the time period of oscillation
  - The time period (and hence, frequency) of a mass-spring system is independent of the force of gravity
- A consequence of this is that oscillations would have the **same time period** on Earth and the Moon

YOUR NOTES

Worked Example

A 200 g toy robot is attached to a pole by a spring, which has a spring constant of 90 N m^{-1} and made to oscillate horizontally.

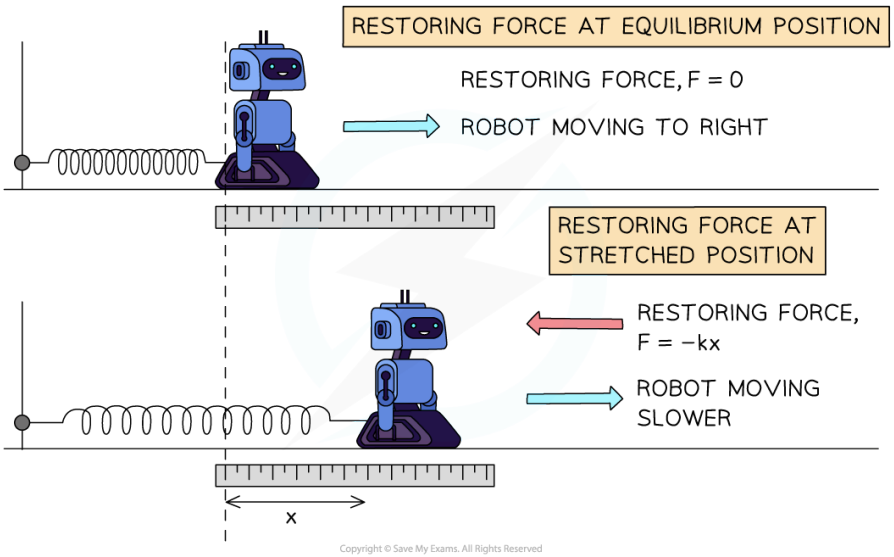
Calculate:

(a)
The force that acts on the robot when the spring is extended by 5 cm.
(b)
The acceleration of the robot whilst at its amplitude position.

Answer:

(a)

- Consider the motion of the robot at the equilibrium and stretched (amplitude) positions:



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- The restoring force is given by

$$F = -kx$$

- Using:
  - Extension, x = 5 cm = 0.05 m
  - Spring constant, k = 90 N m^{-1}

$$F = -90 \times 0.05 = -4.5 \text{ N}$$

- A force of 4.5 N will act on the robot, trying to pull it back towards the equilibrium position.

(b)

YOUR NOTES

- Newton's second law relates force and acceleration by

$$F = ma$$

- Using:
  - Mass, $m = 200\ \text{g} = 0.2\ \text{kg}$

$$a = \frac{F}{m} = \frac{-4.5}{0.2} = -22.5\ \text{m s}^{-2}$$

- The robot will decelerate at a rate of $22.5\ \text{m s}^{-2}$ when at this amplitude position

Worked Example

Calculate the frequency of a mass of 2.0 kg attached to a spring with a spring constant of $0.9\ \text{N m}^{-1}$ oscillating with simple harmonic motion.

Answer:

Step 1: Write down the known quantities

- Mass, $m = 2.0\ \text{kg}$
- Spring constant, $k = 0.9\ \text{N m}^{-1}$

Step 2: Write down the equation for the time period of a mass-spring system

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Step 3: Combine with the equation relating time period *T* and frequency, *f*

$$T = \frac{1}{f}$$

$$\frac{1}{f} = 2\pi\sqrt{\frac{m}{k}} \quad \Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Step 4: Substitute in the values to calculate frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{0.9}{2}}$$

Frequency: $f = 0.11\ \text{Hz}$

Exam Tip

Another area of physics where you may have seen the spring constant k is Hooke's Law, where $F = kx$.

Exam questions commonly merge these topics together, so make sure you're familiar with **Hooke's Law** too.

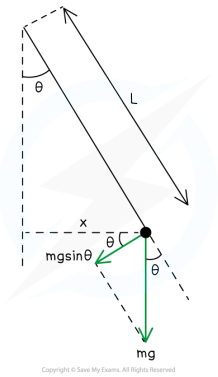
In the second worked example, the frequency calculated is the natural frequency of the mass-spring system, a concept that comes up in the topic of **resonance**.

YOUR NOTES

YOUR NOTES

Period of a Simple Pendulum & a Mass-Spring System

- A **simple pendulum** is a type of simple harmonic oscillator
  - The pendulum consists of a string and a bob (a weight, generally spherical) at the end
- An oscillating pendulum can be modelled as simple harmonic motion when the angle of oscillation is **small**



Forces on a pendulum when it is displaced. Assuming θ< 10°, the small angle approximation can be used to describe the time period of a simple pendulum such as this.

- The restoring force, *F*, which returns an oscillating pendulum bob to the equilibrium position is:

$$F = -mg \sin \theta$$

- Where:
  - *m* = the mass of the pendulum bob (kg)
  - *g* = acceleration due to gravity (m s^{-2})
  - θ = angle between the bob and the vertical (°)
- Using Newton's Second Law:

$$F = ma = -mg \sin \theta$$

- Both sides can be divided by *m* to give an expression for the acceleration, *a*:

$$a = -g \sin \theta$$

- In this case, the small-angle approximation can be used, this is where **sin $\theta \cong \theta$**
  - This is assuming the angle the pendulum makes with the vertical is **less than** 10°
- The expression for acceleration then becomes:

$$a = -g\theta$$

- The displacement x is equal to the length of the arc made by the bob as it moves through an angle θ

$$x = L\theta$$

- Where L = length of the pendulum (m)
- Rearranging this for θ and substituting it into the acceleration equation gives:

$$a = -g\left(\frac{x}{L}\right)$$

- This equation shows:
  - For small values of x, the condition for SHM is satisfied as restoring force, F is proportional to $-x$
  - For large values of x, the acceleration of a simple pendulum is not proportional to the displacement

- Using the defining equation of SHM:

$$a = -\omega^2 x$$

- Where ω = angular frequency (rad s^{-1})
- Equating with the previously derived expression for acceleration leads to:

$$-\omega^2 x = -g\left(\frac{x}{L}\right)$$

$$\omega^2 = \frac{g}{L} \quad \Rightarrow \quad \omega = \sqrt{\frac{g}{L}}$$

- The equation relating angular frequency, ω, and time period, T, is:

$$\omega = \frac{2\pi}{T}$$

- Finally, combining these expressions leads to an equation for the **time period of a simple pendulum**:

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

YOUR NOTES

YOUR NOTES

Worked Example

A swinging pendulum with a length of 80.0 cm has a maximum angle of displacement of 8°.

Determine the angular frequency of the oscillation.

Answer:

Step 1: List the known quantities

- Length of the pendulum, L = 80 cm = 0.8 m
- Acceleration due to gravity, g = 9.81 m s^{-2}

Step 2: Write down the relationship between angular frequency, ω, and period, T

$$T = \frac{2\pi}{\omega}$$

Step 3: Write down the equation for the time period of a simple pendulum

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- This equation is valid for this scenario since the maximum angle of displacement is less than 10°

Step 4: Equate the two equations and rearrange for ω

$$\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \quad \Rightarrow \quad \omega = \sqrt{\frac{g}{L}}$$

Step 5: Substitute the values to calculate ω

$$\omega = \sqrt{\frac{9.81}{0.8}} = 3.50 \text{ rad s}^{-1}$$

Angular frequency: ω = 3.5 rad s^{-1}

Energy Changes in Simple Harmonic Motion

- During simple harmonic motion, energy is constantly exchanged between two forms:
  - **Kinetic** energy
  - **Potential** energy
- The potential energy could be in the form of:
  - Gravitational potential energy (for a pendulum)
  - Elastic potential energy (for a horizontal mass on a spring)
  - Or both (e.g. for a vertical mass on a spring)
- The speed of an oscillator is at a maximum when displacement $(x = 0)$, so:

  The kinetic energy of an oscillator is at a maximum when the displacement is zero

- This is because:
  - Kinetic energy $\propto v^2$
  - So, when the oscillator moves at **maximum velocity** (at the equilibrium position) it also reaches a **maximum** value of kinetic energy
- Therefore, the kinetic energy is **zero** at maximum displacement $(x = x_0)$, so:

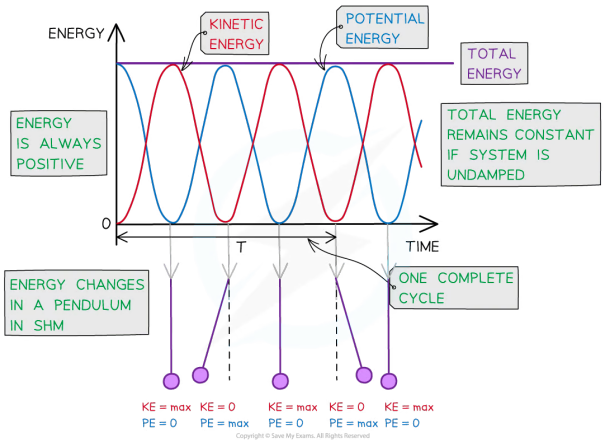
  The potential energy of an oscillator is at a maximum when the displacement (both positive and negative) is at a maximum, $x = \pm x_0$

- This is because:
  - Gravitational potential energy $\propto \Delta h$
  - The kinetic energy is **transferred** to potential energy as the **height** *h* above the equilibrium position increases
- A simple harmonic system is therefore constantly converting between kinetic and potential energy
- When one increases, the other decreases and vice versa, therefore:

  The total energy of a simple harmonic system always remains constant

- The total energy is, therefore, equal to the sum of the kinetic E_k and potential E_p energies

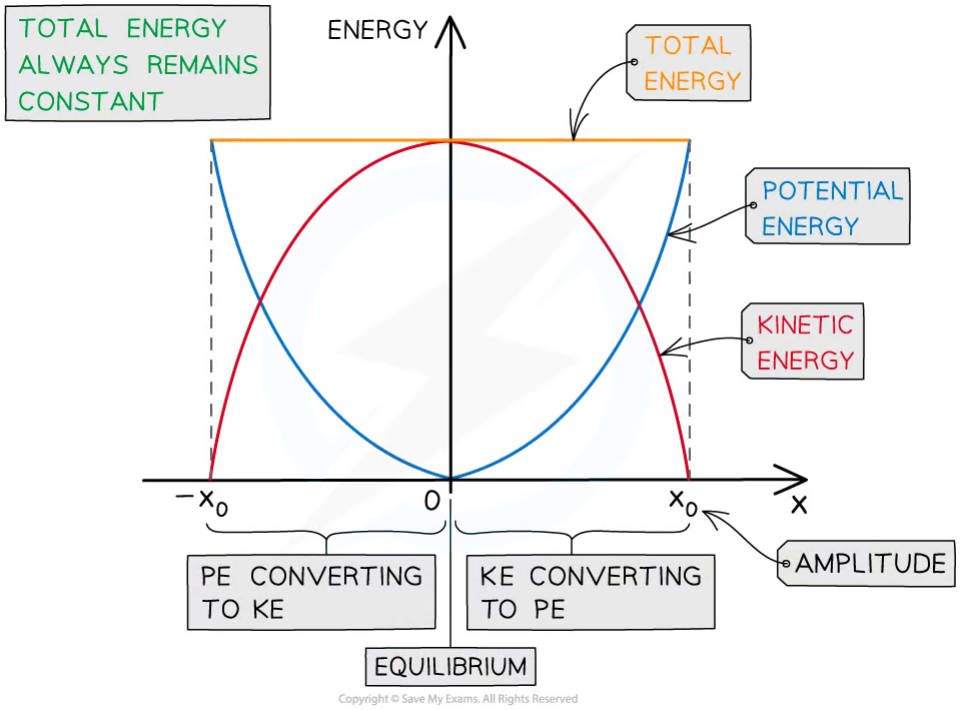
$$\text{Total energy} = E_k + E_p$$



The kinetic and potential energy of an oscillator in SHM vary periodically

The key features of the energy-time graph are...

- When the kinetic energy is **zero**, the potential energy is at a **maximum**
- When the potential energy is **zero**, the kinetic energy is at a **maximum**
- The **total energy** is represented by a **horizontal straight line** directly above the curves, where **total energy = maximum kinetic energy = maximum potential energy**
- Energy is **always positive** so there are no negative values on the y axis
- During **one** complete oscillation, the kinetic and potential energies go through **two** complete cycles
- This is because one complete oscillation reaches the maximum displacement **twice** (positive and negative)



Potential and kinetic energy v displacement in half a period of an SHM oscillation

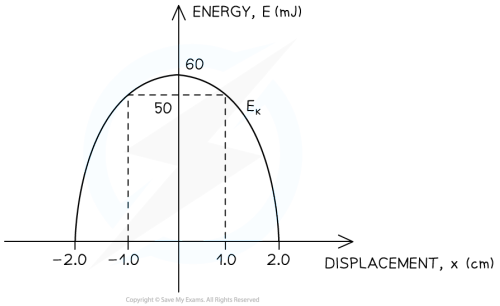
The key features of the energy-displacement graph are...

- Displacement is a **vector**, so, the graph has both **positive** and **negative** x values
- The **potential energy** is always at a maximum at the amplitude positions $(x = x_0)$ and zero at the equilibrium position $(x = 0)$
- This is represented by a **'U' shaped curve**
- The **kinetic energy** is zero at the amplitude positions $(x = x_0)$ and maximum at the equilibrium position $(x = 0)$
- This is represented by a **'n' shaped curve**
- The **total energy** is represented by a **horizontal straight line** directly above the curves, where **total energy = maximum kinetic energy = maximum potential energy**

YOUR NOTES

Worked Example

The following graph shows the variation with the displacement of the kinetic energy of an object of mass 0.50 kg oscillating with simple harmonic motion. Energy losses can be neglected.



Determine:

(a) The total energy of the object

(b) The amplitude of the oscillations

(c) The maximum velocity of the object

(d) The potential energy of the object when the displacement is $x = 1.0$ cm

Answer:

(a)

- From the graph, the maximum value of kinetic energy is 60 mJ
- At the equilibrium position $(x = 0)$, the total energy E is exactly equal to the maximum value of kinetic energy
- Since energy losses can be neglected, the total energy is constant

Total energy: $E = 60$ mJ

(b)

- The amplitude is equal to the maximum displacement on either side of the equilibrium position (where the kinetic energy is zero)

Amplitude: $x_0 = 2.0$ cm

(c)

- The maximum velocity can be found using the maximum kinetic energy in the equation:

$$E_k = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2E_k}{m}}$$

YOUR NOTES

- Using:
  - Mass of the object, $m = 0.50$ kg
  - Maximum kinetic energy, $E_k = 60$ mJ $= 0.06$ J

$$v = \sqrt{\frac{2 \times 0.06}{0.50}}$$

Maximum velocity: $v = 0.49\ \text{m s}^{-1}$

(d)

- From the graph, when the displacement is $x = 1.0$ cm, kinetic energy is $E_K = 50$ mJ
- The relationship between total energy E, kinetic energy E_K and potential energy E_P is:

$$E = E_P + E_K$$

- Therefore, the potential energy is

$$E_P = E - E_K$$

$$E_P = 60 - 50 = 10 \text{ mJ}$$

Exam Tip

You may be expected to draw as well as interpret energy graphs against time or displacement in exam questions. Make sure the sketches of the curves are as even as possible and **use a ruler** to draw straight lines, for example, to represent the total energy.

Calculating Energy Changes in Simple Harmonic Motion

- For a body that begins oscillating from its **equilibrium** position (i.e. $x = 0$ when $t = 0$), its displacement, velocity and acceleration can be described by the equations:

$$x = x_0 \cos \omega t$$

$$v = -\omega x_0 \sin \omega t$$

$$a = -\omega^2 x_0 \cos \omega t$$

- For a body that begins oscillating from its **amplitude** position (i.e. $x = \pm x_0$ when $t = 0$), its displacement, velocity and acceleration can be described by the equations:

$$x = x_0 \sin \omega t$$

$$v = \omega x_0 \cos \omega t$$

$$a = -\omega^2 x_0 \sin \omega t$$

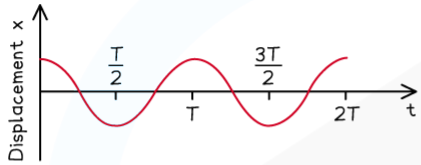
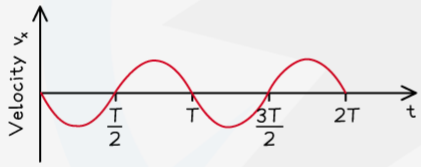
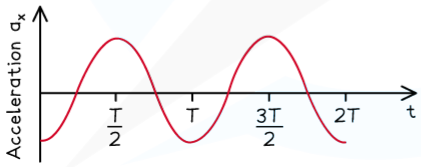
- Where:
  - x = displacement of the oscillator (m)
  - x_0 = maximum displacement, or amplitude (m)
  - v = velocity of the oscillator (m s^{-1})
  - a = acceleration of the oscillator (m s^{-2})
  - ω = angular frequency (rad s^{-1})
  - t = time (s)
- The greatest speed of an oscillator is at the equilibrium position ie. when its displacement $x = 0$
- The variation of an oscillator's velocity with its displacement x is defined by:

$$v = \pm \omega \sqrt{(x_0^2 - x^2)}$$

- This equation shows that the larger the **amplitude** x_0 of an oscillation, the greater the **distance** it must travel in a **given time period**
  - Hence, the **faster** it will travel the closer it is to the equilibrium position

Summary table of equations and graphs for displacement, velocity and acceleration

YOUR NOTES

		Object starts in centre of motion	Object starts at extremes of motion
Displacement		$x = x_0 \sin \omega t$	$x = x_0 \cos \omega t$
Velocity		$v = \omega x_0 \cos \omega t$ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$	$v = -\omega x_0 \sin \omega t$ $v = \pm \omega \sqrt{(x_0^2 - x^2)}$
Acceleration		$a = -x_0 \omega^2 \sin \omega t$ $a = -\omega^2 x$	$a = -x_0 \omega^2 \cos \omega t$ $a = -\omega^2 x$

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Calculating Total Energy in SHM

- Using the expression for the velocity *v* of a simple harmonic oscillator:

$$v = -\omega x_0 \sin(\omega t)$$

- The kinetic energy E_K of an oscillator can be written as:

$$E_K = \frac{1}{2} m v^2$$

$$E_K = \frac{1}{2} m(-\omega x_0 \sin(\omega t))^2$$

$$E_K = \frac{1}{2} m \omega^2 x_0^2 \sin^2(\omega t)$$

- Since the maximum value of $\sin(\omega t)$ or $\cos(\omega t)$ is 1, **maximum kinetic energy** is given by:

$$E_{K(max)} = \frac{1}{2} m \omega^2 x_0^2$$

- When the kinetic energy of the system is at a maximum, the potential energy is zero
  - Hence this represents the **total energy of the system**
- The total energy E_T of a system undergoing simple harmonic motion is, therefore, defined by:

$$E_T = \frac{1}{2} m\omega^2 x_0^2$$

- Where:
  - E_T = total energy of a simple harmonic system (J)
  - m = mass of the oscillator (kg)
  - ω = angular frequency (rad s^{-1})
  - x_0 = amplitude (m)
- **Note:** The same expression for total energy will be achieved if the other expression for velocity is used:

$$v = \omega x_0 \cos(\omega t)$$

Calculating Potential Energy in SHM

- An expression for the potential energy of a simple harmonic oscillator can be derived using the expressions for velocity and displacement

$$x = x_0 \sin(\omega t)$$

$$v = \omega x_0 \cos(\omega t)$$

- The key to deriving this expression is to use the trigonometric identity:

$$\sin^2(\omega t) + \cos^2(\omega t) = 1$$

- In a simple harmonic oscillation, the total energy of the system is equal to:

Total energy = Kinetic energy + Potential energy

$$E_T = E_K + E_P$$

- The **potential energy** of an oscillator can be written as:

$$E_P = E_T - E_K$$

$$E_P = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} mv^2$$

$$E_P = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} m\left(-\omega x_0 \sin(\omega t)\right)^2$$

$$E_P = \frac{1}{2} m\omega^2 x_0^2 - \frac{1}{2} m\omega^2 x_0^2 \sin^2(\omega t)$$

- Taking out a factor of $\frac{1}{2} m\omega^2 x_0^2$ gives:

$$E_P = \frac{1}{2} m\omega^2 x_0^2 (1 - \sin^2(\omega t))$$

$$E_P = \frac{1}{2} m\omega^2 {x_0}^2 \cos^2(\omega t)$$

$$E_P = \frac{1}{2} m\omega^2 \left[x_0 \cos(\omega t)\right]^2$$

- Since $x = x_0 \sin(\omega t)$, the potential energy of the system can be written as:

$$E_P = \frac{1}{2} m\omega^2 x^2$$

- Since the maximum potential energy occurs at the maximum displacement $\left(x = x_0\right)$ of the oscillation:

$$E_{P(max)} = \frac{1}{2} m\omega^2 {x_0}^2$$

- Therefore, it can be seen that:

$$E_T = E_{K(max)} = E_{P(max)}$$

Kinetic Energy–Displacement Relation for SHM

- Using the displacement–velocity relation for SHM:

$$v = \pm\omega\sqrt{{x_0}^2 - x^2}$$

- Substituting into the equation for kinetic energy:

$$E_K = \frac{1}{2} mv^2$$

$$E_K = \frac{1}{2} m\left(\omega\sqrt{{x_0}^2 - x^2}\right)^2$$

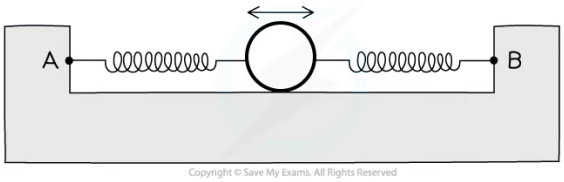
- This leads to the kinetic energy–displacement relation for SHM:

$$E_K = \frac{1}{2} m\omega^2\left({x_0}^2 - x^2\right)$$

YOUR NOTES

Worked Example

A ball of mass 23 g is held between two fixed points A and B by two stretched helical springs, as shown in the diagram below.



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The ball oscillates with simple harmonic motion along line AB. The oscillations are of frequency 4.8 Hz and amplitude 1.5 cm.

Calculate the total energy of the oscillations.

Answer:

Step 1: Write down the known quantities

- Mass, $m = 23\ \text{g} = 23 \times 10^{-3}\ \text{kg}$
- Amplitude, $x_0 = 1.5\ \text{cm} = 0.015\ \text{m}$
- Frequency, $f = 4.8\ \text{Hz}$

Step 2: Write down the equation for the total energy of SHM oscillations:

$$E = \frac{1}{2} m\omega^2 x_0^2$$

Step 3: Write an expression for the angular frequency

$$\omega = 2\pi f = 2\pi \times 4.8$$

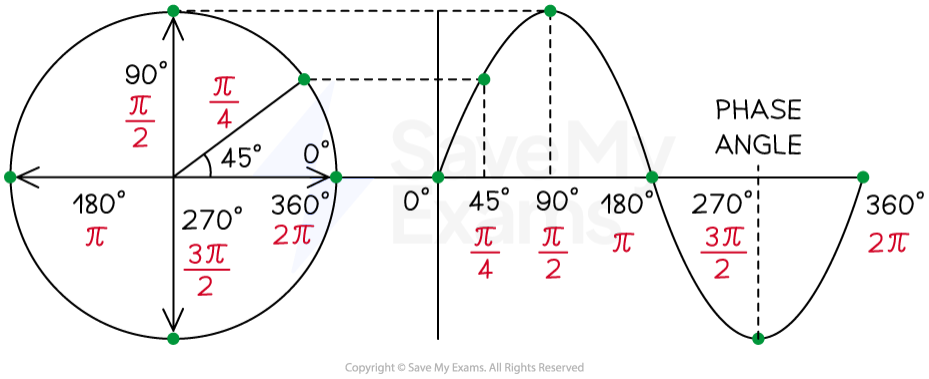
Step 4: Substitute values into the energy equation

$$E = \frac{1}{2} \times (23 \times 10^{-3}) \times (2\pi \times 4.8)^2 \times (0.015)^2$$

Total energy: $E = 2.354 \times 10^{-3} = \mathbf{2.4\ mJ}$

Phase Angles in Simple Harmonic Motion

- Two points on a sine wave, or on different waves, are in phase when they are at the **same point** in their wave cycle
  - The angle between their wave cycles is known as the **phase angle**



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The relationship between a sine wave and phase angle

- If an oscillation does **not** start from the equilibrium position, then it will be out of phase by an angle of ϕ
  - This would be compared to an oscillation which does start from the equilibrium position
- The phase angle ϕ of an oscillation (in SHM) is defined as

  The difference in angular displacement compared to an oscillator which has a displacement of zero initially (i.e. $x = 0$ when $t = 0$)

- The phase angle can vary anywhere from 0 to 2π radians, i.e. one complete cycle
- With the inclusion of the phase angle ϕ, the displacement, velocity and acceleration SHM equations become:

$$x = x_0 \sin(\omega t + \phi)$$

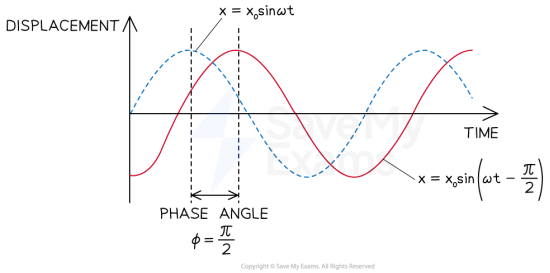
$$v = \omega x_0 \cos(\omega t + \phi)$$

$$a = -\omega^2 x_0 \sin(\omega t + \phi)$$

- If two bodies in simple harmonic motion oscillate with the same frequency and amplitude, but are out of phase by $\frac{\pi}{2}$, then:
  - The displacement of the oscillator starting from the equilibrium position is represented by the equation $x = x_0 \sin \omega t$

- The displacement of the oscillator which leads by $\frac{\pi}{2}$ is represented by the equation

$$x = x_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$



Two oscillators which are out of phase by $\phi = \frac{\pi}{2}$. The blue-dotted wave represents an oscillator starting from the equilibrium position and the red wave represents an oscillator leading by $\frac{\pi}{2}$

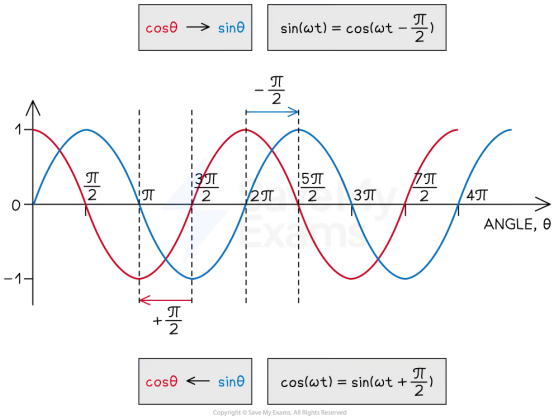
- When a sine wave leads by a phase angle of $\frac{\pi}{2}$, this is equivalent to the cosine of the wave

$$x = x_0 \sin\left(\omega t + \frac{\pi}{2}\right) = x_0 \cos \omega t$$

- Alternatively, a sine wave can be described as a cosine wave that lags by $\frac{\pi}{2}$

$$x = x_0 \cos\left(\omega t - \frac{\pi}{2}\right) = x_0 \sin \omega t$$

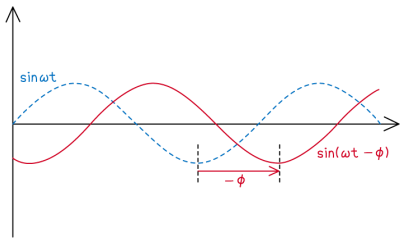
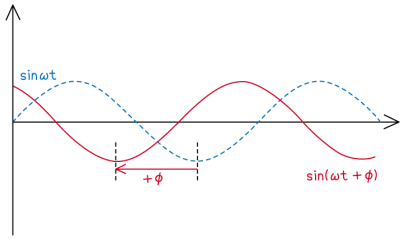
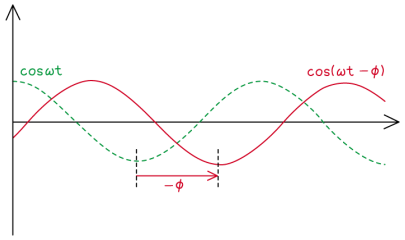
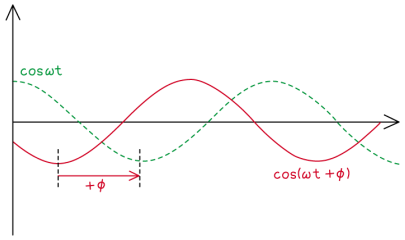
How are sine and cosine functions related?



Sine and cosine functions are simply out of phase by $\frac{\pi}{2}$ radians

YOUR NOTES

- The general rules for phase shifts of sine and cosine functions are shown in the table below

Graph	Equation	Phase shift
	$\sin(\omega t - \phi)$	Shifts by ϕ to the right (positive direction)
	$\sin(\omega t + \phi)$	Shifts by ϕ to the left (negative direction)
	$\cos(\omega t - \phi)$	Shifts by ϕ to the right (positive direction)
	$\cos(\omega t + \phi)$	Shifts by ϕ to the left (negative direction)

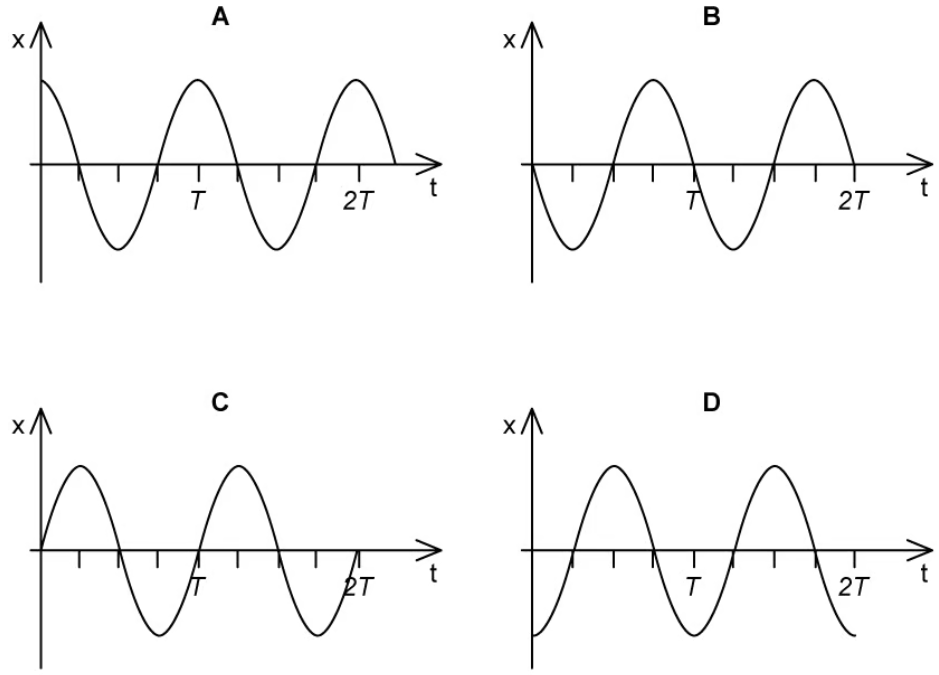
YOUR NOTES

Worked Example

An object oscillates with simple harmonic motion which can be described by the equation

$$x = x_0 \cos\left(\omega t - \frac{\pi}{2}\right)$$

Which of the following graphs correctly represents this equation?



Answer: C

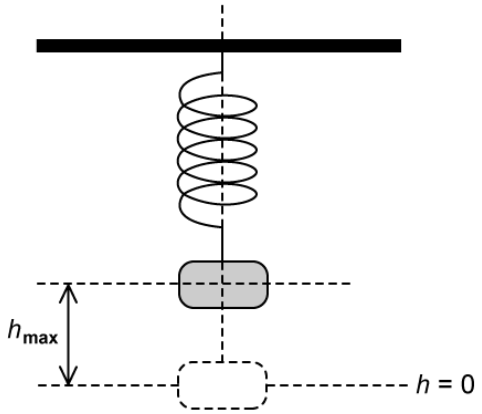
- The equation $x = x_0 \cos\left(\omega t - \frac{\pi}{2}\right)$ is equivalent to $x = x_0 \sin(\omega t)$
- This describes an oscillation where $x = 0$ when $t = 0$
  - Hence, options **A** & **D** are not correct
- When $\sin(\omega t)$ is positive, the oscillation will start moving in the $+x$ direction
  - Hence, option **B** is not correct

YOUR NOTES

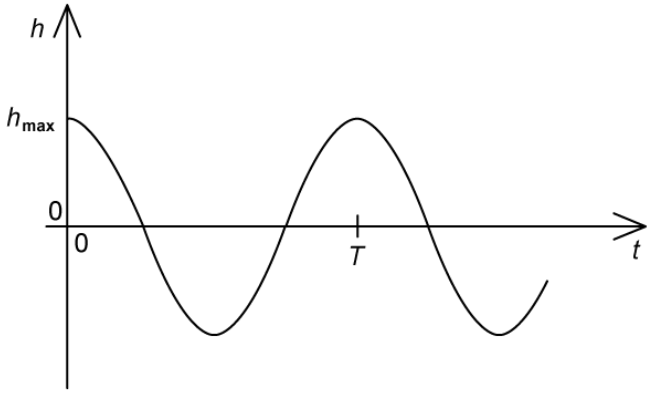
Worked Example

A mass attached to a spring is released from a vertical height of h_{max} at time $t = 0$.

The mass oscillates with a simple harmonic motion of period *T*.



The graph shows the variation of *h* with *t*.



(a)

State the equation of motion for this oscillation.

(b)

A second mass-spring system is set up and made to oscillate with the same frequency but with a phase angle of $\phi = -\frac{\pi}{4}$. On the graph, sketch the variation of *h* with *t* for the second mass-spring system.

Answer:

(a)

- The displacement-time equation for an oscillator released from a maximum displacement has the form

$$x = x_0 \cos \omega t$$

or

$$x = x_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

- Where $x = h$ and $x_0 = h_{max}$
- Angular frequency ω is equal to

$$\omega = \frac{2\pi}{T}$$

- Therefore, the equation of motion for this oscillation is:

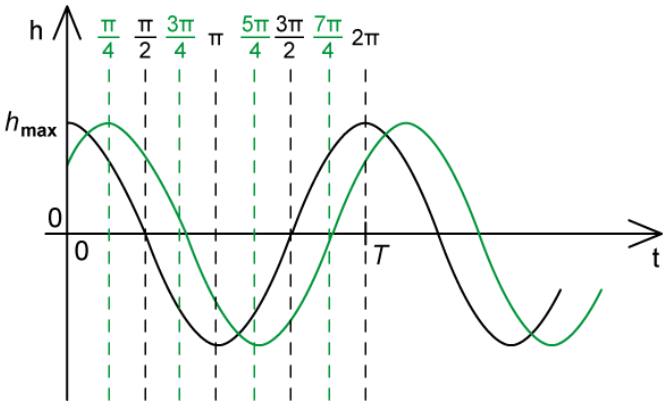
$$h = h_{max} \cos\left(\frac{2\pi}{T} t\right)$$

or

$$h = h_{max} \sin\left(\frac{2\pi}{T} t + \frac{\pi}{2}\right)$$

(b)

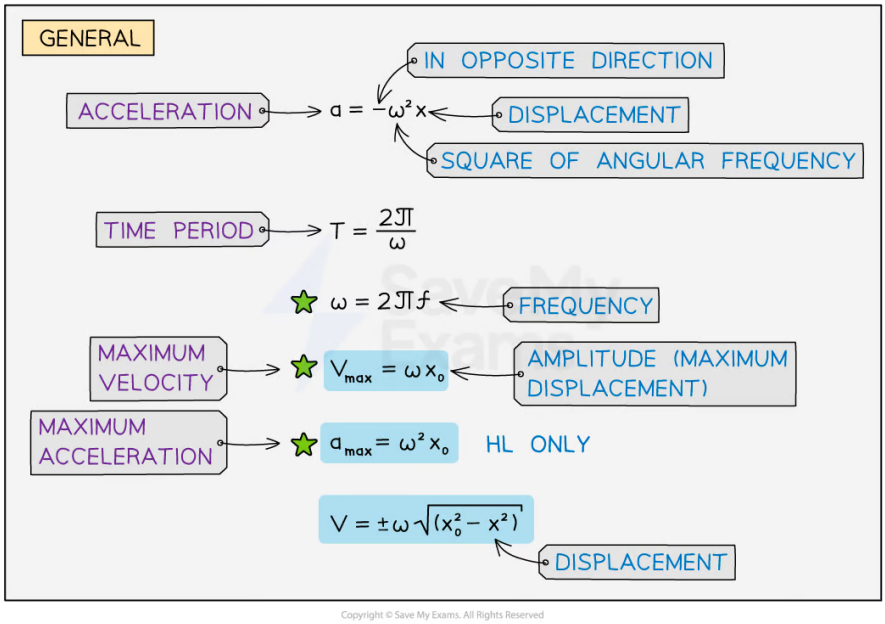
- One complete oscillation is equivalent to 2π rad



- A phase angle of $\phi = -\frac{\pi}{4}$ corresponds to a shift to the right (positive direction)

Equations for Simple Harmonic Motion

- There are many equations to learn in the topic of simple harmonic motion
- The key equations are summarised below:



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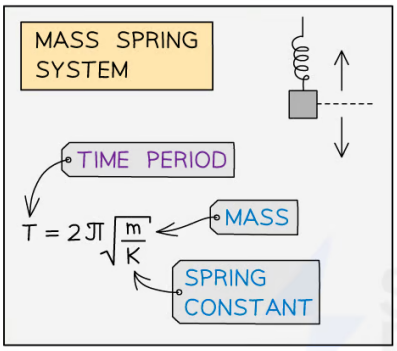

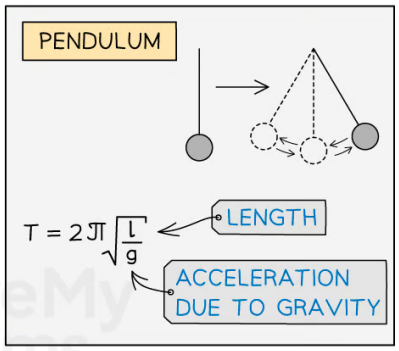

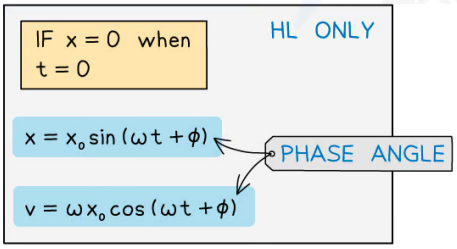

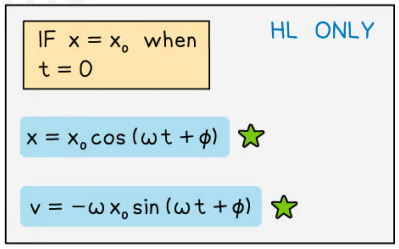

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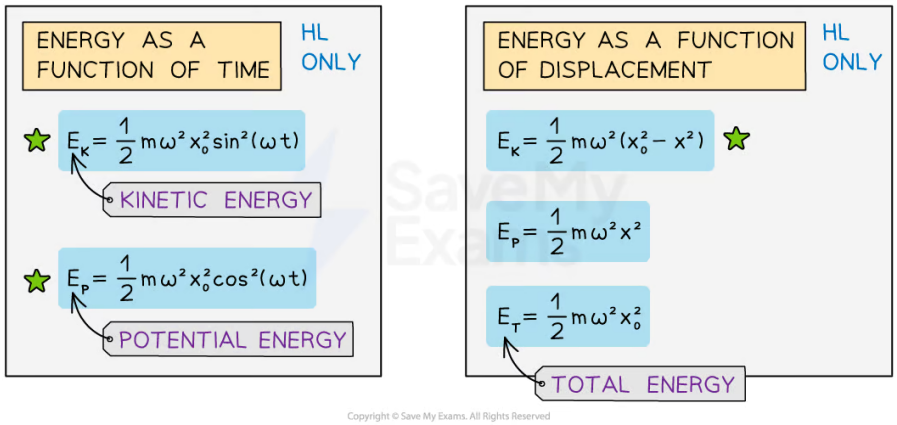

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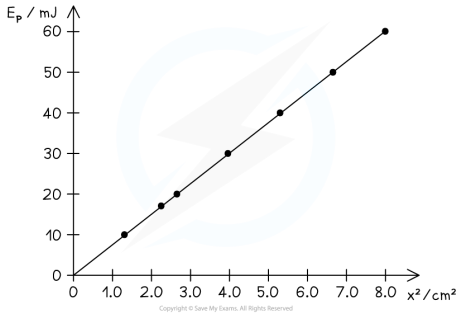
A summary of the equations related to simple harmonic motion. The green stars indicate equations which are not included in the IB data booklet and the highlighted equations are for HL students only

Worked Example

The graph shows the potential energy, E_P, for a particle oscillating with SHM. The particle has mass 45 g.

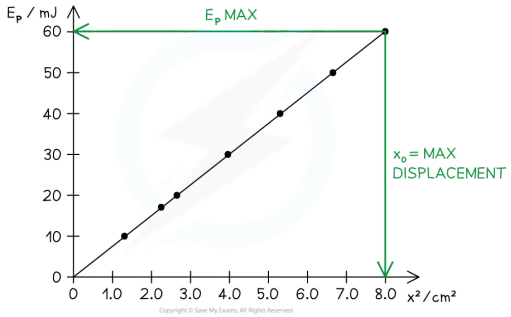


(a)
Use the graph to determine the amplitude and the period of the oscillation.
(b)
Calculate the maximum speed which the particle achieves.

Answer:

(a)

Step 1: Use the graph to determine the maximum potential energy of the particle



- Maximum potential energy, $E_{Pmax} = 60\ \text{mJ} = 60 \times 10^{-3}\ \text{J}$

Step 2: Determine the amplitude of the oscillation

- The amplitude of the motion x_0 is the maximum displacement
- At the maximum displacement, the particle is at its highest point, hence this is the position of maximum potential energy
- From the graph, when $E_P = 60$ mJ:

$$x_0^2 = 8.0\ \text{cm} = 0.08\ \text{m}$$

$$\text{Amplitude: } x_0 = \sqrt{0.08} = 0.28\ \text{m}$$

Step 3: Write down the equation for the potential energy of an oscillator and rearrange for angular velocity ω

$$E_P = \frac{1}{2} m\omega^2 x_0^2$$

$$\omega^2 = \frac{2E_P}{mx_0^2} \quad \Rightarrow \quad \omega = \sqrt{\frac{2E_P}{mx_0^2}}$$

Step 4: Substitute the known values and calculate ω

- Mass of the particle, $m = 45\ \text{g} = 45 \times 10^{-3}\ \text{kg}$

$$\omega = \sqrt{\frac{2 \times (60 \times 10^{-3})}{(45 \times 10^{-3}) \times 0.08}} = 5.77\ \text{rad s}^{-1}$$

Step 5: Determine the time period of the oscillation

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.77} = 1.1\ \text{s}$$

(b)

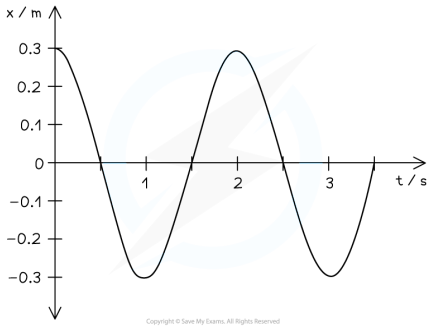
- The maximum speed of the oscillator is equal to

$$v_{max} = \omega x_0$$

$$v_{max} = 5.77 \times 0.28 = 1.6\ \text{m s}^{-1}$$

Worked Example

A student investigated the behaviour of a 200 g mass oscillating on a spring, and produced the graph shown.

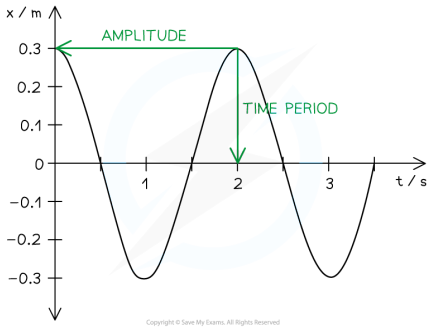


(a)
Determine the values for amplitude and time period
(b)
Hence find the maximum kinetic energy of the oscillating mass.

Answer:

(a)

- Read the values of amplitude and time period from the graph



- Amplitude, x_0 = 0.3 m
- Time period, T = 2.0 s

(b)

Step 1: Write down the equation for the maximum speed of an oscillator

$$v_{max} = \omega x_0$$

Step 2: Write down the equation relating angular speed and time period

$$\omega = \frac{2\pi}{T}$$

YOUR NOTES

Step 3: Combine the two equations and calculate the maximum speed

$$v_{max} = \frac{2\pi x_0}{T} = \frac{2\pi \times 0.3}{2.0}$$

$$v_{max} = 0.942 \text{ m s}^{-1}$$

Step 4: Use the maximum speed to calculate the maximum kinetic energy of the oscillating mass

- Mass of the oscillator, $m = 200 \text{ g} = 0.2 \text{ kg}$

$$E_{Kmax} = \frac{1}{2} m v_{max}^2$$

$$E_{Kmax} = 0.5 \times 0.2 \times 0.942^2 = 0.1 \text{ J}$$

Exam Tip

There are a large number of equations associated with SHM. Most of them are given in the data booklet which you will be given to use in the exam

Make sure you are familiar with the equations, as you will probably need to use several different ones to solve the longer questions.

IB Physics HL

YOUR NOTES

Wave Model

CONTENTS

Properties of Waves
Transverse & Longitudinal Waves
Sound Waves
Electromagnetic Waves

YOUR NOTES

Properties of Waves

- **Travelling waves** are defined as follows:

  > **Oscillations that transfer energy from one place to another without transferring matter**

- Waves transfer **energy**, **not** matter
- Waves are generated by **oscillating sources**
  - These oscillations travel **away** from the source
- Oscillations can propagate through a **medium** (e.g. air, water) or in **vacuum** (i.e. no particles), depending on the type of wave
- The key properties of **travelling waves** are as follows:

Displacement

- Displacement x of a wave is the distance of a point on the wave from its equilibrium position
  - It is a vector quantity; it can be positive or negative
  - Measured in **metres** (m)

Wavelength

- Wavelength λ is the length of one complete oscillation measured from the same point on two consecutive waves
  - For example, two crests, or two troughs
  - Measured in **metres** (m)

Amplitude

- Amplitude A is the maximum displacement of an oscillating wave from its equilibrium position ($x = 0$)
  - Amplitude can be positive or negative depending on the direction of the displacement
  - Measured in **metres** (m)
  - Where the wave has 0 amplitude (the horizontal line) is referred to as the **equilibrium position**

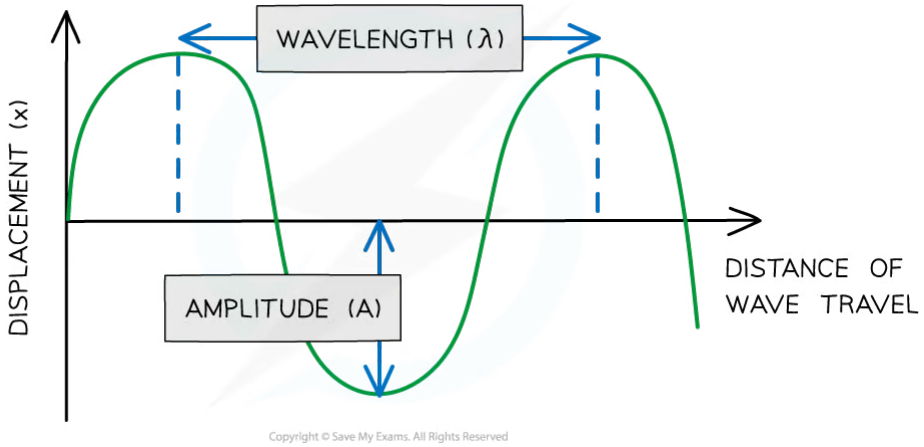


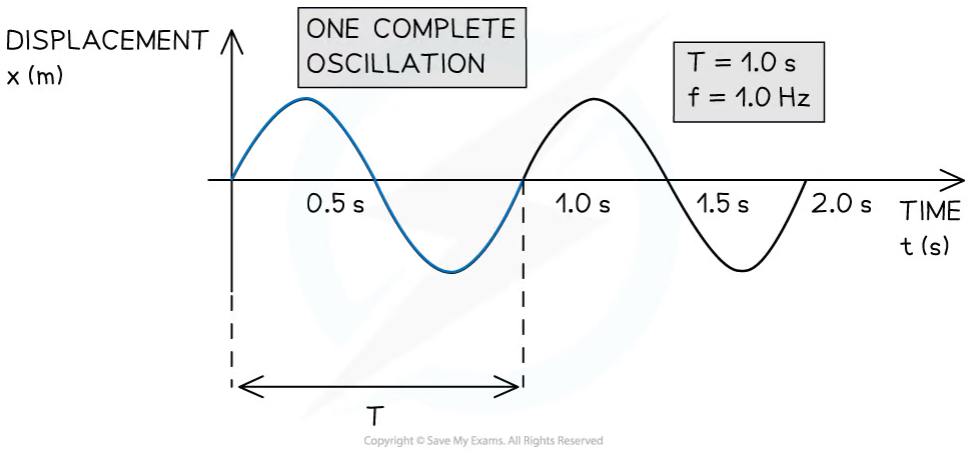
Diagram showing the amplitude and wavelength of a wave

Period & Frequency

- **Period** (*T*) is the time taken for a fixed point on the wave to undergo one complete oscillation
  - Measured in **seconds** (s)
- **Frequency** (*f*) is the number of full oscillations per second
  - Measured in **Hertz** (Hz)
- The frequency, *f*, and the period, *T*, of a travelling wave are related to each other by the equation:

$$f = \frac{1}{T}$$

- Where:
  - *f* = frequency (Hz)
  - *T* = time period (s)



Period T and frequency f of a travelling wave

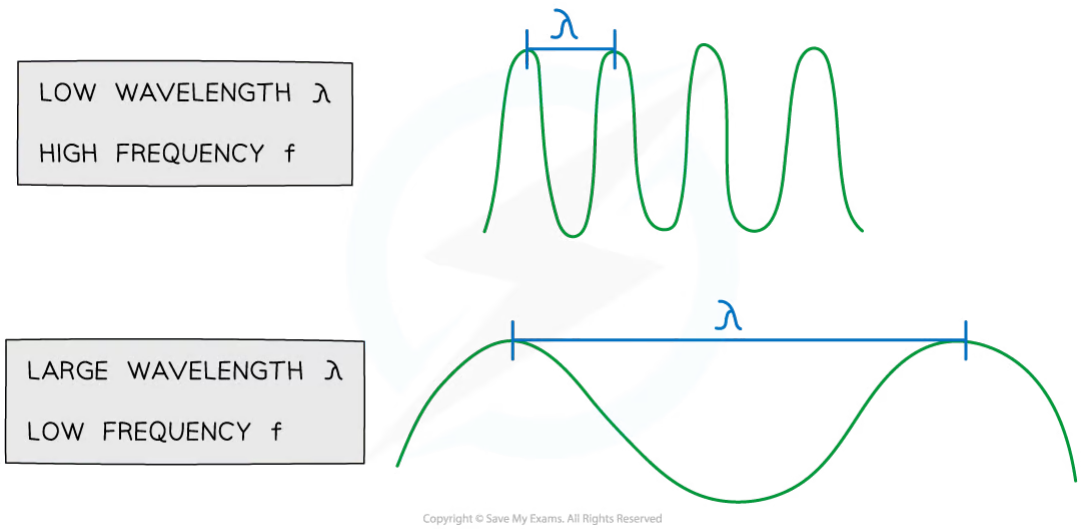
Wave speed

YOUR NOTES

- **Wave speed** (v) is the distance travelled by the wave per unit time
  - Measured in **metres per second** ($m\ s^{-1}$)
- The wave speed is defined by the equation:

$$v = f\lambda = \frac{\lambda}{T}$$

- Where:
  - v = wave speed ($m\ s^{-1}$)
  - λ = wavelength (m)
- This is referred to as the **wave equation**
- It tells us that for a wave of constant speed:
  - As the wavelength **increases**, the frequency **decreases**
  - As the wavelength **decreases**, the frequency **increases**



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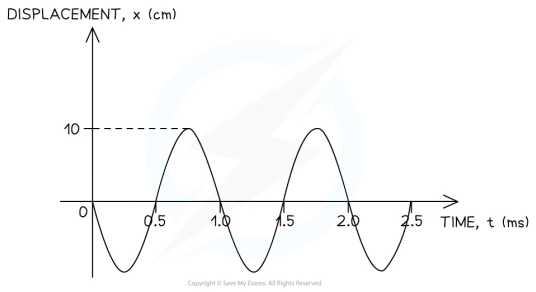
The relationship between the frequency and wavelength of a wave

YOUR NOTES

Worked Example

The graph below shows a travelling wave.



Determine:

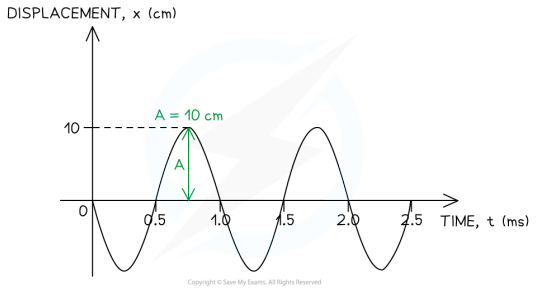
(a) The amplitude *A* of the wave, in m.

(b) The frequency *f* of the wave, in Hz.

Answer:

(a) Identify the amplitude *A* of the wave on the graph

- The amplitude is defined as the maximum displacement from the equilibrium position ($x = 0$)



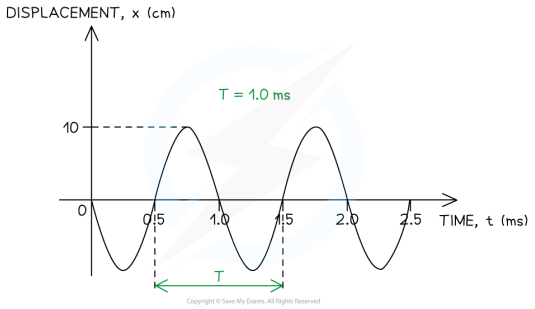
- The amplitude must be converted from centimetres (cm) into metres (m)

$$A = 0.1 \text{ m}$$

(b) Calculate the frequency of the wave

Step 1: Identify the period *T* of the wave on the graph

- The period is defined as the time taken for one complete oscillation to occur



- The period must be converted from milliseconds (ms) into seconds (s)

$$T = 1 \times 10^{-3}\ \text{s}$$

Step 2: Write down the relationship between the frequency *f* and the period *T*

$$f = \frac{1}{T}$$

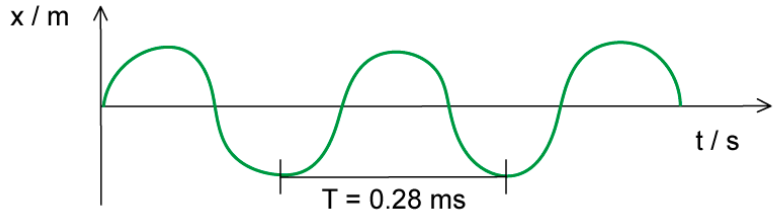
Step 3: Substitute the value of the period determined in Step 1

$$f = \frac{1}{1 \times 10^{-3}} = 1000\ \text{Hz}$$

YOUR NOTES

Worked Example

The wave in the diagram below has a speed of 340 m s^{-1}.



Determine the wavelength of the wave.

Answer:

STEP 1

WAVE EQUATION

$v = f\lambda$

STEP 2

REARRANGE FOR WAVELENGTH

$\lambda = \frac{v}{f}$

STEP 3

CALCULATE f

$f = \frac{1}{T} = \frac{1}{0.28 \times 10^{-3}\ s} = 3571.43\ Hz$

STEP 4

SUBSTITUTE VALUE BACK INTO WAVE EQUATION

$\lambda = \frac{340}{3571.43} = 0.095\ m$ (2 s.f.)

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YOUR NOTES

Worked Example

A travelling wave has a period of 1.0 μs and travels at a velocity of 100 cm s^{-1}.

Calculate the wavelength of the wave, in m.

Answer:

Step 1: Write down the known quantities

- Period, $T = 1.0\ \mu s = 1.0 \times 10^{-6}$ s
- Velocity, $v = 100$ cm $s^{-1} = 1.0$ m s^{-1}

  Note the conversions:

  - The period must be converted from microseconds (μs) into seconds (s)
  - The velocity must be converted from cm s^{-1} into m s^{-1}

Step 2: Write down the relationship between the frequency *f* and the period *T*

$$f = \frac{1}{T}$$

Step 3: Substitute the value of the period into the above equation to calculate the frequency

$$f = \frac{1}{1 \times 10^{-6}} = 1 \times 10^{6}$$

Step 4: Write down the wave equation

$$v = f\lambda$$

Step 5: Rearrange the wave equation to calculate the wavelength λ

$$\lambda = \frac{f}{v}$$

Step 6: Substitute the numbers into the above equation

$$\lambda = \frac{1}{1 \times 10^{6}} = 1 \times 10^{-6}$$

Exam Tip

You must be able to interpret different properties of waves from a variety of graphs. You may recognise calculating the time period and wavelength look very similar (the distance for one full wave). This is the time period if the *x-axis* is **time**. If the *x*-axis is **distance**, then this distance is the wavelength.

Pay very close attention to units. If you want a frequency in Hertz, then the time period must be in **seconds** and not milliseconds etc.

YOUR NOTES

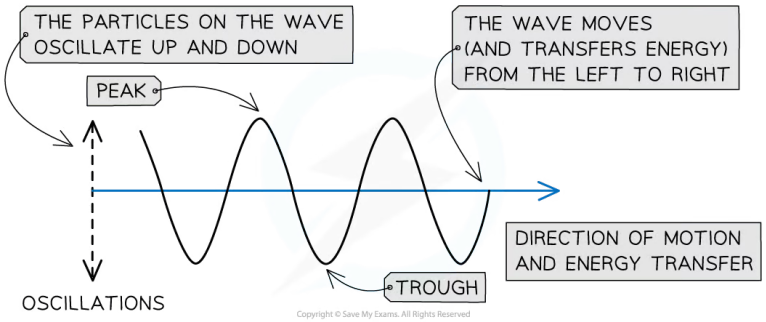
Transverse & Longitudinal Waves

- In mechanical waves, particles **oscillate** about fixed points
- There are two types of wave: **transverse** and **longitudinal**
- The type of wave can be determined by the direction of the oscillations in relation to the direction the wave is travelling

Transverse Waves

- Transverse waves are defined as follows:

> **A wave in which the particles oscillate perpendicular to the direction of motion and energy transfer**



A transverse wave travelling from left to right

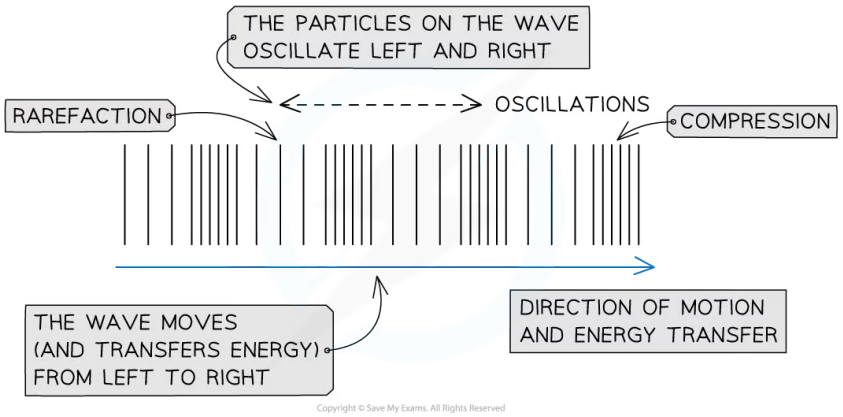
- This means that each particle in the wave vibrates **up** and **down**
- Transverse waves show areas of **peaks** and **troughs**
- Examples of transverse waves include:
  - Electromagnetic waves e.g. radio, visible light, UV
  - Vibrations on a guitar string
- Transverse waves transfer **energy**, even if there is no resultant displacement of the medium
  - This means transverse waves do not need particles to propagate, so they **can travel through a vacuum**
  - This is why we can still feel the UV radiation from the Sun, as it can travel through the vacuum of space

Longitudinal Waves

- Longitudinal waves are defined as follows:

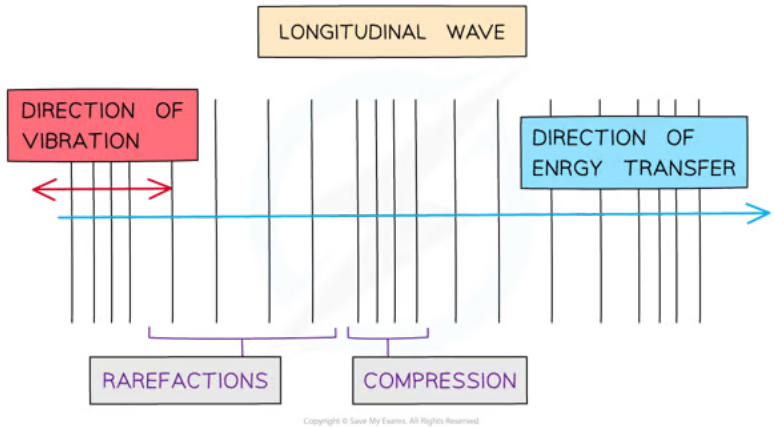
> **A wave in which the particles oscillate parallel to the direction of motion and energy transfer**

YOUR NOTES



A longitudinal wave travelling from left to right

- This means that each particle in the wave only vibrates **left** and **right**
- As a longitudinal wave propagates, areas of low and high pressure can be observed:
  - A **rarefaction** is an area of low pressure, with the particles being further apart from each other
  - A **compression** is an area of high pressure, with the particles being closer to each other



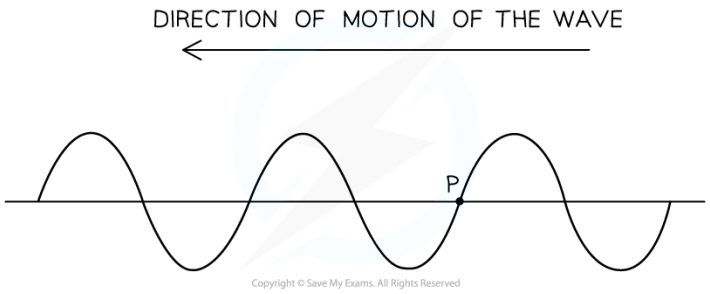
Rarefactions and compressions on a longitudinal wave

- **Sound waves** are an example of longitudinal waves
- Longitudinal waves need **particles** to propagate, so they **cannot travel through a vacuum**
  - This is why you cannot hear anything in the vacuum of outer space

YOUR NOTES

Worked Example

The diagram below represents a transverse wave at time $t = 0$. The direction of motion of the wave is shown. Point **P** is a point on the wave. State in which direction point **P** will move immediately after the time shown.



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Answer:

Step 1: Determine the possible directions that point P can travel in

- In transverse waves, the particles oscillate perpendicular to the direction of motion
- This transverse wave travels from right to left
- Oscillations will either be up or down
- Hence point **P** will either move up or down

Step 2: Determine the next direction of point P

- Since the wave is moving from right to left, a crest (i.e. a point of maximum displacement above the equilibrium position) will be approaching point P immediately after the time shown
- Point P will be moving **upwards**

Exam Tip

Exam questions will focus on the description of the motion of particles of a medium when a wave passes through it, for both types. Make sure you remember the difference between them:

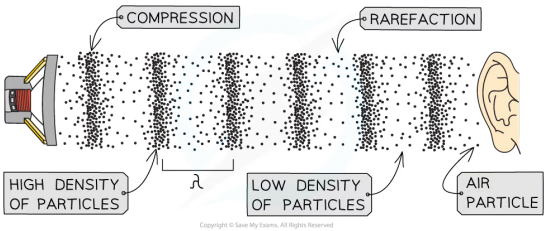
- Particles in a transverse wave move up and down
- Particles in a longitudinal wave move left and right

The particles do **not** travel 'along' the wave, they are just in one position, and can only move either vertically or horizontally. All the particles have the same motion, but are displaced slightly, creating the illusion that the whole **wave** is moving together.

YOUR NOTES

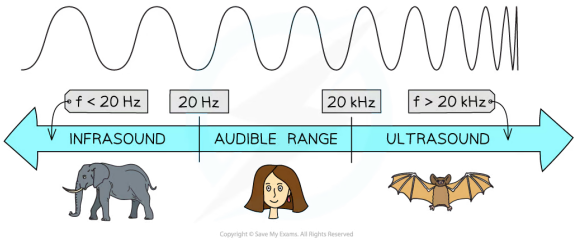
Sound Waves

- Sound waves are **longitudinal waves** and, as such, require a **medium** in which to propagate
- Sound waves are generated by oscillating sources, which produce a change in **density** of the surrounding medium
- The sound wave then travels with a series of **compressions** and **rarefactions**



A sound wave travelling through air

- Sound waves form a continuous **spectrum** based on their frequency

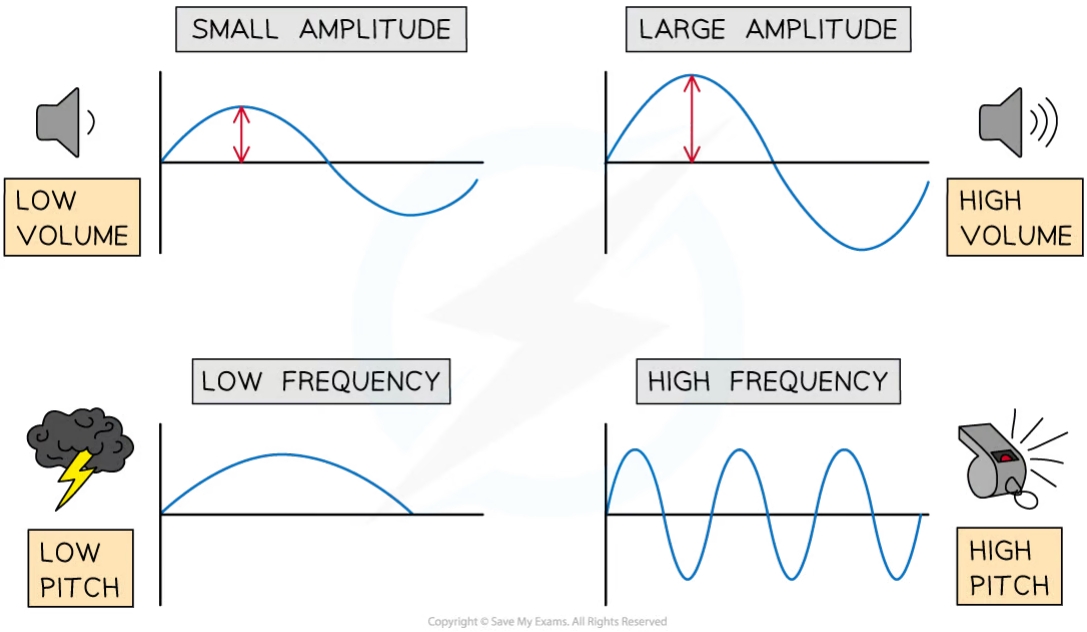


The spectrum of sound waves

- Humans can only hear sounds with frequencies in the range 20 Hz - 20 kHz, known as the **audible range**
- Sounds with frequencies below and above this range cannot be detected by the human ear

Pitch & Volume

- The frequency of a sound wave is related to its **pitch**
  - Sounds with a **high** pitch have a **high** frequency (or short wavelength)
  - Sounds with a **low** pitch have a **low** frequency (or long wavelength)
- The amplitude of a sound wave is related to its **volume**
  - Sounds with a **large** amplitude have a **high** volume
  - Sounds with a **small** amplitude have a **low** volume



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Pitch and amplitude of sound

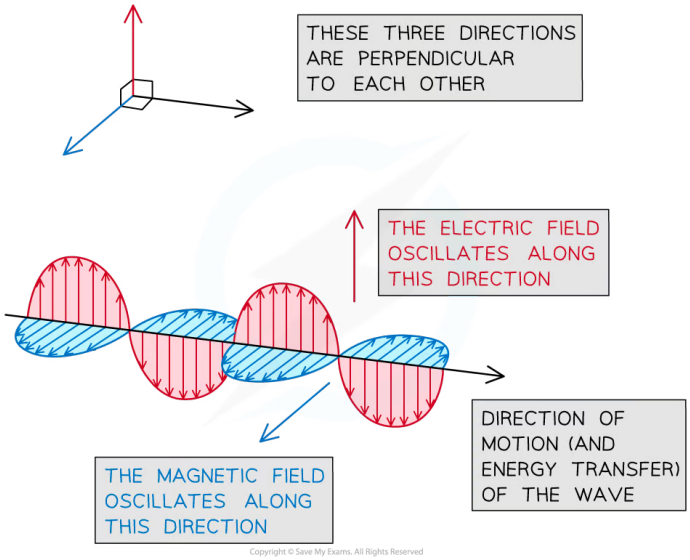
Speed of Sound

- Sound waves travel at a speed of about 340 m s^{-1} in air at room temperature
  - The higher the air temperature, the greater the speed of sound
  - The is because the average kinetic energy of the particles is higher
- Sound travels the **fastest** through **solids**, since solid particles are closely packed and can pass the oscillations onto their neighbours much faster
- Sound travels the **slowest** in **gases**, since gas particles are spread out and less efficient in transferring the oscillations to their neighbours

YOUR NOTES

The Electromagnetic Spectrum

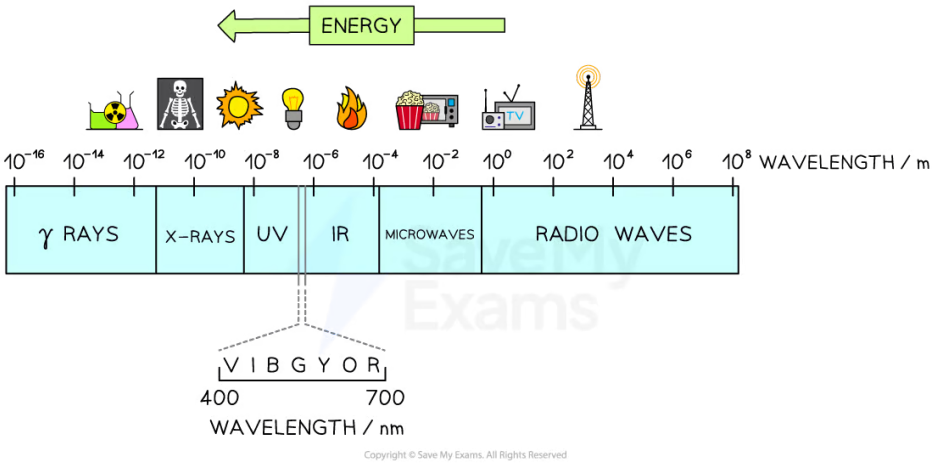
- An electromagnetic wave is generated by the combined oscillation of an **electric** and a **magnetic field**
- These fields oscillate **perpendicular** to each other and to the direction of motion of the wave (i.e. the direction in which energy is transferred)



An electromagnetic wave is generated by the combined oscillation of an electric and a magnetic field

- Electromagnetic waves are **transverse waves** and, as such, they can travel through **vacuum**
- Regardless of their frequency, all electromagnetic waves travel at the **speed of light** $c = 3 \times 10^8$ m s^{-1} in vacuum
- Electromagnetic waves form a continuous **spectrum** based on their frequency (or wavelength)
- The shorter the wavelength, or higher the frequency, the greater the **energy** of the wave

YOUR NOTES



The electromagnetic spectrum

- Humans can only sense electromagnetic waves with wavelengths in the range 700 nm - 400 nm, which are the limits of the so-called **visible spectrum**
  - Electromagnetic waves with longer and shorter wavelengths are invisible to the human eye
- Knowing the wavelengths of electromagnetic waves, their frequencies can be calculated using
  - The wave equation
  - The fact that the speed of light ($c = 3 \times 10^{8}$ m s^{-1}) in a vacuum is constant

YOUR NOTES

Worked Example

:The wavelength of blue light falls within the range 450 nm – 490 nm.

Determine the range of frequencies of blue light.

Answer:

Step 1: Write down the known quantities

- Note that you must convert the values of the wavelength from nanometres (nm) into metres (m)
  - $\lambda_{lower} = 450 \text{ nm} = 4.5 \times 10^{-7} \text{ m}$
  - $\lambda_{higher} = 490 \text{ nm} = 4.9 \times 10^{-7} \text{ m}$

Step 2: Remember that all electromagnetic waves travel at the speed of light in vacuum

- From the data booklet, $c = 3.00 \times 10^{8} \text{ m s}^{-1}$

Step 3: Write down the wave equation

$$v = f\lambda$$

Step 4: Rearrange the above equation to calculate the frequency *f*

$$f = \frac{v}{\lambda}$$

Step 5: Substitute the lower and higher values of the wavelength to calculate the limiting values of the frequency of blue light

- The lower frequency f_{lower} corresponds to the higher value of the wavelength λ_{higher}

$$f_{lower} = \frac{3.00 \times 10^{8}}{4.9 \times 10^{-7}} = 6.1 \times 10^{14} \text{ Hz}$$

- The higher frequency f_{higher} corresponds to the lower value of the wavelength λ_{lower}

$$f = \frac{3.00 \times 10^{8}}{4.5 \times 10^{-7}} = 6.7 \times 10^{14} \text{ Hz}$$

Step 6: Write down the range of frequencies of blue light

$$f = 6.1 \times 10^{14} - 6.7 \times 10^{14} \text{ Hz}$$

Exam Tip

You don't need to memorise the order or the wavelengths of waves in the electromagnetic spectrum, as this is given in your data booklet. However, you must remember all electromagnetic waves travel at the speed of light, *c*.

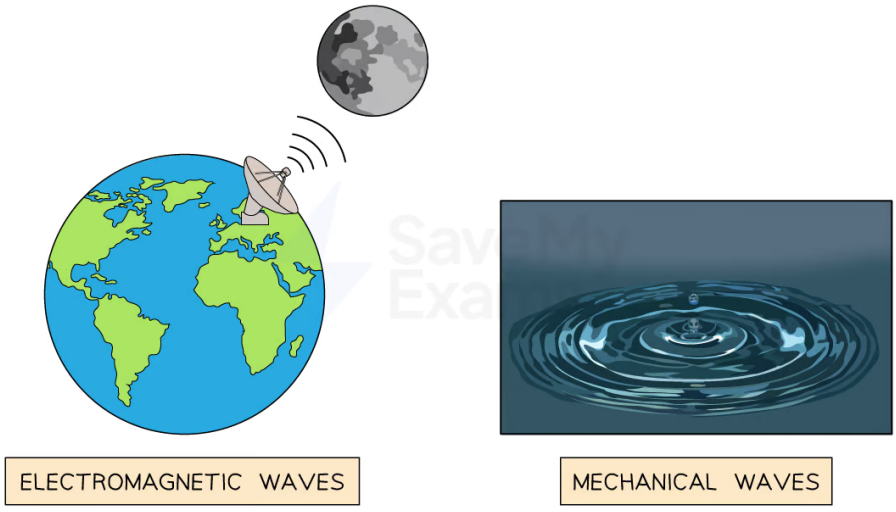
YOUR NOTES

YOUR NOTES

Comparing Mechanical & Electromagnetic Waves

- Travelling waves can be of two types, **mechanical** and **electromagnetic**

Mechanical Waves	Electromagnetic Waves
Require a medium, such as a fluid or solid to propagate through	Do not require a medium
Can be transverse or longitudinal	Are only transverse
Cannot travel through a vacuum	Can travel through a vacuum
Are produced by the oscillation of particles in a medium	Are produced by oscillating charged particles
Examples: Sound waves, waves on the surface of the ocean	Examples: Radio waves, UV rays, X-rays
Travel a lot slower than the speed of light	Travel at the speed of light



Electromagnetic waves can travel in a vacuum (such as space) whilst mechanical waves require a medium (such as water)

Exam Tip

You must be able to identify the features of either mechanical or electromagnetic waves, depending on the question.

A common misconception is that mechanical waves are just longitudinal, like sound waves. However, they can also be transverse waves that travel through a medium such as seismic waves (created during an earthquake) and water waves.

IB Physics HL

YOUR NOTES

Wave Phenomena

CONTENTS

Wavefronts & Rays

Reflection, Refraction & Transmission

Diffraction of Waves

Refraction of Waves

Interference of Waves

Superposition of Waves

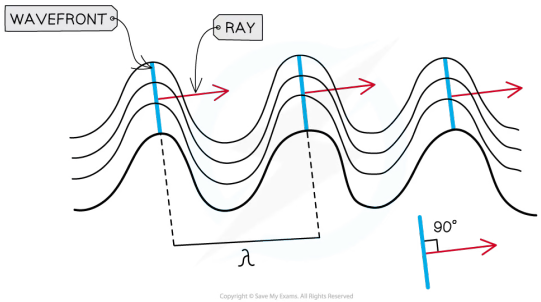
Young's Double-Slit Experiment

Single-Slit Diffraction (HL)

Diffraction Gratings (HL)

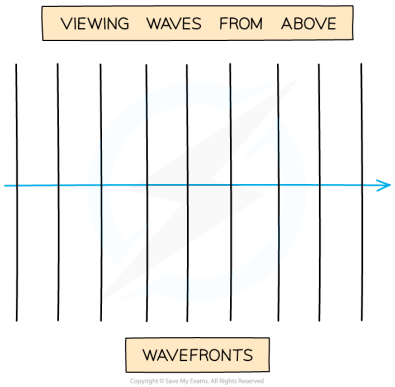
Wavefronts & Rays

- Waves can be represented graphically in two different ways:
  - **Wavefronts** - lines joining all the points that oscillate in phase and are perpendicular to the direction of motion (and energy transfer)
  - **Rays** - lines showing the direction of motion (and energy transfer) of the wave that are perpendicular to the wavefront



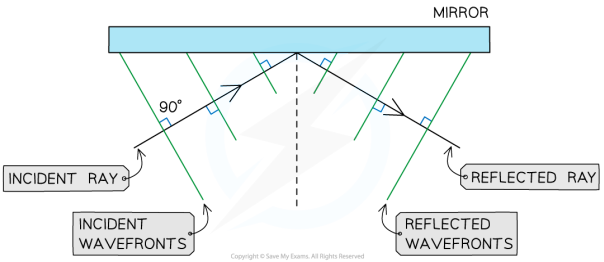
Wavefronts and rays for transverse waves travelling in a horizontal plane

- Wavefronts from above will therefore just look like a series of vertical lines



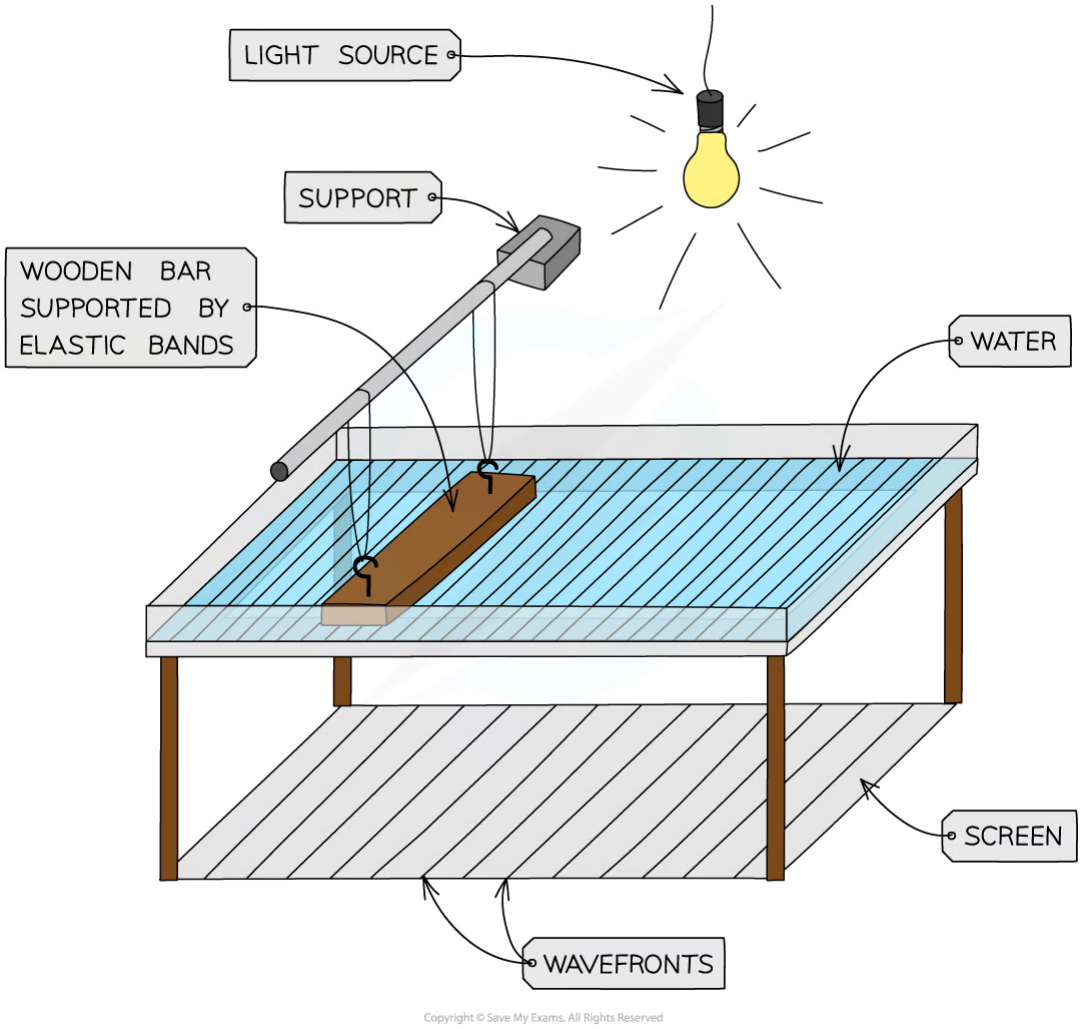
Waves can be represented using wavefronts

- The distance between successive wavefronts is equal to the wavelength of the waves
- Wave phenomena such as reflection can be shown through rays, or the wavefronts
  - Notice the ray is still always **perpendicular** to the wavefronts



Reflection can be shown through rays or wavefronts

- Ripple tanks are used a common experiment to demonstrate diffraction of water waves



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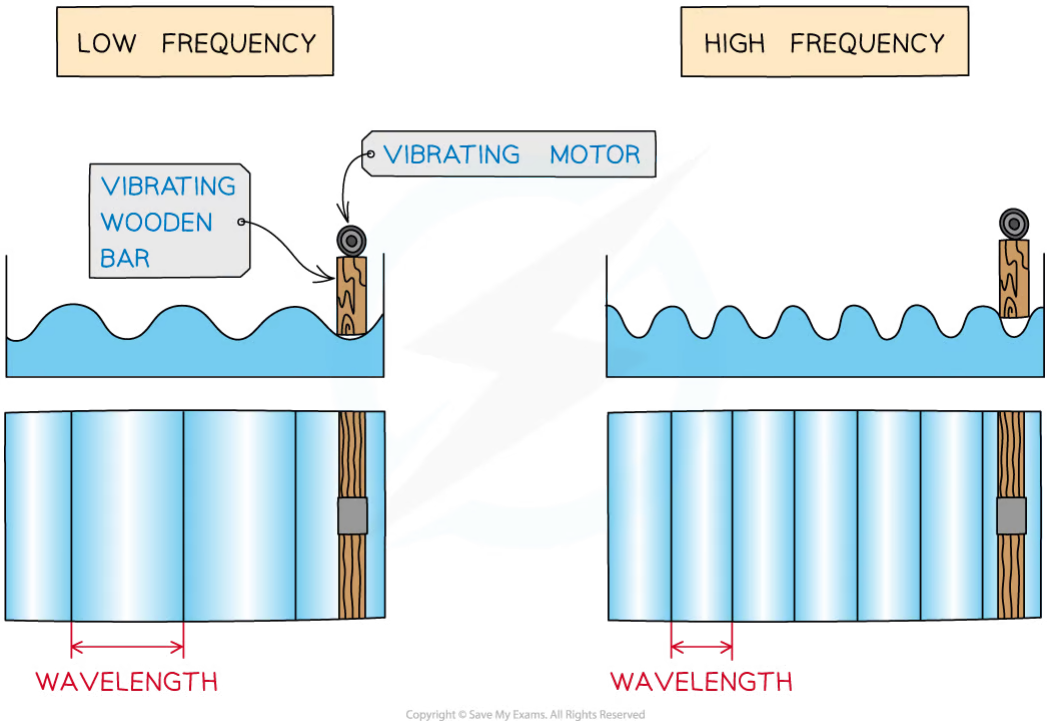
Wave effects may all be demonstrated using a ripple tank

- The diagram below shows how the wavelengths differ with frequency in a ripple tank
  - The **higher** the frequency, the **shorter** the wavelength
  - The **lower** the frequency, the **longer** the wavelength

YOUR NOTES

YOUR NOTES



Ripple tank patterns for low and high frequency vibration

Exam Tip

Exam questions may ask you to sketch or interpret wavefronts and rays. Make sure you draw these with a **ruler** to ensure your lines are straight. Unclear or slopping diagrams are unlikely to get full marks!

Reflection, Refraction & Transmission

- When waves arrive at a boundary between two materials, they can be:
  - Reflected
  - Refracted
  - Transmitted

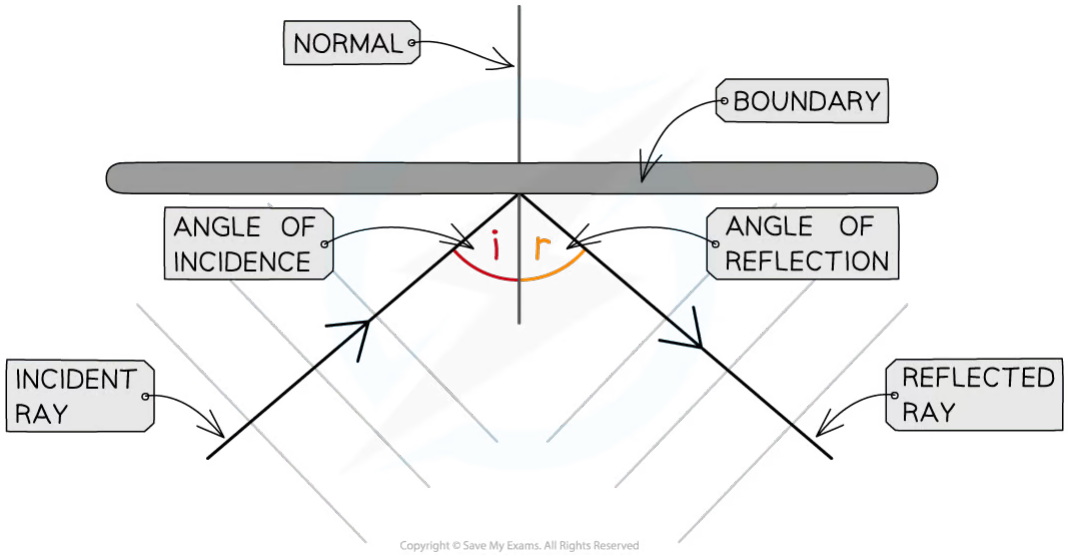
Reflection

- Reflection occurs when:

> **A wave hits a boundary between two media and does not pass through, but instead bounces back to the original medium**

- The **law of reflection** states:

The angle of incidence, *i* = The angle of reflection, *r*



Reflection of a wave at a boundary

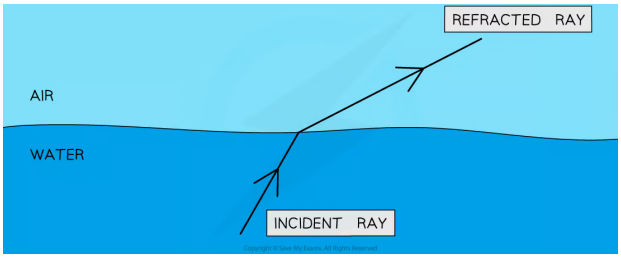
- When a wave is reflected, some of it may also be **absorbed** by the medium, **transmitted** through the medium, or polarised
- At a boundary between two media, the **incident** ray is the ray that travels **towards** the boundary
- During reflection, the frequency, wavelength and speed of the wave does **not** change

Refraction

- When light crosses the boundary between two media with different optical densities, it **refracts**
- Refraction occurs when:

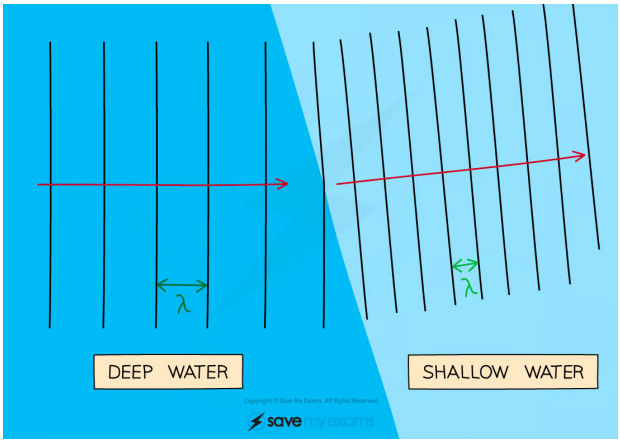
YOUR NOTES

A wave changes speed and direction at the boundary between two media



Refraction of a ray between water and air

- This is due to the **density** of the media
  - If the medium is **more** dense, the wave **slows down**
  - If the medium is **less** dense, the wave **speeds up**
- In this case, water is denser than air
- When a wave refracts, its speed and wavelength change, but its frequency remains the same
  - This is noticeable by the fact that the **colour** of the wave does **not change**
- An example of water waves refracting is when they travel from deeper to shallower water
  - The wavelength of the waves decreases in the shallower water



Water waves being refracted at the boundary between deep and shallow water

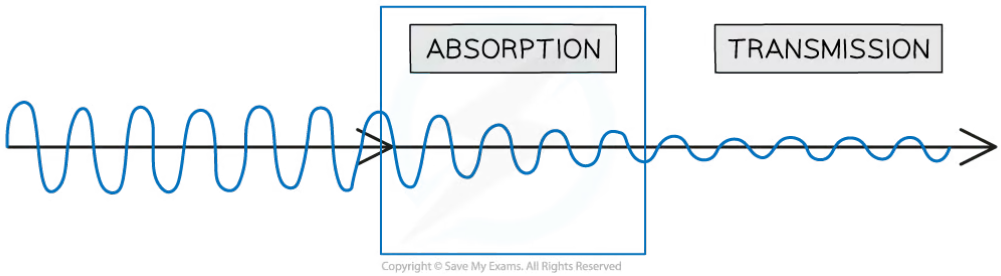
Transmission

- Transmission occurs when:

  A wave passes through a substance

- **Refraction** is a type of transmission
  - Transmission is the more general term for a wave appearing on the opposite side of a boundary (the opposite of reflection)
  - Refraction is specifically the **change in direction** of a wave when it crosses a boundary between two materials that have a different density
- When passing through a material, waves can be partially **absorbed**
- The transmitted wave will have a lower amplitude if some absorption has occurred

YOUR NOTES



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When a wave passes through a boundary it may be absorbed and transmitted

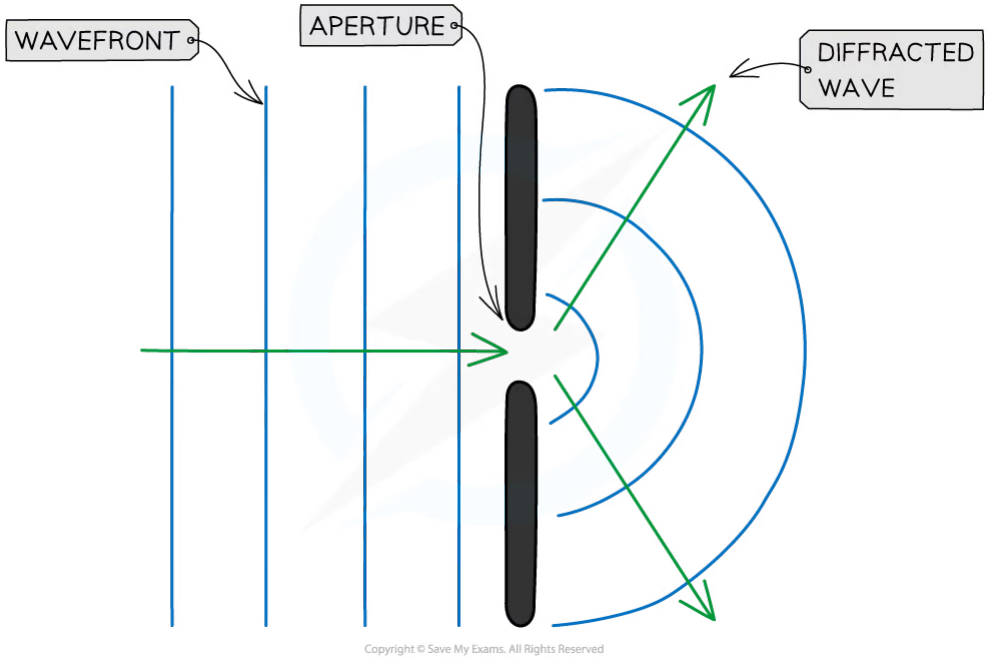
- During transmission, the frequency or speed of the wave does **not** change
- Reflection, refraction and transmission occur for **all** types of waves, both transverse and longitudinal

Exam Tip

You must be able to differentiate between the different types of wave behaviour. Refraction and reflection is commonly mixed up. With refraction, there is always **transmission** and a change in direction in another medium. With reflection however, there is no transmission into another medium.

Diffraction of Waves

- Diffraction is the **spreading out** of waves when they pass an obstruction
  - This obstruction is typically a narrow slit known as an aperture



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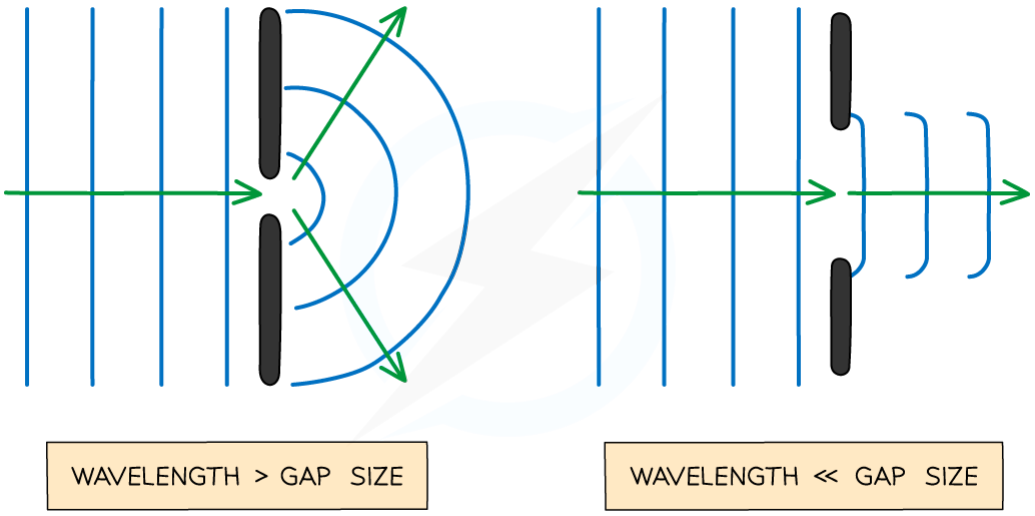
As waves pass through the aperture, they spread out

- The **amplitude** of the diffracted waves is **less** than that of the incident waves, since **energy is distributed over a larger area**
- The extent of diffraction depends on the width of the gap compared to the wavelength of the wave
  - Diffraction is the most prominent when the width of the slit is approximately **equal to or smaller** than the wavelength
  - As the gap size increases, the effect gradually gets less pronounced until, in the case that the gap is much larger than the wavelength, the waves are no longer spread out

YOUR NOTES



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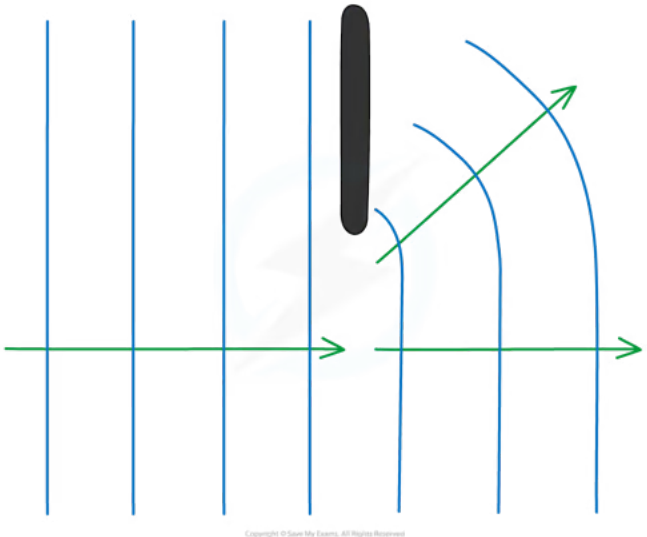
The size of the gap (compared to the wavelength) affects how much the waves spread out

- Any type of wave can be diffracted i.e. sound, light, water
- The only property of a wave that changes when its diffracted is its **amplitude**
  - This is because some energy is **dissipated** when a wave is diffracted through a gap
- Diffraction can also occur when waves curve around an edge:



When a wave goes past the edge of a barrier, the waves can curve around it

YOUR NOTES

Worked Example

An electric guitar student is practising in his room. He has not completely shut the door of his room, and there is a gap of about 10 cm between the door and the door frame.

Determine the frequencies of sound that are best diffracted through the gap.

The speed of sound can be taken to be 340 m s^{-1}

Answer:

Step 1: Optimal diffraction happens when the wavelength of the waves is comparable to (or larger than) the size of the gap

$$\lambda = 10 \text{ cm} = 0.1 \text{ m}$$

Step 2: Write down the wave equation

$$v = f\lambda$$

- where $v = 340 \text{ m s}^{-1}$

Step 3: Rearrange the above equation for the frequency *f*

$$f = \frac{v}{\lambda}$$

Step 4: Substitute the numbers into the above equation

$$f = \frac{340}{0.1} = 3400 \text{ Hz}$$

- The frequencies of sound that are best diffracted through the gap are:

$$f \leq 3400 \text{ Hz}$$

YOUR NOTES

Worked Example

When a wave is travelling through the air, which scenario best demonstrates diffraction?

A. UV radiation through a gate post

B. Sound waves passing a diffraction grating

C. Radio waves passing between human hair

D. X-rays passing through atoms in a crystalline solid

Answer: D

- Diffraction is most prominent when the wavelength is close to the aperture size

Consider option **A**:

- UV waves have a wavelength between (4×10^{-7}) and (1×10^{-8}) m so would **not** be diffracted by a gate post
- Radio waves, microwaves or sound waves would be more likely to be diffracted at this scale

Consider option **B**:

- Sound waves have a wavelength of (1.72×10^{-2}) to 17 m so would **not** be diffracted by the diffraction grating
- Infrared, light and ultraviolet waves would be more likely to be diffracted at this scale

Consider option **C**:

- Radio waves have a wavelength of 0.1 to 10^{6} m so would **not** be diffracted by human hair
  - Infrared, light and ultraviolet waves would be more likely to be diffracted at this scale

Consider option **D**:

- X-rays have a wavelength of (1×10^{-8}) to (4×10^{-13}) m
  - This is a suitable estimate for the size of the gap between atoms in a crystalline solid
  - Hence X-rays could be diffracted by a crystalline solid
- Therefore, the correct answer is **D**

Exam Tip

When drawing diffracted waves, take care to keep the wavelength (the distance between each wavefront) constant.

YOUR NOTES

YOUR NOTES

Snell's Law

- When light crosses the boundary between two media with different optical densities, it **refracts**
- At the boundary, the light undergoes a **change in direction**
- The change in direction depends on the difference in **density** of the media the light rays pass between:

Air → glass (less dense → more dense):

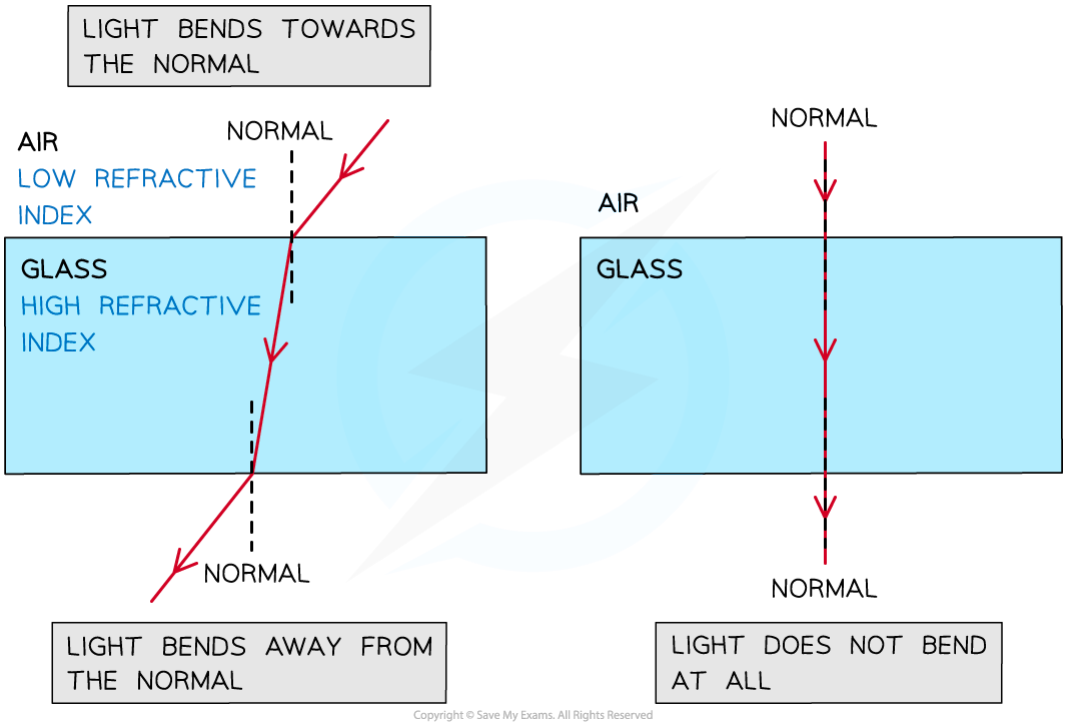
- Light bends **towards** the normal
- The angle of incidence is greater than the angle of refraction $(i > r)$

Glass → air (more dense → less dense):

- Light bends **away** from the normal
- The angle of refraction is greater than the angle of incidence $(i < r)$

When passing along the normal (perpendicular):

- The light **does not bend** at all $(i = r)$
- This would be described as **transmission**



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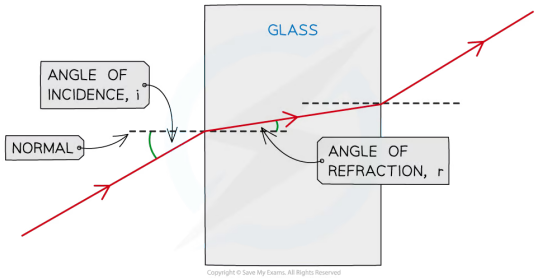
Refraction of light through a glass block

- When light passes from a less dense medium to a more dense medium, (e.g. air → glass):
  - The refracted light has a **lower speed** and a **shorter wavelength** than the incident light

YOUR NOTES

- When light passes from a more dense medium to a less dense medium (e.g. glass → air):
  - The refracted light has a **higher speed** and a **longer wavelength** than the incident light
- When more than one boundary is shown in a refraction ray diagram, (e.g. light passing through a glass block):
  - The new incident ray (at the glass-air boundary) is equal to the angle of refraction (at the air-glass boundary)
  - The refracted ray is equal to the original angle of incidence



A light ray refracted at the air-glass boundary

- Together with refraction, **reflection** might also occur
  - When light travels from a less optically dense medium into an optically denser medium, **both reflection and refraction always occur**
  - Light has to be reflected from the object to the eye in order for objects to be visible
  - Some of the energy in the incident light is reflected back, while some is **transmitted**

Refractive Index

- Transparent media have different optical densities
- Light is **transmitted** through these media at different speeds
- The **refractive index**, ***n***, of a transparent medium is a measure of its optical density, and can be calculated as follows:

$$n = \frac{c}{v}$$

- Where:
  - n = absolute refractive index of the medium
  - c = speed of light in vacuum in metres per second ($m\ s^{-1}$)
  - v = speed of light in the medium in metres per second ($m\ s^{-1}$)
- The value of the speed of light in a vacuum is $c = 3.00 \times 10^8\ m\ s^{-1}$, as given in the data booklet
- This is a property of the material
- Note that, being a ratio, the absolute refractive index is a **dimensionless** quantity
  - This means that it has no units
- The refractive index of **air** can be assumed to be $n = 1$
- Because the speed of light will always be faster than the speed of light in a medium, the refractive index of **any other transparent medium** is $n > 1$

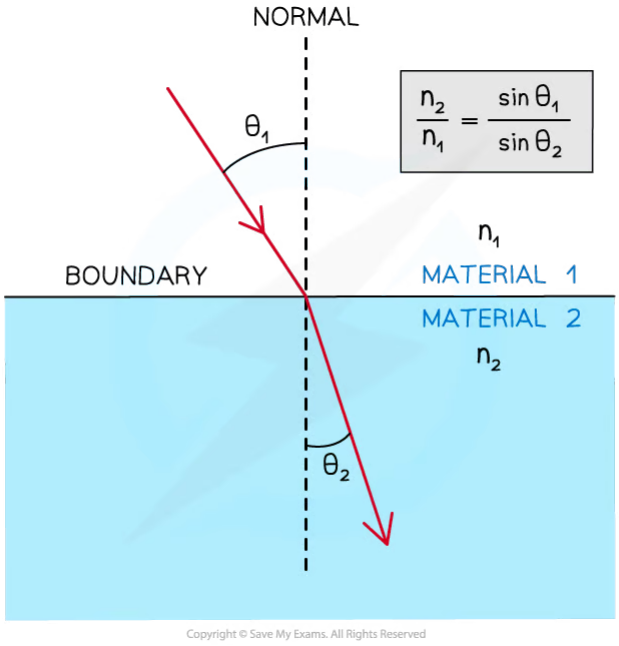
Snell's Law

YOUR NOTES

- Snell's Law is given by:

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

- Where:
  - n = absolute refractive index
  - θ = angles of incidence and refraction
  - v = speed of light in medium
- Snell's Law describes the angle at which light meets the boundary and the angle at which light leaves the boundary, so that the light travels through the media in the least amount of time
- Light can travel through medium 1 at a speed of v_1 due to the optical density n_1 of that medium
  - Light will approach the boundary at angle θ_1
  - This is the angle of **incidence**
- Light can travel through medium 2 at a speed of v_2 due to the optical density n_2 of that medium
  - Light will leave the boundary at angle θ_2
  - This is the angle of **refraction**



Snell's Law

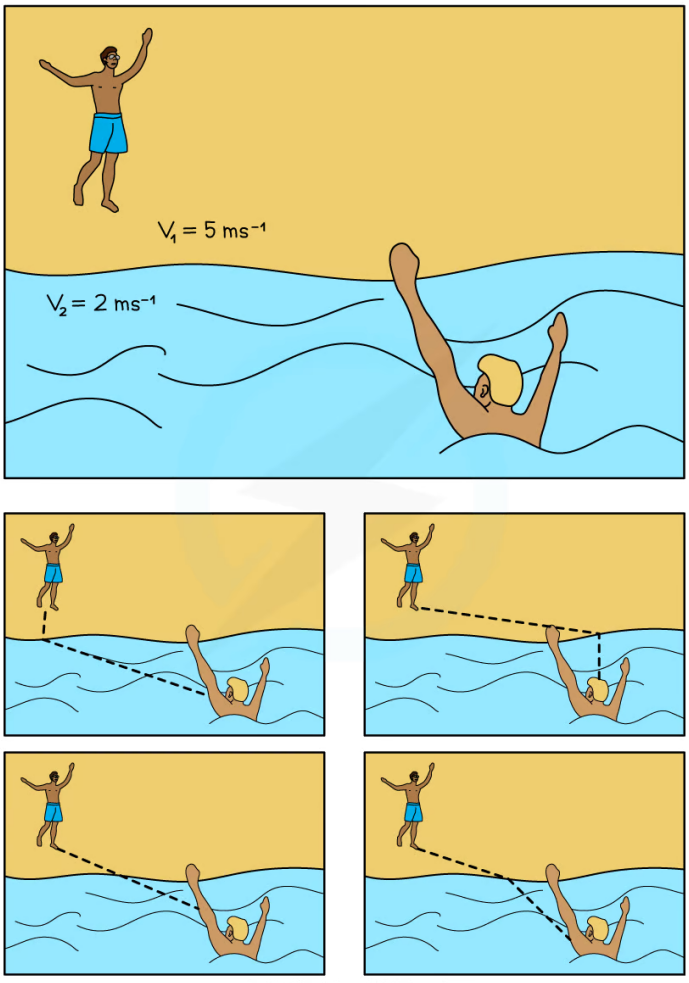
- To illustrate this, there is a classic thought experiment that uses Fermat's Principle of Light

YOUR NOTES

- Fermat's Principle of Light states that light will travel between two points along the path that will take the least amount of time

- A lifeguard on a beach sees a swimmer in need of rescue. They can run at 5 m s^{-1} on the sand and swim at 2 m s^{-1} in the water. What is the fastest path to take?
  - The lifeguard could run on the sand straight to the water and then swim to the person
  - The lifeguard could run on the sand until they are parallel to the person, and then swim directly out to them
  - The lifeguard could run and then swim diagonally in a path that is a straight line from his position in the sand to the position of the swimmer in the water
  - Or, the lifeguard could run diagonally, but so that more of the distance covered is in the sand than in the water

- Some permutation of this answer is where the fastest path will be found



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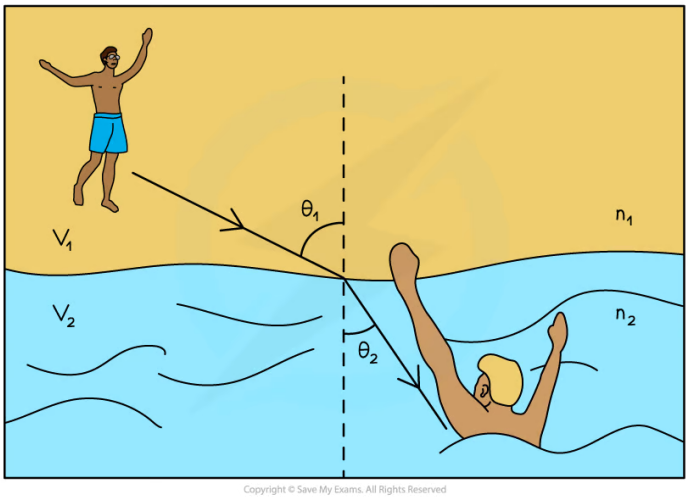
Thought experiment: A lifeguard needs to find the fastest path to a swimmer in trouble

- Using this thought experiment, it can be seen that Snell's Law emerges

YOUR NOTES



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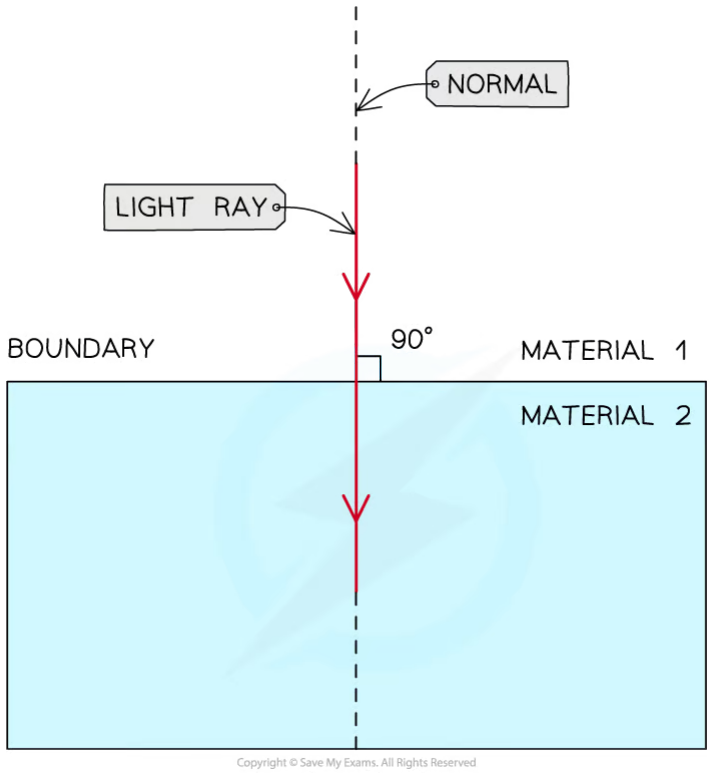
Snell's Law in the context of the thought experiment

- Snell's Law can also be given as:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

- This form of Snell's Law can often be more convenient to use
- When the incident ray is perpendicular to the surface of the boundary, its **speed** changes, but its **direction** does not
- Snell's Law can be used to explain this:
  - The angle of incidence is zero, $\theta_1 = 0$, therefore, $n_1 \sin (0) = n_2 \sin \theta_2$
  - Since, $\sin (0) = 0$
  - Anything multiplied by zero is zero, therefore, $0 = n_2 \sin \theta_2$
  - Hence, the refracted angle θ_2 must also be zero
  - So, there is transmission without refraction

YOUR NOTES



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Light travelling along the normal to the boundary between material 1 and material 2

YOUR NOTES

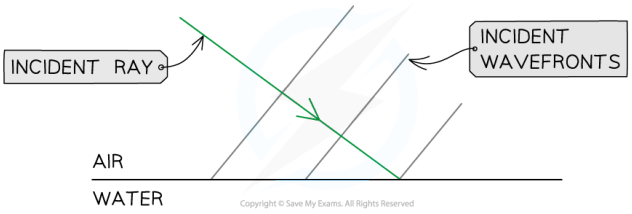
Worked Example

Wavefronts travel from air to water as shown. Add the refracted wavefronts to the diagram.

Answer:

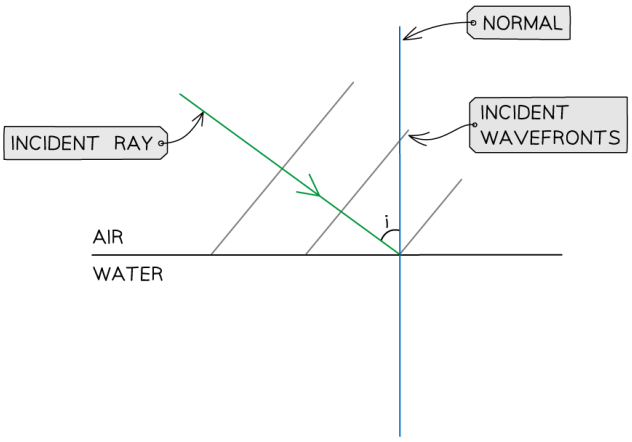
Step 1: Add the incident ray to mark the direction of the incident waves

- The incident ray must be perpendicular to all wavefronts
- Remember to add an arrow pointing towards the air-water boundary



Step 2: Add the normal at the point of incidence

- Mark the angle of incidence (i) between the normal and the incident ray

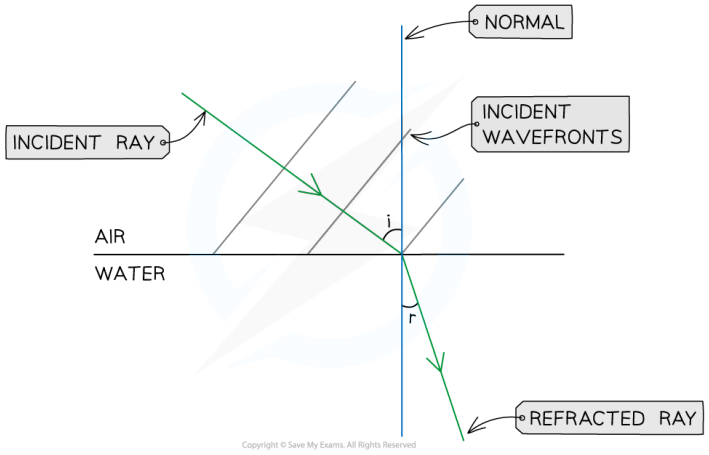


Step 3: Draw the refracted ray into the water

- Water is optically denser than air

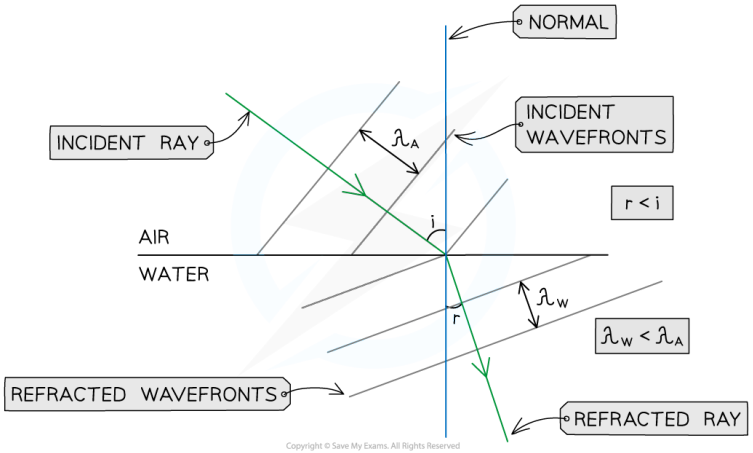
YOUR NOTES

- The refracted ray must bend towards the normal
- Mark the angle of refraction (r) between the normal and the refracted ray
- By eye, i > r



Step 4: Add three equally spaced wavefronts, all perpendicular to the refracted ray

- The refracted wavefronts must be closer to each other than the incident wavefronts, since:
  - The speed *v* of the waves decreases in water
  - The frequency *f* of the waves stays the same
  - The wavelength λ of the waves in water is shorter than the wavelength of the waves in air $\lambda_W < \lambda_A$, since $v = f\lambda$



YOUR NOTES

Worked Example

Light travels from air into glass. Determine the speed of light in glass.

- Refractive index of air, $n_1 = 1.00$
- Refractive index of glass, $n_2 = 1.50$

Answer:

Step 1: Write down the known quantities

- $n_1 = 1.00$
- $n_2 = 1.50$
- From the data booklet, $c = 3 \times 10^8 \text{ m s}^{-1}$ (speed of light in air)

Step 2: Write down the relationship between the refractive indices of air and glass and the speeds of light in air (v_1) and glass (v_2)

$$\frac{n_1}{n_2} = \frac{v_2}{v_1}$$

Step 3: Rearrange the above equation to calculate v_2

$$v_2 = \frac{n_1}{n_2} v_1$$

Step 4: Substitute the numbers into the above equation

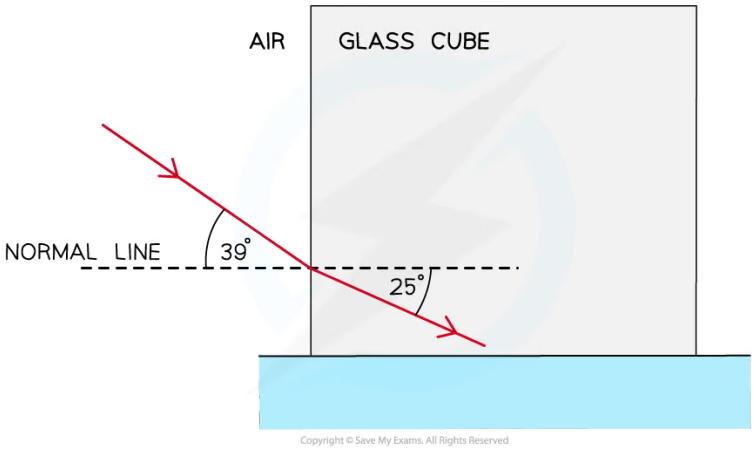
$$v_2 = \frac{1.00}{1.50} \times \left(3 \times 10^8\right)$$

$$v_2 = 2 \times 10^8 \text{ m s}^{-1}$$

YOUR NOTES

Worked Example

A light ray is directed at a vertical face of a glass cube. The angle of incidence at the vertical face is 39° and the angle of refraction is 25° as shown in the diagram.



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Show that the refractive index of the glass is about 1.5.

Answer:

Step 1: Write down the known quantities

- Refractive index of air, $n_1 = 1$
- Refractive index of glass = n_2
- Angle of incidence, $\theta_1 = 39°$
- Angle of refraction, $\theta_2 = 25°$

Step 2: Write out Snell's Law

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2}$$

Step 3: Calculate the refractive index of glass

$$n_2 = \frac{\sin 39}{\sin 25} = 1.489 = 1.5 \text{ (2 s.f.)}$$

- Note that in a "show that" question, the answer should be to at least one more significant figure than the value given in the question

Exam Tip

Always double-check if your calculations for the refractive index are greater than 1. Otherwise, something has definitely gone wrong in your calculation! The refractive index of air might not be given in the question. Always assume that $n_{air} = 1$

Make sure your calculator is always in **degrees** mode for calculating the sine of angles!

Always check that the angle of incidence and refraction are the angles between the normal and the light ray. If the angle between the light ray and the boundary is calculated instead, calculate $90 - \theta$ (since the normal is perpendicular to the boundary) to get the correct angle

YOUR NOTES

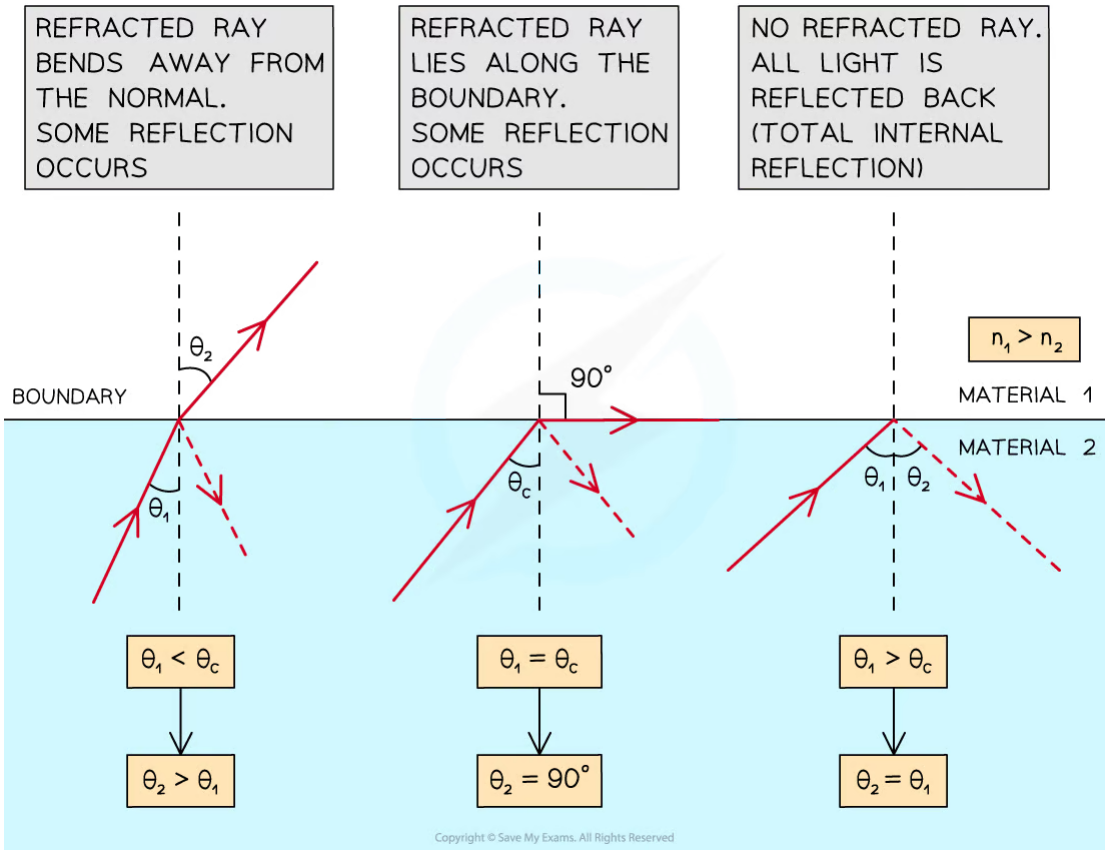
Critical Angle & Total Internal Reflection (TIR)

YOUR NOTES

- Together with refraction, **reflection** might also occur under certain conditions
- According to Snell's law, when light travels from an optically **denser** medium into a **less** optically dense medium, its speed **increases** and it bends **away** from the normal
  - The angle of **refraction** is **greater** than the angle of **incidence**
- As the angle of incidence increases, the angle of refraction eventually reaches 90°
- This is known as the **critical angle** θ_c:

The smallest angle of incidence of which the light ray can totally internally reflect

- For light travelling from an optically denser material 1 into a **less** optically dense material 2
  - If the angle of **incidence** is **smaller** than the critical angle ($\theta_1 < \theta_c$), the refracted ray bends **away** from the normal, and the angle of **refraction** is **greater** than the angle of **incidence** ($\theta_2 > \theta_1$)
  - If the angle of **incidence** is **equal** to the critical angle ($\theta_1 = \theta_c$), the refracted ray lies **along** the boundary between the two materials, and the angle of **refraction** is **equal** to 90° ($\theta_2 = 90°$)
  - If the angle of **incidence** is **greater** than the critical angle ($\theta_1 > \theta_c$), there is no refracted ray, and all light is **reflected** back into material 1 ($\theta_2 = \theta_1$)
    - This is known as **total internal reflection**



Light travelling from the optically denser material 1 into the less optically dense material 2 at different angles of incidence

YOUR NOTES

- The critical angle of material 1 can be calculated as follows:

$$\sin \theta_c = \frac{n_2}{n_1}$$

- Where:
  - θ_c = critical angle of material 1 (°)
  - n_1 = absolute refractive index of material 1
  - n_2 = absolute refractive index of material 2
- The **two conditions** for total internal reflection to occur are:
  - The refractive index of the **second** medium must be **less** than the refractive index of the first, $n_2 < n_1$
  - The angle of **incidence** must be **greater** than the **critical** angle, $\theta_i > \theta_c$

Worked Example

Light travels from a material with refractive index 1.2 into air.

Determine the critical angle of the material.

Answer:

Step 1: Write down the known quantities

- Refractive index of material 1, $n_1 = 1.2$
- Refractive index of air, $n_2 = 1.0$

Step 2: Write down the equation for the critical angle θ_c

$$sin\theta_c = \frac{n_2}{n_1}$$

Step 3: Substitute the numbers into the above equation

$$sin\theta_c = \frac{1.0}{1.2}$$

$$sin\theta_c = 0.83$$

Step 4: Calculate θ_c by taking $\sin^{-1}$ of the above equation

$$\theta_c = \sin^{-1} 0.83$$

$$\theta_c = 56°$$

Exam Tip

You do not need to remember the derivation for the critical angle, but you must understand and remember the critical angle formula, as this is **not** given in your data booklet.

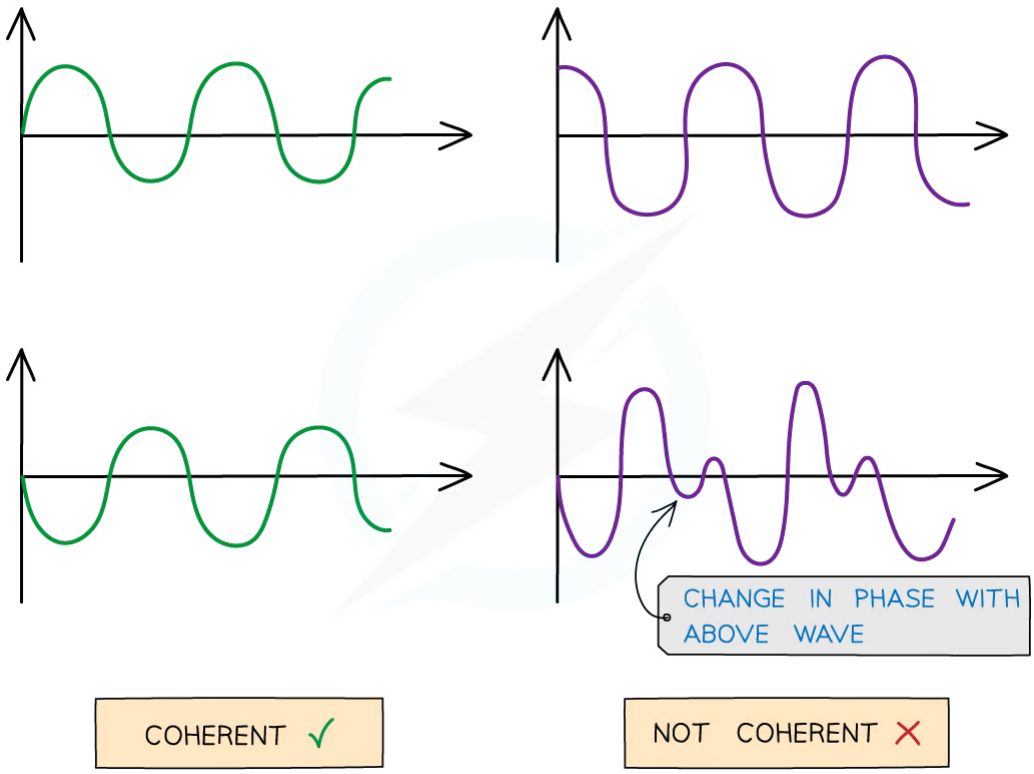
YOUR NOTES

YOUR NOTES

Double-Source Interference

Coherence

- Double source interference is only observable if produced by a **coherent** source
- Waves are said to be coherent if they have:
  - A **constant** phase difference
  - The **same** frequency



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Coherent waves (on the left) and non-coherent waves (on the right). The abrupt change in phase creates an inconsistent phase difference

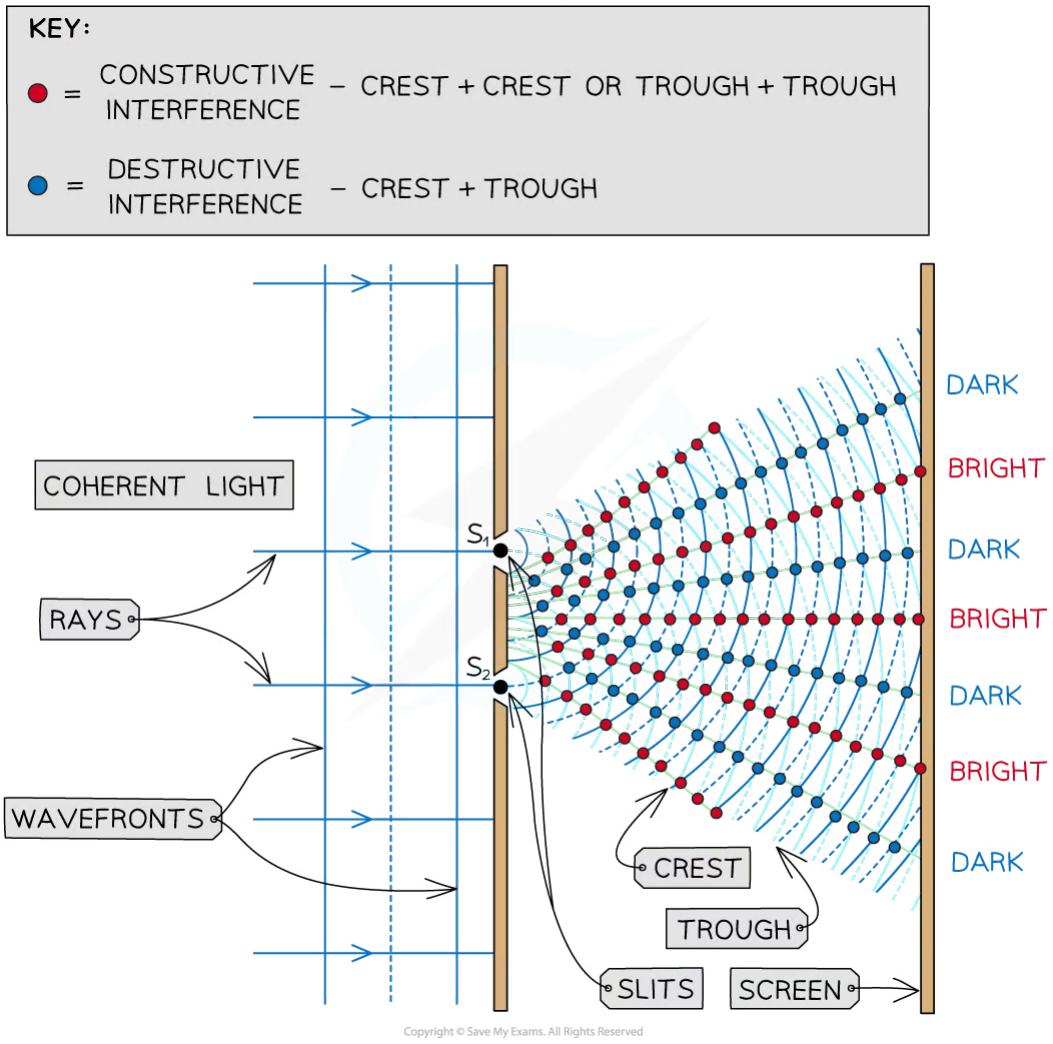
- A coherent beam of light contains light waves that are **monochromatic** and have a constant phase difference
  - Monochromatic light consists of light waves of a **single frequency**
- **Laser** light is an example of a coherent light source
- Filament lamps produce **incoherent** light waves

Double-Slit Interference of Light

- When a **coherent beam of light** is incident on two narrow slits very close together, **diffraction** occurs at each slit (i.e. the waves spread out)
- As the diffracted waves cross, they **interfere** with each other

YOUR NOTES

- If a screen is placed some distance away from the slits, a pattern of equally spaced bright and dark **fringes** is observed on the screen
  - The bright fringes form where the waves interfere constructively (i.e. a crest meets a crest or a trough meets a trough)
  - The dark fringes form where the waves interfere destructively (i.e. a crest meets a trough)



Coherent light waves interfere after passing through two narrow slits. Alternating bright and dark fringes are observed on the screen

Superposition of Waves

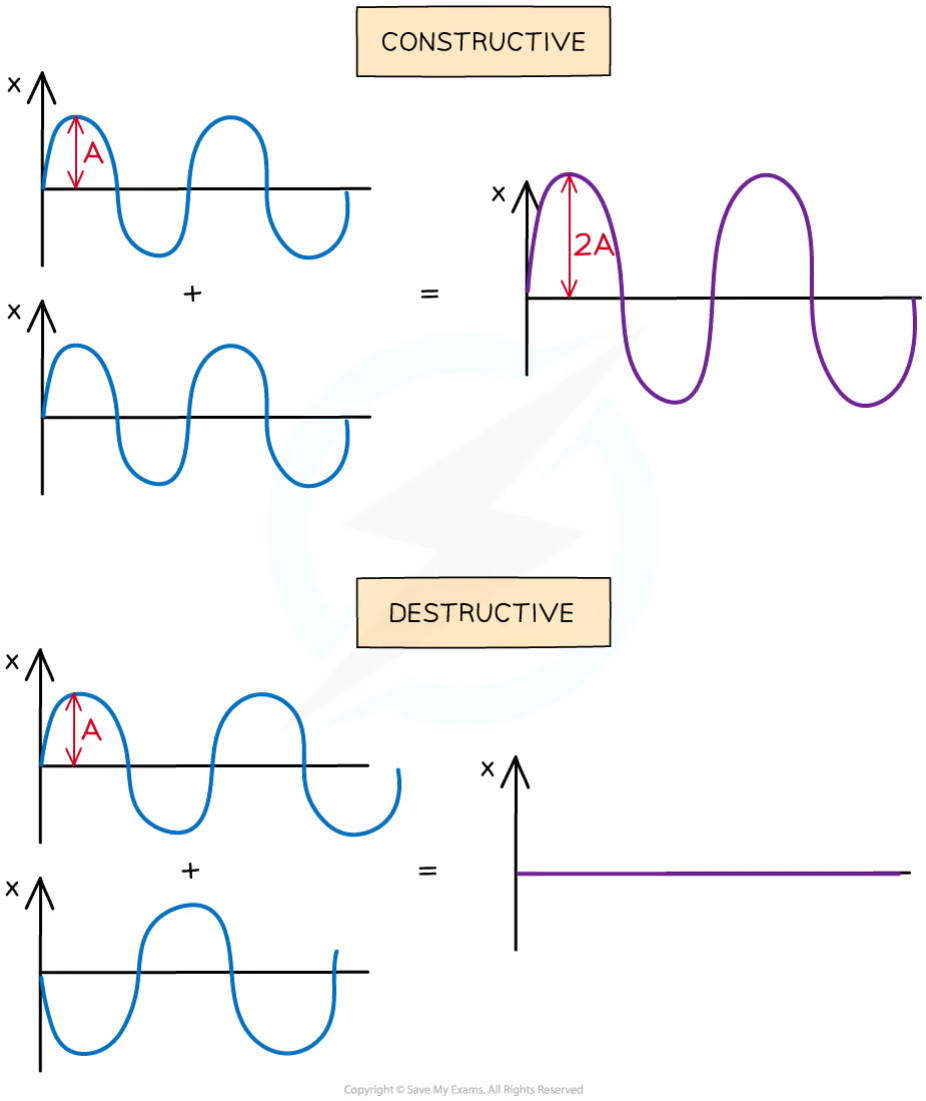
- The **principle of superposition** states that:

  When two or more waves meet, the resultant displacement is the vector sum of the displacements of the individual waves

- The principle of superposition applies to both transverse and longitudinal waves
- **Interference** occurs whenever two or more waves superpose
- For a clear stationary interference pattern, the waves must be of the same:
  - Type
  - Amplitude
  - Frequency
- They must also have a **constant phase difference**

Constructive & Destructive Interference

- **Constructive interference** occurs when
  - Two or more waves superpose and have displacements in the **same direction** (both positive or both negative)
- **Destructive interference** occurs when
  - Two or more waves superimpose and have displacements in **opposite directions** (one positive and one negative)
- When two waves with the same amplitude meet at a point, they can:
  - Be in phase and interfere **constructively**, so that the displacement of the resultant wave is **double** the displacement of each individual wave
  - Be in **anti-phase** and interfere destructively, so that the displacement of the resultant wave is equal to zero



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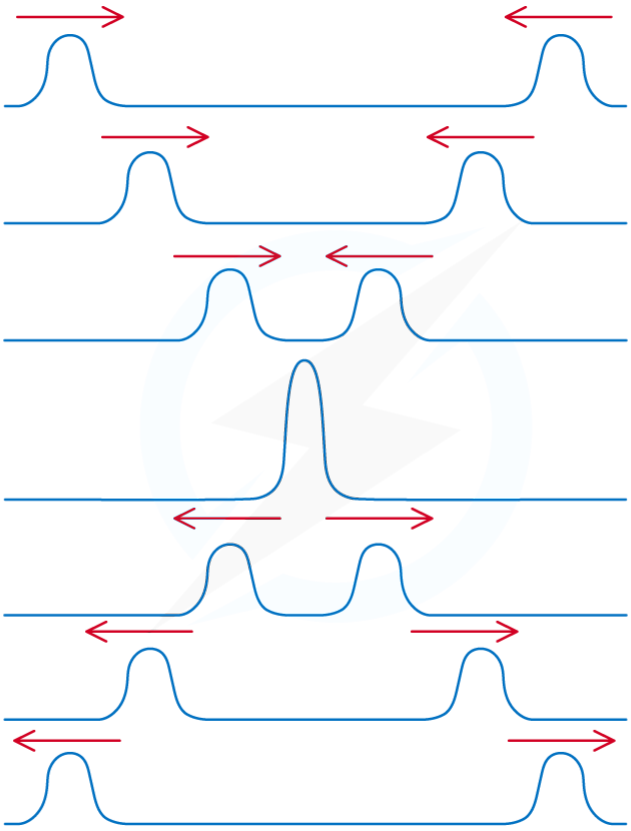
Waves in superposition can undergo constructive or destructive interference

- Superposition occurs for any two waves or pulses that overlap and can result in a combination of both constructive and destructive interference
  - For example, the peak of one wave superposes with the peak of another wave with a smaller displacement
  - The resultant peak will have a displacement that is in the middle of the displacement of both waves
- Superposition can also be demonstrated with two pulses
  - When the pulses meet, the resultant displacement is the algebraic sum of the displacement of the individual pulses
  - After the pulses have interacted, they then carry on as normal

YOUR NOTES



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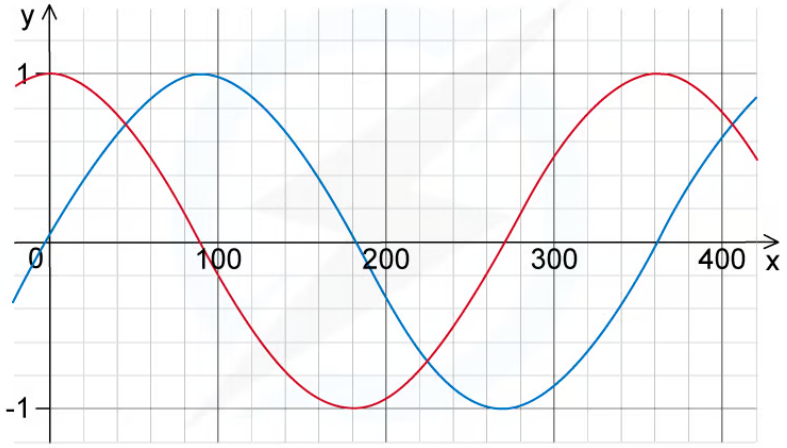
YOUR NOTES

Worked Example

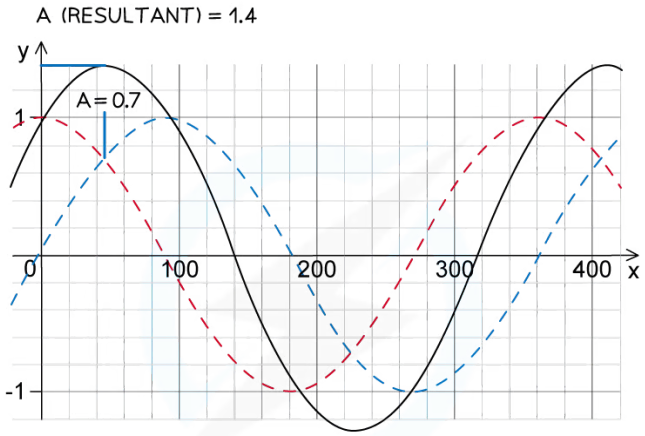
Two overlapping waves of the same types travel in the same direction. The variation with x and y displacement of the wave is shown in the figure below.



Use the principle of superposition to sketch the resultant wave.

Answer:

THE GRAPH OF THE SUPERPOSITION OF BOTH WAVES IS SHOWN IN BLACK BELOW:



TO PLOT THE CORRECT AMPLITUDE AT EACH POINT, SUM THE AMPLITUDE OF BOTH GRAPHS AT THAT POINT.

e.g. AT POINT A – EACH GRAPH HAS A VALUE OF 0.7. THEREFORE THE SAME POINT WITH THE RESULTANT SUPERPOSITION IS 0.7 x 2 = 1.4

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Constructive & Destructive Interference

YOUR NOTES

- **Interference** occurs whenever two or more waves combine to produce a resultant wave with a new resultant **displacement**
  - The waves combine according to the principle of superposition
- When constructive interference occurs:

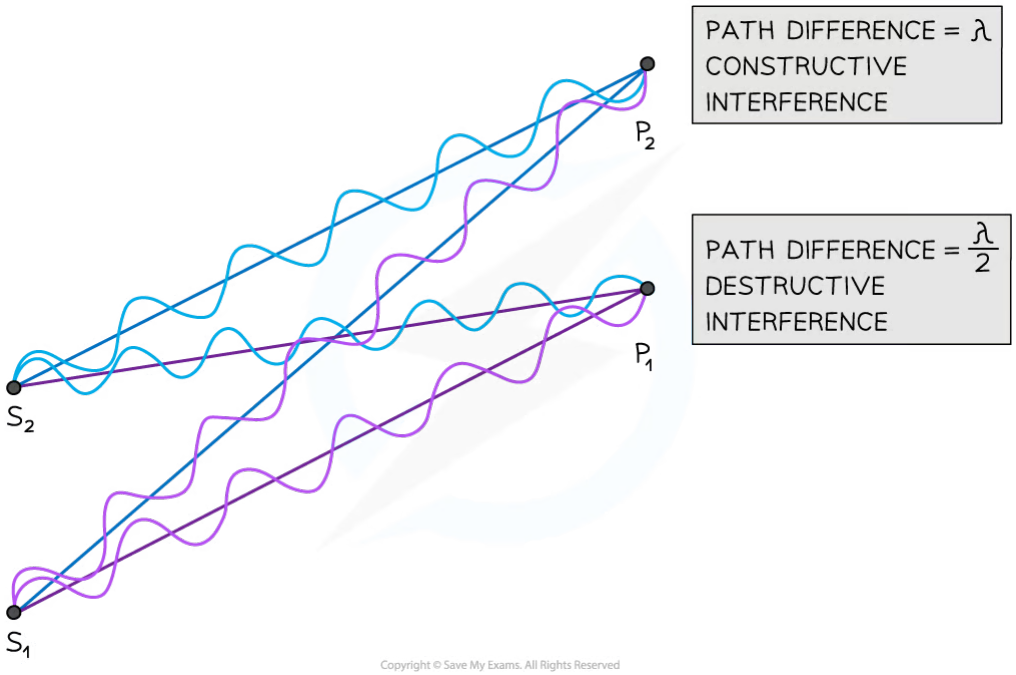
  Two coherent waves combine by adding their displacements to create a resultant wave with a larger amplitude
- When destructive interference occurs:

  Two coherent waves combine by cancel out their displacements to create a resultant wave with a smaller amplitude
- The type of interference occurring at a given point (i.e. constructive or destructive) depends on the **path difference** of the overlapping waves

Path Difference

- Path difference is defined as:

  The difference in distance travelled by two waves from their sources to the point where they meet
- Path difference is generally expressed in multiples of wavelength



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At point P_2 the waves have a path difference of a whole number of wavelengths resulting in constructive interference. At point P_1 the waves have a path difference of an odd number of half wavelengths resulting in destructive interference

YOUR NOTES

- In the diagram above, the number of wavelengths between:
  - $S_1 \rightarrow P_1 = 6\lambda$
  - $S_2 \rightarrow P_1 = 6.5\lambda$
  - $S_1 \rightarrow P_2 = 7\lambda$
  - $S_2 \rightarrow P_2 = 6\lambda$
- The path difference is:
  - $(6.5\lambda - 6\lambda) = \frac{\lambda}{2}$ at point P_1
  - $(7\lambda - 6\lambda) = \lambda$ at point P_2
- Hence:
  - **Destructive interference** occurs **at** point $\mathbf{P_1}$
  - **Constructive interference** occurs **at** point $\mathbf{P_2}$

Conditions for Constructive and Destructive Interference

- In general, for waves emitted by two coherent sources very close together:
- The condition for **constructive interference** is:

$$\textbf{path difference} = n\lambda$$

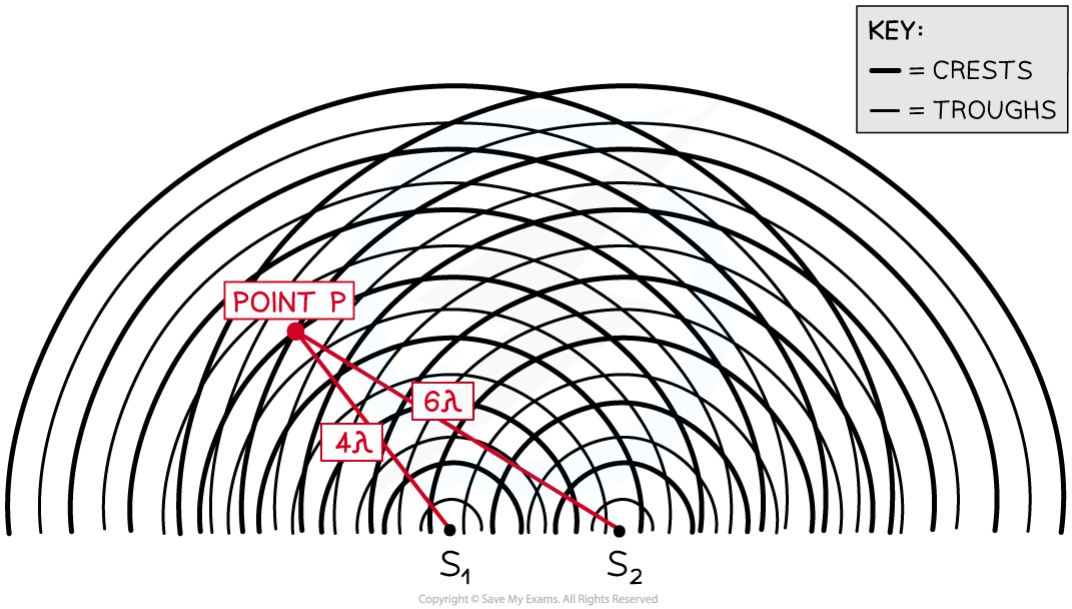
- The condition for **destructive interference** is:

$$\textbf{path difference} = \left(n + \frac{1}{2}\right)\lambda$$

- Where:
  - λ = wavelength of the waves in metres (m)
  - n = 0, 1, 2, 3... (any other integer)
- The same conditions apply to waves emitted by a single coherent source and diffracted by two narrow slits very close together

Path Difference and Wavefronts

YOUR NOTES



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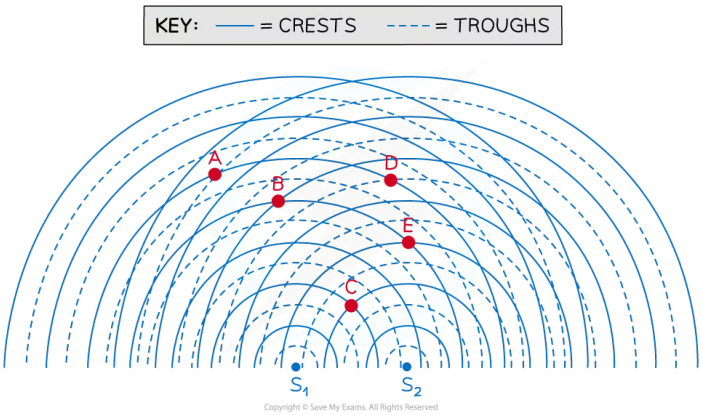
At point P the waves have a path difference of a whole number of wavelengths resulting in constructive interference

- Another way to represent waves spreading out from two sources is shown in the diagram above
- At point **P**, the number of **crests** from:
  - Source $S_1 = 4\lambda$
  - Source $S_2 = 6\lambda$
- The path difference at **P** is $(6\lambda - 4\lambda) = \mathbf{2\lambda}$
- This is a whole number of wavelengths ($n = 2$), hence **constructive interference** occurs **at** point **P**

YOUR NOTES

Worked Example

The diagram below is a snapshot of overlapping wavefronts resulting from the interference of coherent waves diffracted by two narrow slits S_1 and S_2.



For each of the points A, B, C, D and E, determine:

- The path difference from the sources
- The value of *n* in the path difference formula
- Whether they are locations of constructive or destructive interference

Answer:

Step 1: Count the number of wavelengths between each source and the desired point

- For example, the number of wavelengths between:
  - S_1 ➜ A = 5λ
  - S_2 ➜ A = 6.5λ

Step 2: Determine the path difference by subtracting the distances of the point from the two sources

- For example, path difference at A = (6.5λ – 5λ) = 1.5λ

Step 3: Compare the path difference calculated in Step 2 with the condition for constructive or destructive interference and give the value of *n*

- For example, path difference at A = 1.5λ = $(n + ½)\lambda$ ➜ $n = 1$

Step 4: Decide whether the point is a location of constructive or destructive interference

- For example, at A, destructive interference occurs
- Point **A**:
  - Path difference = (6.5λ – 5λ) = 1.5λ
  - $n = 1$
  - Destructive interference

YOUR NOTES

- Point **B**:
  - Path difference = $(5\lambda - 4\lambda) = \lambda$
  - $n = 1$
  - Constructive interference
- Point **C**:
  - Path difference = $(2\lambda - 2\lambda) = 0$
  - $n = 0$
  - Constructive interference
- Point **D**:
  - Path difference = $(5\lambda - 4.5\lambda) = 0.5\lambda$
  - $n = 0$
  - Destructive interference
- Point **E**:
  - Path difference = $(4\lambda - 3\lambda) = \lambda$
  - $n = 1$
  - Constructive interference

Exam Tip

You are not required to memorise the conditions for constructive and destructive interference, as these are given in the data booklet.

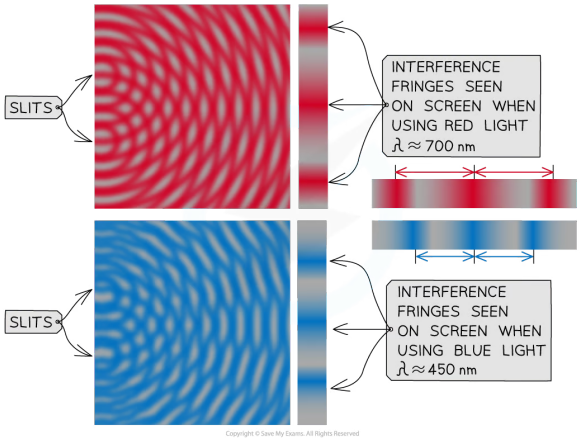
You must be able to determine the path difference of waves from two sources (or two narrow slits) at a given point. You can then compare this with the given conditions for constructive and destructive interference, in order to decide which type of interference occurs at the point you are considering.

YOUR NOTES

Young's Double-Slit Experiment

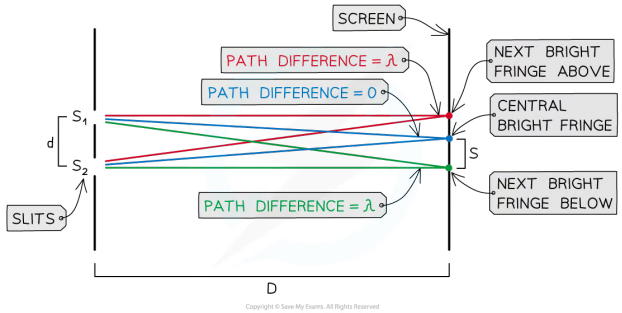
- Interference patterns depend on:
  - The coherent light incident on the slits
  - The **distance between** the **slits** and the **screen** where the interference pattern is observed
  - The **separation** between the slits

- In particular, it can be seen that:
  - The **longer** the wavelength λ, the **further apart** the bright fringes on the screen become
  - The **further** the screen is placed from the slits, the **further apart** the bright fringes on the screen become
  - The greater the separation between the slits, the **closer** the bright fringes on the screen become



Differences between the interference pattern of red and blue light. Red light has a longer wavelength than blue light, so the fringes are more spaced out

- Two slits are separated by a length *d*, and a screen is placed a distance *D* away



Light rays of wavelength λ incident on two slits a distance S apart interfere and form bright fringes on a screen placed a distance D away from the slits

- When a light ray of wavelength λ is incident on the two slits, the observations are:

YOUR NOTES

1. A central bright fringe forms

- This is where the two waves have travelled the **same** distance to the screen and their path difference is **zero**

2. Bright fringes are formed on either side of the central fringe

- This is where the path difference between the waves is equal to **exactly one wavelength**
- This is **constructive** interference

3. Dark fringes are formed between the bright fringes

- This is where the path difference between the waves is some number of wavelengths **plus half a wavelength**
- This is **destructive** interference

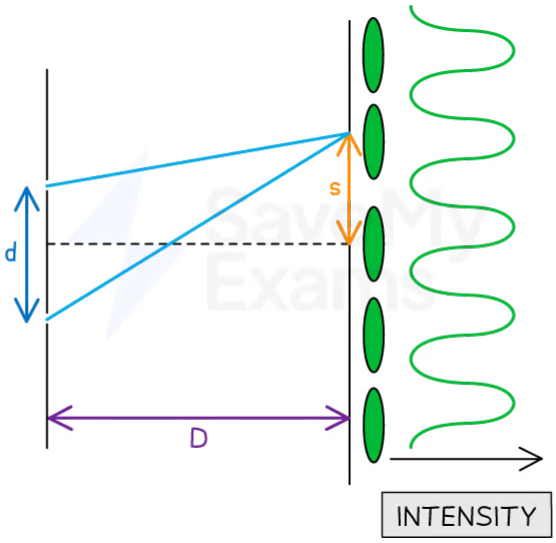
- Further bright fringes will be located at each position on the screen where the path difference is exactly equal to $n\lambda$
  - For example: 2λ, 3λ, 4λ...
  - This is **constructive** interference
- Dark fringes are located in between the bright ones, where the path difference is exactly equal to $\left(n + \frac{1}{2}\right)\lambda$
  - For example, $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, $\frac{5\lambda}{2}$...
  - This is **destructive** interference

Double-Slit Equation

- The separation s of successive fringes is given by:

$$s = \frac{\lambda D}{d}$$

- Where:
  - s = separation between successive fringes on the screen (m)
  - λ = wavelength of the waves incident on the slits (m)
  - D = distance between the screen and the slits (m)
  - d = separation between the slits (m)



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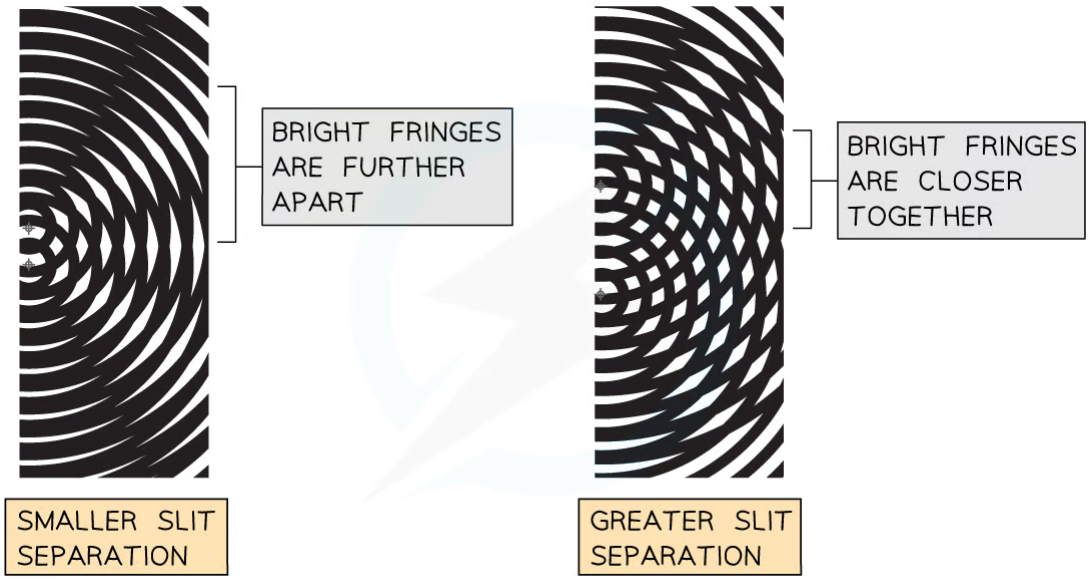
- Note that s is the separation between **two successive** bright fringes or **two** successive dark ones
- The above equation shows that the separation between the fringes will **increase** if:
  - The **wavelength** of the incident light **increases**
  - The **distance** between the screen and the slits **increases**
  - The **separation** between the slits **decreases**



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Dependence of the interference pattern on the separation between the slits. The further apart the slits, the closer together the bright fringes

YOUR NOTES

Worked Example

In a double-slit experiment, two slits are placed a distance of 0.40 mm apart and a screen is located 0.50 m away from the slits.

Coherent electromagnetic waves incident on the slits produce an interference pattern on the screen. The separation between dark fringes is 0.50 cm.

Determine the wavelength and state the type of electromagnetic waves used in the experiment.

Answer:

Step 1: List the known quantities

- $d = 0.40\ \text{mm} = 4.0 \times 10^{-4}\ \text{m}$
- $D = 0.50\ \text{m}$
- $s = 0.50\ \text{cm} = 5.0 \times 10^{-3}\ \text{m}$
- Note that you must convert all lengths into metres (m)

Step 2: Write down the double-slit equation

$$s = \frac{\lambda D}{d}$$

Step 3: Rearrange the above equation to calculate the wavelength λ

$$\lambda = \frac{sd}{D}$$

Step 4: Substitute the numbers into the above equation

$$\lambda = \frac{\left(5.0 \times 10^{-3}\right) \times (4.0 \times 10^{-4})}{0.50}$$

$$\lambda = 4.0 \times 10^{-6}\ \text{m} = 4.0\ \mu\text{m}$$

- This corresponds to the **infrared** area of the electromagnetic spectrum

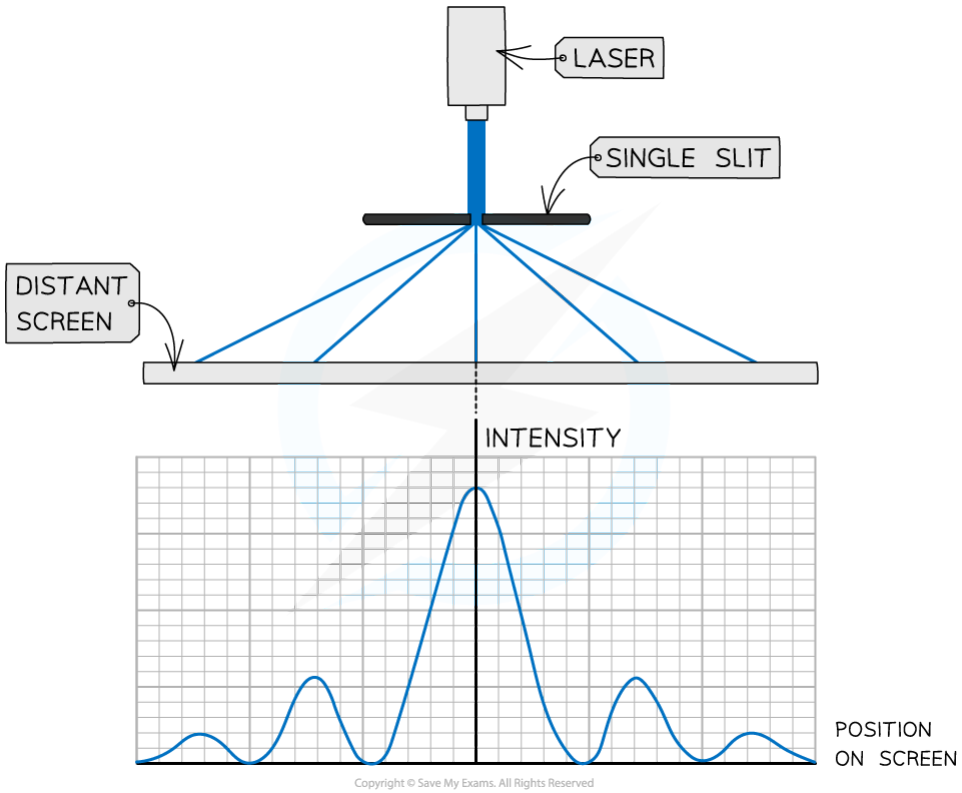
Exam Tip

Remember that the separation between dark fringes is exactly the **same** as the separation between bright fringes. Whether a question gives you or asks about the separation between dark fringes instead of bright ones it makes no difference.

Some tricky questions might give you the separation between a bright and a dark fringe. This is equal to half the value of s!

Single Slit Intensity Patterns

- The **intensity pattern** of light can be represented as a series of light and dark fringes which show the areas of maximum and minimum intensity
- If a laser emitting blue light is directed at a **single** slit, where the slit width is larger than the wavelength of the light, it will spread out as follows:



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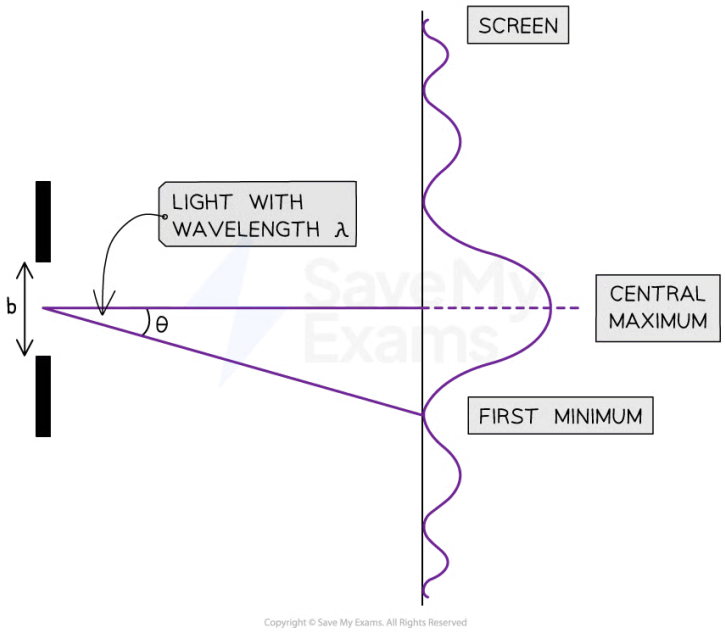
The intensity pattern of blue laser light diffracted through a single slit

- The features of the single slit diffraction pattern are:
  - A **central** maximum with a **high** intensity
  - **Subsidiary** maxima **equally spaced**, successively **smaller** in **intensity** and **half** the **width** of the central maximum
- The width of the central maximum changes depending on the wavelength of the light
- This is calculated by the equation:

$$\theta = \frac{\lambda}{b}$$

- Where:
  - θ = angle between the central maximum and first minimum (°)
  - λ = wavelength of incoming light (m)
  - b = width of the single slit (m)



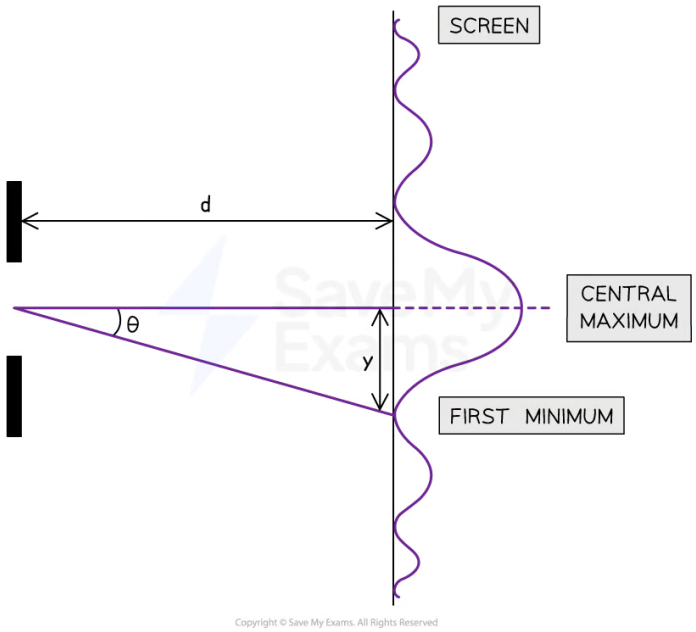
Single-slit diffraction

- The angle θ tells us how **spread out** the diffraction pattern is
- This equation shows us that the pattern is more spread out if:
  - The width of the slit is reduced
  - The wavelength of the incoming light is increased

- This can also be compared to distances
- Since θ is a very small angle, we can approximate it to:

$$\theta = \frac{y}{d}$$

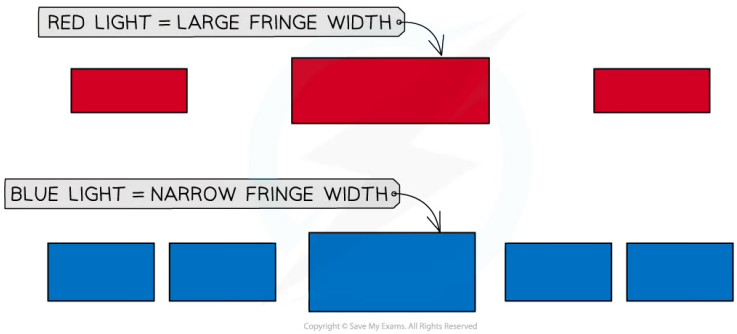
- Where:
  - y = distance between the central maximum and first minimum (m)
  - d = distance between single-slit and screen (m)



Single slit diffraction in terms of distances

Different Wavelengths of Light

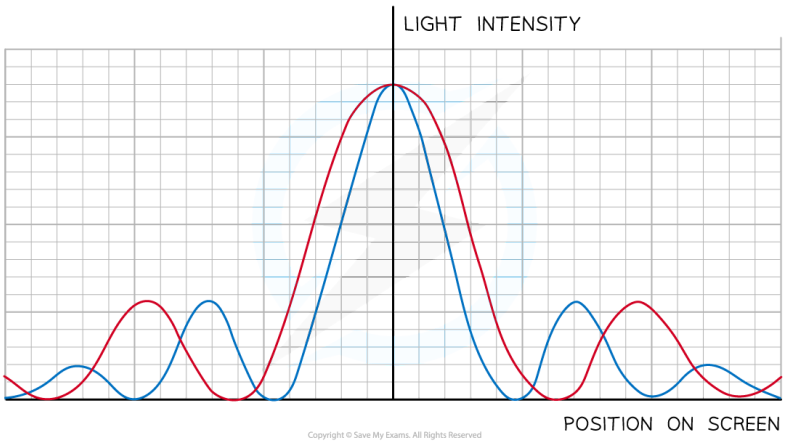
- The angle of diffraction, θ is directly proportional to the **wavelength** of the light, λ
  - This means that the **width** of the **bright maxima**, or fringe, is also proportional to the wavelength
- **Red** light – which has the **longest wavelength** of visible light – will produce a diffraction pattern with **wide fringes**
- **Blue** light – which has a much **shorter wavelength** – will produce a diffraction pattern with **narrow fringes**



Fringe width depends on the wavelength of the light

- Therefore, if the blue laser were to be replaced with a red laser:
  - The **wavelength** of red light is **longer** so the light would diffract **more**
  - The **intensity** fringes would therefore be **wider**

YOUR NOTES

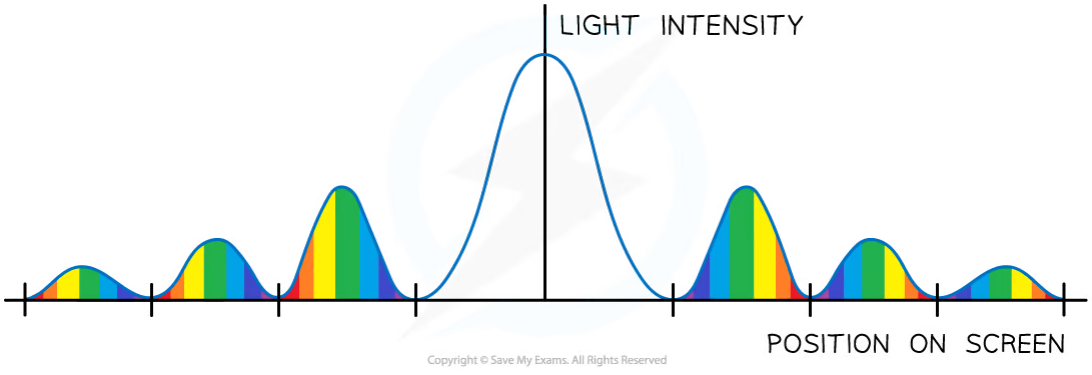


The intensity pattern of red laser light shows longer wavelengths diffract more than shorter blue wavelengths

- If the slit was made narrower:
  - The intensity would **decrease**
  - The fringe spacing would be **wider**

Spectrum for White Light

- If the laser were to be replaced by a non-laser source emitting white light:
  - The central maximum would be **white**
  - All maxima would be composed of a **spectrum**
  - The shortest wavelength (violet / blue) would appear **nearest** to the central maximum
  - The longest wavelength (red) would appear **furthest** from the central maximum
  - The fringe spacing would be smaller and the maxima would be wider



Qualitative treatment of the variation of the width of the central diffraction maximum with wavelength and slit width

YOUR NOTES

Worked Example

Light of wavelength 560 nm is passed through a slit of size 0.030 mm.

Calculate the width of the central maximum formed on a screen 4.0 m away.

Answer:

Step 1: List the known quantities

- Wavelength of light, $\lambda = 560 \text{ nm} = 560 \times 10^{-9} \text{ m}$
- Wdith of slit, $b = 0.030 \text{ mm} = 0.030 \times 10^{-3} \text{ m}$
- Distance between slit and screen, $d = 4.0 \text{ m}$

Step 2: Calculate the angle between central maximum and first minimum

$$\theta = \frac{\lambda}{b} = \frac{560 \times 10^{-9}}{0.030 \times 10^{-3}} = 0.019°$$

Step 3: Determine an equation for the width of the central maximum

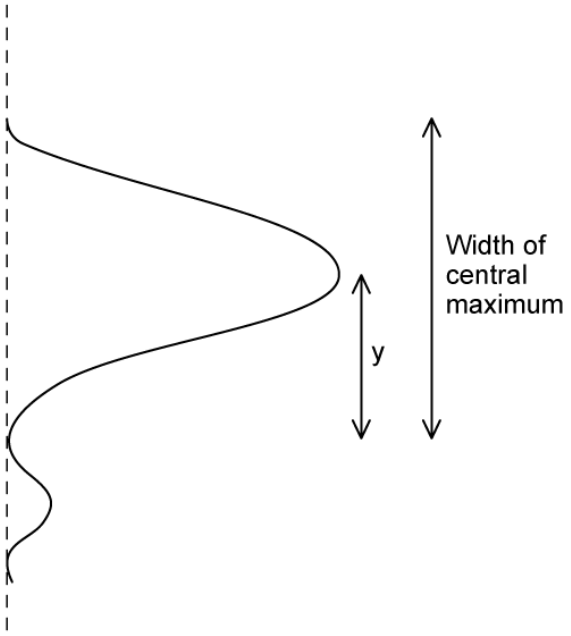
$$\theta = \frac{y}{d} \quad \Rightarrow \quad y = \theta d$$

Step 4: Substitute in the values

$$y = 0.019 \times 4 = 0.076 \text{ m}$$

Step 5: Calculate the width of the central maximum

$$\text{Width} = 2y = 2 \times 0.076 = 0.152 \text{ m}$$



Exam Tip

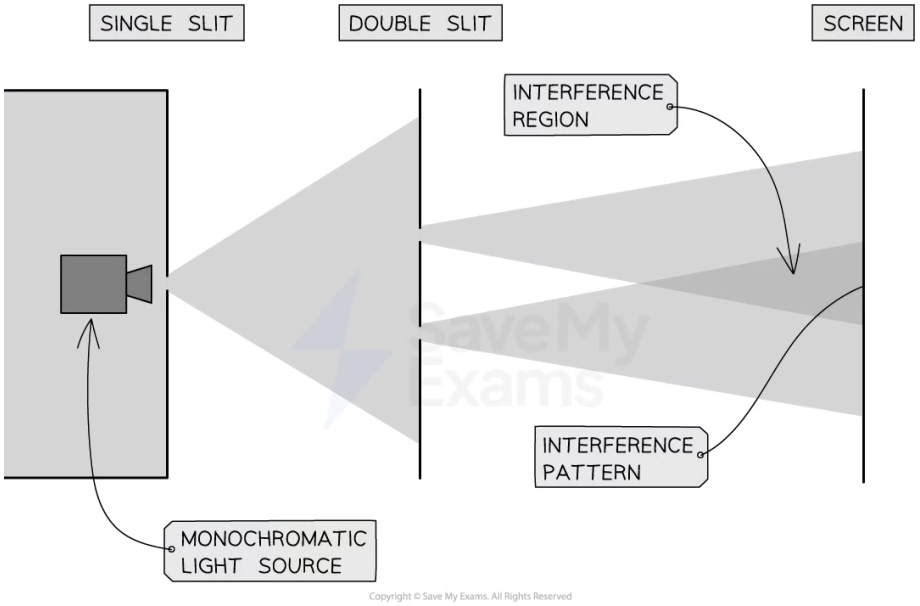
When drawing diffracted waves, take care to keep the wavelength (the distance between each wavefront) constant. It is only the amplitude of the wave that changes when diffracted.

Remember θ is the angle between the central maxima and the first minimum. If you want the **angle** of the whole central maxima, you then need to **double** this value!

Single-Slit Diffraction Pattern

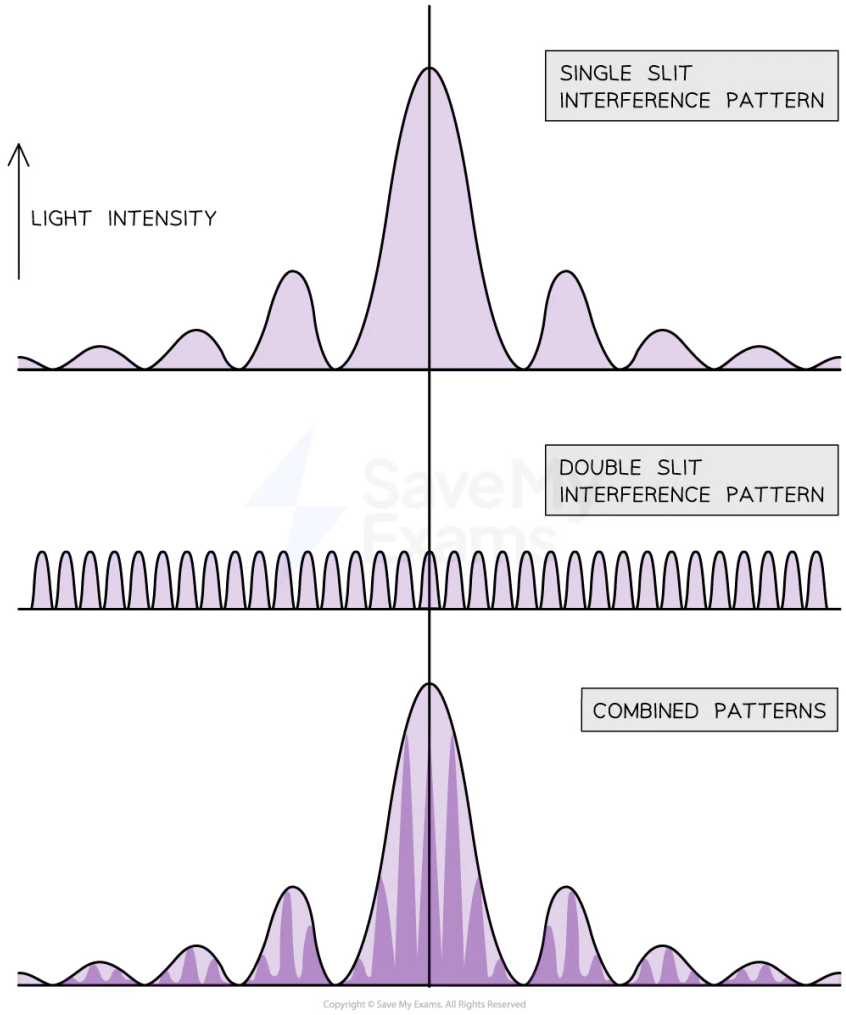
YOUR NOTES

- When light is passed through both a single slit, then a double slit, the resultant pattern is one in which the single-slit pattern **modulates** the double-slit pattern



Light travelling through both a single then double slit

- The slit-slit interference pattern has a distinctive **central** maxima, and **subsequent** minima at lower intensity
- The double-slit interference pattern has **equally** spaced lines at **equal** maxima
- Together, the combined pattern has equally spaced lines but now in a single slit 'envelope'



A modulated double-slit interference pattern

- The pattern now **decreases** in intensity the further it is from the centre

Exam Tip

You should be able to confidently recognise and sketch both single and double-slit interference patterns. Always make them as symmetrical as possible.

YOUR NOTES

Diffraction Gratings

- A diffraction grating is a **plate** on which there is a very large number of parallel, identical, close-spaced **slits**
- When monochromatic light is incident on a grating, a pattern of narrow bright fringes is produced on a screen

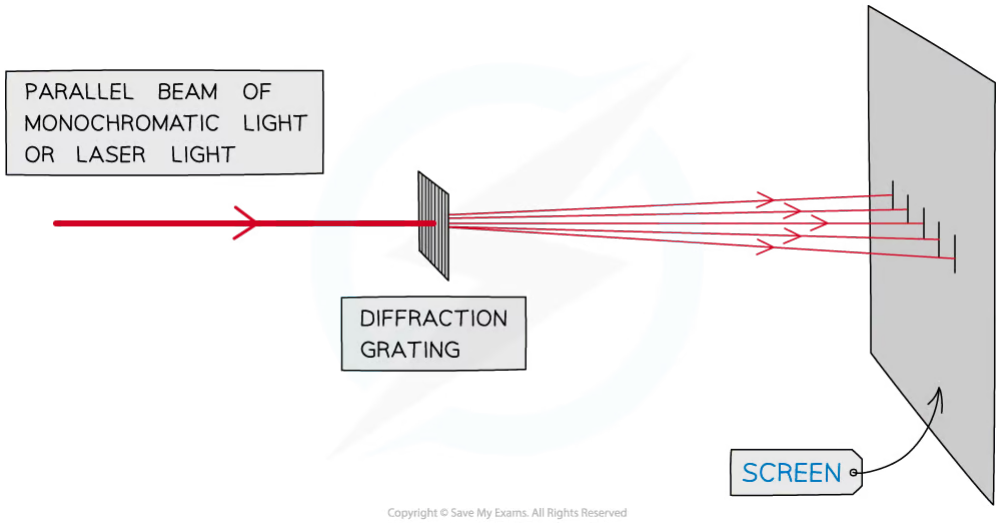


Diagram of diffraction grating used to obtain a fringe pattern

- The angles at which the maxima of intensity (constructive interference) are produced can be deduced by the diffraction grating equation

$$n\lambda = d\sin\theta$$

- Where:
  - n = order of maxima (0, 1, 2, 3)
  - λ = wavelength (m)
  - d = spacing between adjacent slits (m)
  - θ = angular separation between the order of maxima (°)
- Exam questions sometimes state the **lines per m** (or per mm, per nm etc.) on the grating which is represented by the symbol N
- d can be calculated from N using the equation

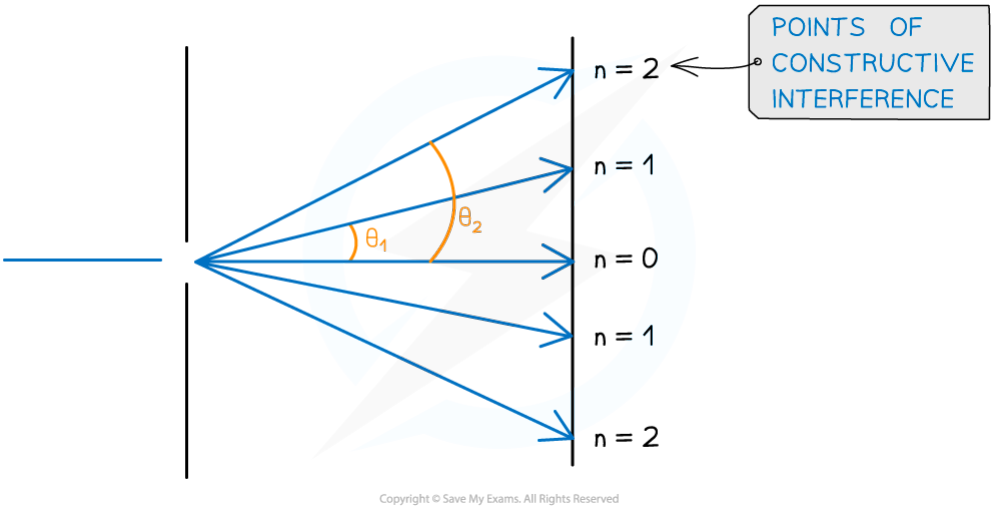
$$d = \frac{1}{N}$$

Angular Separation

- The angular separation of each maxima is calculated by rearranging the grating equation to make θ the subject

YOUR NOTES

- The angle θ is taken from the centre meaning the higher orders are at greater angles



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Angular separation

- The angular separation between two angles is found by subtracting the smaller angle from the larger one
- The angular separation between the first and second maxima n_1 and n_2 is $\theta_2 - \theta_1$

Highest Visible Order of Maxima

- The maximum angle to see orders of maxima is when the beam is at right angles to the diffraction grating
  - This means $\theta = 90^\circ$ and $\sin\theta = 1$
- The highest order of maxima visible is therefore calculated by the equation:

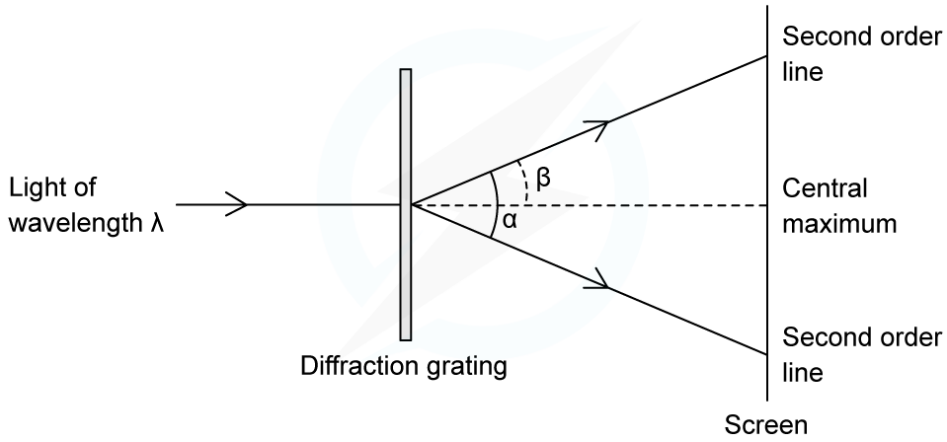
$$n = \frac{d}{\lambda}$$

- Note that since n must be an integer, if the value is a decimal it must be rounded **down**
  - E.g If n is calculated as 2.7 then $n = 2$ is the highest order visible

YOUR NOTES

Worked Example

An experiment was set up to investigate light passing through a diffraction grating with a slit spacing of 1.7 μm. The fringe pattern was observed on a screen. The wavelength of the light is 550 nm.



Calculate the angle α between the two second-order lines.

STEP 1

DIFFRACTION GRATING EQUATION

$d \sin(\theta) = n\lambda$

$n = 2$ FOR THE SECOND ORDER LINE

$D = 1.7\ \mu m$

$\lambda = 550\ nm$

STEP 2

REARRANGE FOR $\sin(\theta)$

$$\sin(\theta) = \frac{n\lambda}{d}$$

STEP 3

SUBSTITUTE IN VALUES

$$\sin(\theta) = \frac{2 \times 550 \times 10^{-9}}{1.7 \times 10^{-6}} = 0.64705... = 0.65 \text{ (2 s.f.)}$$

STEP 4

FIND θ THROUGH THE INVERSE SINE

$\sin^{-1}(0.65) = 40.54°$

STEP 5

θ IS ANGLE FROM THE CENTRE TO THE SECOND ORDER LINE (β ON THE DIAGRAM)

$\alpha = \theta \times 2 = 81°$ (2 s.f.)

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Exam Tip

Take care that the angle θ is the correct angle taken from the **centre** and not the angle taken **between** two orders of maxima.

YOUR NOTES

IB Physics HL

Standing Waves & Resonance

YOUR NOTES

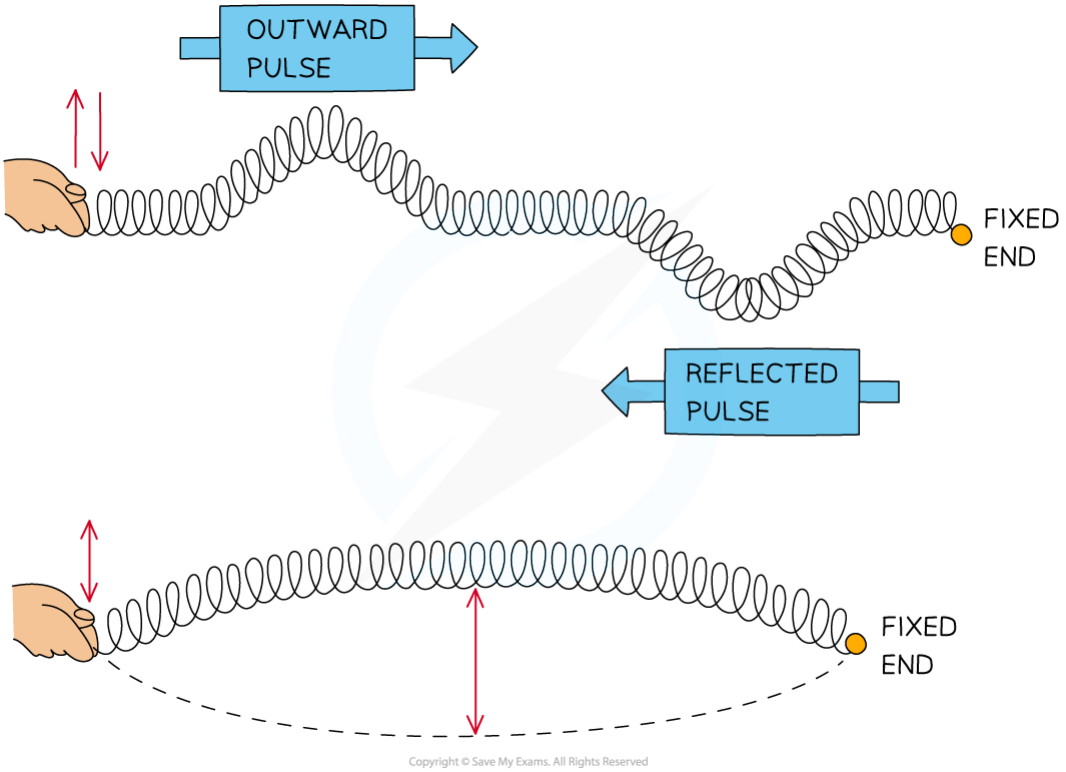
CONTENTS

Standing Waves
Nodes & Antinodes
Boundary Conditions for Standing Waves
Harmonics in Strings & Pipes
The Nature of Resonance
The Effect of Damping

YOUR NOTES

Standing Waves

- **Standing waves** are produced by two waves as they travel in **opposite directions**
- This is usually achieved when a travelling wave superimposes its reflection
  - The superposition produces a wave pattern where the crests and troughs only move vertically



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Formation of a stationary wave on a stretched spring fixed at one end

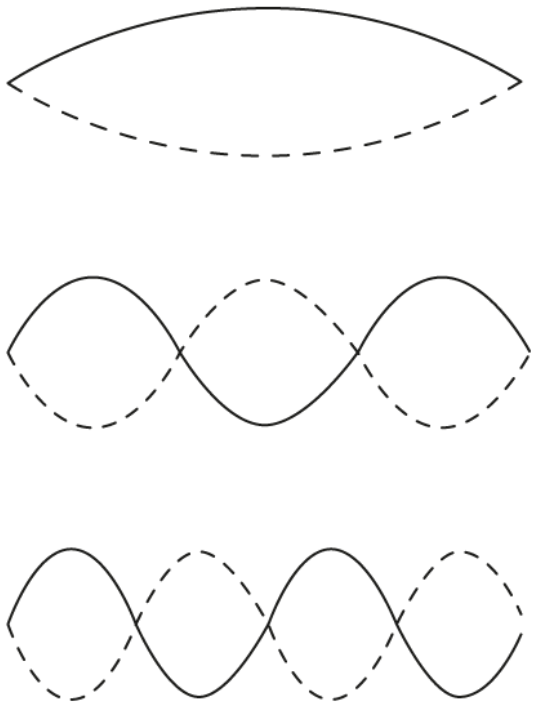
Formation of Standing Waves

- Standing waves are formed from the principle of superposition. This is when:

  Two waves travelling in opposite directions along the same line with the same frequency superpose

- The principle of superposition applies to **all** types of waves i.e. transverse and longitudinal, progressive and stationary
- The waves must have:
  - The same wavelength
  - A similar amplitude
- As a result of superposition, a resultant wave is produced

YOUR NOTES



Standing waves produced at varying frequencies

Comparing Progressive and Standing Waves

- Standing waves (or stationary waves) **store** energy
- **Progressive waves** (or travelling waves) **transfer** energy
- The table below outlines the main differences between progressive and stationary waves

Table of Differences Between Progressive and Stationary Waves

YOUR NOTES

Progressive Waves	Stationary Waves
All points have the same amplitude (in turn)	Each point has a different amplitude depending on the amount of superposition
Points exactly a wavelength apart are in phase. The phase of points within one wavelength can be between 0 to 360°	Points between nodes are in phase. Points on either side of a node are out of phase
Energy is transferred along the wave	Energy is stored, not transferred
Does not have nodes or antinodes	Has nodes and antinodes
The wave speed is the speed at which the wave moves through a medium	Each point on the wave oscillates at a different speed. The overall wave does not move

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Worked Example

A travelling wave is incident on a barrier. The wave profile is shown below.

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The travelling wave reflects off the barrier. The reflected and incident waves superimpose.

State and explain whether or not a standing wave is formed.

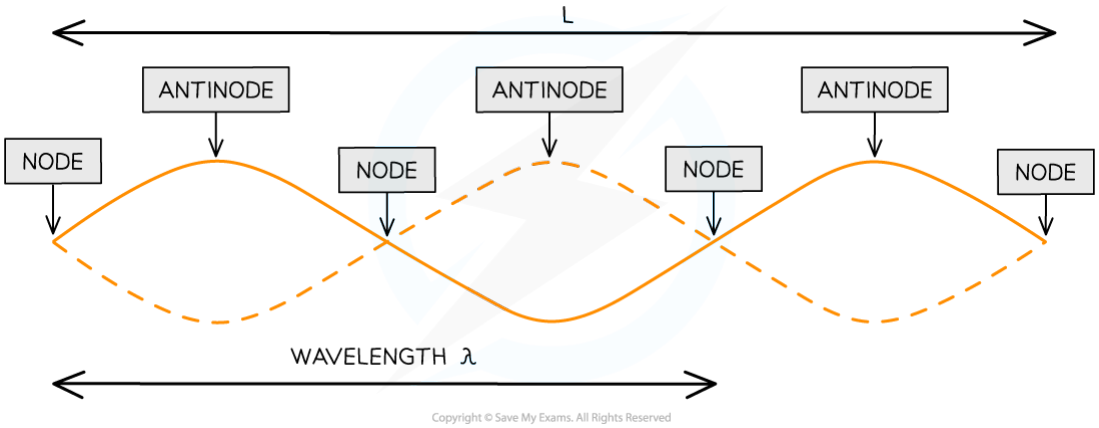
Answer:

- For standing waves to be formed, the half-cycles of the wave profile must be symmetrical (i.e. the same but inverted)
- For this wave, the half-cycles are not symmetrical
  - The leading edge is straight
  - The trailing edge is sinusoidal
- When the incident and reflected waves superimpose, they will not cancel out at any point
- Therefore, a **standing wave is not formed**

YOUR NOTES

Nodes & Antinodes

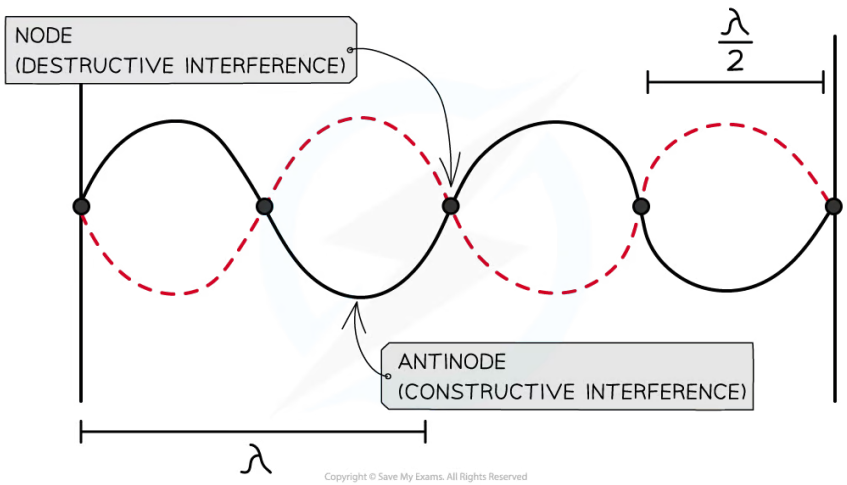
- A standing wave is made up **nodes** and **antinodes**
  - **Nodes** are locations of **zero amplitude** and they are **separated by half a wavelength** ($\lambda/2$)
  - **Antinodes** are locations of **maximum amplitude**
- The nodes and antinodes **do not** move along the wave
  - Nodes are fixed and antinodes only oscillate in the vertical direction



Nodes and antinodes of a stationary wave of wavelength λ on a string of length L at a point in time

The Formation of Nodes and Antinodes

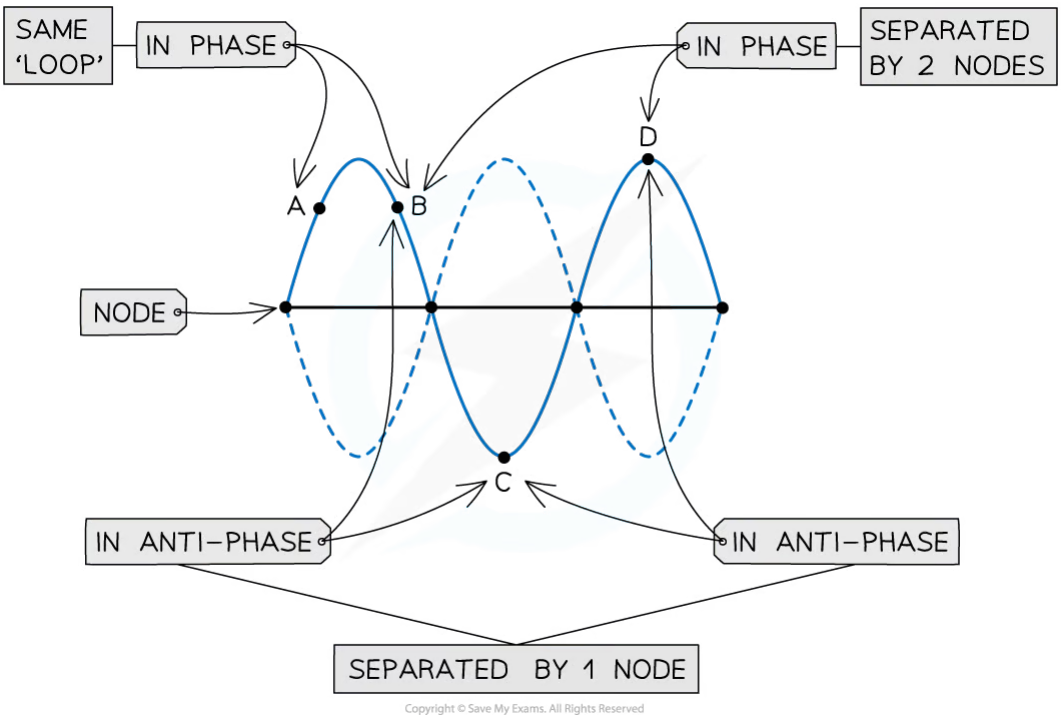
- At the **nodes**:
  - The waves are in anti-phase meaning **destructive** interference occurs
  - The crest of one wave meets the trough of another
  - This causes the two waves to **cancel** each other out
- At the **antinodes**:
  - The waves are in phase meaning **constructive** interference occurs
  - The crest of one wave meets the crest of another (same for troughs)
  - This causes the waves to **add** together



Nodes and antinodes are a result of destructive and constructive interference respectively

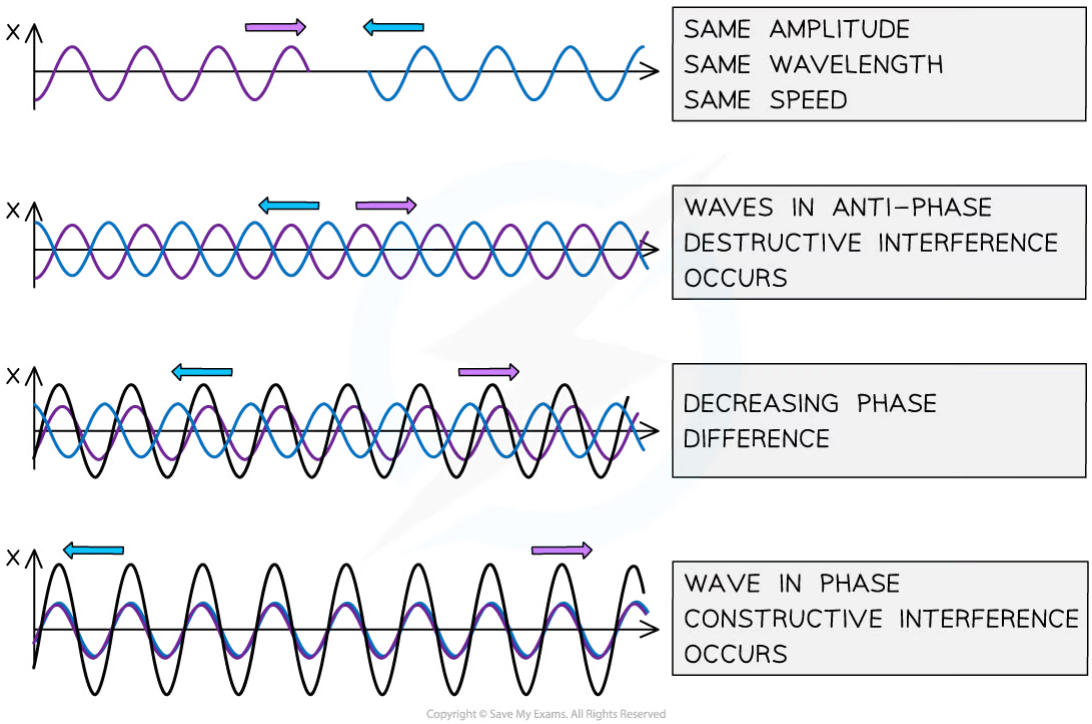
Phase on a Standing Wave

- Two points on a standing wave are either in phase or in anti-phase
  - Points that have an **odd** number of nodes between them are in **anti-phase**
  - Points that have an **even** number of nodes between them are **in phase**
  - All points within a **"loop"** are **in phase**



Points A, B and D are all in phase. While points A and D are in antiphase with point C

- Constructive and destructive interference can be seen from the phase differences between two waves



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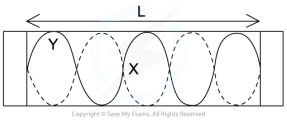
A graphical representation of how stationary waves are formed - the black line represents the resulting wave

YOUR NOTES

Worked Example

Which row in the table correctly describes the length of L and the name of **X** and **Y**?



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	Length L	Point X	Point Y
A	5 wavelengths	Node	Antinode
B	$2\frac{1}{2}$ wavelengths	Antinode	Node
C	$2\frac{1}{2}$ wavelengths	Node	Antinode
D	5 wavelengths	Antinode	Node

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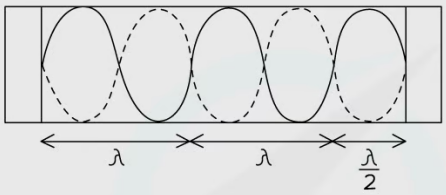
Answer: C

STEP 1

CALCULATE HOW MANY WAVELENGTHS IN THE LENGTH OF THE STRING



$= 2\frac{1}{2}$ WAVELENGTHS

THIS RULES OUT A AND D

STEP 2

X IS A POINT OF 0 DISPLACEMENT – A NODE

STEP 3

Y IS A POINT OF MAXIMUM DISPLACEMENT – AN ANTINODE

STEP 4

THE CORRECT ROW IS C

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Exam Tip

Phase difference on standing waves is different to travelling (progressive) waves.

Phase differences between two points on travelling waves can be anything from 0 to 2π. Between two points on a standing wave can **only** be in-phase (0 phase difference) or anti-phase (π out of phase).

YOUR NOTES

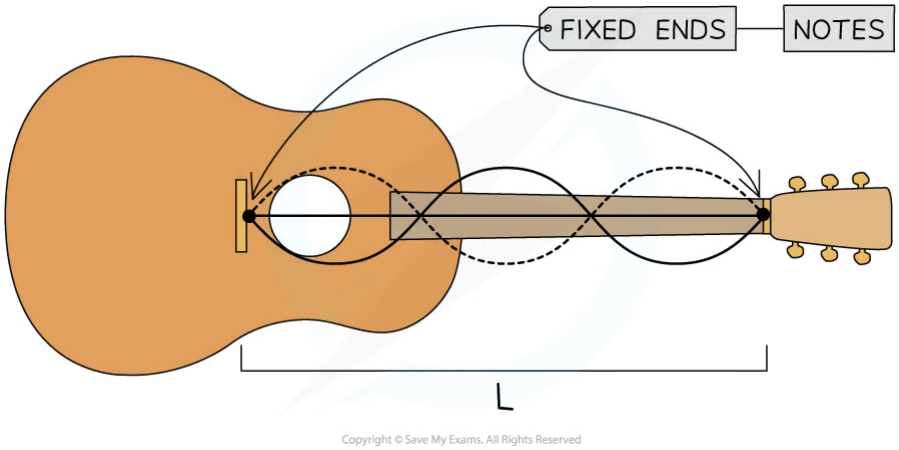
YOUR NOTES

Boundary Conditions

- Stationary waves can form on strings or in pipes
- In both cases, progressive waves travel in a medium (i.e. the string or air) and superimpose with their reflections
- The number of nodes and antinodes that fit within the available length of medium depends on:
  - The frequency of the progressive waves
  - The **boundary conditions** (i.e. whether the progressive waves travel between two fixed ends, two free ends or a fixed and a free end)

Standing Waves on Stretched Strings

- When guitar strings are plucked, they can vibrate with different frequencies
- The frequency with which a string vibrates depends on:
  - The tension, which is adjusted using rotating 'tuning pegs'
  - The mass per unit length, which is the reason why a guitar has strings of different thicknesses



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Standing wave on a guitar string

- For a string, the boundary condition can be
  - Fixed at both ends
  - Free at both ends
  - One end fixed, the other free
- At specific frequencies, known as **natural frequencies**, an **integer number of half wavelengths** will fit on the length of the string
  - As progressive waves of different natural frequencies are sent along the string, standing waves with different numbers of nodes and antinodes form

Standing Waves in Pipes

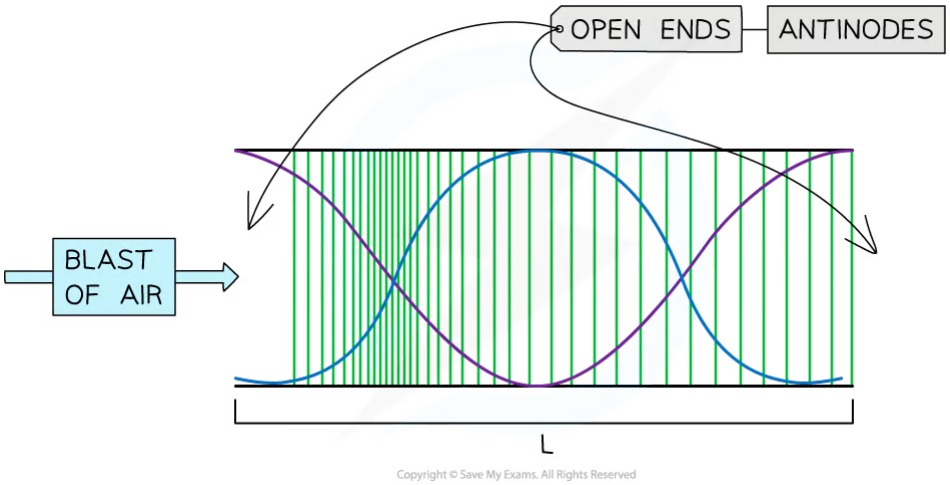
- When the air within a pipe vibrates, longitudinal waves travel along the pipe

- Simply blowing across the open end of a pipe can produce a standing wave in the pipe
- For a pipe, there is more than one possible boundary condition, theses are pipes that are:
  - Closed at both ends
  - Open at both ends
  - Closed at one end and open on the other

YOUR NOTES

Nodes & Antinodes

- When a progressive wave travels towards a free end for a string, or open end for a pipe:
  - The **reflected wave** is **in phase** with the incident wave
  - The amplitudes of the incident and reflected waves add up
  - A free end is a location of maximum displacement - i.e. an **antinode**



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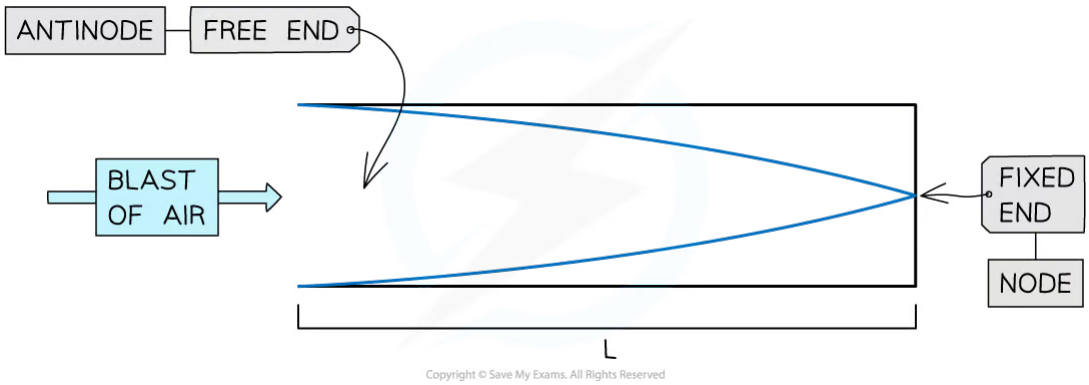
Standing wave inside a pipe open at both ends

- When a progressive wave travels towards a fixed end for a string, or closed end for a pipe:
  - The **reflected wave** is in **anti-phase** with the incident wave
  - The two waves cancel out
  - A **fixed** end is a location of zero displacement - i.e. a **node**
  - The **open** end is therefore a location of maximum displacement - i.e. an **antinode**



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Standing wave inside a pipe open at both ends

YOUR NOTES

Harmonics

- Stationary waves can have different wave patterns, known as **harmonics**
  - These depend on the **frequency** of the vibration and the **boundary conditions** (i.e. fixed and/or free ends)
- The harmonics are the only frequencies and wavelengths that will form standing waves on strings or in pipes

Harmonics on Strings

- The **boundary condition** is that **both ends are fixed**
- The simplest wave pattern is a single loop made up of two nodes (i.e. the two fixed ends) and an antinode
  - This is called the **first harmonic**
  - The wavelength of this harmonic is $\lambda_1 = 2L$
  - Using the wave equation, the frequency is $f_1 = \frac{v}{2L}$, where *v* is the wave speed of the travelling waves on the string (i.e. the incident wave and the reflected wave)
- As the vibrating **frequency** increases, more complex patterns arise
  - The **second** harmonic has three nodes and two antinodes
  - The **third** harmonic has four nodes and three antinodes

YOUR NOTES

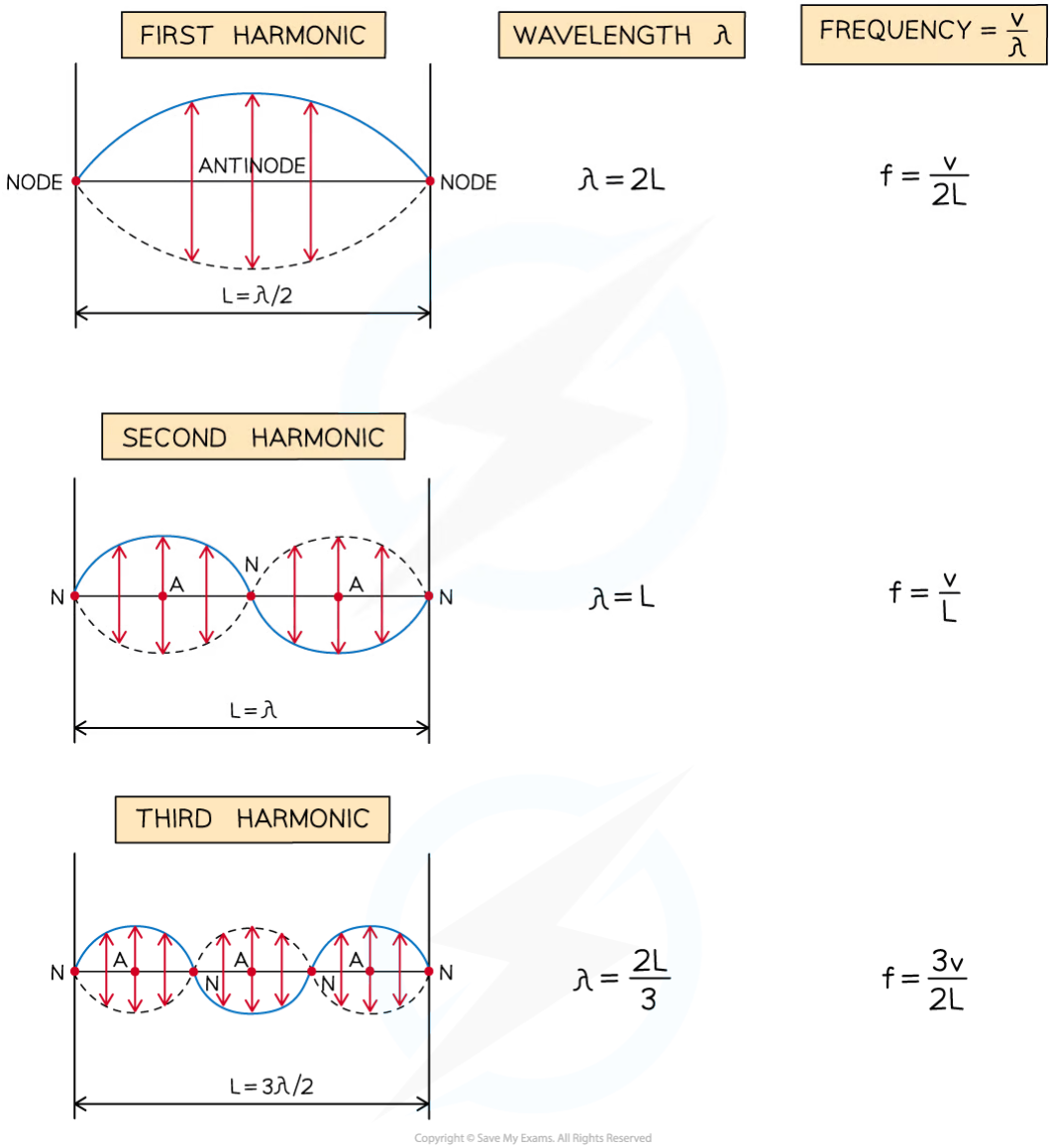

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Diagram showing the first three harmonics on a stretched string fixed at both ends

- The *n*th harmonic will have (*n* + 1) nodes and *n* antinodes
- The general expression for the **wavelength of the *n*th harmonic** on a string that is fixed at both ends is:

$$\lambda_n = \frac{2L}{n}$$

- Where:
  - λ_n = wavelength in metres (m)
  - L = length of the string in metres (m)
  - n = integer number greater than zero - i.e. 1, 2, 3...

YOUR NOTES

- Knowing the wavelength λ_n of the standing wave and the speed v of the travelling waves (i.e. incident and reflected), the **natural frequency** f_n of any harmonic can be calculated using the wave equation $v = f\lambda_n$, so that:

$$f_n = \frac{nv}{2L}$$

Harmonics in Pipes

- The **boundary conditions** vary, since pipes can have:
  - **two open ends**
  - only **one open end**

- For a pipe that is **open at both ends**:
  - The simplest wave pattern is one central node and two antinodes
  - The second harmonic consists of two nodes and three antinodes
  - The nth harmonic will have $(n + 1)$ antinodes and n nodes
  - The expression for the wavelength of the nth harmonic in a pipe of length L is the same as that given above for nth harmonic on a string

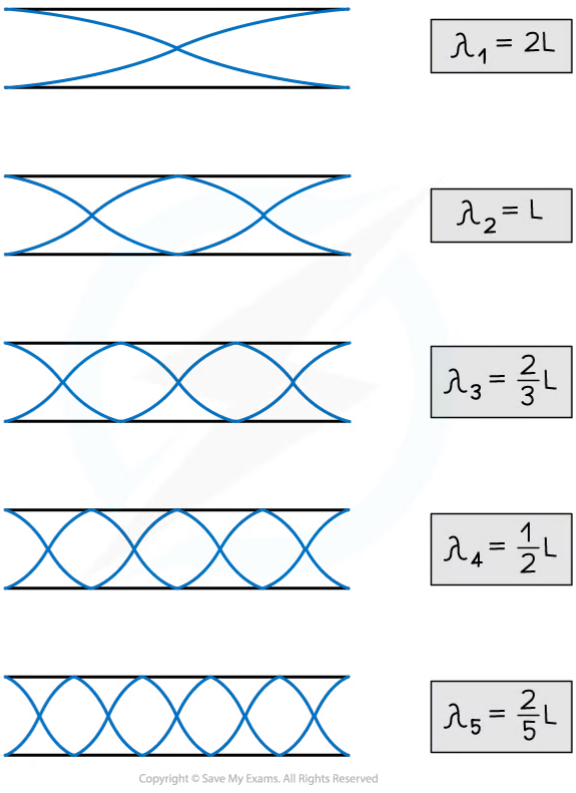

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Diagram showing the first five harmonics in a pipe open at both ends

- For a pipe that is **open at one end**:
  - The lowest harmonic is a "**half-loop**" with one node and one antinode
  - The next possible harmonic will have two nodes and two antinodes

- This is the **third** harmonic, **not** the second one
- Since **only odd harmonics can exist** under this boundary condition

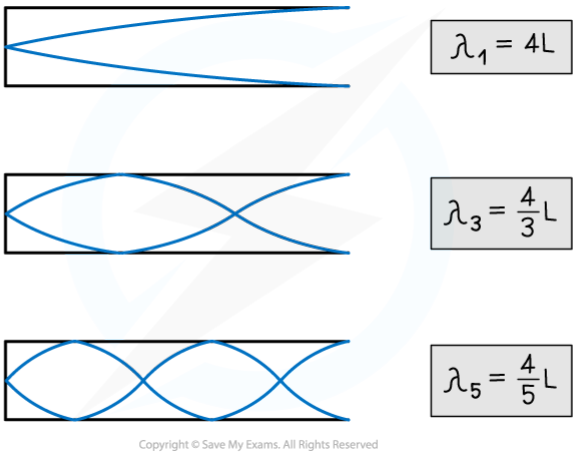

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Diagram showing the first three possible harmonics in a pipe open at one end. Only the odd harmonics can form in this case

- The expression for the wavelength of the *n*th harmonic in a pipe of length *L* is:

$$\lambda_n = \frac{4L}{n}$$

- Where this time, *n* is an odd number - i.e. 1, 3, 5...
- Under both boundary conditions, the natural frequencies are once again calculated from the **wavelength** of the standing wave and the **speed** *v* of the travelling waves using the wave equation

YOUR NOTES

Worked Example

Transverse waves travel along a stretched wire 100 cm long. The speed of the waves is 250 m s^{-1}.

Determine the maximum harmonic detectable by a person who can hear up to 15 kHz.

Answer:

Step 1: Write down the known quantities

- Length of the wire, L = 100 cm = 1.00 m
- Speed of the waves, v = 250 m s^{-1}
- Maximum frequency of human hearing, f_n = 15 kHz = 15 000 Hz

Step 2: Write down the equation for the frequency of the *n*th harmonic and rearrange for n

$$f_n = \frac{nv}{2L} \quad \Rightarrow \quad n = \frac{2Lf_n}{v}$$

Step 3: Substitute the numbers into the above equation

$$n = \frac{2 \times 1.00 \times 15\,000}{250} = 120$$

- The person can hear up to the 120th harmonic

Exam Tip

Before carrying out any calculation on standing waves, you should look carefully at the **boundary conditions**, since these will determine the **wavelengths** and **natural frequencies** of the harmonics.

The expressions for the wavelength of the *n*th harmonic on strings fixed at both ends (or in pipes open at both ends) and in pipes open at one end are **not** given in the data booklet and you must be able to recall them to make calculations easier.

Remember that *n* can take any **integer** value **greater** than **zero** in the case of standing waves on strings and in pipes open at both ends. For pipes open at **one** end, instead, *n* can only be an **odd** integer.

YOUR NOTES

Free & Forced Oscillations

Free Oscillations

- Free oscillations occur when there is no transfer of energy to or from the surroundings
  - This happens when an oscillating system is displaced and then left to oscillate
- In practice, this only happens in a vacuum. However, anything vibrating in air is still considered a free vibration as long as there are **no external forces** acting upon it
- Therefore, a **free oscillation** is defined as:

> **An oscillation where there are only internal forces (and no external forces) acting and there is no energy input**

- A free vibration always oscillates at its **resonant frequency**

Forced Oscillations

- In order to sustain oscillations in a simple harmonic system, a periodic force must be applied to replace the energy lost in damping
  - This periodic force **does work** on **resistive** forces (i.e. the force that decreases the amplitude of the oscillations), such as air resistance
  - It is sometimes known as an **external driving** force
- This period force creates **forced oscillations** (or vibrations) is are defined as:

> **Oscillations which are produced by a periodic external force**

- Forced oscillations are made to oscillate at the same frequency as the external oscillator creating the external, periodic driving force
  - This means the driving force can change the frequency of the oscillator

Resonance

- The frequency of the forced oscillations on a system is referred to as the **driving frequency** f
- All oscillating systems have a **natural frequency** f_0 which is defined as

> **The frequency of an oscillation when the oscillating system is allowed to oscillate freely**

- Oscillating systems can exhibit a property known as **resonance** when

$$\textbf{driving frequency } f = \textbf{natural frequency } f_0$$

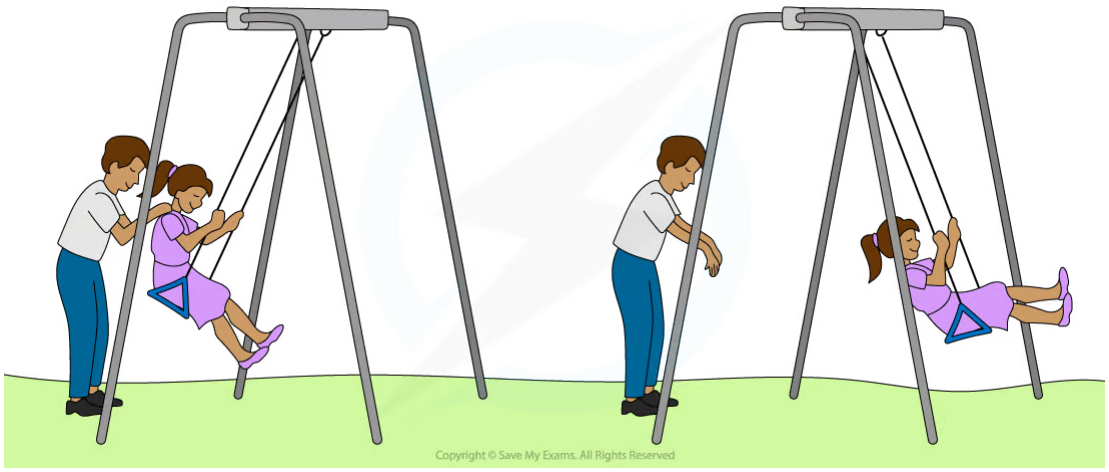
- When the driving frequency approaches the natural frequency of an oscillator, the system **gains more energy** from the driving force
  - Eventually, when they are equal, the oscillator vibrates with its **maximum amplitude**
  - This is **resonance**
- Resonance is defined as:

When the frequency of the applied force to an oscillating system is equal to its natural frequency, the amplitude of the resulting oscillations is at its maximum

YOUR NOTES

Example of resonance: a child pushed on a swing

- Every system (in this case, the swing and the child) has a **fixed natural frequency**
- A small push (the **driving force**) after each cycle increases the **amplitude** of the oscillations, resulting in the swing's motion back and forth
- The frequency at which the swing is pushed is the **driving frequency**
- When the **driving** frequency is **equal** to the **natural** frequency of the swing, **resonance** occurs
- If the driving frequency is slightly lower or higher than the natural frequency, the amplitude will increase but to a **lesser** extent than if they were equal
- This is because, at resonance, **energy** is transferred from the **driver** to the **oscillating** system **most efficiently**
  - Therefore, at resonance, the driving force transfers the **maximum kinetic energy** to the system
  - In this case, the child will swing the highest when resonance occurs



Pushing a child on a swing is an example of how forced oscillations can produce resonance

YOUR NOTES

Worked Example

State and explain whether the following scenarios are examples of free or forced oscillations:

(a)
Striking a tuning fork
(b)
Breaking a glass from a high pitched sound
(c)
The interior of a car vibrating when travelling at a high speed
(d)
Playing the clarinet

Answer:

(a) Striking a tuning fork

- This is a free vibration
- When a tuning fork is struck, it will vibrate at its natural frequency and there are no other external forces

(b) Breaking a glass from a high-pitched sound

- This is a forced vibration
- The glass is forced to vibrate at the same frequency as the sound until it breaks (when it equals the natural frequency of the glass)
- The frequency of the high-pitched sound is the external driving frequency

(c) The interior of a car vibrating when travelling at a particular speed

- This is a forced vibration
- The interior of the car vibrates at the same frequency as the wheels travelling over a rough surface at a high speed

(d) Playing the clarinet

- This is a forced vibration
- The air from the player's lungs is used to sustain the vibration in the air column in a clarinet to create and hold a sound
- The air column inside the clarinet mimics the vibrations at the same frequency as the air forced into the mouthpiece of the clarinet (the reed). This creates the sound

Exam Tip

Avoid writing 'a free oscillation is not forced to oscillate'. Mark schemes are mainly looking for a reference to **internal** and **external** forces and **energy** transfers. Make sure to include about the **amplitude** of the oscillations too.

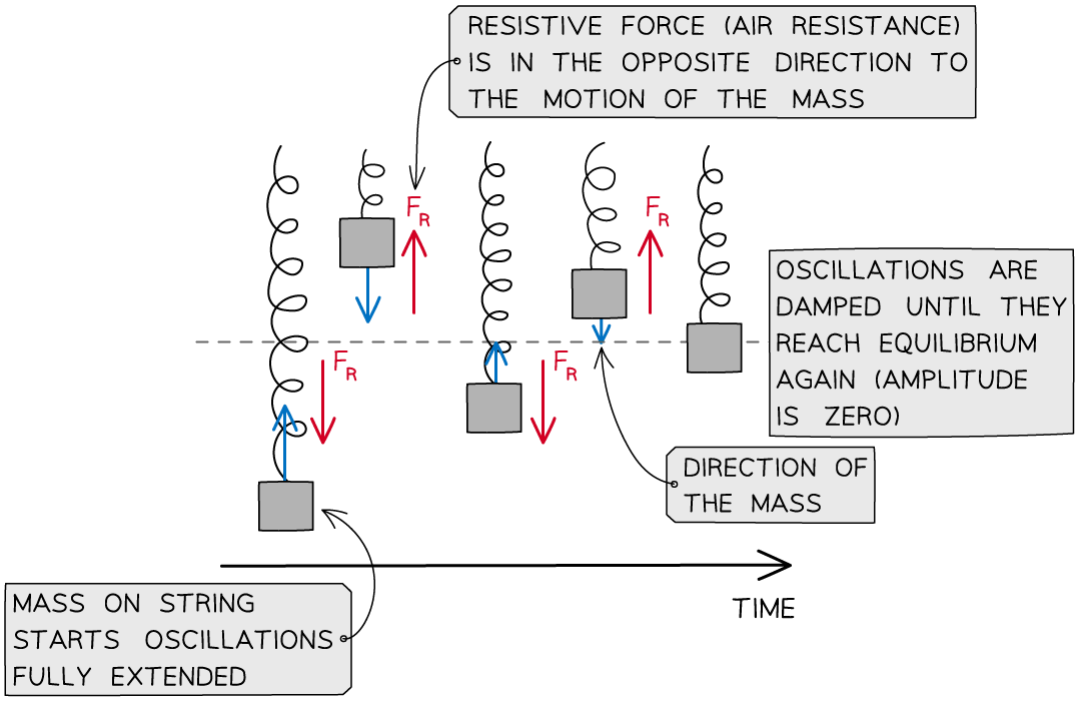
YOUR NOTES

Types of Damping

- In practice, all oscillators eventually stop oscillating
  - Their amplitudes decrease rapidly, or gradually
- This happens due to **resistive forces**, such as friction or air resistance, which act in the **opposite** direction to the motion of an oscillator
- Resistive forces acting on an oscillating simple harmonic system cause **damping**
  - These are known as **damped** oscillations
- Damping is defined as:

> ***The reduction in energy and amplitude of oscillations due to resistive forces on the oscillating system***

- Damping continues until the oscillator comes to rest at the equilibrium position
- A key feature of simple harmonic motion is that the **frequency** of damped oscillations **does not change** as the amplitude decreases
  - For example, a child on a swing can oscillate back and forth once every second, but this time remains the same regardless of the amplitude



Damping on a mass on a spring is caused by a resistive force (F_R) acting in the opposite direction to the motion (blue arrow). This continues until the amplitude of the oscillations reaches zero

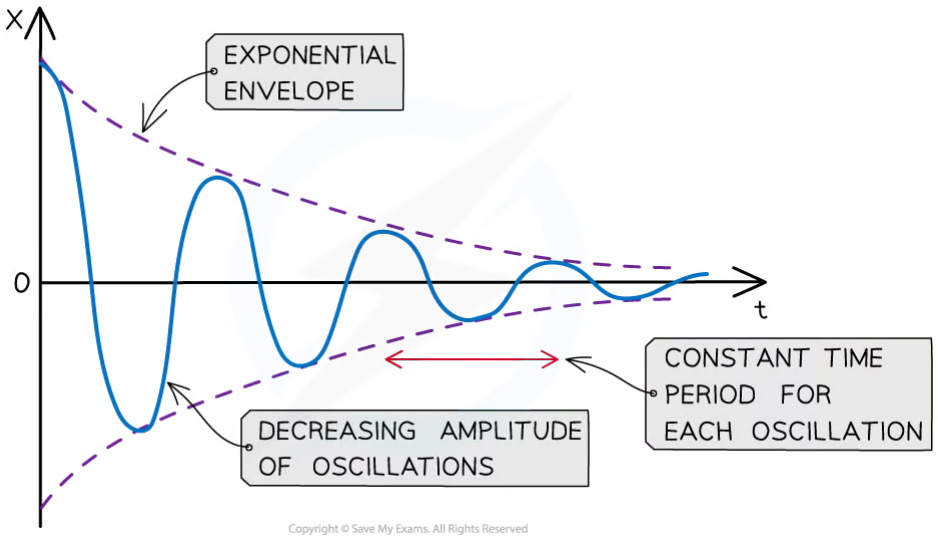
YOUR NOTES

Types of Damping

- There are three degrees of damping depending on how quickly the amplitude of the oscillations decreases:
  - **Light** damping
  - **Critical** damping
  - **Heavy** damping

Light Damping

- When oscillations are lightly damped, the amplitude **does not decrease linearly**
  - It decays exponentially with time
- When a lightly damped oscillator is displaced from the equilibrium, it will oscillate with **gradually decreasing** amplitude
  - For example, a swinging pendulum decreasing in amplitude until it comes to a stop

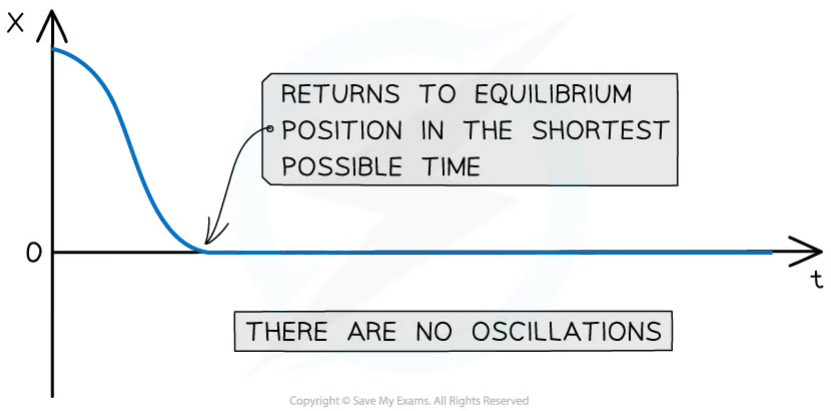


A graph for a lightly damped system consists of oscillations decreasing exponentially

- **Key features of a displacement-time graph for a lightly damped system:**
  - There are many oscillations represented by a sine or cosine curve with gradually decreasing amplitude over time
  - This is shown by the height of the curve decreasing in both the positive and negative displacement values
  - The amplitude decreases **exponentially**
  - The frequency of the oscillations remains constant, this means the time period of oscillations must stay the same and each peak and trough are equally spaced

Critical Damping

- When a critically damped oscillator is displaced from the equilibrium, it will **return** to rest at its equilibrium position in the shortest possible time **without** oscillating
  - For example, car suspension systems prevent the car from oscillating after travelling over a bump in the road



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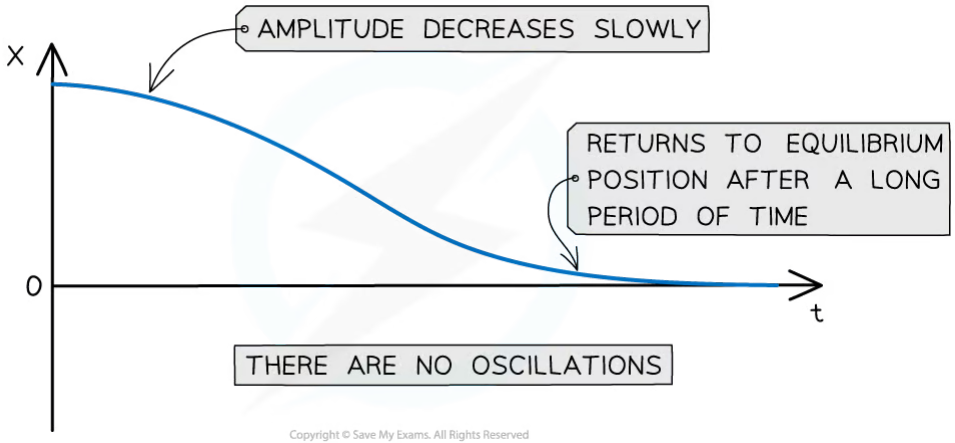
The graph for a critically damped system shows no oscillations and the displacement returns to zero in the quickest possible time

- **Key features of a displacement-time graph for a critically damped system:**
  - This system does **not** oscillate, meaning the displacement falls to 0 straight away
  - The graph has a fast decreasing gradient when the oscillator is first displaced until it reaches the x axis
  - When the oscillator reaches the equilibrium position (x = 0), the graph is a horizontal line at x = 0 for the remaining time

Heavy Damping

- When a heavily damped oscillator is displaced from the equilibrium, it will take a **long time** to return to its equilibrium position **without** oscillating
- The system returns to equilibrium more slowly than the critical damping case
  - For example, door dampers to prevent them from slamming shut



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A heavy damping curve has no oscillations and the displacement returns to zero after a long period of time

- **Key features of a displacement-time graph for a heavily damped system:**
  - There are no oscillations. This means the displacement does not pass 0

- The graph has a **slow decreasing gradient** from when the oscillator is first displaced until it reaches the x-axis
- The oscillator reaches the equilibrium position ($x = 0$) after a **long** period of time, after which the graph remains a **horizontal** line for the remaining time

YOUR NOTES

Worked Example

A mechanical weighing scale consists of a needle which moves to a position on a numerical scale depending on the weight applied.

Sometimes the needle moves to the equilibrium position after oscillating slightly, making it difficult to read the number on the scale to which it is pointing.

Suggest, with a reason, whether light, critical or heavy damping should be applied to the mechanical weighing scale to read the scale more easily.

Answer:

- Ideally, the needle should not oscillate before settling - this means the scale should have either **critical** or **heavy damping**
- Since the scale is read straight away after a weight is applied, ideally the needle should settle as quickly as possible
- Heavy damping would mean the needle will take some time to settle on the scale
- Therefore, **critical damping** should be applied to the weighing scale so the **needle can settle as quickly as possible** to read from the scale

Exam Tip

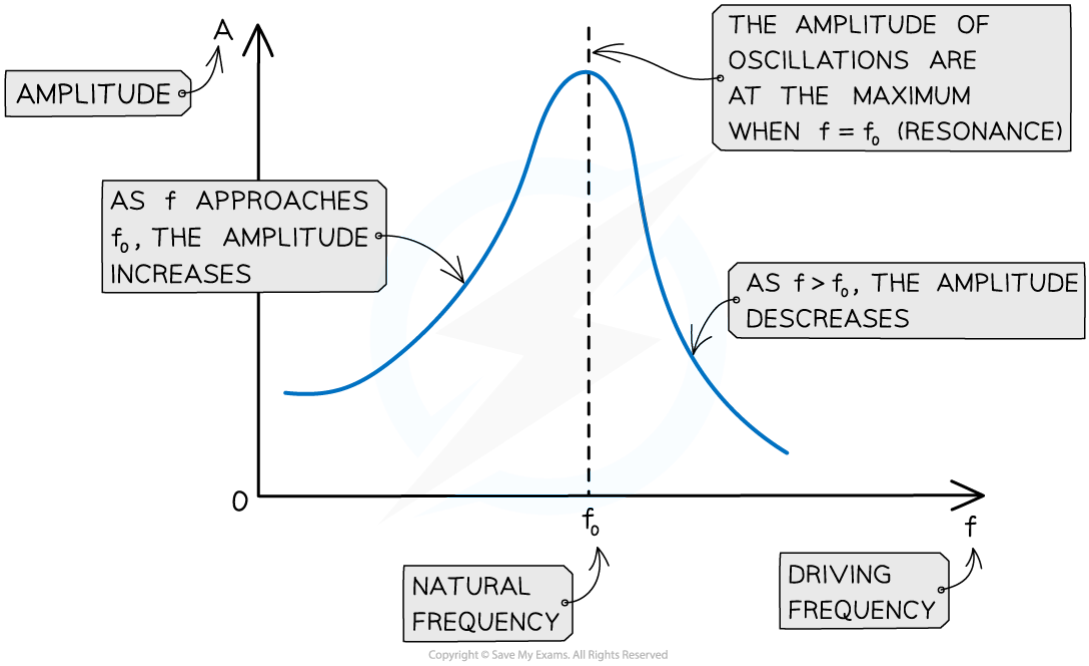
Make sure not to confuse **resistive** force and **restoring** force:

- Resistive force is what **opposes the motion** of the oscillator and causes damping
- Restoring force is what brings the oscillator **back to the equilibrium position**

Effects of Damping

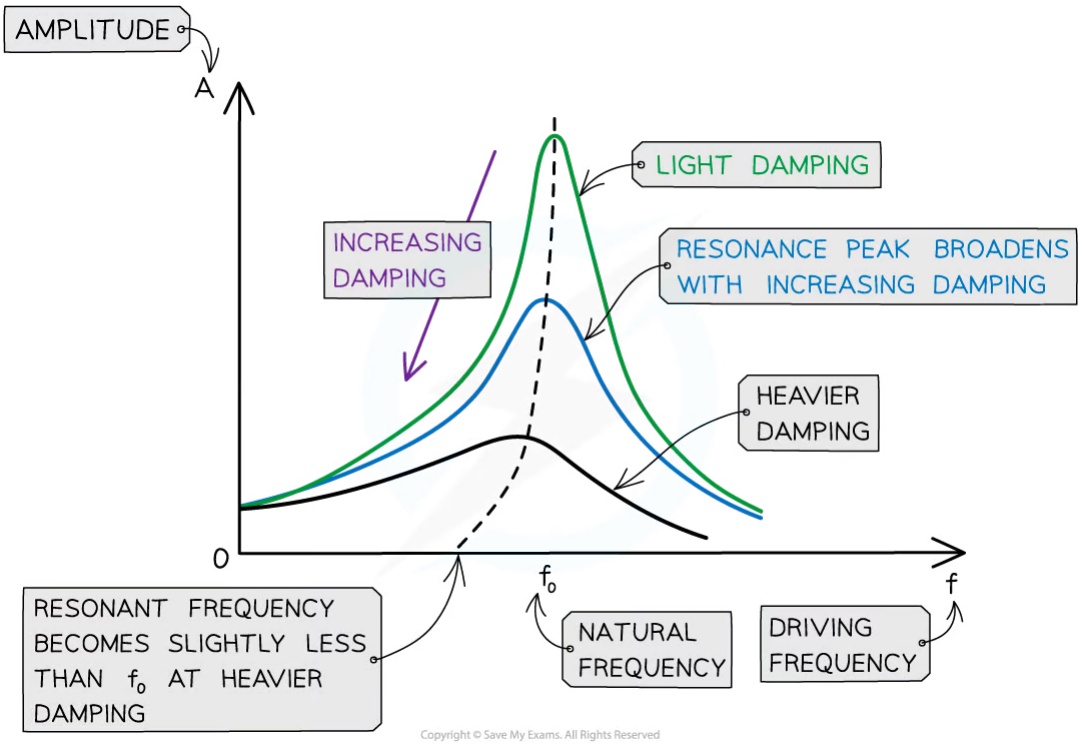
YOUR NOTES

- The effects of damping can be seen on a **resonance curve**
- This is a graph of driving frequency *f* against amplitude *A* of oscillations
- It has the following key features:
  - When $f < f_0$, the amplitude of oscillations **increases**
  - At the peak where $f = f_0$, the amplitude is at its **maximum**
    - This is **resonance**
  - When $f > f_0$, the amplitude of oscillations starts to **decrease**



The maximum amplitude of the oscillations occurs when the driving frequency is equal to the natural frequency of the oscillator

- Damping **reduces** the amplitude of resonance vibrations
- The height and shape of the resonance curve will therefore change slightly depending on the degree of damping
  - **Note:** the natural frequency f_0 of the oscillator will remain the same
- As the degree of damping is increased, the resonance graph is altered in the following ways:
  - The amplitude of resonance vibrations **decrease**, meaning the peak of the curve lowers
  - The resonance peak **broadens**
  - The resonance peak moves slightly to the **left** of the natural frequency when heavily damped
- Therefore, damping reduced the sharpness of resonance and reduces the amplitude at resonant frequency



As damping is increased, resonance peak lowers, the curve broadens and moves slightly to the left

Exam Tip

An exam question can include any damping scenario. Therefore, make sure you can interpret the amplitude-frequency graph depending gon the type of oscillation.

Most importantly, the resonant frequency **does not change** with damping, because it is a property of the object itself. Only the driving frequency can change the amplitude of oscillations.

IB Physics HL

YOUR NOTES

Doppler Effect

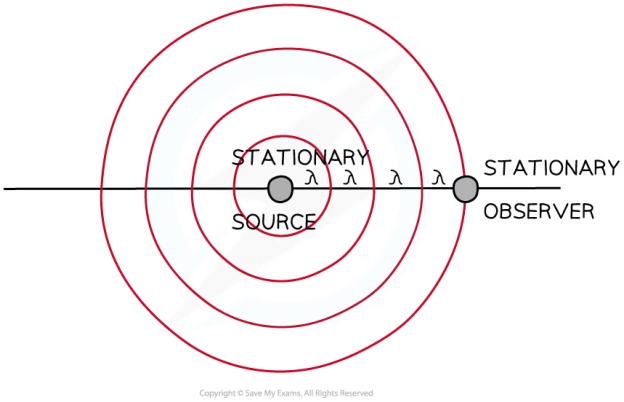
CONTENTS

The Doppler Effect
The Doppler Effect of Light
Galactic Redshift
Equations for the Doppler Effect of Sound (HL)

YOUR NOTES

The Doppler Effect

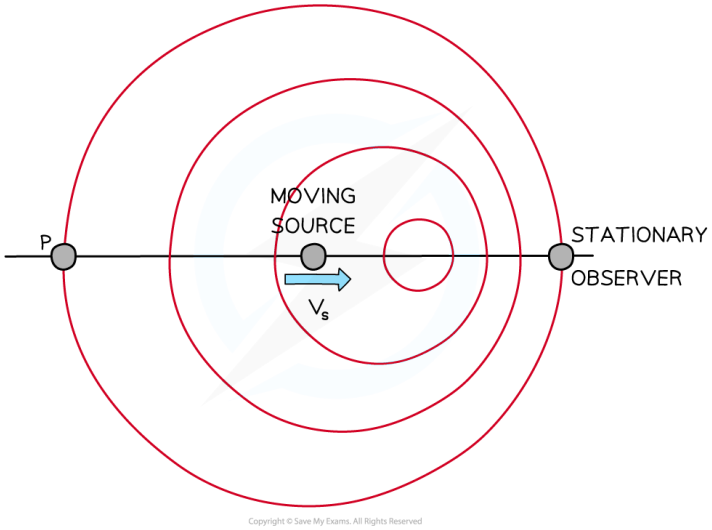
- When a source of sound, such as the whistle of a train or the siren of an ambulance, moves **away** from an observer:
  - It appears to **decrease** in frequency, i.e. it sounds **lower** in pitch
  - The **source** of the sound however, remains at a **constant** frequency
- This frequency change due to the relative motion between a source of sound or light and an observer is known as the **Doppler effect** (or **Doppler shift**)
- When the observer and the source of sound (e.g. ambulance siren) are both **stationary**:
  - The waves appear to remain at the **same** frequency for both the observer and the source
- When the observer and the source of sound (e.g. ambulance siren) are **moving** relative to each other
  - The waves appear to have a **different** frequency for both the observer and the source



Stationary source and observer. The wavelength of the waves are the same for both observers

- When the source starts to move **towards** the observer, the wavelength of the waves is **shortened**
  - The sound, therefore, appears at a **higher** frequency to the observer

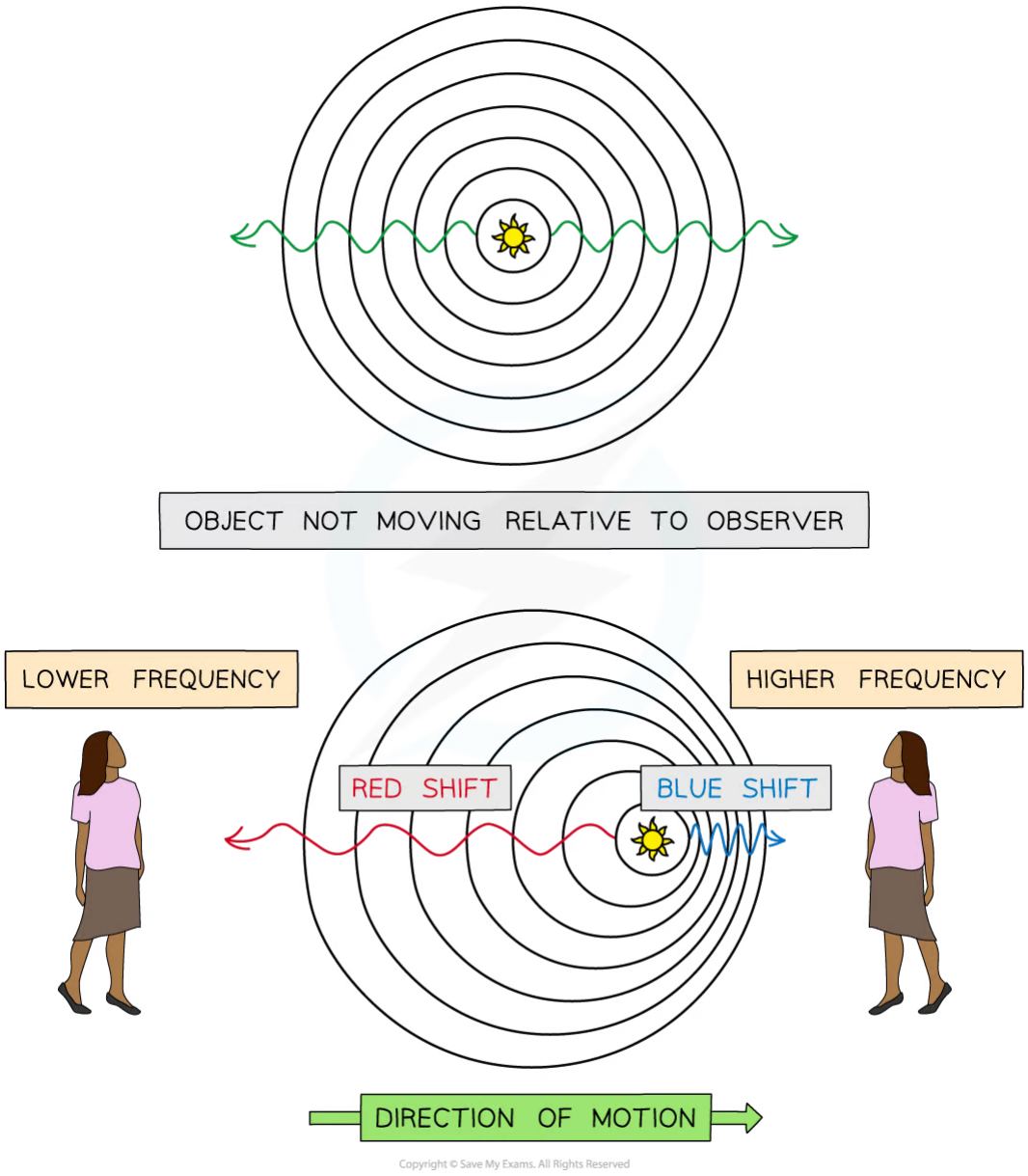
YOUR NOTES



Moving source at speed v_s and stationary observer. The waves are closer together closer near the stationary observer

- Notice how the waves are **closer** together between the source and the observer compared to point **P** and the source
- This also works if the source is moving **away** from the observer
  - If the observer was at point **P** instead, they would hear the sound at a lower frequency due to the wavelength of the waves **broadening**
- The frequency is **increased** when the source is moving **towards** the observer
- The frequency is **decreased** when the source is moving **away** from the observer
- The same phenomena occurs for electromagnetic waves, such as light
- Waves moving **away** from the observer are **red-shifted**
  - Their wavelengths shift to the red end of the electromagnetic spectrum
  - This is equivalent to sound waves appearing at a **lower** frequency to the observer
- Waves moving **towards** the observer are **blue-shifted**
  - Their wavelengths shift to the blue end of the electromagnetic spectrum
  - This is equivalent to sound waves appearing at a **higher** frequency to the observer
- This is because red light has a **longer wavelength** than blue light

YOUR NOTES



Red shift and blue shift for electromagnetic waves

YOUR NOTES

Worked Example

A cyclist rides a bike ringing their bell past a stationary observer.

Which row correctly describes the Doppler shift caused by the sound of the bell?

	Wavelength	Frequency	Sound pitch
A	Shorter	Higher	Lower
B	Longer	Lower	Higher
C	Shorter	Lower	Higher
D	Longer	Lower	Lower

Answer: D

- If the cyclist is riding past the observer, the wavelength of sound waves are going to become longer
  - This rules out options **A** and **C**
- A longer wavelength means a lower frequency (from $v = f\lambda$)
- Lower frequency creates a lower sound pitch
  - Therefore, the answer is row **D**

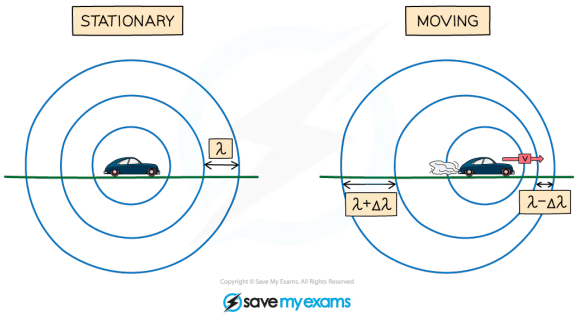
Exam Tip

The relationship between frequency and wavelength is determined by the wave equation, which is given in your data booklet. The speed *v* of the wave does not change.

Representing The Doppler Effect

YOUR NOTES

- Wavefront diagrams help visualise the Doppler effect for moving wave sources and stationary observers



Wavefronts are even in a stationary object but are squashed in the direction of the moving wave source

- $\Delta\lambda$ is the **change in** wavelength
  - The bigger the change, the bigger the doppler shift
- A moving object will cause the **wavelength**, λ, (and frequency) of the waves to change:
  - The **wavelength** of the waves **in front** of the source **decreases** ($\lambda - \Delta\lambda$) and the **frequency increases**
  - The wavelength **behind** the source **increases** ($\lambda + \Delta\lambda$) and the **frequency decreases**
- The Doppler shift is observed by **all** waves including **sound** and **light**

The Doppler Effect of Light

- The Doppler shift for a light-emitting non-relativistic source can be described using the equation:

$$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} \approx \frac{\Delta v}{c}$$

- Where:
  - Δf = change in frequency (Hz)
  - f = reference frequency (frequency of the source) (Hz)
  - $\Delta \lambda$ = change in wavelength (m)
  - λ = reference wavelength (wavelength of the source) (m)
  - Δv = relative velocity of the source and observer ($m\ s^{-1}$)
  - c = the speed of light ($m\ s^{-1}$)
- The sign ≈ means 'approximately equals to'
- This equation only works if $v << c$
- The change in wavelength $\Delta \lambda$ is equal to:

$$\Delta \lambda = \lambda' - \lambda$$

- Where:
  - λ' = **observed** wavelength of the source (m)
- Since the fractions have the **same** units on the numerator (top number) and denominator (bottom number), the Doppler shift has **no units**
- The **relative speed** between the source and observer along the line joining them is given by:

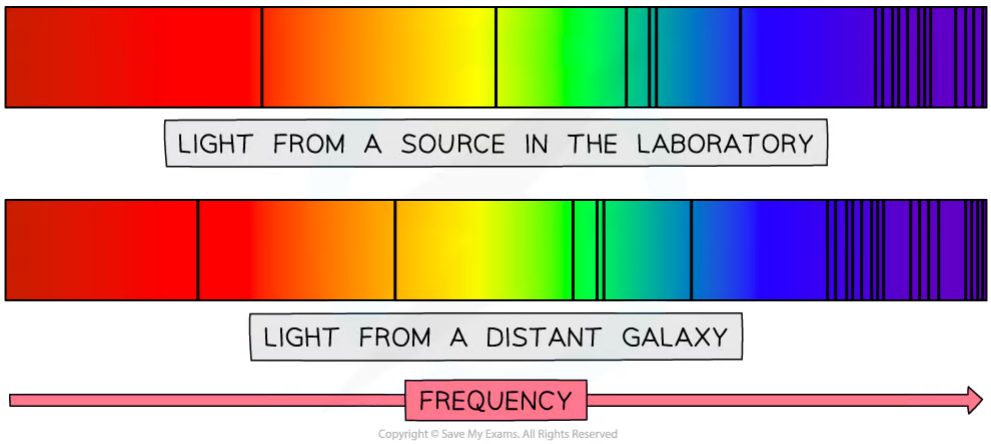
$$\Delta v = v_s - v_o$$

- Where:
  - v_s = velocity of the **source** of the light ($m\ s^{-1}$)
  - v_o = velocity of the **observer** ($m\ s^{-1}$)
- Usually, we are calculating the speed of the source of electromagnetic waves **relative** to an observer which we assume to be **stationary**
  - Therefore $v_o = 0$, hence $\Delta v = v_s = v$
  - Where v is the velocity at which the source of the electromagnetic waves is moving from the observer
- Sometimes the reference frequency is written as f_0 and the frequency wavelength as λ_0
- Hence, the Doppler shift equation can therefore also be written as:

$$\frac{\Delta f}{f} = \frac{\lambda' - \lambda}{\lambda} = \frac{\Delta \lambda}{\lambda} \approx \frac{\Delta v}{c}$$

Spectral Lines

- Doppler shift can easily be seen in atomic spectral lines from planets and stars



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Spectral lines showing red shift

- Each line represents an element making up the composition of the galaxy
- The lines are identical in that predicted in lab and the light measured from the distant galaxy
- Since the lines all move to the left (the red end of the spectrum) this means the galaxy is travelling **away** from Earth

YOUR NOTES

YOUR NOTES

Worked Example

A stationary source of light is found to have a spectral line of wavelength 438 nm. The same line from a distant star that is moving away from us has a wavelength of 608 nm.

Calculate the speed at which the star is travelling away from Earth.

Answer:

Step 1: List the known quantities

- Unshifted wavelength, $\lambda = 438$ nm
- Shifted wavelength, $\lambda' = 608$ nm
- Change in wavelength, $\Delta\lambda = (608 - 438)$ nm $= 170$ nm
- Speed of light, $c = 3.0 \times 10^8$ m s^{-1}

Step 2: Write down the Doppler equation and rearrange for velocity v

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

$$v = \frac{c\Delta\lambda}{\lambda}$$

Step 3: Substitute values to calculate v

$$v = \frac{(3.0 \times 10^8) \times 170}{438} = 1.16 \times 10^8 \text{ m s}^{-1}$$

YOUR NOTES

Worked Example

The stars in a distant galaxy can be seen to orbit about a galactic centre. The galaxy can be observed 'edge-on' from the Earth.

Light emitted from a star on the left-hand side of the galaxy is measured to have a wavelength of 656.44 nm. The same spectral line from a star on the right-hand side is measured to have a wavelength of 656.12 nm.

The wavelength of the same spectral line measured on Earth is 656.28 nm.

(a)
State and explain which side of the galaxy is moving towards the Earth.
(b)
Calculate the rotational speed of the galaxy.

Answer:

(a)

- The light from the right-hand side (656.12 nm) is observed to be at a shorter wavelength than the reference line (656.28 nm)
- Therefore, the right-hand side has been blue-shifted and must be moving towards the Earth

(b)

Step 1: List the known quantities

- Observed wavelength on LHS, $\lambda_{LHS} = 656.44$ nm
- Observed wavelength on RHS, $\lambda_{RHS} = 656.12$ nm
- Reference wavelength, $\lambda = 656.28$ nm
- Speed of light, $c = 3.0 \times 10^8$ m s^{-1}

Step 2: Calculate the average change in wavelength

$$\Delta\lambda = \frac{\lambda_{LHS} - \lambda_{RHS}}{2} = \frac{656.44 - 656.12}{2}$$

$$\Delta\lambda = 0.32 \text{ nm}$$

Step 3: Write down the Doppler equation and rearrange for velocity v

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

$$v = \frac{c\Delta\lambda}{\lambda}$$

Step 4: Substitute values into the velocity equation

$$v = \frac{(3 \times 10^8) \times 0.32}{656.28}$$

Rotational speed: $v = 1.46 \times 10^5 \text{ m s}^{-1} = 146 \text{ km s}^{-1}$

Exam Tip

You need to know that in the visible light spectrum **red light** has the **longest wavelength** and the **smallest frequency** compared to **blue light** which has a **shorter wavelength** and **higher frequency**.

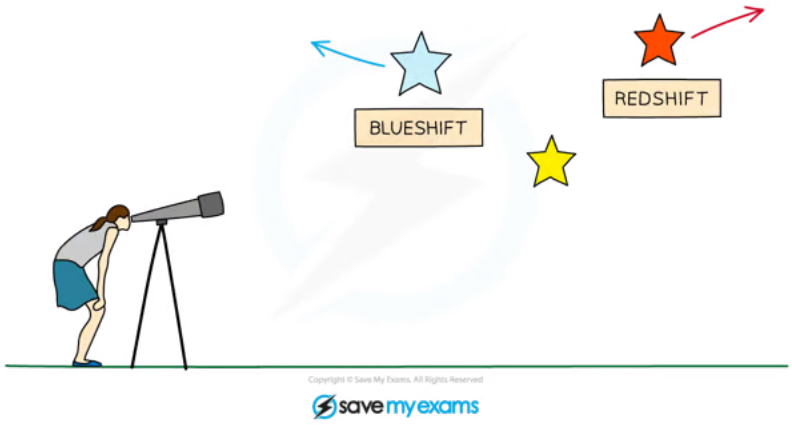
The second worked example didn't change the wavelengths from nm into m, since it doesn't matter in the equation as the units will cancel out.

YOUR NOTES

YOUR NOTES

Galactic Redshift

- In space, the Doppler effect of **light** can be observed when spectra of distant stars and galaxies are observed, this is known as:
  - **Redshift** if the object is moving **away** from the Earth
  - **Blueshift** if the object is moving **towards** the Earth

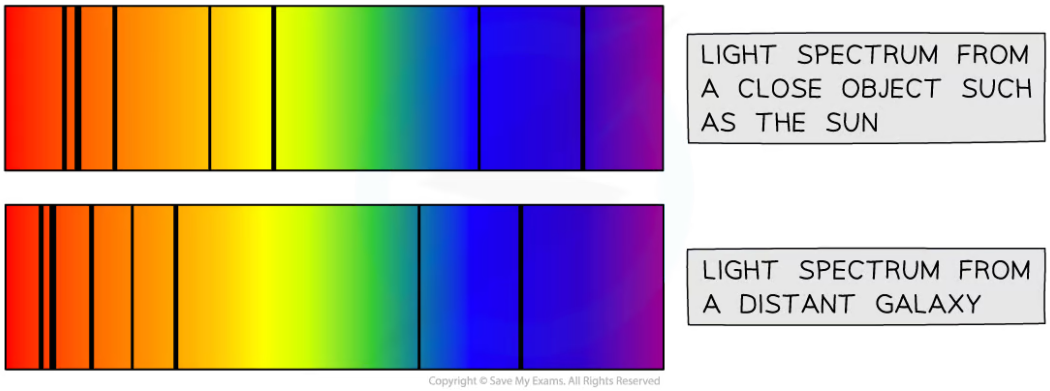


Stars and galaxies can be red or blueshifted to an observer on Earth

- Redshift is defined as:

  The fractional increase in wavelength (or decrease in frequency) due to the source and observer receding from each other

- Redshift can be observed by comparing the light spectrum produced from a close object, such as our Sun, with that of a distant galaxy
  - The light from the distant galaxy is shifted towards the **red** end of the spectrum (compared to the Sun's spectra)
  - This provides evidence that the universe is **expanding**

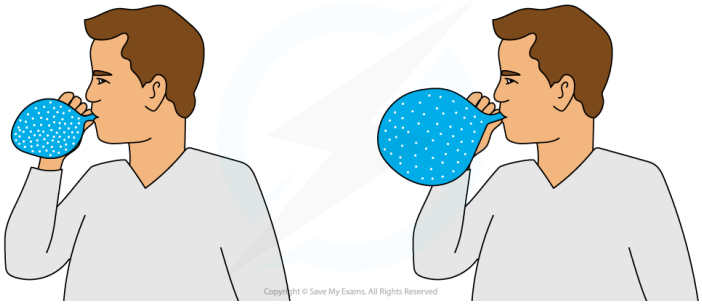


Comparing the light spectrum produced by the Sun with light from a distant galaxy

An Expanding Universe

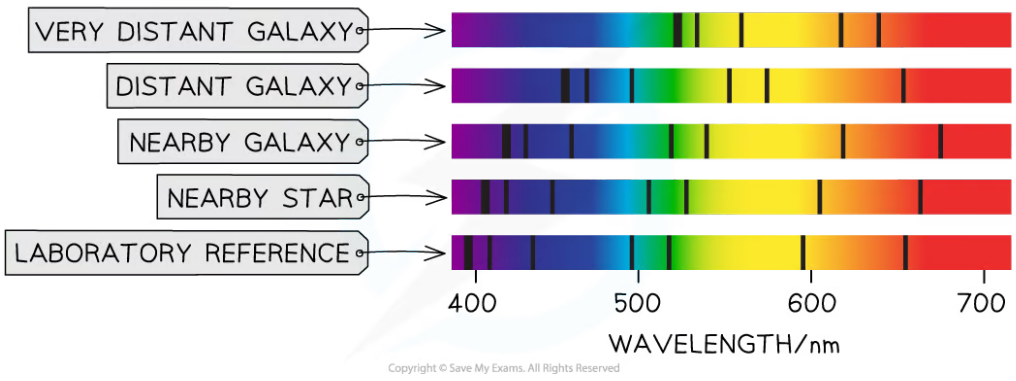
YOUR NOTES

- After the discovery of Doppler redshift, astronomers began to realize that almost all the galaxies in the universe are receding
- This led to the idea that the space between the Earth and the galaxies must be **expanding**
- This expansion stretches out the light waves as they travel through space, shifting them towards the red end of the spectrum



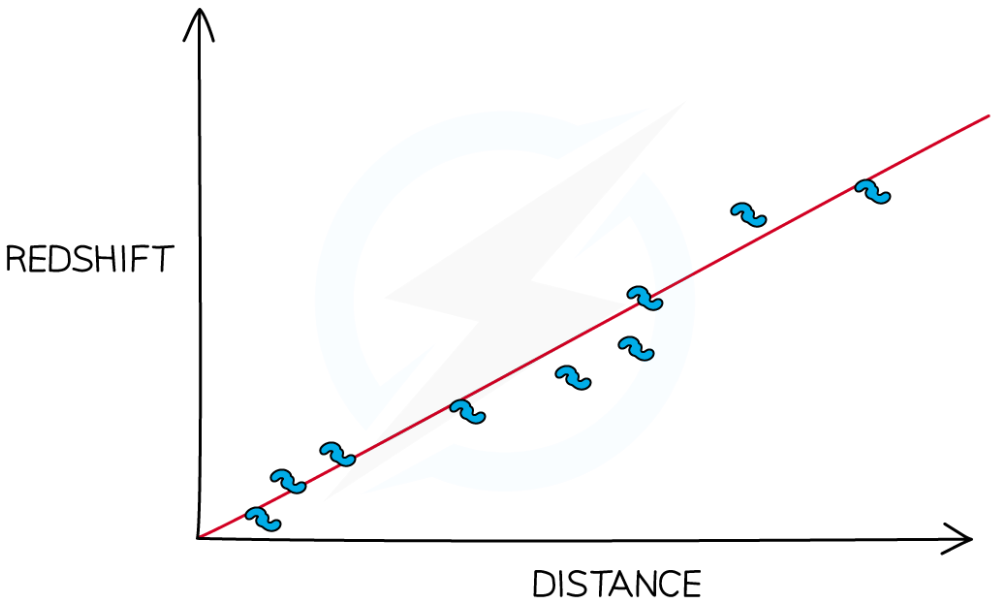
- The expansion of the universe can be compared to dots on an inflating balloon
  - As the balloon is inflated, the dots all move away **from each other**
  - In the same way, as the rubber stretches when the balloon is inflated, space itself is **stretching out between galaxies**
  - Just like the dots, the galaxies move away from each other, however, **they themselves** do not move

- Another observation from looking at the light spectra produced by distant galaxies is that the **greater** the **distance** to the galaxy, the **greater** the **redshift**
  - This means that the greater the degree of redshift, the **faster** the galaxy is moving away from Earth



The further a galaxy is from Earth, the greater its redshift tends to be

YOUR NOTES



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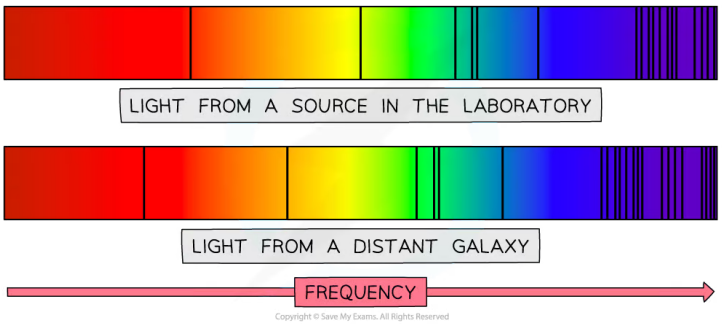
The furthest galaxies appear to be redshifted the most and are receding the fastest

YOUR NOTES

Worked Example

The spectra below show dark absorption lines against a continuous visible spectrum.



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A particle line in the spectrum of light from a source in the laboratory has a frequency of 4.570×10^{14} Hz. The same line in the spectrum of light from a distant galaxy has a frequency of 4.547×10^{14} Hz.

Calculate the speed of the distant galaxy in relation to the Earth. Determine whether it is moving towards or away from the Earth.

Answer:

Step 1: Write down the known quantities

- Received frequency, $f' = 4.547 \times 10^{14}$ Hz
- Original frequency, $f = 4.570 \times 10^{14}$ Hz
- Shift in frequency, $\Delta f = (4.547 - 4.570) \times 10^{14} = -2.3 \times 10^{12}$ Hz
- Speed of light, $c = 3.0 \times 10^{8}$ m s^{-1}

Step 2: Write down the Doppler redshift equation

$$\frac{\Delta f}{f} = \frac{v}{c}$$

Step 3: Rearrange for speed *v*, and calculate

$$v = \frac{c\Delta f}{f} = \frac{(3.0 \times 10^{8}) \times (-2.3 \times 10^{12})}{4.570 \times 10^{14}} = -1.5 \times 10^{6} \text{ m s}^{-1}$$

Step 4: Write a concluding sentence

- The velocity is **negative**, so the source is moving **away** from the Earth

OR

- The observed frequency is **less** than the emitted frequency (the light from a laboratory source), therefore, the source is **receding**, or moving away, from the Earth at $\mathbf{1.5 \times 10^6\ m\ s^{-1}}$

YOUR NOTES

Exam Tip

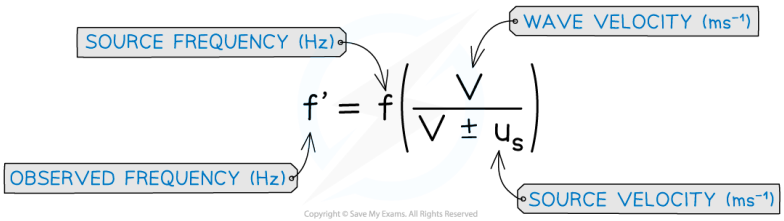
Keep travel of the minus signs in your calculation, as this gives you information about whether the object is moving away or towards the observer.

The speed of light is given in your data booklet, you will not need to memorise this value.

YOUR NOTES

The Doppler Effect of Sound

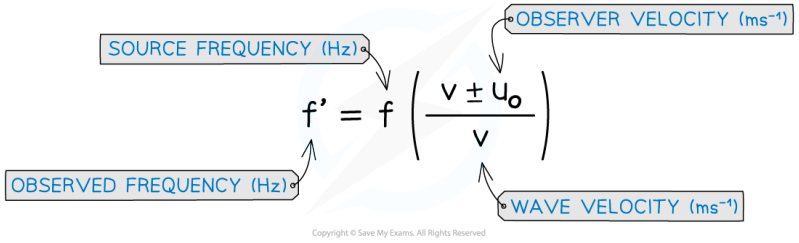
- When a source of sound waves moves relative to a stationary observer, the observed frequency can be calculated using the equation below:



Doppler shift equation for a moving source

- The wave velocity for sound waves is 340 ms^{-1}
- The ± depends on whether the source is moving towards or away from the observer
  - If the source is moving **towards** the observer, the denominator is $v - u_s$
  - If the source is moving **away** from the observer, the denominator is $v + u_s$

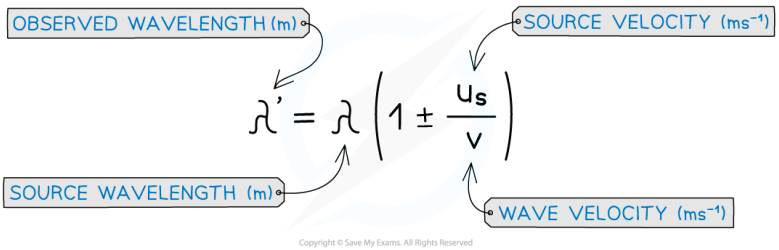
- When a source of sound waves remains stationary, but the observer is moving relative to the source, the observed frequency can be calculated using the equation below:



Doppler shift equation for a moving observer

- The ± depends on whether the observer is moving towards or away from the source
  - If the observer is moving **towards** the source, the numerator is $v + u_o$
  - If the observer is moving **away** from the source, the numerator is $v - u_o$

- These equations can also be written in terms of wavelength
  - For example, the equation for a moving source is shown below:



Doppler shift equation for a moving source in terms of wavelength

- The ± depends on whether the source is moving **towards or away** from the observer
  - If the source is moving **towards**, the term in the brackets is $1 - \frac{u_s}{v}$
  - If the source is moving **away**, the term in the brackets is $1 + \frac{u_s}{v}$

Worked Example

A police car siren emits a sound wave with a frequency of 450 Hz. The car is travelling away from an observer at a speed of 45 m s^{-1}.

The speed of sound is 340 m s^{-1}.

What frequency of sound does the observer hear?

A. 519 Hz **B.** 483 Hz **C.** 397 Hz **D.** 358 Hz

ANSWER: C

STEP 1

DOPPLER SHIFT EQUATION

$$f_o = f_s \left(\frac{V}{V \pm V_s} \right)$$

STEP 2

SUBSTITUTE VALUES INTO THE EQUATION

f_s = 450 Hz
V = SPEED OF SOUND = 340 ms^{-1}
V_s = VELOCITY OF THE POLICE CAR (SOURCE) = 45 ms^{-1}

THE SOURCE IS MOVING AWAY FROM THE OBSERVER, SO WE USE $V + V_s$

$$f_o = 450 \left(\frac{340}{340 + 45} \right) = 397 \text{ Hz} \quad (3 \text{ s.f.})$$

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YOUR NOTES

Worked Example

A bank robbery has occurred and the alarm is sounding at a frequency of 3 kHz. The robber jumps into a car which accelerates and reaches a constant speed.

As he drives away at a constant speed, he hears the frequency of the alarm decrease to 2.85 kHz.

Determine the speed at which the robber must be driving away from the bank.

Speed of sound = 340 m s^{-1}

Answer:

Step 1: List the known quantities

- Source frequency, f = 3 kHz
- Observed frequency, f' = 2.85 kHz
- Speed of sound, v = 340 m s^{-1}

Step 2: Write down the Doppler shift equation

- The observer is moving away from a stationary source of sound, so the equation to use is

$$f' = f\left(\frac{v - u_o}{v}\right)$$

Step 3: Rearrange to find the desired quantity

$$\frac{f'}{f} = \left(\frac{v - u_o}{v}\right) \quad \Rightarrow \quad \frac{vf'}{f} = v - u_o$$

$$\frac{vf'}{f} + u_o = v \quad \Rightarrow \quad u_o = v - \frac{vf'}{f}$$

$$u_o = v\left(1 - \frac{f'}{f}\right)$$

Step 4: Substitute the values into the equation

$$u_o = 340 \times \left(1 - \frac{2.85}{3}\right) = 17 \text{ m s}^{-1}$$

- The robber must be driving away at a constant speed of **17 m s^{-1}** based on the change in frequency heard

Exam Tip

Pay careful attention as to whether you need to + or – sign in the relevant equation! If it helps, label the 'observer' and 'source' on in your question on the exam paper.

YOUR NOTES

IB Physics HL

YOUR NOTES

Gravitational Fields

CONTENTS

Newton's Law of Gravitation
Gravitational Field Strength
Gravitational Field Lines
Gravitational Potential (HL)
Gravitational Potential Energy in a Non-Uniform Field (HL)
Gravitational Potential Energy Equation (HL)
Gravitational Potential Gradient (HL)
Gravitational Equipotential Surfaces (HL)
Kepler's Laws of Planetary Motion
Escape Speed (HL)
Orbital Motion, Speed & Energy (HL)
Effects of Drag on Orbital Motion (HL)

YOUR NOTES

Newton's Law of Gravitation

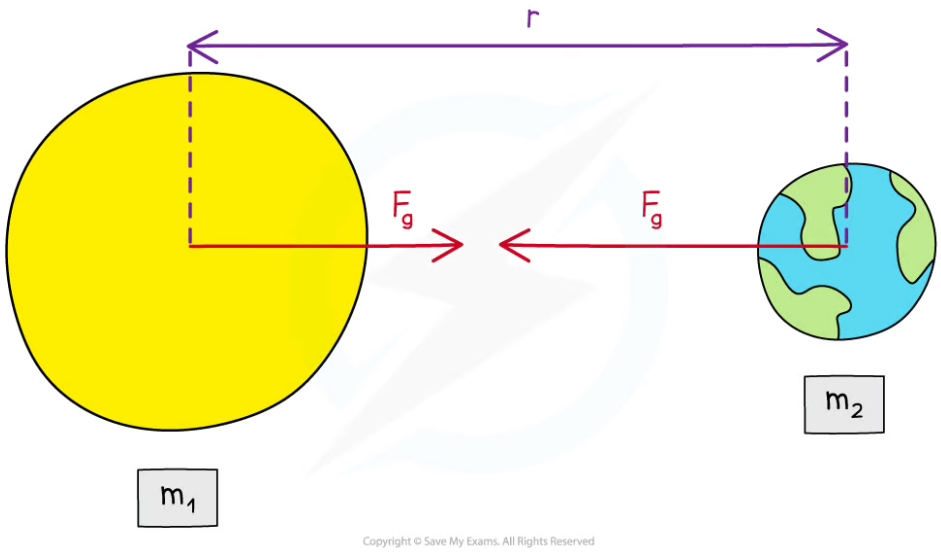
- The gravitational force between two bodies outside a uniform field, e.g. between the Earth and the Sun, is defined by Newton's Law of Gravitation
- Newton's Law of Gravitation states that:

> **The gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square of their separation**

- All planets and stars are assumed to be point masses
- In equation form, this can be written as:

$$F = \frac{Gm_1 m_2}{r^2}$$

- Where:
  - F = gravitational force between two masses (N)
  - G = Newton's Gravitational Constant
  - m_1 and m_2 = mass of body 1 and mass of body 2 (kg)
  - r = distance between the centre of the two masses (m)



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The gravitational force between two masses outside a uniform field is defined by Newton's Law of Gravitation

- Although planets are not point masses, their separation is much larger than their radius
  - Therefore, Newton's law of gravitation applies to planets orbiting the Sun
- The $F \propto \frac{1}{r^2}$ relation is called the inverse square law
- This means that when a mass is twice as far away from another, its force due to gravity reduces by $(½)^2 = ¼$

YOUR NOTES

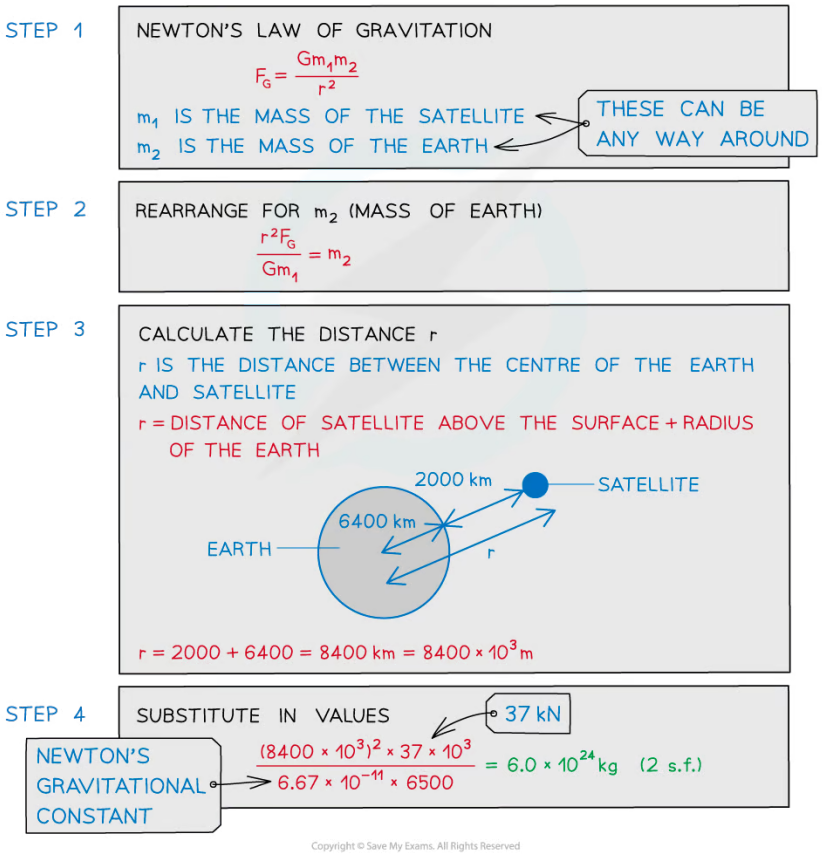
Worked Example

A satellite of mass 6500 kg orbits at 2000 km above the Earth's surface. The gravitational force between the Earth and the satellite is 37 kN.

Calculate the mass of the Earth.

Radius of the Earth = 6400 km

Answer:



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Exam Tip

A common mistake in exams is to forget to **add together** the distance from the surface of the planet and its radius to obtain the value of *r*. The distance *r* is measured from the **centre** of the mass, which is from the **centre** of the planet.

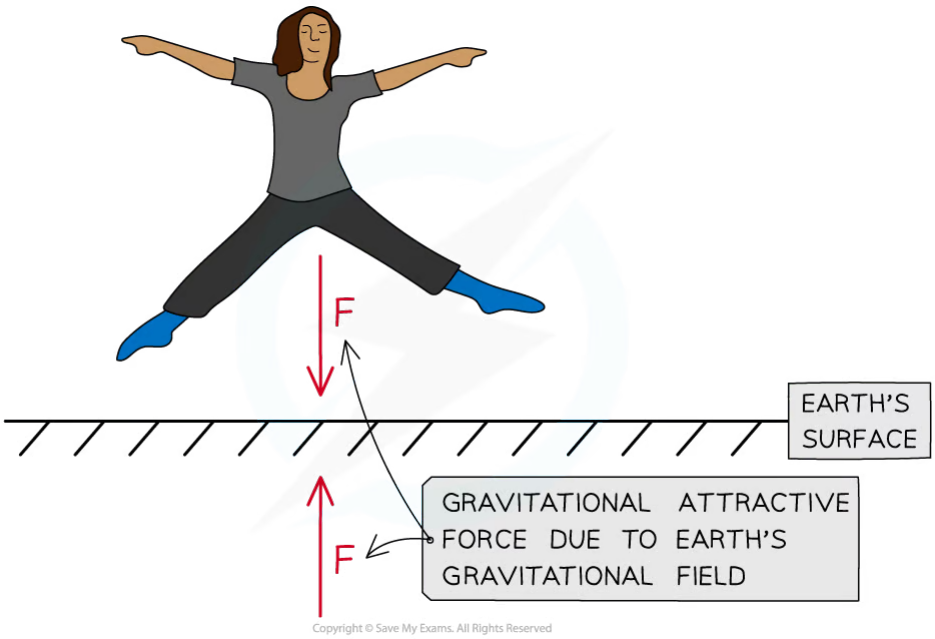
Make sure to **square** the separation *r* in the equation!

Gravitational Field Strength

- There is a universal force of attraction between all matter with **mass**
  - This force is known as the 'force due to gravity' or the **weight**
- The Earth's gravitational field is responsible for the weight of all objects on Earth
- A gravitational field is defined as:

  A region of space where a test mass experiences a force due to the gravitational attraction of another mass

- The direction of the gravitational field is always towards the centre of the mass causing the field
  - Gravitational forces are **always** attractive
- Gravity has an infinite range, meaning it affects **all** objects in the universe
  - There is a **greater** gravitational force around objects with a **large mass** (such as planets)
  - There is a **smaller** gravitational force around objects with a **small mass** (almost negligible for atoms)



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The Earth's gravitational field produces an attractive force. The force of gravity is always attractive

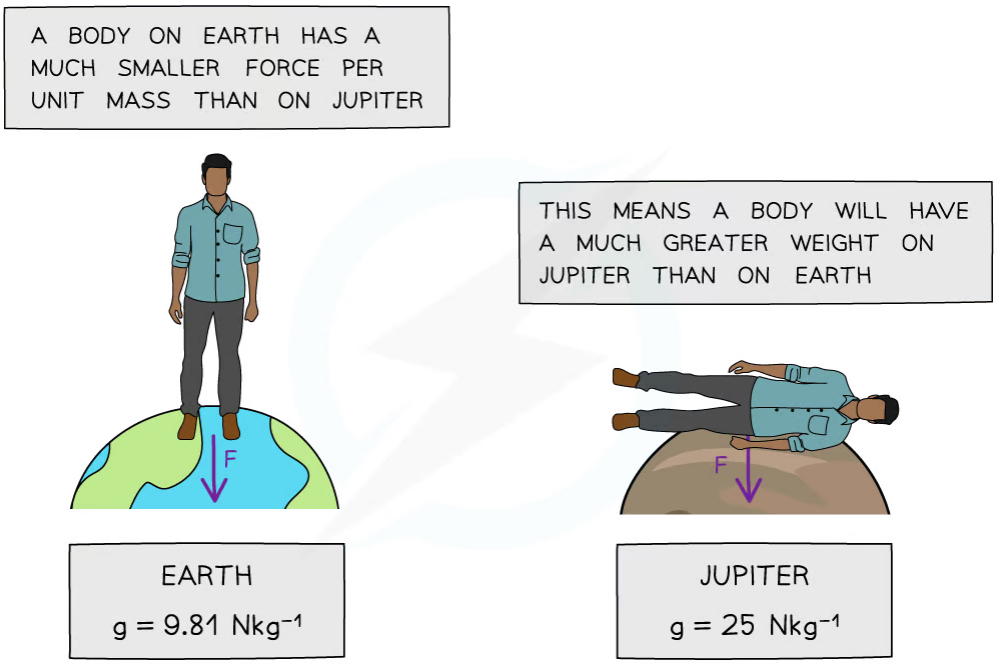
- The gravitational field strength at a point is defined as:

  The force per unit mass experienced by a test mass at that point

- This can be written in equation form as:

$$g = \frac{F}{m}$$

- Where:
  - g = gravitational field strength (N kg^{-1})
  - F = force due to gravity, or weight (N)
  - m = mass of test mass in the field (kg)
- This equation shows that:
  - On planets with a large value of g, the gravitational force per unit mass is **greater** than on planets with a smaller value of g
- An object's mass remains the **same** at all points in space
  - However, on planets such as Jupiter, the **weight** of an object will be greater than on a less massive planet, such as Earth
  - This means the gravitational force would be so high that humans, for example, would not be unable to fully stand up



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A person's weight on Jupiter would be so large that a human would be unable to fully stand up

- Factors that affect the gravitational field strength at the surface of a planet are:
  - The **radius** r (or diameter) of the planet
  - The **mass** M (or density) of the planet
- This can be shown by equating the equation $F = mg$ with Newton's law of gravitation:

$$F = \frac{GMm}{r^2}$$

YOUR NOTES

- Substituting the force F with the gravitational force mg leads to:

$$mg = \frac{GMm}{r^2}$$

- Cancelling the mass of the test mass m leads to the equation:

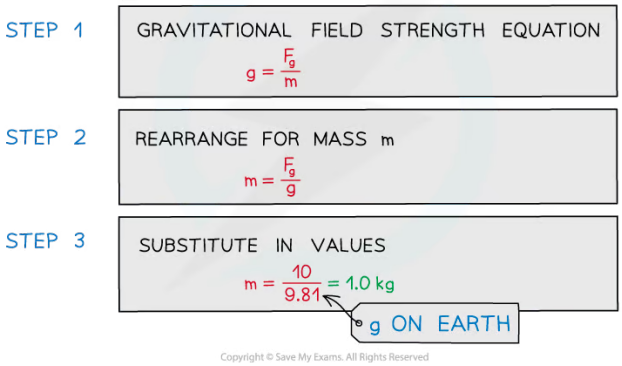
$$g = \frac{GM}{r^2}$$

- Where:
  - G = Newton's Gravitational Constant
  - M = *mass of the body causing the field (kg)*
  - r = distance from the mass where you are calculating the field strength (m)
- This equation shows that:
  - The gravitational field strength g depends only on the mass of the body M causing the field
  - Hence, objects with any mass m in that field will experience the **same gravitational field strength**
  - The gravitational field strength g is **inversely proportional** to the **square** of the radial distance, r^2

Worked Example

Calculate the mass of an object with weight 10 N on Earth.

Answer:



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Worked Example

The mean density of the Moon is $\frac{3}{5}$ times the mean density of the Earth. The gravitational field strength on the Moon is $\frac{1}{6}$ the gravitational field strength on Earth.

Determine the ratio of the Moon's radius r_M to the Earth's radius r_E.

Answer:

Step 1: Write down the known quantities

- g_M = gravitational field strength on the Moon, ρ_M = mean density of the Moon
- g_E = gravitational field strength on the Earth, ρ_E = mean density of the Earth

$$\rho_M = \frac{3}{5}\rho_E$$

$$g_M = \frac{1}{6}g_E$$

Step 2: Write down the equations for the gravitational field strength, volume and density

Gravitational field strength: $g = \frac{GM}{r^2}$

Volume of a sphere: $V = \frac{4}{3}\pi r^3 \quad \Rightarrow \quad V \propto r^3$

Density: $\rho = \frac{M}{V} \quad \Rightarrow \quad M = \rho V = \frac{4}{3}\pi\rho r^3 \quad \Rightarrow \quad M \propto \rho r^3$

Step 3: Substitute the relationship between M and r into the equation for g

$$g \propto \rho\frac{(r^3)}{r^2} \quad \Rightarrow \quad g \propto \rho r$$

Step 4: Find the ratio of the gravitational field strength

$$g_M \propto \rho_M r_M$$

$$g_E \propto \rho_E r_E$$

$$g_M = \frac{1}{6}g_E \quad \Rightarrow \quad \rho_M r_M = \frac{1}{6}\rho_E r_E$$

Step 5: Substitute the ratio of the densities into the equation

$$\left(\frac{3}{5}\rho_E\right) r_M = \frac{1}{6}\rho_E r_E$$

$$\frac{3}{5} r_M = \frac{1}{6} r_E$$

Step 6: Calculate the ratio of the radii

$$\frac{r_M}{r_E} = \frac{1}{6} \div \frac{3}{5} = \frac{5}{18} = 0.28$$

Exam Tip

There is a big difference between g and G (sometimes referred to as 'little g' and 'big G' respectively), g is the gravitational field strength and G is Newton's gravitational constant. Make sure not to use these interchangeably!

Remember the equation $density = \frac{mass}{volume}$, which may come in handy with some calculations. The equation for the volume of common shapes is in your data booklet.

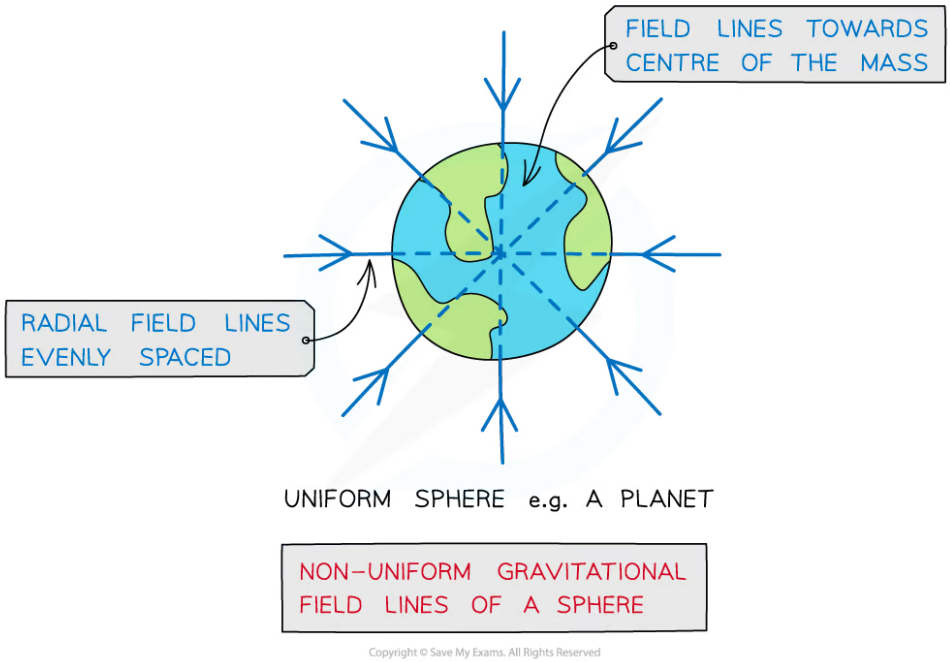
YOUR NOTES

Point Mass Approximation

- For a point outside a uniform sphere, the mass of the sphere may be considered to be a **point mass** at its centre
  - A uniform sphere is one where its mass is **distributed evenly**
- The gravitational field lines around a uniform sphere are therefore **identical to those around a point mass**
- An object can be regarded as a point mass when:

> **A body covers a very large distance compared to its size, so, to study its motion, its size or dimensions can be neglected**

- An example of this is field lines around planets



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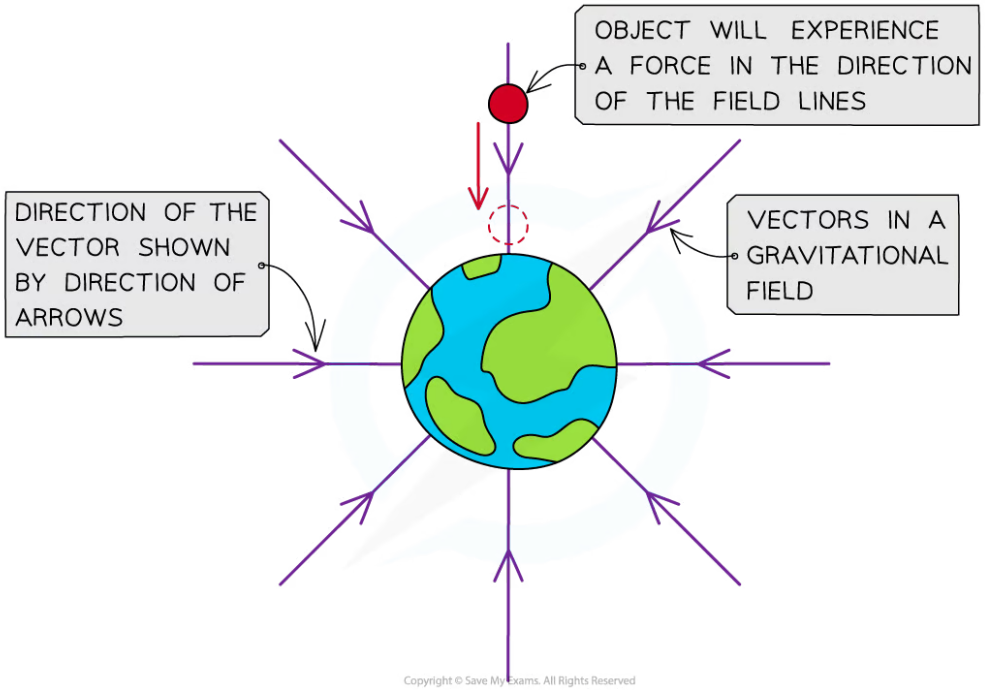
Gravitational field lines around a uniform sphere are identical to those on a point mass

- Radial fields are considered **non-uniform** fields
  - So, the gravitational field strength *g* is different depending on how far an object is from the centre of mass of the sphere
- Newton's universal law of gravitation is extended to spherical masses of uniform density by assuming that their mass is concentrated at their centre i.e point masses

Representing Gravitational Fields

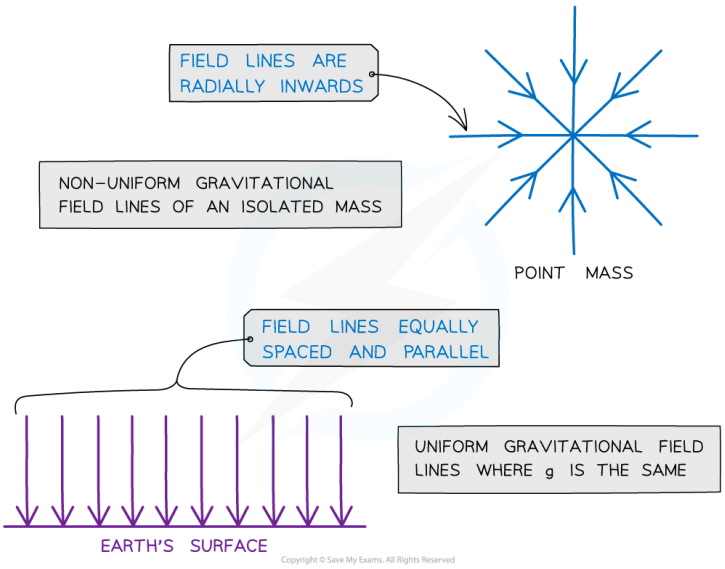
YOUR NOTES

- Gravitational fields represent the **action** of gravitational forces between masses, the direction of these forces can be shown using vectors
  - The direction of the **vector** shows the direction of the **gravitational force** that would be exerted on a **mass** if it was placed at that position in the field
  - These vectors are known as **field lines** (or 'lines of force')
- The direction of a gravitational field is represented by gravitational field lines
  - Therefore, gravitational field lines also show the direction of **acceleration** of a mass placed in the field
- Gravitational field lines are always directed toward the centre of mass of a body
  - This is because gravitational forces are **attractive only** (they are never repulsive)
  - Therefore, masses **always** attract each other via the gravitational force
- The gravitational field around a point mass will be **radial** in shape and the field lines will always point towards the centre of mass



The direction of the gravitational field is shown by the vector field lines

- The gravitational field lines around a point mass are **radially inwards**
- The gravitational field lines of a uniform field, where the field strength is the same at all points, is represented by **equally spaced parallel lines**
  - For example, the fields lines on the Earth's surface



Gravitational field lines for a point mass and a uniform gravitational field

- Radial fields are considered **non-uniform fields**
  - The gravitational field strength *g* is different depending on how far you are from the centre
- Parallel field lines on the Earth's surface are considered a **uniform field**
  - The gravitational field strength *g* is the same throughout

Exam Tip

Always label the arrows on the field lines! Gravitational forces are attractive only. Remember:

- For a radial field: it is towards the centre of the sphere or point charge
- For a uniform field: towards the surface of the object e.g. Earth

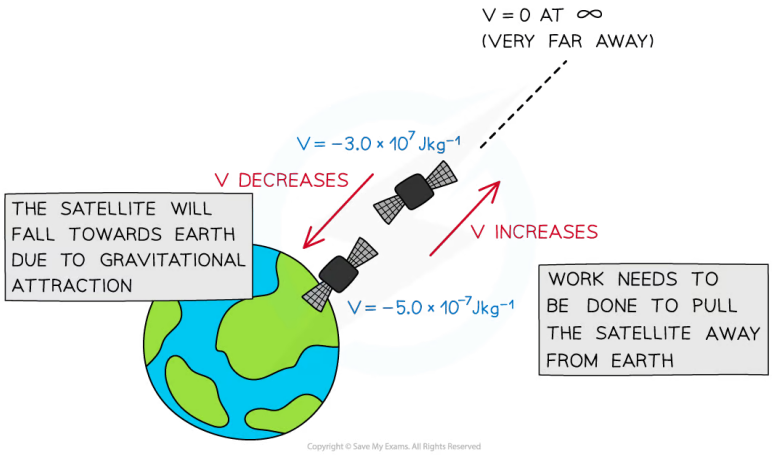
YOUR NOTES

Gravitational Potential

- The gravitational potential *V* at a point can, therefore, be defined as:

  The work done per unit mass in bringing a test mass from infinity to a defined point

- Gravitational potential is measured in J kg^{-1}
- It is always has a **negative** value because:
  - It is defined as having a value of **zero** at **infinity**
  - Since the gravitational force is **attractive**, work must be done **on** a mass to reach infinity
- On the surface of a mass (such as a planet), gravitational potential has a negative value
  - The value becomes less negative, i.e. it increases, with distance from that mass
- Work has to be done **against** the gravitational pull of the planet to take a unit mass away from the planet
- The gravitational potential at a point depends on:
  - The **mass** of the object
  - The **distance** from the centre of mass of the object to the point



Gravitational potential decreases as the satellite moves closer to the Earth

Calculating Gravitational Potential

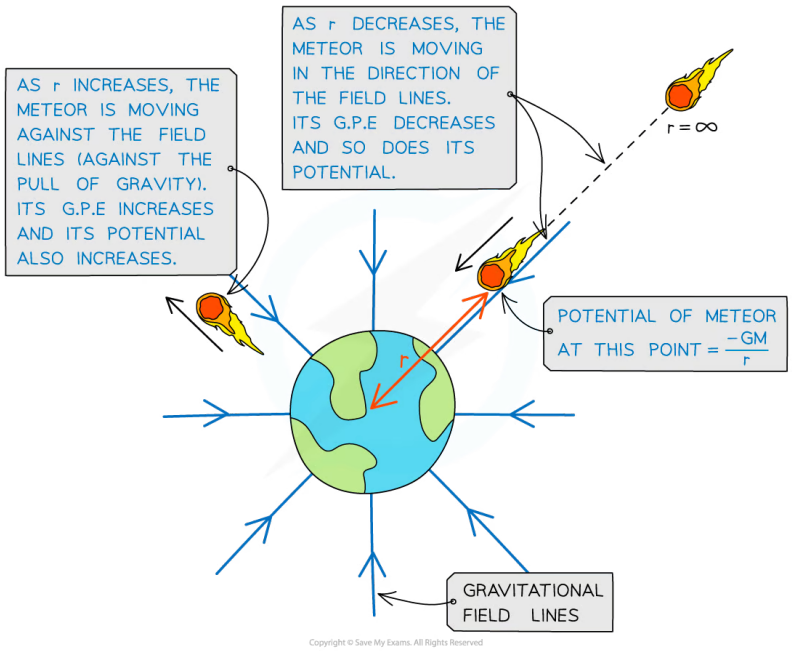
- The equation for gravitational potential *V* is defined by the mass *M* and distance *r*:

$$V_g = -\frac{GM}{r}$$

- Where:
  - V_g = gravitational potential (J kg^{-1})
  - *G* = Newton's gravitational constant
  - *M* = mass of the body producing the gravitational field (kg)
  - *r* = distance from the centre of the mass to the point mass (m)

YOUR NOTES

- The gravitational potential always is negative near an isolated mass, such as a planet, because:
  - The potential when *r* is at infinity (∞) is defined as zero
  - Work must be done to move a mass away from a planet (V becomes less negative)
- It is also a scalar quantity, unlike the gravitational field strength which is a vector quantity
- Gravitational forces are always **attractive**, this means as *r* decreases, positive work is done by the mass when moving from infinity to that point
  - When a mass is closer to a planet, its gravitational potential becomes smaller (more negative)
  - As a mass moves away from a planet, its gravitational potential becomes larger (less negative) until it reaches 0 at infinity
- This means when the distance *r* becomes very large, the gravitational force tends rapidly towards zero the further away the point is from a planet



Gravitational potential increases and decreases depending on whether the object is travelling towards or against the field lines from infinity

YOUR NOTES

Worked Example

A planet has a diameter of 7600 km and a mass of 3.5×10^{23} kg. A meteor of mass 6000 kg accelerates towards the planet from infinity.

Calculate the gravitational potential of the rock at a distance of 400 km above the planet's surface.

Answer:

- The gravitational potential at a point is

$$V_g = -\frac{GM}{r}$$

- Where *r* is the distance from the centre of the planet to the point i.e. the radius of the planet + the height above the planet's surface

$$r = \frac{7600}{2} + 400 = 4200\,\text{km}$$

- And *M* is the mass of the larger mass, i.e. the planet (not the meteor)

$$V_g = -\frac{(6.67 \times 10^{-11}) \times (3.5 \times 10^{23})}{4200 \times 10^3} = -5.6 \times 10^6\,\text{J kg}^{-1}$$

Exam Tip

Notice the red herring in the worked example. You do not need the mass *m* of the meteor, as *M* in the equation for gravitational potential is only the mass of the **object creating the gravitational field**. *m* will come into play with gravitational potential **energy**.

YOUR NOTES

Gravitational Potential Energy in a Non-Uniform Field

- In a radial field, gravitational potential energy (GPE) describes the energy an object possesses due to its **position** in a gravitational field
- The gravitational potential energy of a system is defined as:

  The work done to assemble the system from infinite separation of the components of the system

- Similarly, the gravitational potential energy of a point mass is defined as:

  The work done in bringing a mass from infinity to a point

Near the Earth's Surface

- The gravitational potential energy near the Earth's surface is equal to

$$E_p = mg\Delta h$$

- The GPE on the surface of the Earth is taken to be zero
  - This means work is done to **lift** the object
- This equation can **only** be used for objects that are **near the Earth's surface**
  - This is because, near Earth's surface, the gravitational field is approximated to be **uniform**
  - Far away from the Earth's surface, the gravitational field is **radial** because the Earth is a **sphere**

Exam Tip

You should be able to interpret areas under curves by thinking about what the **product** of the quantities on the axes would represent. Since, in this case, **force × distance = work done**, then it follows that the **area** under the curve represents the **change** in **energy** between two points. Specifically, this would be a change in **gravitational potential energy**.

The equation GPE = $mg\Delta h$ is very **rarely** used in this topic. This is only relevant for objects **on a planet's surface**.

The only difference between GPE and g is GPE = mg where m is the mass of the object in the gravitational field of mass M.

This equation is **not** given on your data booklet, but you must understand its significance

YOUR NOTES

Work Done on a Mass

- When a mass is moved against the force of gravity, work is required
  - This is because gravity is **attractive**, therefore, energy is needed to work against this attractive force
- The work done in moving a mass *m* is given by:

$$\Delta W = m\Delta V_g$$

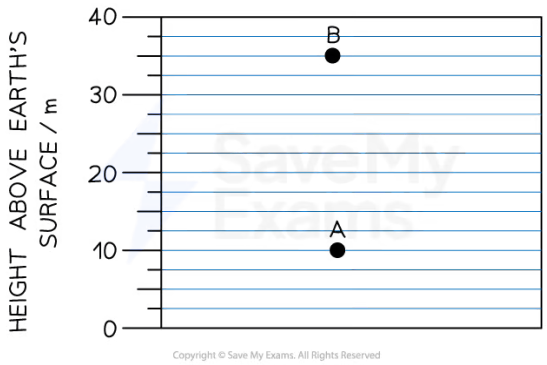
- Where:
  - ΔW = change in work done (J)
  - m = mass (kg)
  - ΔV_g = change in gravitational potential ($J\,kg^{-1}$)

YOUR NOTES

Worked Example

A particle of mass 50 g is moved vertically from point A to point B, as shown in the diagram.

Take the gravitational field strength to be 10 N kg^{-1}.



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Determine

(a)
the potential difference between A and B
(b)
the work done in moving the mass from A to B

Answer:

(a)

- The work done in moving a mass in a gravitational field is:

$$W = m\Delta V \text{ and } W = mg\Delta h \text{ (close to the Earth's surface)}$$

$$m\Delta V = mg\Delta h \quad \Rightarrow \quad \Delta V = g\Delta h$$

- Where the change in height is Δh = 45 – 10 = 35 m
- Therefore, the potential difference between A and B is:

$$\Delta V = 10 \times 35 = 350 \text{ J kg}^{-1}$$

(b)

- The work done in moving the mass from A to B is:

$$W = m\Delta V$$

$$W = (50 \times 10^{-3}) \times 350 = 17.5 \text{ J}$$

Gravitational Potential Energy

- In a radial field, gravitational potential energy (GPE) describes the energy an object possesses due to its **position** in a gravitational field
- The gravitational potential energy of a system is defined as:

  The work done to assemble the system from infinite separation of the components of the system

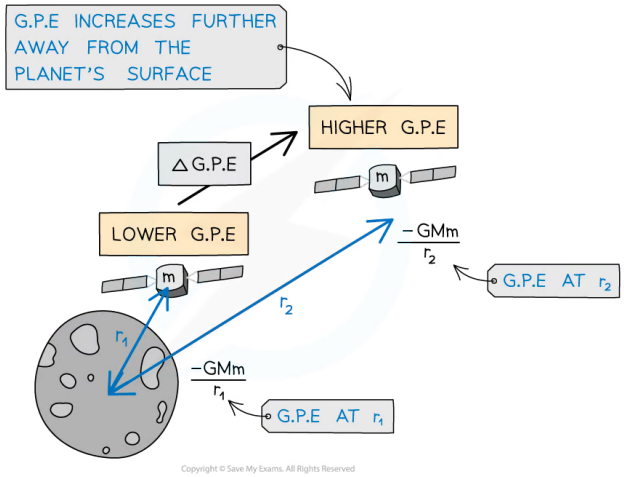
- Similarly, the gravitational potential energy of a point mass is defined as:

  The work done in bringing a mass from infinity to a point

- The equation for GPE of two point masses *m* and *M* at a distance *r* is:

$$E_p = -\frac{Gm_1m_2}{r}$$

- Where:
  - G = universal gravitational constant (N m^2 kg^{-2})
  - m_1 = larger mass producing the field (kg)
  - m_2 = mass moving within the field of M (kg)
  - r = distance between the centre of m and M (m)



Gravitational potential energy increases as a satellite leaves the surface of the Moon (of mass M)

- Recall that Newton's Law of Gravitation relates the magnitude of the force F between two masses M and m:

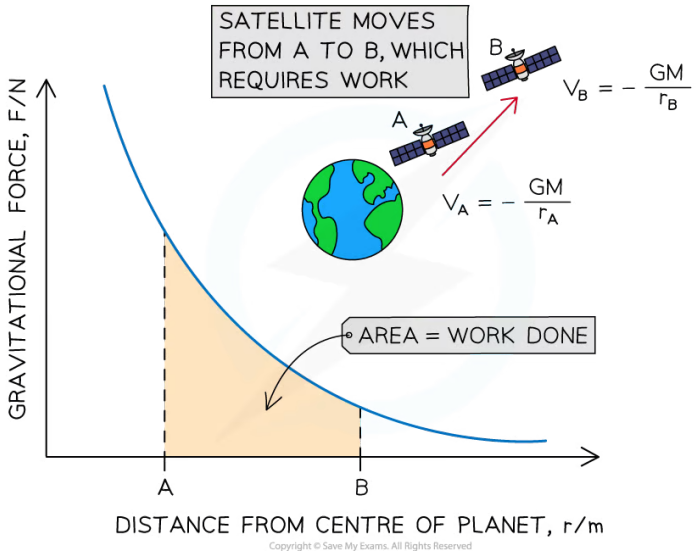
$$F = \frac{Gm_1m_2}{r^2}$$

YOUR NOTES

- Therefore, a **force-distance** graph would be a curve, because *F* is **inversely proportional** to r^2, or:

$$F \propto \frac{1}{r^2}$$

- The product of **force** and **distance** is equal to work done (or energy transferred)
- Therefore, the **area** under the **force-distance** graph for gravitational fields is equal to the **work done**
  - In the case of a mass *m* moving further away from a mass *M*, the potential **increases**
  - Since gravity is attractive, this requires **work to be done** on the mass *m*
  - The area between two points under the **force-distance** curve, therefore, gives the change in **gravitational potential energy** of mass *m*



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Work is done on the satellite of mass m to move it from A to B, because gravity is attractive. The area under the curve represents the magnitude of energy transferred

Change in Gravitational Potential Energy

- Two points at different distances from a mass will have **different** gravitational potentials
  - This is because the gravitational potential **increases** with distance from a mass
- Therefore, there will be a **gravitational potential difference** ΔV between the two points

$$\Delta V = V_f - V_i$$

- Where:
  - V_i = initial gravitational potential (J kg^{-1})
  - V_f = final gravitational potential (J kg^{-1})
- The change in work done against a gravitational field is equal to the change in **gravitational potential energy** (GPE)
  - When $V = 0$, then the GPE = 0

YOUR NOTES

- It is usually more useful to find the **change** in the GPE of a system
  - For example, a satellite lifted into space from the Earth's surface
- The change in GPE when a mass moves towards, or away from, another mass is given by:

$$\Delta E_p = -\frac{Gm_1 m_2}{r_2} - \left(-\frac{Gm_1 m_2}{r_1}\right)$$

$$\Delta E_p = Gm_1 m_2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Where:
  - m_1 = mass that is producing the gravitational field (e.g. a planet) (kg)
  - m_2 = mass that is moving in the gravitational field (e.g. a satellite) (kg)
  - r_1 = first distance of m from the centre of M (m)
  - r_2 = second distance of m from the centre of M (m)
- The change in potential ΔV_g is the same, without the mass of the object m_2:

$$\Delta V_g = -\frac{Gm_1}{r_2} - \left(-\frac{Gm_1}{r_1}\right)$$

$$\Delta V_g = Gm_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Work is done when an object in a planet's gravitational field moves **against** the gravitational field lines i.e. **away** from the planet

YOUR NOTES

Worked Example

A spacecraft of mass 300 kg leaves the surface of Mars up to an altitude of 700 km.

Calculate the work done by the spacecraft.

- Radius of Mars = 3400 km
- Mass of Mars, $m_1 = 6.40 \times 10^{23}$ kg

Answer:

- The change in GPE is equal to

$$\Delta E_p = Gm_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

- Where
  - r_1 = radius of Mars = 3400 km
  - r_2 = radius + altitude = 3400 + 700 = 4100 km

$$\Delta E_p = (6.67 \times 10^{-11}) \times (6.40 \times 10^{23}) \times 300 \times \left(\frac{1}{3400 \times 10^3} - \frac{1}{4100 \times 10^3} \right)$$

Work done by satellite: $\Delta E_p = 643.1 \times 10^6 = 640$ MJ (2 s.f.)

YOUR NOTES

Worked Example

A satellite of mass 1450 kg moves from an orbit of 980 km above the Earth's surface to a lower orbit of 480 km.

Calculate the change in gravitational potential energy of the satellite.

- Mass of the Earth = 5.97×10^{24} kg
- Radius of the Earth = 6.38×10^{6} m

Answer:

Step 1: Write down the known quantities

- Initial height above Earth's surface, $h_1 = 980$ km
- Final height above Earth's surface, $h_2 = 480$ km
- Mass of the satellite, $m_1 = 1450$ kg
- Mass of the Earth, $m_2 = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $R = 6.38 \times 10^{6}$ m

Step 2: Write down the equation for change in gravitational potential energy

$$\Delta E_p = G m_1 m_2 \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Step 3: Convert distances into standard units and include Earth radius

- Distance from centre of Earth to higher orbit:

$$r_1 = h_1 + R$$

$$r_1 = (980 \times 10^3) + (6.38 \times 10^6) = 7.36 \times 10^6 \text{ m}$$

- Distance from centre of Earth to lower orbit:

$$r_2 = h_2 + R$$

$$r_2 = (480 \times 10^3) + (6.38 \times 10^6) = 6.86 \times 10^6 \text{ m}$$

Step 4: Substitute values into the equation

$$\Delta E_p = (6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times 1450 \times \left(\frac{1}{7.36 \times 10^6} - \frac{1}{6.86 \times 10^6} \right)$$

Change in gravitational potential energy: $\Delta E_p = 5.72 \times 10^9$ J

Exam Tip

Make sure to not confuse the ΔE_p equation with $\Delta E = mg\Delta h$, they look similar but refer to quite **different situations.**

The more familiar equation is only relevant for an object lifted in a **uniform gravitational field**, meaning very close to the Earth's surface, where we can model the field as uniform.

The new equation for E_p does not include g. The gravitational field strength, which is different on different planets, does not remain constant as the distance from the surface increases. Gravitational field strength **falls away** according to the inverse square law.

The change in gravitational potential energy **is** the work done.

Gravitational Potential Gradient

- A gravitational field can be defined in terms of the variation of gravitational potential at different points in the field:

  The gravitational field at a particular point is equal to the negative gradient of a potential-distance graph at that point

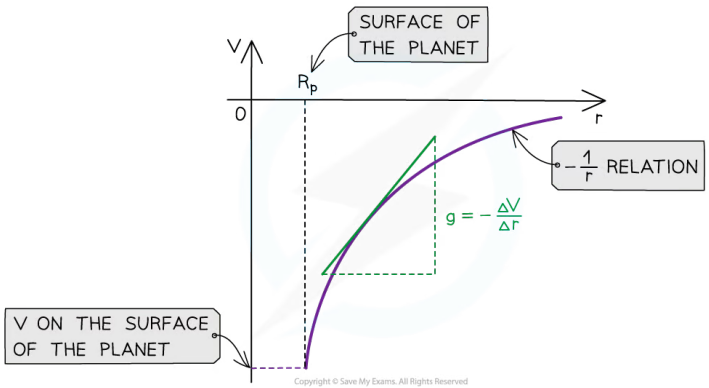
- The potential gradient is defined by the **equipotential lines**
  - These demonstrate the gravitational potential in a gravitational field and are always drawn **perpendicular** to the field lines
- The potential gradient in a gravitational field is defined as:

  The rate of change of gravitational potential with respect to displacement in the direction of the field

- Gravitational field strength, g and the gravitational potential, V can be graphically represented against the distance from the centre of a planet, r

$$g = -\frac{\Delta V_g}{\Delta r}$$

- Where:
  - g = gravitational field strength (N kg^{-1})
  - ΔV_g = change in gravitational potential (J kg^{-1})
  - Δr = distance from the centre of a point mass (m)
- The graph of V_g against r for a planet is:



The gravitational potential and distance graphs follow a -1/r relation

- **The key features of this graph are:**
  - The values for V_g are all negative
  - As r increases, V_g against r follows a $-\frac{1}{r}$ relation
  - The **gradient** of the graph at any particular point is the value of g at that point
  - The graph has a shallow increase as r increases

YOUR NOTES

- To calculate *g*, draw a tangent to the graph at that point and calculate the gradient of the tangent
- This is a graphical representation of the gravitational potential equation:

$$V_g = -\frac{GM}{r}$$

where *G* and *M* are constant

Worked Example

Determine the change in gravitational potential when travelling from 3 Earth radii (from Earth's centre) to the surface of the Earth.

Take the mass of the Earth to be 5.97×10^{24} kg and the radius of the Earth to be 6.38×10^{6} m.

Answer:

Step 1: List the known quantities

- Mass of the Earth, $M_E = 5.97 \times 10^{24}$ kg
- Radius of the Earth, $r_E = 6.38 \times 10^{6}$ m
- Initial distance, $r_1 = 3r_E = 3 \times (6.38 \times 10^{6})$ m $= 1.914 \times 10^{7}$ m
- Final distance, $r_2 = r_E = 6.38 \times 10^{6}$ m
- Gravitational constant, $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}

Step 2: Write down the equation for potential difference

$$\Delta V_g = -GM_E\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Step 3: Substitute the values into the equation

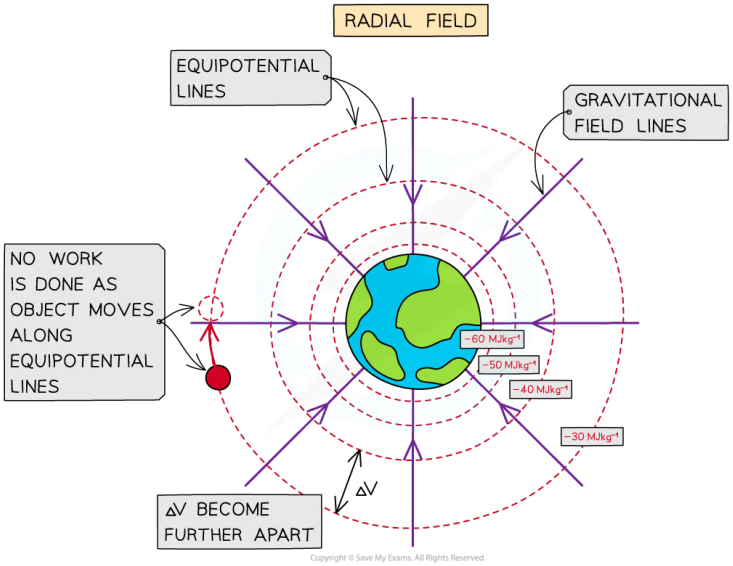
$$\Delta V_g = -(6.67 \times 10^{-11}) \times (5.97 \times 10^{24}) \times \left(\frac{1}{6.38 \times 10^{6}} - \frac{1}{1.914 \times 10^{7}}\right)$$

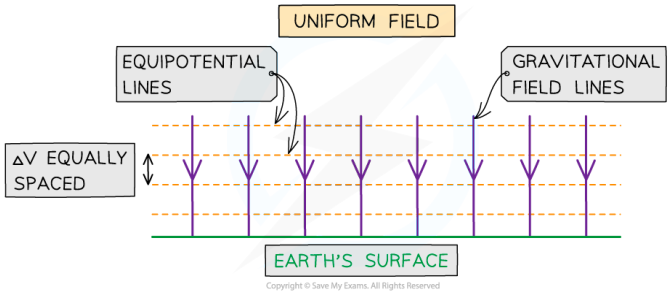
$$\Delta V_g = -4.16 \times 10^{7} \text{ J kg}^{-1}$$

YOUR NOTES

Gravitational Equipotential Surfaces

- Equipotential lines (when working in 2D) and surfaces (when working in 3D) join together points that have the same gravitational potential
- These are always:
  - **Perpendicular** to the gravitational field lines in both radial and uniform fields
  - Represented by **dotted** lines (unlike field lines, which are solid lines with arrows)
- In a radial field (eg. a planet), the equipotential lines:
  - Are concentric circles around the planet
  - Become further apart further away from the planet
- In a uniform field (eg. near the Earth's surface), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced
- Potential gradient is defined by the **equipotential lines**
- **No work is done** when moving along an equipotential line or surface, only **between** equipotential lines or surfaces
  - This means that an object travelling along an equipotential doesn't lose or gain energy and $\Delta V = 0$





Gravitational equipotential lines in a non-uniform and uniform gravitational field

- The distinction between radial and uniform fields is an important one
- In a **radial field** (eg. a point charge), the equipotential lines:
  - Are concentric circles around the charge
  - Become further apart further away from the charge
  - Remember: **radial** field is made up of lines which follow the **radius** of a circle
- In a **uniform field** (eg. between charged parallel plates), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced
  - Remember: **uniform** field is made up of lines which are a **uniform** distance apart

Exam Tip

Remember equipotential lines do **not** have arrows, since they have no particular direction and are not vectors.

Make sure to draw any straight lines with a ruler or a straight edge.

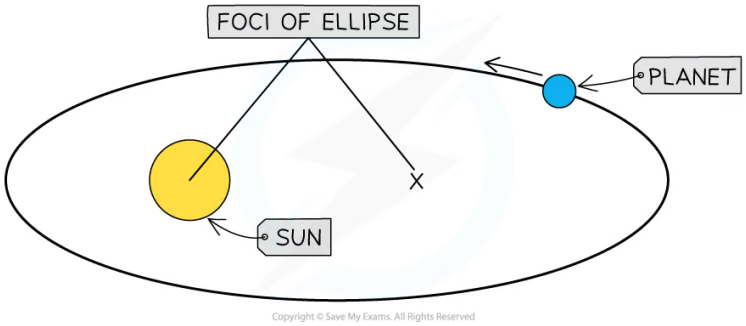
YOUR NOTES

Kepler's Laws of Planetary Motion

Kepler's First Law

- Kepler's First Law describes the **shape** of planetary orbits
- It states:

The orbit of a planet is an ellipse, with the Sun at one of the two foci



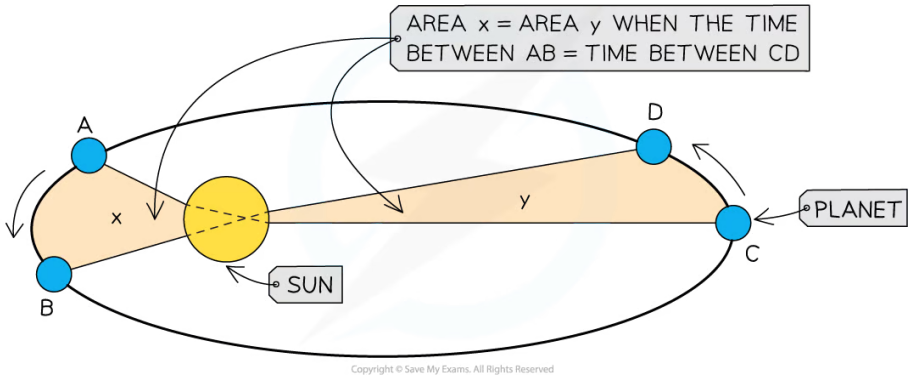
The orbit of all planets are elliptical, and with the Sun at one focus

- An ellipse is just a 'squashed' circle
  - Some planets, like Pluto, have highly elliptical orbits around the Sun
  - Other planets, like Earth, have near circular orbits around the Sun

Kepler's Second Law

- Kepler's Second Law describes the **motion** of all planets around the Sun
- It states:

A line segment joining the Sun to a planet sweeps out equal areas in equal time intervals



- The consequence of Kepler's Second Law is that planets move **faster** nearer the Sun and **slower** further away from it

Kepler's Third Law

YOUR NOTES

- Kepler's Third Law states

> **For planets or satellites in a circular orbit about the same central body, the square of the time period is proportional to the cube of the radius of the orbit**

- This law describes the relationship between the **time** of an orbit and its **radius**

$$T^2 \propto r^3$$

- Where:
  - T = orbital time period (s)
  - r = mean orbital radius (m)

Time Period & Orbital Radius Relation

- Since a planet or a satellite is travelling in circular motion when in order, its orbital time period T to travel the circumference of the orbit $2\pi r$, the linear speed v is:

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation, speed = distance / time and first introduced in the circular motion topic
- Substituting the value of the linear speed v from equating the gravitational and centripetal force into the above equation gives:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

- Squaring out the brackets and rearranging for T^2 gives the equation relating the time period T and orbital radius r:

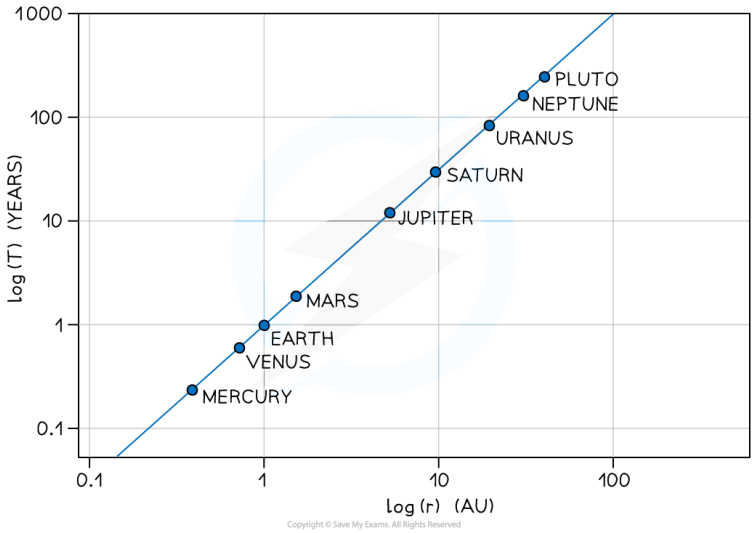
$$T^2 = \frac{4\pi^2 r^3}{GM}$$

- Where:
  - T = time period of the orbit (s)
  - r = orbital radius (m)
  - G = Gravitational Constant
  - M = mass of the object being orbited (kg)

- The relationship between T and r can be shown using a logarithmic plot

$$T^2 \propto r^3 \quad \Rightarrow \quad 2 \log T \propto 3 \log r$$

- The graph of log T in years against log r in AU (astronomical units) for the planets in our solar system is a straight-line graph:



The logarithmic graph of log T against log r gives a straight line

- The graph does not go through the origin since it has a negative y-intercept
  - Only the graph of log *T* and log *r* will produce a straight-line graph, a graph of T vs r would not

Worked Example

Planets A and B orbit the same star.

Planet A is located an average distance *r* from the star. Planet B is located an average distance 6*r* from the star

What is $\frac{orbital\ period\ of\ planet\ A}{orbital\ period\ of\ planet\ B}$?

A. $\frac{1}{\sqrt[3]{6}}$ **B.** $\frac{1}{\sqrt{6}}$ **C.** $\frac{1}{\sqrt[3]{6^2}}$ **D.** $\frac{1}{\sqrt{6^3}}$

Answer: D

- Kepler's third law states $T^2 \propto r^3$
- The orbital period of planet A: $T_A \propto \sqrt{r^3}$
- The orbital period of planet B: $T_B \propto \sqrt{(6r)^3}$
- Therefore the ratio is equal to:

$$\frac{T_A}{T_B} = \frac{\sqrt{r^3}}{\sqrt{(6r)^3}} = \frac{1}{\sqrt{6^3}}$$

Exam Tip

You are expected to be able to describe Kepler's Laws of Motion, so make sure you are familiar with how they are worded.

YOUR NOTES

Escape Speed

- To escape a gravitational field, a mass must travel at, or above, the minimum **escape speed**
  - This is dependent on the mass and radius of the object creating the gravitational field, such as a planet, a moon or a black hole
- Escape speed is defined as:

  The minimum speed that will allow an object to escape a gravitational field with no further energy input

- It is the same for all masses in the same gravitational field
  - For example, the escape speed of a rocket is the same as a tennis ball on Earth
- The escape speed of an when all its kinetic energy has been transferred to gravitational potential energy
- This is calculated by equating the equations:

$$\frac{1}{2}mv_{esc}^{2} = \frac{GMm}{r}$$

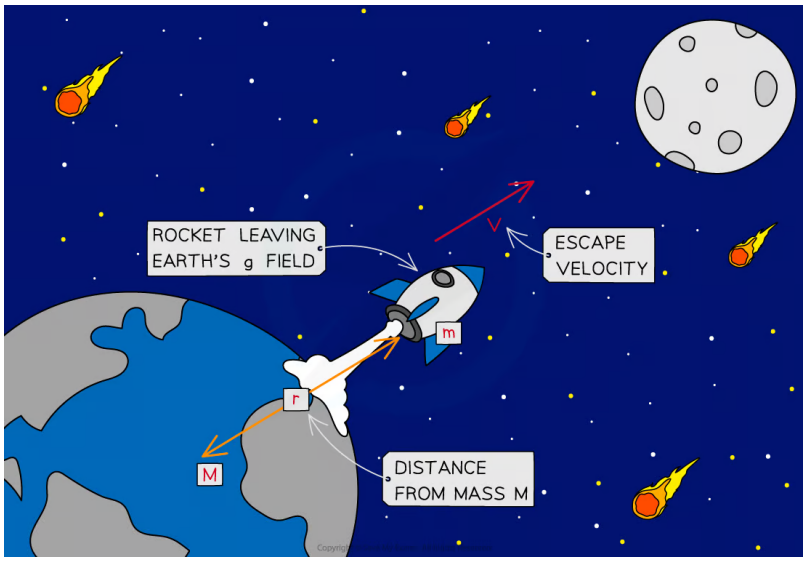
- Where:
  - m = mass of the object in the gravitational field (kg)
  - v_{esc} = escape velocity of the object (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object to be escaped from (i.e. a planet) (kg)
  - r = distance from the centre of mass M (m)
- Since mass m is the same on both sides of the equations, it can cancel on both sides of the equation:

$$\frac{1}{2}v_{esc}^{2} = \frac{GM}{r}$$

- Multiplying both sides by 2 and taking the square root gives the equation for escape velocity v_{esc}:

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

YOUR NOTES



For an object to leave the Earth's gravitational field, it will have to travel at a speed greater than the Earth's escape velocity, v

- Rockets launched from the Earth's surface do **not** need to achieve escape velocity to reach their orbit around the Earth
- This is because:
  - They are continuously given energy through fuel and thrust to help them move
  - Less energy is needed to achieve orbit than to escape from Earth's gravitational field
- The escape velocity is **not** the velocity needed to escape the planet but to escape the planet's **gravitational field** altogether
  - This could be quite a large distance away from the planet

YOUR NOTES

Worked Example

Calculate the escape speed at the surface of the Moon.

- Density of the Moon = 3340 kg m^{-3}
- Mass of the Moon = 7.35 × 10^{22} kg

Answer:

Step 1: List the known quantities

- Gravitational constant, G = 6.67 × 10^{-11} N m^2 kg^{-2}
- Density of the Moon, ρ = 3340 kg m^{-3}
- Mass of the Moon, M = 7.35 × 10^{22} kg

Step 2: Rearrange the density equation for radius *r*

Density: $\rho = \frac{M}{V}$ and volume of a sphere: $V = \frac{4}{3}\pi r^3$

$$\rho = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3M}{4\pi r^3}$$

$$r = \sqrt[3]{\frac{3M}{4\pi\rho}}$$

Step 3: Calculate the radius by substituting in the values

$$r = \sqrt[3]{\frac{3 \times (7.35 \times 10^{22})}{4\pi \times 3340}} = 1.7384 \times 10^6 \text{ m}$$

Step 4: Substitute r into the escape speed equation

$$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (7.35 \times 10^{22})}{1.7384 \times 10^6}}$$

Escape speed of the Moon: v_{esc} = 2.37 km s^{-1}

Exam Tip

When writing the definition of **escape velocity**, avoid terms such as 'gravity' or the 'gravitational pull / attraction' of the planet. It is best to refer to its **gravitational field.**

This equation is **not** given on the datasheet. Be sure to memorise how to derive it if you're stuck.

YOUR NOTES

Orbital Motion, Speed & Energy

- Since most planets and satellites have near-circular orbits, the gravitational force F_G between two bodies (e.g. planet & star, planet & satellite) provides the centripetal force needed to stay in an orbit
  - Both the gravitational force and centripetal force are **perpendicular** to the direction of travel of the planet
- Consider a satellite with mass *m* orbiting Earth with mass *M* at a distance *r* from the centre travelling with linear speed *v*

$$F_G = F_{circ}$$

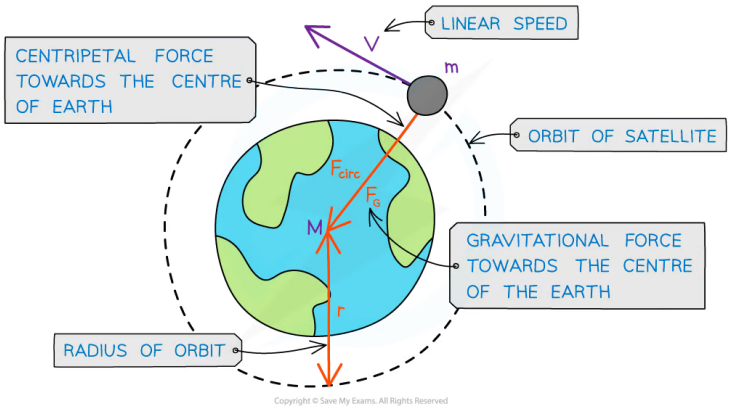
- Equating the gravitational force to the centripetal force for a planet or satellite in orbit gives:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- The mass of the satellite *m* will cancel out on both sides to give:

$$v^2 = \frac{GM}{r} \quad \Rightarrow \quad v_{orbital} = \sqrt{\frac{GM}{r}}$$

- Where:
  - $v_{orbital}$ = orbital speed of the smaller mass (m s^{-1})
  - *G* = Newton's Gravitational Constant
  - *M* = mass of the larger mass being orbited (kg)
  - *r* = orbital radius (m)
- This means that all satellites, **whatever their mass**, will travel at the same speed *v* in a particular orbit radius *r*
  - Since the direction of a planet orbiting in circular motion is constantly changing, the **centripetal acceleration** acts towards the planet

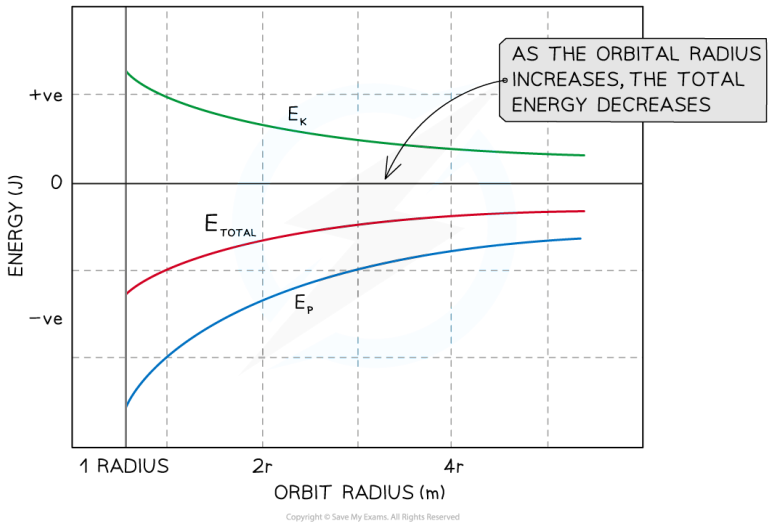


A satellite in orbit around the Earth travels in circular motion

Energy of an Orbiting Satellite

YOUR NOTES

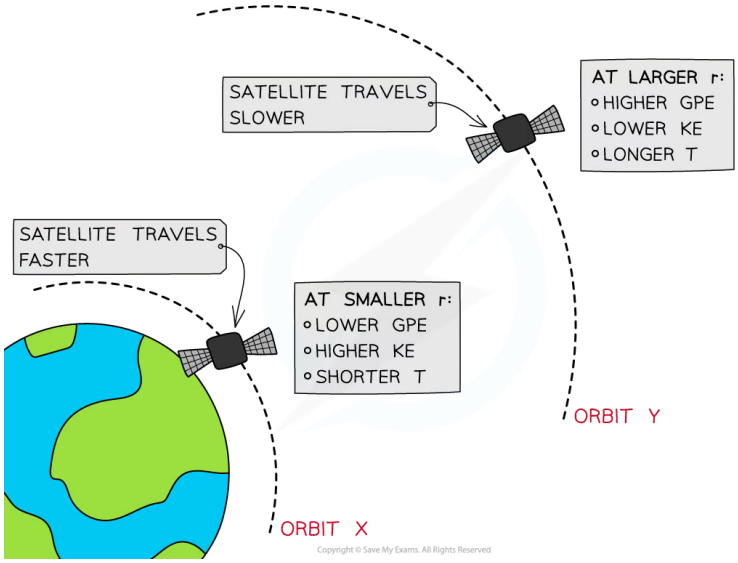
- An orbiting satellite follows a circular path around a planet
- Just like an object moving in circular motion, it has both kinetic energy (E_k) **and** gravitational potential energy (E_p) and its **total** energy is always **constant**
- An orbiting satellite's total energy is calculated by:



A graph showing the kinetic, potential and total energy for a mass at varying orbital distances from a massive body

Total energy = Kinetic energy + Gravitational potential energy

- This means that the satellite's E_k and E_p are also both constant in a particular orbit
  - If the orbital radius of a satellite **decreases** its E_k **increases** and its E_p **decreases**
  - If the orbital radius of a satellite **increases** its E_k **decreases** and its E_p **increases**



At orbit Y, the satellite has greater GPE and less KE than at at orbit X

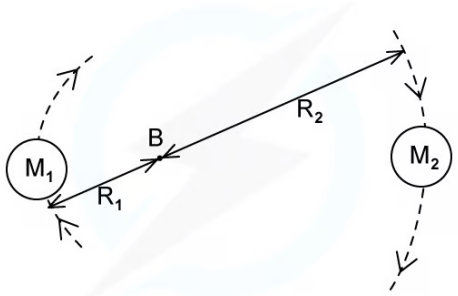
- A satellite is placed in two orbits, X and Y, around Earth
- At orbit X, where the radius of orbit *r* is smaller, the satellite has a:
  - Larger gravitational force on it
  - Higher speed
  - Higher E_k
  - Lower E_p
  - Shorter orbital time period, *T*
- At orbit Y, where the radius of orbit *r* is larger, the satellite has a:
  - Smaller gravitational force on it
  - Smaller speed
  - Lower E_k
  - Higher E_p
  - Longer orbital time period, *T*

YOUR NOTES

Worked Example

A binary star system constant of two stars orbiting about a fixed point **B**.

The star of mass M_1 has a circular orbit of radius R_1 and mass M_2 has a radius of R_2. Both have linear speed v and an angular speed ω about **B**.



In terms of G, M_2, R_1 and R_2, write an expression for

(a)
the angular speed ω of mass M_1
(b)
the time period T of each star

Answer:

(a) Angular speed:

- The centripetal force on mass M_1 is:

$$F = \frac{M_1 v_1^2}{R_1} = \frac{M_1(\omega R_1)^2}{R_1} = M_1 R_1 \omega^2$$

- The gravitational force between the two masses is:

$$F = \frac{GM_1 M_2}{(R_1 + R_2)^2}$$

- Equating these expressions gives:

$$M_1 R_1 \omega^2 = \frac{GM_1 M_2}{(R_1 + R_2)^2}$$

- Rearrange for angular velocity

$$\omega = \sqrt{\frac{GM_2}{R_1(R_1 + R_2)^2}}$$

(b) Orbital period:

YOUR NOTES

- The relation between angular speed and orbital period is

$$\omega = \frac{2\pi}{T} \quad \Rightarrow \quad T = \frac{2\pi}{\omega}$$

- Using the expression for angular velocity from part (a)

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1 + R_2)^2}} = 2\pi \sqrt{\frac{R_1(R_1 + R_2)^2}{GM_2}}$$

Worked Example

Two identical satellites, X and Y, orbit a planet at radii *R* and 3*R* respectively.

Which one of the following statements is **incorrect**?

A. Satellite X has more kinetic energy and less potential energy than satellite Y

B. Satellite X has a shorter orbital period and travels faster than satellite Y

C. Satellite Y has less kinetic energy and more potential energy than satellite X

D. Satellite Y has a longer orbital period and travels faster than satellite X

Answer: D

- Satellite Y is at a larger orbital radius, therefore it will have a **longer** orbital period, since $T^2 \propto R^3$
- Being at a larger orbital radius means the gravitational force will be weaker for Y than for X
- So, satellite Y will travel much **slower** than X as centripetal force: $F \propto v^2$
- Travelling at a slower speed means satellite Y will have **less** kinetic energy, as $E_K \propto v^2$, and, therefore, **more** potential energy than X

	Satellite X	Satellite Y
orbital radius	smaller	larger
orbital period	shorter	longer
orbital speed	faster	slower
kinetic energy	greater	lower
potential energy	lower	greater

- Therefore, all statements are correct except in **D** where it says 'Satellite Y travels faster than satellite X'

Exam Tip

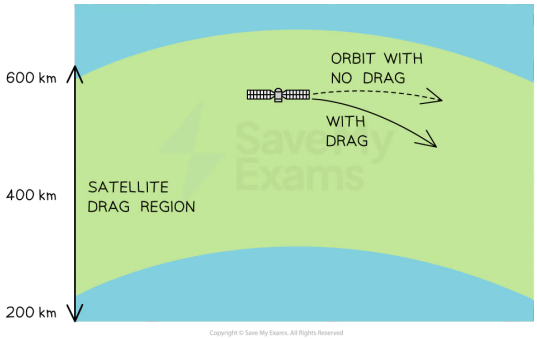
If you can't remember which way around the kinetic and potential energy increases and decreases, think about the velocity of a satellite at different orbits.

When it is orbiting close to a planet, it experiences a larger gravitational pull and therefore orbits faster. Since the kinetic energy is proportional to v^2, it, therefore, has higher kinetic energy closer to the planet. To keep the total energy constant, the potential energy must decrease too.

YOUR NOTES

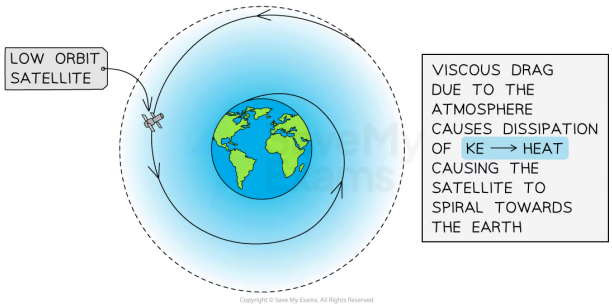
Effects of Drag on Orbital Motion

- Satellites in low orbits (<600 km) may be slightly affected by viscous drag, or air resistance
- The effects of drag on the motion of the satellite are usually very small, but over time, it can have a significant effect on the height and speed of the satellite's orbit



Viscous drag can affect the height and speed of a low-orbit satellite as a result of energy dissipation

- The density of the air in the very upper layers of the atmosphere is very low, but not zero
- As a result, satellites travelling through these thin layers of air will experience a small dissipation of kinetic energy into thermal energy
  - This heating is due to the friction between the air particles and the surface of the satellite



As a low-orbit satellite loses energy, it spirals towards the Earth as its orbital radius decreases

- As some of the kinetic energy is dissipated into the surroundings, the satellite's total energy is reduced
  - When a satellite loses energy, its **orbital radius decreases**
  - However, as the satellite's orbit becomes lower, some of its **potential** energy is transferred to **kinetic** energy
- Overall, its speed increases and the effects of air resistance become even greater in its lower orbit resulting in greater dissipation of kinetic energy into thermal energy
- If the overall **decrease** in potential energy is **larger** than the overall **increase** in kinetic energy, the **total energy will decrease**

$\Delta E_{total} < 0$ if $\Delta E_p > \Delta E_k$

YOUR NOTES

IB Physics HL

YOUR NOTES

Electric & Magnetic Fields

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Electric Potential (HL)
Electric Potential Energy (HL)
Electric Potential Energy Equation (HL)
Electric Potential Gradient (HL)
Electric Equipotential Surfaces (HL)
Magnetic Fields

YOUR NOTES

Electric Charge

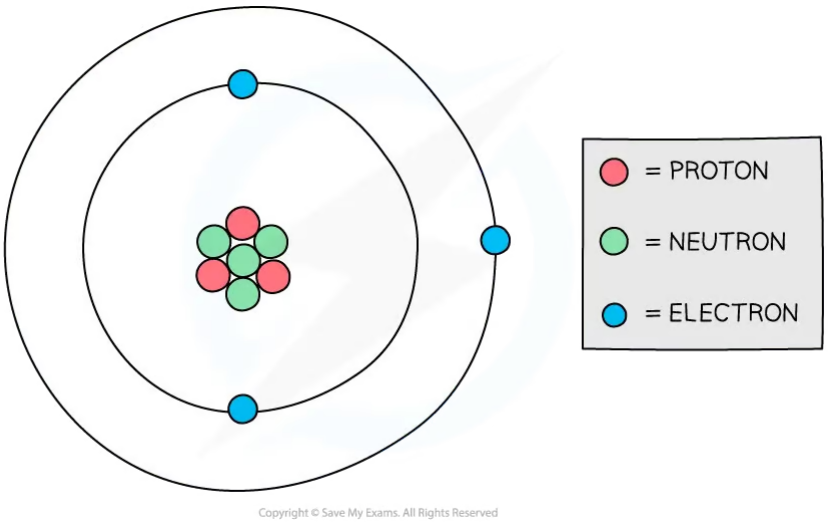
- Charge is measured in **coulombs** (**C**). One coulomb is defined as:

  The charge carried by an electric current of one ampere in one second

- Charge is a scalar quantity
  - **Electrons** have a **negative charge**
  - **Protons** have a **positive charge**

The Structure of an Atom

- In **neutral** (i.e. **uncharged**) atoms and objects the number of electrons and the number of protons are equal



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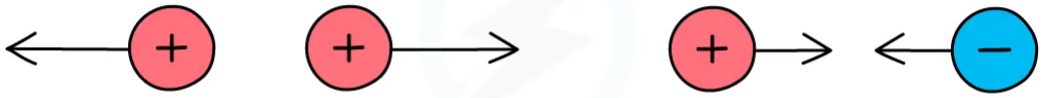
The number of negative electrons in an atom balances the number of positive protons

The Direction of Forces

- When two charged particles or objects are close together, they also exert a **force** on each other
- This force could be:
  - **Attractive** (the objects get closer together)
  - **Repulsive** (the objects move further apart)
- Whether two objects attract or repel depends on their **charge**
  - If the charges are the **opposite**, they will **attract**
  - If the charges are the **same**, they will **repel**



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Opposite charges attract, like charges repel

Attraction or Repulsion Summary Table

Charge of Object 1	Charge of Object 2	Attract or repel?
Positive	Positive	Repel
Positive	Negative	Attract
Negative	Positive	Attract
Negative	Negative	Repel

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- Attraction and repulsion between two charged objects are examples of a **non-contact force**
  - This is a force that acts on an object without being physically in contact with it

Exam Tip

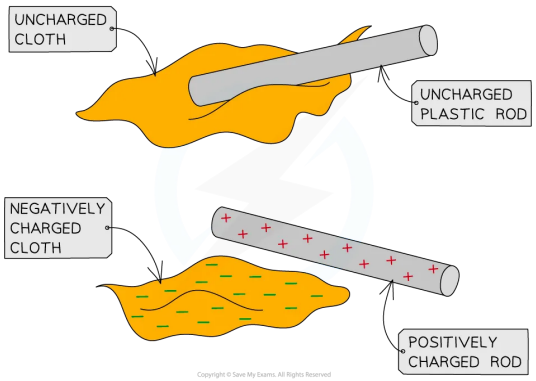
Remember the saying: "**Opposites attract**".

Conservation of Electric Charge

- All objects are initially electrically **neutral**, meaning the negative (electrons) and positive charges are evenly distributed
- However, when the electrons are transferred through friction, one object becomes **negatively** charged and the other **positively** charged
  - The object to which the electrons are transferred **to** becomes **negatively charged**
  - The object from which the electrons **leave from** becomes **positively charged**
- This difference in charges leads to a force of **attraction** between itself and other objects which are also electrically neutral
  - This is done by attracting the opposite charge to the surface of the objects they are attracted to
- A stable is atom is always neutral, meaning it has the same amount of positive charge as negative charge
  - This means a stable atom has the same number of **protons** (positive charge) and **electrons** (negative charge)

Rubbing the Cloth with the Rod

- In the example below, when the cloth and rod are rubbed together, the electrons are **transferred to** the **cloth** and **leave from** the **rod**
- The charge is **conserved.** It is **transferred** but **never created** or **destroyed**



Electrons are rubbed onto the cloth leaving the cloth negatively charged and the rod positively charged

Exam Tip

Materials only become positively charged because of the **loss** of electrons, rather than the 'gain' of any positive charge, which is a common misconception.

YOUR NOTES

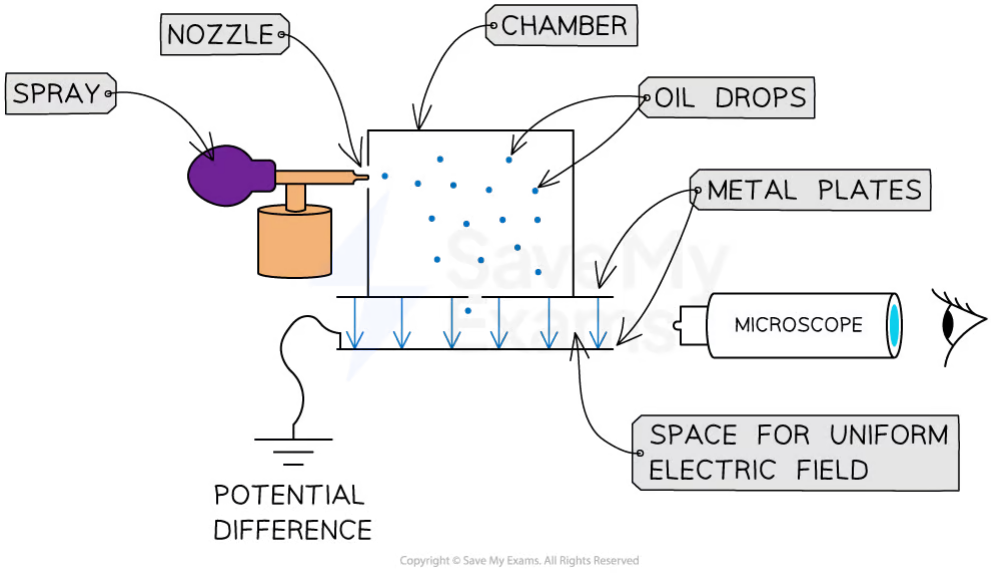
Milikan's Oil Drop Experiment

- This experiment was conducted by Milikan and Fletcher in 1909
- It determined the value of fundamental or elementary charge

Method for Milikan's Oil Drop Experiment

- A fine mist of **atom-sized oil drops** is sprayed into a chamber
  - Oil is used instead of water because it does not evaporate quickly
  - This means the **mass** of the drops will remain **constant**
- As the drops pass out of the spray nozzle they are ionised by **X-Rays**
  - This consequently changes their charge from neutral
  - They will become positively charged if they lose electrons
  - They will become negatively charged if they gain electrons
- The drops pass into a region between **two metal plates** and are viewed using a **microscope**

Equipment Set Up for Milikan's Oil Drop Experiment



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In Milikan's Oil Drop Experiment oil is sprayed into a chamber before passing between metal plates where the electric and gravitational forces are compared

Electric vs Gravitational Force

No Electric Field

- The oil drops fall **under gravity** between the metal plates
- They reach a **terminal velocity** when the air resistance and gravitational force acting on the drop are **equal**

Electric Field

- A **potential difference** is applied between the metal plates which creates an electric field
- The charged oil drops begin to **rise** when the electric field is strong enough
- This means the upward electrical force is greater than the gravitational force
    - The electrical force equation is:

$$F = Eq$$

    - Where:
        - E = electric field strength (N C^{-1})
        - F = electrostatic force on the charge (N)
        - q = charge (C)
- The distance the drops rise depends upon their **mass**
    - The bigger the mass, then the shorter the distance they rise
    - The gravitational force equation, which comes from Newton's second law is:

$$W = mg$$

    - Where:
        - W = weight of drop (N)
        - m = mass of drop (kg)
        - g = gravitational field strength (m s^{-2})
- Through balancing the electric and gravitational forces on the drops the value of fundamental charge was determined to be 1.60×10^{-19} C
- The magnitude of charge on any object is found to be a **multiple** of 1.60×10^{-19} C
- This is called the **quantisation of charge**

YOUR NOTES

YOUR NOTES

Methods of Electric Charge Transfer

- There are several methods by which electric charge can be transferred
  - Charging by friction
  - Charging by electrostatic induction
  - Charging by contact

Charging by Friction

- When certain insulating materials are rubbed against each other they become **electrically charged**
  - This is called **charging by friction**
- The charges remain on the insulators and cannot immediately flow away
  - One becomes positive and the other negative
- An example of this is a plastic or polythene rod being charged by rubbing it with a cloth
  - Both the rod and cloth are insulating materials

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A polythene rod may be given a charge by rubbing it with a cloth

- This occurs because negatively charged electrons are **transferred** from one material to the other
  - The material, in this case, the rod, **loses** electrons
- Since electrons are negatively charged, the rod becomes **positively** charged
  - As a result, the cloth has **gained** electrons and therefore is left with an equal **negative** charge

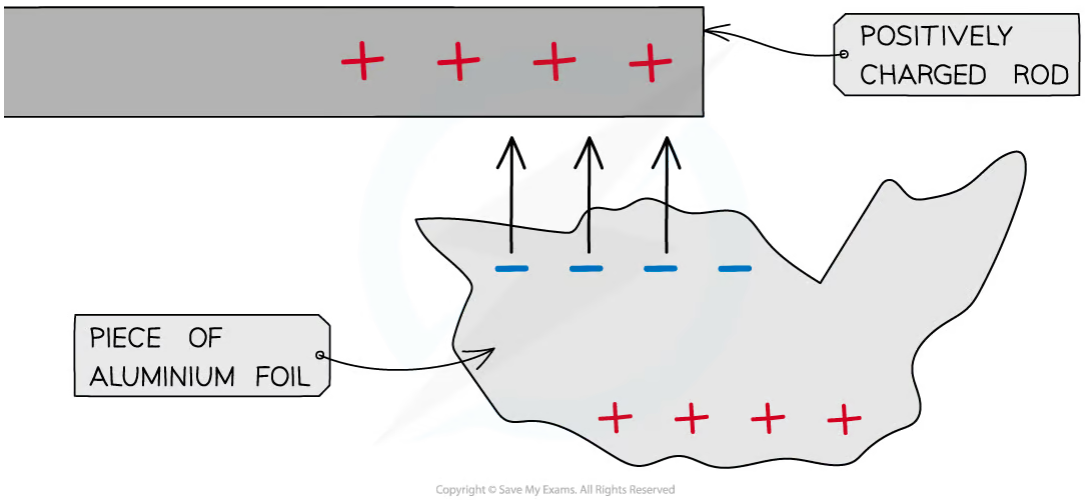
Charging by Electrostatic Induction

YOUR NOTES

- **Electrostatic Induction** is defined as:

  A process which generates an electrostatic force in a material by bringing an electrically charged object near it

- This causes the electrical charges within the material to be **redistributed**
  - So, one side has an excess of either positive or negative charges
- When a charged object is held close to a conductor, electrons in the conductor are able to move towards (or away from) the charged object:

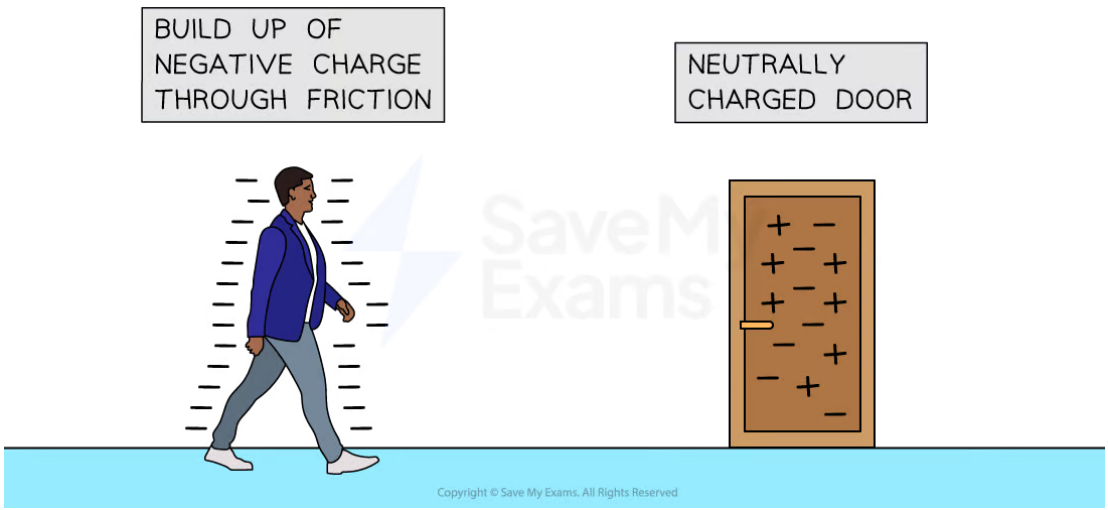


Negative electrons in the aluminium are attracted to the positive rod

- In the diagram above, electrons in the aluminium foil are attracted to the positively charged rod
- This causes the top of the foil to become **negatively** charged, whilst the bottom edge of it will be left with a **positive** charge
- The **electrostatic attraction** between the positive rod and the negative charges on the top surface of the foil causes the foil to be attracted to the rod

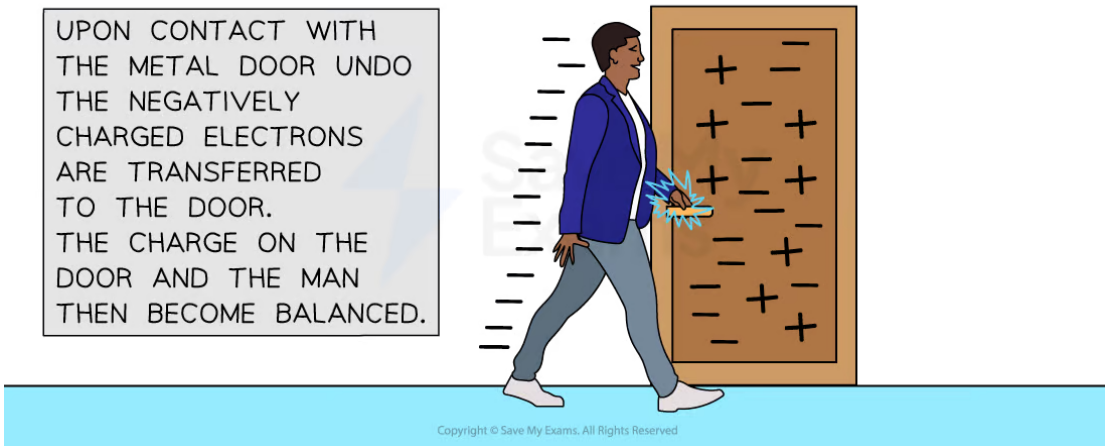
Charging by Contact

- Charge can also be transferred via **contact**
- For this to happen one of the two bodies involved must have a considerably **higher net** positive or negative **charge** build-up
- An **insulator** must prevent this charge from flowing out into neighbouring bodies
  - When the two bodies touch **electrons flow** from one to the other to reduce the difference between the charges of the two objects
- An example of charge transfer via contact is a 'shock' from a doorknob
- As you walk across a carpet the friction creates a build-up of negative charge on your shoes
  - Now you have a high net negative charge



A negative charge builds up on the bottom of your shoes as you walk across the carpet to the neutrally charged door

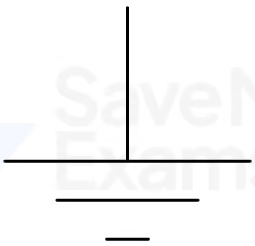
- During this process, you are insulated, as you are not in contact with anything that can conduct electricity
  - However, when you touch a metal doorknob, electrons flow from you to the doorknob to reduce the difference between your charge and the doorknob's charge
- This is because metal conducts electricity, and you are no longer insulated



Upon contact with the metal door knob, the negative charge is transferred to the door and the charge balances out

Earthing

- To prevent a transfer of charge through contact, both bodies can be **grounded**
- This means they are connected electrically to the **earth**
  - This is called **earthing**



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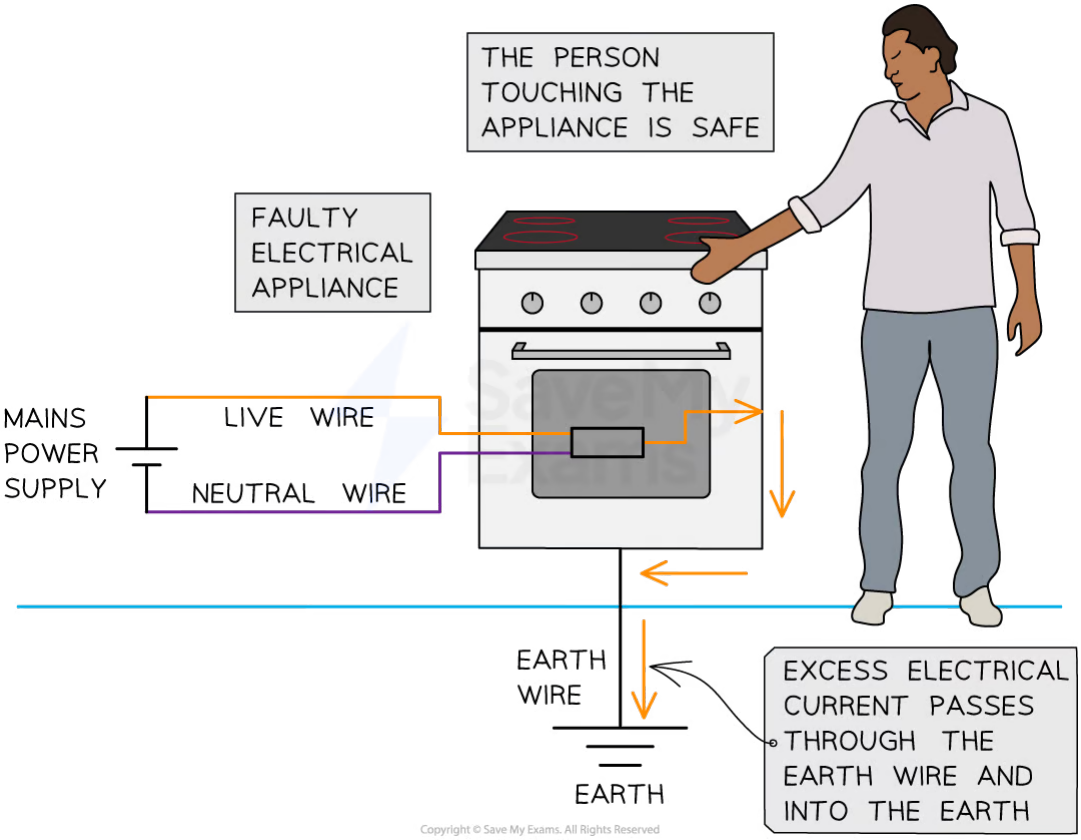
The electrical symbol for Earth

- Many electrical appliances have **metal cases**
  - If a live wire (inside the appliance) came into contact with the case, the case would become electrified and anyone who touched it would risk being electrocuted
- The electrical appliance can be connected to an **earthing system** via an **earth wire**



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A problem in the electric appliance causes a surge of current which passes through the earth wire and is grounded into the earth. The person in contact with the appliance does not get an electric shock.

- If there is a sudden surge of current within the system:
  - The earth wire provides a **direct low resistance path to the earth**

- So the current flows through **the earth wire**
- Into the **ground**

- It makes the appliance **safe** to touch and fix

Exam Tip

If asked to explain how things gain or lose charge, you must discuss **electrons** and explain whether something has gained or lost them. Remember when charging by friction, it is only the **electrons** that can move, not any 'positive' charge, therefore if an object gains a negative charge, something else must have gained a positive charge.

It can be harder than you anticipate to explain the process of earthing and the transfer of charge. Practice explaining it to someone to check you don't get tangled up and that they understand what you are saying.

YOUR NOTES

YOUR NOTES

Coulomb's Law

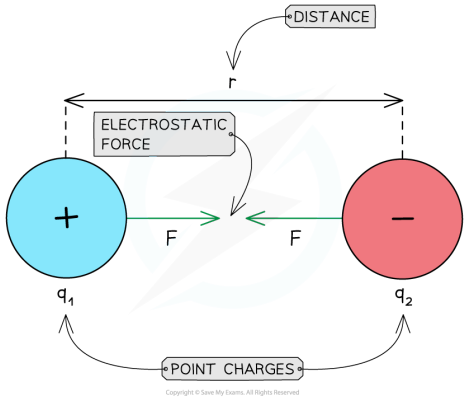
- Any charged particle generates an electric field
  - This field exerts a force on any other charged particle within range
- The electrostatic force between two charges is defined by **Coulomb's Law**, which states that:

> **The attractive or repulsive electrostatic force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of their separation**

- According to Coulomb's law, the electrostatic force between two point charges is calculated as follows:

$$F = k\frac{q_1 q_2}{r^2}$$

- Where:
  - F = electrostatic force (N)
  - q_1, q_2 = magnitudes of the charges (C)
  - r = distance between the centres of the two charges (m)
  - k = **Coulomb constant** (N m^2 C^{-2})



Attractive electrostatic force between two opposite charges

- Coulomb's constant is given by:

$$k = \frac{1}{4\pi\varepsilon_0}$$

- The value of k depends on the material between the charges
  - In a **vacuum**, $k = 8.99 \times 10^9$ N m^2 C^{-2}
- ε_0 is the **permittivity of free space**
  - $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2} and refers to charges in a vacuum

- The value of the permittivity of air is taken to be the same as ε_0
- Any other material has a higher permittivity $\varepsilon > \varepsilon_0$
- ε is a measure of the resistance offered by a material in creating an electric field within it

YOUR NOTES

Repulsive & Attractive Forces

- For **like charges**:
  - The product q_1q_2 is positive
  - F is **positive**
  - The charges **repel** each other
- For **opposite charges**:
  - The product q_1q_2 is negative
  - F is **negative**
  - The charges **attract** each other

YOUR NOTES

Worked Example

An alpha particle is situated 2.0 mm away from a gold nucleus in a vacuum.

Assuming them to be point charges, calculate the magnitude of the electrostatic force acting on each of the charges.

- Atomic number of helium = 2
- Atomic number of gold = 79

Answer:

Step 1: Write down the known quantities

- Distance, $r = 2.0 \text{ mm} = 2.0 \times 10^{-3} \text{ m}$
- Elementary charge, $e = 1.60 \times 10^{-19} \text{ C}$ (from the data booklet)

  Note that you must convert the distance from millimetres (mm) into metres (m)

Step 2: Calculate the charges of the alpha particle and gold nucleus

- The elementary charge e (when taken to be positive) is the charge of a proton
- An alpha particle (helium nucleus) has 2 protons
  - So its charge is:

$$q_1 = 2 \times (1.60 \times 10^{-19}) = 3.2 \times 10^{-19} \text{ C}$$

- A gold nucleus has 79 protons
  - So its charge is:

$$q_2 = 79 \times (1.60 \times 10^{-19}) = 1.264 \times 10^{-17} \text{ C}$$

Step 3: Write down the equation to calculate the electrostatic force

$$F = k \frac{q_1 q_2}{r^2}$$

Step 4: Substitute the numbers into the equation

$$F = (8.99 \times 10^{9}) \times \frac{(3.2 \times 10^{-19}) \times (1.264 \times 10^{-17})}{(2.0 \times 10^{-3})^2}$$

$$F = 9.1 \times 10^{-21} \text{ N}$$

Exam Tip

You do not need to memorise the numerical value of the Coulomb's constant k or that of the permittivity of free space ε_0. They will both be given in the data booklet.

Unless specified in the question, you should assume that charges are located in a vacuum.

You should note that Coulomb's law can only be applied to charged spheres whose size is much smaller than their separation. Only in this case, the point charge approximation is valid. You must remember that the separation r must be taken from the centres of the spheres.

You cannot use Coulomb's law to calculate the electrostatic force between charges distributed on irregularly-shaped objects.

Different Values of Permittivity

YOUR NOTES

Permittivity

- Permittivity is the measure of how easy it is to generate an electric field in a certain material
- The relativity permittivity ε_r is sometimes known as the **dielectric constant**
- For a given material, it is defined as:

The ratio of the permittivity of a material to the permittivity of free space

- This can be expressed as:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

- Where:
  - ε_r = relative permittivity
  - ε = permittivity of a material (F m^{-1})
  - ε_0 = permittivity of free space (F m^{-1})
- The relative permittivity has **no** units because it is a ratio of two values with the same unit

Examples of Relative Permittivity

- It is helpful to understand the values of relative permittivity for different materials
- Some examples are below:

Material	ε_0
Vacuum	1
Air	1.00054
Paper	4
Polystyrene	3
Ceramic	100 - 15 000
Paraffin	2.3
Pure Water	80

YOUR NOTES

Worked Example

Calculate the permittivity of a material that has a relative permittivity of 4.5×10^{11}. State an appropriate unit for your answer.

Answer:

Step 1: Write down the relative permittivity equation

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Step 2: Rearrange for permittivity of the material ε

$$\varepsilon = \varepsilon_r \varepsilon_0$$

Step 3: Substitute in the values

$$\varepsilon = (4.5 \times 10^{11}) \times (8.85 \times 10^{-12}) = 3.9825 = 4 \text{ F m}^{-1}$$

Electric Field Strength

- An electric field is a region of space in which an electric charge "feels" a force
- The **electric field strength** at a point is defined as:

The electrostatic force per unit positive charge acting on the charge at that point

- The electric field strength can be calculated using the equation:

$$E = \frac{F}{Q}$$

- Where:
  - E = electric field strength (N C^{-1})
  - F = electrostatic force on the charge (N)
  - Q = charge (C)

- It is important to use a positive test charge in this definition, as this determines the direction of the electric field
- The electric field strength is a **vector** quantity, it is always directed:
  - **Away** from a positive charge
  - **Towards** a negative charge

- Recall that **opposite charges** (positive and negative) charges **attract** each other
- Conversely, **like charges** (positive and positive or negative and negative) **repel** each other

YOUR NOTES

Worked Example

A charged particle is in an electric field with electric field strength 3.5×10^4 N C^{-1} where it experiences a force of 0.3 N.

Calculate the charge of the particle.

Answer:

Step 1: Write down the equation for electric field strength

$$E = \frac{F}{Q}$$

Step 2: Rearrange for charge Q

$$Q = \frac{F}{E}$$

Step 3: Substitute in values and calculate

$$Q = \frac{0.3}{3.5 \times 10^4} = 8.571 \times 10^{-6} = \mathbf{8.6 \times 10^{-6}\ C\ (2\ s.f)}$$

YOUR NOTES

Electric Field Between Parallel Plates

- The magnitude of the electric field strength in a **uniform** field between two charged parallel plates is defined as:

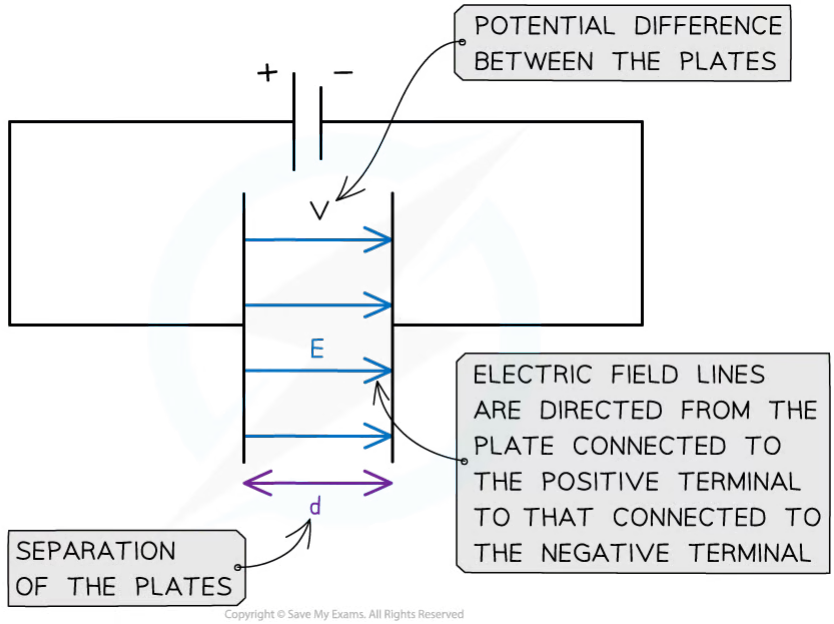
$$E = \frac{V}{d}$$

- Where:
  - E = electric field strength ($V\ m^{-1}$)
  - V = potential difference between the plates (V)
  - d = separation between the plates (m)

- **Note:** the electric field strength is now also defined by the units **$V\ m^{-1}$**
- The equation shows:
  - The greater the **voltage** between the plates, the **stronger** the field
  - The greater the **separation** between the plates, the **weaker** the field

- Remember this equation cannot be used to find the electric field strength around a point charge (since this would be a radial field)
- The direction of the electric field is from the plate connected to the **positive** terminal of the cell to the plate connected to the **negative** terminal

Uniform Electric Field Between two Parallel Plates



The E field strength between two charged parallel plates is the ratio of the potential difference and separation of the plates

YOUR NOTES

- **Note:** if one of the parallel plates is **earthed**, it has a voltage of 0 V

Worked Example

Two parallel metal plates are separated by 3.5 cm and have a potential difference of 7.9 kV.

Calculate the electric force acting on a stationary charged particle between the plates that has a charge of 2.6×10^{-15} C.

Answer:

Step 1: Write down the known values

- Potential difference, $V = 7.9 \text{ kV} = 7.9 \times 10^{3} \text{ V}$
- Distance between plates, $d = 3.5 \text{ cm} = 3.5 \times 10^{-2} \text{ m}$
- Charge, $Q = 2.6 \times 10^{-15} \text{ C}$

Step 2: Calculate the electric field strength between the parallel plates

$$E = \frac{V}{d}$$

$$E = \frac{7.9 \times 10^{3}}{3.5 \times 10^{-2}} = 2.257 \times 10^{5} \text{ V m}^{-1}$$

Step 3: Write out the equation for electric force on a charged particle

$$F = QE$$

Step 4: Substitute electric field strength and charge into electric force equation

$$F = QE = (2.6 \times 10^{-15}) \times (2.257 \times 10^{5}) = 5.87 \times 10^{-10} \text{ N} = 5.9 \times 10^{-10} \text{ N (2 s.f.)}$$

Exam Tip

Remember the equation for electric field strength with V and d is only used for **parallel plates**, and not for point charges (where you would use $E = \frac{F}{Q}$)

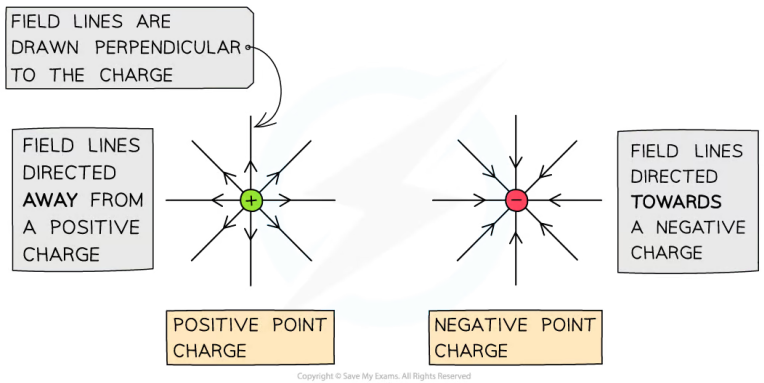
YOUR NOTES

Representing Electric Fields

- The direction of electric fields is represented by electric field lines
- Electric field lines are directed from positive to negative
  - Therefore, the field lines must be pointed **away** from the **positive** charge and **towards** the **negative** charge
- A radial field spreads uniformly to or from the charge in all directions
  - e.g. the field around a point charge or sphere

Electric Fields Around Point Charges

- Around a point charge, the electric field lines are directly radially inwards or outwards:
  - If the charge is **positive** (+), the field lines are radially **outwards**
  - If the charge is **negative** (-), the field lines are radially **inwards**

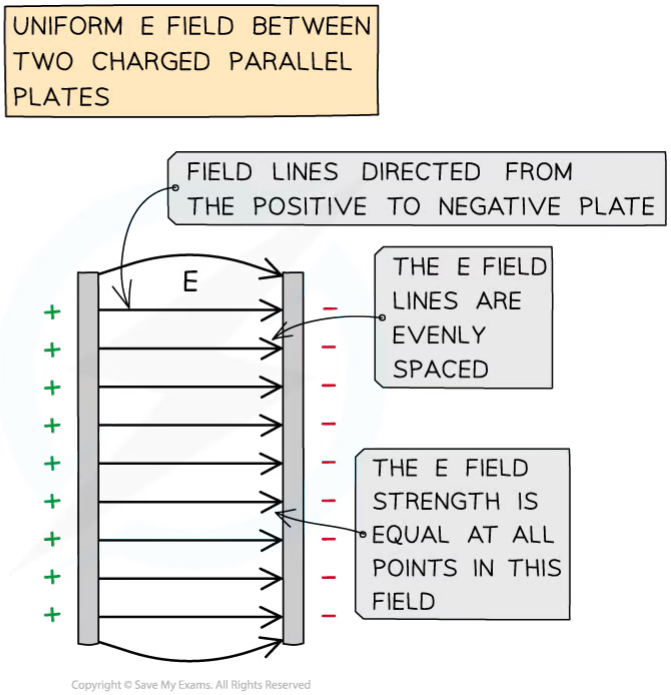


Electric field lines point away from a positive charge and point towards a negative charge

- This shares many similarities to radial gravitational field lines around a point mass
  - Since gravity is only an attractive force, the field lines will look similar to the negative point charge, whilst electric field lines can be in either direction
- A uniform electric field has the same electric field strength throughout the field
  - For example, the field between oppositely charged parallel plates
  - This is represented by **equally spaced** field lines and shares many similarities to uniform gravitational field lines on the surface of a planet
- A **non-uniform** electric field has varying electric field strength throughout
  - The strength of an electric field is represented by the spacing of the field lines:
  - A **stronger** field is represented by the field lines which are **closer** together
  - A **weaker** field is represented by the field lines which are **further** apart

Electric Field Lines Between Two Parallel Plates

- The electric field lines are directed from the **positive** to the **negative** plate



The electric field between two parallel plates is directed from the positive to the negative plate. A uniform E field has equally spaced field lines

- A radial field is considered a **non-uniform** field
  - Electric field strength *E* varies with distance from a charged particle

YOUR NOTES

Worked Example

Sketch the electric field lines between the two point charges in the diagram below.

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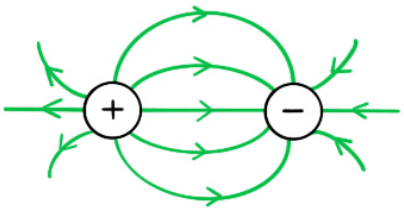
Answer:

- Electric field lines around point charges have arrows which point radially outwards for positive charges and radially inwards for negative charges
- Arrows (representing force on a positive test charge) point **from the positive charge to the negative charge**



Exam Tip

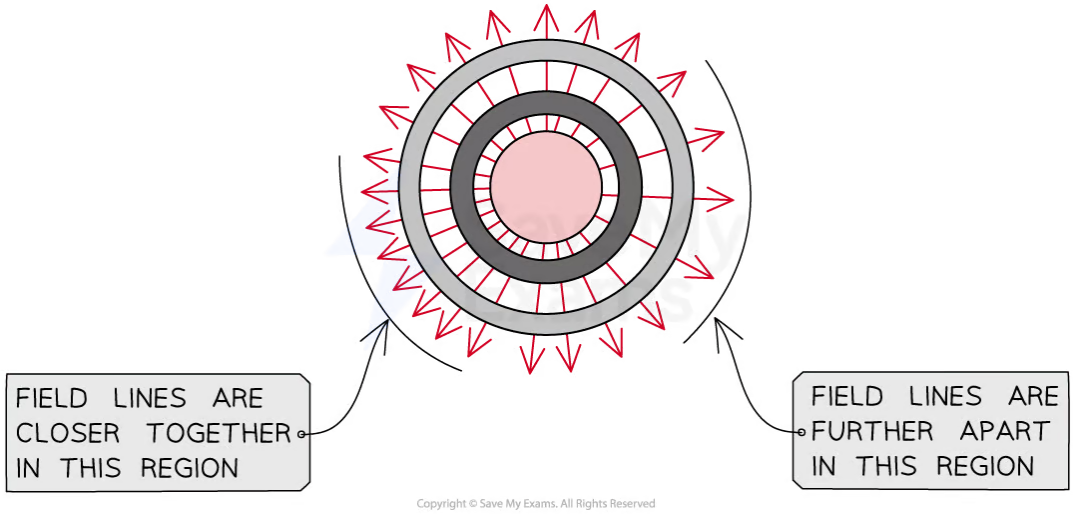
Always label the arrows on the field lines! The lines must also touch the surface of the source charge or plates.

YOUR NOTES

Electric Field Strength and Line Density

- The **density** of electric **field lines** determines the **strength** of an electric field
  - Regions where the electric field lines are **closer together**, mean the electric field is **stronger**
  - Regions where the field lines are **further apart** mean the electric field is **weaker**

Example of Different Field Line Density



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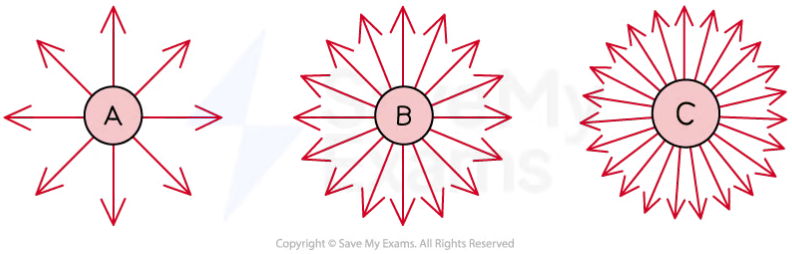
The field lines on the left-hand side are much closer together than the field lines on the right-hand side. This means that the electric field is stronger on the left than it is on the right.

Density of Field Lines Around Point Charges

- The **stronger** an electric point **charge** then the **stronger** the electric **field** around it and the **higher** the **density** of its field lines



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Electric field strength A > Electric Field Strength B > Electric Field Strength C

So, Magnitude of Charge A > Magnitude of Charge B > Magnitude of Charge C

YOUR NOTES

Electric Potential

- In order to move a positive charge closer to another positive charge, work must be done to overcome the force of repulsion between them
- Energy is therefore transferred to the charge that is being pushed upon
  - This means its **potential energy** increases
- If the positive charge is free to move, it will start to move away from the repelling charge
  - As a result, its potential energy decreases back to 0
- This is analogous to the gravitational potential energy of a mass increasing as it is being lift upwards and decreasing and it falls
- The electric potential at a point is defined as:

  The work done per unit positive charge in bringing a small test charge from infinity to a defined point

- Electric potential is a **scalar** quantity
  - This means it doesn't have a direction
- However, you will still see the electric potential with a positive or negative sign. This is because the electric potential is:
  - **Positive** when near an isolated positive charge
  - **Negative** when near an isolated negative charges
  - **Zero** at infinity
- Positive work is done by the mass from infinity to a point around a positive charge and negative work is done around a negative charge. This means:
  - When a test charge moves closer to a **negative** charge, its electric potential **decreases**
  - When a test charge moves closer to a **positive** charge, its electric potential **increases**
- To find the potential at a point caused by multiple charges, add up each potential separately

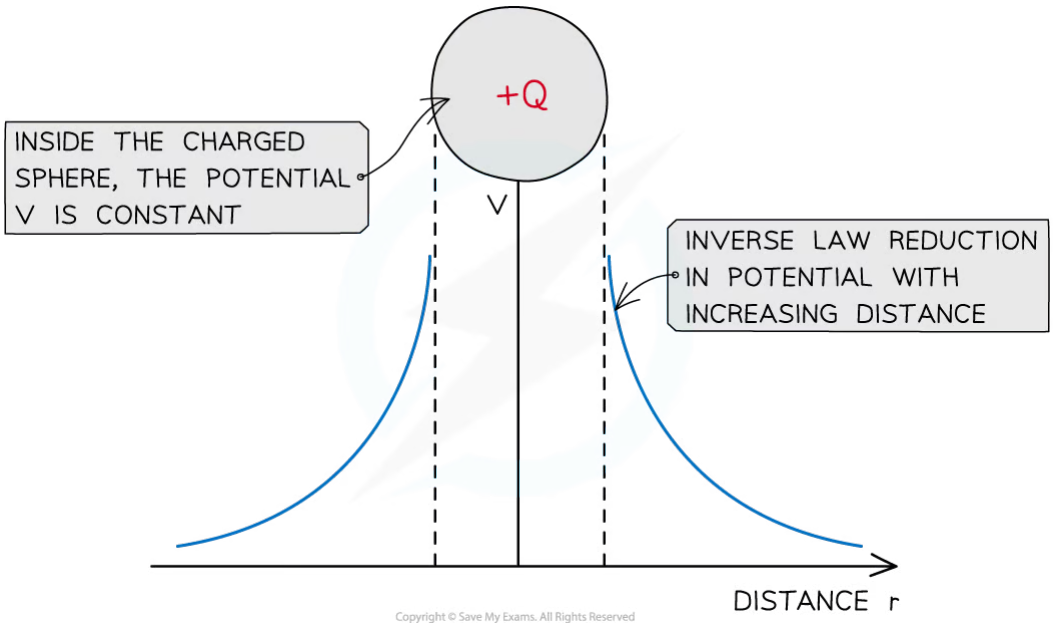
Calculating Electric Potential

YOUR NOTES

- The electric potential in the field due to a point charge is defined as:

$$V_e = \frac{Q}{4\pi\varepsilon_0 r}$$

- Where:
  - V_e = the electric potential (V)
  - Q = the point charge producing the potential (C)
  - ε_0 = permittivity of free space (F m^{-1})
  - r = distance from the centre of the point charge (m)
- This equation shows that for a positive (+) charge:
  - As the distance from the charge r **decreases**, the potential V_e **increases**
  - This is because more work has to be done on a positive test charge to overcome the repulsive force
- For a negative (–) charge:
  - As the distance from the charge r **decreases**, the potential V_e **decreases**
  - This is because less work has to be done on a positive test charge since the attractive force will make it easier
- Unlike the gravitational potential equation, the minus sign in the electric potential equation will be included in the charge
- The electric potential changes according to an inverse square law with distance



The potential changes as an inverse law with distance near a charged sphere

YOUR NOTES

- **Note:** this equation still applies to a conducting sphere. The charge on the sphere is treated as if it concentrated at a point in the sphere from the point charge approximation

Worked Example

A Van de Graaf generator has a spherical dome of radius 15 cm. It is charged up to a potential of 240 kV. Calculate

(a) How much charge is stored on the dome

(b) The potential a distance of 30 cm from the dome

Answer:

Part (a)

Step 1: List down the known quantities

- Radius of the dome, $r = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$
- Potential difference, $V = 240 \text{ kV} = 240 \times 10^{3} \text{ V}$

Step 2: Write down the equation for the electric potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 3: Rearrange for charge Q

$$Q = V4\pi\varepsilon_0 r$$

Step 4: Substitute in values

$$Q = (240 \times 10^{3}) \times (4\pi \times 8.85 \times 10^{-12}) \times (15 \times 10^{-2}) = 4.0 \times 10^{-6} \text{ C} = 4.0 \ \mu\text{C}$$

Part (b)

Step 1: Write down the known quantities

- Q = charge stored in the dome = $4.0 \ \mu\text{C} = 4.0 \times 10^{-6} \text{ C}$
- r = radius of the dome + distance from the dome = $15 + 30 = 45 \text{ cm} = 45 \times 10^{-2} \text{ m}$

Step 2: Write down the equation for electric potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Step 3: Substitute in values

$$V = \frac{(4.0 \times 10^{-6})}{(4\pi \times 8.85 \times 10^{-12}) \times (45 \times 10^{-2})} = 79.93 \times 10^{3} = \mathbf{80 \text{ kV}} \text{ (2 s.f.)}$$

Electric Potential Energy

- **Electric field strength**, *E* and the **electric potential**, *V* can be graphically represented against the distance from the centre of a charge, *r*
- *E*, *V* and *r* are related by the equation:

$$E = \frac{\Delta V}{\Delta r}$$

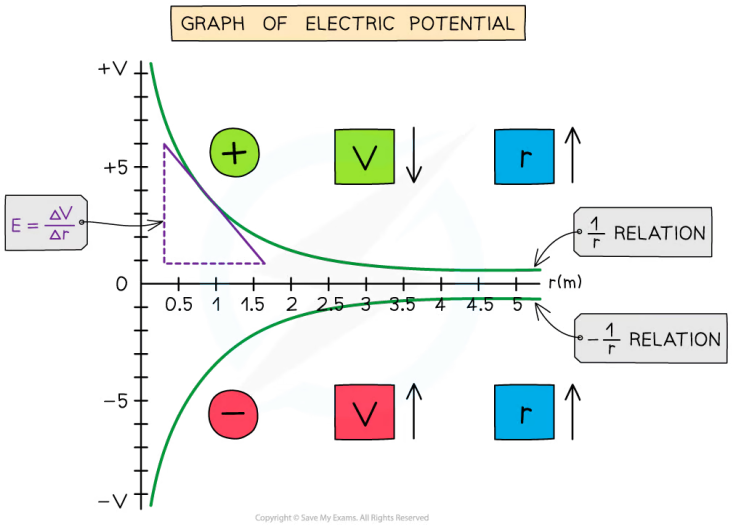
- Where:
  - *E* = electric field strength (V m^{-1})
  - ΔV = change in potential (V)
  - Δr = displacement in the direction of the field (m)
- An electric field can be defined in terms of the variation of **electric potential** at different points in the field**:**

  The electric field at a particular point is equal to the gradient of a potential–distance graph at that point

- The potential gradient in an electric field is defined as:

  The rate of change of electric potential with respect to displacement in the direction of the field

- The graph of potential *V* against distance *r* for a negative or positive charge is:



The electric potential around a positive charge decreases with distance and increases with distance around a negative charge

- **The key features of this graph are:**
  - The values for *V* are all negative for a negative charge
  - The values for *V* are all positive for a positive charge

YOUR NOTES

- As *r* increases, *V* against *r* follows a 1/*r* relation for a positive charge and -1/*r* relation for a negative charge
- The **gradient** of the graph at any particular point is the value of *E* at that point
- The graph has a shallow increase (or decrease) as *r* increases

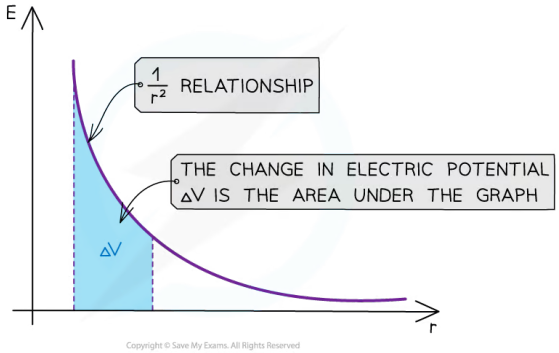
- The electric potential changes according to the charge creating the potential as the distance *r* increases from the centre:
  - If the charge is **positive**, the potential **decreases** with distance
  - If the charge is **negative**, the potential **increases** with distance

- To calculate *E*, draw a tangent to the graph at that point and calculate the gradient of the tangent
- This is a graphical representation of the equation:

$$V = \frac{Q}{4\pi\varepsilon_o r}$$

- Where Q and $4\pi\varepsilon_0$ are constants

- The graph of *E* against *r* for a charge is:



The electric field strength E has a 1/r² relationship

- **The key features of this graph are**:
  - The values for *E* are all positive
  - As *r* increases, *E* against *r* follows a $1/r^2$ relation (inverse square law)
  - The **area** under this graph is the change in electric potential Δ*V*
  - The graph has a steep decline as *r* increases

- The area under the graph can be estimated by counting squares, if it is plotted on squared paper, or by splitting it into trapeziums and summing the area of each trapezium
- The inverse square law relation means that as the distance *r* doubles, *E* decreases by a factor of **4**

- This is a graphical representation of the equation:

$$E = \frac{Q}{4\pi\varepsilon_o r^2}$$

- Where Q and $4\pi\varepsilon_0$ are constants

YOUR NOTES

Exam Tip

Drawing, interpreting or calculating from inverse square law graphs are common exam questions - there are lots of similarities between gravitational and electric field graphs:

Graphs of field strength against distance should start off steeper and decrease rapidly compared to that of potential graphs against distance, to distinguish it as an inverse square law ($1/r^2$) relation instead of just an inverse relation ($1/r$)

There are plenty of differences too:

For example, gravitational potential **always increases** with respect to distance whereas electric potential can increase or decrease

One way to remember whether the electric potential increases or decreases with respect to the distance from the charge is by the direction of the electric field lines - the potential always **decreases** in the **same** direction as the field lines and vice versa.

YOUR NOTES

Work Done on a Charge

- When a mass with charge moves through an electric field, work is done
- The work done in moving a charge q is given by:

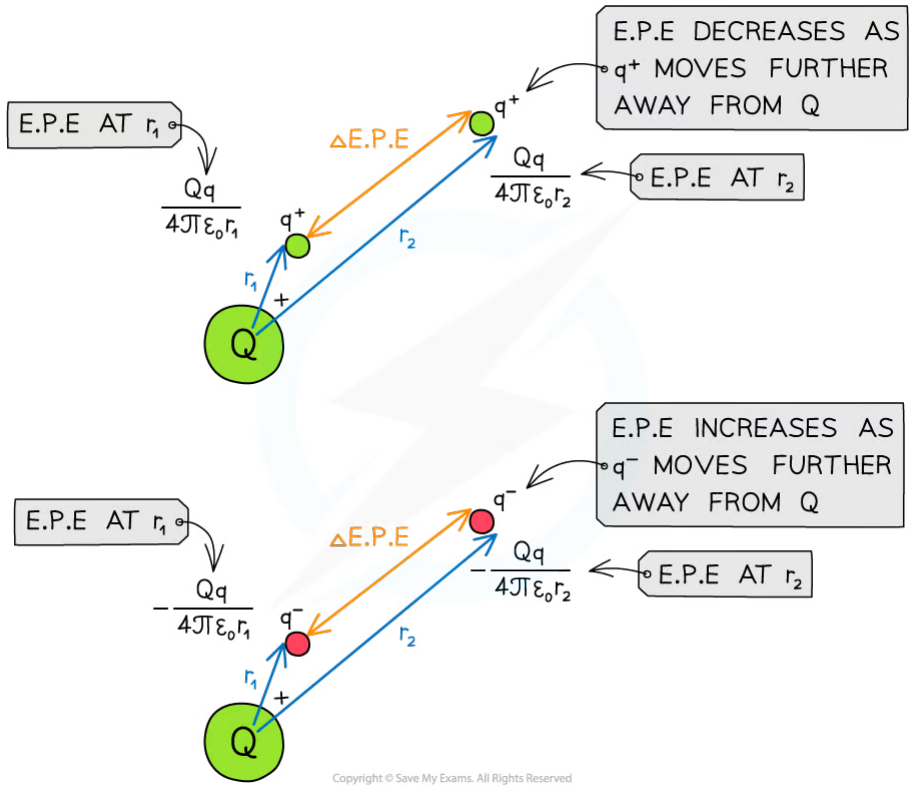
$$\Delta W = q\Delta V$$

- Where:
  - ΔW = change in work done (J)
  - q = charge (C)
  - ΔV = change in electric potential (J C^{-1})
- This change in work done is equal to the change in **electric potential energy** (E.P.E)
  - When $V = 0$, then the E.P.E = 0
- The change in E.P.E, or work done, for a point charge q at a distance r_1 from the centre of a larger charge Q, to a distance of r_2 further away can be written as:

$$\Delta E.P.E = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

- Where:
  - Q = charge that is producing the electric field (C)
  - q = charge that is moving in the electric field (C)
  - r_1 = first distance of q from the centre of Q (m)
  - r_2 = second distance of q from the centre of Q (m)

YOUR NOTES



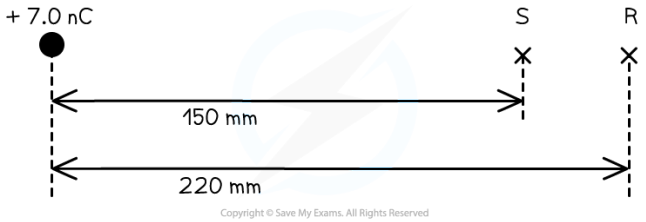
Work is done when moving a point charge away from another charge

- Work is done when a positive charge in an electric field moves **against** the electric field lines or when a negative charge moves **with** the electric field lines

YOUR NOTES

Worked Example

The potentials at points **R** and **S** due to the +7.0 nC charge are 675 V and 850 V respectively.



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Calculate how much work is done when a +3.0 nC charge is moved from **R** to **S**.

Answer:

Step 1: Write down the known quantities

- p.d. at **R**, $V_1 = 675$ V
- p.d. at **S**, $V_2 = 850$ V
- Charge, $q = +3.0$ nC $= +3.0 \times 10^{-9}$ C

Step 2: Write down the work done equation

$$W = q\Delta V$$

Step 3: Substitute in the values into the equation

$$W = (3.0 \times 10^{-9}) \times (850 - 675) = 5.3 \times 10^{-7} \text{ J}$$

Exam Tip

Remember that q in the work done equation is the charge that is being moved, whilst Q is the charge which is producing the potential. Make sure not to get these two mixed up, as both could be given in the question (like the worked example) and you will be expected to choose the correct one

YOUR NOTES

Electrical Potential Difference

- Two points at different distances from a charge will have different electric potentials
  - This is because the electric potential increases with distance from a negative charge and decreases with distance from a positive charge
- Therefore, there will be an **electric potential difference** between the two points
  - This is represented by the symbol ΔV
- ΔV is normally given as the equation

$$\Delta V = V_f - V_i$$

- Where:
  - V_f = final electric potential ($J\,C^{-1}$)
  - V_i = initial electric potential ($J\,C^{-1}$)

Electrical Potential Energy

- The electric potential energy E_p at point in an electric field is defined as:

The work done in bringing a charge from infinity to that point

- The electric potential energy of a pair of point charges Q_1 and Q_2 is defined by:

$$E_p = k\frac{Q_1 Q_2}{r}$$

- Where:
  - E_p = electric potential energy (J)
  - $k = \frac{1}{4\pi\varepsilon_0}$
  - r = separation of the charges Q_1 and Q_2 (m)
  - ε_0 = permittivity of free space ($F\,m^{-1}$)
- The potential energy equation is defined by the work done in moving point charge Q_2 from infinity towards a point charge Q_1

YOUR NOTES

Worked Example

An alpha particle $^{4}_{2}\mathrm{He}$ is moving directly towards a stationary gold nucleus $^{197}_{79}\mathrm{Au}$.

At a distance of 4.7×10^{-15} m, the alpha particle momentarily comes to rest.

Calculate the electric potential energy of the particles at this instant.

Answer:

Step 1: Write down the known quantities

- Distance, $r = 4.7 \times 10^{-15}$ m
- The charge of one proton = $+1.60 \times 10^{-19}$ C

  An alpha particle (helium nucleus) has 2 protons

  - Charge of alpha particle, $Q_1 = 2 \times 1.60 \times 10^{-19} = +3.2 \times 10^{-19}$ C

The gold nucleus has 79 protons

  - Charge of gold nucleus, $Q_2 = 79 \times 1.60 \times 10^{-19} = +1.264 \times 10^{-17}$ C

Step 2: Write down the equation for electric potential energy

$$E_p = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}$$

Step 3: Substitute values into the equation

$$E_p = \frac{(1.264 \times 10^{-17}) \times (3.2 \times 10^{-19})}{(4\pi \times 8.85 \times 10^{-12}) \times (4.7 \times 10^{-15})} = \mathbf{7.7 \times 10^{-12}\ J}\ (2\ \text{s.f})$$

Exam Tip

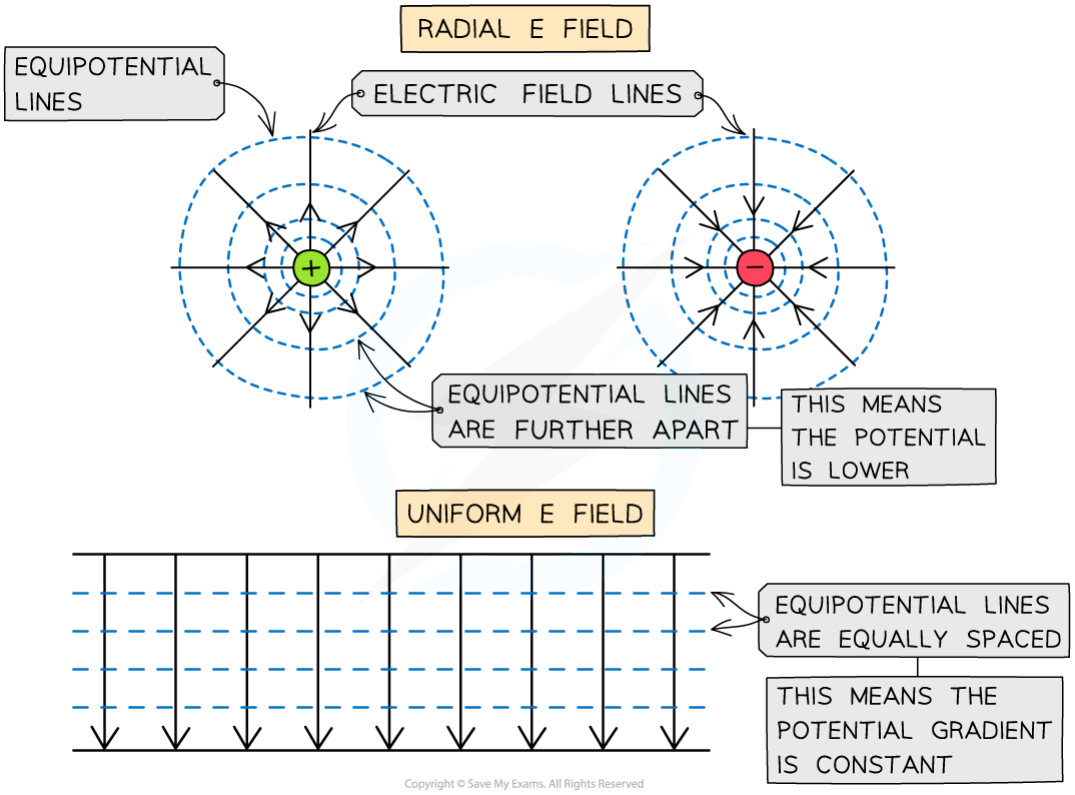
When calculating electric potential energy, make sure you **do not** square the distance!

Electric Potential Gradient

- An electric field can be defined in terms of the variation of electric potential at different points in the field:

  The electric field at a particular point is equal to the negative gradient of a potential-distance graph at that point

- The potential gradient is defined by the **equipotential lines**
  - These demonstrate the electric potential in an electric field and are always drawn **perpendicular** to the field lines



Equipotential lines around a radial field or uniform field are perpendicular to the electric field lines

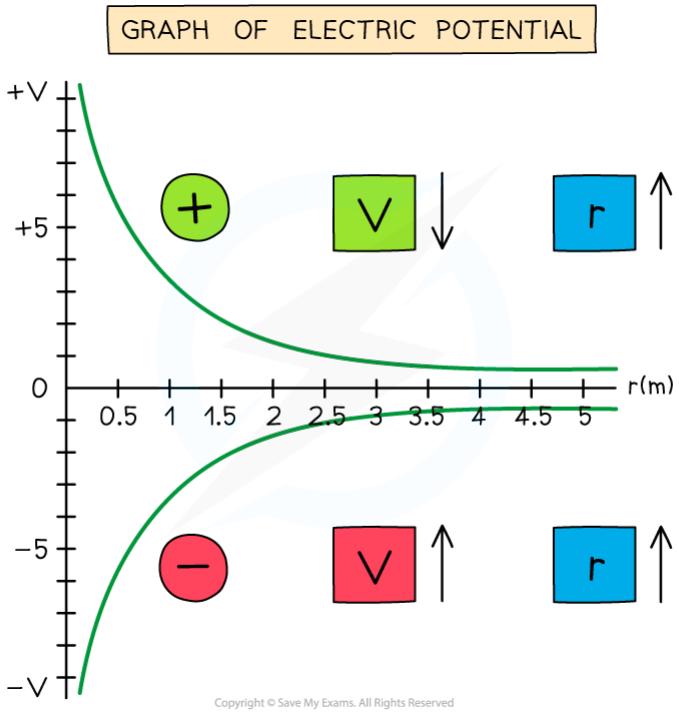
- Equipotential lines are lines of **equal electric potential**
  - Around a radial field, the equipotential lines are represented by concentric circles around the charge with increasing radius
  - The equipotential lines become further away from each other
  - In a uniform electric field, the equipotential lines are equally spaced
- The potential gradient in an electric field is defined as:

The rate of change of electric potential with respect to displacement in the direction of the field

- The electric field strength is equivalent to this, except with a negative sign:

$$E = -\frac{\Delta V}{\Delta r}$$

- Where:
  - E = electric field strength (V m^{-1})
  - ΔV = change in potential (V)
  - Δr = displacement in the direction of the field (m)
- The minus sign is important to obtain an attractive field around a negative charge and repulsive field around a positive charge



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The electric potential around a positive charge decreases with distance and increases with distance around a negative charge

- The electric potential changes according to the charge creating the potential as the distance *r* increases from the centre:
  - If the charge is **positive**, the potential **decreases** with distance
  - If the charge is **negative**, the potential **increases** with distance
- This is because the test charge is positive

Exam Tip

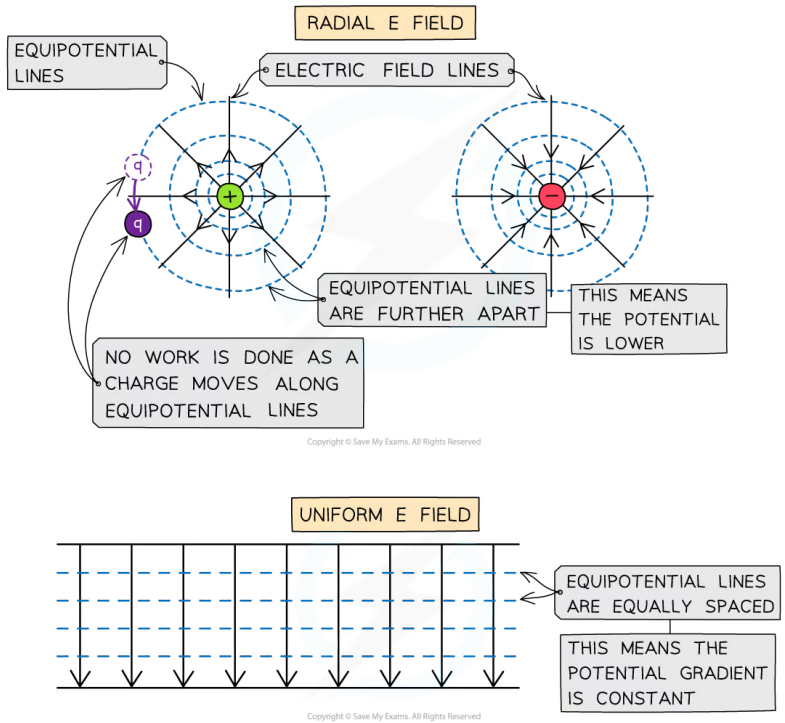
One way to remember whether the electric potential increases or decreases with respect to the distance from the charge is by the direction of the electric field lines. The potential always **decreases** in the **same** direction as the field lines and vice versa.

YOUR NOTES

YOUR NOTES

Electric Equipotential Surfaces

- Equipotential lines (when working in 2D) and surfaces (when working in 3D) join together points that have the same electric potential
- These are always:
  - **Perpendicular** to the electric field lines in both radial and uniform fields
  - Represented by **dotted** lines (unlike field lines, which are solid lines with arrows)
- In a radial field (eg. a point charge), the equipotential lines:
  - Are concentric circles around the planet
  - Become further apart further away from the planet
- In a uniform field (eg. between two metal plates of equal but opposite charge), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced
- Potential gradient is defined by the **equipotential lines**
- **No work is done** when moving along an equipotential line or surface, only **between** equipotential lines or surfaces
  - This means that an object travelling along an equipotential doesn't lose or gain energy and $\Delta V = 0$



Equipotential lines around a radial field or uniform field are perpendicular to the electric field lines

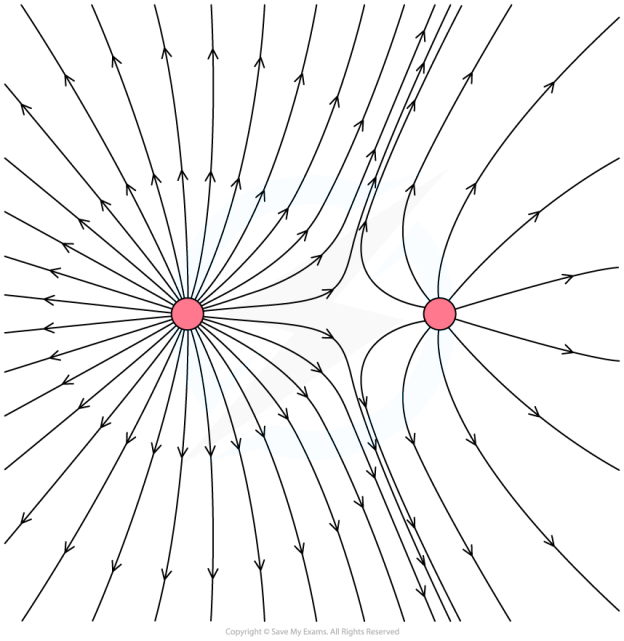
- The distinction between radial and uniform fields is an important one
- In a **radial field** (eg. a point charge), the equipotential lines:
  - Are concentric circles around the charge
  - Become further apart further away from the charge
  - Remember: **radial** field is made up of lines which follow the **radius** of a circle
- In a **uniform field** (eg. between charged parallel plates), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced
  - Remember: **uniform** field is made up of lines which are a **uniform** distance apart

YOUR NOTES

YOUR NOTES

Worked Example

In the following diagram, two electric charges are shown which include the electric field lines



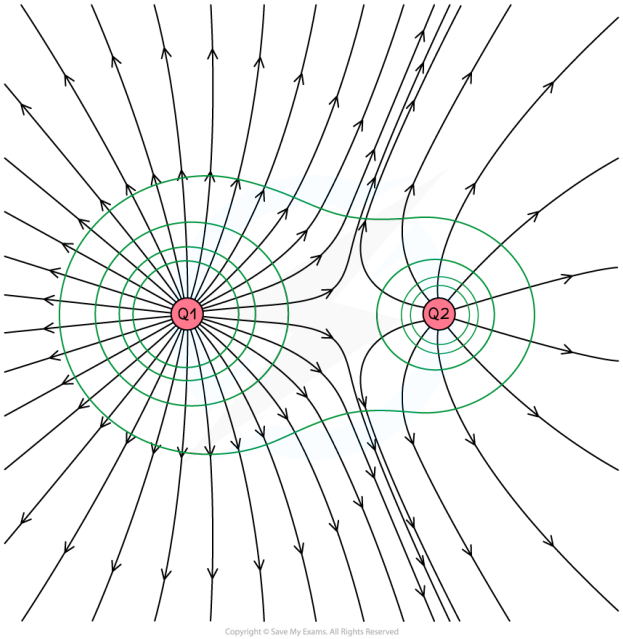
(a) Draw the lines of equipotential including at least four lines and at least one that encircles both charges

(b) By considering the field lines and equipotentials from part (a), state what can be assumed about the two charges

Answer:

Part (a)

- The lines of equipotential need to be perpendicular to the field lines at all times
- These lines are almost circular when they are near the charges
- And when moving out further the lines of equipotential cover both charges.
- The lines of equipotential can be seen below



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Part (b)

- It can be assumed that both charges are positive since the field lines point outwards.
- It can also be assumed that the charge on the left has a larger charge than the charge on the right since:
  - It has a greater density of field lines
  - It has a larger sphere of influence shown by the lines of equipotential
  - The point of zero electric field strength between the two charges is closer to the right charge

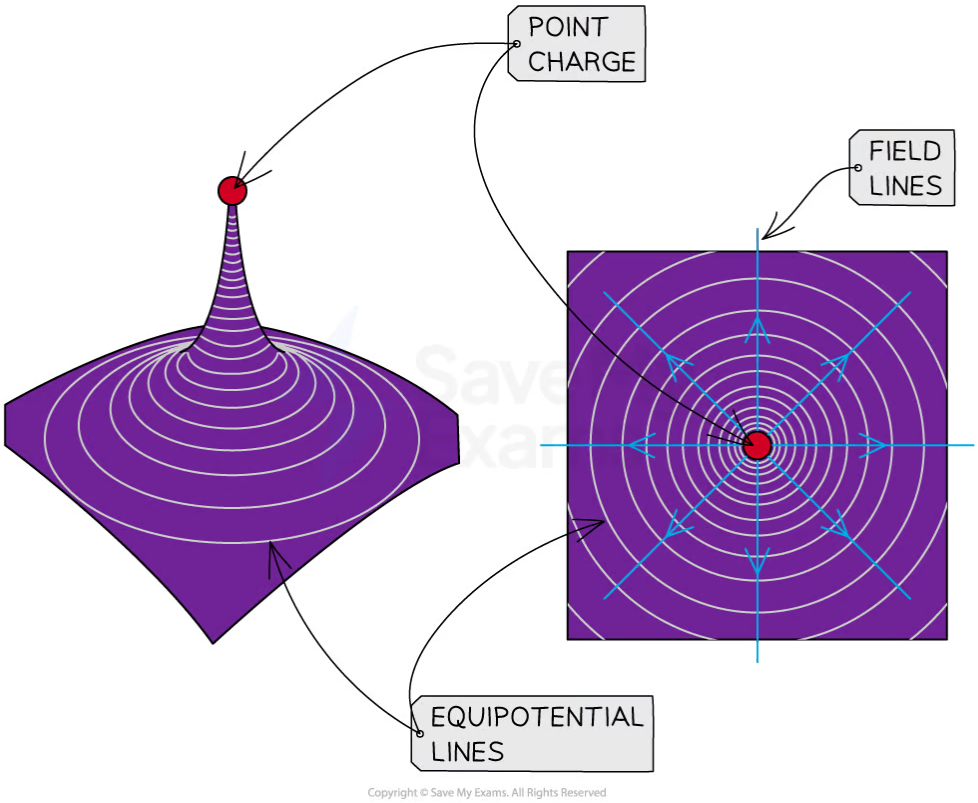
Exam Tip

Remember equipotential lines do **not** have arrows, since they have no particular direction and are not vectors.

Make sure to draw any straight lines with a ruler or a straight edge.

Equipotential Surfaces & Electric Field Lines

- The **electric field** is always **perpendicular** to the lines of **equipotential**
- If a charge moves along a line of equipotential the force acting on it is perpendicular to the direction of motion
- This means no work is done, so the particle's potential energy does not change



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The left side image shows the equipotential lines for a point charge acting like contours on a map. They are perpendicular to the field lines, as shown on the right side image.

YOUR NOTES

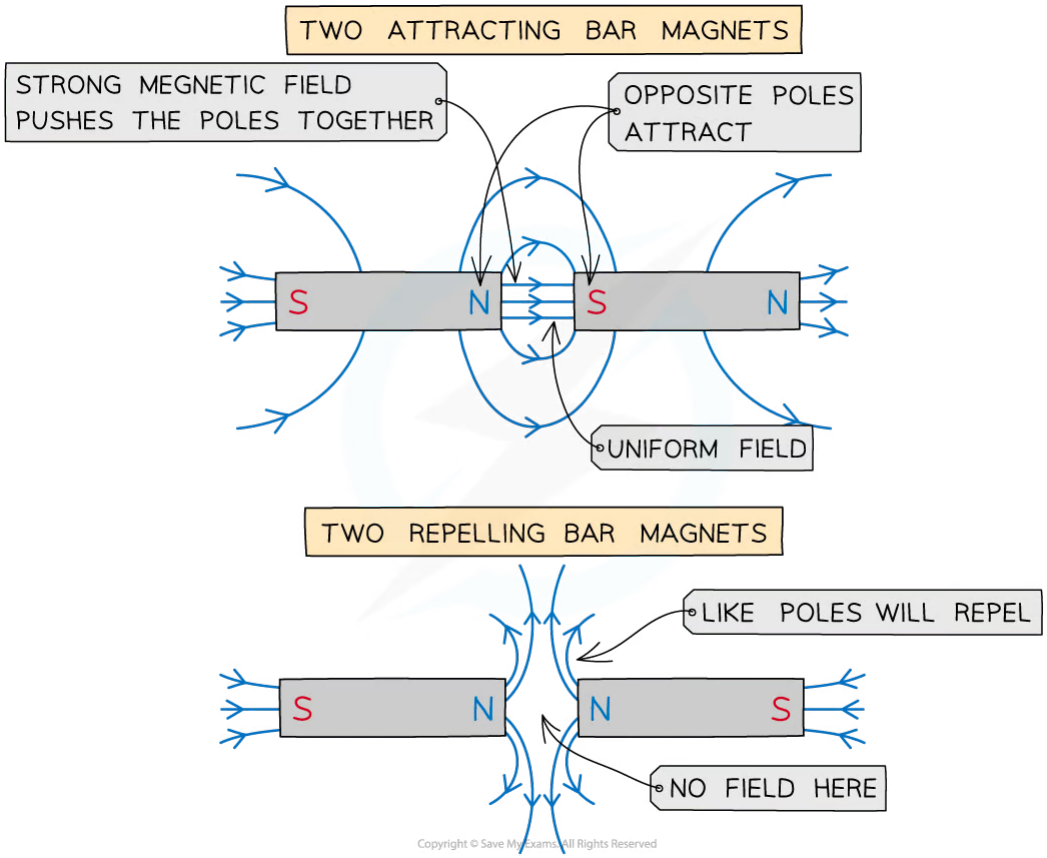
Representing Magnetic Fields

Magnetic Field Definition

- A magnetic field is a **field of force** that is created either by:
  - **Moving** electric charge
  - **Permanent** magnets
- Permanent magnets are materials that produce a magnetic field
- A stationary charge will **not** produce a magnetic field
- A magnetic field is sometimes referred to as a **B-field**
- A magnetic field is created around a **current carrying wire** due to the movement of electrons
- Although magnetic fields are invisible, they can be observed by the force that pulls on magnetic materials, such as iron or the movement of a needle in a plotting compass

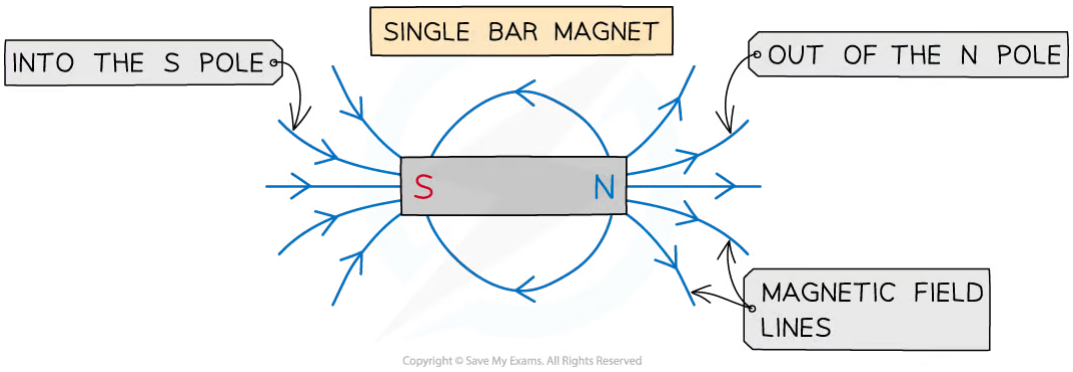
Representing Magnetic Fields

- Magnetic fields are represented by magnetic field lines
  - These can be shown using iron filings or plotting compasses
- Field lines are best represented on **bar magnets**, which consist of a north pole on one end and south pole on the other
- The magnetic field is produced on a bar magnet by the movement of electrons within the atoms of the magnet
- This is a result of the electrons circulating around the atoms, representing a tiny current and hence setting up a magnetic field
- The direction of a magnetic field on a bar magnet is always from **north to south**

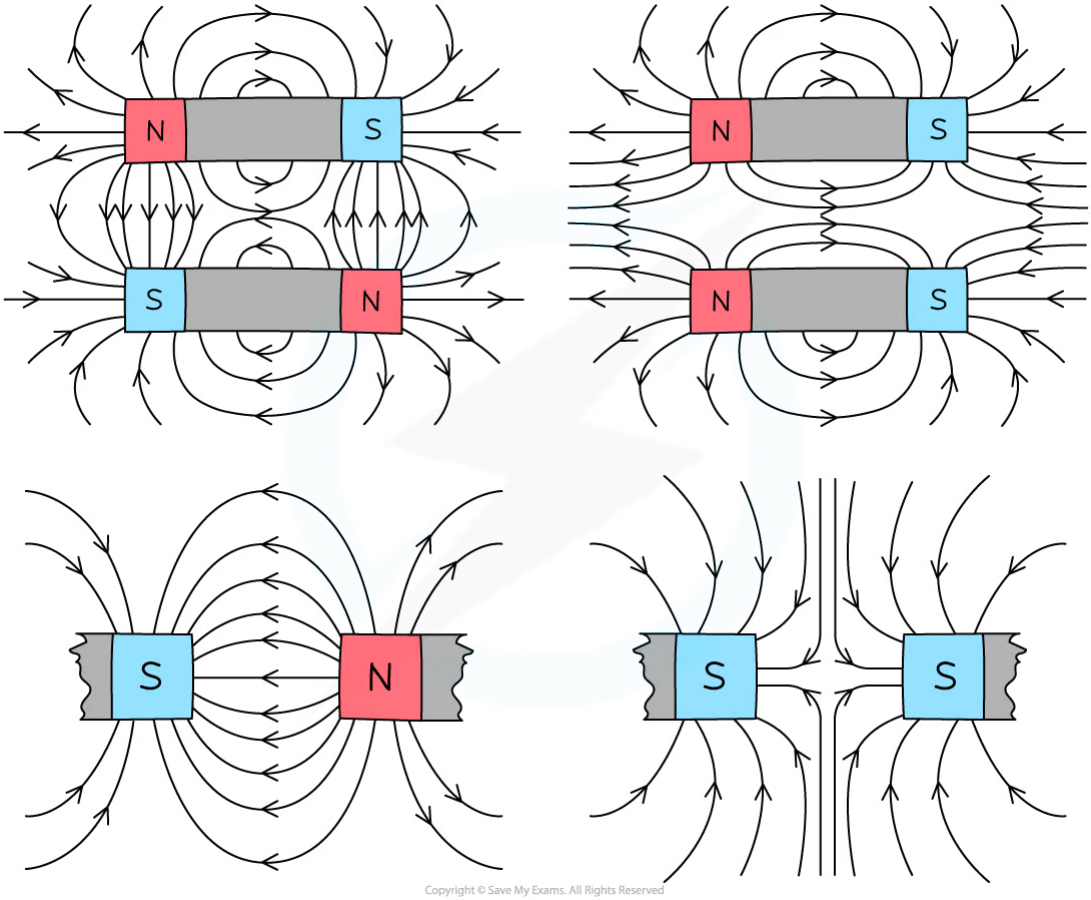


Magnetic field lines are directed from the north pole to the south pole

- When two bar magnets are pushed together, they either attract or repel each other:
  - Two **like** poles (north and north or south and south) **repel** each other
  - Two **opposite** poles (north and south) **attract** each other



Two opposite poles attract each other and two like poles repel each other



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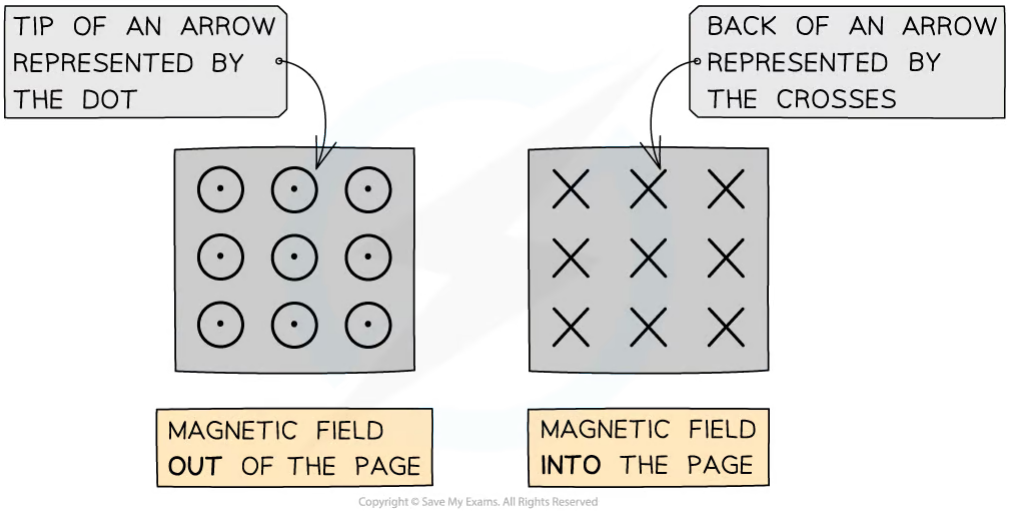
Magnetic field lines between two bar magnets

- **The key aspects of drawing magnetic field lines:**
  - The lines come **out** from the north poles and **into** the south poles
  - The direction of the field line shows the direction of the force that a free magnetic north pole would experience at that point
  - The field lines are **stronger** the **closer** the lines are together
  - The field lines are **weaker** the **further apart** the lines are
  - Magnetic field lines **never** cross since the magnetic field is unique at any point
  - Magnetic field lines are **continuous**
- A uniform magnetic field is where the magnetic field strength is the same at all points
  - This is represented by equally spaced parallel lines, just like electric fields
- The direction of the magnetic field into or out of the page in 3D is represented by the following symbols:
  - Dots (sometimes with a circle around them) represent the magnetic field directed **out** of the plane of the page
  - Crosses represent the magnetic field directed **into** the plane of the page



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The magnetic field into or out of the page is represented by circles with dots or crosses

Exam Tip

The best way to remember which way around to draw magnetic fields in 3D is by imagining an arrow coming towards or away from you

- When the head of an arrow is coming towards you, you see the tip as a dot representing the arrow coming **'out'** of the page
- When an arrow is travelling away from you, you see the cross at the back of the arrow representing the arrow going **'into'** the page

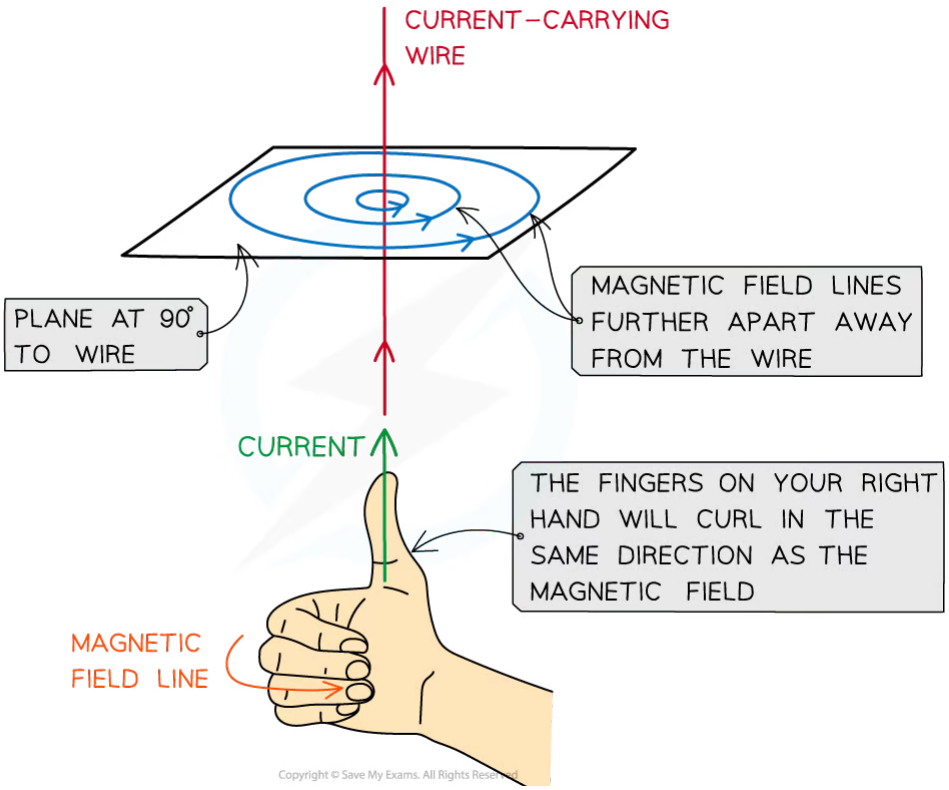
Right Hand Grip Rule

Magnetic Fields in Wires, Coils & Solenoids

- Magnetic field patterns are not only observed around bar magnets, magnetic fields are formed wherever current is flowing, such as in:
  - Long straight wires
  - Long solenoids
  - Flat circular coils

Field Lines in a Current-Carrying Wire

- Magnetic field lines in a current carrying wire are circular rings, centered on the wire
- The field lines are strongest near the wire and become further part away from the wire
- Reversing the current reverses the direction of the field



The direction of magnetic field lines on a current carrying wire can be determined by the right hand thumb rule

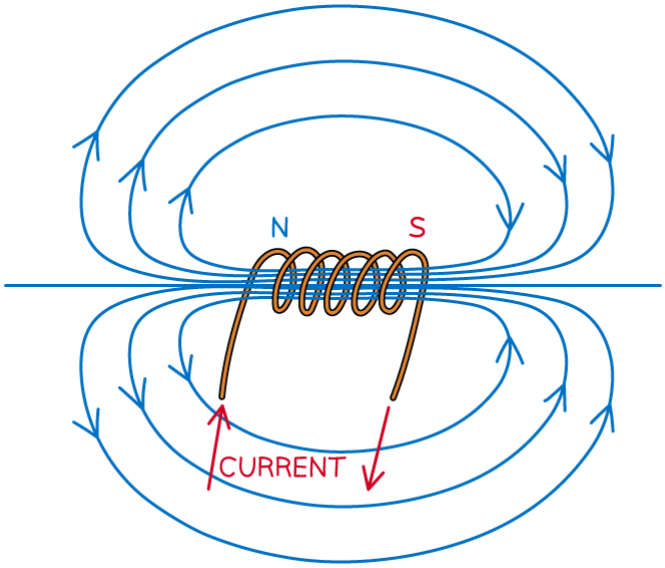
- The field lines are clockwise or anticlockwise around the wire, depending on the direction of the current
- The direction of the magnetic field is determined by **Maxwell's right hand screw rule**
  - This is determined by pointing the **right-hand** thumb in the direction of the current in the wire and curling the fingers onto the palm
  - The direction of the curled fingers represents the direction of the magnetic field around the wire

YOUR NOTES

- For example, if the current is travelling vertically upwards, the magnetic field lines will be directed anticlockwise, as seen from directly above the wire
- **Note:** the direction of the current is taken to be the conventional current ie. from **positive** to **negative**, **not** the direction of electron flow

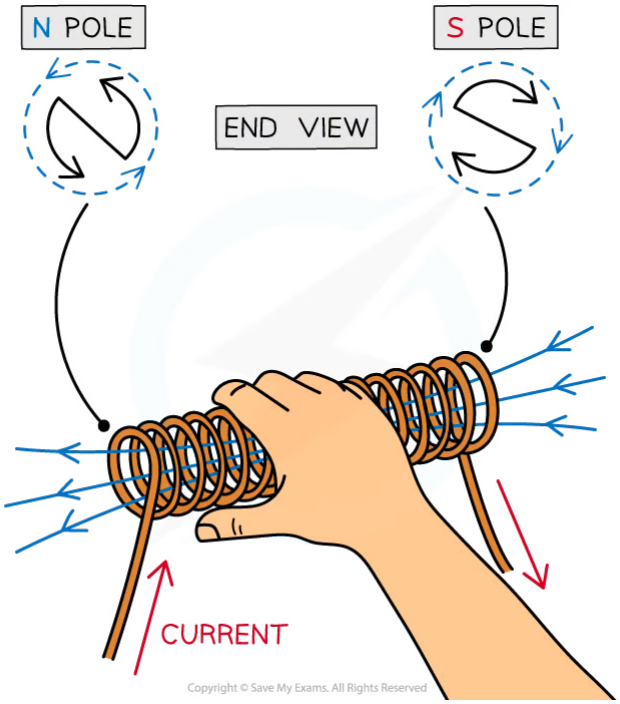
Field Lines in a Solenoid

- As seen from a current carrying wire, an electric current produces a magnetic field
- An electromagnetic makes use of this by using a coil of wire called a **solenoid** which concentrates the magnetic field
- One ends becomes a north pole and the other the south pole



Magnetic field lines around a solenoid are similar to a bar magnet

- Therefore, the magnetic field lines around a solenoid are very similar to a bar magnet
  - The field lines **emerge** from the **north** pole
  - The field lines **return** to the **south** pole
- Which is the north or south pole depends on the direction of the current
  - This is found by the **right-hand grip rule**
- This involves gripping the electromagnet so the fingers represent the direction of the current flow of the wire
- The thumb points in the direction of the field lines inside the coil, or in other words, point towards the electromagnet's **north pole**

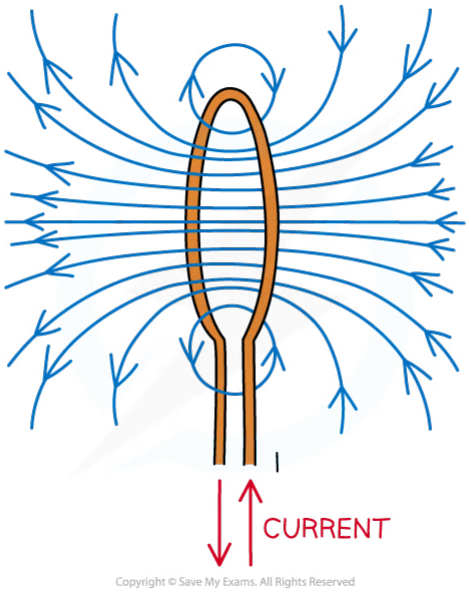


Poles of a Solenoid

Field Lines in a Flat Circular Coil

- A flat circular coil is equal to one of the coils of a solenoid
- The field lines will emerge through one side of the circle (north pole) and leave the other (south pole)
- As before, the direction of the north and south pole depends on the direction of the current
  - This can be determined by using the **right-hand thumb rule**
  - It is easier to find the direction of the magnetic field on the straight part of the circular coil to determine which direction the field lines are passing through

YOUR NOTES



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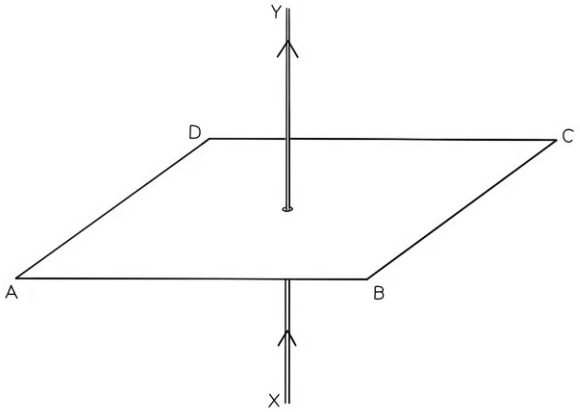
Magnetic field lines of a single circular coil are added up together to make to make the field lines of a solenoid

YOUR NOTES

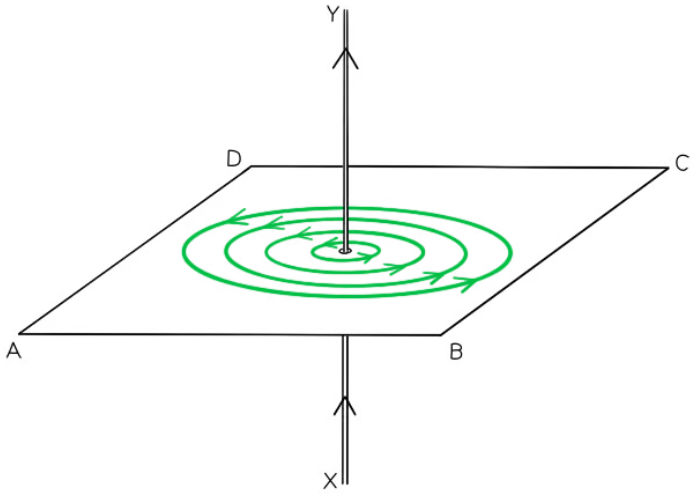
Worked Example

The current in a long, straight vertical wire is in the direction XY, as shown in the diagram.



Sketch the pattern of the magnetic flux in the horizontal plane ABCD due to the current-carrying wire. Draw at least four flux lines.

Answer:



✓Concentric circles

✓Increasing separation between each circle

✓Arrows drawn in anticlockwise direction

Exam Tip

Remember to draw the arrows showing the direction of the field lines on every single field line you draw. Also, ensure that in a uniform magnetic field, the field lines are equally spaced.

YOUR NOTES

IB Physics HL

Motion in Electromagnetic Fields

YOUR NOTES

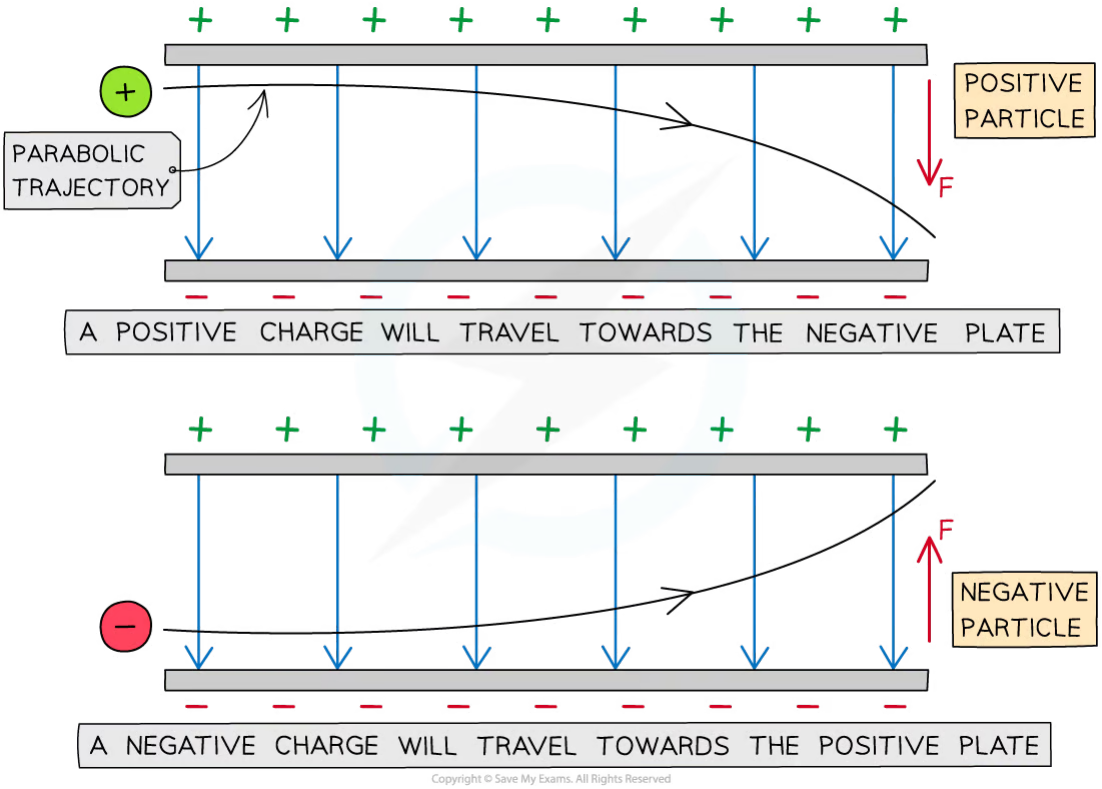
CONTENTS

Charged Particles in Electric Fields
Charged Particles in Magnetic Fields
Charged Particles in Electric & Magnetic Fields
Magnetic Force on a Charge
Magnetic Force on a Current-Carrying Conductor
Magnetic Force between Two Parallel Conductors

YOUR NOTES

Charged Particles in Electric Fields

- A **charged** particle in an **electric** field will experience a **force** on it that will cause it to **move**
- If a charged particle remains **stationary** in a uniform electric field, it will move **parallel** to the electric field lines (along or against the field lines depending on its charge)
- If a charged particle is in **motion** through a uniform electric field (e.g. between two charged parallel plates), it will experience a constant electric force and travel in a **parabolic trajectory**



The parabolic path of charged particles in a uniform electric field

- The direction of the parabola will depend on the charge of the particle
  - A **positive** charge will be deflected towards the **negative** plate
  - A **negative** charge will be deflected towards the **positive** plate
- The force on the particle is the same at all points and is always in the same direction
- **Note:** an uncharged particle, such as a neutron experiences **no** force in an electric field and will therefore travel **straight through** the plates undeflected
- The amount of deflection depends on the following properties of the particles:
  - **Mass** – the **greater** the mass, the **smaller** the deflection and vice versa
  - **Charge** – the **greater** the magnitude of the charge of the particle, the **greater** the deflection and vice versa

- **Speed** – the **greater** the speed of the particle, the **smaller** the deflection and vice versa

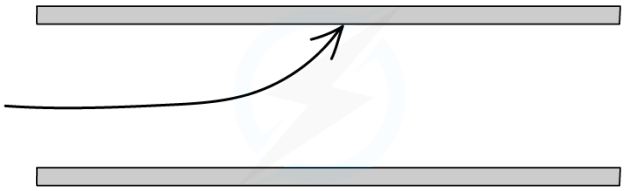
YOUR NOTES

YOUR NOTES

Worked Example

A single proton travelling with a constant horizontal velocity enters a uniform electric field between two parallel charged plates.

The diagram shows the path taken by the proton.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Draw the path taken by a boron nucleus that enters the electric field at the same point and with the same velocity as the proton.

Atomic number of boron = 5

Mass number of boron = 11

Answer:

Step 1:

- Compare the charge of the boron nucleus to the proton
  - Boron has 5 protons, meaning it has a charge 5 × greater than the proton
  - The force on boron will therefore be 5 × greater than on the proton
    - This is from the equation for electric force, F which is proportional to the charge q

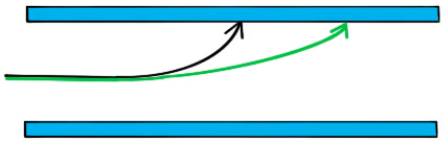
$$F = Eq$$

Step 2:

- Compare the mass of the boron nucleus to the proton
  - The boron nucleus has a mass of 11 nucleons meaning its mass is 11 × greater than the proton
  - The boron nucleus will therefore be **less** deflected than the proton

Step 3:

- Draw the trajectory of the boron nucleus
  - Since the mass comparison is much greater than the charge comparison, the boron nucleus will be **much less deflected** than the proton
  - The nucleus is positively charged since the neutrons in the nucleus have no charge
    - Therefore, the shape of the path will be the same as the proton



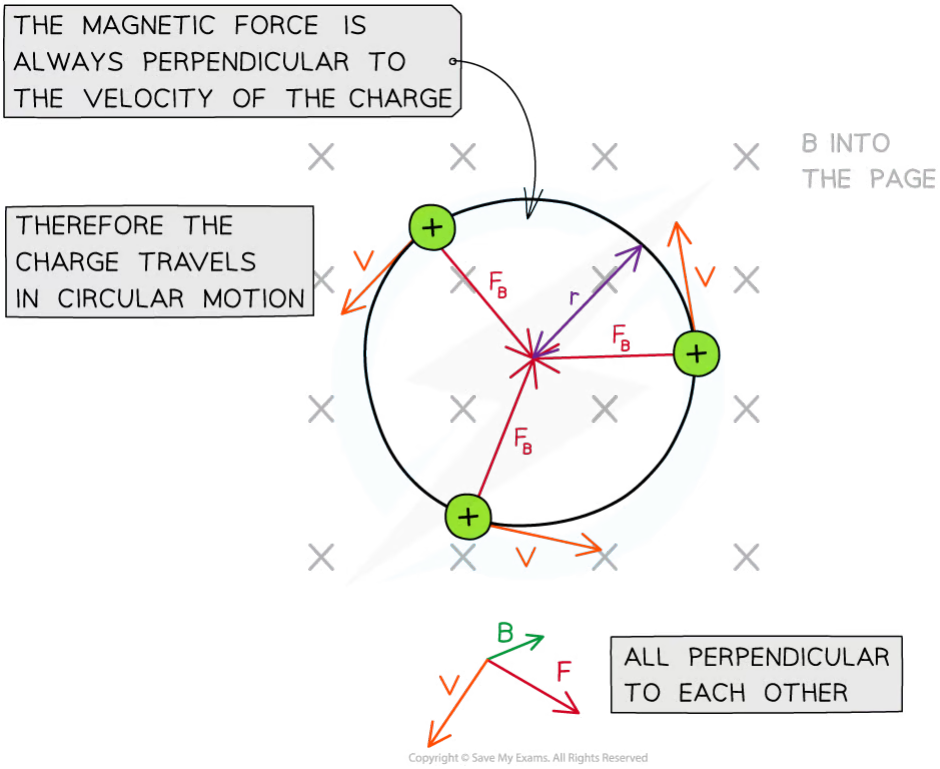
Exam Tip

Remember **less** deflection (e.g. from a higher mass particle) means the particle hits the plate **later** and the path has a smaller **curve**. Think of it as the particle being heavier so it is harder to steer it towards the plate.

YOUR NOTES

Charged Particles in Magnetic Fields

- A charged particle in uniform magnetic field which is perpendicular to its direction of motion travels in a **circular** path
- This is because the magnetic force F_B will always be perpendicular to its velocity v
  - F_B will always be directed towards the centre of the path



A charged particle moves travels in a circular path in a magnetic field

- The magnetic force F_B provides the **centripetal force** on the particle
- Recall the equation for centripetal force:

$$F = \frac{mv^2}{r}$$

- Where:
  - m = mass of the particle (kg)
  - v = linear velocity of the particle (m s^{-1})
  - r = radius of the orbit (m)

- Equating this to the force on a moving charged particle gives the equation:

$$\frac{mv^2}{r} = Bqv$$

YOUR NOTES

- Rearranging for the radius *r* obtains the equation for the radius of the orbit of a charged particle in a perpendicular magnetic field:

$$r = \frac{mv}{Bq}$$

- This equation shows that:
  - Faster moving particles with speed *v* move in larger circles (larger *r*): $\mathbf{r \propto v}$
  - Particles with greater mass *m* move in larger circles: $\mathbf{r \propto m}$
  - Particles with greater charge *q* move in smaller circles: $\mathbf{r \propto 1 / q}$
  - Particles moving in a strong magnetic field *B* move in smaller circles: $\mathbf{r \propto 1 / B}$

Worked Example

An electron with charge-to-mass ratio of 1.8×10^{11} C kg^{-1} is travelling at right angles to a uniform magnetic field of flux density 6.2 mT. The speed of the electron is 3.0×10^{6} m s^{-1}.

Calculate the radius of the circle path of the electron.

Answer:

Step 1: Write down the known quantities

- Charge-to-mass ratio, $\frac{q}{m} = 1.8 \times 10^{11}$ C kg^{-1}
- Magnetic flux density, B = 6.2 mT
- Electron speed, $v = 3.0 \times 10^{6}$ m s^{-1}

Step 2: Write down the equation for the radius of a charged particle in a perpendicular magnetic field

$$r = \frac{mv}{Bq}$$

Step 3: Substitute in values

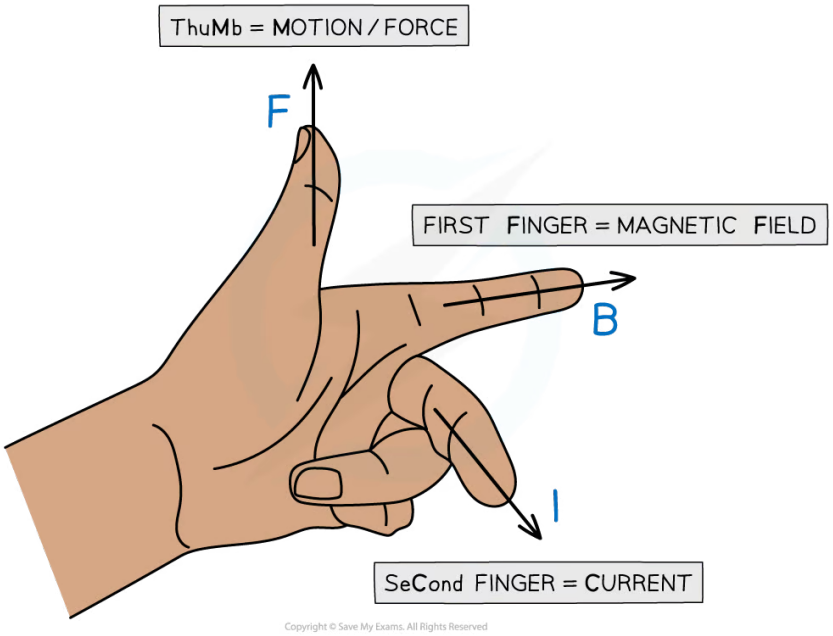
$$\frac{m}{q} = \frac{1}{1.8 \times 10^{11}}$$

$$r = \frac{(3.0 \times 10^{6})}{(1.8 \times 10^{11})(6.2 \times 10^{-3})} = 2.688 \times 10^{-3} \text{ m} = \mathbf{2.7 \text{ mm}} \text{ (2 s.f.)}$$

Flemming's Left-Hand Rule

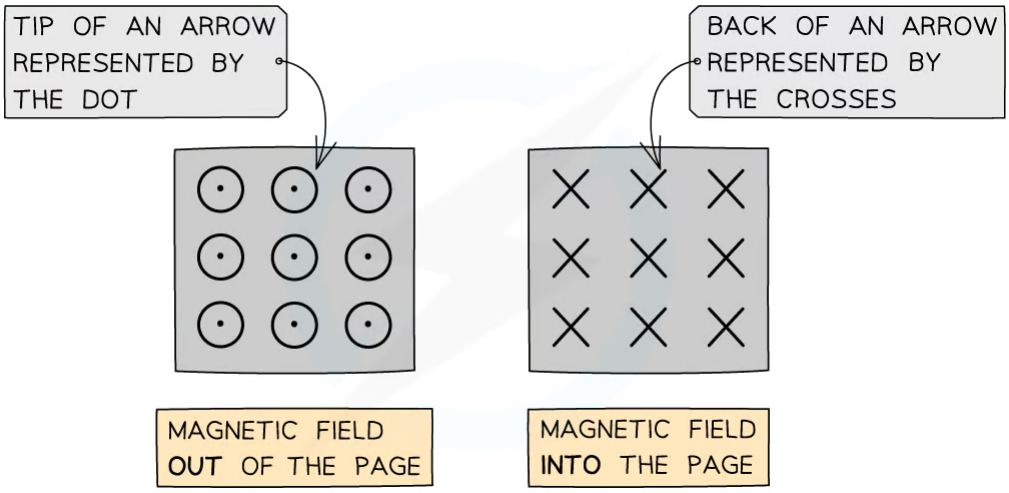
- The **direction** of the force on a charge moving in a **magnetic field** is determined by the direction of the magnetic field and the current
  - Recall that the direction of the current is in the direction of **conventional** current flow (positive to negative)
- When the force, the magnetic field and the current are all mutually perpendicular to each other, the directions of each can be interpreted by **Fleming's left-hand rule**

- The **thumb** points in the direction of **motion** of the rod (or the direction of the force) (*F*)
- The **first** finger points in the direction of the external **magnetic field** (*B*)
- The **second** finger points in the direction of conventional **current** flow (*I*)On the left hand, with the thumb pointed upwards, first finger forwards and the second finger to the right, ie. all three are perpendicular to each other



Fleming's left-hand rule

- Since this is represented in 3D space, sometimes the current, force or magnetic field could be directed into or out of the page, not just left, right, up and down
- The direction of the magnetic field into or out of the page in 3D is represented by the following symbols:
  - Dots (sometimes with a circle around them) represent the magnetic field directed **out** of the plane of the page
  - Crosses represent the magnetic field directed **into** the plane of the page



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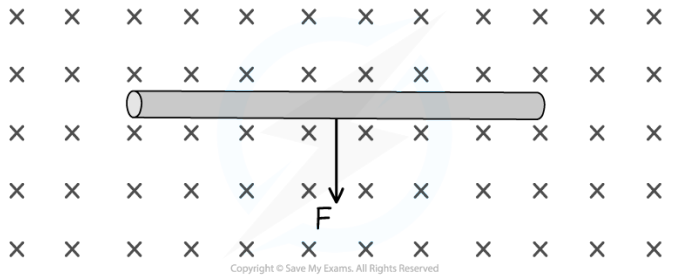
The magnetic field into or out of the page is represented by circles with dots or crosses

- The way to remember this is by imagining an arrow used in archery or darts:
  - If the arrow is approaching **head-on**, such as out of a page, only the very tip of the arrow can be seen (a dot)
  - When the arrow is **receding away**, such as into a page, only the cross of the feathers at the back can be seen (a cross)

YOUR NOTES

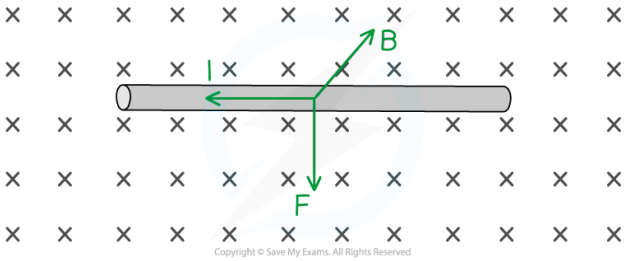
Worked Example

State the direction of the current flowing in the wire in the diagram below.



Answer:

- Using Fleming's left-hand rule:
  - *B* = into the page
  - *F* = vertically downwards
  - *I* = from right to left



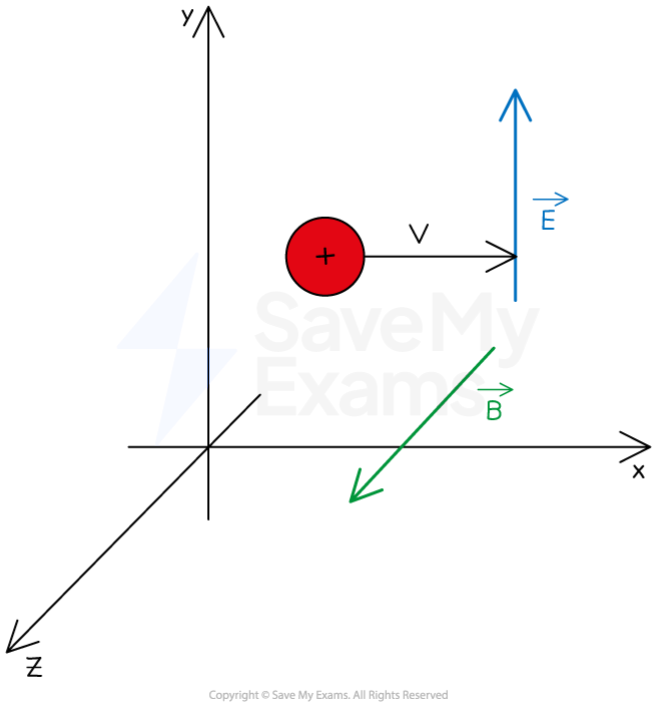
Exam Tip

Don't be afraid to use Fleming's left-hand rule during an exam. Although, it is best to do it subtly in order not to give the answer away to other students!

YOUR NOTES

Charged Particles in Electric & Magnetic Fields

- A **charged particle** moving at a **constant speed** in **electric** and **magnetic fields** orientated at right angles to each other will continue to travel at a **constant speed**
- One particular orientation is:
  - Motion of particle, *v* to the right
  - Magnetic field *B* coming out of the page on the *z-axis*
  - Electric field *E* going up the page on the *y*-axis
- Hence, the three vectors are **perpendicular** to each other



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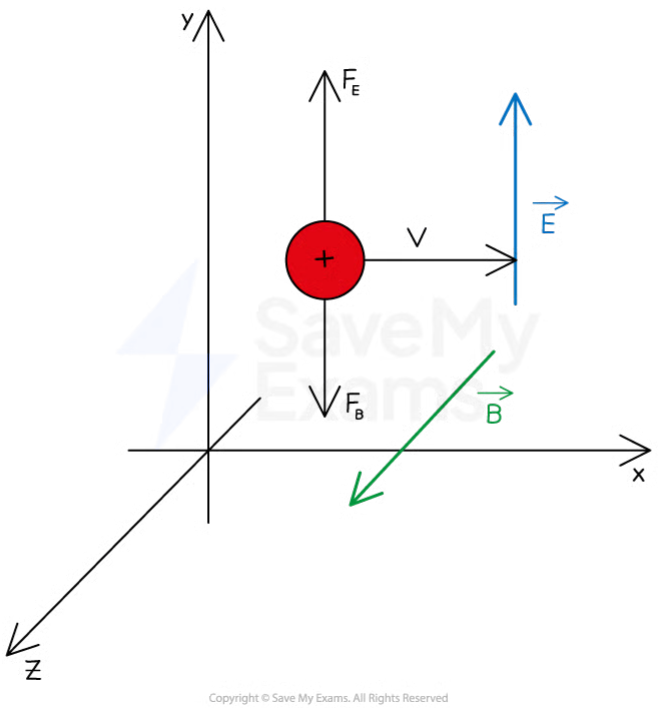
An example of the orientation of an electric field perpendicular to a magnetic field

Situation 1: Moving Positively Charged Particle

- The **electric force** is **parallel** to the **electric field**, so it acts up the page
- The **magnetic force** is **perpendicular** to the **magnetic field** according to Fleming's left hand rule
- When applied to the orientation above:
  - The positive charge moves to the right, this is the **second** finger
  - The magnetic field comes out of the page, this is the **first** finger
  - So, the magnetic force is acting down the page, this is the **thumb**
- Hence, the electric force and magnetic force act in **opposite directions** on the charge



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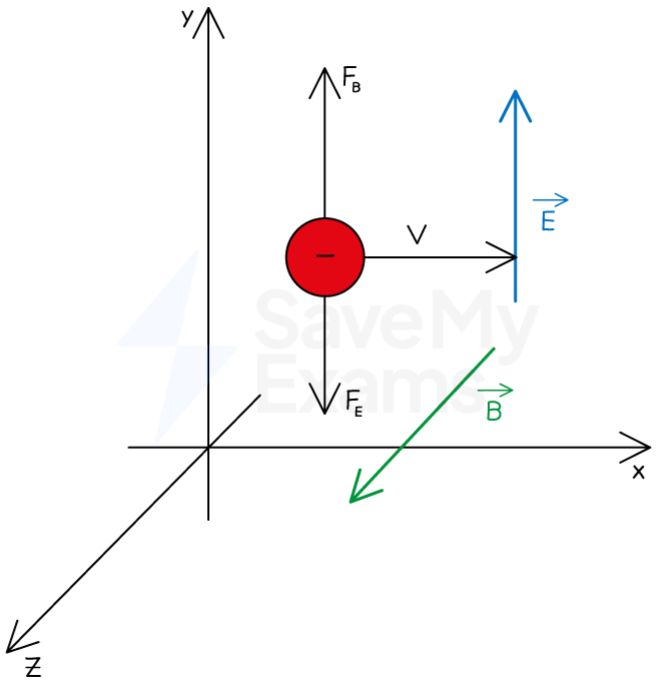
The electric force acts up the page and the magnetic force acts in the opposite direction down the page

Situation 2: Moving Negatively Charged Particle

- The charge is **negative**, so the **electric force** acts in the **opposite direction** to the electric field, down the page
- The magnetic force is perpendicular to the magnetic field according to the Fleming's left hand rule
- When applied to the orientation above:
  - A positive charge moves to the **left,** this is the **second** finger (since the negative moves to the right, in the opposite direction)
  - The magnetic field comes **out** of the page, this is the **first** finger
  - So, the magnetic force is acting **up** the page, this is the **thumb**



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The magnetic force acts up the page and the electric force acts in the opposite direction down the page

Strength of Forces

- Adjusting the strengths of the two fields the magnitude of the electric and magnetic forces can be made equal
- So the forces cancel each other out
- Hence, the particle continues its original motion in a straight line with its constant speed *v*

Calculating Particle Velocity

- When the two forces are equal then electric force, F_E = magnetic force, F_B

$$\boldsymbol{F_E = Eq} \text{ and } \boldsymbol{F_B = Bqv}$$

$$\text{So, } \boldsymbol{Eq = Bqv}$$

$$\text{and } \boldsymbol{Bv = E}$$

$$\text{Hence, } v = \frac{E}{B}$$

- The equation for the magnetic force F_B on charge is introduced in this revision note

Worked Example

An electron passes between two parallel metal plates moving with a constant velocity of 2.1×10^7 m s^{-1}. The potential difference between the plates is 3100 V. A uniform magnetic field of magnitude 0.054 T acts perpendicular to the electric field and the movement of the electron.

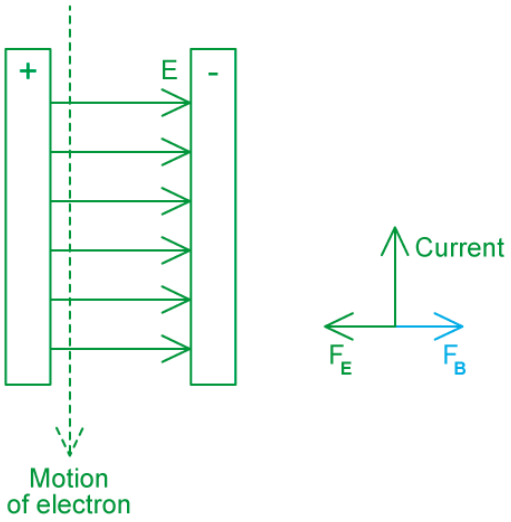
The electric field acts to the right and the electron is moving downwards.

(a)
Determine the direction of the magnetic field
(b)
Calculate the separation of the plates

Answer:

(a) The direction of the magnetic field:

Step 1: Draw a diagram of the situation



- The electric field goes (from the positive plate to the negative plate), to the right
- The electron is moving vertically downwards
- So, the current is moving upwards in the opposite direction to the electron
- The electric force is acting in the opposite direction to the electric field because the particle is an electron

The answer is:

- The magnetic force acts in an equal and opposite direction to the electric force
- Hence the magnetic force acts to the right

Step 2: Determine the direction of the magnetic field

- The electron is moving at a constant speed, so the magnetic and electric forces are equal and opposite
  - Hence, the magnetic force acts to the right

(b) Calculate the separation of the plates:

Step 1: Calculate the magnitude of the electric field, *E*

$$v = \frac{E}{B} \quad \Rightarrow \quad E = vB$$

$$E = (2.1 \times 10^7) \times 0.054 = 1.134 \times 10^6 \text{ N C}^{-1}$$

Step 2: Calculate the separation of the plates

- Use the electric field strength equation:

$$E = \frac{V}{d} \quad \Rightarrow \quad d = \frac{V}{E}$$

$$d = \frac{3100}{1.134 \times 10^6}$$

$$d = 2.73 \times 10^{-3} \text{ m}$$

Exam Tip

Take time to consider the direction of all components of the electric and magnetic fields.

Remember that the electric and magnetic forces act in the opposite direction for negatively charged particles compared to positively charged.

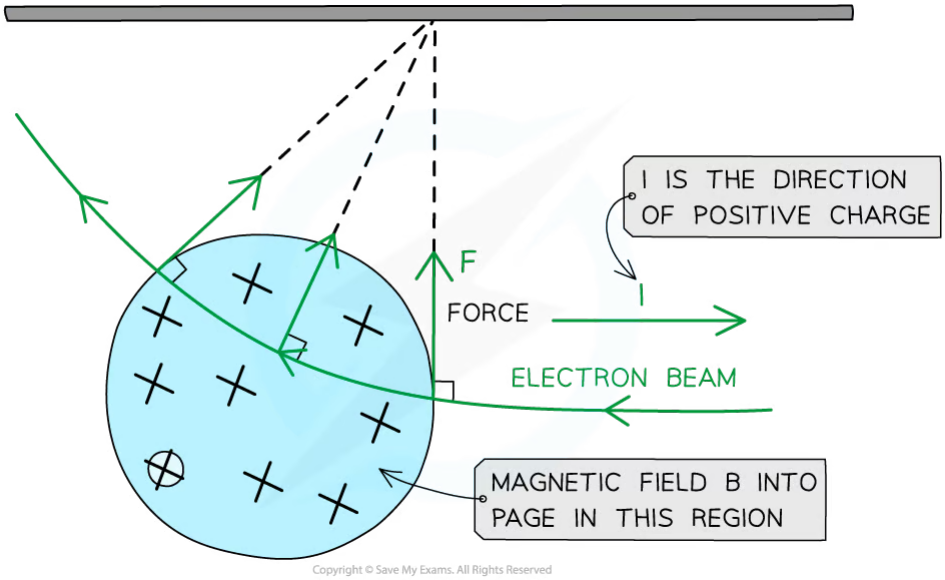
The direction of the charge in Fleming's left hand rule is always the direction of **positive** charge. This should be in the **opposite** direction if the particle has a negative charge!

Magnetic Force on a Charge

- The magnetic force on an isolating moving charge, such an electron, is given by the equation:

$$F = BQv \sin\theta$$

- Where:
  - F = force on the charge (N)
  - B = magnetic flux density (T)
  - Q = charge of the particle (C)
  - v = speed of the charge ($m\ s^{-1}$)
  - θ = angle between charge's velocity and magnetic field (degrees)



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The force on an isolated moving charge is perpendicular to its motion and the magnetic field B

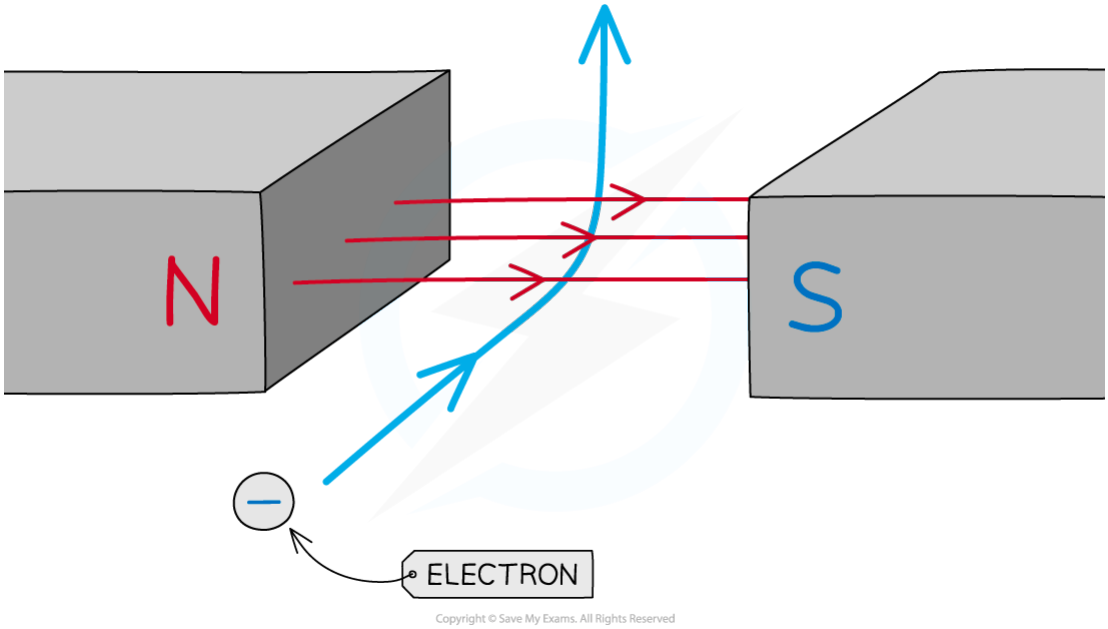
- Equivalent to the force on a wire, if the magnetic field *B* is perpendicular (to the direction of the charge's velocity, the equation simplifies to:

$$F = BQv$$

- This is because $\sin(90) = 1$
- According to Fleming's left hand rule:
  - When an electron enters a magnetic field from the **left**, and if the magnetic field is directed **into the page**, then the force on it will be directed **downwards**
- This equation shows that
- The force due to the magnetic field is always perpendicular to the velocity of the electron
  - **Note:** this is equivalent to circular motion

YOUR NOTES

- Fleming's left-hand rule can be used again to find the direction of the force, magnetic field and velocity
  - The key difference is that the second finger representing current *I* (direction of positive charge) is now the **direction of velocity *v*** of the positive charge



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The electron experiences a force upwards when it travels through the magnetic field between the two poles

YOUR NOTES

Worked Example

An electron is moving at 5.3×10^7 m s^{-1} in a uniform magnetic field of flux density 0.2 T.

Calculate the force on the electron when it is moving at 30° to the field, and state the factor it increases by compared to when it travels perpendicular to the field.

Answer:

Step 1: Write out the known quantities

- Speed of the electron, $v = 5.3 \times 10^7$ m s^{-1}
- Charge of an electron, $Q = 1.60 \times 10^{-19}$ C
- Magnetic flux density, $B = 0.2$ T
- Angle between electron and magnetic field, $\theta = 30^\circ$

Step 2: Write down the equation for the magnetic force on an isolated particle

$$F = BQv \sin\theta$$

Step 3: Substitute in values, and calculate the force on the electron at 30°

$$F = (0.2) \times (1.60 \times 10^{-19}) \times (5.3 \times 10^7) \times \sin(30) = 8.5 \times 10^{-13} \text{ N}$$

Step 4: Calculate the electron force when travelling perpendicular to the field

$$F = BQv = (0.2) \times (1.60 \times 10^{-19}) \times (5.3 \times 10^7) = 1.696 \times 10^{-12} \text{ N}$$

Step 5: Calculate the ratio of the perpendicular force to the force at 30°

$$\frac{1.696 \times 10^{-12}}{8.5 \times 10^{-13}} = 1.995 = \mathbf{2}$$

- Therefore, the force on the electron is twice as strong when it is moving perpendicular to the field than when it is moving at 30° to the field

Exam Tip

Remember not to mix this up with F = BIL!

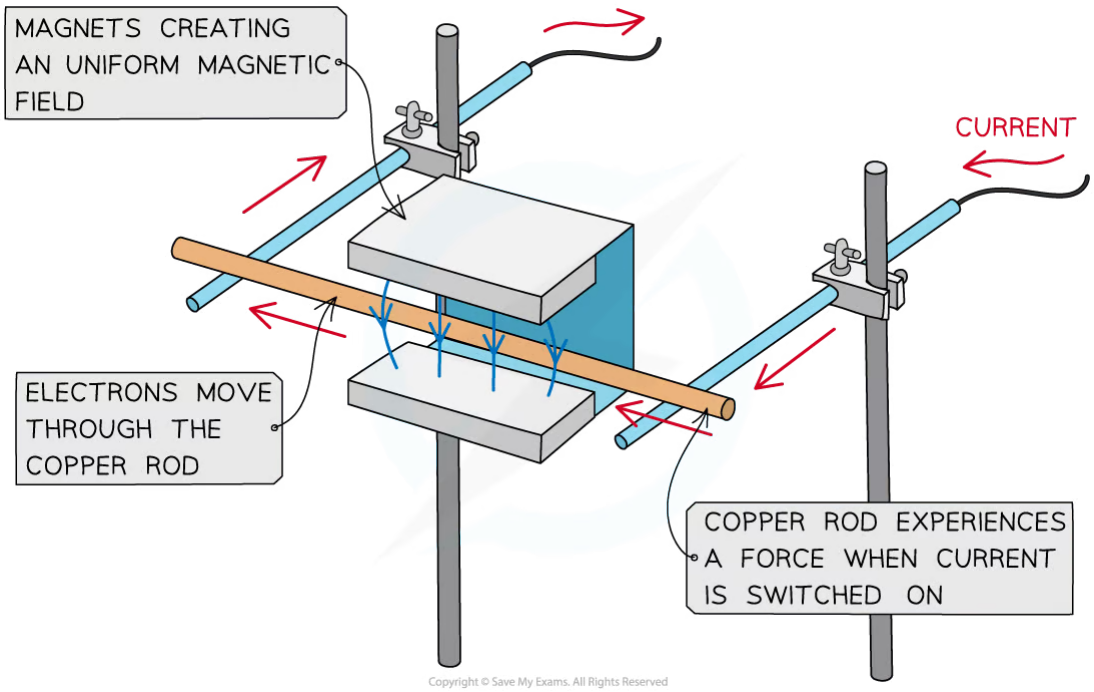
- **F = BIL** is for a current carrying conductor
- **F = Bqv** is for an isolated moving charge (which may be inside a conductor)

It is important to note that when the **moving charge** is travelling **along the field direction** - precisely with or against the field lines - then there is **no magnetic force** on that charge! There is only a force if the charge if not parallel to the field.

YOUR NOTES

Magnetic Force on a Current-Carrying Conductor

- A current-carrying conductor produces its own magnetic field
  - When interacting with an external magnetic field, it will experience a **force**
- A current-carrying conductor will only experience a force if the current through it is **perpendicular** to the direction of the magnetic field lines
- A simple situation would be a copper rod placed within a uniform magnetic field
- When current is passed through the copper rod, it experiences a force that makes it move



A copper rod moves within a magnetic field when current is passed through it

Calculating Magnetic Force on a Current-Carrying Conductor

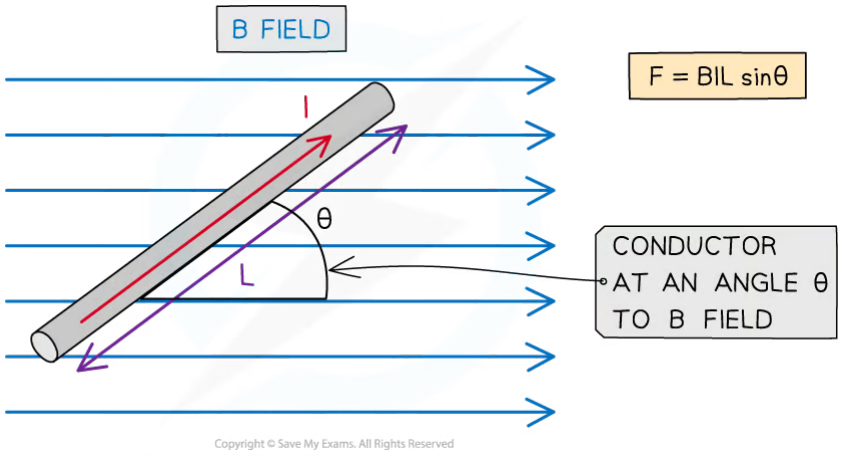
- The strength of a magnetic field is known as the **magnetic flux density, B**
  - This is also known as the magnetic field strength
  - It is measured in units of **Tesla (T)**
- The force *F* on a conductor carrying current *I* at right angles to a magnetic field with flux density *B* is defined by the equation

$$F = BIL\sin\theta$$

- Where:
  - F = force on a current carrying conductor in a magnetic field (N)
  - B = magnetic flux density of external magnetic field (T)
  - I = current in the conductor (A)

- L = length of the conductor (m)
- θ = angle between the conductor and external magnetic field (degrees)

- This equation shows that the greater the current or the magnetic field strength, the greater the force on the conductor



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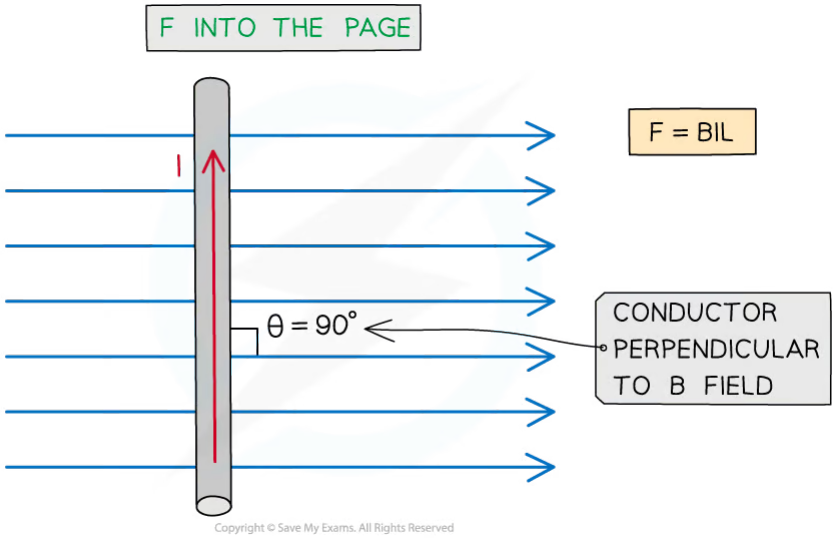

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Magnitude of the force on a current carrying conductor depends on the angle of the conductor to the external B field

- The **maximum** force occurs when sin θ = 1
  - This means θ = 90° and the conductor is **perpendicular** to the B field
  - This equation for the magnetic force now becomes:

$$F = BIL$$

- The **minimum** force (0) is when sin θ = 0
  - This means θ = 0° and the conductor is **parallel** to the B field

- It is important to note that a current-carrying conductor will experience **no** force if the current in the conductor is parallel to the field

YOUR NOTES

Worked Example

A current of 0.87 A flows in a wire of length 1.4 m placed at 30° to a magnetic field of flux density 80 mT.

Calculate the force on the wire.

Answer:

Step 1: Write down the known quantities

- Magnetic flux density, B = 80 mT = 80 × 10^{-3} T
- Current, I = 0.87 A
- Length of wire, L = 1.4 m
- Angle between the wire and the magnetic field, $\theta = 30°$

Step 2: Write down the equation for force on a current-carrying conductor

$$F = BIL \sin\theta$$

Step 3: Substitute in values and calculate

$$F = (80 \times 10^{-3}) \times (0.87) \times (1.4) \times \sin(30) = 0.04872 = 0.049 \text{ N (2 s.f)}$$

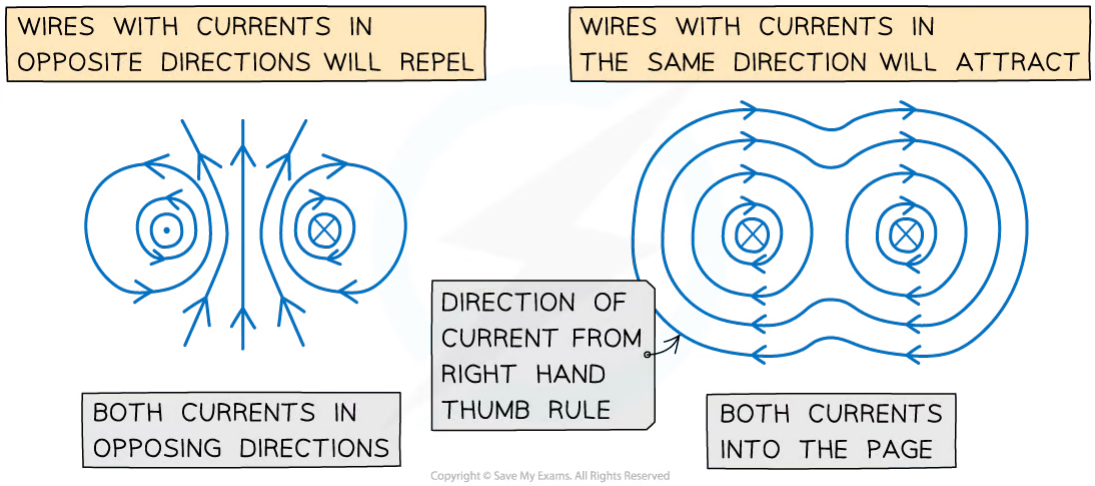
Exam Tip

Remember that the direction of current flow is the flow of **positive** charge (positive to negative), and this is in the **opposite direction** to the flow of electrons

YOUR NOTES

Magnetic Force Between Two Parallel Conductors

- A current carrying conductor, such as a wire, produces a magnetic field around it
- The direction of the field depends on the direction of the current through the wire
  - This is determined by the **right hand thumb rule**
- Parallel current-carrying conductors will therefore either attract or repel each other
  - If the currents are in the **same** direction in both conductors, the magnetic field lines between the conductors cancel out – the conductors will **attract** each other
  - If the currents are in the **opposite** direction in both conductors, the magnetic field lines between the conductors push each other apart – the conductors will **repel** each other



Both wires will attract if their currents are in the same direction and repel if in opposite directions

- When the conductors **attract**, the direction of the magnetic forces will be **towards** each other
- When the conductors **repel**, the direction of the magnetic forces will be **away** from each other
- The magnitude of each force depends on the amount of current and the length of the wire

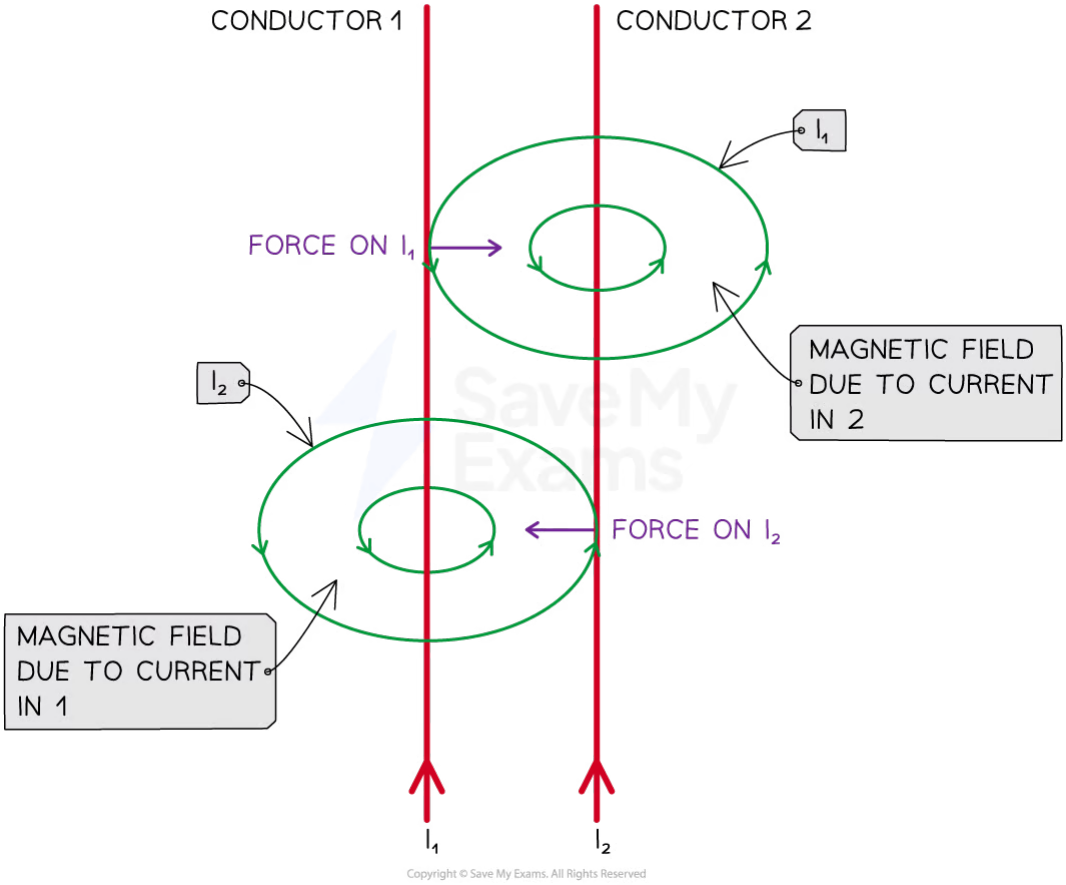
Force per Unit Length Between Two Parallel Conductors

- $\frac{F}{L}$ is the force per unit length between two parallel currents I_1 and I_2 separated by a distance r
- The force is **attractive** if the currents are in the **same direction** and **repulsive** if they are in **opposite directions**
- It is calculated using the equation:

$$\frac{F}{L} = \mu_0 \frac{I_1 I_2}{2\pi r}$$

YOUR NOTES

- Where:
  - F is the force applied between the two parallel wires (N)
  - L is the length of each parallel conductor (m)
  - μ_0 is the constant for the magnetic permeability of free space = $4\pi \times 10^{-7}$ N A^{-2}
  - I_1 is the current through the first conducting wire (A)
  - I_2 is the current through the second conducting wire (A)
  - r is the separation between the two conducting wires (m)



The forces on each of the current-carrying wires are equal and opposite in direction

Obtaining the Equation

- The force from wire 2 on wire 1, $F_2 = B_2 I_1 L \sin(\theta)$
- In this situation the magnetic field is perpendicular to the current in the wire, so $\sin(\theta) = 1$
- $F_2 = -F_1$ so the force between them is F

YOUR NOTES

- The force on a unit length of the wires is then given by $\frac{F}{L} = \frac{B_2 I_1 L}{L}$
  - Hence, $\frac{F}{L} = B_2 I_1$

- The magnitude of the magnetic field at a radial distance, *r* away from the current conducting wire is: $B = \frac{\mu_0 I}{2\pi r}$
  - In this case the magnetic field strength from B_2 at a distance *r* away from wire 2 is:

    $$B_2 = \frac{\mu_0 I_2}{2\pi r}$$

- Substituting for B_2 into the force per unit length equation gives us: $\frac{F}{L} = \left(\frac{\mu_0 I_2}{2\pi r}\right) I_1$

YOUR NOTES

Worked Example

Two long, straight, current-carrying conductors, WX and YZ, are held at a close distance, as shown in diagram 1.

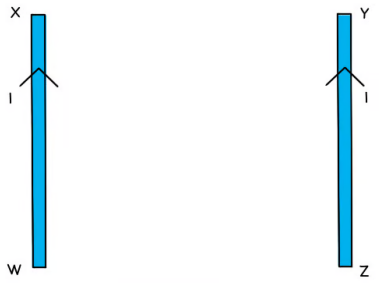


diagram 1

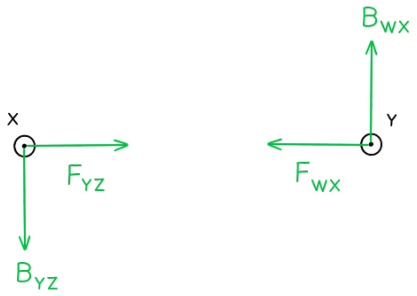
The conductors each carry the same magnitude current in the same direction. A plan view from above the conductors is shown in diagram 2.

diagram 2

On diagram 2, draw arrows, one in each case, to show the direction of:

- The magnetic field at X due to the current in wire YZ (label this arrow B_{YZ})
- The force at X as a result of the magnetic field due to the current in the wire YZ (label this arrow F_{YZ})
- The magnetic field at Y due to the current in wire WX (label this arrow B_{WX})
- The force at Y as a result of the magnetic field due to the current in the wire WX (label this arrow F_{WX})

Answer:



- Newton's Third Law states:

- When two bodies interact, the force on one body is **equal but opposite** in direction to the force on the other body

- Therefore, the forces on the wires act in equal but opposite directions

YOUR NOTES

IB Physics HL

YOUR NOTES

Induction

CONTENTS

Induced Emf (HL)
Magnetic Flux (HL)
Faraday's Law of Induction (HL)
Lenz's Law (HL)
AC Generators (HL)

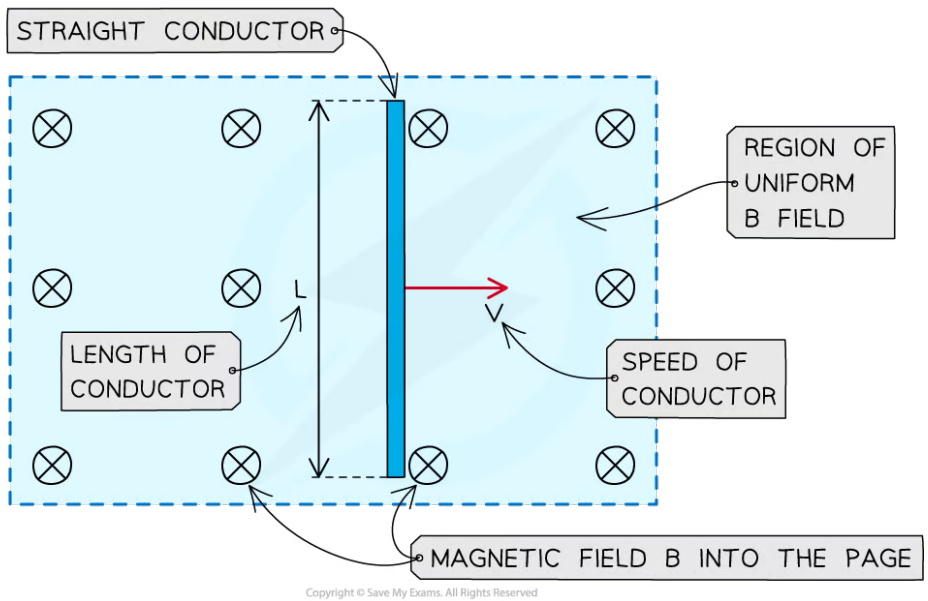
YOUR NOTES

Induced Emf

- Electromagnetic induction is a phenomenon which occurs when an e.m.f. is induced when a conductor moves through a magnetic field
- When the conductor cuts through the magnetic field lines:
  - This causes a change in magnetic flux ($\Delta\Phi$)
  - Which causes **work to be done**
  - This work is then transformed into **electrical energy**
- Therefore, if attached to a complete circuit, a **current** will be **induced**
- This is known as **electromagnetic induction** and is defined as:

> **The process in which an e.m.f is induced in a closed circuit due to changes in magnetic flux**

- An example of this is moving **conductors** in a magnetic field
  - Similar to a coil or a solenoid, a straight conducting rod moving through a magnetic field will have an e.m.f induced in it
  - This is only when the conductor moves **perpendicular** to the magnetic field lines to **cut through** the magnetic field lines



Conducting rod moving perpendicular to a magnetic field directed into the page

- The induced e.m.f in the conductor, as it moves through the magnetic field, is:

$$\varepsilon = BLv$$

- Where:
  - ε = induced e.m.f. (V)

- B = magnetic flux density (T)
- L = length of the conductor in the field (m)
- v = velocity of the conductor travelling through the field (m s^{-1})

- This equation shows that the size of the induced e.m.f increases if:
  - The **length** of the conductor in the field is increased
  - The magnetic field **strength** is increased
  - The conductor cuts through the field lines **faster**

- Coiling a wire to form many loops or turns, will also increase the size of the induced e.m.f.
- For a wire in a magnetic field with N turns, the induced e.m.f. will become

$$\varepsilon = BLvN$$

- The direction of the magnetic field is **into** the page

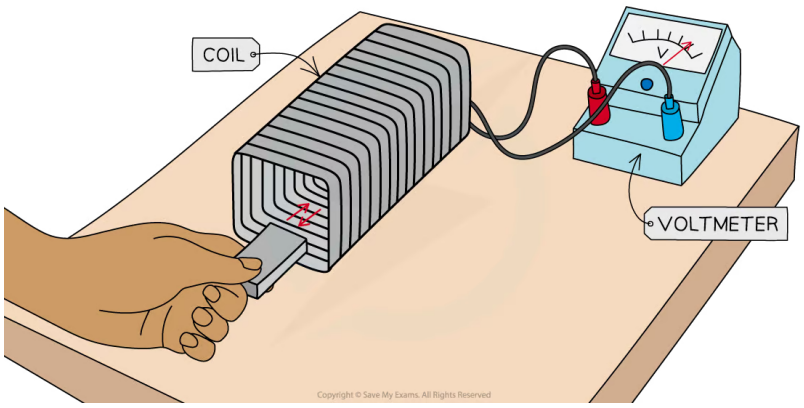
- This can occur either when:
  - A conductor cuts through a magnetic field
  - The direction of a magnetic field through a coil changes

- Electromagnetic induction is used in:
  - Electrical **generators** which convert mechanical energy to electrical energy
  - **Transformers** which are used in electrical power transmission

- This phenomenon can easily be demonstrated with a magnet and a coil, or a wire and two magnets

Experiment 1: Moving a magnet through a coil

- When a coil is connected to a sensitive voltmeter, a bar magnet can be moved in and out of the coil to induce an e.m.f



A bar magnet is moved through a coil connected to a voltmeter to induce an e.m.f

The expected results are...

1. When the bar magnet is not moving, the voltmeter shows a zero reading

- When the bar magnet is held still inside, or outside, the coil, the rate of change of flux is zero, so, there is **no e.m.f induced**

2. When the bar magnet begins to move inside the coil, there is a reading on the voltmeter

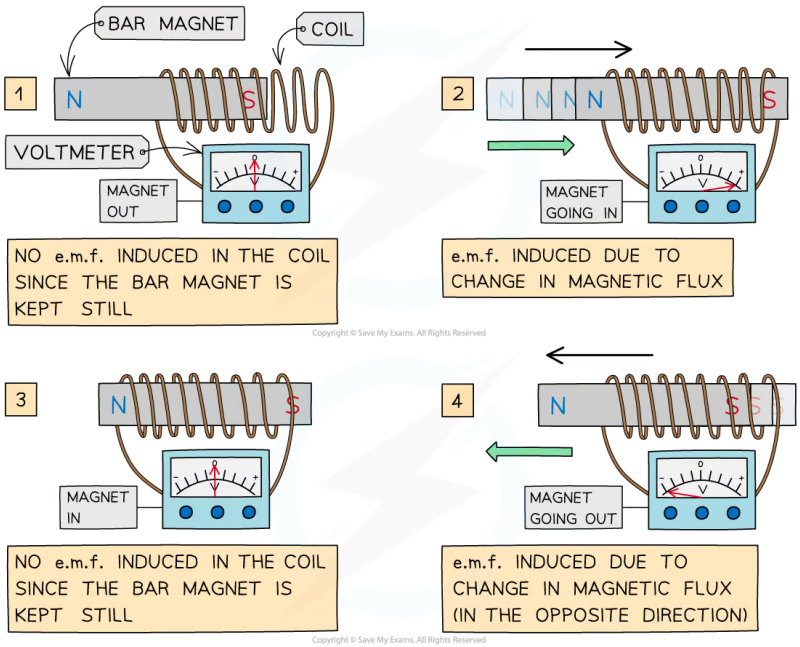
- As the bar magnet moves, its magnetic field lines 'cut through' the coil, generating a **change in magnetic flux** ($\Delta\Phi$)
- This induces an **e.m.f** within the coil, shown momentarily by the reading on the voltmeter

3. When the bar magnet is taken back out of the coil, an e.m.f is induced in the opposite direction

- As the magnet changes direction, the direction of the current changes
- The voltmeter will momentarily show a reading with the opposite sign

4. Increasing the speed of the magnet induces an e.m.f with a higher magnitude

- As the speed of the magnet increases, the rate of change of flux increases

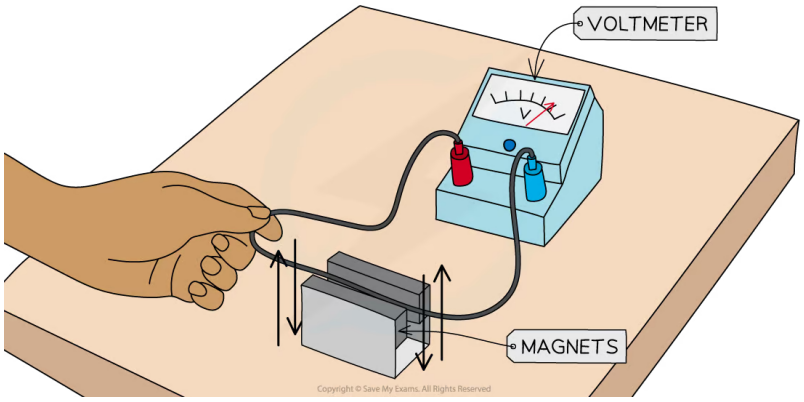


An e.m.f is induced only when the bar magnet is moving through the coil

- Factors that will increase the induced e.m.f are:
  - Moving the magnet **faster** through the coil
  - Adding more **turns** to the coil
  - Increasing the **strength** of the bar magnet

Experiment 2: Moving a wire through a magnetic field

- When a long wire is connected to a voltmeter and moved between two magnets, an e.m.f is induced
  - **Note:** there is no current flowing through the wire to start with



A wire is moved between two magnets connected to a voltmeter to induce an e.m.f

The expected results are...

1. When the wire is not moving, the voltmeter shows a zero reading

- When the wire is held still inside, or outside, the magnets the rate of change of flux is zero so there is **no e.m.f induced**

2. As the wire moves between the magnets, an e.m.f is induced within the wire

- This is shown momentarily by the reading on the voltmeter
- As the wire moves through the magnetic field, it 'cuts through' the magnetic field lines, generating a **change in magnetic flux**

3. When the wire is moved back out of the field, an e.m.f is induced in the opposite direction

- As the wire changes direction, the direction of the current changes
- The voltmeter will momentarily show a reading with the opposite sign

- Factors that will increase the induced e.m.f are:
  - Increasing the **length** of the wire
  - Moving the wire between the magnets **faster**
  - Increasing the **strength** of the magnets

YOUR NOTES

Emf, Magnetic Flux & Magnetic Flux Linkage

- Magnetic flux is a quantity which signifies how much of a magnetic field passes **perpendicularly** through an area
- It is defined as:

> **The product of the magnetic flux density and the cross-sectional area perpendicular to the direction of the magnetic flux density**

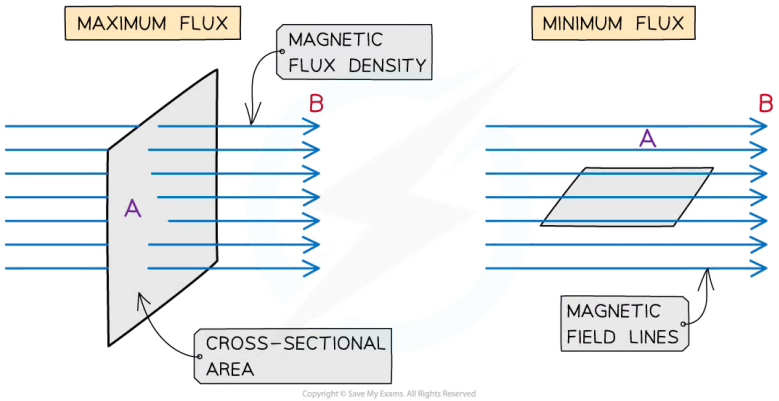
- Magnetic flux when the field and motion are at 90° can be calculated using the simple equation:

$$\Phi = BA$$

- Where:
  - Φ = magnetic flux (Wb)
  - B = magnetic flux density (T)
  - A = cross-sectional area (m^2)
- it is defined by the symbol Φ (greek letter 'phi') and measured in units of **Webers** (Wb)

How does magnetic flux change with angle?

- Since the flux is the total magnetic field that passes through a given **area**
  - It is a maximum when the magnetic field lines are **perpendicular** to the plane of the area
  - It is zero when the magnetic field lines are **parallel** to the plane of the area
- For a coil, the amount of magnetic flux varies as the coil rotates within the field



The magnetic flux is maximum when the magnetic field lines and the area they are travelling through are perpendicular

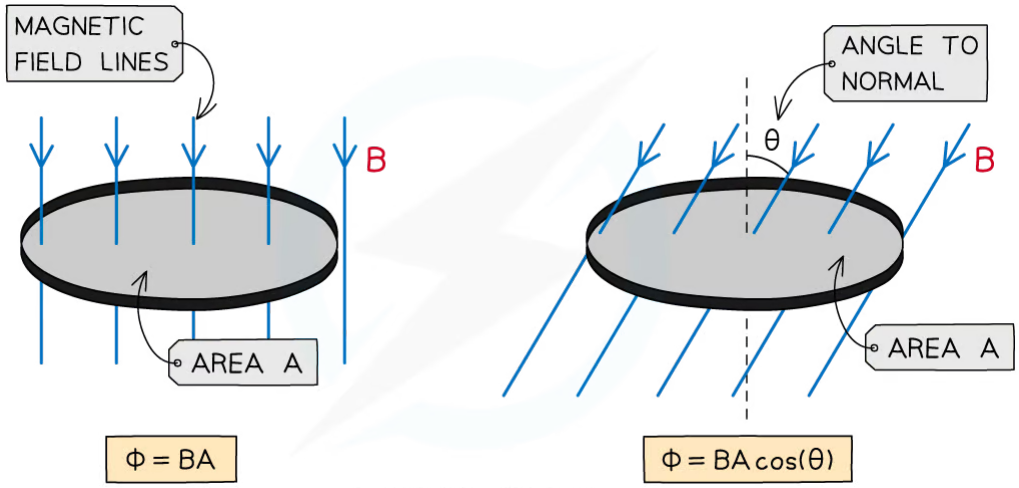
- In other words, magnetic flux is the **number of magnetic field lines through a given area**

YOUR NOTES

- When the magnetic field lines are **not** completely perpendicular to the area *A*, then the **component** of magnetic flux density *B* is **perpendicular** to the area is taken
- The equation then becomes:

$$\Phi = BA\cos(\theta)$$

- Where:
  - Φ = magnetic flux (Wb)
  - B = magnetic flux density (T)
  - A = cross-sectional area (m^2)
  - θ = angle between magnetic field lines and the line perpendicular to the plane of the area (often called the normal line) (degrees)



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The magnetic flux decreases as the angle between the field lines and plane decrease

- This means the magnetic flux is:
  - **Maximum** = BA when cos(θ) =1 therefore **θ = 0°**. The magnetic field lines are perpendicular to the plane of the area
  - **Minimum** = 0 when cos(θ) = 0 therefore **θ = 90°**. The magnetic field lines are parallel to the plane of the area

Magnetic Flux Linkage

- More coils in a wire mean a **larger** e.m.f is induced
- The **magnetic flux linkage** is a quantity commonly used for solenoids which are made of *N* turns of wire
- The flux linkage is defined as:

The product of the magnetic flux and the number of turns of the coil

- It is calculated using the equation:

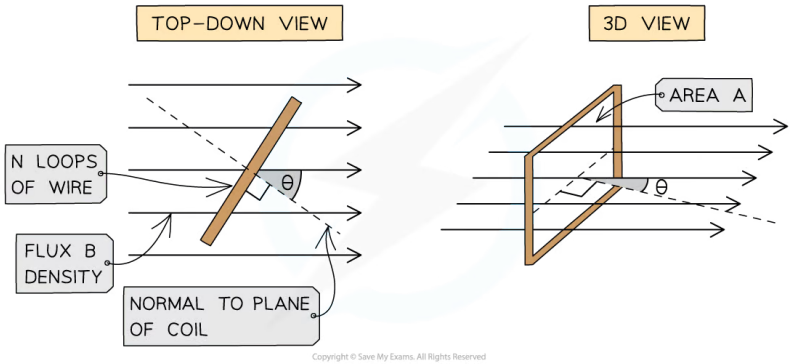
$$\text{Magnetic flux linkage} = \Phi N = BAN$$

- Where:

- Φ = magnetic flux (Wb)
- N = number of turns of the coil
- B = magnetic flux density (T)
- A = cross-sectional area (m^2)

- The flux linkage ΦN has the units of **Weber turns (Wb turns)**

- An e.m.f is induced in a circuit when the magnetic flux linkage changes with respect to time
- This means an e.m.f is induced when there is:
  - A changing magnetic flux density B
  - A changing cross-sectional area A
  - A change in angle θ



The magnetic flux through a rectangular coil decreases as the angle between the field lines and plane decrease

- Magnetic flux linkage also **changes** with the rotation of the coil
  - It is at a **maximum** when the field lines are **perpendicular** to the plane of the area they are passing through
  - This is the same pattern as the magnetic flux
- Therefore, the component of the flux density which is perpendicular is equal to:

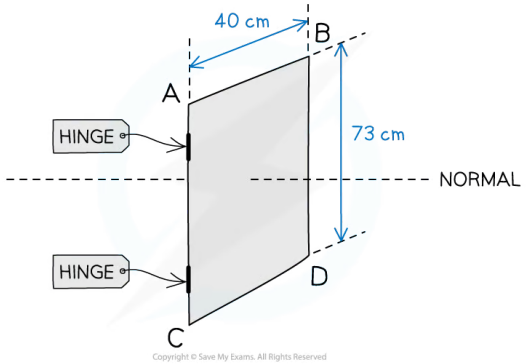
$$\Phi N = BAN\cos(\theta)$$

- Where:
  - N = number of turns of the coil

YOUR NOTES

Worked Example

An aluminium window frame has width of 40 cm and length of 73 cm.



The frame is hinged along the vertical edge AC. When the window is closed, the frame is normal to the Earth's magnetic field with magnetic flux density 1.8×10^{-5} T.

(a)
Calculate the magnetic flux through the window when it is closed.
(b)
Sketch the graph of the magnetic flux against angle between the field lines and the normal when the window is opened and rotated by 180°.

Answer:

(a)

Step 1: Write out the known quantities

- Cross-sectional area, $A = 40 \text{ cm} \times 73 \text{ cm} = (40 \times 10^{-2}) \times (73 \times 10^{-2}) = 0.292 \text{ m}^2$
- Magnetic flux density, $B = 1.8 \times 10^{-5}$ T

Step 2: Write down the equation for magnetic flux

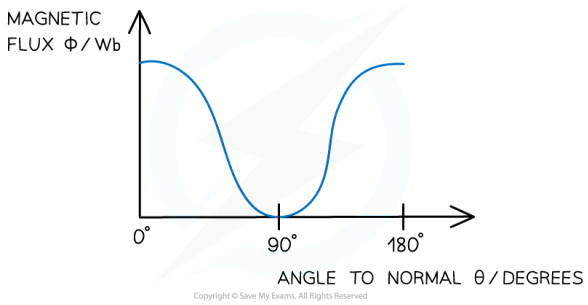
$$\Phi = BA$$

Step 3: Substitute in values

$$\Phi = (1.8 \times 10^{-5}) \times 0.292 = 5.256 \times 10^{-6} = 5.3 \times 10^{-6} \text{ Wb}$$

(b)

- The magnetic flux will be at a **minimum** when the window is opened by 90° and a **maximum** when fully closed or opened to 180°



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YOUR NOTES

Exam Tip

Watch out for area given in cm^2 or mm^2. For the magnetic flux to be in Webers, the flux density needs to be Tesla and the area in **m^2.**

This area may not always be a square, but a circle as well. The areas of common 2D shapes are given in your data booklet.

YOUR NOTES

Faraday's Law of Induction

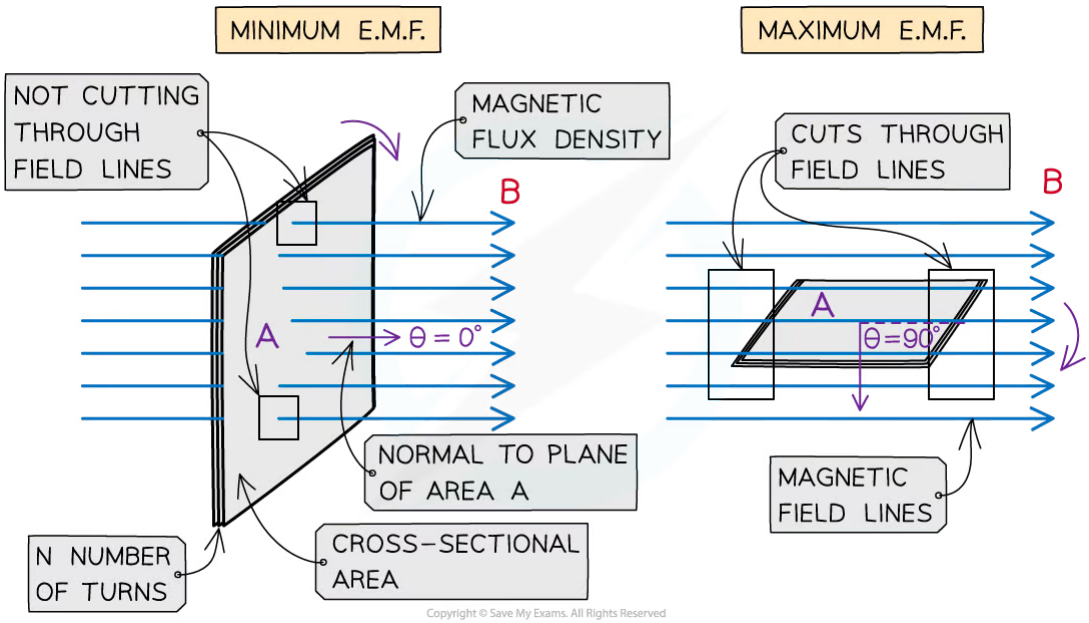
- Faraday's Law connects the **rate** of change of magnetic flux linkage with induced e.m.f
- It is defined in words as:

> **The magnitude of an induced e.m.f is directly proportional to the rate of change of magnetic flux linkage**

- Faraday's Law of induction is defined by the equation:

$$\varepsilon = \frac{N(\Delta \Phi)}{\Delta t}$$

- Where:
  - ε = induced e.m.f (V)
  - $N\Delta(\Phi)$ = change in magnetic flux linkage (Wb turns)
  - Δt = time interval (s)
- When a coil is completely vertical relative to the magnetic field lines:
  - The change in magnetic flux linkage is at a **maximum** - the field lines are travelling through the area of the coil
  - There is **no e.m.f** induced - there is no cutting of field lines i.e. there is no change in magnetic flux linkage
- When a coil is completely horizontal relative to the magnetic field lines:
  - The change in magnetic flux linkage is **zero** - there are no field lines travelling through the area of the coil
  - **Maximum e.m.f** is induced - there is the maximum cutting of field lines

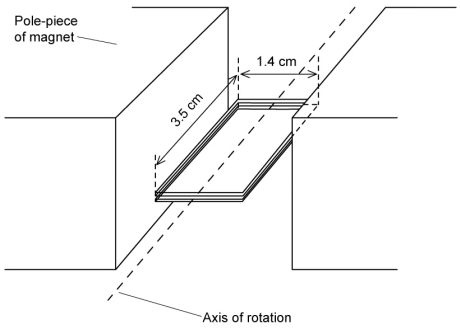


YOUR NOTES

YOUR NOTES

Worked Example

A small rectangular coil contains 350 turns of wire. The longer sides are 3.5 cm and the shorter sides are 1.4 cm.



The coil is held between the poles of a large magnet so that the coil can rotate about an axis through its centre. The magnet produces a uniform magnetic field of flux density 80 mT between its poles.

The coil is positioned horizontally and then turned through an angle of 40° over a time interval of 0.18 s.

Calculate the magnitude of the average e.m.f induced in the coil.

Answer:

Step 1: Write down the known quantities

- Magnetic flux density, $B = 80\ \text{mT} = 80 \times 10^{-3}\ \text{T}$
- Area, $A = 3.5 \times 1.4 = (3.5 \times 10^{-2}) \times (1.4 \times 10^{-2}) = 4.9 \times 10^{-4}\ \text{m}^2$
- Number of turns, $N = 350$
- Angle of rotation, $\theta = 40°$
- Time interval, $\Delta t = 0.18\ \text{s}$

Step 2: Write down the equation for Faraday's law:

$$\varepsilon = \frac{N(\Delta \Phi)}{\Delta t}$$

Step 3: Write out the equation for the change in flux linkage:

- The number of turns N and the coil area A stay constant
- The flux through the coil changes as $B \cos \theta$ as it rotates
- Therefore, the equation to use is:

$$N(\Delta \Phi) = NBA\cos\theta$$

Step 4: Determine the change in magnetic flux linkage

- The **initial** flux through the coil is zero (flux lines are parallel to the coil face)

- The **final** flux through the coil is 80 mT (flux lines are more perpendicular to the coil face)
- This is because the coil begins horizontally in the field and is rotated by 40°
- Therefore, the change in flux linkage is:

$$N\Delta(\Phi) = NBA\cos\theta = 350 \times (4.9 \times 10^{-4}) \times (80 \times 10^{-3}) \times \cos 40$$

$$N(\Delta\Phi) = 0.011 \text{ Wb turns}$$

Step 5: Substitute the change in flux linkage and time into Faraday's law equation:

$$\varepsilon = \frac{0.011}{0.18} = 0.061 \text{ V}$$

Exam Tip

The important point to notice is that an emf is induced in a conductor in a magnetic field if there is **change** in flux linkage. This means, the conductor (e.g. a coil) must **cut through** the field lines to have an emf (and hence a current) induced.

YOUR NOTES

YOUR NOTES

Lenz's Law

- Lenz's Law is used to predict the **direction** of an induced e.m.f in a coil or wire
  - It is a consequence of the principle of conservation of energy
- Lenz's Law is summarised below:

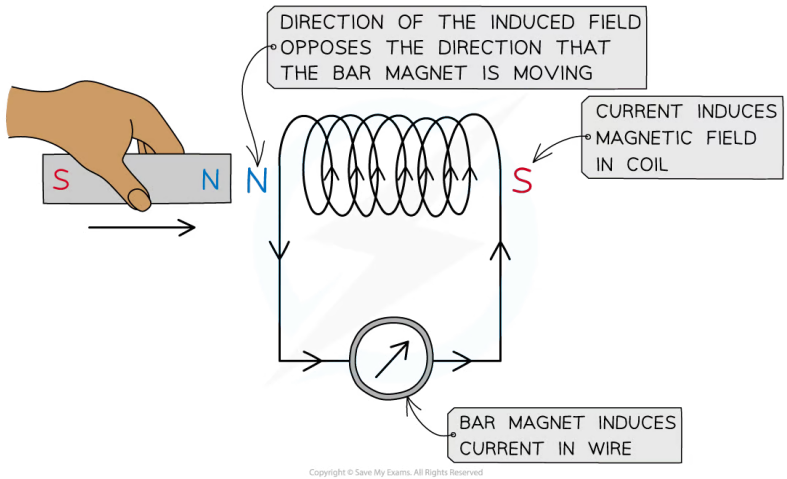
  The induced e.m.f is such that it will oppose the change causing it
- Lenz's law combined with Faraday's law is given by the equation:

$$\varepsilon = -N\frac{\Delta \Phi}{\Delta t}$$

- This equation shows:
  - When a bar magnet goes through a coil, an e.m.f is induced within the coil due to a change in magnetic flux
  - A current is also induced which means the coil now has its own magnetic field
  - The coil's magnetic field acts in the **opposite direction** to the magnetic field of the bar magnet (shown by the minus sign)
- If a direct current (d.c) power supply is replaced with an alternating current (a.c) supply, the e.m.f induced will also be alternating with the same frequency as the supply

Experimental Evidence for Lenz's Law

- To verify Lenz's Law, the only apparatus needed is:
  - A bar magnet
  - A coil of wire
  - A sensitive ammeter
- Note: a cell is **not** required

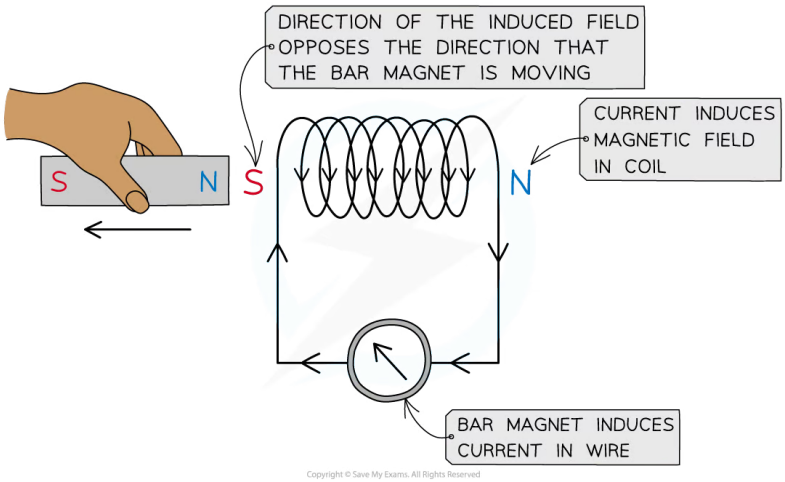


Lenz's law can be verified using a coil connected in series with a sensitive ammeter and a bar magnet

YOUR NOTES

- A known pole (either north or south) of a bar magnet is pushed into the coil
  - This induces an e.m.f in the coil
  - The induced e.m.f drives a current (because it is a complete circuit)
- Lenz's Law dictates:
  - The direction of the e.m.f, and hence the current, must be set up to **oppose** the incoming magnet
  - Since a **north pole** approaches the coil face, the e.m.f must be set up to create an induced **north pole**
  - This is because the two north poles will **repel** each other

- The direction of the current is therefore as shown in the image above
  - The direction of the current can be verified using the right-hand grip rule
  - Fingers curl around the coil in the direction of current and the thumb points along the direction of the flux lines, from **north** to **south**
  - Therefore, the **current** flows in an **anti-clockwise** direction in the image shown, in order to induce a north pole opposing the incoming magnet

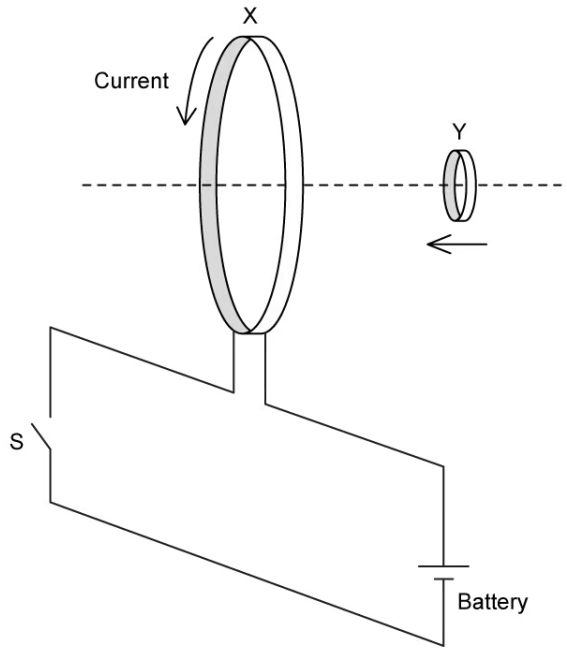


- **Reversing** the magnet direction would give an **opposite** deflection on the voltmeter
  - Lenz's Law would then predict a **south** pole induced at the coil entrance (next to the bar magnet)
  - This would create a north pole at the exist, **attracting** the bar's south pole attempting to leave
  - Therefore, the induced e.m.f **always** produces effects to **oppose** the changes causing it
- This means:
  - The coil will try and **push** the bar magnet to stop it from entering
  - The coil will try and **pull** the bar magnet to stop it from leaving
  - This means the poles of the coil **swaps around** as the bar magnet travels through

- Lenz's Law is a direct consequence of the **principle of conservation of energy**
  - Electromagnetic effects will not create electrical energy out of nothing
  - In order to induce and sustain an e.m.f, for instance, **work** must be done in order to overcome the repulsive effect due to Lenz's Law

Worked Example

Two conducting loops, X and Y, are positioned so that their planes are parallel and their centres sit on the same line, as shown in the diagram.



When the switch S is closed, a constant counterclockwise current flows in X. Loop X is stationary and loop Y is moved towards X at a constant speed.

State and explain:

(a)
how the magnetic flux in loop Y varies with time.
(b)
how the size and direction of the induced current in loop Y varies with time.

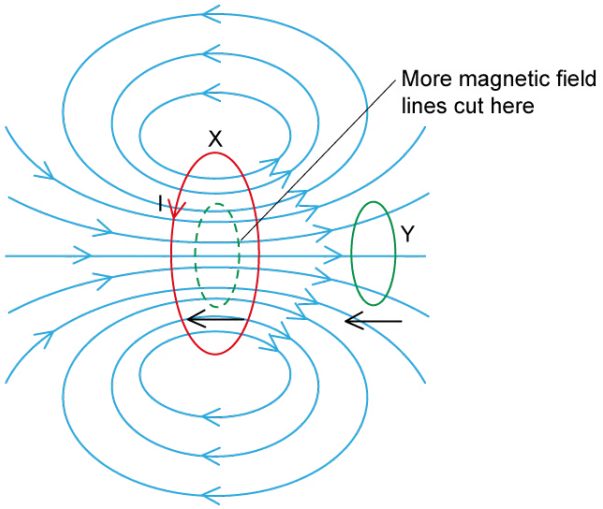
Work must be done on loop Y to maintain its constant speed towards X

(c)
Explain why.

Answer:

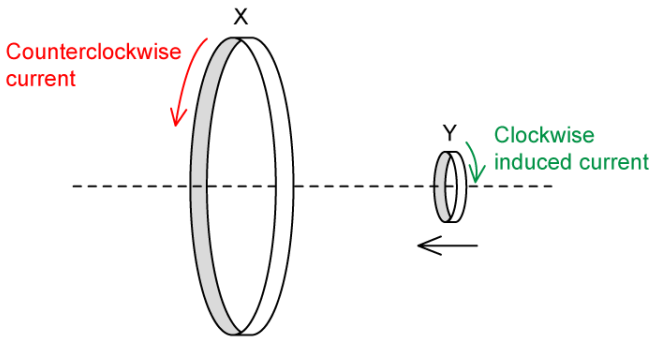
(a) Variation of magnetic flux in Y:

- As Y approaches X, it cuts an increasing number of magnetic field lines
- Therefore, the magnetic flux in Y **increases** as it approaches X



(b) Direction of the induced current in Y:

- Lenz's law states that the induced e.m.f will be such that it will oppose the change producing it
- Hence, the induced current in Y will flow in a constant **clockwise** direction



Size of the induced current in Y:

- Faraday's law states that the induced e.m.f. increases with the rate of change of flux linkage
- The rate of change of magnetic flux in Y increases as it approaches X
- Potential difference and current are related by $V = IR$
- The resistance of the loop is constant, hence, the size of the induced current in Y will **increase** with time

(c) Why work must be done to maintain a constant speed:

- The current induced in Y produces a magnetic field opposing that of X
- So, according to Lenz's law, there will be a magnetic force opposing the motion of Y
- Hence, work must be done on Y to **overcome** this opposing force

YOUR NOTES

Exam Tip

You should remember that the negative sign is representative of **Lenz's Law** (without out it, it is just **Faraday's Law**) which says that the induced e.m.f ε is set up **to oppose the change causing it.** The negative sign indicates motion in an opposing direction. This is the form of the equation given on your data booklet.

Self Induction & Mutual Induction

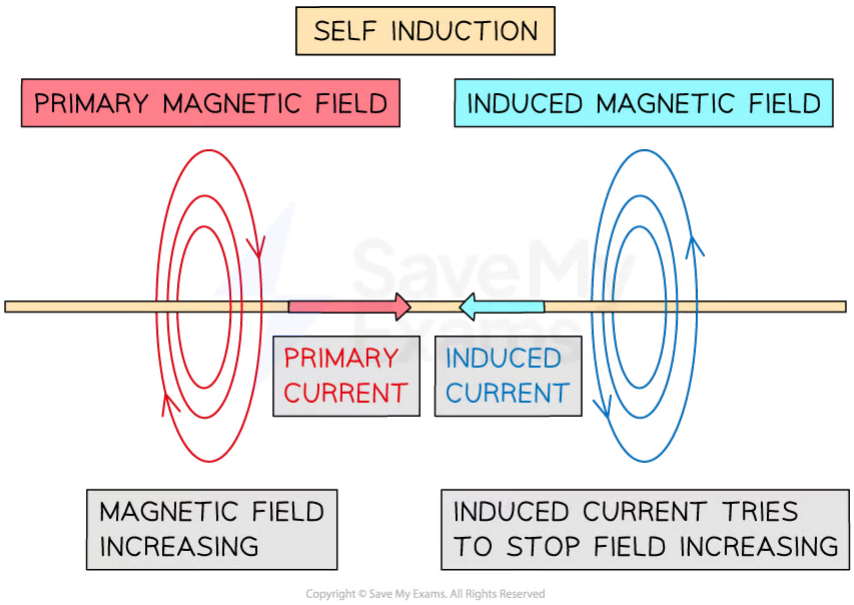
- When changes in current within a circuit occur, particularly when the current is alternating at high frequencies, changes in magnetic flux will also occur
- Two types of induction can be observed
  - **Self-induction** - induction within the **same** circuit
  - **Mutual induction** - induction **between** circuits
- These both occur as a consequence of **Lenz's law**

Self Induction

- When induction occurs within the same circuit, this is known as self-induction
- Self-induction is defined as

> **The effect in which a change in the current tends to produce an induced emf which opposes the change of current in the same circuit**

- The induced current is produced by a **back e.m.f.** i.e. an induced e.m.f. that opposes a change of current (in the same circuit)
  - Note: this is the same induced e.m.f. as described by Lenz's law



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Self-induction occurs within the same circuit. The primary magnetic field induces a current which opposes the primary current.

- The back e.m.f. is proportional to minus the rate of current change

$$\text{back e.m.f.} \propto -\frac{\Delta I}{\Delta t}$$

Mutual Induction

YOUR NOTES

- When induction occurs in a separate circuit from the one producing the change, this is known as mutual induction
- Mutual induction is defined as

> **The effect in which a change in the current in one circuit tends to produce an induced emf which opposes the change of current in a neighbouring circuit**

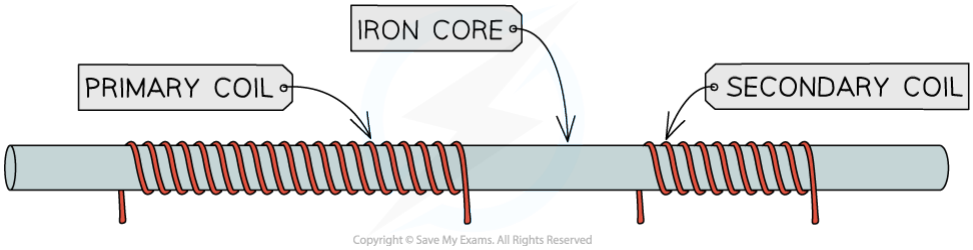
- An important application of mutual induction is **transformers**

Transformers

- A transformer is

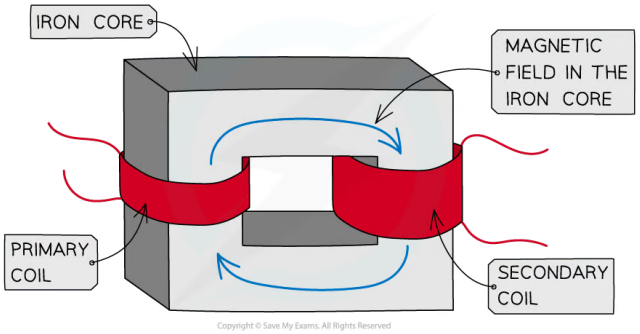
> **A device that changes high alternating voltage at low current to low alternating voltage at high current, and vice versa**

- A transformer is designed to reduce heat energy loss whilst transmitting electricity through power lines from power stations to the national grid
- A transformer is made up of:
  - A primary coil
  - A secondary coil
  - An iron core



Components of a transformer

- The primary and secondary coils are wound around the **soft iron core**
  - The soft iron core is necessary because it enhances the strength of the magnetic field from the primary to the secondary coil
  - Soft iron is used because it can easily be magnetised and demagnetised



A step-up transformer has more turns in the secondary coil than primary

- In the primary coil, an alternating current producing an alternating voltage is applied

YOUR NOTES

- This creates an **alternating magnetic field** inside the iron core and therefore a changing magnetic flux linkage

- A changing magnetic field passes through to the secondary coil through the iron core
  - This results in a changing magnetic flux linkage in the secondary coil and from Faraday's Law, an **e.m.f is induced**

- An e.m.f produces an alternating output voltage from the secondary coil
  - The output alternating voltage is at the **same** frequency as the input voltage

- In a **step-up** transformer
  - The secondary coil has **more turns** than the primary coil
  - Hence, the secondary voltage is **larger** than the primary voltage

- In a **step-down** transformer
  - The primary coil has **more turns** than the secondary coil
  - Hence, the primary voltage is **larger** than the secondary voltage

Exam Tip

You don't need to remember the equation for back e.m.f, but you must be clear on self and mutual induction and the differences between them.

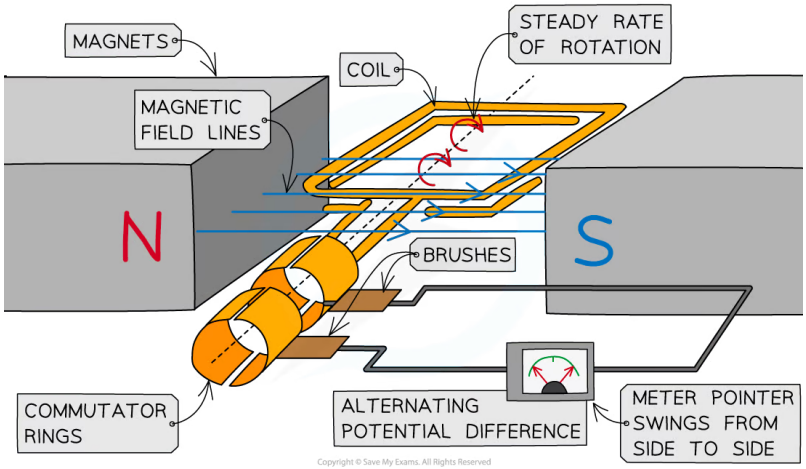
YOUR NOTES

AC Generators

- If a coil of wire is rotated inside a magnetic field by an external force, an emf will be generated in the wire which causes current to flow within the coil
- The **generator effect** can be used to:
  - Generate **a.c.** in an **alternator**
  - Generate **d.c.** in a **dynamo**

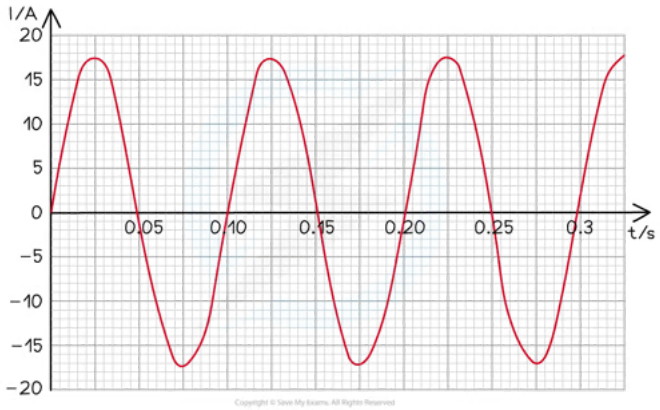
Alternators

- A simple alternator is a type of generator that converts mechanical energy to electrical energy in the form of alternating current



An alternator is a rotating coil in a magnetic field connected to commutator rings

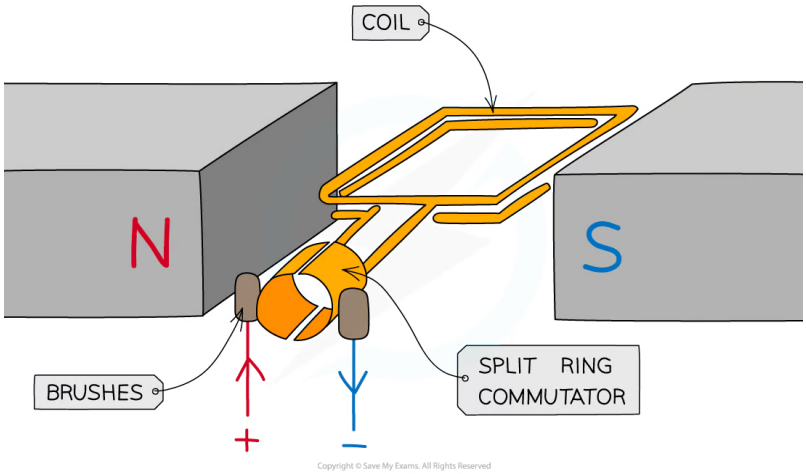
- A rectangular coil that is forced to spin in a **uniform magnetic field**
- The coil is connected to a centre-reading meter by **metal brushes** that press on two metal **slip rings** (or commutator rings)
  - The slip rings and brushes provide a continuous connection between the coil and the meter
- When the coil turns in one direction:
  - The pointer defects first one way, then the opposite way, and then back again
  - This is because the coil **cuts through** the magnetic field lines and a **potential difference,** and therefore current, is **induced** in the coil
- The pointer deflects in both directions because the current in the circuit repeatedly **changes direction** as the coil spins
  - This is because the induced potential difference in the coil repeatedly changes its direction
  - This continues on as long as the coil keeps turning in the **same** direction
- The induced potential difference and the current **alternate** because they repeatedly **change direction**



An example current output from an alternator

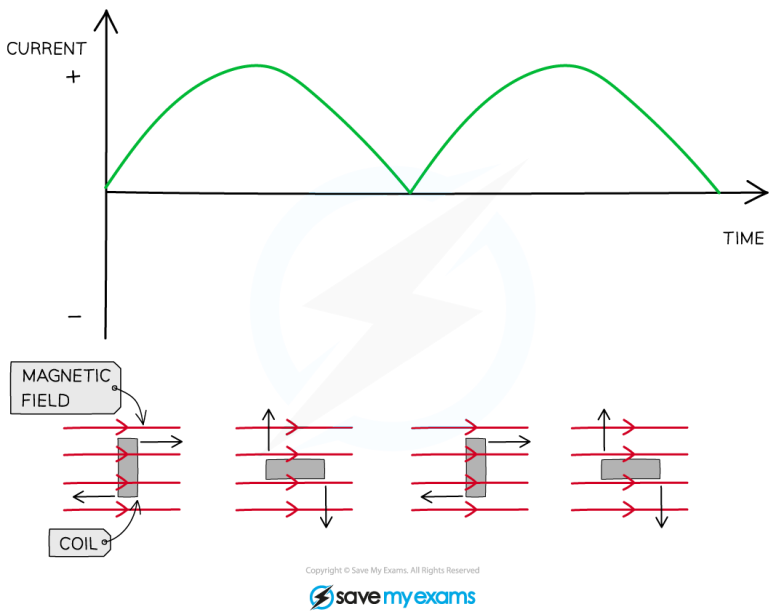
Dynamos

- A dynamo is a direct-current generator
- A simple dynamo is the **same** as an **alternator** except that the dynamo has a **split-ring commutator** instead of two separate slip rings



A dynamo is a rotating coil in a magnetic field connected to a split ring commutator

- As the coil rotates, it **cuts** through the field lines
  - This **induces a potential difference** between the end of the coil
- The split ring commutator changes the connections between the coil and the brushes every half turn in order to keep the current leaving the dynamo in the **same direction**
  - This happens each time the coil is perpendicular to the magnetic field lines



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D.C output from a dynamo – the current is only in the positive region of the graph

- Therefore, the induced potential difference **does not reverse** its direction as it does in the alternator
- Instead, it varies from zero to a maximum value twice each cycle of rotation, and never changes polarity (positive to negative)
    - This means the current is always **positive** (or always **negative**)

Emf Induction in a Rotating Coil

YOUR NOTES

- When a coil rotates in a uniform magnetic field, the flux through the coil will vary as it rotates
- Since e.m.f is the rate of change of flux linkage, this means the e.m.f will also change as it rotates
  - The maximum e.m.f is when the coil **cuts through** the most field lines
  - The e.m.f induced is an **alternating** voltage
- Flux linkage is given by

$$N\Phi = BAN\cos\theta$$

- Angular speed ω is defined as the rate of change of angular displacement, so

$$\omega = \frac{\theta}{t}$$

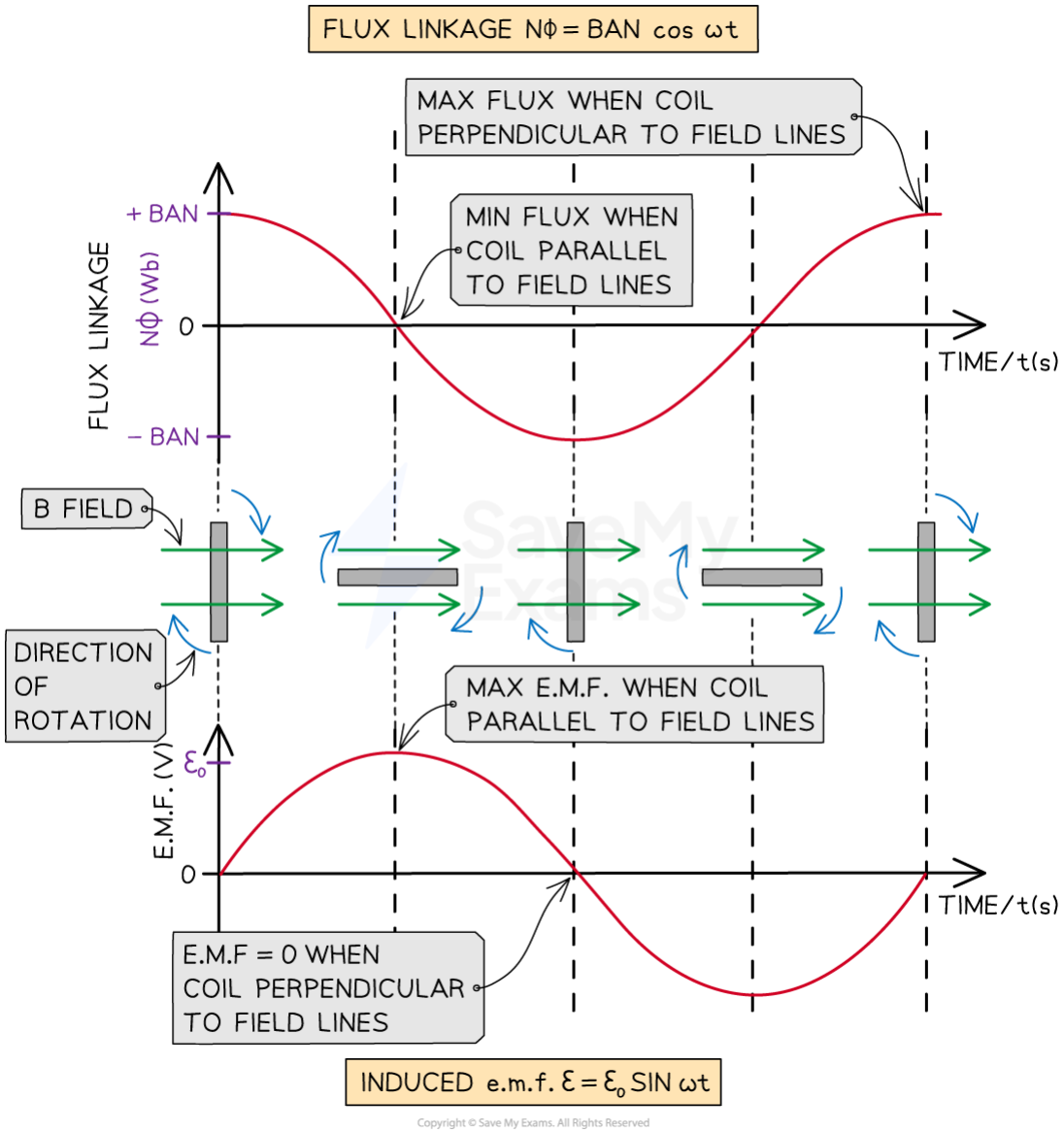
- Therefore, for a rotating coil, the angle θ depends on the angular speed of the coil ω:

$$\theta = \omega t$$

- Hence, flux linkage can also be written as:

$$N\Phi = BAN\cos\omega t$$

- Where:
  - $N\Phi$ = flux linkage (Wb turns)
  - B = magnetic flux density (T)
  - A = cross-sectional area of the coil (m^2)
  - N = number of turns of coil
  - ω = angular speed of the coil (rad s^{-1})
  - t = time (s)



Flux linkage and induced e.m.f. in a rotating coil

- The graph shows that
  - The induced e.m.f varies sinusoidally and it is 90° out of phase with the flux linkage
- Mathematically, the induced e.m.f. can also be written as:

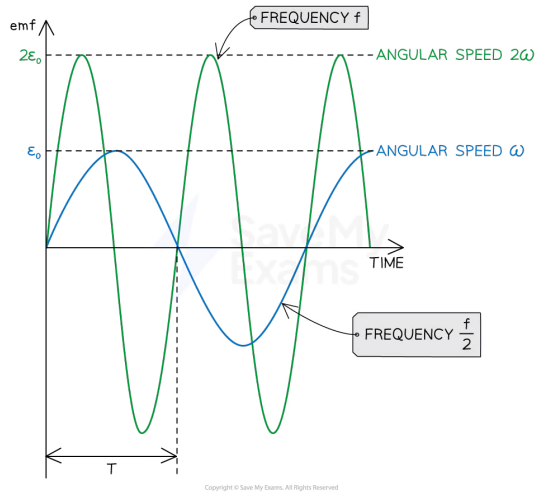
$$\varepsilon = \varepsilon_0 \sin \omega t$$

$$\varepsilon = BAN\omega \sin \omega t$$

- Where:
  - ε = e.m.f induced in the coil (V)
  - ε_0 = maximum e.m.f induced in the coil (V)

YOUR NOTES

- The size of the induced e.m.f. in a rotating coil can be increased by **increasing the frequency of rotation of the coil**
- Increasing the coil's frequency of rotation increases:
  - The **frequency** of the alternating voltage
  - The **amplitude** of the alternating voltage



Doubling the angular speed of the rotating coil in a magnetic field doubles the size of the induced e.m.f. (double the amplitude) and the frequency of the rotation (half the time period)

YOUR NOTES

Worked Example

An alternating current generator induces an e.m.f. of ε at a frequency f.

The rotational speed of the coil in the generator is doubled.

Which row correctly identifies the new output e.m.f. and the new frequency?

	e.m.f.	frequency
A.	2ε	$2f$
B.	$\sqrt{2}\varepsilon$	$2f$
C.	2ε	$\frac{f}{2}$
D.	$\sqrt{2}\varepsilon$	$\frac{f}{2}$

Answer: A

- Angular speed, time period and frequency are related by

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- Therefore, $\omega \propto f$, so if angular speed doubles, the frequency will also double
- If the coil rotates at twice the frequency, the rate of change of magnetic flux linkage will double
- Hence, induced e.m.f. and angular speed are directly proportional $\varepsilon \propto \omega$
- This means the induced e.m.f. will double if angular speed doubles

new e.m.f. = 2ε, new frequency = $2f$

Exam Tip

Remember not to get mixed up with when the e.m.f or the flux linkage is at its maximum:

- When the plane of the coil is **perpendicular** to the field lines
  - The flux linkage is at its **maximum**
  - The e.m.f = **zero**
- When the plane of the coil is **parallel** to the field lines
  - The flux linkage is **zero**
  - The e.m.f is at its **maximum**

Since ω is in units of rads s^{-1}, make sure your calculator is in **radians** mode before entering any values into $\sin(\omega t)$ or $\cos(\omega t)$.

The equation of e.m.f with $\sin(\omega t)$ is **not** given in your data booklet - you must be able to recognise this in exam questions!

IB Physics HL

YOUR NOTES

Structure of the Atom

CONTENTS

Rutherford's Gold Foil Experiment

Nuclear Notation

Emission & Absorption Spectrum

Photon Energy

Rutherford Scattering & Nuclear Radius (HL)

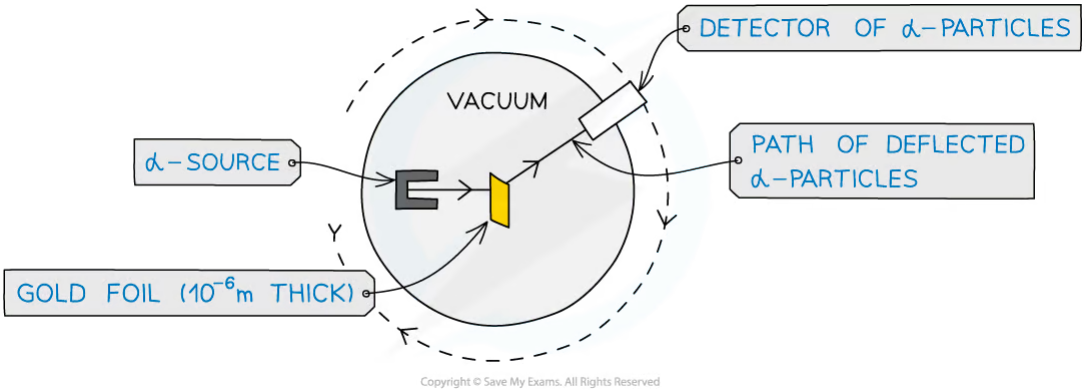
Deviations from Rutherford Scattering (HL)

The Bohr Model of Hydrogen (HL)

YOUR NOTES

Rutherford's Gold Foil Experiment

- Evidence for the structure of the atom was discovered by Ernest Rutherford at the beginning of the 20th century from the study of **alpha particle scattering**
- The experiment consisted of beams of high-energy alpha particles fired at thin gold foil and a detector on the other side to determine
  - The different **angles of deflection** of the alpha particles
  - The **number of alpha particles** that were deflected at each angle



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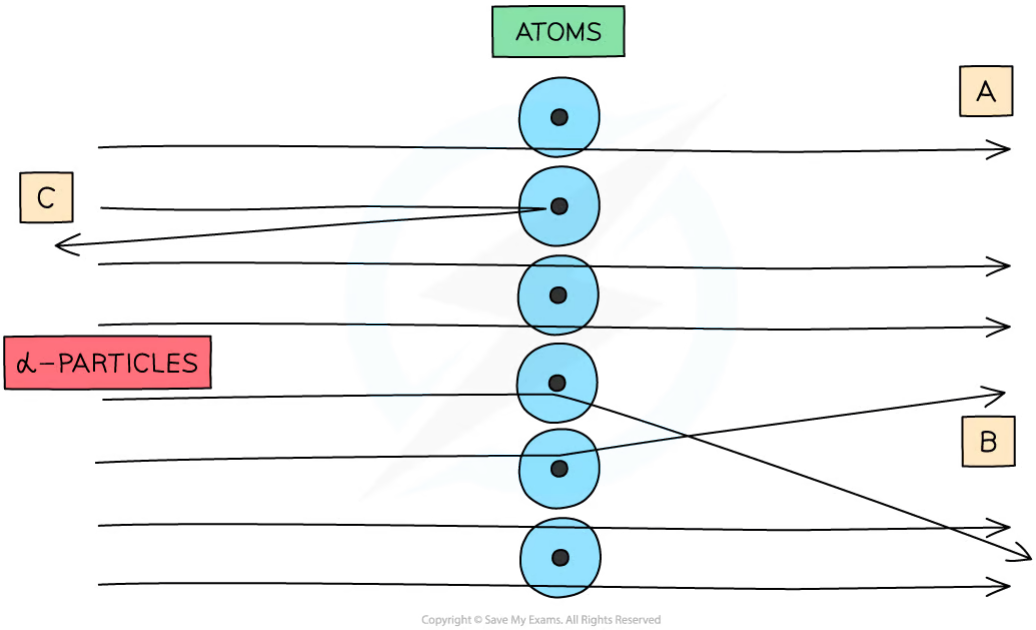
α-particle scattering experiment set up

- α-particles are the nucleus of a helium atom and are positively charged



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When α-particles are fired at thin gold foil, most of them go straight through but a small number bounce straight back

YOUR NOTES

- The observations from Rutherford's experiment were:

A. The majority of α-particles went straight through

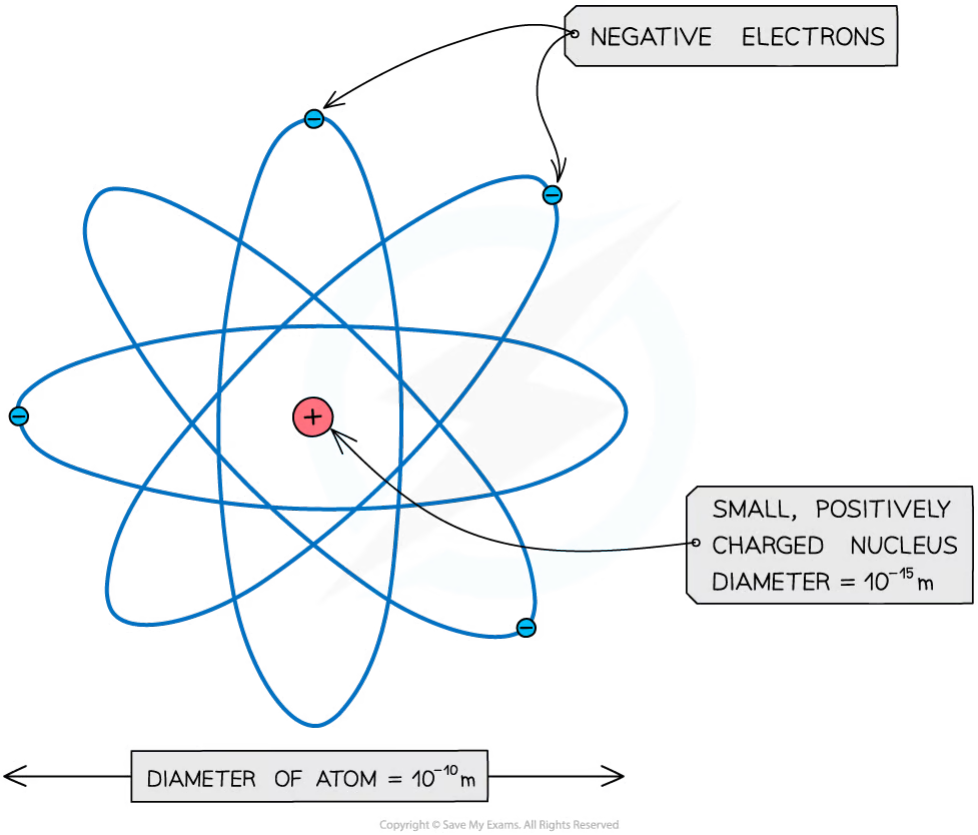
  - This suggested the atom is mainly empty space

B. Some α-particles deflected through small angles of <10°

  - This suggested there is a positive nucleus at the centre (since two positive charges would repel)

C. Only a small number of α-particles deflected straight back at angles of >90°

  - This suggested the nucleus is extremely small and this is where the mass and charge of the atom is concentrated
  - It was therefore concluded that atoms consist of small dense positively charged nuclei, surrounded by negatively charged electrons



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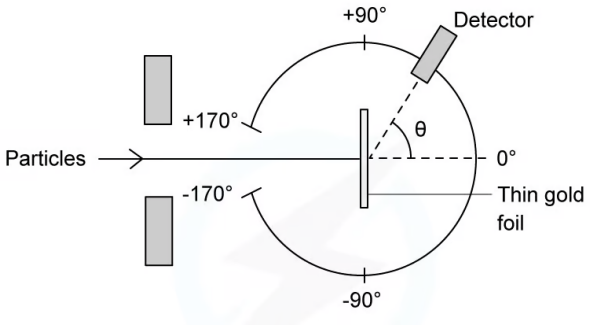
An atom: a small positive nucleus, surrounded by negative electrons

- (Note: The atom is around 100,000 times larger than the nucleus!)

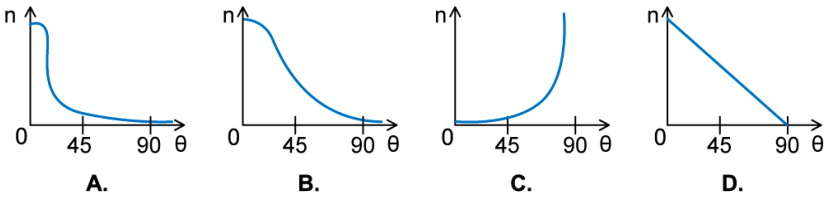
YOUR NOTES

Worked Example

In an α-particle scattering experiment, a student set up the apparatus below to determine the number of α-particles, *n*, incident per unit time on a detector held at various angles θ.



Which of the following graphs best represents the variation of n with θ from 0 to 90°?



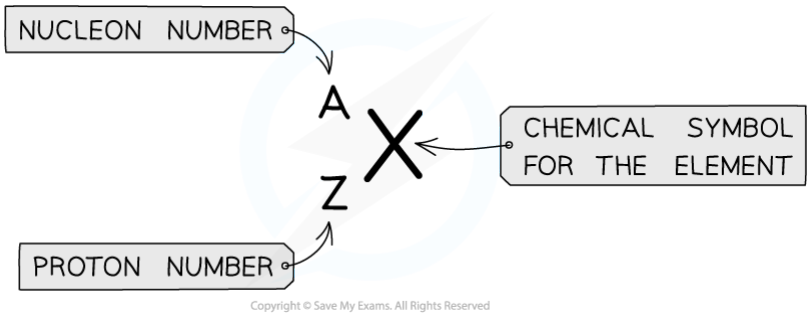
Answer: A

- The Rutherford scattering experience directed parallel beams of α-particles at gold foil
- The observations were:
  - Most of the α-particles went straight through the foil
  - The largest value of n will therefore be at small angles
  - Some of the α-particles were deflected through small angles
  - n drops quickly with increasing angle of deflection θ
- These observations fit with graph A

YOUR NOTES

Nuclear Notation

- A nuclide is a group of atoms containing the same number of protons and neutrons
  - For example, 5 atoms of oxygen are all the same nuclide but are 5 separate atoms
- Atomic symbols are written in a specific notation called **nuclide** or **AZX notation**



Atomic symbols in AZX Notation describe the constituents of nuclei

- The top number A represents the **nucleon** number or the **mass** number
  - **Nucleon number (A)** = total number of **protons and neutrons** in the nucleus
- The lower number Z represents the **proton** or **atomic** number
  - **Proton number (Z)** = total number of **protons** in the nucleus

Exam Tip

In Chemistry, the nucleon number is usually referred to as the mass number and the proton number as the atomic number

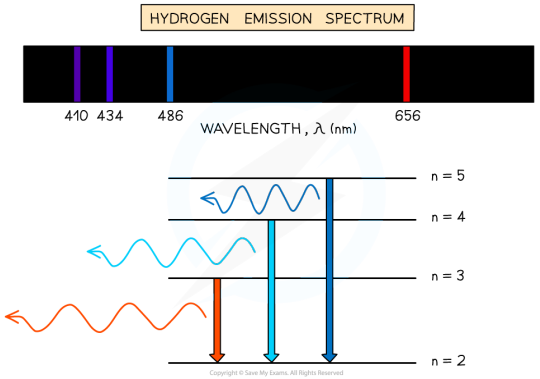
Make sure you know that the periodic table is ordered by **atomic number**

YOUR NOTES

Spectra & Atomic Energy Levels

Emission Spectra

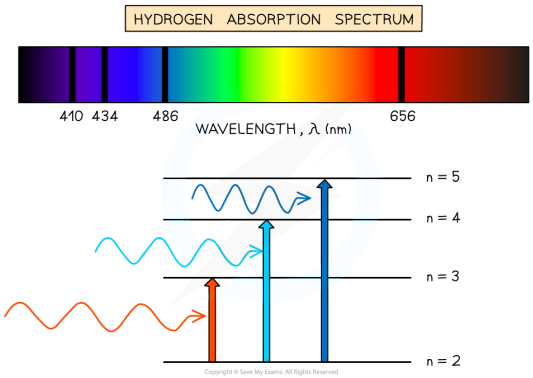
- When an electron **transitions** from a **higher** energy level to a **lower** energy level, this results in the **emission** of a photon
  - Each transition corresponds to a **specific wavelength** of light which correlates to an observable spectral line
- The resulting **emission spectrum** contains a set of discrete wavelengths, represented by coloured lines on a black background
- Each emitted photon has a wavelength which is associated with a discrete change in energy



Emission spectrum of hydrogen gas

Absorption Spectra

- An atom can be raised to an excited state by the absorption of a photon
- When white light passes through one side of a **cool, low-pressure gas** it is found that certain wavelengths of light are missing when detected out the other side
  - This type of spectrum is called an absorption spectrum
- An absorption spectrum consists of a continuous spectrum of all wavelengths (i.e. white light) with dark lines where specific wavelengths have been absorbed by the gas
  - These dark lines correspond exactly to the **differences in energy levels** in an atom
- When electrons return to lower energy levels, the photons are emitted in all directions, rather than in the original direction of the white light
  - Therefore, some wavelengths appear to be missing
- The wavelengths missing from an absorption spectrum correspond to the coloured lines on an emission spectrum for the **same** element



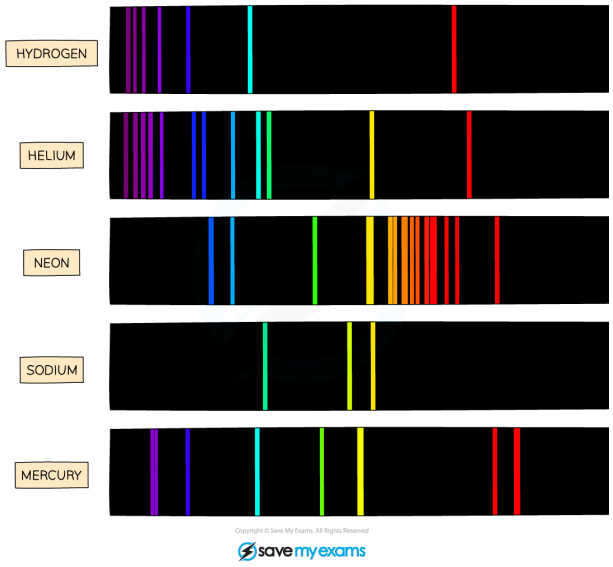
Absorption spectrum of hydrogen gas

YOUR NOTES

Spectra & Chemical Composition

YOUR NOTES

- The **chemical composition** of a substance can be investigated using emission and absorption spectra
- Each element produces a unique set of spectral lines
- No two elements emit the same set of spectral lines, therefore, elements can be identified by their line spectrum



Emission line spectra are unique to each element, like a fingerprint

- For example:
  - **Hydrogen** is known to produce strong spectral lines in the **red** portion of the visible spectrum, at **656 nm**
  - When sodium is burned, a characteristic **yellow** flame is observed due to it producing strong spectral lines in the yellow portion of the spectrum, at **589 nm**
  - When mercury is burned, most of the emission lines are below **450 nm**, which produces a characteristic **blue** light

- Elements such as **sodium** and **mercury** are known for their use in street lights, as well as **neon** for its use in colourful signs
- This can be achieved when
  - An electrical discharge is applied to the vapourised substance
  - The energy supplied excites orbital electrons within individual atoms to a higher energy state
  - When the electrons move back down to the ground state, a specific wavelength of light is emitted

YOUR NOTES

Photons & Atomic Transitions

The Photon Model

- Photons are fundamental particles that make up all forms of electromagnetic radiation
- A photon is defined as

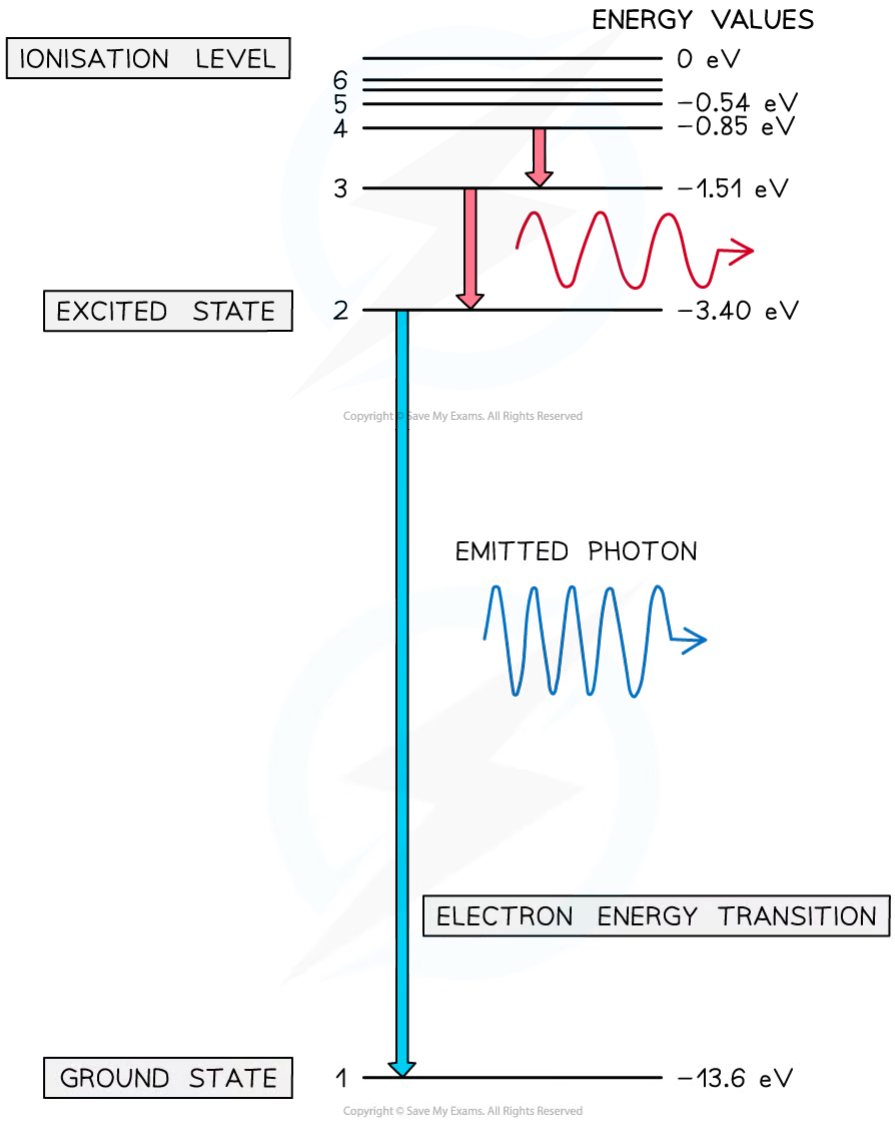
> **A massless "packet" or a "quantum" of electromagnetic energy**

- This means that the energy transferred by a photon is not continuous but as discrete packets of energy
  - In other words, each photon carries a specific amount of energy and transfers this energy all in one go
  - This is in contrast to waves which transfer energy continuously

Atomic Energy Levels

- Electrons in an atom can have only certain specific energies
  - These energies are called **electron energy levels**
- They can be represented as a series of stacked horizontal lines increasing in energy
- Normally, electrons occupy the lowest energy level available, this is known as the **ground state**
- Electrons can gain energy and move up the energy levels if they absorb energy via:
  - Collisions with other atoms or electrons
  - Absorbing a photon
  - A physical source, such as heat
- This is known as **excitation**, and when electrons move up an energy level, they are said to be in an **excited state**
- If the electron gains enough energy to be removed from the atom entirely, this is known as **ionisation**
- When an electron returns to a lower energy state from a higher excited state, it releases energy in the form of a photon

YOUR NOTES



Electron energy levels in atomic hydrogen. Photons are emitted when an electron moves from a higher energy state to a lower energy state

YOUR NOTES

Worked Example

Explain how atomic spectra provide evidence for the quantisation of energy in atoms.

Answer:

Step 1: Outline the meaning of atomic spectra

- Atomic spectra show the spectrum of discrete wavelengths emitted or absorbed by a specific atom

Step 2: Describe the relationship between energy and wavelength

- Photon energy is related to frequency and wavelength
- Therefore, photons with discrete wavelengths have discrete energies equal to the difference between two energy levels

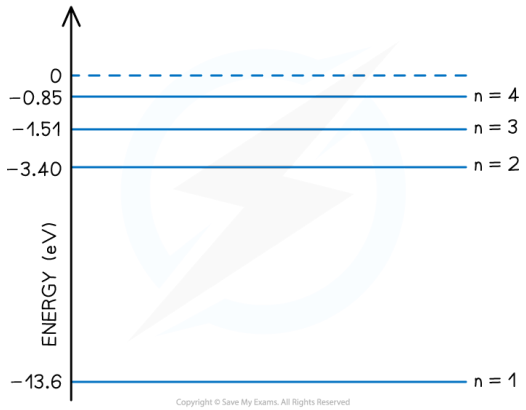
Step 3: Explain how atomic spectra give evidence for the quantisation of energy

- Photons arise from electron transitions between energy levels
- This happens when an electron is excited, or de-excited, from one energy level to another, by either emitting or absorbing light of a specific wavelength
- Since atomic spectra are made up of discrete wavelengths, this shows that atoms must contain discrete, or quantised, energy levels

YOUR NOTES

Worked Example

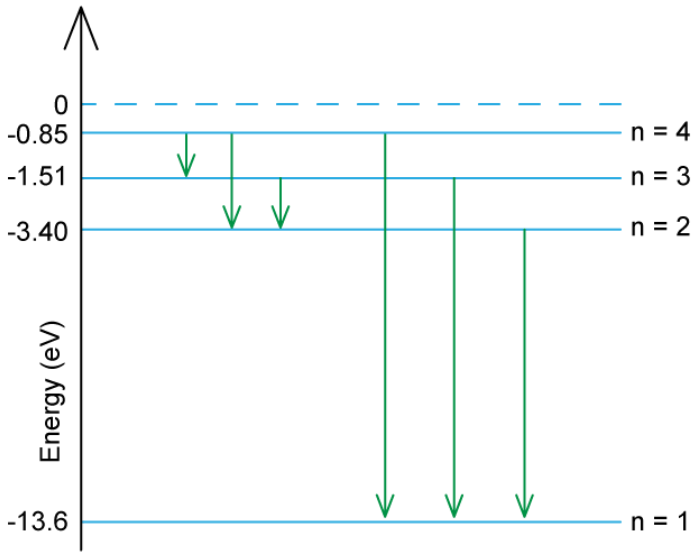
The diagram shows the electron energy levels in an atom of hydrogen.



Determine the number of possible wavelengths that can be produced from transitions between the n = 4 excited state and the n = 1 ground state.

Answer:

- There are **six** possible wavelengths that could be produced from the different energy level transitions



- The possible transitions are:
  - n = 4 to n = 3
  - n = 4 to n = 2
  - n = 4 to n = 1
  - n = 3 to n = 2
  - n = 3 to n = 1

- n = 2 to n = 1

YOUR NOTES

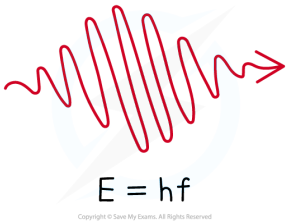
Exam Tip

Make sure you learn the definition for a photon: *discrete quantity / packet / quantum of electromagnetic energy* are all acceptable definitions

Calculating Photon Energy

YOUR NOTES

- Each line of the emission spectrum corresponds to a different **energy level transition** within the atom
  - Electrons can transition between energy levels absorbing or emitting a **discrete amount of energy**
  - An excited electron can transition down to the next energy level or move to a further level closer to the ground state
- For example, if an atom has **six** energy levels:
  - At low temperatures, most electrons will occupy the ground state n = 1
  - At high temperatures, electrons may be excited to the most excited state n = 6



Energy and frequency of a photon are directly proportional

- The energy of a photon can be calculated using the formula:

$$E = hf$$

- Using the wave equation, energy can also be equal to:

$$E = \frac{hc}{\lambda}$$

- Where:
  - E = energy of the photon (J)
  - h = Planck's constant (J s)
  - c = the speed of light (m s^{-1})
  - f = frequency (Hz)
  - λ = wavelength (m)
- This equation tells us:
  - The higher the frequency of EM radiation, the higher the energy of the photon
  - The energy of a photon is inversely proportional to the wavelength
  - A long-wavelength photon of light has a lower energy than a shorter-wavelength photon

Difference in discrete energy levels

- The difference between two energy levels is equal to a specific photon energy
- The energy of the photon is given by:

$$\Delta E = hf = E_2 - E_1$$

- Where:
  - E_1 = energy of the lower level (J)
  - E_2 = energy of the higher level (J)
- Using the wave equation, the wavelength of the emitted, or absorbed, radiation can be related to the energy difference by the equation:

$$\lambda = \frac{hc}{E_2 - E_1}$$

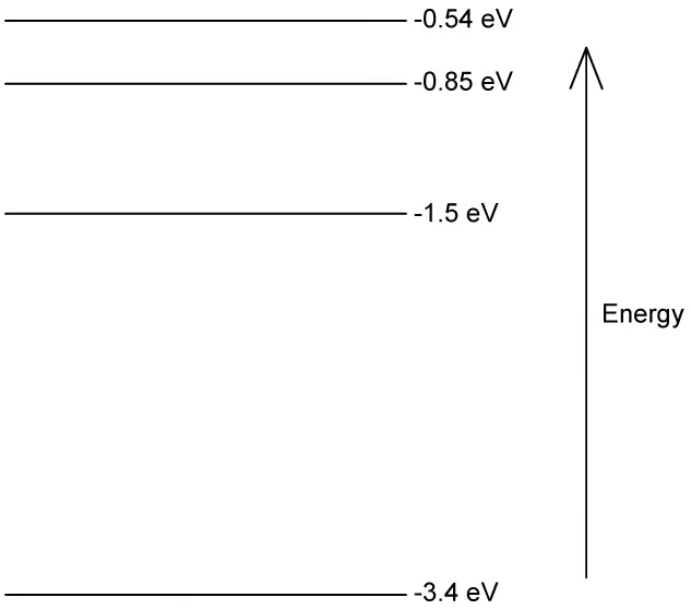
- This equation shows that:
  - The larger the difference in energy between two levels ΔE, the shorter the wavelength λ and vice versa

YOUR NOTES

YOUR NOTES

Worked Example

Some electron energy levels in atomic hydrogen are shown below.



The longest wavelength produced as a result of electron transitions between two of the energy levels is 4.0×10^{-6} m.

(a)
Draw an arrow to show the transition that would produce:

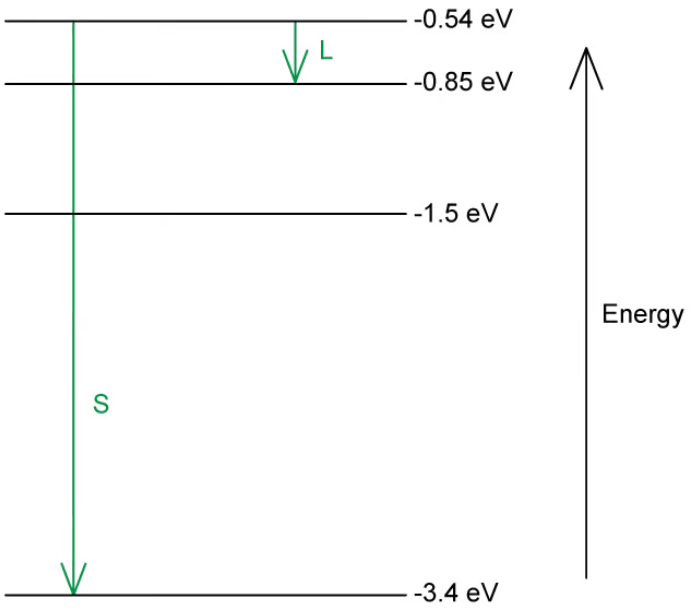
- A photon of wavelength 4.0×10^{-6} m. Mark with the letter **L**.
- The photon with the shortest wavelength. Mark with the letter **S**.

(b)
Calculate the wavelength for the transition giving rise to the shortest wavelength.

Answer:

(a)

- Photon energy and wavelength are inversely proportional
- Therefore, the largest energy change corresponds to the shortest wavelength (line **S**)
- The smallest energy change corresponds to the longest wavelength (line **L**)



(b)

Step 1: Write down the equation linking wavelength and energy

$$\lambda = \frac{hc}{\Delta E} = \frac{hc}{E_2 - E_1}$$

Step 2: Identify the energy levels that give rise to the shortest wavelength

- The shortest wavelength photon will come from a transition between the energy levels that have the largest difference:
  - $E_2 = -0.54\,\text{eV}$
  - $E_1 = -3.4\,\text{eV}$
- Therefore, the greatest possible difference in energy is

$$\Delta E = E_2 - E_1 = -0.54 - (-3.4) = 2.86\,\text{eV}$$

Step 3: Calculate the wavelength

- To convert from eV to J: multiply by 1.6×10^{-19} J

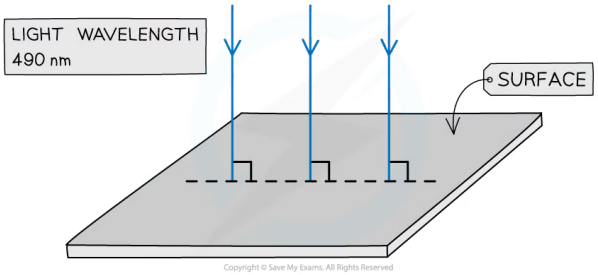
$$\lambda = \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{2.86 \times (1.6 \times 10^{-19})}$$

$$\lambda = 4.347 \times 10^{-7}\,\text{m} = 435\,\text{nm}$$

YOUR NOTES

Worked Example

Light of wavelength 490 nm is incident normally on a surface, as shown in the diagram.



The power of the light is 3.6 mW. The light is completely absorbed by the surface.

Calculate the number of photons incident on the surface in 30 s.

Answer:

Step 1: Write down the known quantities

- Wavelength, $\lambda = 490\ \text{nm} = 490 \times 10^{-9}\ \text{m}$
- Power, $P = 3.6\ \text{mW} = 3.6 \times 10^{-3}\ \text{W}$
- Time, $t = 30\ \text{s}$

Step 2: Write the equation for photon energy and write in terms of wavelength

$$E = hf \quad \Rightarrow \quad E = \frac{hc}{\lambda}$$

Step 3: Calculate the energy of one photon

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34})(3.0 \times 10^{8})}{490 \times 10^{-9}} = 4.06 \times 10^{-19}\ \text{J}$$

Step 4: Calculate the number of photons hitting the surface every second

$$\frac{power\ of\ light\ source}{energy\ of\ one\ photon} = \frac{3.6 \times 10^{-3}}{4.06 \times 10^{-19}} = 8.87 \times 10^{15}\ \text{s}^{-1}$$

Step 5: Calculate the number of photons that hit the surface in 30 s

number of photons in 30 s = $(8.87 \times 10^{15}) \times 30 =$ **2.7×10^{17}**

Exam Tip

The values of Planck's constant and the speed of light will be included in the data booklet, however, it helps to memorise them to speed up calculations!

Nuclear Radius

- The radius of a nucleus depends on the nucleon number *A* of the atom
  - The greater the number of nucleons a nucleus has, the greater the space the nucleus occupies, hence giving it a larger radius
- The exact relationship between the **radius** and **nucleon number** can be determined from experimental data
- By doing this, physicists were able to deduce the following relationship:

$$R = R_0 A^{\frac{1}{3}}$$

- Where:
  - R = nuclear radius (m)
  - A = nucleon / mass number
  - R_0 = constant of proportionality = 1.20 fm

Nuclear Density

- Assuming that the nucleus is spherical, its volume is equal to:

$$V = \frac{4}{3}\pi R^3$$

- Combining this with the expression for nuclear radius gives:

$$V = \frac{4}{3}\pi\left(R_0 A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi {R_0}^3 A$$

- This tells us that the nuclear volume V is proportional to the mass of the nucleus m, which is equal to

$$m = Au$$

- Where u = atomic mass unit (kg)
- Using the definition for density, nuclear density is equal to:

$$\rho = \frac{m}{V}$$

$$\rho = \frac{Au}{\frac{4}{3}\pi {R_0}^3 A} = \frac{3u}{4\pi {R_0}^3}$$

- Since the mass number A cancels out, the remaining quantities in the equation are all **constants**
- Therefore, this shows the density of the nucleus is:

- The **same** for all nuclei
- Independent of the **radius**

- The fact that nuclear density is constant shows that nucleons are **evenly separated** throughout the nucleus regardless of their size
- The accuracy of nuclear density depends on the accuracy of the constant R_0
  - As a guide, nuclear density should always be of the order 10^{17} kg m^{-3}
- Nuclear density is significantly **larger** than atomic density which suggests:
  - The **majority** of the atom's mass is contained in the nucleus
  - The nucleus is very **small** compared to the atom
  - Atoms must be predominantly **empty space**

Worked Example

Determine the value of nuclear density.

You may take the constant of proportionality R_0 to be 1.20 fm.

Answer:

Step 1: Derive an expression for nuclear density

- Using the equation derived above, the density of the nucleus is:

$$\rho = \frac{3u}{4\pi {R_0}^3}$$

Step 2: List the known quantities

- Atomic mass unit, $u = 1.661 \times 10^{-27}$ kg
- Constant of proportionality, $R_0 = 1.20$ fm $= 1.20 \times 10^{-15}$ m

Step 3: Substitute the values to determine the nuclear density

$$\rho = \frac{3 \times (1.661 \times 10^{-27})}{4\pi (1.20 \times 10^{-15})^3} = 2.3 \times 10^{17} \text{ kg m}^{-3}$$

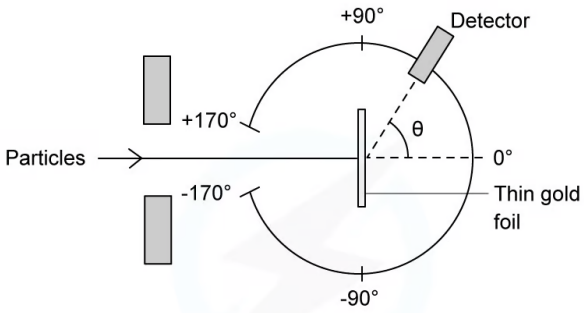
Rutherford Scattering Experiment

YOUR NOTES

- In the Rutherford scattering experiment, alpha particles are fired at a thin gold foil
- Initially, before interacting with the foil, the particles have kinetic energy equal to

$$E_k = \frac{1}{2}mv^2$$

- Some of the alpha particles are found to come straight back from the gold foil
- This indicates that there is **electrostatic repulsion** between the alpha particles and the gold nucleus



Experimental set-up of the Rutherford alpha scattering experiment

- At the point of closest approach *d*, the repulsive force **reduces** the speed of the alpha particles to **zero** momentarily, before any change in direction
- At this point, the **initial kinetic energy** E_k of the alpha particle is equal to the **electric potential energy** E_p of the target nucleus:

$$E_k = E_p$$

- Where the electric potential energy is given by

$$E_p = k\frac{Qq}{d}$$

- Where:
  - Charge of an alpha particle, Q = 2e
  - Charge of a target nucleus, *q* = Ze
  - *Z* = proton (atomic) number
  - e = elementary charge (C)
  - k = Coulomb constant
- This gives an expression for the **potential energy** at the point of **repulsion**:

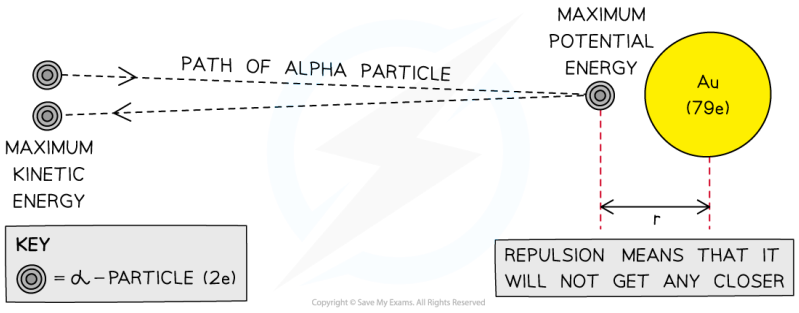
$$E_p = k\frac{(2e)(Ze)}{d} = k\frac{2Ze^2}{d}$$

- Which, due to the conservation of energy also gives the **initial kinetic energy** possessed by the alpha particle

- Rearranging for the distance of closest approach *d*

$$d = k\frac{2Ze^2}{E_p} = k\frac{2Ze^2}{E_k}$$

- This gives a value for the **radius** of the nucleus, assuming the alpha particle is fired at a high energy



The closest approach method of determining the size of a gold nucleus

Exam Tip

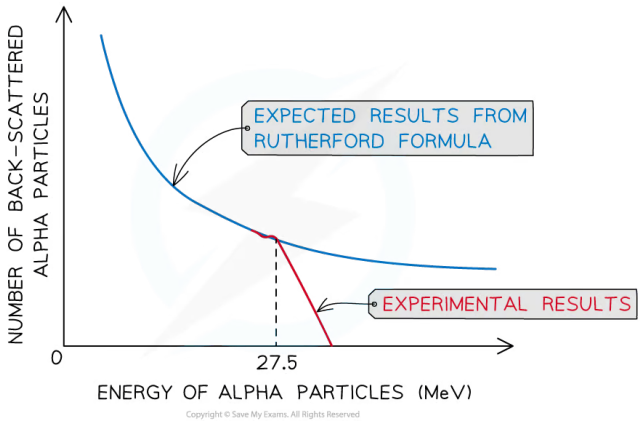
Make sure you're comfortable with the calculations involved with the alpha particle closest approach method, as this is a common exam question.

You will be expected to remember that the charge of an α is the charge of 2 protons (2 x the charge of an electron)

YOUR NOTES

Deviations from Rutherford Scattering

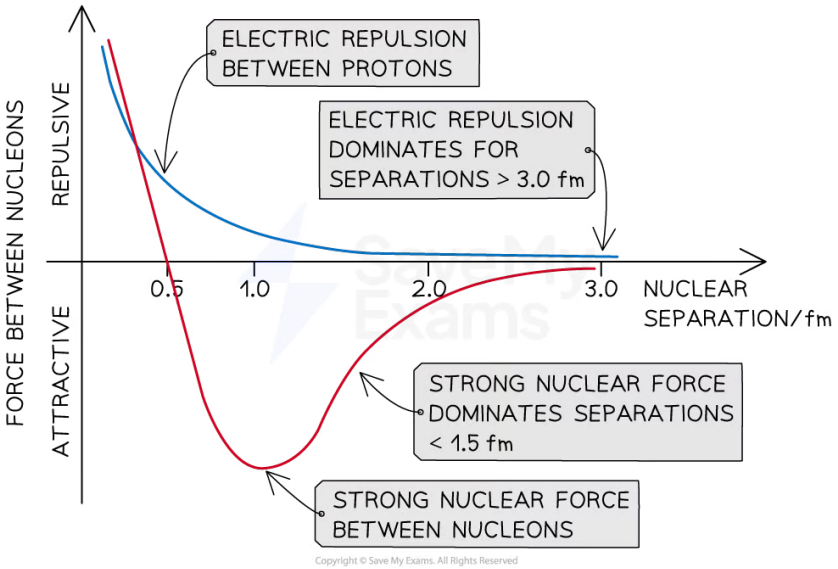
- Rutherford's scattering experiment predicted that
  - As the scattering angle **increases**, the number of alpha particles scattered at that angle **decreases**
- This was found to be correct at low to moderately high energies
- However, at **very high energies** (>27.5 MeV) significant **deviations** from Rutherford's predictions were observed
- Instead of the number decreasing at the expected rate, the number of alpha particles being back-scattered **sharply decreases** to zero



The observed back-scattering from alpha particles strongly deviates from the predicted relationship based only on electromagnetic repulsion at 27.5 MeV

- Rutherford's alpha scattering experiment originally assumed that the alpha particles **only** interact through **electrostatic repulsion**
- However, if the energy of the alpha particles exceeds 27.5 MeV, then they will be close enough to **interact** with the nucleus via the strong nuclear force
- Factoring in the interactions due to the strong nuclear force explains the scattering pattern observed in the experimental results
- Therefore, deviations from Rutherford scattering provide **evidence** for the **strong nuclear force**

YOUR NOTES



Very high-energy alpha particles can get close enough to the nucleons that the effect of the strong nuclear force becomes significant

YOUR NOTES

Worked Example

Alpha particles undergo scattering after being fired at a thin gold $^{197}_{79}\text{Au}$ foil. The gold is then replaced to make a comparison.

Describe the predicted difference in the scattering pattern when the foil is replaced with aluminium $^{30}_{13}\text{Al}$ foil of the same thickness.

Answer:

Step 1: Compare the relative charges of the nuclei

- The force between nuclei due to the electric repulsion is

$$F = k\frac{qQ}{r^2} \quad \Rightarrow \quad r^2F = kqQ$$

- Therefore, the charge of a nucleus is proportional to the square of the distance between it and an alpha particle

$$Q \propto r^2$$

- Gold has 79 protons, so $Q_{gold} = +79e$
- Aluminium has 13 protons, so $Q_{aluminium} = +13e$
- Therefore, an alpha particle will get closer to the nucleus with less charge i.e. the aluminium nucleus than the gold nucleus

Step 2: Predict the patterns and deviations from Rutherford scattering

- Deviations from Rutherford scattering occur when alpha particles get close enough for the strong nuclear force to begin to become more significant than the electric force
- At very small separations (<1.5 fm) the effect of the strong nuclear force becomes significant
- Alpha particles will be able to get closer to aluminium nuclei at lower energies than the gold nuclei
- Therefore, alpha particles will be less affected by electric repulsion and able to get close enough for interactions with the strong nuclear force
- Hence, more deviation will be seen with aluminium foil than with gold foil

Exam Tip

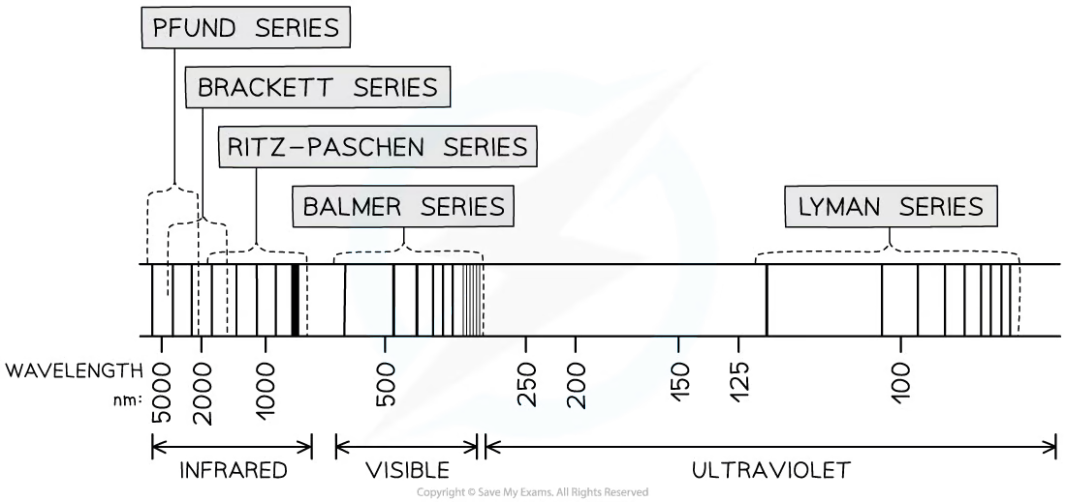
Make sure you can explain when the greatest deviations from Rutherford scattering will occur i.e.

- When the alpha particles have **high energies**
- The target nuclei have a **low nucleon number**

YOUR NOTES

The Bohr Model of Hydrogen

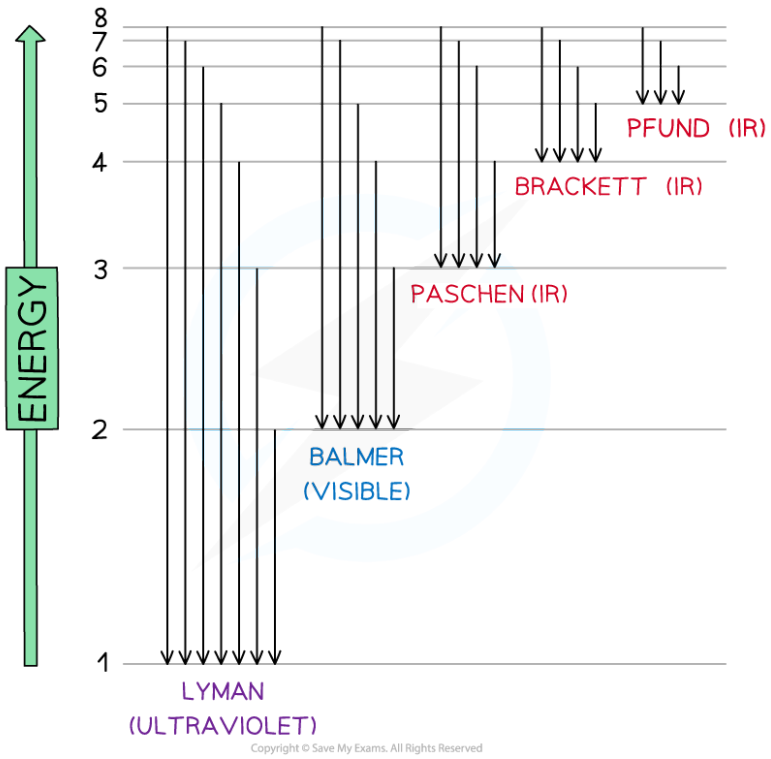
- Hydrogen is the simplest atom in existence, making it ideal for experiments investigating the nature of electron energy levels
- Line spectra produced by hydrogen atoms showed that
  - Electrons are able to jump, or **transition**, between specific energy levels producing specific energy photons
  - Different transitions can be categorised into **series**, or families, of lines
- The Lyman series converges on the **ground state** ($n = 1$) for electrons
  - The Balmer series converges on the **second** energy level ($n = 2$)
  - The Ritz-Paschen converges on the **third** energy level ($n = 3$) and so on



The transitions observed in hydrogen line spectra can be classed into series

- The Lyman series photons will have the **highest** energies since they have the **shortest** wavelength
  - These transitions tend to produce **ultraviolet** photons
- The Pfund series photons will have the **lowest** energies since they have the **longest** wavelength
  - These transitions tend to produce **infrared** photons

YOUR NOTES



Representing the electron jumps in the hydrogen spectra

- The finding of these electron transitions helped scientists to understand how electrons work to produce photons of specific wavelength and energy
- This led to the development of the **Bohr model of hydrogen**, which states that
  - Electrons can only move in fixed orbits
  - The orbital radius of electrons is restricted to certain values
- The **discrete energy** of the transitions in the Bohr model for hydrogen are described by the equation:

$$E = -\frac{13.6}{n^2}\ \text{eV}$$

- Where
  - E = photon energy (J)
  - n = an integer 1, 2, 3 etc. to describe the energy level of an atom

YOUR NOTES

Worked Example

Determine the frequency of an emitted photon from a hydrogen atom when an electron makes a transition between levels n = 4 and n = 2.

Answer:

Step 1: List the known quantities

- Transition between n = 4 and n = 2
- Planck's constant, $h = 6.63 \times 10^{-34}$ J s
- Electronvolt, $eV = 1.6 \times 10^{-19}$ J

Step 2: Determine an equation for the change in energy ΔE

$$E_n = -\frac{13.6\ eV}{n^2}$$

$$\Delta E = E_4 - E_2$$

$$\Delta E = \left(-\frac{13.6\ eV}{4^4}\right) - \left(-\frac{13.6\ eV}{2^4}\right)$$

Step 3: Calculate the change in energy, in eV, for the photon using the given equation

$$\Delta E = -13.6\left(\frac{1}{4^2} - \frac{1}{2^2}\right) = 2.55\ eV$$

Step 4: Rearrange the photon energy equation for frequency f

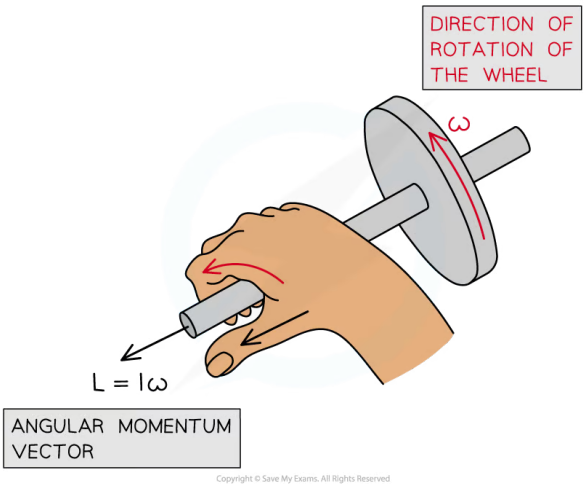
$$E = hf \quad \Rightarrow \quad f = \frac{E}{h}$$

Step 5: Substitute the known values into the equation for frequency

$$f = \frac{2.55 \times (1.6 \times 10^{-19})}{6.63 \times 10^{-34}} = 6.15 \times 10^{14}\ \text{Hz}$$

Quantisation of Angular Momentum

- Angular momentum is a property of any spinning or rotating body, very similar to linear momentum
  - In linear motion, momentum is the product of mass and velocity
  - In rotational motion, the momentum is the product of moment of inertia and angular speed
- Angular momentum is a vector, which means:
  - The **magnitude** is equal to the momentum of the particle times its radial distance from the centre of its circular orbit
  - The **direction** of the angular momentum vector is normal to the plane of its orbit with the direction being given by the corkscrew rule



Angular momentum acts at right angles to the direction of rotation

- Niels Bohr proposed that the angular momentum *L* of an electron in an energy level is quantised in integer multiples of Planck's constant over 2π:

$$L = n\frac{h}{2\pi}$$

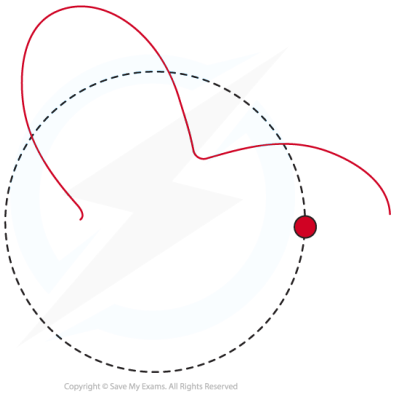
- Where:
  - *n* = an integer (n = 1, 2, 3...)
  - *h* = Planck's constant
- Hence the angular momentum for an electron in a circular orbit is **constant**
- De Broglie proposed that an electron with momentum *p* = *mv* has a wavelength λ given by

$$\lambda = \frac{h}{p}$$

- For an electron moving in a straight line, the matter wave takes a familiar wave shape consisting of peaks and troughs
  - Although the electron itself isn't oscillating up and down, only the matter wave is

de Broglie matter wave for an electron moving in a straight line at constant speed

- For the same electron moving in a circle, the matter wave still has a sinusoidal shape but is wrapped into a circle

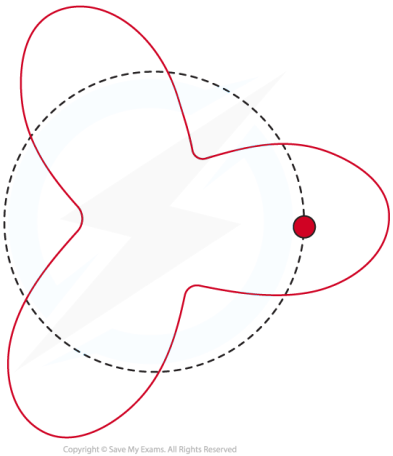


de Broglie matter wave for an electron moving in a circular orbit at constant speed

- As the electron continues to orbit in a circle two possibilities may occur:

1. On completing one oscillation, the waves overlap in phase

- The waves will continue in phase over many orbits giving rise to constructive interference and a standing wave

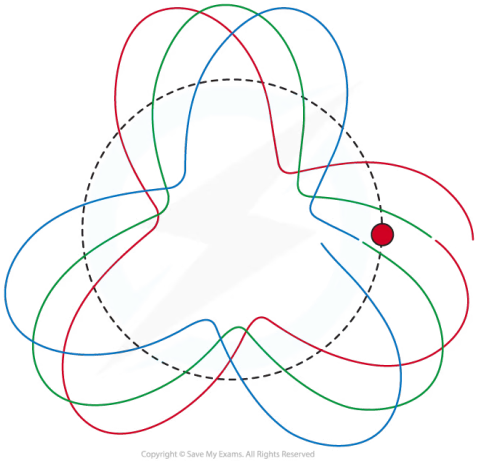


de Broglie matter wave where n = 3. Here the circumference of the circular orbit is 3λ

2. On completing one oscillation, the waves overlap but they are not in phase

YOUR NOTES

- In other words, peak overlaps with peak, trough with trough
- This means that where the waves overlap, destructive interference occurs and as a result, no such electron orbit is allowed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de Broglie matter wave where 3λ is less than the orbit's circumference

- Hence, the circumference of the orbit $(2\pi r)$ must equal an integer number of wavelengths $(n\lambda)$ for a standing wave to form:

$$n\lambda = 2\pi r$$

- Using the de Broglie relation:

$$\lambda = \frac{h}{p}$$

$$n\left(\frac{h}{p}\right) = 2\pi r$$

- Since momentum is equal to $p = mv$:

$$n\left(\frac{h}{mv}\right) = 2\pi r$$

- Rearranging for angular momentum $(L = mvr)$:

$$\frac{nh}{2\pi} = mvr$$

- This is known as the **Bohr Condition**
- Where:
  - n = integer number of energy level
  - h = Planck's constant (J s)
  - m = mass of an electron (kg)
  - v = velocity of electron (m s^{-1})
  - r = radius of orbit (m)

YOUR NOTES

Worked Example

Determine the velocity of the electron in the first Bohr orbit of the hydrogen atom.

You may use the following values:

- Mass of an electron = 9.1×10^{-31} kg
- Radius of the orbit = 0.529×10^{-10} m
- Planck's constant = 6.63×10^{-34} kg m^2 s^{-1}

Answer:

Step 1: List the known quantities

- First orbital level, n = 1
- Mass of an electron, $m = 9.1 \times 10^{-31}$ kg
- Radius of the orbit, $r = 0.529 \times 10^{-10}$ m
- Planck's constant, $h = 6.63 \times 10^{-34}$ kg m^2 s^{-1}

Step 2: Write the Bohr Condition equation and rearrange for velocity, *v*

$$\frac{nh}{2\pi} = mvr \quad \Rightarrow \quad v = \frac{nh}{2\pi mr}$$

Step 3: Substitute the values in and calculate the velocity *v*

$$v = \frac{1 \times (6.63 \times 10^{-34})}{2\pi \times (9.1 \times 10^{-31})(0.529 \times 10^{-10})}$$

Step 4: Write the final answer

Velocity of an electron ($n = 1$): $v = 2.2 \times 10^6$ m s^{-1}

IB Physics HL

YOUR NOTES

Quantum Physics

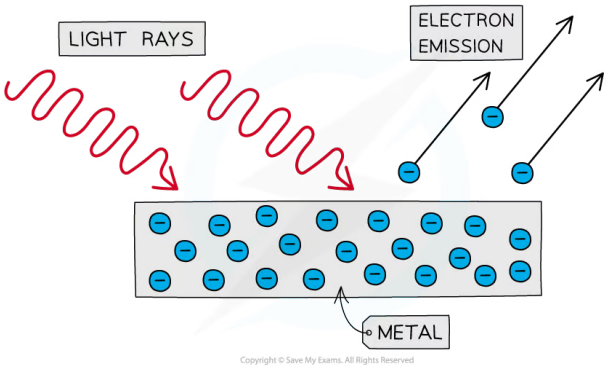
CONTENTS

The Photoelectric Effect (HL)
The Photoelectric Equation (HL)
The Particle Nature of Light (HL)
The de Broglie Wavelength (HL)
Wave-Particle Duality (HL)
Compton Scattering (HL)

YOUR NOTES

The Photoelectric Effect

- The **photoelectric effect** is a phenomenon in which electrons are emitted from the surface of a metal upon the **absorption** of electromagnetic radiation
  - Electrons removed from the surface of a metal in this manner are known as **photoelectrons**
- The photoelectric effect provides important evidence that light behaves as a **particle** i.e. it is quantised, or carried in discrete packets
  - This is shown by the fact each electron can absorb only a single photon



Photoelectrons are emitted from the surface of metal when light shines onto it

Threshold Frequency

- Photoelectrons are not emitted from the surface of a metal when any light is shone on it
- The photoelectric effect only occurs when the frequency of light is above a **threshold frequency**
- The threshold frequency of a metal is defined as:

> **The minimum frequency of incident electromagnetic radiation required to remove a photoelectron from the surface of a metal**

- Threshold frequency and wavelength are properties of a material and vary from metal to metal

Threshold frequencies and wavelengths for different metals

Metal	Threshold Frequency f_0 / Hz	Threshold Wavelength λ_0 / nm
sodium	4.40×10^{14}	682
potassium	5.56×10^{14}	540
zinc	1.02×10^{15}	294

iron	1.04×10^{15}	289
copper	1.13×10^{15}	266
gold	1.23×10^{15}	244
silver	9.71×10^{15}	30.9

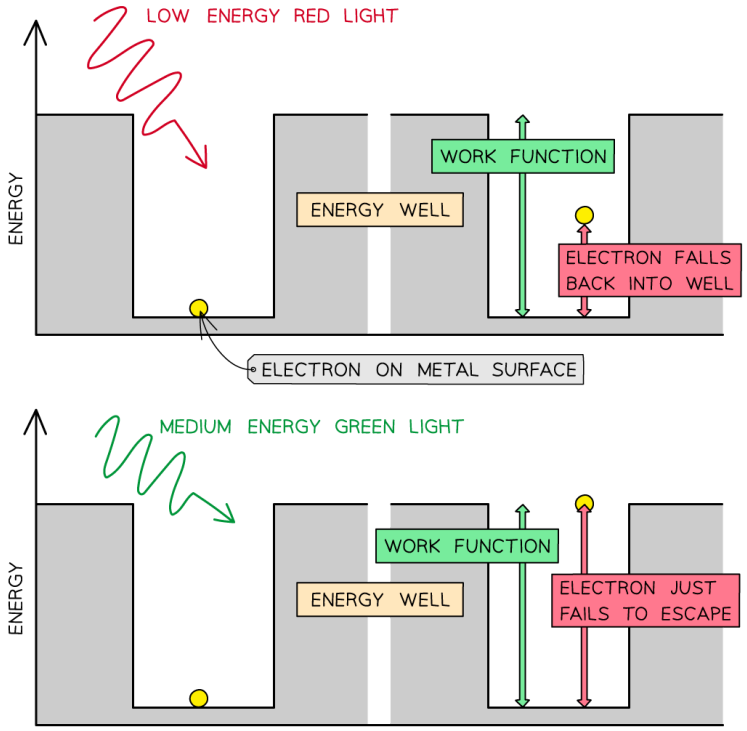
YOUR NOTES

The Work Function

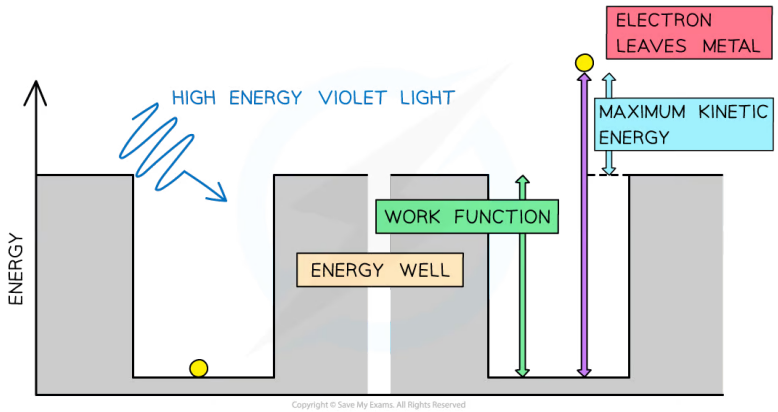
- The work function Φ, or threshold energy, of a material is defined as:

  The minimum energy required to release a photoelectron from the surface of a metal

- Consider the electrons in a metal as trapped inside an 'energy well' where the energy between the surface and the top of the well is equal to the work function Φ
  - **One** electron absorbs **one** photon
  - Therefore, an electron can only escape from the surface of the metal if it absorbs a photon which has an energy equal to the work function Φ or higher



YOUR NOTES



In the photoelectric effect, a single photon may cause a surface electron to be released if it has sufficient energy

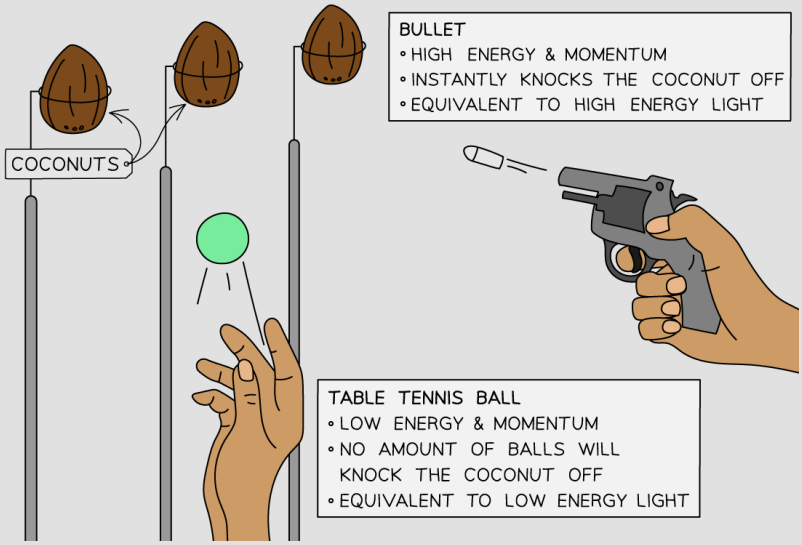
- Different metals have different threshold frequencies and hence different work functions
- Using the well analogy:
  - A more tightly bound electron requires **more** energy to reach the top of the well
  - A less tightly bound electron requires **less** energy to reach the top of the well
- Alkali metals, such as sodium and potassium, have threshold frequencies in the **visible light region**
  - This is because the attractive forces between the surface electrons and positive metal ions are relatively weak
- Transition metals, such as zinc and iron, have threshold frequencies in the **ultraviolet region**
  - This is because the attractive forces between the surface electrons and positive metal ions are much stronger

YOUR NOTES

Exam Tip

A useful analogy for threshold frequency is a fairground coconut shy:

- One person is throwing table tennis balls at the coconuts, and another person has a pistol
- No matter how many of the table tennis balls are thrown at the coconut it will still stay firmly in place – this represents the **low frequency quanta**
- However, a single shot from the pistol will knock off the coconut immediately – this represents the **high frequency quanta**



The Photoelectric Equation

- The energy possessed by a photon is equal to

$$E = hf$$

- The fraction of the energy transferred to the electron to release it from the metal is equal to the work function Φ
- The remaining amount is given as kinetic energy to the emitted photoelectron

$$E_{K\,max} = \frac{1}{2} m_e v_{max}^2$$

- Since energy is always conserved, the energy of a photon incident on the surface of a metal is equal to:

The work function + the maximum kinetic energy of the photoelectron

- This equation is known as the **photoelectric equation**:

$$E = hf = \Phi + \frac{1}{2} m_e v_{max}^2$$

- Where:
  - h = Planck's constant (J s)
  - f = frequency of the incident radiation (Hz)
  - Φ = work function of the metal (J)
  - $E_{K\,max}$ = maximum kinetic energy of a photoelectron (J)
  - m_e = mass of an electron (kg)
  - v_{max} = maximum velocity of a photoelectron (m s^{-1})
- This equation demonstrates:
  - If the incident photons do not have a high enough frequency and energy to overcome the work function (Φ), then no electrons will be emitted
  - When $E = hf_0 = \Phi$, where f_0 = threshold frequency, photoelectric emission only just occurs
  - The maximum kinetic energy $E_{K\,max}$ depends only on the frequency of the incident photon, and **not** the intensity of the radiation
  - The majority of photoelectrons will have kinetic energies **less than** $E_{K\,max}$

Graphical Representation of Work Function

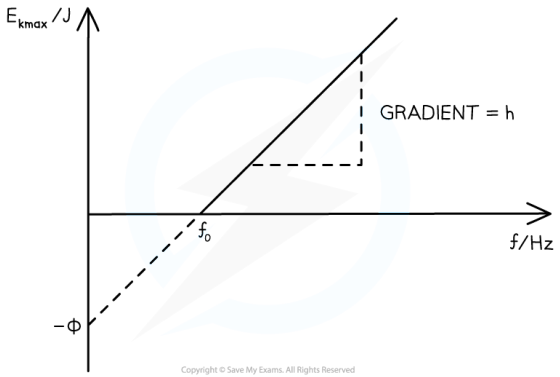
- The photoelectric equation can be rearranged into the straight-line equation:

$$y = mx + c$$

- Comparing this to the photoelectric equation:

$$E_{K\,max} = hf - \Phi$$

- A graph of maximum kinetic energy $E_{K\,max}$ against frequency f can be obtained



- The key elements of the graph are:
  - The work function Φ is the **y-intercept**
  - The threshold frequency f_0 is the **x-intercept**
  - The **gradient** is equal to Planck's constant h
  - There are no electrons emitted below the threshold frequency f_0

Kinetic Energy & Intensity

- The **kinetic energy** of the photoelectrons is **independent of the intensity** of the incident radiation
  - This is because **each electron can only absorb one photon**
  - Kinetic energy is only dependent on the **frequency** of the incident radiation
- Intensity is a measure of the **number** of photons **incident** on the surface of the metal
  - So, increasing the number of electrons striking the metal will not increase the kinetic energy of the electrons, it will increase the **number** of photoelectrons emitted

Why Kinetic Energy is a Maximum

- Each electron in the metal acquires the **same** amount of energy from the photons in the incident monochromatic radiation.
- However, the energy required to remove an electron from the metal varies because some electrons are on the surface whilst others are deeper in the metal
  - The photoelectrons with the maximum kinetic energy will be those on the **surface** of the metal since they do not require much energy to leave the metal
  - The photoelectrons with lower kinetic energy are those deeper within the metal since some of the energy absorbed from the photon is used to approach the metal surface (and overcome the work function)
  - There is less kinetic energy available for these photoelectrons once they have left the metal

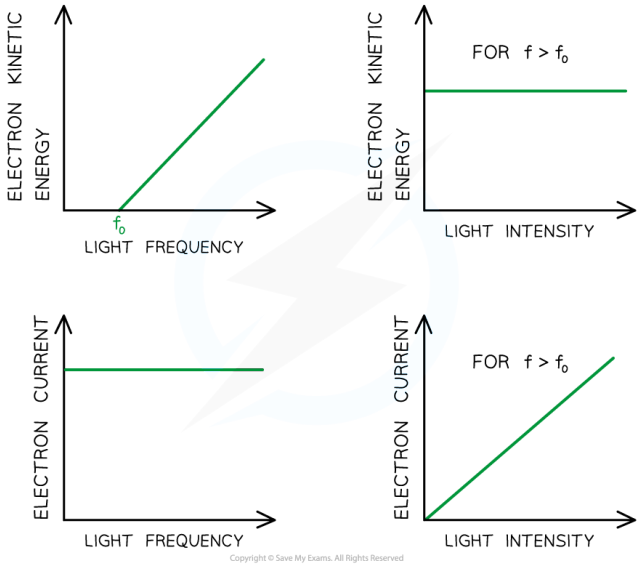
Photoelectric Current

- The photoelectric current is the number of photoelectrons emitted per second

YOUR NOTES

YOUR NOTES

- **Photoelectric current** is proportional to the **intensity** of the radiation incident on the surface of the metal
- This is because the intensity is proportional to the number of photons striking the metal per second
- Since each photoelectron absorbs a single photon, the photoelectric current must be proportional to the intensity of the incident radiation



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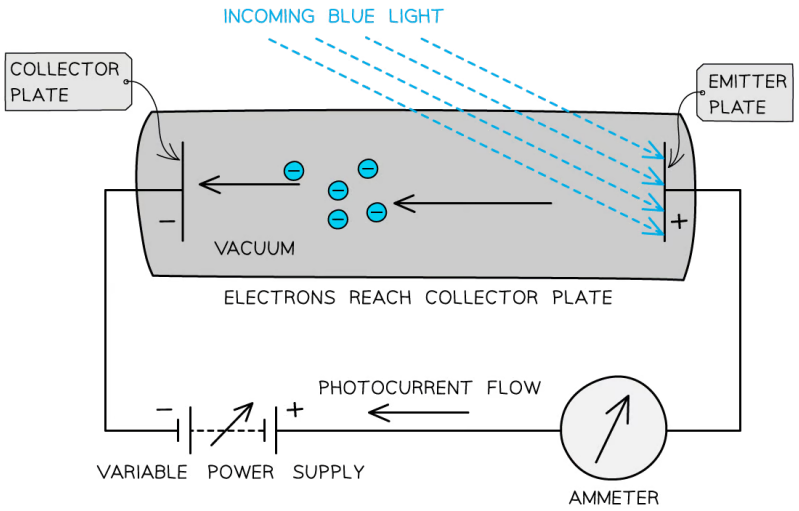
Graphs showing the variation of electron KE and photocurrent with the frequency of the incident light

Stopping Voltage

- Stopping voltage V_S is defined as:

The voltage required to stop photoelectron emission from occurring

- The photons arriving at the metal plate cause photoelectrons to be emitted
  - This is called the **emitter plate**
- The electrons that cross the gap are collected at the other metal plate
  - This is called the **collector plate**



This setup can be used to determine the maximum kinetic energy of the emitted photoelectrons

- The flow of electrons across the gap sets up an e.m.f. between the plates that allows a current to flow around the rest of the circuit
  - Effectively, it becomes a photoelectric cell which produces a **photoelectric current**
- If the e.m.f. of the variable power supply is initially **zero**, the circuit operates **only** on the photoelectric current
- As the supply is turned up, the emitter plate becomes more **positive**
  - This is because it is connected to the positive terminal of the supply
- As a result, electrons leaving the emitter plate are **attracted** back towards it
  - This is because the p.d. across the tube **opposes** the motion of the electrons between the plates
- If any electrons escape with high enough kinetic energy, they can **overcome** this attraction and cross to the collector plate
  - And if they don't have enough energy, they can't cross the gap
- By **increasing** the e.m.f. of the supply, eventually, a p.d. will be reached at which no electrons will be able to cross the gap
  - This value of e.m.f. is equal to the stopping voltage V_S
- At this point, the energy needed to cross the gap is equal to the maximum kinetic energy $E_{K\,max}$ of the electrons

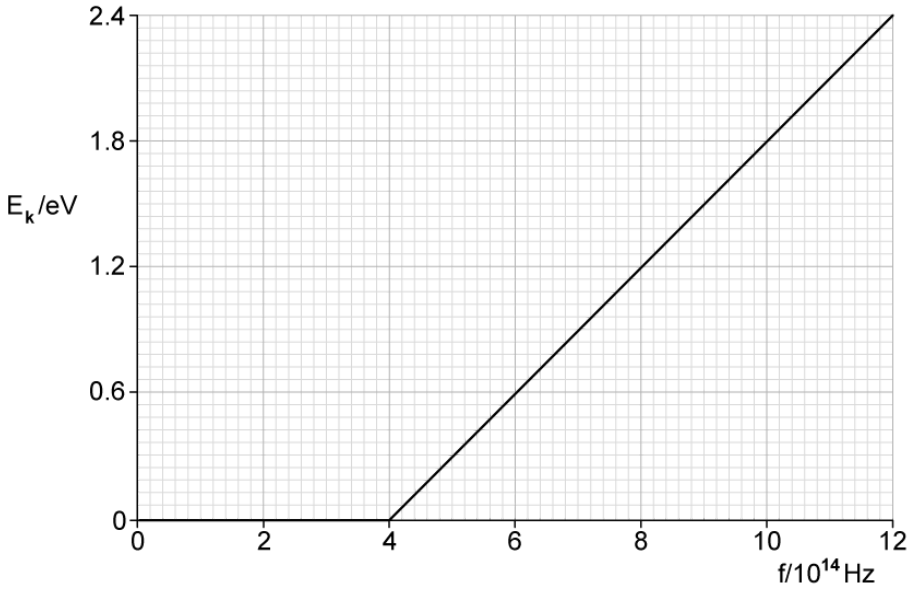
$$E_{K\,max} = eV_S$$

- Where:
  - $E_{K\,max}$ = maximum kinetic energy of the electrons (J)
  - e = elementary charge (C)
  - V_S = stopping voltage (V)

YOUR NOTES

Worked Example

The graph below shows how the maximum kinetic energy E_k of electrons emitted from the surface of sodium metal varies with the frequency f of the incident radiation.



Calculate the work function of sodium in eV.

Answer:

Step 1: Write out the photoelectric equation and rearrange it to fit the equation of a straight line

$$hf = \Phi + E_{K\,max}$$

$$E_{K\,max} = hf - \Phi$$

$$y = mx + c$$

- Therefore, when $E_K = 0$, $hf = \Phi$ and $f = f_0$

Step 2: Identify the threshold frequency from the x-axis of the graph

- From the graph:
  - When $E_K = 0$, threshold frequency: $f = f_0 = 4 \times 10^{14}$ Hz

Step 3: Calculate the work function

$$\Phi = hf_0 = (6.63 \times 10^{-34}) \times (4 \times 10^{14})$$

Work function: $\Phi = 2.652 \times 10^{-19}$ J

Step 4: Convert the work function into eV

- To convert from J to eV: divide by 1.6×10^{-19} J

$$E = \frac{2.652 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.66 \text{ eV}$$

YOUR NOTES

YOUR NOTES

Worked Example

Monochromatic light of wavelength λ_1 is incident on the surface of a metal. The stopping voltage for this light is V_1.

When another monochromatic light of wavelength λ_2 is incident on the same surface, the stopping voltage is V_2.

What is the quantity $\frac{(V_2 - V_1)}{\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)}$ equal to?

A. $\frac{h}{2\pi}$ **B.** $\frac{h}{c}$ **C.** $\frac{hc}{e}$ **D.** $\frac{h}{e}$

Answer: C

- The photoelectric equation for light 1 is

$$hf_1 = \Phi + E_{K\,max(1)}$$

Where $E_{K\,max(1)} = eV_1$ and $E_1 = hf_1 = \frac{hc}{\lambda_1}$

- The photoelectric equation for light 2 is

$$hf_2 = \Phi + E_{K\,max(2)}$$

Where $E_{K\,max(2)} = eV_2$ and $E_2 = hf_2 = \frac{hc}{\lambda_2}$

- Since the metal is the same, the work function Φ is the same for both, so:

$$\Phi = \frac{hc}{\lambda_1} - eV_1 = \frac{hc}{\lambda_2} - eV_2$$

- Collecting the terms together and simplifying gives

$$eV_2 - eV_1 = \frac{hc}{\lambda_2} - \frac{hc}{\lambda_1}$$

$$e(V_2 - V_1) = hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$\frac{(V_2 - V_1)}{\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)} = \frac{hc}{e}$$

YOUR NOTES

Exam Tip

When using the photoelectric effect equation, hf, Φ and $E_{k(max)}$ must all have the **same** units; Joules.

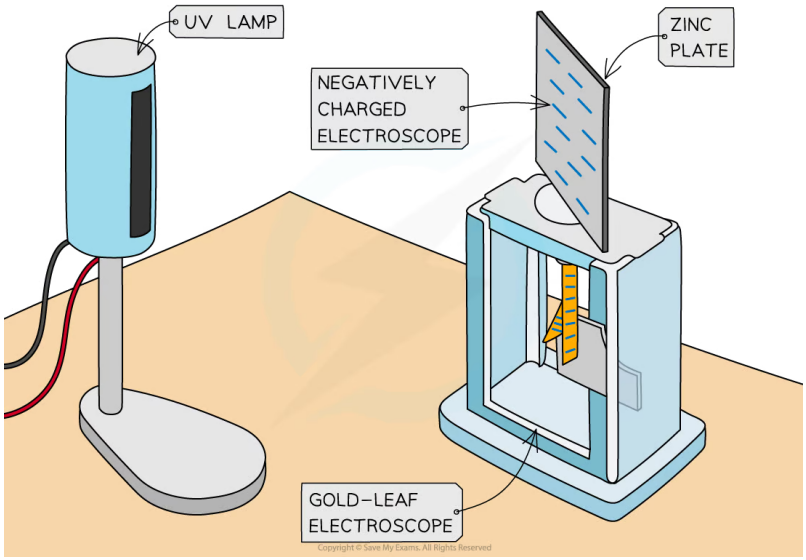
All values given in eV need to be converted into Joules by multiplying by 1.6×10^{-19}. Do this right away, in the same way as you would convert into SI units before calculating.

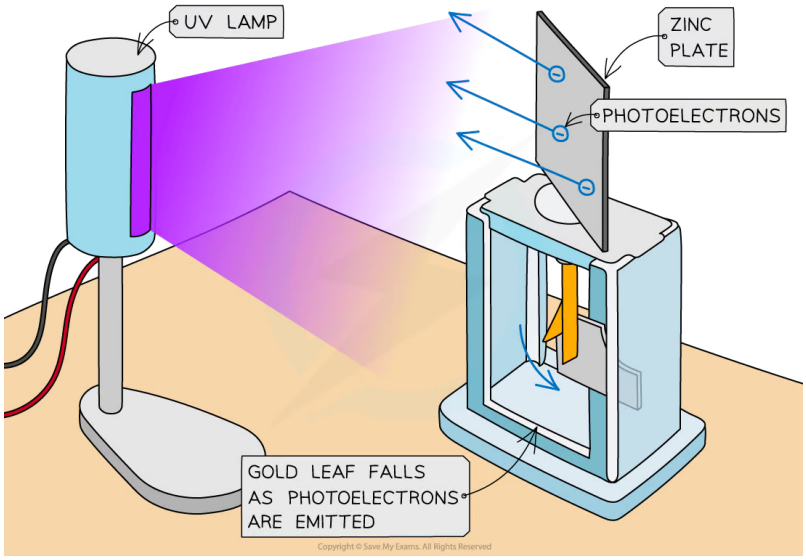
It is important to note that the stopping voltage actually holds a **negative value**, but since we use it to determine the maximum kinetic energy of the emitted electrons, its sign is not important in calculations, it's acceptable to just quote its magnitude.

YOUR NOTES

Evidence for the Particle Nature of Light

- The photoelectric effect can be observed on a **gold leaf electroscope**
- A plate of metal, usually **zinc**, is attached to a gold leaf, which initially has a negative charge, causing it to be repelled by a central negatively charged rod
  - This causes negative charge, or electrons, to build up on the zinc plate
- **UV light** is shone onto the metal plate, leading to the **emission** of photoelectrons
- This causes the extra electrons on the central rod and gold leaf to be removed, so, the gold leaf begins to fall back towards the central rod
  - This is because they become less negatively charged, and hence repel less
- Some notable observations:
  - Placing the UV light source **closer** to the metal plate causes the gold leaf to fall at a **faster rate**
  - Using a light source of a **higher frequency** does **not** affect the **rate** at which the gold leaf falls
  - Using a **filament** light source causes **no change** in the gold leaf's position
  - Using a **positively** charged plate also causes **no change** in the gold leaf's position





Typical set-up of the gold leaf electroscope experiment

Laws of Photoelectric Emission

- The photoelectric effect is **evidence** for the **particle nature** of light, as demonstrated by the following observations

1. **Placing the UV light source closer to the metal plate causes the gold leaf to fall more quickly**

- Placing the UV source closer to the plate increases the intensity incident on the surface of the metal
- Increasing the intensity, or brightness, of the incident radiation increases the number of photoelectrons emitted per second
- Therefore, the gold leaf loses negative charge more rapidly

2. **Using a higher frequency light source does not change how quickly the gold leaf falls**

- The maximum kinetic energy of the emitted electrons increases with the frequency of the incident radiation
- In the case of the photoelectric effect, energy and frequency are **independent** of the intensity of the radiation
- So, the intensity of the incident radiation affects how quickly the gold leaf falls, not the frequency

3. **Using a filament light source causes no change in the gold leaf's position**

- If the incident frequency is below a certain threshold frequency, no electrons are emitted, no matter the intensity of the radiation
- A filament light source has a frequency below the threshold frequency of the metal, so, no photoelectrons are released

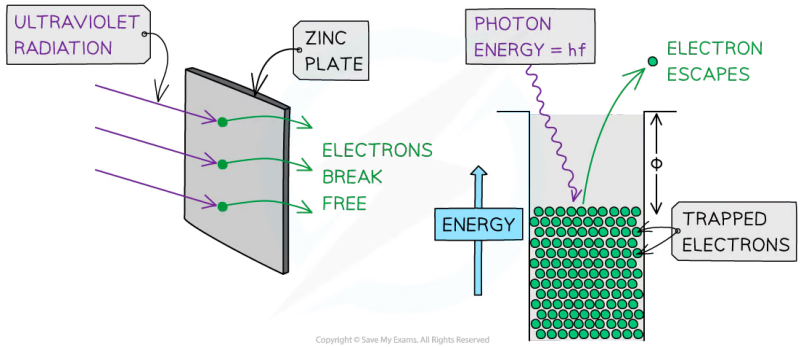
4. **Using a positively charged plate causes no change in the gold leaf's position**

YOUR NOTES

- If the plate is positively charged, that means there is an excess of positive charge on the surface of the metal plate
- Electrons are negatively charged, so they will not be emitted unless they are on the surface of the metal
- Any electrons emitted will be attracted back by positive charges on the surface of the metal

5. **Emission of photoelectrons happens as soon as the radiation is incident on the surface of the metal**

- A single photon interacts with a single electron
- If the energy of the photon is equal to the work function of the metal, photoelectrons will be released instantaneously



In the photoelectric effect, a single photon may cause a surface electron to be released if it has sufficient energy

YOUR NOTES

Worked Example

Describe how the photoelectric effect proves the particulate nature of light.

Answer:

Step 1: Outline what wave theory predicts about the photoelectric effect

- Wave theory predicts that...
  - Energy should be transferred to the electrons continuously until they have enough energy to be ejected
  - The kinetic energy of the emitted photoelectrons should depend on the intensity of the incident wave

Step 2: Outline the observations of the photoelectric effect experiment

- However, observations show that...
  - When $f < f_0$ (below threshold frequency): no electrons are emitted
  - When $f > f_0$ (above threshold frequency): as f increases, the max KE of the emitted electrons increases
  - Increasing the intensity of the radiation does not increase the kinetic energy of the emitted electron

Step 3: Suggest how the observations support the particulate nature of light

- These observations support the particle nature of light because...
  - One photon interacts with one electron, so each photon must have an energy greater than Φ
  - Photons carry energy which is proportional to the frequency of the radiation and not intensity

Exam Tip

The observations and explanations of the photoelectric effect are key findings in Physics, which led to a whole new branch of discovery. As such, questions involving them are favourites with Examiners. Make sure you have revised them thoroughly!

The de Broglie Wavelength

- De Broglie proposed that electrons travel through space as a wave
  - This would explain why they can exhibit behaviour such as **diffraction**
- He, therefore, suggested that electrons must also hold wave properties, such as **wavelength**
  - This came to be known as the **de Broglie wavelength**
- However, he realised **all particles** can show wave-like properties, not just electrons
  - He hypothesised that all moving particles have a **matter wave** associated with them
- The definition of a de Broglie wavelength is:

  The wavelength associated with a moving particle

- De Broglie suggested that the momentum p of a particle and its associated wavelength λ are related by the equation:

$$\lambda = \frac{h}{p}$$

- Since momentum $p = mv$, the de Broglie wavelength can be related to the speed of a moving particle v by the equation:

$$\lambda = \frac{h}{mv}$$

Kinetic Energy & de Broglie Wavelength

- Kinetic energy is defined as

$$E_K = \frac{1}{2}mv^2$$

$$E_K = \frac{p^2}{2m} \quad \Rightarrow \quad p = \sqrt{2mE_K}$$

- Combining this with the previous equation gives relationship between the de Broglie wavelength of a particle to its kinetic energy:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_K}}$$

- Where:
  - λ = the de Broglie wavelength (m)
  - h = Planck's constant (J s)
  - p = momentum of the particle (kg m s^{-1})
  - E_K = kinetic energy of the particle (J)
  - m = mass of the particle (kg)

- v = speed of the particle ($m\ s^{-1}$)

YOUR NOTES

Worked Example

A proton and an electron are each accelerated from rest through the same potential difference.

Determine the ratio: $\frac{\text{de Broglie wavelength of the proton}}{\text{de Broglie wavelength of the electron}}$

- Mass of a proton = 1.67×10^{-27} kg
- Mass of an electron = 9.11×10^{-31} kg

Answer:

Step 1: Consider how the proton and electron can be related via their masses

The proton and electron are accelerated through the same p.d., therefore, they both have the same kinetic energy

Step 2: Write the equation relating the de Broglie wavelength of a particle to its kinetic energy

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

Step 3: Calculate the ratio

$$\frac{\text{de Broglie wavelength of the proton}}{\text{de Broglie wavelength of the electron}} = \frac{1}{\sqrt{m_p}} \div \frac{1}{\sqrt{m_e}}$$

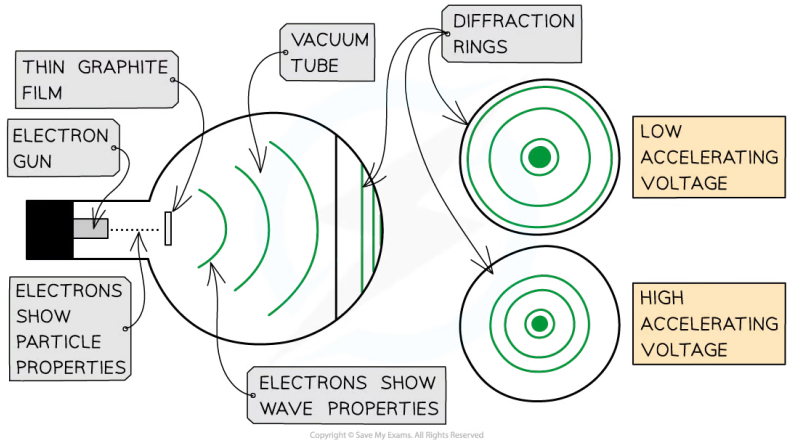
$$\sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{9.11 \times 10^{-31}}{1.67 \times 10^{-27}}} = 2.3 \times 10^{-2}$$

This means the de Broglie wavelength of the proton is 0.023 times smaller than that of the electron **OR** the de Broglie wavelength of the electron is about 40 times larger than that of the proton

Evidence for the Wave Nature of Matter

YOUR NOTES

- The majority of the time, and for everyday objects travelling at normal speeds, the de Broglie wavelength is far too small for any quantum effects to be observed
  - A typical electron in a metal has a de Broglie wavelength of about 10 nm
- Therefore, quantum mechanical effects will only be observable when the width of the sample is around that value



Electrons accelerated through a high potential difference demonstrate wave-particle duality

- The electron diffraction tube can be used to investigate how the wavelength of electrons depends on their speed
  - The smaller the radius of the rings, the smaller the de Broglie wavelength of the electrons
- As the voltage is increased:
  - The energy of the electrons increases
  - The radius of the diffraction pattern decreases
- This shows as the **speed of the electrons increases**, the **de Broglie wavelength of the electrons decreases**
- Electron diffraction was the first clear evidence that matter can behave like light and has wave properties
  - This is demonstrated using the electron diffraction tube
- The electrons are accelerated in an electron gun to a high potential, such as 5000 V, and are then directed through a thin film of graphite
  - The lattice structure of the graphite acts like the slits in a diffraction grating
- The electrons diffract from the gaps between carbon atoms and produce a circular pattern on a fluorescent screen made from phosphor
- In order to observe the diffraction of electrons, they must be focused through a gap similar to their size, such as an atomic lattice
- Graphite film is ideal for this purpose because of its crystalline structure

- The gaps between neighbouring planes of the atoms in the crystals act as slits, allowing the electron waves to spread out and create a diffraction pattern

- The diffraction pattern is observed on the screen as a series of concentric rings
  - This phenomenon is similar to the diffraction pattern produced when light passes through a diffraction grating
  - If the electrons acted as particles, a pattern would not be observed, instead, the particles would be distributed uniformly across the screen

- It is observed that a larger accelerating voltage reduces the diameter of a given ring, while a lower accelerating voltage increases the diameter of the rings

YOUR NOTES

Wave-Particle Duality

- Light can behave as a particle (i.e. photons) **and** a wave
- This phenomenon is called the wave-particle nature of light or **wave-particle duality**
- Light interacts with matter, such as electrons, **as a particle**
  - The evidence for this is provided by the photoelectric effect
- Light propagates through space **as a wave**
  - The evidence for this comes from the diffraction and interference of light in Young's Double Slit experiment

Light as a Particle

- Einstein proposed that light can be described as a quanta of energy that behave as particles, called photons
- The photon model of light explains that:
  - Electromagnetic waves carry energy in discrete packets called photons
  - The energy of the photons are quantised according to the equation ***E = hf***
  - In the photoelectric effect, each electron can absorb only a single photon - this means only the frequencies of light above the threshold frequency will emit a photoelectron
- The wave theory of light does **not** support the idea of a threshold frequency
  - The wave theory suggests any frequency of light can give rise to photoelectric emission if the exposure time is long enough
  - This is because the wave theory suggests the energy absorbed by each electron will increase gradually with each wave
  - Furthermore, the kinetic energy of the emitted electrons should increase with radiation intensity
  - However, in the photoelectric effect, this is not what is observed
- If the frequency of the incident light is above the threshold and the intensity of the light is increased, **more photoelectrons are emitted per second**
- Although the wave theory provides good explanations for phenomena such as interference and diffraction, it fails to explain the photoelectric effect

Compare wave theory and particulate nature of light

The wave theory of light suggests...	This is wrong because...
Any frequency of light can give rise to photoelectric emission if the exposure time is long enough	Photoelectrons will be released immediately if the frequency is above the threshold for that metal
The energy absorbed by each electron will increase gradually with each wave	Energy is absorbed instantaneously – photoelectrons are either emitted or not emitted after exposure to light
The kinetic energy of the emitted electrons should increase with radiation intensity	If the intensity of the light is increased, more photoelectrons are emitted per second

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Development of the Theory of Wave-Particle Duality

- Ideas about the nature of light were contested by modern science for around 300 years
- The evidence to prove both theories was available
  - Some prominent scientists argued light was a wave
  - Others contested that light was a particle
- It was not until the early 20th century that scientists settled on a theory of duality

YOUR NOTES

YOUR NOTES

CORPUSCULAR THEORY OF LIGHT (1672)

NEWTON ARGUED THAT LIGHT WAS MADE UP OF PARTICLES CALLED CORPUSCLES

WAVELETS (1690)

HUYGENS DEVELOPED WAVE THEORY OF LIGHT

YOUNG'S DOUBLE SLIT DIFFRACTION (1803)

DIFFRACTION PATTERNS PROVE LIGHT HAS WAVE BEHAVIOUR

PHOTOELECTRIC EFFECT (1887)

HERTZ OBSERVES PHOTOELECTRIC EFFECT

PROVES THAT LIGHT HAS PARTICLE BEHAVIOUR

QUANTISED ENERGY THEORY (1900 – 05)

PLANCK IDENTIFIES THE PLANCK CONSTANT

EINSTEIN DETERMINED THE QUANTUM OF ENERGY WHICH BECAME KNOWN AS THE PHOTON

WAVE PARTICLE DUALITY (1924)

DE BROGLIE WROTE IN HIS PhD THAT IF LIGHT COULD SHOW PARTICLE PROPERTIES, THEN SURELY PARTICLES COULD SHOW WAVE PROPERTIES

THEORISED THE DE BROGLIE MATTER WAVELENGTH

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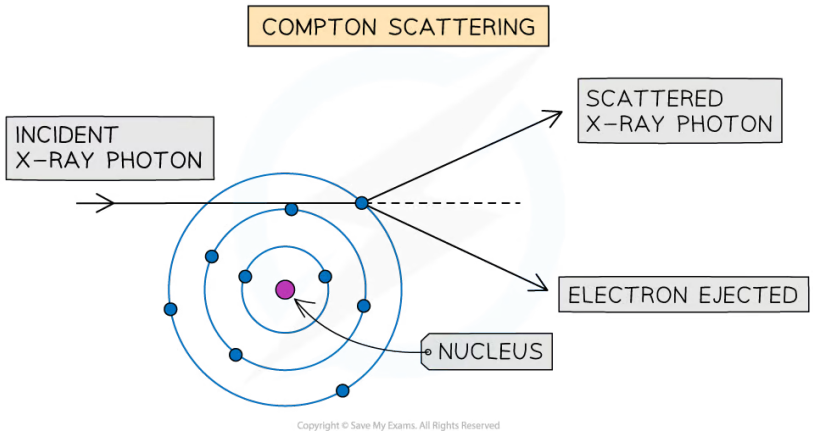
YOUR NOTES

Compton Scattering

- Compton scattering can be observed when a high energy photon (typically X-ray or gamma) interacts with an orbital electron
  - This phenomenon is further **evidence** of the particle nature of light



Compton scattering of an X-ray photon with an orbital electron

- The Compton Effect is defined as:

> **The interaction of a high-energy photon with an orbital electron which causes an increase in the wavelength of the photon and the ejection of the electron**

- During the collision, the photon imparts some of its energy to the orbital electron
- Because of this exchange of energy:
  - The photon is **deflected** from its initial path
  - The photon's **wavelength** increases, as its energy decreases
  - The electron involved is **ejected** from the atom involved in the interaction
- The electron and photon are deflected in different directions due to **conservation of momentum**

The Compton Formula

YOUR NOTES

- The Compton scattering formula is given by

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

- Where:
  - $\Delta\lambda$ = change in the wavelength of the photon $\left(\lambda_f - \lambda_i\right)$ (m)
  - h = Planck constant
  - m_e = mass of an electron (kg)
  - c = speed of light (m s^{-1})
  - θ = scattering angle of the photon (°)

- The constant $\frac{h}{m_e c}$ is known as the **Compton wavelength**

- The equation tells us:
  - The reduced wavelength of the photon depends on the scattering angle
  - The greater the scattering angle, the longer the wavelength

- This equation assumes that the electron is initially at rest before the interaction

Worked Example

An X-ray photon collides with a stationary orbital electron. The scattered photon has an energy of 120 keV and the recoiling electron has an energy of 40 keV.

Determine

(a)
the wavelength of the incident X-ray photon.
(b)
the change in wavelength of the photon.
(c)
the scattering angle of the photon.

Answer:

(a) Initial photon wavelength

- Photon energy and wavelength are related by

$$E_i = hf_i = \frac{hc}{\lambda_i} \quad \Rightarrow \quad \lambda_i = \frac{hc}{E_i}$$

- Where:
  - Energy of the incident photon, $E_i = 120 + 40 = 160$ keV
  - $hc = 1.99 \times 10^{-25}$ J m $= 1.24 \times 10^{-6}$ eV m

$$\text{Initial wavelength: } \lambda_i = \frac{1.24 \times 10^{-6}}{160 \times 10^3} = 0.0075 \text{ nm}$$

(b) Change in photon wavelength

- Energy of the scattered photon, $E_f = 120$ keV

$$\text{Scattered wavelength: } \lambda_f = \frac{1.24 \times 10^{-6}}{120 \times 10^3} = 0.0103 \text{ nm}$$

- Therefore, the change in wavelength is

$$\Delta\lambda = \lambda_f - \lambda_i$$

$$\Delta\lambda = 0.0103 - 0.0075 = 0.0028 \text{ nm}$$

(c) Photon scattering angle

- The Compton formula is

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta) \quad \Rightarrow \quad \cos\theta = \left(1 - \frac{m_e c \Delta\lambda}{h}\right)$$

YOUR NOTES

- The scattering angle is therefore:

$$\cos\theta = 1 - \frac{(9.11\times10^{-31})\times(3.0\times10^{8})\times(0.0028\times10^{-9})}{6.63\times10^{-34}} = -0.1542$$

scattering angle: $\theta = 98.871 = 99°$

Worked Example

Deduce the scattering angle at which

(a)
no change in photon wavelength is observed
(b)
the largest change in photon wavelength is observed

Answer:

(a) No change in photon wavelength

- From the Compton formula:

$$\Delta\lambda \propto (1 - \cos\theta)$$

- When $\theta = 0°$, $\cos\theta = 1$

So, $(1 - \cos\theta) = 0$ when $\theta = 0°$

- Therefore, when $\theta = 0°$, the change in photon wavelength will be zero

(b) Maximum change in photon wavelength

- When $\theta = 90°$, $\cos\theta = 0$

So, $(1 - \cos\theta) = 1$ when $\theta = 90°$

- When $\theta = 180°$, $\cos\theta = -1$

So, $(1 - \cos\theta) = 2$ when $\theta = 180°$

- Therefore, when $\theta = 180°$, the change in photon wavelength will be **twice** the Compton wavelength of the electron

IB Physics HL

YOUR NOTES

Radioactive Decay

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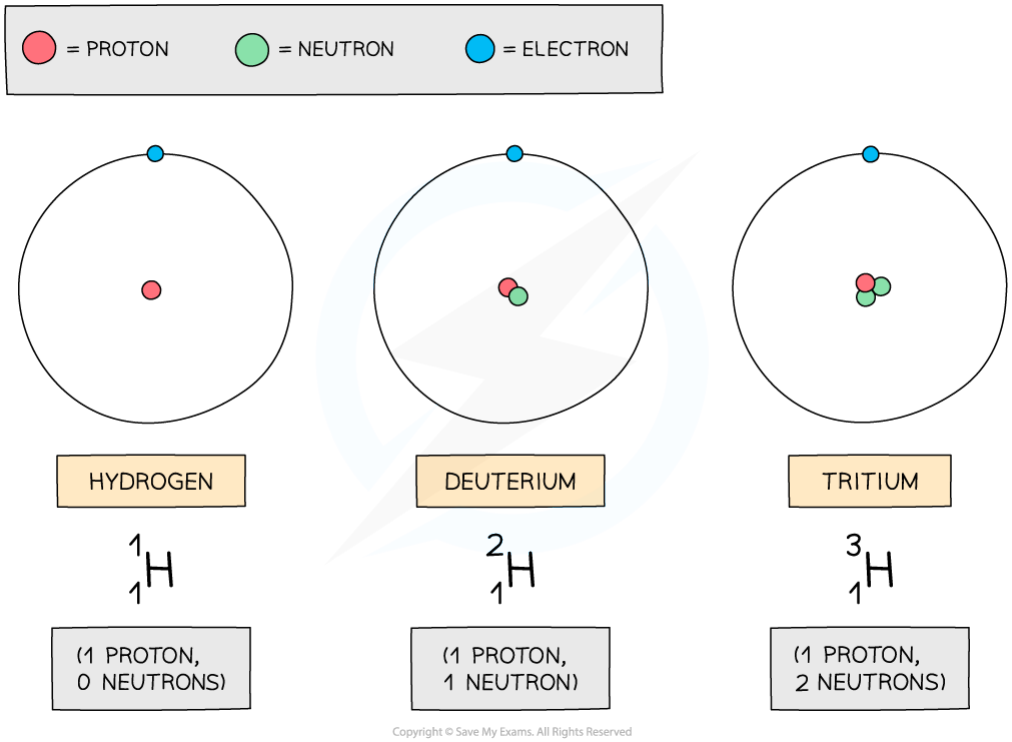
YOUR NOTES

Isotopes

- Although all atoms of the same element always have the same number of protons (and hence electrons), the number of neutrons can vary
- An isotope is defined as:

> **An atom (of the same element) that has an equal number of protons but a different number of neutrons**

- The element hydrogen has two other isotopes: **deuterium** and **tritium**



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The three atoms shown above are all forms of hydrogen, but they each have different numbers of neutrons

- The neutron number of an atom is found by subtracting the proton number from the nucleon number
- Since nucleon number includes the number of neutrons, an isotope of an element will also have a **different nucleon / mass number**
- Some isotopes have an imbalance of neutrons and protons, they are **unstable**
  - This means they constantly decay and emit radiation to achieve a more stable form
  - This can happen from anywhere between a few nanoseconds to 100,000 years

Differences between isotopes

YOUR NOTES

- The number of neutrons in an atom does not affect the chemical properties of an atom, such as its charge, but only its mass
  - This is because neutrons have no charge but do have mass
- The charge of the nucleus of a particular element is **always the same**
- In the periodic table, the mass number of Chlorine is often given as 35.5

			4 He HELIUM 2
14 N NITROGEN 7	16 O OXYGEN 8	19 F FLUORINE 9	20 Ne NEON 10
31 P PHOSPHORUS 15	32 S SULFUR 16	35.5 Cl CHLORINE 17	40 Ar ARGON 18

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This section of the periodic table shows chlorine as having a mass number of 35.5, but other elements have an integer mass number

- The mass number of Chlorine is given as 35.5 because it has **two** isotopes with mass numbers of 35 and 37, and these occur in a ratio of 3:1 respectively so 35.5 is the **average** nucleon / mass number
- The number of electrons and protons in different isotopes remains the **same**
- Less common isotopes tend to be more **unstable** due to the imbalance of protons and neutrons

Isotopic Data

- Isotopic data is defined as:

The relative amounts of different isotopes of an element found within a substance

- It is used to identify an isotopic signature within organic and inorganic materials
- Isotopic data is often used for determining the age of archaeological findings and is used in **radioactive dating**
- Carbon-14 is a naturally occurring isotope most often used for this since it is present in all living beings and undergoes radioactive decay
- When a plant or animal dies, the natural **radioactive** decay of this isotope means the concentration of the carbon-14 in its tissue gradually reduces
- Since carbon-14 has a long half-life of around 6000 years, the half-life can be used to determine the age of the plant or animal when it died

YOUR NOTES

Worked Example

Which of the following rows shows a pair of nuclei that are isotopes of one another?

		nucleon number	number of neutrons
A.	nucleus 1	39	19
	nucleus 2	35	22
B.	nucleus 1	37	20
	nucleus 2	35	18
C.	nucleus 1	37	18
	nucleus 2	35	20
D.	nucleus 1	35	20
	nucleus 2	35	18

Answer: B

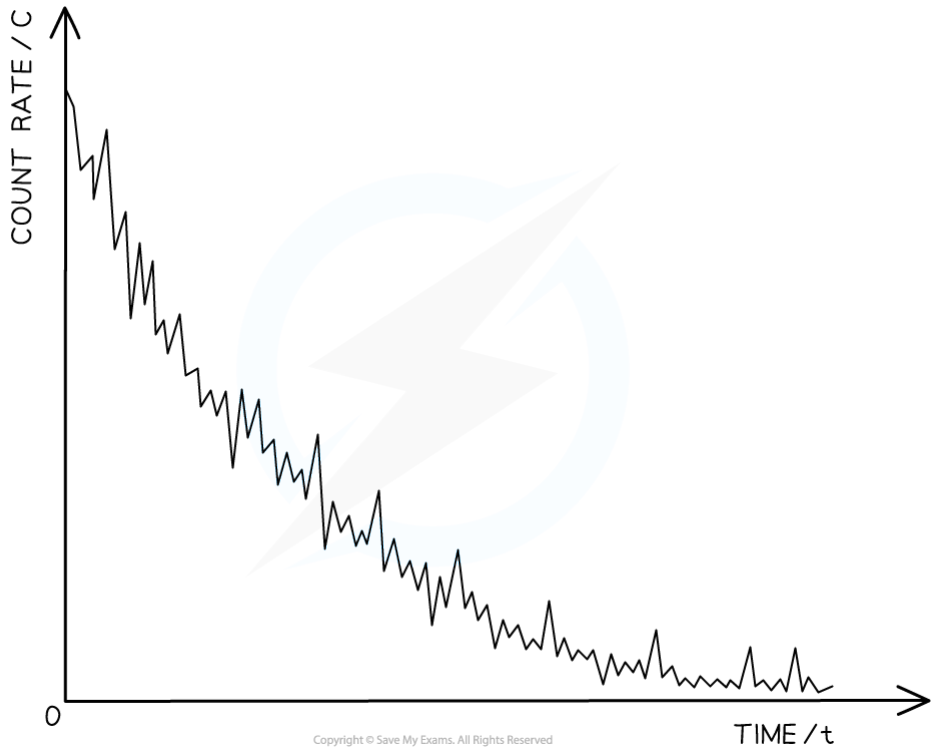
- In nucleus 1:
  - Nucleon number: 37
  - Neutrons: 20
  - Protons = 37 – 20 = 17
- In nucleus 2:
  - Nucleon number: 35
  - Neutrons: 18
  - Protons = 35 – 18 = 17
- They have the **same number of protons** but different numbers of neutrons hence, they are isotopes of each other

Radioactive Decay

- Radioactive decay is defined as:

  The spontaneous disintegration of a nucleus to form a more stable nucleus, resulting in the emission of an alpha, beta or gamma particle

- The **random nature** of radioactive decay can be demonstrated by observing the count rate of a Geiger-Muller (GM) tube
  - When a GM tube is placed near a radioactive source, the counts are found to be irregular and cannot be predicted
  - Each count represents a decay of an unstable nucleus
- These fluctuations in count rate on the GM tube provide **evidence** for the **randomness of radioactive decay**



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The variation of count rate over time of a sample radioactive gas. The fluctuations show the randomness of radioactive decay

Characteristics of Radioactive Decay

- Radioactive decay is both **spontaneous** and **random**
- A spontaneous process is defined as:

  A process which cannot be influenced by environmental factors

- This means radioactive decay cannot be affected by environmental factors such as:
  - Temperature
  - Pressure

- Chemical conditions

YOUR NOTES

- A random process is defined as:

A process in which the exact time of decay of a nucleus cannot be predicted

- Instead, the nucleus has a constant probability, ie. the same chance, of decaying in a given time
- Therefore, with large numbers of nuclei, it is possible to statistically predict the behavior of the entire group

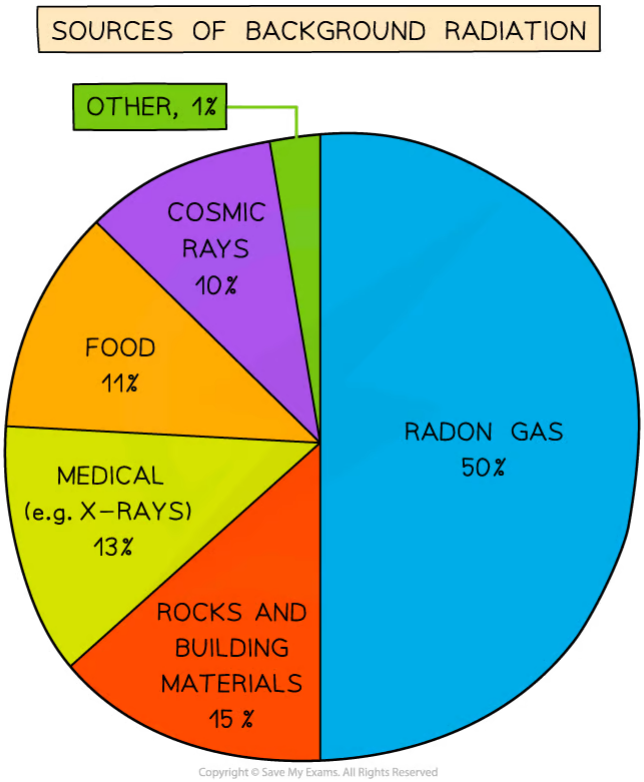
YOUR NOTES

Background Radiation

- It is important to remember that radiation is a natural phenomenon
- Radioactive elements have **always** existed on Earth and in outer space
  - However, human activity has added to the amount of radiation that humans are exposed to on Earth
- Background radiation is defined as:

  Background radiation describes the low level of radiation present in the surroundings at all times

- There are two types of background radiation:
  - Natural sources
  - Man-made sources



Background radiation is the radiation that is present all around in the environment. Radon gas is given off from some types of rock

- Every second of the day there is some radiation emanating from **natural sources** such as:
  - Rocks
  - Cosmic rays from space
  - Foods

Origins of Background Radiation

YOUR NOTES

- Background radiation can come from natural sources on Earth or space and man-made sources

Natural Sources

- **Radon gas from rocks and soil**
  - Heavy radioactive elements, such as uranium and thorium, occur naturally in rocks in the ground
  - Uranium decays into radon gas, which is an alpha emitter
  - This is particularly dangerous if inhaled into the lungs in large quantities
- **Cosmic rays from space**
  - The sun emits an enormous number of protons every second
  - Some of these enter the Earth's atmosphere at high speeds
  - When they collide with molecules in the air, this leads to the production of gamma radiation
  - Other sources of cosmic rays are supernovae and other high energy cosmic events
- **Carbon-14 in biological material**
  - All organic matter contains a tiny amount of carbon-14
  - Living plants and animals constantly replace the supply of carbon in their systems hence the amount of carbon-14 in the system stays almost constant
- **Radioactive material in food and drink**
  - Naturally occurring radioactive elements can get into food and water since they are in contact with rocks and soil containing these elements
  - Some foods contain higher amounts such as potassium-40 in bananas
  - However, the amount of radioactive material is minuscule and is not a cause for concern

Man-Made Sources

- **Medical sources**
  - In medicine, radiation is utilised all the time
  - Uses include X-rays, CT scans, radioactive tracers, and radiation therapy
- **Nuclear waste**
  - While nuclear waste itself does not contribute much to background radiation, it can be dangerous for the people handling it
- **Nuclear fallout from nuclear weapons**
  - Fallout is the residue radioactive material that is thrown into the air after a nuclear explosion, such as the bomb that exploded at Hiroshima
  - While the amount of fallout in the environment is presently very low, it would increase significantly in areas where nuclear weapons are tested
- **Nuclear accidents**

YOUR NOTES

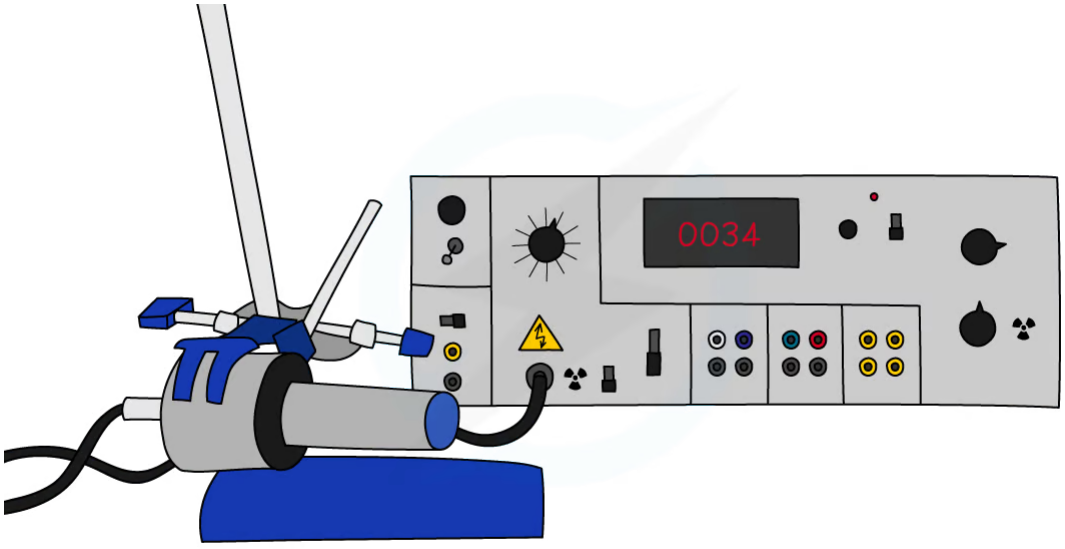
- Accidents such as that in Chernobyl contributed a large dose of radiation into the environment
- While these accidents are now extremely rare, they can be catastrophic and render areas devastated for centuries

Corrected Count Rate

- Background radiation must be accounted for when taking readings in a laboratory
- This can be done by taking readings with no radioactive source present and then subtracting this from readings with the source present
  - This is known as the **corrected count rate**

Detecting Radiation

- When radiation passes close to an atom, it knocks out electrons, ionising the atom
- Radiation detectors work by detecting the presence of these ions or the chemical changes that they produce
- Examples of radiation detectors include:
  - **Photographic film** (often used in badges)
  - **Geiger-Muller** (GM) tubes
  - **Ionisation chambers**
  - **Scintillation counters**
  - **Spark counters**



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A Geiger-Muller tube (or Geiger counter) is a common type of radiation detector

YOUR NOTES

Worked Example

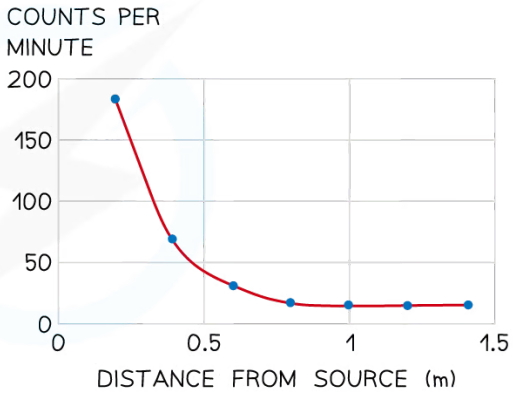
A student is using a Geiger counter to measure the counts per minute at different distances from a source of radiation.

Their results and a graph of the results are shown below.

RESULTS TABLE

Distance from source (m)	Counts per minute
0.2	180
0.4	67
0.6	29
0.8	17
1.0	15
1.2	15
1.4	15

GRAPH



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Determine the background radiation count.

Answer:

- The background radiation is the amount of radiation detected in the absence of a radioactive source
- When the source is moved back far enough, all radiation is absorbed by the air before reaching the Geiger-counter
- This is shown at 1.0 m where the count rate becomes constant
- Therefore, the readings after 1.0 m are due to background radiation only
- Background radiation count = **15 counts per minute**

YOUR NOTES

Alpha, Beta & Gamma Decay

- Some elements have nuclei that are unstable
  - This tends to be when the number of protons and neutrons in a nucleus is unbalanced
- In order to become more **stable**, they emit particles and/or electromagnetic radiation
  - These nuclei are said to be **radioactive**
- There are **three** main types of radiation:
  - Alpha
  - Beta (minus & plus)
  - Gamma

Alpha Decay

- **Alpha (α) particles** are high-energy particles made up of **2 protons and 2 neutrons** (the same as a helium nucleus)
- They are usually emitted from nuclei that are too large

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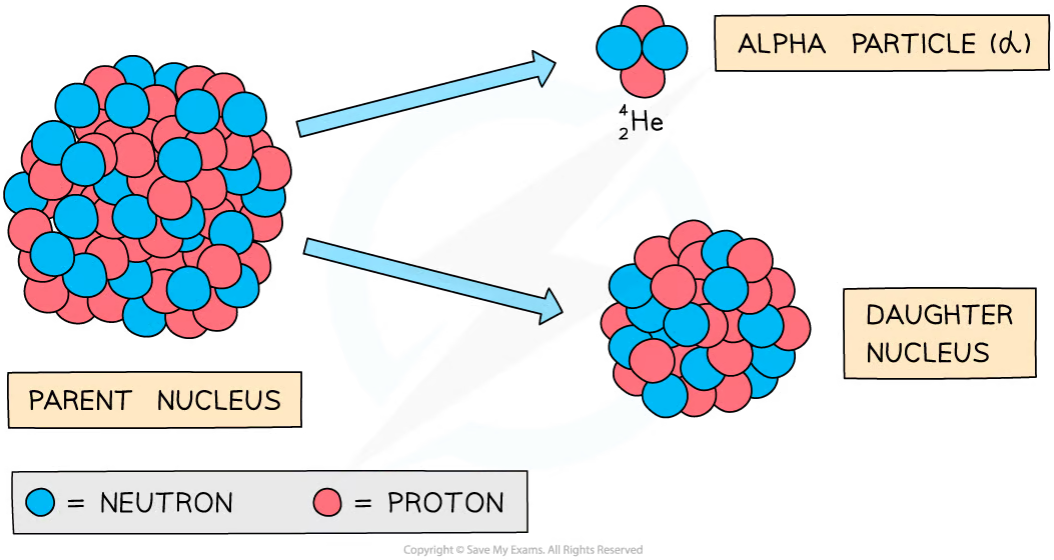
- Alpha decay is common in large, unstable nuclei with too many protons
- The decay involves a nucleus emitting an alpha particle and decaying into a different nucleus



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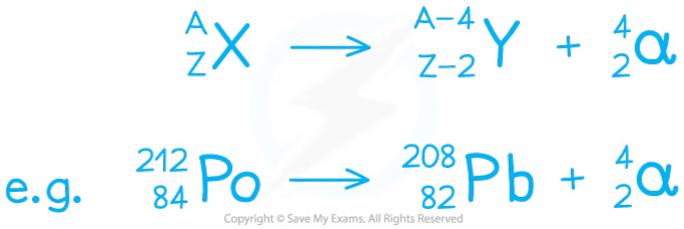
Alpha decay produces a daughter nucleus and an alpha particle (helium nucleus)

YOUR NOTES

- When an unstable nucleus (the parent nucleus) emits radiation, the composition of its nucleus changes
- As a result, the isotope will change into a different element (the daughter nucleus)
- Alpha decay can be represented by the following radioactive decay equation:



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- When an alpha particle is emitted from a nucleus:
  - The nucleus loses 2 protons: **proton number decreases by 2**
  - The nucleus loses 4 nucleons: **nucleon number decreases by 4**

Beta-Minus Decay

- Beta-minus (β^-) particles are **high-energy electrons** emitted from the nucleus
  - Beta-minus particles are emitted by nuclei that have too many **neutrons**

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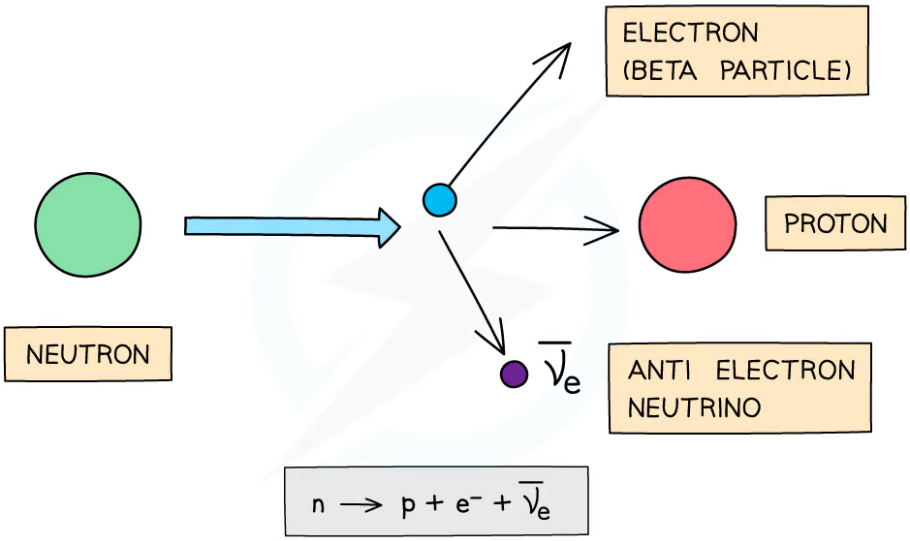
- Beta-minus decay is when a **neutron** turns into a **proton** and emits an **electron** and an **anti-electron neutrino**
  - Therefore, the **nucleon number** of the decaying nuclei **remains the same**
- Electrons have a proton number of -1, so overall:
  - The **proton number increases by 1**
  - The **nucleon number remains the same**

YOUR NOTES



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Beta-minus decay often happens in unstable nuclei that have too many neutrons. The nucleon number stays the same, but the proton number increases by one

- When a β^- particle is emitted from a nucleus, such as carbon-14, the decay equation is as follows:

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}_e$$

e.g. $$^{14}_{6}C \longrightarrow {}^{14}_{7}N + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}_e$$

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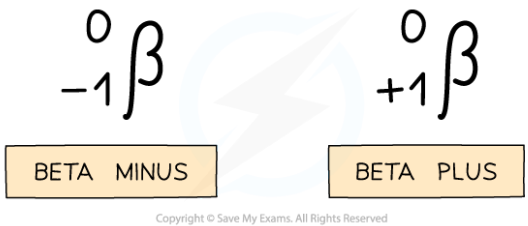
- The new nucleus formed from the decay is called the "daughter" nucleus (nitrogen in the example above)

Beta-Plus Decay

- Beta-plus (β^+) particles are high-energy positrons (antimatter particle of electrons) emitted from the nucleus
  - Beta-plus particles are emitted by nuclei that have too many **protons**



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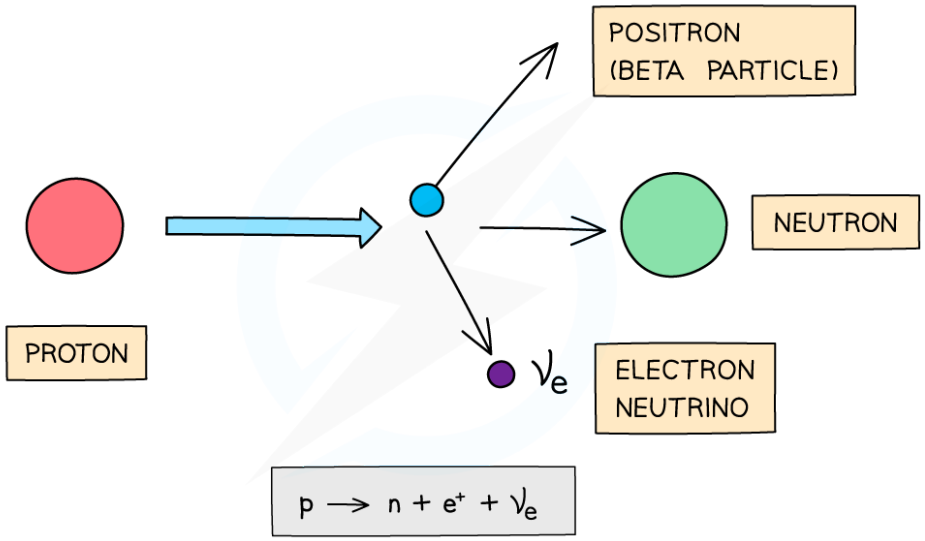
- Beta-plus decay is when a **proton** turns into a **neutron** and emits a **positron** and an **electron neutrino**

YOUR NOTES

- Positrons have a proton number of +1, so overall:
  - The **proton number decreases by 1**
  - The **nucleon number remains the same**



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Beta-plus decay often happens in unstable nuclei that have too many protons. The nucleon number stays the same, but the proton number decreases by one

- When a β^+ particle is emitted from a nucleus, such as carbon-6, the decay equation is as follows:

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{+1}\beta + {}^{0}_{0}\nu_e$$

e.g. $$^{6}_{6}C \rightarrow {}^{6}_{5}B + {}^{0}_{+1}\beta + {}^{0}_{0}\nu_e$$

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Gamma Radiation

- **Gamma (γ) rays are high energy electromagnetic waves**
- They are emitted by nuclei that need to lose some energy

$$^{0}_{0}\gamma$$

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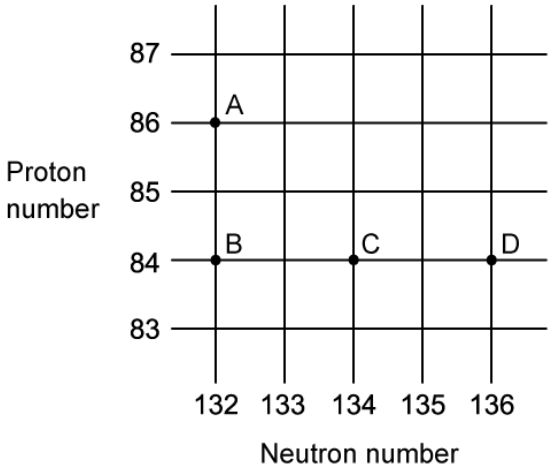
- Gamma particles are **photons**, so they have a proton number of 0, so overall:
  - The **proton number remains the same**
  - The **nucleon number remains the same**

YOUR NOTES

Worked Example

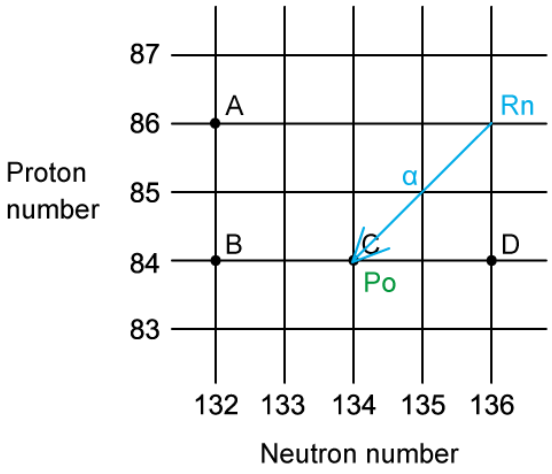
The radioactive nucleus $^{222}_{86}Rn$ undergoes alpha decay into a daughter nucleus Po.



(a)
Which letter in the diagram represents the daughter product?
(b)
What is the nucleon number and proton number of Po?

(a) Answer: C

- The number of neutrons in $^{222}_{86}Rn$ is 222 – 86 = 136
- In alpha decay, the parent nucleus loses a helium nucleus (2 protons, 2 neutrons)
  - Proton number: 86 decreases to 84
  - Neutron number: 136 decreases to 134



- Therefore, the correct answer is **C**

(b)

YOUR NOTES

- The equation for alpha decay is as follows:

$$^{A}_{Z}X \longrightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}Y$$

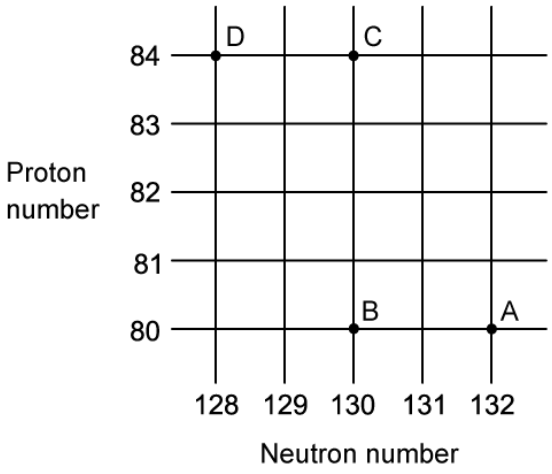
$$^{222}_{86}Rn \longrightarrow {}^{4}_{2}\alpha + {}^{218}_{84}Po$$

- Hence the daughter nucleus Po has
  - Nucleon number = 222 – 4 = 218
  - Proton number = 86 – 2 = 84

YOUR NOTES

Worked Example

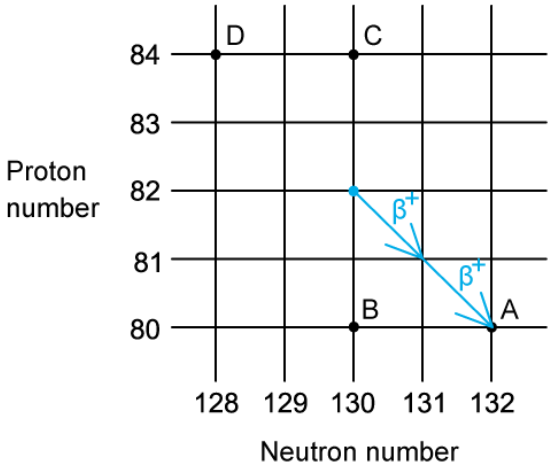
A radioactive substance with a nucleon number of 212 and a proton number of 82 decays by β-plus emission into a daughter product which further decays by β-plus emission into a granddaughter product.



Which letter in the diagram represents the granddaughter product?

Answer: A

- The number of neutrons in the parent nucleus is 212 – 82 = 130
- In beta-plus decay, a proton turns into a neutron
  - Proton number: 82 decreases to 80
  - Neutron number: 130 increases to 132



- Therefore, the correct answer is **A**

Exam Tip

Remember to avoid the common mistake of confusing the number of neutrons with the nucleon number. In alpha decay, the nucleon (protons and neutrons) number decreases by 4 but the number of neutrons only decreases by 2.

Properties of Alpha, Beta & Gamma

Properties of Alpha Radiation

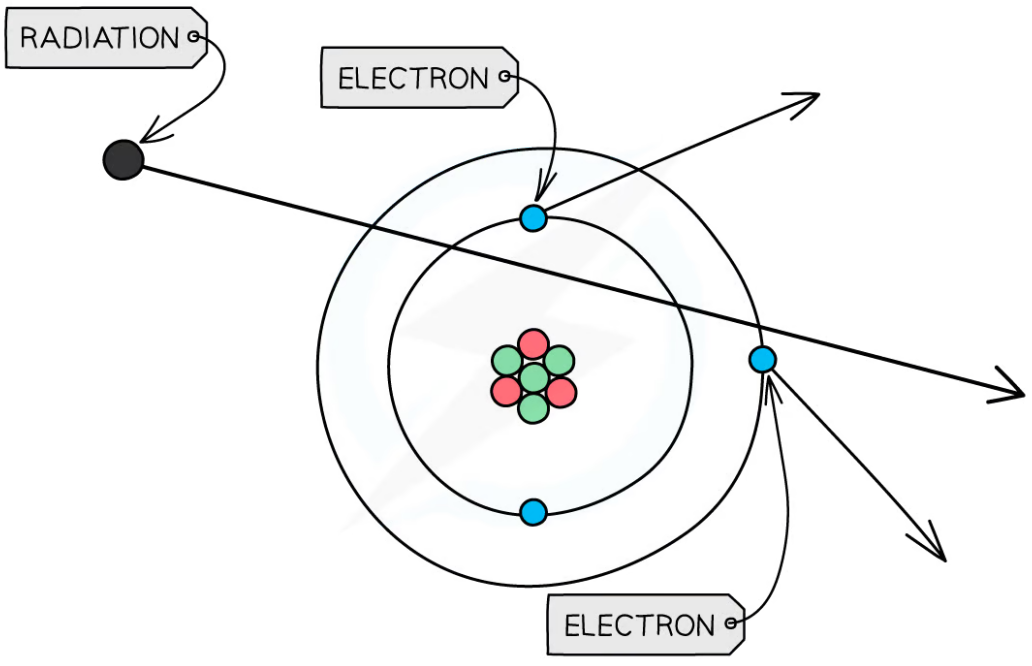
- Alpha is the **most** ionising type of radiation
  - This is due to it having the highest charge of +2e
  - This means it produces the greatest number of ion pairs per mm in air
  - This also means it is able to do more damage to cells than the other types of radiation
- Alpha is the **least** penetrating type of radiation
  - This means it travels the shortest distance in air before being absorbed
  - Alpha particles have a range of around 3–7 cm in air
- Alpha can be stopped by a single **piece of paper**

Properties of Beta Radiation

- Beta is a **moderately** ionising type of radiation
  - This is due to it having a charge of ±1e
  - This means it is able to do some slight damage to cells (less than alpha but more than gamma)
- Beta is a **moderately** penetrating type of radiation
  - Beta particles have a range of around 20 cm - 3 m in air, depending on their energy
- Beta can be stopped by a few millimetres of **aluminium** foil

Properties of Gamma Radiation

- If gamma radiation collides with an atom, it can knock out electrons, **ionising the atom**
- This can cause chemical changes in materials and can damage or kill living cells



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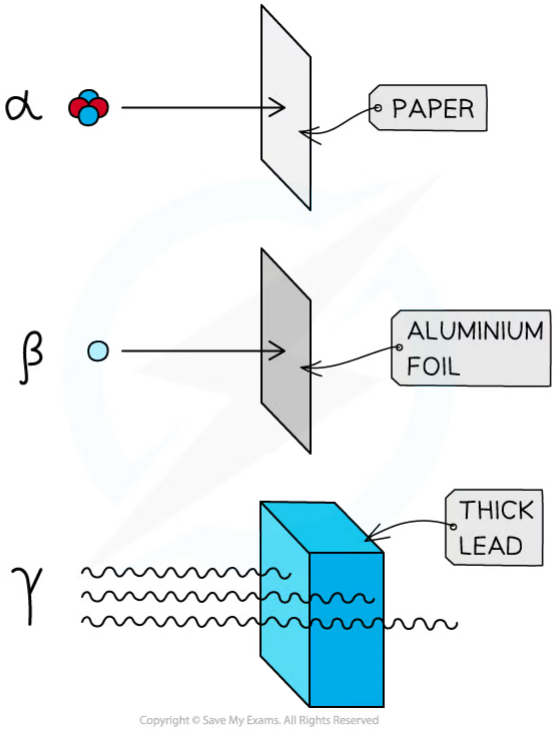
YOUR NOTES

When radiation passes close to atoms, it can knock out electrons, ionising the atom

- Gamma is the **least** ionising type of radiation
  - This is because it is an electromagnetic wave with no charge
  - This means it produces the least number of ion pairs per mm in air
  - It can still cause damage to cells, but not as much as alpha or beta radiation. This is why it is used for cancer radiotherapy
- Gamma is the **most** penetrating type of radiation
  - This means it travels the furthest distance in air before being absorbed
  - Gamma radiation has an infinite range and follows an **inverse square law**
- Gamma can be stopped by several **metres of concrete** or several **centimetres of lead**



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Different types of radiation are stopped by different materials

- The properties of the different types of radiation are summarised in the table below
- Note that charge is often described as 'relative charge'

YOUR NOTES

Particle	Composition	Mass / u	Charge / e	Speed / c
Alpha (α)	2 protons + 2 neutrons	4	+2	0.05
Beta minus (β^-)	Electron (e^-)	0.0005	−1	> 0.99
Beta plus (β^+)	Positron (e^+)	0.0005	+1	> 0.99
Gamma (γ)	Electromagnetic wave	0	0	1

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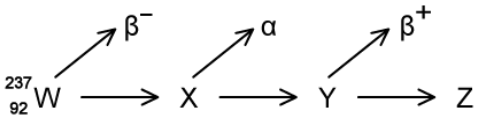
- Where:
  - u = atomic mass unit
  - e = charge of the electron: 1.60×10^{-19} C
  - c = speed of light: 3×10^{8} m s^{-1}

YOUR NOTES

Worked Example

Three successive radioactive decays are shown in the diagram below. Each decay results in a particle being emitted.



The first decay results in the emission of a beta-minus particle.

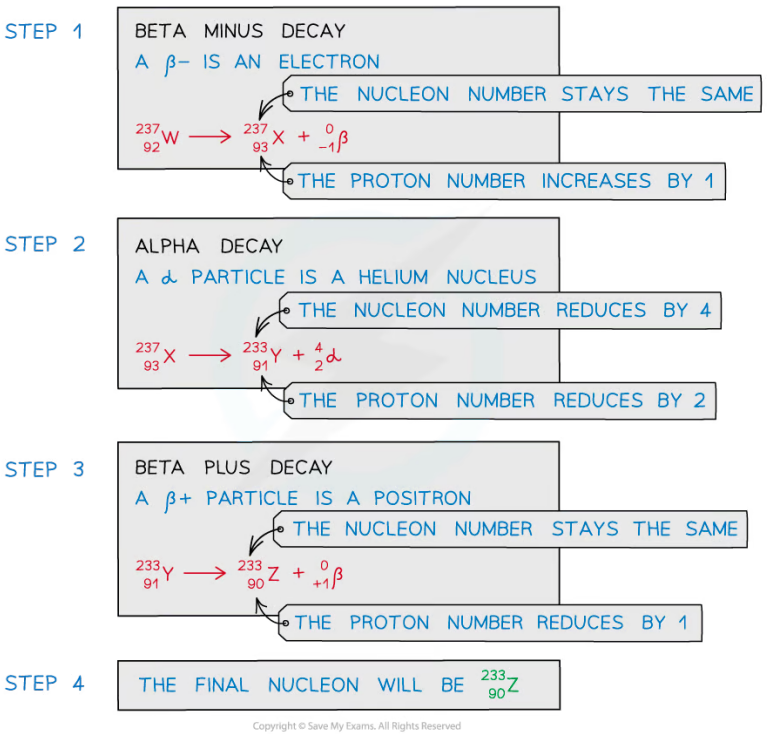
The second decay results in the emission of an alpha particle.

The third decay results in the emission of a beta-plus particle.

What is nuclide Z?

A. $^{237}_{90}Z$ **B.** $^{233}_{92}Z$ **C.** $^{237}_{89}Z$ **D.** $^{233}_{90}Z$

Answer: D



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Exam Tip

One way to remember which particle decays into which depends on the type of beta emission, think of beta '**p**lus' as the '**p**roton' that turns into the neutron (plus an electron neutrino)

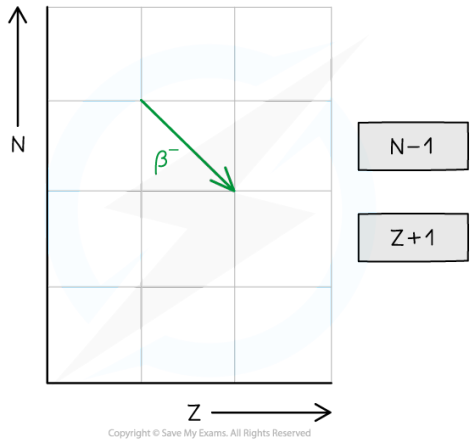
Radioactive Decay Equations

- There are four reasons why a nucleus might become unstable, and these determine which decay mode will occur
  - Too many neutrons
  - Too many protons
  - Too many nucleons
  - Too much energy

Too many neutrons

- Decays through **beta-minus (β^-) emission**
- One of the **neutrons** in the nucleus changes into a **proton** and a β^- particle (an electron) and antineutrino is released
- The nucleon number is constant
  - The neutron number (N) decreases by 1
  - The proton number (Z) increases by 1
- The general decay equation for β^- emission is:

$$^{A}_{Z}X \rightarrow {}^{0}_{-1}\beta + {}^{A}_{Z+1}Y + \overline{\nu_e}$$



The y axis represents number of neutrons, N, and the x axis represents number of protons, Z

Too many protons

- Decays through **beta-plus (β^+) emission** or **electron capture**
- In beta-plus decay, a **proton** changes into a **neutron** and a β^+ particle (a positron) and neutrino are released
- In electron capture, an orbiting electron is taken in by the nucleus and combined with a proton causing the formation of a neutron and neutrino
- In both types of decay, the nucleon number stays constant
  - The neutron number (N) increases by 1
  - The proton number (Z) decreases by 1

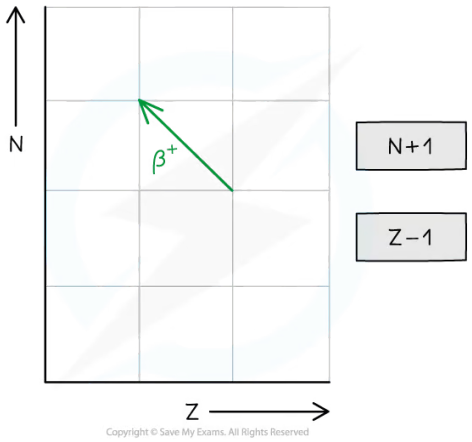
YOUR NOTES

- The general decay equation for β^+ emission is:

$$^{A}_{Z}X \rightarrow {}^{0}_{+1}\beta + {}^{A}_{Z-1}Y + \nu_e$$

- The equation for electron capture is:

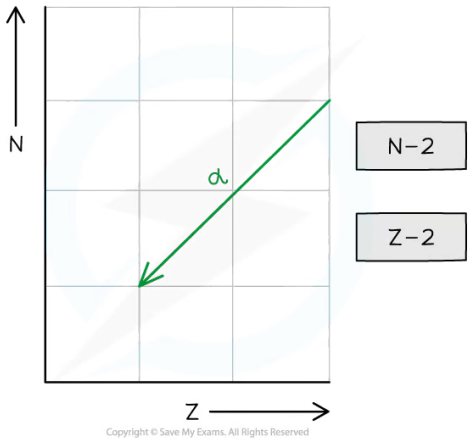
$$^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}Y + \nu_e$$



Too many nucleons

- Decays through **alpha (α) emission**
- An α particle is a helium nucleus
- The nucleon number decreases by 4 and the proton number decreases by 2
  - The neutron number (N) decreases by 2
  - The proton number (Z) decreases by 2
- The general decay equation for α emission is:

$$^{A}_{Z}X \rightarrow {}^{4}_{2}\alpha + {}^{A-4}_{Z-2}Y$$



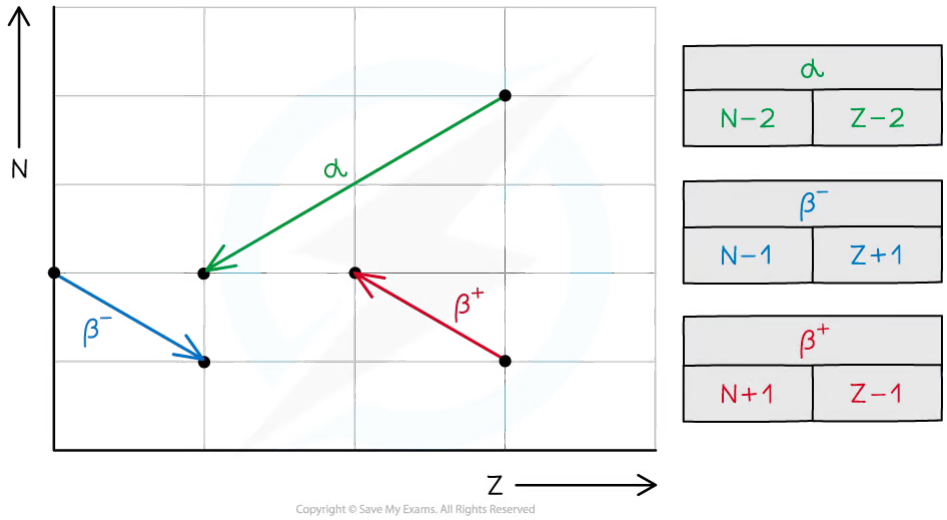
Too much energy

- Decays through **gamma (γ) emission**

YOUR NOTES

- A gamma particle is a high-energy electromagnetic radiation
- This usually occurs after a different type of decay, such as alpha or beta decay
- This is because the nucleus becomes excited and has excess energy
- This process does not change the number of nucleons

- In summary, alpha decay, beta decay and electron capture can be represented on an N–Z graph as follows:



α	
N−2	Z−2

β^-	
N−1	Z+1

β^+	
N+1	Z−1

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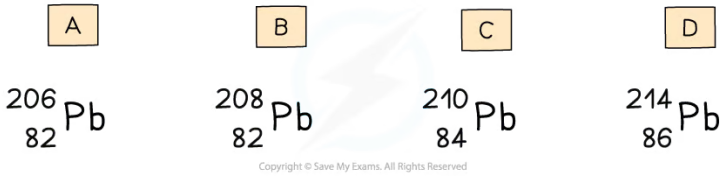
YOUR NOTES

Worked Example

A nucleus with 84 protons and 126 neutrons undergoes alpha decay. It forms lead, which has the element symbol Pb.



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Which of the isotopes of lead pictured is the correct one formed during the decay?

Answer: A

Step 1: Calculate the mass number of the original nucleus

- The mass number is equal to the number of protons plus the number of neutrons
- The original nucleus has 84 protons and 126 neutrons

$$84 + 126 = 210$$

- The mass number of the original nucleus is 210

Step 2: Calculate the new atomic number

- The alpha particle emitted is made of two protons and two neutrons
- Protons have an atomic number of 1, and neutrons have an atomic number of 0
- Removing two protons and two neutrons will reduce the atomic number by 2

$$84 - 2 = 82$$

- The new nucleus has an atomic number of **82**

Step 3: Calculate the new mass number

- Protons and neutrons both have a mass number of 1
- Removing two protons and two neutrons will reduce the mass number by 4

$$210 - 4 = 206$$

- The new nucleus has a mass number of **206**

YOUR NOTES

Worked Example

Plutonium-239 is a radioactive isotope that contains 94 protons and emits α particles to form a radioactive isotope of uranium. This isotope of uranium emits α particles to form an isotope of thorium which is also radioactive.

Write two equations to represent the decay of plutonium-239 and the subsequent decay of uranium.

Answer:

Step 1: Write down the general equation of alpha decay

$$^{A}_{Z}X \rightarrow ^{4}_{2}\alpha + ^{A-4}_{Z-2}Y$$

Step 2: Write down the decay equation of plutonium into uranium

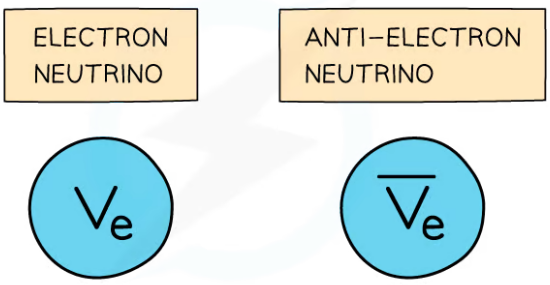
$$^{239}_{94}Pu \rightarrow ^{4}_{2}\alpha + ^{235}_{92}U$$

Step 3: Write down the decay equation of uranium into thorium

$$^{235}_{92}U \rightarrow ^{4}_{2}\alpha + ^{231}_{90}Th$$

Neutrinos & Antineutrinos

- An electron neutrino is a type of subatomic particle with no charge and negligible mass which is also emitted from the nucleus
- The anti-neutrino is the antiparticle of a neutrino
  - Electron anti-neutrinos are produced during $\beta-$ decay
  - Electron neutrinos are produced during $\beta+$ decay



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YOUR NOTES

Activity & Half-Life

- The **activity** of a radioactive sample is defined as

  The rate at which radioactive particles are emitted

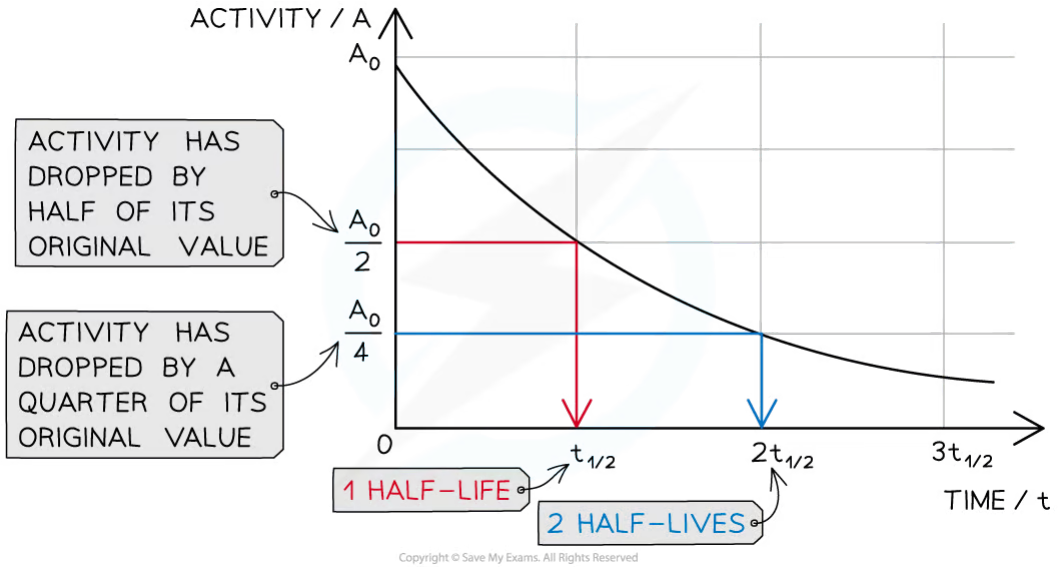
- It is impossible to know when a particular unstable nucleus will decay
- But the **rate** at which the activity of a sample decreases can be known
  - This is known as the **half-life**
- Half-life is defined as:

  The time taken for half the undecayed nuclei to decay or the activity of a source to decay by half

- In other words, the time it takes for the activity of a sample to fall to half its original level
- Different isotopes have different half-lives and half-lives can vary from a fraction of a second to billions of years in length

Using Half-life

- Scientists can measure the half-lives of different isotopes accurately:
- Uranium-235 has a half-life of 704 million years
  - This means it would take 704 million years for the activity of a uranium-235 sample to decrease to half its original amount
- Carbon-14 has a half-life of 5700 years
  - So after 5700 years, there would be 50% of the original amount of carbon-14 remaining
  - After two half-lives, or 11 400 years, there would be just 25% of the carbon-14 remaining
- With each half-life, the amount remaining **decreases by half**



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Graph showing how the activity of a radioactive sample changes over time. Each time the original activity halves, another half-life has passed

- The time it takes for the activity of the sample to decrease from 100 % to 50 % is the half-life
- It is the same length of time as it would take to decrease from 50 % activity to 25 % activity
- The half-life is **constant** for a particular isotope
- The following table shows that as the number of half-life increases, the proportion of the isotope remaining **halves**

NUMBER OF HALF-LIVES	PROPORTION OF ISOTOPE REMAINING
0	1
1	1/2
2	1/4
3	1/8
4	1/16
...	...

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Worked Example

A radioactive sample has a half-life of 3 years. What is the ratio of decayed : original nuclei, after 15 years?

Answer:

Step 1: Calculate the number of half-lives

- The time period is 15 years
- The half-life is 3 years

$$\text{half-life} = 15 / 3 = 5$$

- There have been 5 half-lives

Step 2: Raise 1/2 to the number of half-lives

- The proportion of nuclei remaining is

$$(1/2)^5 = 1/32$$

- So 1/32 of the original nuclei are remaining

Step 3: Write the ratio correctly

- If 1/32 of the original nuclei are remaining, then 31/32 must have decayed
- Therefore, the ratio is **31 decayed : 32 original**, or **31:32**

YOUR NOTES

Worked Example

A particular radioactive sample contains 2 million un-decayed atoms. After a year, there are only 500 000 atoms left un-decayed.

Determine the half-life of the material.

Answer:

Step 1: Calculate how many times the number of un-decayed atoms has halved

- There were 2 000 000 atoms to start with
- **1 000 000** atoms would remain after **1 half-life**
- **500 000** atoms would remain after **2 half-lives**
- Therefore, the sample has undergone 2 half-lives

Step 2: Divide the time period by the number of half-lives

- The time period is a year
- The number of half-lives is 2
- 1 year divided by 2 is half a year or 6 months
- Therefore, the half-life is **6 months**

Decay Curves

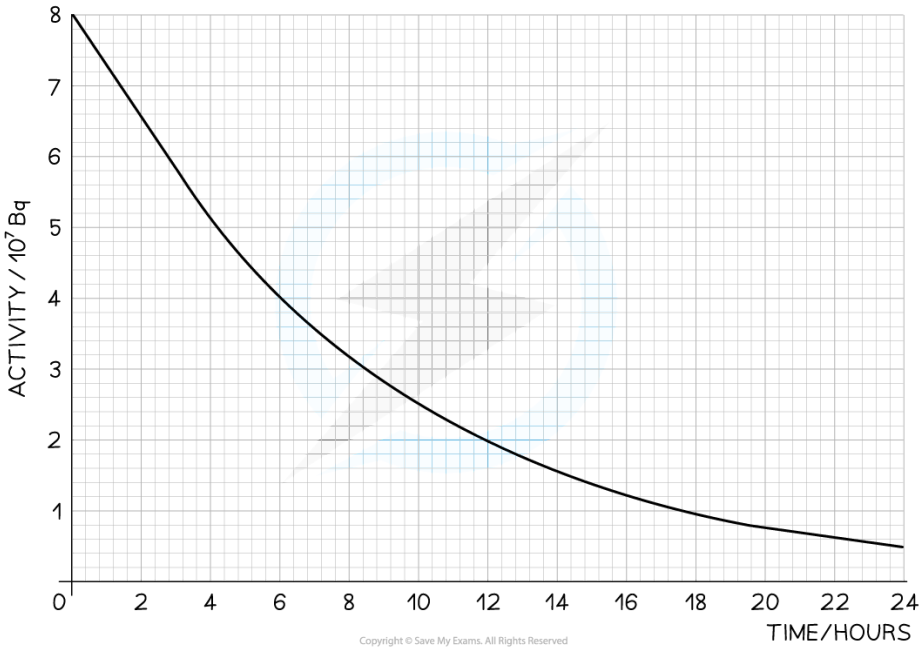
- To calculate the half-life of a sample, the procedure is:
  - Measure the initial activity, A_0, of the sample
  - Measure how the activity changes with time
  - Determine the half-life of this original activity
- The time taken for the activity to decrease to half its original value is the **half-life**

YOUR NOTES

YOUR NOTES

Worked Example

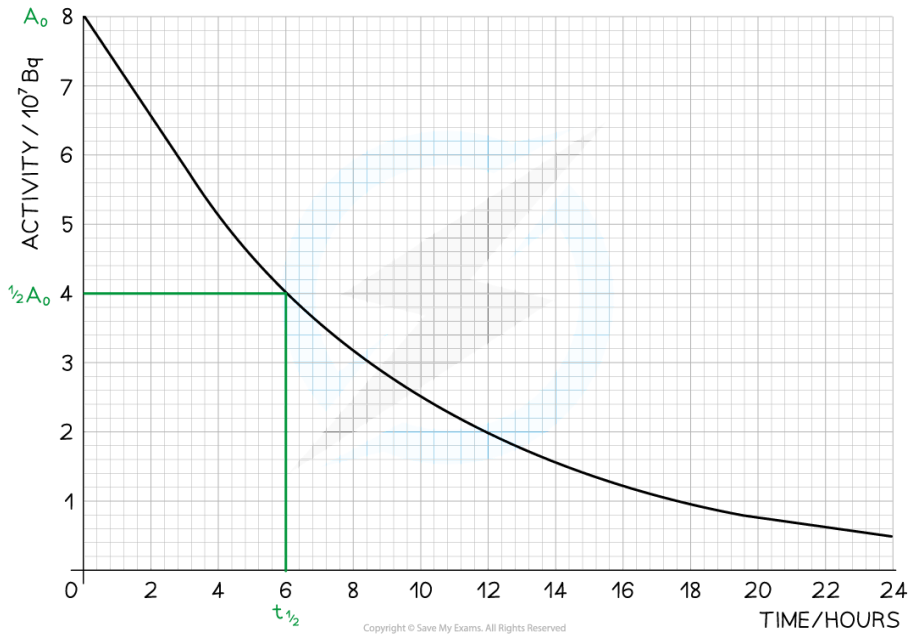
The radioisotope technetium is used extensively in medicine. The graph below shows how the activity of a sample varies with time.



Determine the half-life of this material.

Answer:

Step 1: Draw lines on the graph to determine the time it takes for technetium to drop to half of its original activity



Step 2: Read the half-life from the graph

- In the diagram above the initial activity, A_0, is 8×10^7 Bq
- The time taken to decrease to 4×10^7 Bq, or $\frac{1}{2}A_0$, is 6 hours
- The time taken to decrease to 2×10^7 Bq is 6 **more** hours
- The time taken to decrease to 1×10^7 Bq is 6 **more** hours
- Therefore, the half-life of this isotope is **6 hours**

YOUR NOTES

YOUR NOTES

Decay Constant & Half-Life

- Since radioactive decay is spontaneous and random, it is useful to consider the average number of nuclei that are expected to decay per unit time
  - This is known as the **average decay rate**
- As a result, each radioactive element can be assigned a **decay constant**
- The decay constant λ is defined as:

The probability that an individual nucleus will decay per unit of time

- When a sample is highly radioactive, this means the number of decays per unit time is very high
  - This suggests it has a high level of **activity**
- Activity, or the number of decays per unit time can be calculated using:

$$A = -\frac{\Delta N}{\Delta t} = \lambda N$$

- Where:
  - A = activity of the sample (Bq)
  - ΔN = number of decayed nuclei
  - Δt = time interval (s)
  - λ = decay constant (s^{-1})
  - N = number of nuclei remaining in a sample
- The activity of a sample is measured in **Becquerels** (Bq)
  - An activity of 1 Bq is equal to one decay per second, or 1 s^{-1}
- This equation shows:
  - The greater the decay constant, the **greater the activity** of the sample
  - The activity depends on the number of **undecayed nuclei remaining** in the sample
  - The minus sign indicates that the number of nuclei remaining **decreases** with time
- Half-life and decay constant can be linked, using an equation called the **exponential decay equation** showing how the number of active nuclei, N, changes over time, t, where N_0 is the initial number of active nuclei

$$N = N_0 e^{-\lambda t}$$

- When time t is equal to the half-life $t_{½}$, the number of active nuclei in the sample, N, will be half of its original value, so $N = ½ N_0$

$$\frac{1}{2} N_0 = N_0 e^{-\lambda t_{½}}$$

- The formula linking half-life and decay constant can then be derived as follows:

Divide both sides by N_0: $\frac{1}{2} = e^{-\lambda t_{½}}$

YOUR NOTES

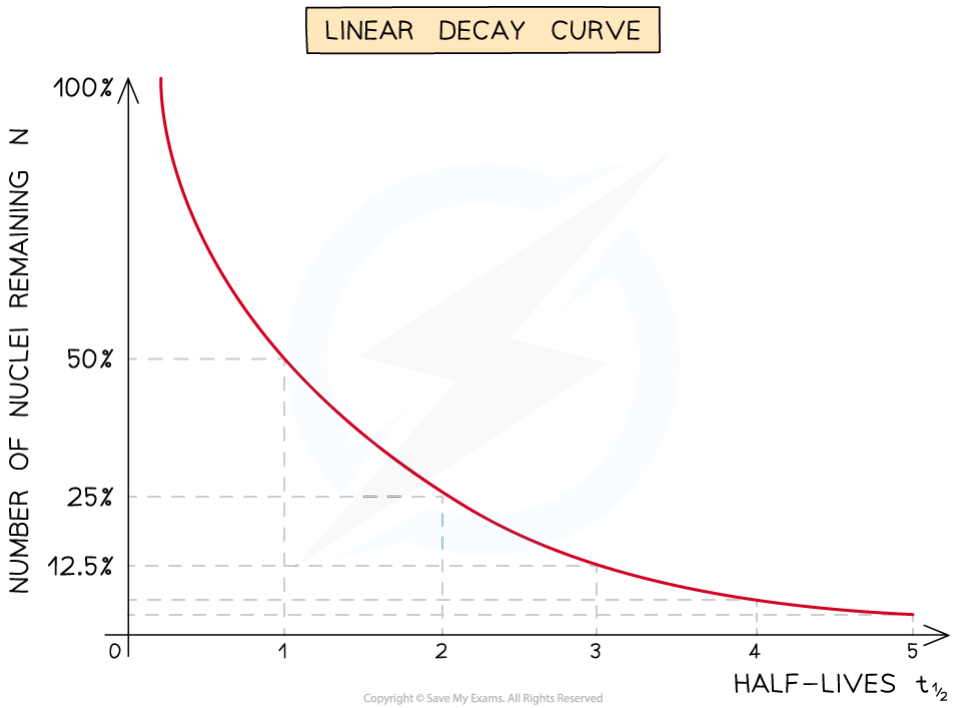
Take the natural log of both sides: $\ln\left(\frac{1}{2}\right) = -\lambda t_{½}$

Apply properties of logarithms: $\lambda t_{½} = \ln(2)$

- Therefore, half-life $t_{½}$ can be calculated using the equation:

$$t_{½} = \frac{ln\,2}{\lambda} \simeq \frac{0.693}{\lambda}$$

- This equation shows that half-life $t_{½}$ and the radioactive decay rate constant λ are inversely proportional
  - Therefore, the **shorter** the half-life, the **larger** the decay constant and the **faster** the decay
- The half-life of a radioactive substance can be determined from decay curves and log graphs
- Since half-life is the **time taken for the initial number of nuclei, or activity, to reduce by half**, it can be found by
  - Drawing a line to the curve at the point where the activity has dropped to half of its original value
  - Drawing a line from the curve to the time axis, this is the half-life



A linear decay curve. This represents the relationship: $\frac{\Delta N}{\Delta t} = -\lambda N$

- Straight-line graphs tend to be more useful than curves for interpreting data
  - Due to the exponential nature of radioactive decay logarithms can be used to achieve a straight line graph
- Take the exponential decay equation for the number of nuclei

$$N = N_0 e^{-\lambda t}$$

- Taking the natural logs of both sides

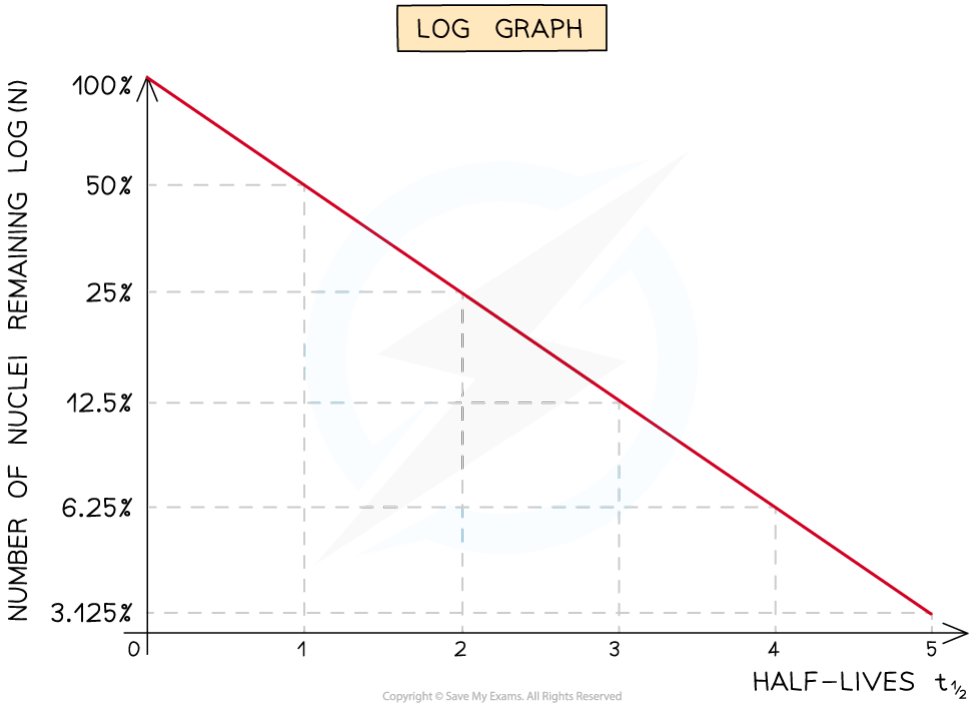
$$\ln N = \ln (N_0) - \lambda t$$

$$ln\, N = -\lambda t + ln\, (N_0)$$

- In this form, this equation can be compared to the equation of a straight line

$$y = mx + c$$

- Where:
  - ln (*N*) is plotted on the y-axis
  - *t* is plotted on the x-axis
  - gradient = –λ
  - y-intercept = ln (N_0)
- Half-lives can be found in a similar way to the decay curve but the intervals will be regular as shown below:



A logarithmic graph. This represents the relationship: $\ln N = -\lambda t + \ln N_0$

- Note: experimentally, the measurement generally taken is the count rate of the source
  - Since count rate ∝ activity ∝ number of nuclei, the graphs will all take the same shapes when plotted against time (or number of half-lives) linearly or logarithmically

YOUR NOTES

YOUR NOTES

Worked Example

Radium is a radioactive element first discovered by Marie and Pierre Curie.

They used the radiation emitted from radium-226 to define a unit called the Curie (Ci) which they defined as the activity of 1 gram of radium.

It was found that in a 1 g sample of radium, 2.22×10^{12} atoms decayed in 1 minute.Another sample containing 3.2×10^{22} radium-226 atoms had an activity of 12 Ci.

(a) Determine the value of 1 Curie

(b) Determine the decay constant for radium-226

Answer:

(a)

Step 1: Write down the known quantities

- Number of atoms decayed, $\Delta N = 2.22 \times 10^{12}$
- Time, $\Delta t = 1$ minutes = 60 s

Step 2: Write down the activity equation

$$A = \frac{\Delta N}{\Delta t}$$

Step 3: Calculate the value of 1 Ci

$$A = \frac{2.22 \times 10^{12}}{60} = 3.7 \times 10^{10} \text{ decays s}^{-1}$$

(b)

Step 1: Write down the known quantities

- Number of atoms, $N = 3.2 \times 10^{22}$
- Activity, $A = 12$ Ci $= 12 \times (3.7 \times 10^{10}) = 4.44 \times 10^{11}$ Bq

Step 2: Write down the activity equation

$$A = \lambda N$$

Step 3: Calculate the decay constant of radium

$$\lambda = \frac{A}{N} = \frac{4.44 \times 10^{11}}{3.2 \times 10^{22}} = 1.388 \times 10^{-11} \text{ s}^{-1}$$

- Therefore, the decay constant of radium-226 is 1.4×10^{-11} s^{-1}

YOUR NOTES

Worked Example

Strontium-90 is a radioactive isotope with a half-life of 28.0 years. A sample of Strontium-90 has an activity of 6.4×10^9 Bq.

(a)
Calculate the decay constant λ, in year^{-1}, of Strontium-90.
(b)
Determine the fraction of the sample remaining after 50 years.

Answer:

(a)

Step 1: List the known quantities

- Half-life, $t_{½}$ = 28 years

Step 2: Write the equation for half-life

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Step 3: Rearrange for λ and calculate

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{28} = 0.025 \text{ year}^{-1}$$

(b)

Step 1: List the known quantities

- Decay constant, $\lambda = 0.025$ year^{-1}
- Time passed, $t = 50$ years

Step 2: Write the equation for exponential decay

$$N = N_0 \, e^{-\lambda t}$$

Step 3: Rearrange for $\frac{N}{N_0}$ and calculate

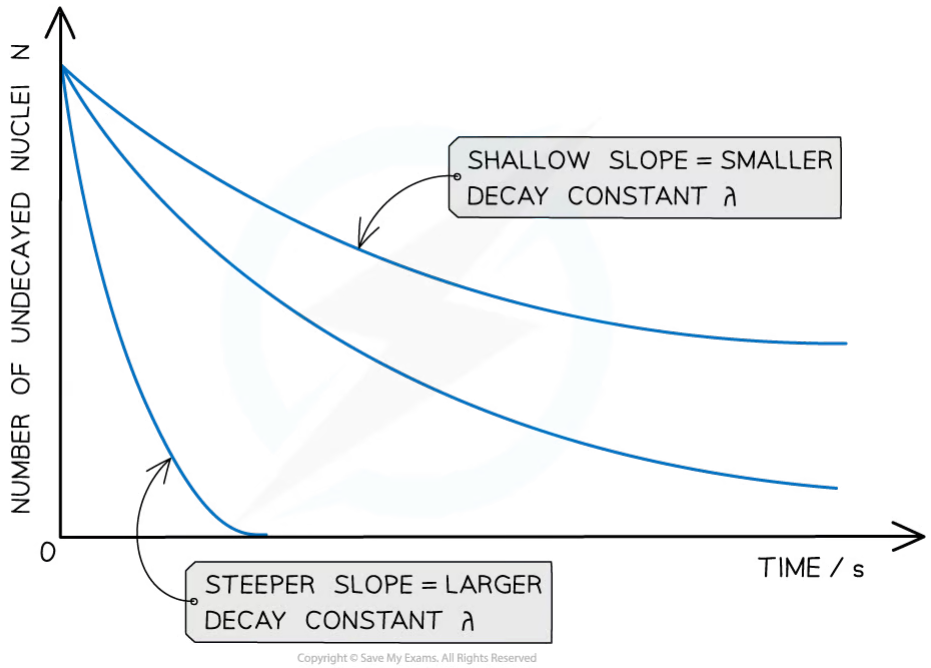
$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-(0.025) \times 50} = 0.287$$

- Therefore, **28.7%** of the sample will remain after 50 years

The Law of Radioactive Decay

- In radioactive decay, the number of undecayed nuclei falls very rapidly, without ever reaching zero
  - Such a model is known as **exponential decay**
- The graph of number of undecayed nuclei against time has a very distinctive shape:



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Radioactive decay follows an exponential pattern. The graph shows three different isotopes each with a different rate of decay

- **The key features of this graph are:**
  - The steeper the slope, the larger the decay constant λ (and vice versa)
  - The decay curves always start on the y-axis at the initial number of undecayed nuclei (N_0)

Equations for Radioactive Decay

- The number of undecayed nuclei *N* can be represented in exponential form by the equation:

$$N = N_0 e^{-\lambda t}$$

- Where:
  - N_0 = the initial number of undecayed nuclei (when $t = 0$)
  - N = number of undecayed nuclei at a certain time t
  - λ = decay constant (s^{-1})
  - t = time interval (s)
- The number of nuclei can be substituted for other quantities.

YOUR NOTES

- For example, the activity *A* is directly proportional to *N*, so it can also be represented in exponential form by the equation:

$$A = A_0 e^{-\lambda t}$$

- Where:
  - *A* = activity at a certain time *t* (Bq)
  - A_0 = initial activity (Bq)
- The received count rate *C* is related to the activity of the sample, hence it can also be represented in exponential form by the equation:

$$C = C_0 e^{-\lambda t}$$

- Where:
  - *C* = count rate at a certain time *t* (counts per minute or cpm)
  - C_0 = initial count rate (counts per minute or cpm)

The exponential function e

- The symbol e represents the exponential constant
  - It is approximately equal to e = 2.718
- On a calculator, it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the natural logarithmic function
  - This is because, if $e^x = y$, then $x = \ln(y)$

Worked Example

Strontium-90 decays with the emission of a β-particle to form Yttrium-90.The decay constant of Strontium-90 is 0.025 year $^{-1}$.

Determine the activity A of the sample after 5.0 years, expressing the answer as a fraction of the initial activity A_0.

Answer:

Step 1: Write out the known quantities

- Decay constant, $\lambda = 0.025$ year $^{-1}$
- Time interval, $t = 5.0$ years
- Both quantities have the same unit, so there is no need for conversion

Step 2: Write the equation for activity in exponential form

$$A = A_0 e^{-\lambda t}$$

Step 3: Rearrange the equation for the ratio between A and A_0

$$\frac{A}{A_0} = e^{-\lambda t}$$

Step 4: Calculate the ratio A/A_0

$$\frac{A}{A_0} = e^{-(0.025 \times 5)} = 0.88$$

- Therefore, the activity of Strontium-90 decreases by a factor of 0.88, or 12%, after 5 years

YOUR NOTES

Mass Defect & Nuclear Binding Energy

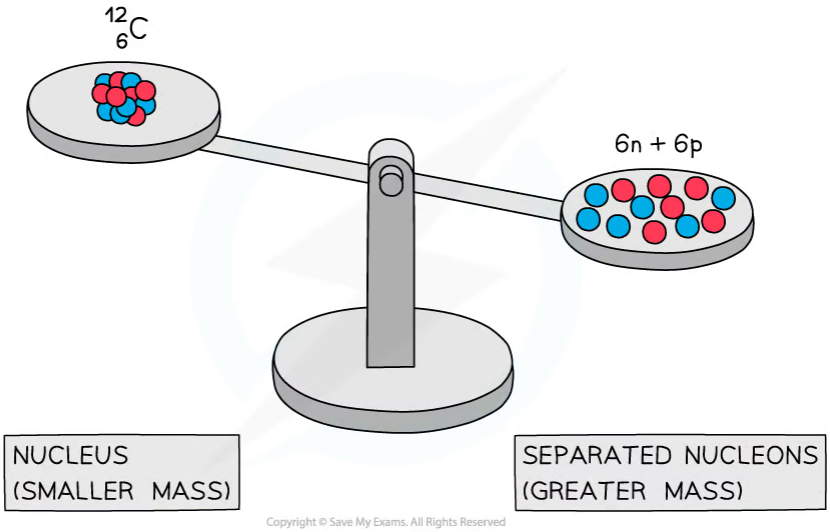
- Experiments into nuclear structure have found that the total mass of a nucleus is **less** than the sum of the masses of its constituent nucleons
  - In other words, the combined mass of 6 separate protons and 6 separate neutrons is more than the mass of a carbon-12 nucleus
  - This difference in mass is known as the **mass defect**
- Mass defect is defined as:

  The difference between the measured mass of a nucleus and the sum total of the masses of its constituents

- The mass defect Δm of a nucleus can be calculated using:

$$\Delta m = Zm_p + (A - Z)m_n - m_{total}$$

- Where:
  - Z = proton number
  - A = nucleon number
  - m_p = mass of a proton (kg)
  - m_n = mass of a neutron (kg)
  - m_{total} = measured mass of the nucleus (kg)



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A system of separated nucleons has a greater mass than a system of bound nucleons

- Due to mass-energy equivalence, a decrease in mass infers that energy must be released
- Energy and mass are proportional, so, the total energy of a nucleus is **less than the sum of the energies** of its constituent nucleons
- Binding energy is defined as:

The energy required to break a nucleus into its constituent protons and neutrons

- The formation of a nucleus from a system of isolated protons and neutrons **releases energy**

Worked Example

The binding energy per nucleon is 7.98 MeV for an atom of Oxygen-16 (^{16}O).

Determine an approximate value for the energy required, in MeV, to completely separate the nucleons of this atom.

Answer:

Step 1: List the known quantities

- Binding energy per nucleon, $E = 7.98$ MeV

Step 2: State the number of nucleons

- The number of nucleons is 8 protons and 8 neutrons, therefore 16 nucleons in total

Step 3: Find the total binding energy

- The binding energy for oxygen-16 is:

$$7.98 \times 16 = 127.7 \text{ MeV}$$

Step 4: State the final answer

- The approximate total energy needed to completely separate this nucleus is **127.7 MeV**

Exam Tip

Binding energy is named in a confusing way, so be careful!

Avoid describing the binding energy as the energy stored in the nucleus – this is not correct – it is energy that must be put **into** the nucleus to **pull it apart.**

YOUR NOTES

Mass-Energy Equivalence

YOUR NOTES

- Einstein showed in his Theory of Relativity that matter can be considered a form of energy and hence, he proposed:
  - Mass can be converted into energy
  - Energy can be converted into mass
- This is known as **mass-energy equivalence**, and can be summarised by the equation:

$$\Delta E = \Delta mc^2$$

- Where:
  - E = energy (J)
  - m = mass (kg)
  - c = the speed of light ($m\ s^{-1}$)
- Some examples of mass-energy equivalence are:
  - The **fusion** of hydrogen into helium in the centre of the sun
  - The **fission** of uranium in nuclear power plants
  - Nuclear **weapons**
  - High-energy **particle collisions** in particle accelerators

YOUR NOTES

Worked Example

Calculate the binding energy per nucleon, in MeV, for the radioactive isotope potassium-40 $\left({}^{40}_{19}\mathrm{K}\right)$.

You may use the following data:

- Nuclear mass of potassium-40 = 39.953 548 u
- Mass of one neutron = 1.008 665 u
- Mass of one proton = 1.007 276 u

Answer:

Step 1: Identify the number of protons and neutrons in potassium-40

- Proton number, $Z = 19$
- Neutron number, $N = 40 - 19 = 21$

Step 2: Calculate the mass defect, Δm

- Proton mass, $m_p = 1.007\,276$ u
- Neutron mass, $m_n = 1.008\,665$ u
- Mass of potassium-40, $m_{\text{total}} = 39.953\,548$ u

$$\Delta m = Zm_{\text{p}} + Nm_{\text{n}} - m_{\text{total}}$$

$$\Delta m = (19 \times 1.007276) + (21 \times 1.008665) - 39.953\,548$$

$$\Delta m = 0.36666 \text{ u}$$

Step 3: Convert mass units from u to kg

- $1 \text{ u} = 1.661 \times 10^{-27}$ kg

$$\Delta m = 0.36666 \times (1.661 \times 10^{-27}) = 6.090 \times 10^{-28} \text{ kg}$$

Step 4: Write down the equation for mass-energy equivalence

$$E = \Delta m c^2$$

- Where $c = 3.0 \times 10^8$ m s^{-1}

Step 5: Calculate the binding energy, E

$$E = 6.090 \times 10^{-28} \times (3.0 \times 10^8)^2 = 5.5 \times 10^{-11} \text{ J}$$

Step 6: Determine the binding energy per nucleon and convert J to MeV

- Take the binding energy and divide it by the number of nucleons
- $1 \text{ MeV} = 1.6 \times 10^{-13}$ J

$$\text{Binding energy per nucleon} = \frac{5.5 \times 10^{-11}}{40} = 1.375 \times 10^{-12} \text{ J}$$

$$\text{Binding energy per nucleon} = \frac{1.375 \times 10^{-12}}{1.6 \times 10^{-13}} = 8.594 \text{ MeV}$$

YOUR NOTES

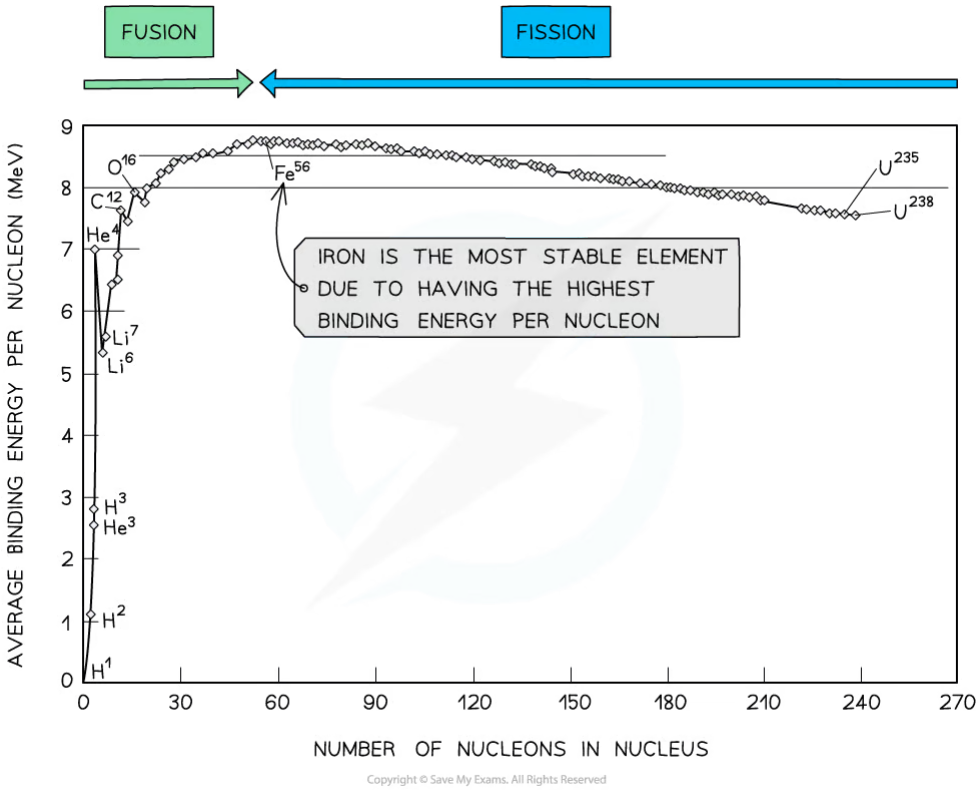
YOUR NOTES

Binding Energy per Nucleon Curve

- In order to compare nuclear stability, it is more useful to look at the **binding energy per nucleon**
- The binding energy per nucleon is defined as:

The binding energy of a nucleus divided by the number of nucleons in the nucleus

- A higher binding energy per nucleon indicates a higher stability
  - In other words, it requires more energy to pull the nucleus apart
- Iron (A = 56) has the highest binding energy per nucleon, which makes it the **most stable** of all the elements



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By plotting a graph of binding energy per nucleon against nucleon number, the stability of elements can be inferred

Key Features of the Graph

- At low values of A:
  - Nuclei tend to have a lower binding energy per nucleon, hence, they are generally less stable

- This means the lightest elements have weaker electrostatic forces and are the most likely to undergo **fusion**

- Helium (^{4}He), carbon (^{12}C) and oxygen (^{16}O) do not fit the trend
  - Helium-4 is a particularly stable nucleus hence it has a **high** binding energy per nucleon
  - Carbon-12 and oxygen-16 can be considered to be three and four helium nuclei, respectively, bound together

- At high values of A:
  - The general binding energy per nucleon is high and gradually decreases with A
  - This means the heaviest elements are the most unstable and likely to undergo **fission**

Comparing Fusion & Fission

- Fusion occurs at low values of A because:
  - Attractive nuclear forces between nucleons dominate over repulsive electrostatic forces between protons

- In fusion, the mass of the nucleus that is created is slightly **less** than the total mass of the original nuclei
  - The mass defect is equal to the binding energy that is released since the nucleus that is formed is more stable

- Fission occurs at high values of A because:
  - Repulsive electrostatic forces between protons begin to dominate, and these forces tend to break apart the nucleus rather than hold it together

- In fission, an unstable nucleus is converted into more stable nuclei with a smaller total mass
  - This difference in mass, the mass defect, is equal to the binding energy that is released

- Fusion releases much more energy per kg than fission
- The energy released is the difference in binding energy caused by the difference in mass between the reactant and products
  - Hence, the greater the increase in binding energy, the greater the energy released

- At small values of A (fusion region), the gradient is **much steeper** compared to the gradient at large values of A (fission region)
- This corresponds to a larger binding energy per nucleon being released

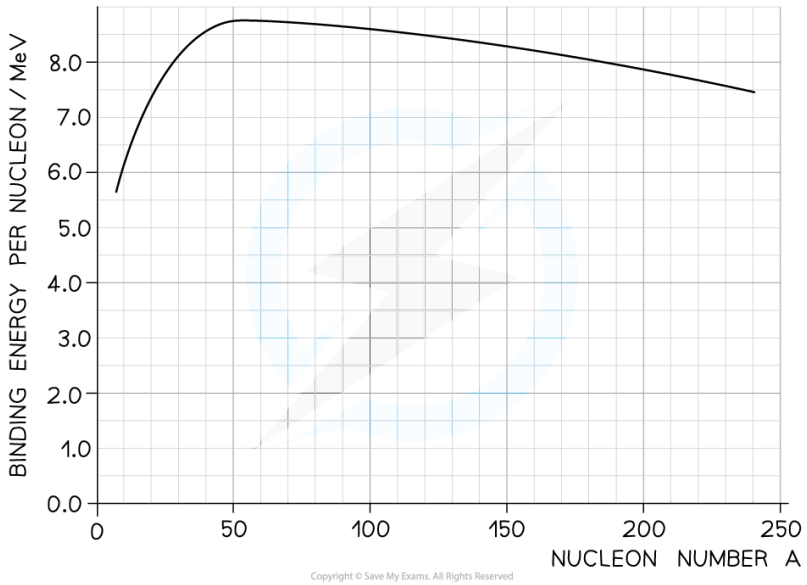
YOUR NOTES

Worked Example

The equation below represents one possible decay of the induced fission of a nucleus of uranium-235.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{98}_{38}Sr + ^{135}_{54}Xe + 3^{1}_{0}n$$

The graph shows the binding energy per nucleon plotted against nucleon number A.



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Calculate the energy released:

a) By the fission process represented by the equation

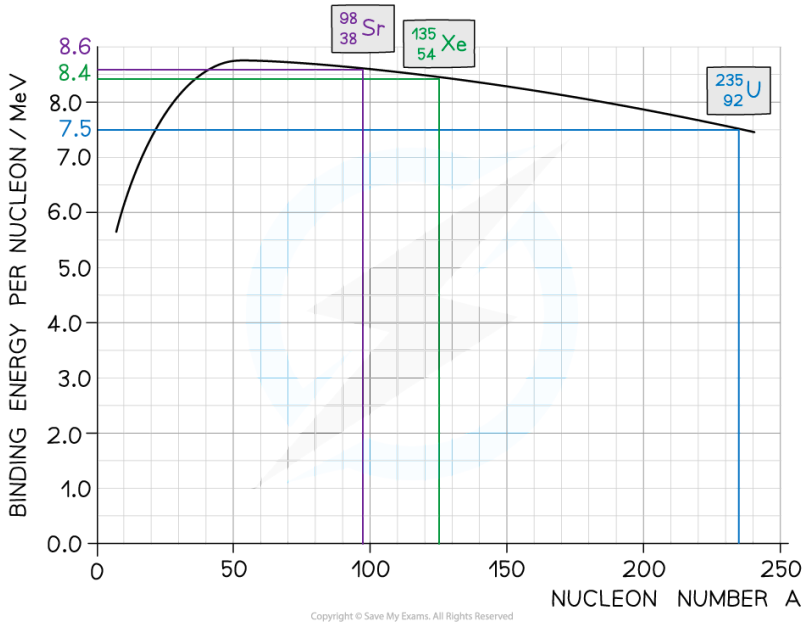
b) When 1.0 kg of uranium, containing 3% by mass of U-235, undergoes fission

Answer:

Part (a)

Step 1: Use the graph to identify each isotope's binding energy per nucleon

YOUR NOTES



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- Binding energy per nucleon (U-235) = 7.5 MeV
- Binding energy per nucleon (Sr-98) = 8.6 MeV
- Binding energy per nucleon (Xe-135) = 8.4 MeV

Step 2: Determine the binding energy of each isotope

Binding energy = Binding Energy per Nucleon × Mass Number

- Binding energy of U-235 nucleus = (235 × 7.5) = 1763 MeV
- Binding energy of Sr-98 = (98 × 8.6) = 843 MeV
- Binding energy of Xe-135 = (135 × 8.4) = 1134 MeV

Step 3: Calculate the energy released

Energy released = Binding energy after (Sr + Xe) – Binding energy before (U)

Energy released = (1134 + 843) – 1763 = 214 MeV

Part (b)

Step 1: Calculate the energy released by 1 mol of uranium-235

- There are N_A (the Avogadro constant) atoms in 1 mol of U-235, which is equal to a mass of 235 g
- Energy released by 235 g of U-235 = $(6 \times 10^{23}) \times 214$ MeV

Step 2: Convert the energy released from MeV to J

- 1 MeV = 1.6×10^{-13} J
- Energy released = $(6 \times 10^{23}) \times 214 \times (1.6 \times 10^{-13}) = 2.05 \times 10^{13}$ J

Step 3: Work out the proportion of uranium-235 in the sample

- 1 kg of uranium which is 3% U-235 contains 0.03 kg or 30 g of U-235

- 1 mol of U-235 has a mass of 235 g so 30 g divided by 235 g gives the number of moles of U-235 in 30 g

Step 4: Calculate the energy released by the sample

- Energy released from 1 kg of uranium,

$$E = (2.05 \times 10^{13}) \times \frac{30}{235} = 2.6 \times 10^{12} \text{ J}$$

YOUR NOTES

Exam Tip

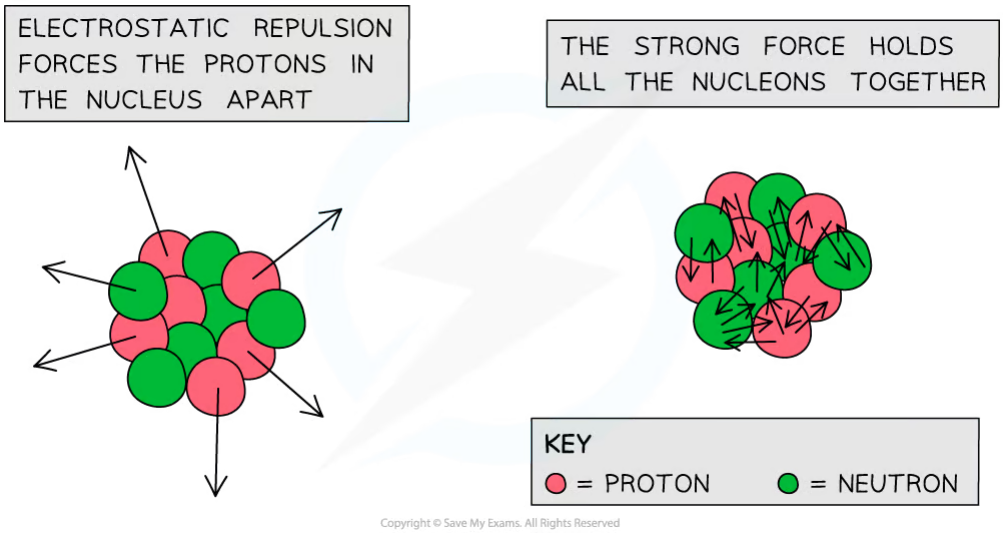
Checklist on what to include (and what not to include) in an exam question asking you to draw a graph of binding energy per nucleon against nucleon number:

- Do not begin your curve at A = 0, this is not a nucleus!
- Make sure to correctly label both axes AND units for binding energy per nucleon
- You will be expected to include numbers on the axes, mainly at the peak to show the position of iron (^{56}Fe)

YOUR NOTES

The Strong Nuclear Force

- In the nucleus, there are electrostatic forces between the protons due to their electric charge and gravitational forces due to their mass
- Comparatively, gravity is a very weak force and the electrostatic repulsion between protons is therefore much stronger than their gravitational attraction
- If these were the only forces, the nucleus wouldn't hold together
- Therefore, the force that does hold the nucleus together is called the **strong nuclear force**
- The strong nuclear force keeps the nucleus stable since it holds quarks together
- Since protons and neutrons are made up of quarks, the strong force keeps them bound within a nucleus



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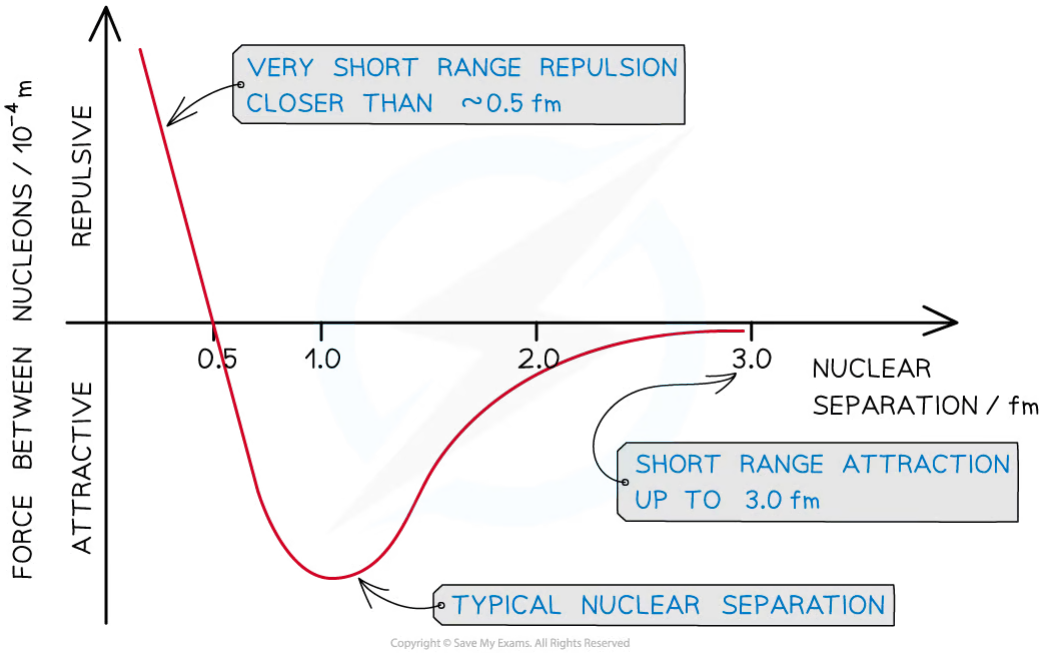
Whilst the electrostatic force is a repulsive force in the nucleus, the strong nuclear force holds the nucleus together

Range of the Strong Nuclear Force

- The strength of the strong nuclear force between two nucleons varies with the separation between them
- This can be plotted on a graph which shows how the force changes with separation

YOUR NOTES



The strong nuclear force is repulsive before a separation of ~ 0.5 fm and attractive up till ~ 3.0 fm

- The key features of this graph are that the strong nuclear force is:
  - **Repulsive** closer than around 0.5 fm (femtometres, 10^{-15} m)
  - **Attractive** up to around 3.0 fm
  - Reaches a **maximum** attractive value at around 1.0 fm (the typical nuclear separation)
  - Becomes **zero** after 3.0 fm
- In comparison to other fundamental forces, the strong force has a **very small range** (only up to 3.0 fm)

The Strong Nuclear Force & Fusion

- For two nuclei to fuse, both nuclei must have **high kinetic energy**
  - This is because nuclei must be able to get close enough to fuse
- However, two forces acting within the nuclei make this difficult to achieve
- **Electrostatic Repulsion**
  - Protons inside the nuclei are positively charged, which means that they **electrostatically** repel one another
- **Strong Nuclear Force**
  - The strong nuclear force, which binds nucleons together, acts at very short distances within nuclei
  - Therefore, nuclei must get very close together for the strong nuclear force to take effect
- It takes a great deal of energy to overcome the repulsive electrostatic forces, hence fusion can only be achieved in an **extremely hot** environment where particles have a great deal of **kinetic energy**, such as the core of a star

Exam Tip

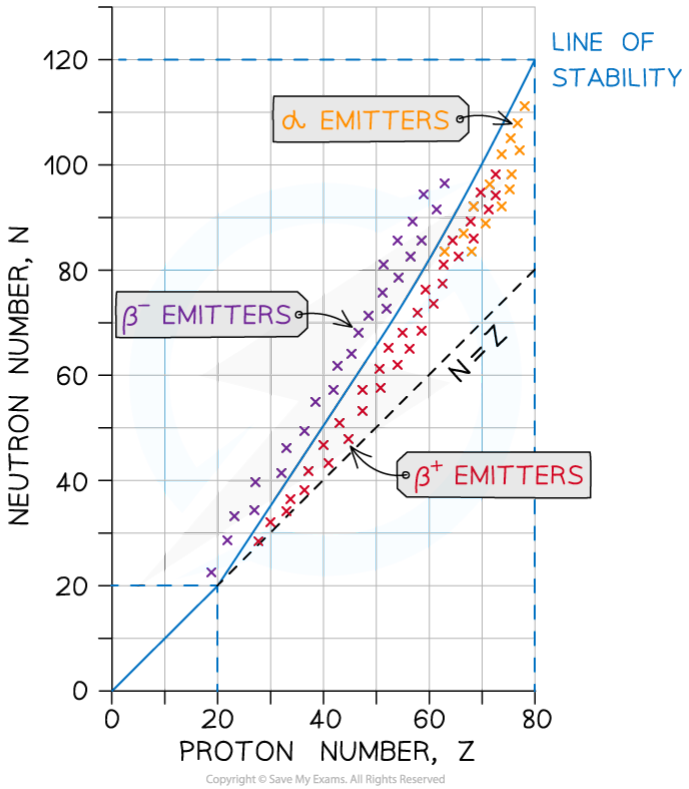
- You may see the strong nuclear force also referred to as the **strong interaction**
- Remember to write that after 3 fm, the strong force becomes 'zero' or 'has no effect' rather than it is 'negligible'.
- Recall that 1 fm = 1×10^{-15} m

YOUR NOTES

YOUR NOTES

Nuclear Stability

- The most common elements in the universe all tend to have values of N and Z less than 20 (plus iron which has Z = 26, N = 30)
- Where:
  - N = number of neutrons
  - Z = number of protons / atomic number
- This is because lighter elements (with fewer protons) tend to be much **more stable** than heavier ones (with many protons)
- Nuclear stability becomes vastly clearer when viewed on a graph of N against Z



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This nuclear stability curve shows the line of stable isotopes and which unstable isotopes will emit alpha or beta particles

- The line of stability shows N and Z values that produce stable nuclei
  - If a nucleus on this line were to have more neutrons, for example, it would move above the line and become an unstable β^- emitter
- A nucleus will be unstable if it has:
  - Too many neutrons
  - Too many protons
  - Too many nucleons ie. too heavy
  - Too much energy

- An unstable atom wants to become stable
- For light isotopes, Z < 20:
  - All these nuclei tend to be very stable
  - They follow the straight-line N = Z
- For heavy isotopes, Z > 20:
  - The neutron-proton ratio increases
  - Stable nuclei must have more neutrons than protons

YOUR NOTES

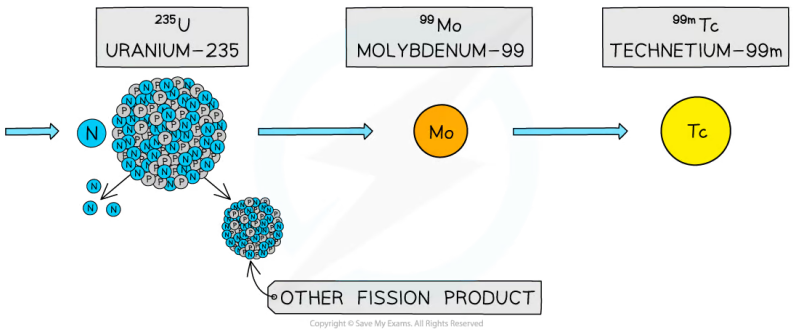
Evidence for the Strong Nuclear Force

- The imbalance in the neutron-proton ratio is very significant to the stability of nuclei
  - At a short range (around 1–4 fm), nucleons are bound by the **strong nuclear force**
  - Below 1 fm, the strong nuclear force is **repulsive** in order to prevent the nucleus from collapsing
  - At longer ranges, the electromagnetic force acts between protons, so **more protons cause more instability**
  - Therefore, as more protons are added to the nucleus, more neutrons are needed to add distance between protons to **reduce** the electrostatic repulsion
  - Also, the extra neutrons increase the amount of binding force which helps to **bind the nucleons together**

YOUR NOTES

Nuclear Energy Levels

- In the same way that electrons can exist in excited states, nuclei can also exist in **excited states**
- After an unstable nucleus emits an alpha particle, beta particle or undergoes electron capture, it may emit any remaining energy in the form of a **gamma photon** (γ)
  - Emission of a γ photon does not change the number of protons or neutrons in the nucleus, it only allows the nucleus to **lose energy**
- This happens when a daughter nucleus is in an excited state after a decay
- This excited state is usually very **short-lived**, and the nucleus quickly moves to its **ground state**, either directly or via one or more lower-energy excited states

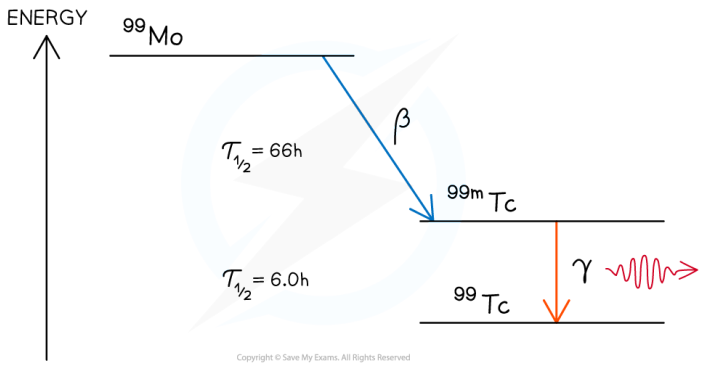


- One common application of this is the use of technetium-99m as a γ source in medical diagnosis
  - The 'm' stands for **metastable** which means the nucleus exists in a particularly stable excited state
- Technetium-99m is the decay product of molybdenum-99, which can be found as a product in nuclear reactors
- The decay of molybdenum-99 is shown below:

$$^{99}_{42}Mo \rightarrow {}^{99m}_{43}Tc + {}^{0}_{-1}\beta + \overline{\nu_e}$$

$$^{99m}_{43}Tc \rightarrow {}^{99}_{43}Tc + \gamma$$

- Nuclear energy levels are similar to electron energy levels
- The nuclear energy level diagram of molybdenum-99 can be represented as follows:



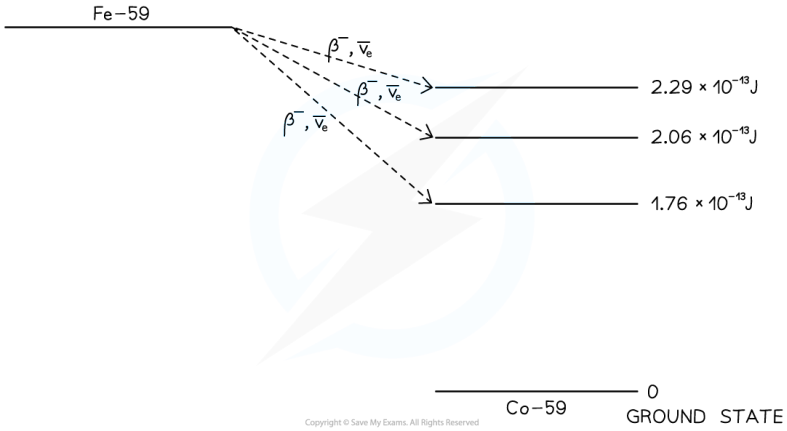
- The decay mode (usually alpha or beta) is represented by a diagonal line
- The excited state, or states, are generally stacked in descending energy order to the right of the decay

YOUR NOTES

Worked Example

A nucleus of iron Fe-59 decays into a stable nucleus of cobalt Co-59. It decays by β^- emission followed by the emission of γ-radiation as the Co-59 nucleus de-excites into its ground state.

The total energy released when the Fe-59 nucleus decays is 2.52×10^{-13} J. The Fe-59 nucleus can decay to one of three excited states of the cobalt-59 nucleus as shown below. The energies of the excited states are shown relative to the ground state.



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Following the production of excited states of Co-59, γ-radiation of discrete wavelengths is emitted.

(a) Calculate the maximum possible kinetic energy of the $\beta-$ particle emitted in MeV

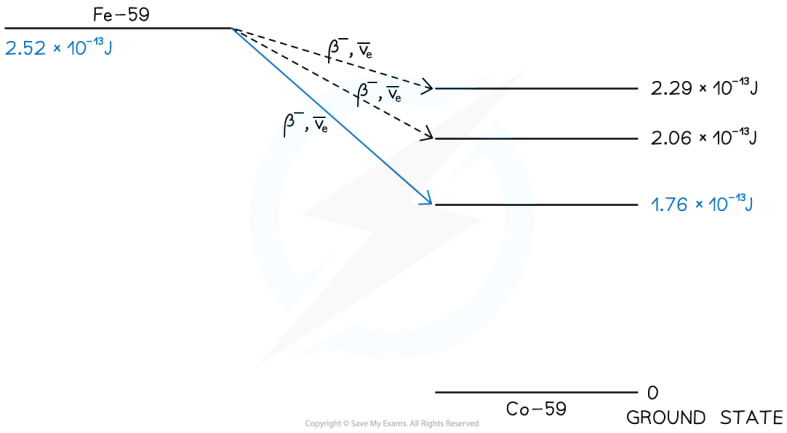
(b) State the maximum number of discrete wavelengths that could be emitted

(c) Calculate the longest wavelength of the emitted γ-radiation

Answer:

Part (a)

Step 1: Identify the beta emission with the largest energy gap



Step 2: Calculate the energy difference

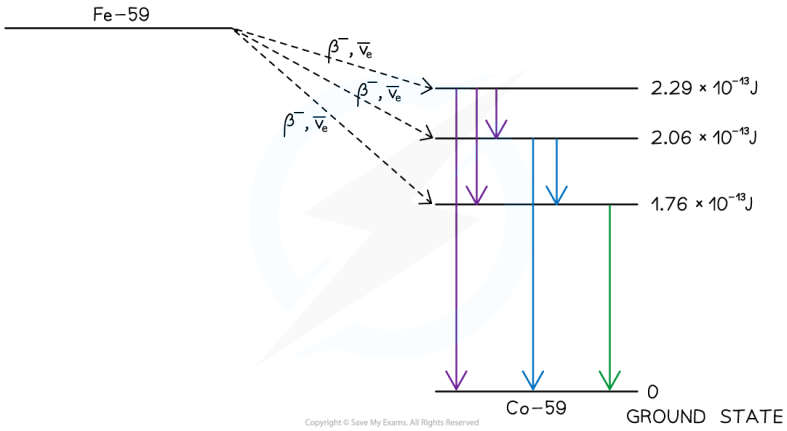
$$\Delta E = (2.52 - 1.76) \times 10^{-13} = 7.6 \times 10^{-14} \text{ J}$$

Step 3: Convert from J to MeV

$$\Delta E = \frac{7.6 \times 10^{-14}}{1.6 \times 10^{-13}} = 0.475 = 0.48 \text{ MeV}$$

- $1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}$

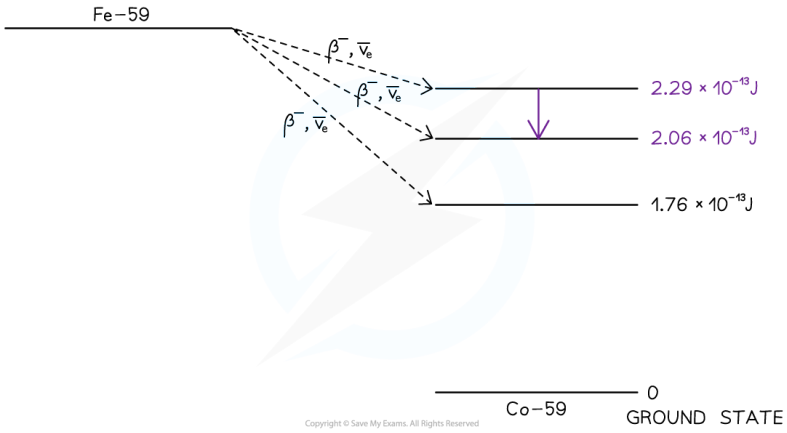
Part (b)



- There are 6 possible transitions, hence 6 discrete wavelengths could be emitted

Part (c)

Step 1: Identify the emission with the longest wavelength / smallest energy gap



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- Longest wavelength = lowest frequency = smallest energy

Step 2: Calculate the energy difference

$$\Delta E = (2.29 - 2.06) \times 10^{-13} = 2.3 \times 10^{-14}\ \text{J}$$

Step 3: Relate photon energy and wavelength

$$E = \frac{hc}{\lambda}$$

Where:

- h = Planck's constant
- c = speed of light

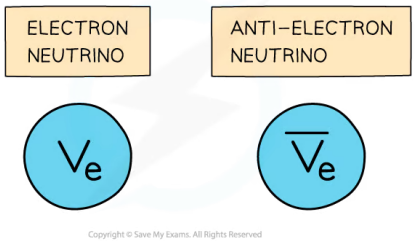
Step 4: Calculate the wavelength associated with the energy change

$$\lambda = \frac{hc}{E} = \frac{\left(6.63 \times 10^{-34}\right) \times \left(3.0 \times 10^{8}\right)}{2.3 \times 10^{-14}} = 8.6 \times 10^{-12}\ \text{m}$$

YOUR NOTES

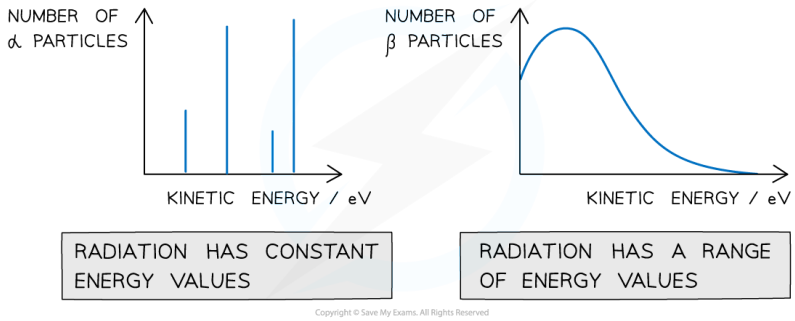
Evidence for the Neutrino

- An electron neutrino is a type of subatomic particle with no charge and negligible mass which is also emitted from the nucleus
- The anti-neutrino is the antiparticle of a neutrino
  - Electron anti-neutrinos are produced during β− decay
  - Electron neutrinos are produced during β+ decay



- Although the neutrino has no charge and negligible mass, its existence was hypothesised to account for the conservation of **energy** in beta decay

- When the number of α particles is plotted against kinetic energy, there are clear spikes that appear on the graph
- This demonstrates that **α-particles have discrete energies** (only certain values)



Alpha particles have discrete energy levels whilst beta particles have a continuous range of energies

- When the number of β particles is plotted against kinetic energy, the graph shows a curve
- This demonstrates that **beta particles (electrons or positrons) have a continuous range of energies**
- This is because the energy released in beta decay is shared between the **beta particles** (electrons or positrons) and **neutrinos** (or anti-neutrinos)
- This was one of the first clues of the neutrino's existence
- The principle of conservation of momentum and energy applies in both alpha and beta emission

Exam Tip

Remember that beta minus particles **are** electrons and beta plus are protons. The names are interchangeable.

YOUR NOTES

IB Physics HL

YOUR NOTES

Fission

CONTENTS

Spontaneous & Induced Fission
Energy Released in Fission Reactions
Chain Reactions from Fission
Operation of a Nuclear Reactor
Radioactive Waste Management

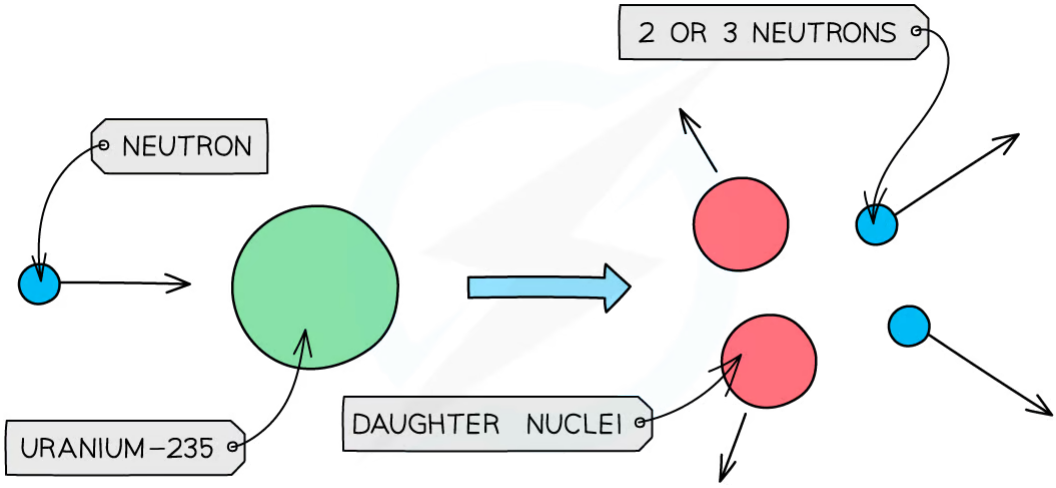
YOUR NOTES

Spontaneous & Induced Fission

- There is a lot of energy stored within the nucleus of an atom
  - This energy can be released in a nuclear reaction such as **fission** or **fusion**
- Nuclear fission is defined as:

  The splitting of a large, unstable nucleus into two smaller nuclei

- Isotopes of **uranium** and **plutonium** both undergo fission and are used as fuels in nuclear power stations to convert nuclear energy into electrical energy
- During fission, when a neutron collides with an unstable nucleus, the nucleus splits into **two smaller nuclei** (called daughter nuclei) as well as **two or three neutrons**
  - Gamma rays are also emitted



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Large nuclei can decay by fission to produce smaller nuclei and neutrons with a lot of kinetic energy

- The products of fission move away very **quickly**
  - Energy transferred is from **nuclear potential energy** to kinetic energy

Spontaneous Fission

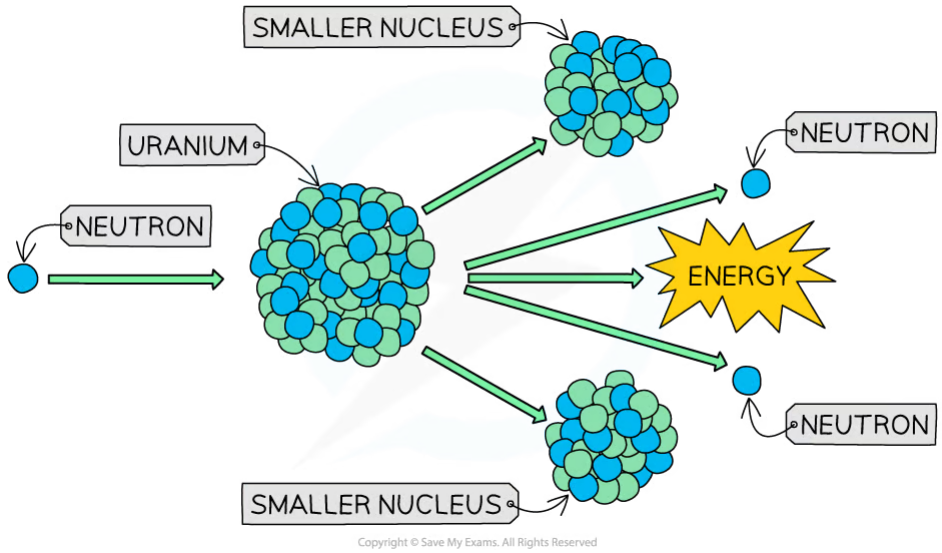
- It is **rare** for nuclei to undergo fission without additional energy being put into the nucleus
- When nuclear fission occurs in this way it is called **spontaneous fission**

Induced Fission

- Usually, for fission to occur the unstable nucleus must first **absorb** a **neutron**
  - This will be slow moving, often called a 'thermal' neutral

YOUR NOTES

- Take, for example, uranium-235, which is commonly used as a fuel in nuclear reactors
- It has a very long half-life of 700 million years
- This means that it would have low activity and energy would be released only if it was to have additional neutrons added



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- During induced fission, a **neutron** is absorbed by the uranium-235 nucleus to make uranium-236
  - This is very unstable and splits by nuclear fission almost immediately

YOUR NOTES

Worked Example

During a particular spontaneous fission reaction, plutonium-239 splits as shown in the equation below:

$$^{239}_{94}Pu \longrightarrow {}^{112}_{46}Pd + {}^{124}_{48}Cd + \ldots$$

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Which answer shows the section missing from this equation?

A	$^{3}_{0}n$
B	$^{0}_{0}\gamma$
C	$^{4}_{2}\alpha$
D	$3^{1}_{0}n$

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ANSWER: D

Step 1: Identify the different mass and atomic numbers

- Pu (Plutonium) has mass number 239 and atomic number 94
- Pd (Palladium) has mass number 112 and atomic number 46
- Cd (Cadmium) has mass number 124 and atomic number 48

Step 2: Calculate the mass and atomic number of the missing section

- Mass number is equal to the difference between the mass numbers of the reactants and the products

$$239 - (112 + 124) = 3$$

- Atomic number is equal to the difference between the atomic numbers of the reactants and the products

$$94 - (46 + 48) = 0$$

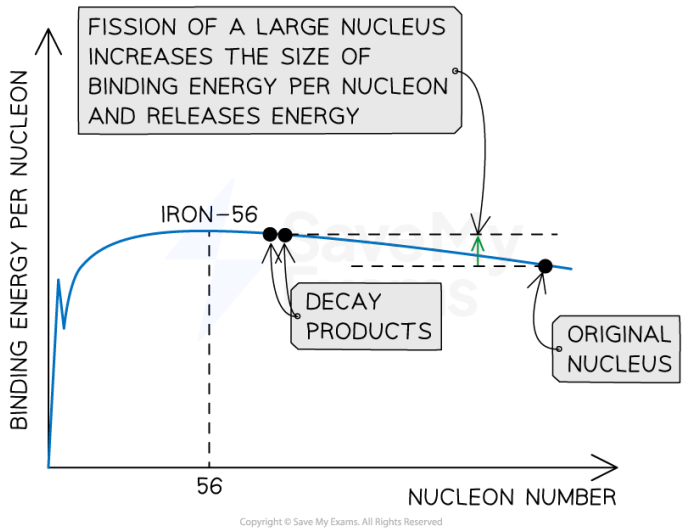
- The answer is therefore not **B** or **C**

Step 3: Determine the correct notation

- Neutrons have a mass number of 1
- The answer is therefore not **A**
- Therefore, this must be three neutrons, which corresponds to **D**

Energy Released in Fission Reactions

- When a large (parent) nucleus, such as uranium-235, undergoes a fission reaction, the daughter nuclei produced as a result will have a **higher binding energy per nucleon** than the parent nucleus
- As a result of the mass defect between the parent nucleus and the daughter nuclei, **energy is released**



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Energy can be extracted from fission reactions due to the mass defect between parent and daughter nuclei

- Nuclear fission is well-regarded as having the fuel source with the **highest energy density** of any fuel that is currently available to us (until fusion reactions become feasible)

Examples of Common Fuels: Energy Density and Specific Energy Table

Fuel	Specific energy (MJ kg^{-1})	Energy density (MJ m^{-3})
Coal	35	2×10^4
Hydrogen	130	10
Kerosene	48	3.3×10^4
Gasoline (petrol)	45	3.4×10^4
Wood	15.5	1×10^4
Uranium–235 (fission)	7.5×10^7	4×10^{13}

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- Calculations involving energy released in fission reactions often require the use of equations found in an array of previous topics, such as

$$density\ (kg\ m^{-3}) = \frac{energy\ density\ (J\ m^{-3})}{specific\ energy\ (J\ kg^{-1})}$$

$$no.\ of\ atoms = \frac{mass\ (g) \times Avogadro's\ number\ N_A\ (mol^{-1})}{molar\ mass\ (g\ mol^{-1})}$$

YOUR NOTES

YOUR NOTES

Worked Example

When a uranium-235 nucleus absorbs a slow-moving neutron and undergoes a fission reaction, one possible pair of fission fragments is technetium-112 and indium-122.

The equation for this process, and the binding energy per nucleon for each isotope, are shown below.

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{112}_{43}\mathrm{Tc} + {}^{122}_{49}\mathrm{In} + 2{}^{1}_{0}\mathrm{n}$$

nucleus	binding energy per nucleon / MeV
$^{235}_{92}\mathrm{U}$	7.59
$^{112}_{43}\mathrm{Tc}$	8.36
$^{122}_{49}\mathrm{In}$	8.51

(a)
Calculate the energy released per fission of uranium-235, in MeV.
(b)
Determine the mass of uranium-235 required per day to run a 500 MW power plant at 35% efficiency.
(c)
The energy density of coal is approximately 35 MJ kg^{-1}.

For the same power plant, estimate the ratio

$$\frac{mass\ of\ coal\ required\ per\ day}{mass\ of\ ^{235}U\ required\ per\ day}$$

Answer:

(a) Energy released per fission of uranium-235

Step 1: Determine the binding energies of the nuclei before and after the reaction

- Binding energy is equal to binding energy per nucleon × mass number
- Binding energy before $\left({}^{235}\mathrm{U}\right)$ = 235 × 7.59 = 1784 MeV
- Binding energy after $\left({}^{112}\mathrm{Tc} + {}^{122}\mathrm{In}\right)$ = (112 × 8.36) + (122 × 8.51) = 1975 MeV

Step 2: Find the difference to obtain the energy released per fission reaction

- Therefore, the energy released per fission = 1975 – 1784 = 191 MeV

(b) Mass of uranium-235 required per day

Step 1: List the known quantities

- Avogadro's number, $N_A = 6.02 \times 10^{23}$ mol^{-1}
- Molar mass of U-235, $m_r = 235$ g mol^{-1}
- Power output, $P_{out} = 500$ MW $= 500 \times 10^6$ J s^{-1}
- Efficiency, e = 35% = 0.35
- Time, $t = 1$ day $= 60 \times 60 \times 24 = 86\,400$ s

Step 2: Determine the number of atoms in 1 kg of U-235

- There are N_A (Avogadro's number) atoms in 1 mol of U-235, which is equal to a mass of 235 g

$$\text{number of atoms} = \frac{mass\,(g) \times N_A\,(mol^{-1})}{m_r\,(g\ mol^{-1})}$$

- A mass of 1 kg (1000 g) of U-235 contains $\frac{1000 \times (6.02 \times 10^{23})}{235} = 2.562 \times 10^{24}$ atoms kg^{-1}

Step 3: Determine the energy density of U-235

- Energy density of U-235 = total amount of energy released by 1 kg of U-235
- Energy density of U-235 = (number of atoms per kg) × (energy released per atom) = energy released per kg
- Energy released per atom of U-235 = 191 MeV
- Therefore, energy density of U-235 = $(2.562 \times 10^{24}) \times 191 = 4.893 \times 10^{26}$ MeV kg^{-1}
- To convert 1 MeV = $10^6 \times (1.6 \times 10^{-19})$ J
- Energy density of U-235 = $(4.893 \times 10^{26}) \times 10^6 \times (1.6 \times 10^{-19}) = 7.83 \times 10^{13}$ J kg^{-1}

Step 4: Use the relationship between power, energy and efficiency to determine the mass

- The input power required is:

efficiency & power: $e = \frac{P_{out}}{P_{in}} \quad \Rightarrow \quad P_{in} = \frac{P_{out}}{e}$

input power: $P_{in} = \frac{500}{0.35} = 1429\ \text{MW}$

input power: $P_{in} = \frac{E_{in}}{t} = 1429 \times 10^6\ \text{J s}^{-1}$

YOUR NOTES

- Therefore, the mass of U-235 required in a day is:

$$mass\ of\ ^{235}U\ (kg\ s^{-1}) = \frac{E_{in}\ (J)}{1\ s} \times \frac{1\ kg}{energy\ density\ of\ ^{235}U\ (J)}$$

mass of U-235 (per second) =

$$\frac{1429 \times 10^{6}}{1} \times \frac{1}{7.83 \times 10^{13}} = 1.82 \times 10^{-5}\ \text{kg s}^{-1}$$

mass of U-235 (per day) = $(1.82 \times 10^{-5}) \times 86\,400 = 1.58$ kg

- Therefore, **1.58 kg of uranium-235** is required per day to run a 500 MW power plant at 35% efficiency

(c) Ratio of the masses of coal and U-235

- Since energy density $\propto \frac{1}{mass}$

$$\frac{energy\ density\ of\ ^{235}U}{energy\ density\ of\ coal} \propto \frac{mass\ of\ coal\ required\ per\ day}{mass\ of\ ^{235}U\ required\ per\ day}$$

- Where the energy density of coal = 35 MJ kg^{-1}

$$\frac{mass\ of\ coal\ required\ per\ day}{mass\ of\ ^{235}U\ required\ per\ day} = \frac{7.83 \times 10^{13}}{35 \times 10^{6}} = 2.24 \times 10^{6}$$

- Over **2 million times** (~3.5 x 10^6 kg) more coal is required than uranium-235 to achieve the same power output in a day (or second, or month or year)

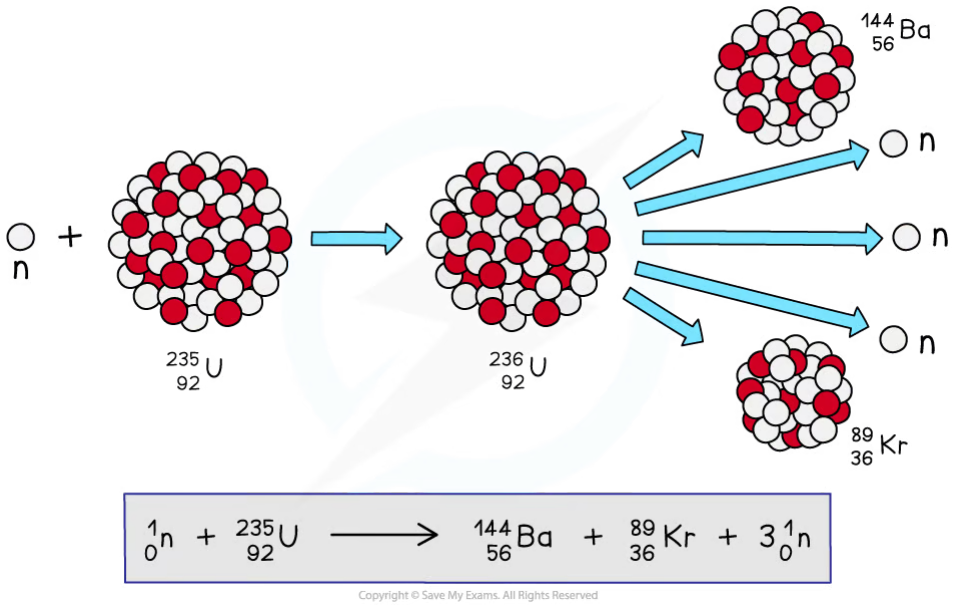
Exam Tip

If you need to brush up on binding energy calculations, take a look at the Mass Defect & Nuclear Binding Energy revision notes.

YOUR NOTES

Chain Reactions

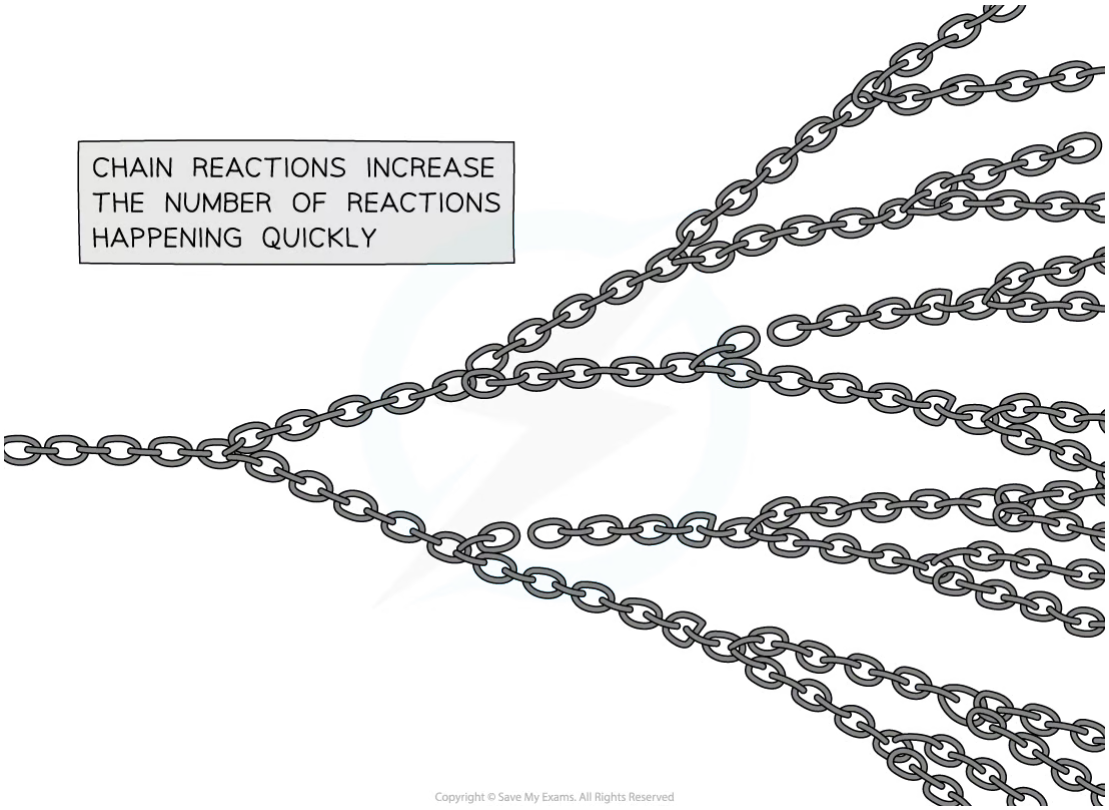
- One of the many decay reactions uranium-235 can undergo is shown below:



Uranium-235 decay chain from nuclear fission

- Neutrons involved in induced fission are known as **thermal neutrons**
- Thermal neutrons have low energy and speed meaning they can induce fission
  - This is important as neutrons with too much energy will rebound away from the uranium-235 nucleus and fission will not take place
- Only one extra neutron is required to induce a Uranium-235 nucleus to split by fission
- During the fission, it produces two or three neutrons which move away at high speed
- Each of these new neutrons can start another fission reaction, which again creates further **excess neutrons**
- This process is called a **chain reaction**

YOUR NOTES



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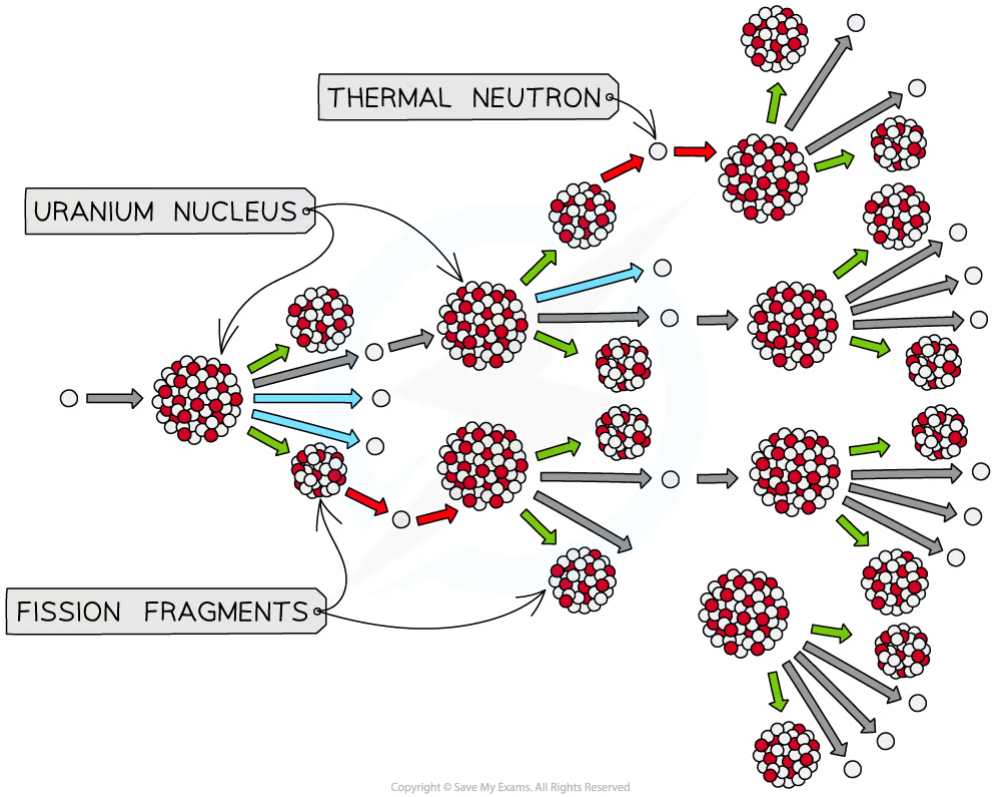
The neutrons released by each fission reaction can go on to create further fissions, like a chain that is linked several times – from each chain comes two more

- The products of fission are two daughter nuclei and **at least one neutron**
- The neutrons released during fission go on to cause more fission reactions leading to a **chain reaction**, where each fission goes on to cause at least one more fission

YOUR NOTES



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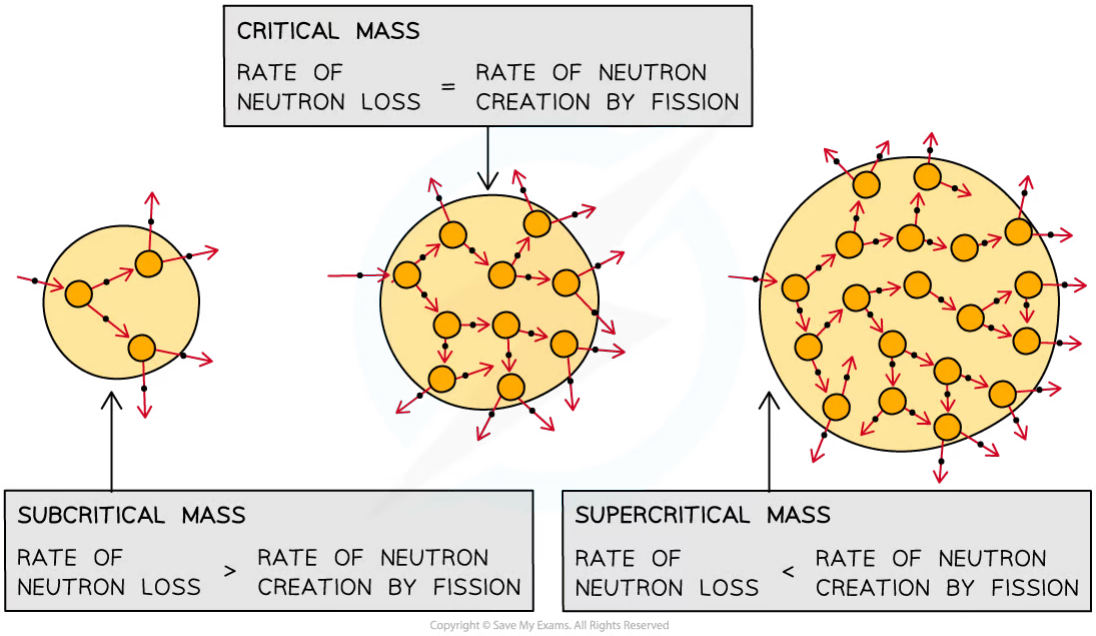
Only one thermal neutron is used to create another fission reaction in a controlled chain reaction

- Nuclear reactions are designed to be self-sustaining yet very controlled
- This can be achieved by using a precise amount of uranium fuel, known as the **critical mass**
- The critical mass is defined as:

  The minimum mass of fuel required to maintain a steady chain reaction

- Using exactly the critical mass of fuel will mean that a single fission reaction follows the last
  - Using **less** than the critical mass (**subcritical mass**) would lead the reaction to eventually stop
  - Using **more** than the critical mass (**supercritical mass**) would lead to a runaway reaction and eventually an explosion



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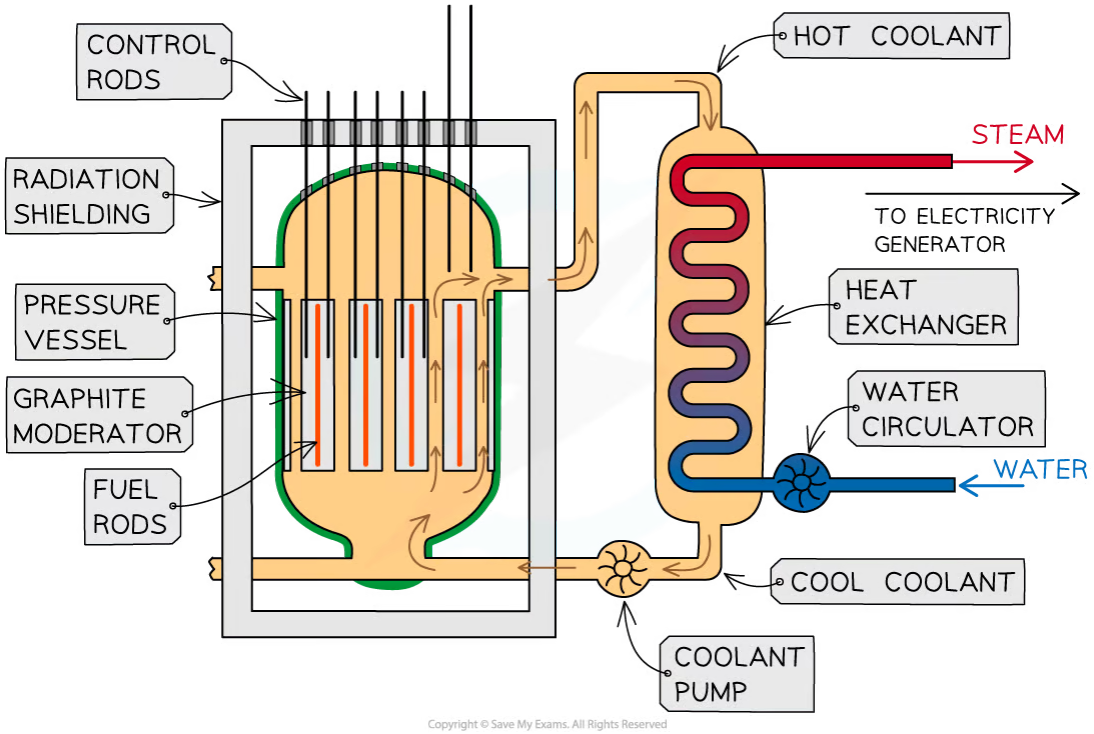
Subcritical, critical and supercritical mass

YOUR NOTES

Operation of a Nuclear Reactor

- In a nuclear reactor, a chain reaction is required to keep the reactor running
- When the reactor is producing energy at the required rate, two factors must be controlled:
  - The **number** of free neutrons in the reactor
  - The **energy** of the free neutrons
- The main components of a nuclear reactor are:
  - Control rods
  - Moderators
  - Heat exchangers
  - Shielding



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The overall purpose of a nuclear reactor is to collect the heat energy produced from nuclear reactions

Control Rods

Purpose of a control rod: To absorb neutrons

- Control rods are made of a material which absorbs neutrons without becoming dangerously unstable themselves
- The number of neutrons absorbed is controlled by varying the depth of the control rods in the fuel rods

YOUR NOTES

- Lowering the rods further **decreases** the rate of fission, as more neutrons are absorbed
- Raising the rods **increases** the rate of fission, as fewer neutrons are absorbed

- This is adjusted automatically so that exactly one fission neutron produced by each fission event goes on to cause another fission
- In the event the nuclear reactor needs to shut down, the control rods can be lowered all the way so no reaction can take place

Moderator

The purpose of a moderator: To slow down neutrons

- The moderator is a material that surrounds the **fuel rods** and **control rods** inside the reactor core
- The fast-moving neutrons produced by the fission reactions **slow down** by colliding with the molecules of the moderator, causing them to lose some momentum
- The neutrons are slowed down so that they are in **thermal equilibrium** with the moderator, hence the term 'thermal neutron'
  - This ensures neutrons can react efficiently with the uranium fuel

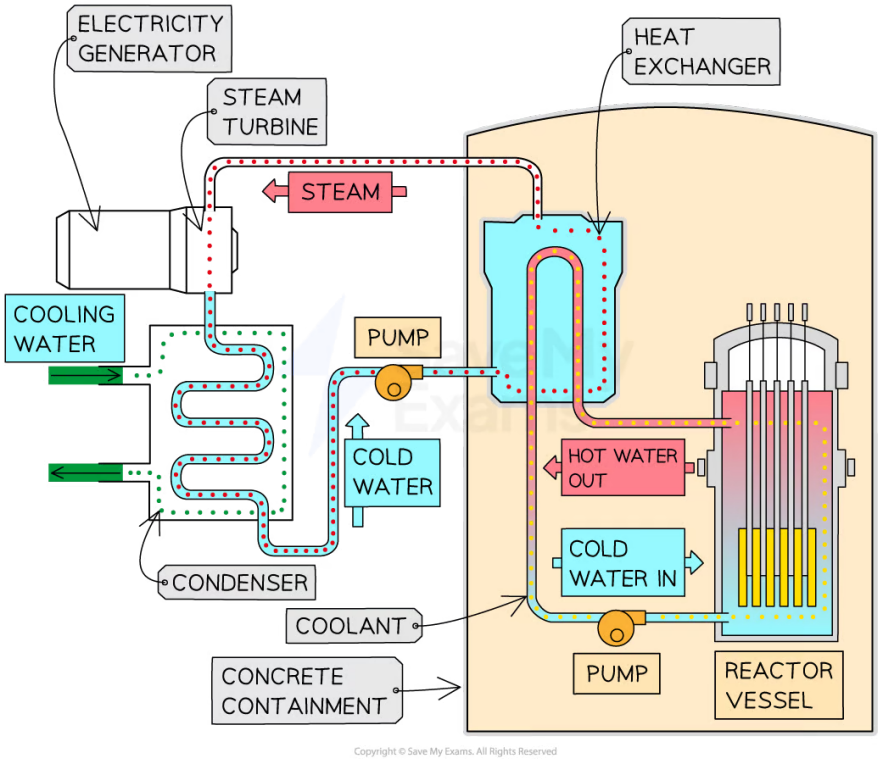
Heat exchangers

The purpose of a heat exchanger: To transfer thermal energy efficiently between the water systems of a nuclear power plant

- There are multiple water systems in a nuclear power plant that need regulating
  - The coolant (usually water) used in the reactor vessel
  - The water and steam that drives the turbine
  - The condenser that cools the steam

- The heat exchanger mediates the thermal energy exchanges between these water systems
- The **coolant** is a substance, such as water, that is pumped into the reactor at a cold temperature to extract the heat released by the fission reactions
- In the heat exchanger, the coolant transfers the heat to water that is pumped in externally to produce steam
- This steam then goes on to power electricity-generating turbines

YOUR NOTES

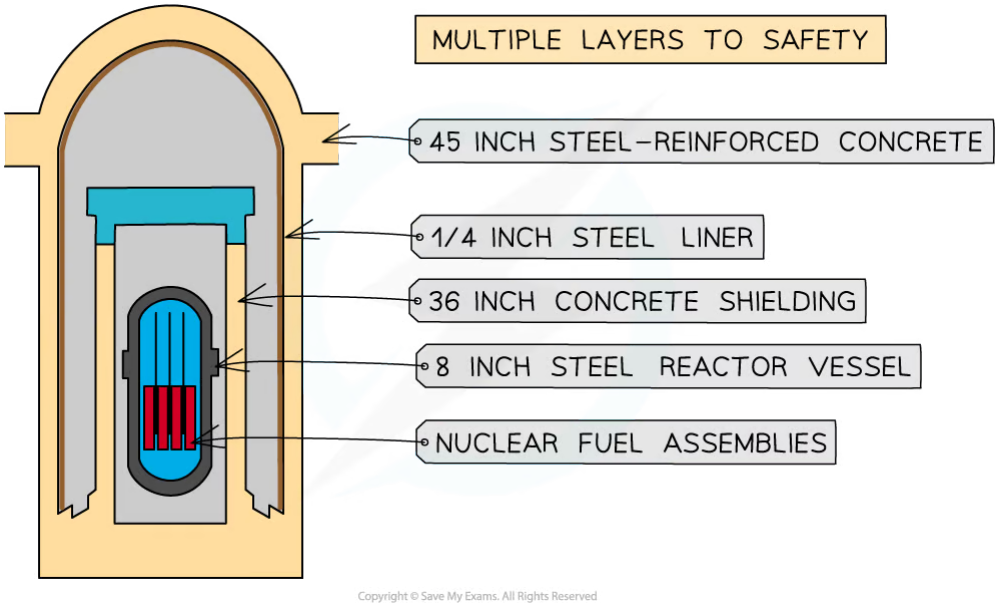


Water systems in a nuclear power plant are regulated by a heat exchanger

Shielding

Purpose: To house the reactor and absorb hazardous radiation

- The entire nuclear reactor is surrounded by **shielding** materials
- The daughter nuclei formed during fission, and the neutrons emitted, are radioactive
- The reactor is surrounded by a **steel** and **concrete** wall that can be nearly 2 metres thick
- This absorbs the emissions from the reactions
  - It ensures that the environment around the reactor is **safe**



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Shielding metals in a nuclear reactor

Safety Issues in Nuclear Power

Types of Radioactive Waste

- There are three main types of nuclear waste:
  - Low level
  - Intermediate level
  - High level

Low-level waste

- This is waste such as clothing, gloves and tools which may be lightly contaminated
- This type of waste will be radioactive for a few years, so must be encased in concrete and stored a few metres underground until it can be disposed of with regular waste

Intermediate-level waste

- This is everything between daily used items and the fuel rods themselves
- Usually, this is the waste produced when a nuclear power station is decommissioned and taken apart
- This waste will have a longer half-life than the low-level waste, so must be encased in cement in steel drums and stored securely underground

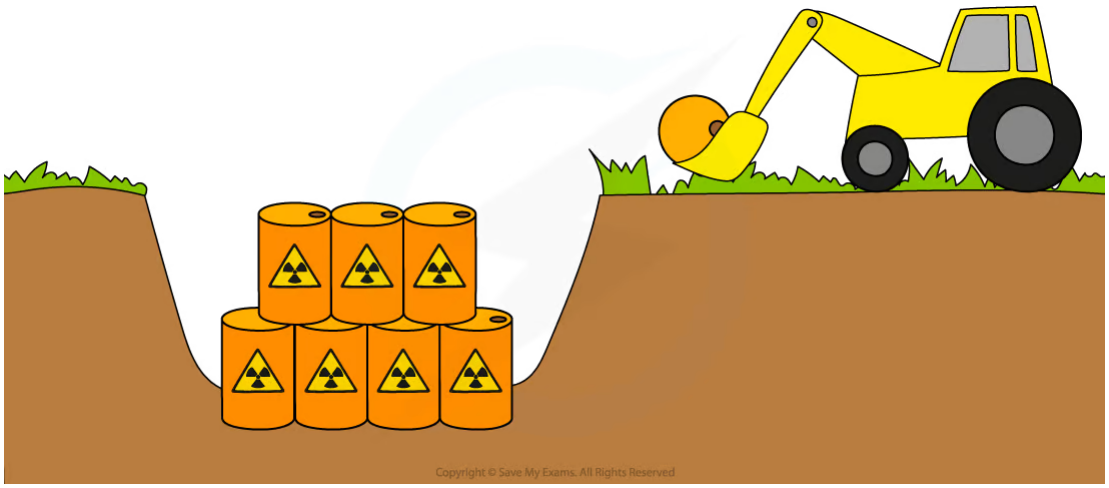
High-level waste

- High-level waste refers to the unusable fission products from the fission of uranium-235 or from spent fuel rods
- This is by far the **most dangerous** type of waste as it will remain radioactive for thousands of years
- As well as being highly radioactive, the spent fuel rods are **extremely hot** and must be handled and stored much more carefully than the other types of waste

- The **issues** with high-level waste are:
  - Within the fuel rods, nuclei of **uranium-238** quickly decay into nuclei of **plutonium-239**
  - Plutonium-239 is classified as **high-level radioactive waste**
  - This is because its nuclei are extremely radioactive and have a **very long half-life** of 24 000 years
  - This presents a long-term risk of **contamination**

- The **treatment** of high-level waste is as follows:
  - The waste is initially placed in cooling ponds of water close to the reactor for a number of years
  - Isotopes of plutonium and uranium are harvested to be used again
  - Waste is mixed with molten glass and made solid (this is known as **vitrification**)
  - Then it is encased in containers made from steel, lead, or concrete
  - This type of waste must be stored very **deep** underground

YOUR NOTES



Depending on the activity of radioactive waste, it is buried in different ways

Advantages & Disadvantages of Nuclear Power

Advantages of using nuclear power:

- **Climate change friendly:** Nuclear power stations produce **no greenhouse gases**
- **High energy density:** Uranium provides far **more energy per kg** compared to coal and other fossil fuels
- **Availability of fuel:** The reserves of fissionable materials are much higher compared to fossil fuel reserves
- **High reliability & safety:** Despite some serious incidents in the past, nuclear power is now regarded as one of the safest and most reliable processes for the production of electricity

Disadvantages of using nuclear power:

- **Hazardous waste products:** The production of radioactive waste is very dangerous and expensive to deal with and stays at hazardous levels of activity for a very long time (>1000s of years)
- **Potential for catastrophic accidents:** A nuclear meltdown, such as at Chornobyl, could have catastrophic consequences on the environment and for the people living in the surrounding area
- **Potential for misuse:** There is a danger of misuse of nuclear material and infrastructure in nuclear weapons and terrorist attacks
- **Dangers with mining fuel:** There are many issues associated with mining uranium, from the people handling it to the detrimental effects it can have on the environment

Environmental Considerations

- Isotopes with long half-lives must not enter our water and food supplies
- Burial locations must be geologically stable, secure from attack, and designed for safety

- Space for such locations is limited

YOUR NOTES

Safety Measures for Workers

- Several measures must be put in place to reduce the worker's exposure to radiation
- The fuel rods are **handled remotely** i.e. by machines
- The nuclear reactor is surrounded by a very thick lead or concrete **shielding** to limit exposure to radiation
- In an emergency, the control rods are fully lowered into the reactor core to stop fission reactions by absorbing all the free neutrons in the core, this is known as an **emergency shut-down**

Nuclear Energy in Society

- Nuclear power can scare people if they do not understand it
- It is dangerous if not handled properly, yet it is invisible which can be difficult for some people to comprehend
- However, with increased education on nuclear energy, society can use this knowledge to inform their own decisions and opinions

IB Physics HL

YOUR NOTES

Fusion & Stars

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Fusion Reactions in Stars

Energy Released in Fusion Reactions

Star Formation

Life Cycle of a Star

The Hertzsprung–Russell (HR) Diagram

Emission & Absorption Spectra in Stars

Stellar Parallax

Determination of Stellar Radii

Fusion Reactions in Stars

- In the centre of a stable star, hydrogen atoms undergo **nuclear fusion** to form helium
- The equation for this reaction is:

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n$$

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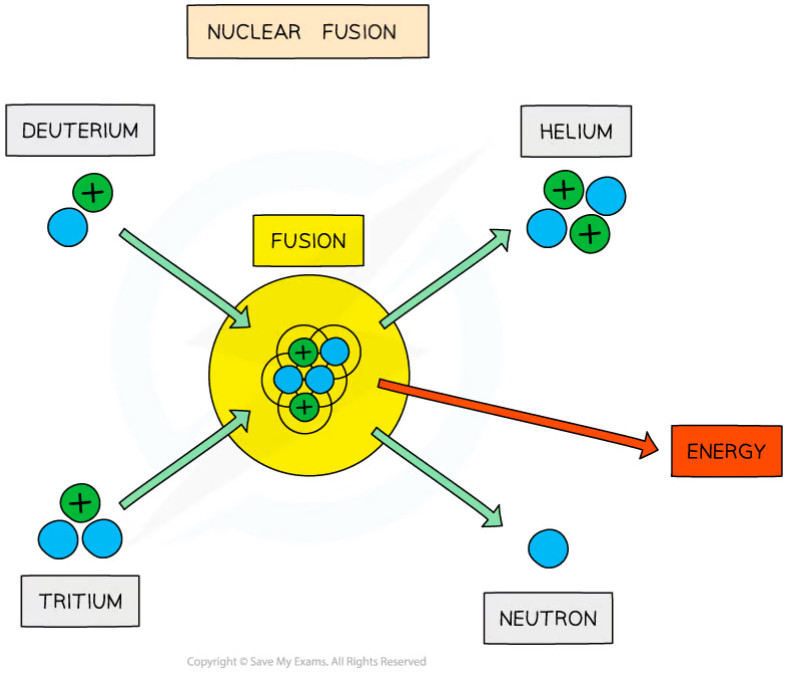
Deuterium and tritium are both isotopes of hydrogen. They can be formed through other fusion reactions in the star

- Fusion is defined as:

The joining of two small nuclei to produce a larger nucleus

- Low-mass nuclei (such as hydrogen and helium) can undergo fusion and release energy
- A **huge** amount of energy is released in the reaction
- This provides a pressure that prevents the star from collapsing under its gravity



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The fusion of deuterium and tritium to form helium with the release of energy

- For two nuclei to fuse, both nuclei must have high kinetic energy
  - This is because the protons inside the nuclei are positively charged, which means that they repel one another
- It takes a great deal of energy to overcome the electrostatic force between protons
  - This is why fusion reactions can only be achieved in an extremely high-energy environment, such as a star's core

- When two protons fuse, the element deuterium is produced
- In the centre of stars, the deuterium combines with a tritium nucleus to form a helium nucleus, plus the release of energy, which provides fuel for the star to continue burning

YOUR NOTES

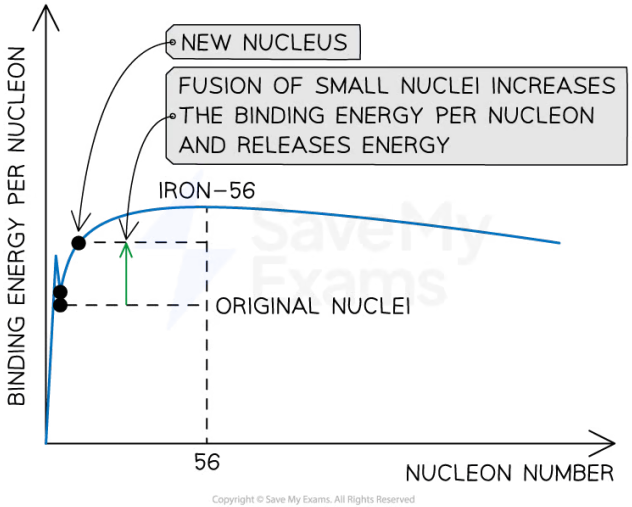
Exam Tip

In the fusion process, the mass of the new heavier nucleus is less than the mass of the constituent parts of the nuclei fused together, as some mass is converted into energy.

Not all of this energy is used as binding energy for the new larger nucleus, so energy will be released from this reaction. The binding energy per nucleon afterwards is higher than at the start.

Energy Released in Fusion Reactions

- When two small nuclei undergo a fusion reaction, the single larger nucleus produced as a result will have a **higher binding energy per nucleon** than the original two nuclei
- As a result of the mass defect between the parent nuclei and the daughter nucleus, **energy is released**



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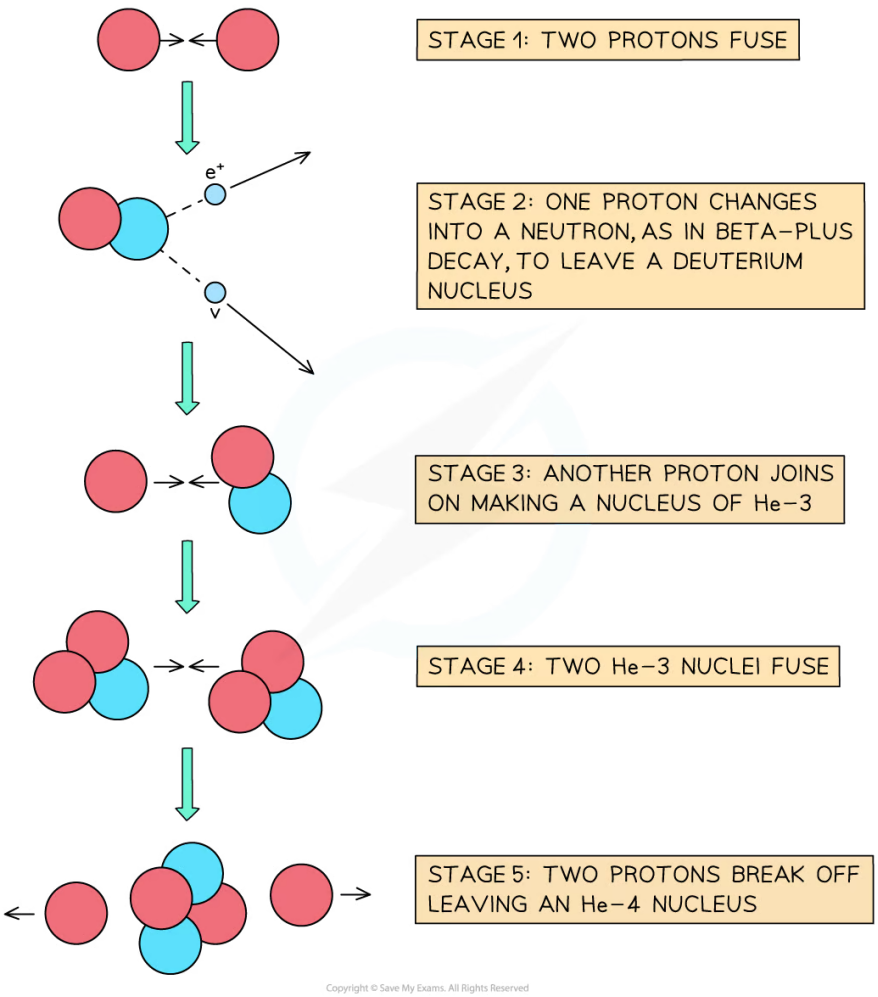
The energy released from fusion reactions is due to the mass defect between parent and daughter nuclei

- When two protons fuse, the element deuterium is produced
- In the centre of stars, the deuterium combines with a tritium nucleus to form a helium nucleus, plus the release of energy, which provides fuel for the star to continue burning

YOUR NOTES



The five-stage process of fusion in stars

YOUR NOTES

Worked Example

In the Sun, fusion occurs via a process known as the proton-proton cycle.

It is predicted that 80% of the total power output of the Sun is produced through the following cycle:

$$\left.\begin{aligned} {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} &\rightarrow {}^{2}_{1}\mathrm{H} + {}^{0}_{1}\mathrm{e}^{+} + \nu_{e} \\ {}^{1}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} &\rightarrow {}^{2}_{1}\mathrm{He} + \gamma \\ {}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} &\rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \end{aligned}\right\} 4\,{}^{1}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{0}_{1}\mathrm{e}^{+} + 2\nu_{e} \text{ (overall reaction)}$$

nucleus	rest mass / u
hydrogen-1	1.007825
helium-4	4.002603

The neutrinos produced in the first step carry away 2% of the energy released by the process.

Determine the mass of hydrogen-1 that must be fused each second to produce this output.

Luminosity of the Sun = 3.85×10^{26} W.

Answer:

Step 1: Determine the energy released per overall fusion reaction

- In the overall reaction, 4 hydrogen-1 nuclei fuse into a helium-4 nucleus, so the mass defect is

$$\text{mass defect: } \Delta m = 4(1.007825\text{u}) - 4.002603\text{u} = 0.028697\text{u}$$

- Where atomic mass unit, $u = 1.66 \times 10^{-27}$ kg
- Using mass-energy equivalence, the energy released by one reaction is

$$\Delta E = \Delta m c^2 = 0.028697 \times (1.66 \times 10^{-27}) \times (3 \times 10^{8})^2$$

$$\text{energy released: } \Delta E = 4.287 \times 10^{-12}\text{ J}$$

Step 2: Determine the energy released minus the energy that is carried away by neutrinos

- Per reaction, neutrinos carry away 2% of 4.287×10^{-12} J, so 98% of the energy contributes to the luminosity of the Sun

$$\text{energy released: } \Delta E = 0.98 \times (4.287 \times 10^{-12}) = 4.201 \times 10^{-12}\text{ J}$$

Step 3: Determine the number of fusion reactions that happen each second

- This process accounts for 80% of the luminosity of the Sun,
- So, the total power output of the reaction = $0.8 \times (3.85 \times 10^{26})$ W
- The number of fusion reactions each second is:

$$\text{number of reactions} = \frac{power\ output}{energy\ released\ per\ reaction} = \frac{0.8 \times (3.85 \times 10^{26})}{4.201 \times 10^{-12}}$$

$$\text{number of reactions} = 7.332 \times 10^{37}\ s^{-1}$$

Step 4: Determine the mass of hydrogen that fuses each second

- Every reaction fuses 4 hydrogen-1 nuclei, so the mass per reaction is 4 × 1.007825u
- The mass of hydrogen-1 that fuses each second in this process is

$$= 4u \times \text{number of reactions}$$

$$\text{mass of hydrogen-1} = 4 \times 1.007825 \times (1.66 \times 10^{-27}) \times (7.332 \times 10^{37})$$

$$\text{mass of hydrogen-1} = \mathbf{4.91 \times 10^{11}\ kg\ s^{-1}}$$

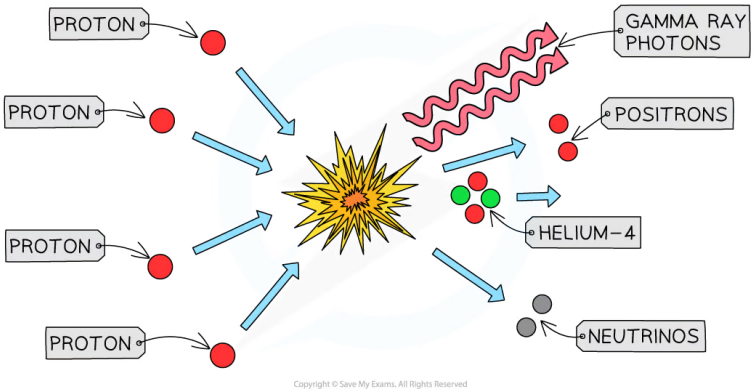
YOUR NOTES

YOUR NOTES

Conditions for Fusion

- For nuclear fusion to occur, both nuclei must have sufficiently **high kinetic energy** to overcome the electrostatic repulsion between protons
- The conditions required to achieve this are:
  - **Very high temperature** (on the scale of 100 million Kelvin)
  - **Very high pressure and density**

- Four hydrogen nuclei (protons) are fused into one helium nucleus, producing two gamma-ray photons, two neutrinos and two positrons
  - Massive amounts of energy are released
  - The momentum of the gamma-ray photons results in an outward acting pressure called radiation pressure

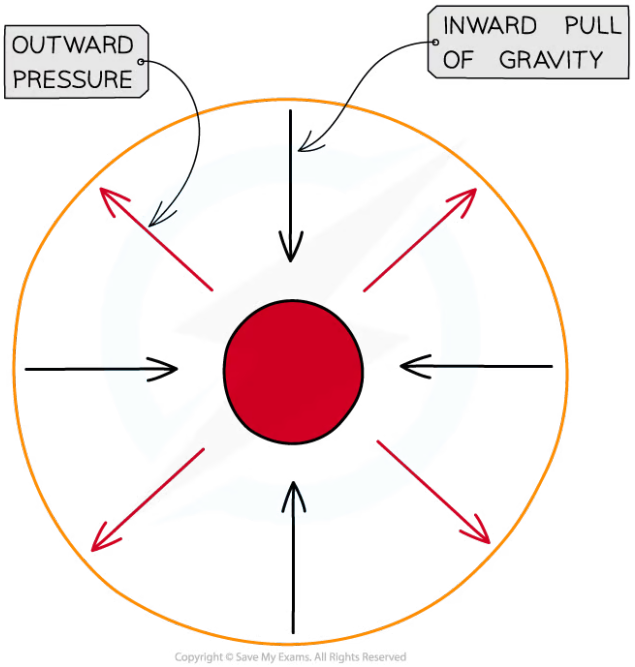


Nuclear fusion of hydrogen nuclei to form helium nuclei

Equilibrium in Stars

YOUR NOTES

- Once the core temperature of a star reaches millions of degrees kelvin and the fusion of hydrogen nuclei to helium nuclei begins
  - The protostar's gravitational field continues to attract more gas and dust, increasing the temperature and pressure of the core
  - With more frequent collisions, the kinetic energy of the particles increases, increasing the probability that fusion will occur
  - Eventually, when the core becomes **hot** enough and fusion reactions can occur, they will begin to produce an **outward radiation pressure** which balances the inward pull of gravity
- The star reaches a **stable state** where the inward and outward **forces** are in **equilibrium**
  - As the temperature of the star increases and its volume decreases due to gravitational collapse, the gas pressure increases
  - The gas pressure and the radiation pressure act **outwards** to balance the gravitational force (weight, $F = mg$) acting **inwards**



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Equilibrium in stars occurs when the outward radiation pressure is balanced with the inward gravitational force

- If the **temperature** of a star **increases**, the **outward pressure** will also **increase**
  - If outward pressure > gravitational force, the star will **expand**
- If the **temperature drops** the **outward pressure** will also **decrease**
  - If outward pressure < gravitational force, the star will **contract**
- As long as these two forces are **balanced**, the star will remain **stable**

YOUR NOTES

The Life Cycle of Stars

- The life cycle of a star follows predictable stages
- The exact route a star's development takes depends on its initial mass

Initial Stages for All Masses

- The first four stages in the life cycle of stars are the same for stars of all masses
- After these stages, the life-cycle branches depending on whether the star is:
  - **Low mass**: stars with a **core mass** of **less** than about 1.4 times the mass of the Sun (< 1.4 M_{Sun})
  - **High mass**: stars with a **core mass** of **more** than about 1.4 times the mass of the Sun (> 1.4 M_{Sun})

1. Nebula

- All stars form from a giant cloud of **hydrogen gas** and **dust** called a **nebula**
  - **Gravitational attraction** between individual atoms forms denser clumps of matter
  - This inward movement of matter is called gravitational collapse

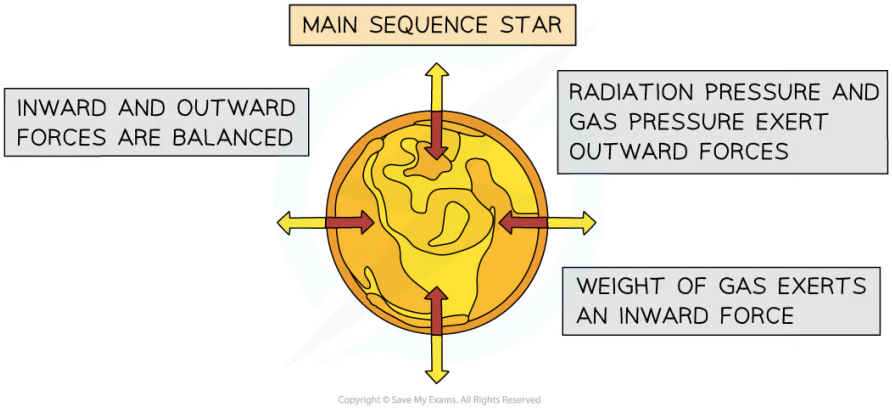
2. Protostar

- The gravitational collapse causes the gas to heat up and glow, forming a **protostar**
  - Work done on the particles of gas and dust by collisions between the particles causes an increase in their kinetic energy, resulting in an increase in **temperature**
  - Protostars can be detected by telescopes that can observe **infrared radiation**
- Eventually the temperature will reach millions of **degrees Kelvin** and the fusion of hydrogen nuclei to helium nuclei begins
  - The protostar's gravitational field continues to attract more gas and dust, increasing the temperature and pressure of the core
  - With more frequent collisions, the kinetic energy of the particles increases, increasing the probability that fusion will occur

3. Main Sequence Star

- The star reaches a **stable state** where the inward and outward **forces** are in **equilibrium**
  - As the temperature of the star increases and its volume decreases due to gravitational collapse, the **gas pressure** increases

YOUR NOTES



Forces acting within a main sequence star. The balanced inward and outward forces will remain that way for millions, or even billions of years

- A star will spend most of its life on the **main sequence**
  - 90% of stars are on the main sequence
  - Main sequence stars can vary in mass from ~10% of the mass of the Sun to 200 times the mass of the Sun
  - The Sun has been on the main sequence for 4.6 billion years and will remain there for an estimated 6.5 billion years

Next Stages for Low Mass Stars

- The fate of a star beyond the main sequence depends on its **mass**
  - The cut-off point is **1.4 times the mass of the Sun**
  - A star is classed as a low-mass star if it has a mass less than 1.4 times the mass of the Sun
  - A low-mass star will become a **red giant** before turning into a **white dwarf**

4. Red Giant

- Hydrogen fuelling the star begins to run out
  - Most of the hydrogen nuclei in the core of the star have been fused into helium
  - Nuclear fusion **slows**
  - Energy released by fusion decreases
- The star initially shrinks and then swells and cools to form a **red giant**
- Fusion continues in the **shell** around the core

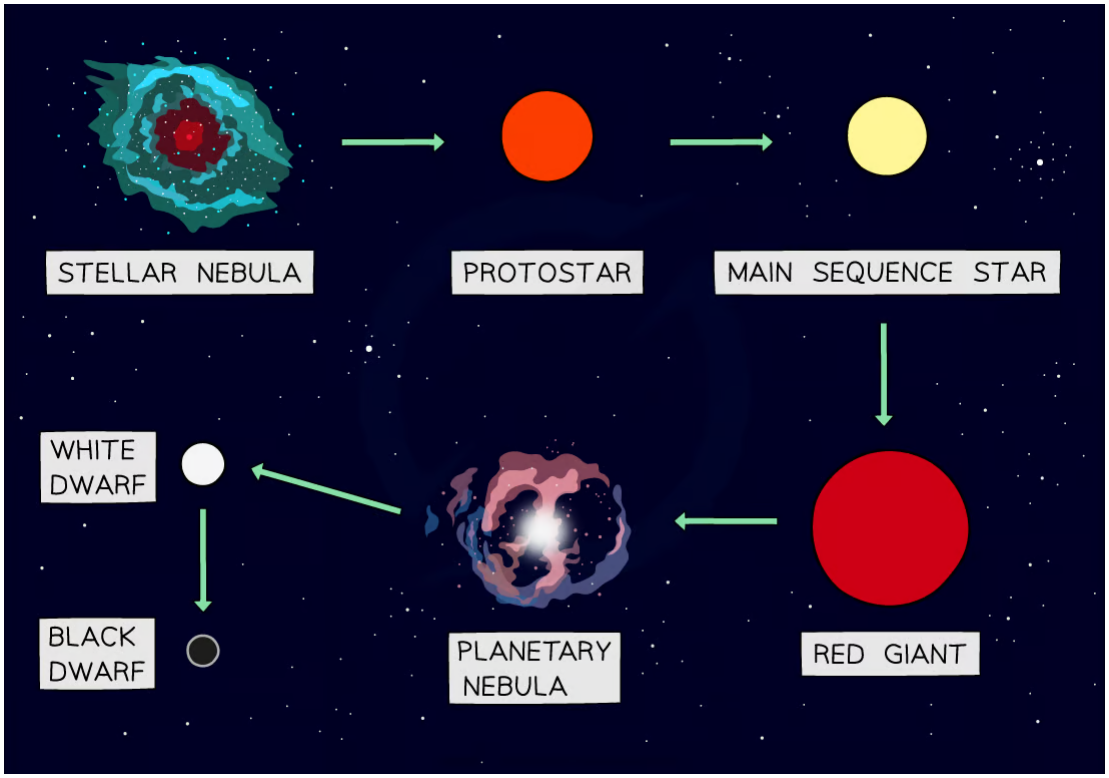
5. Planetary Nebula

- The **outer layers** of the star are **released**
  - Core helium burning releases massive amounts of energy in the fusion reactions

6. White Dwarf

YOUR NOTES

- The solid **core collapses** under its own mass, leaving a very hot, dense core called a **white dwarf**



The lifecycle of a low mass star

Next Stages for Massive Stars

4. Red Super Giant

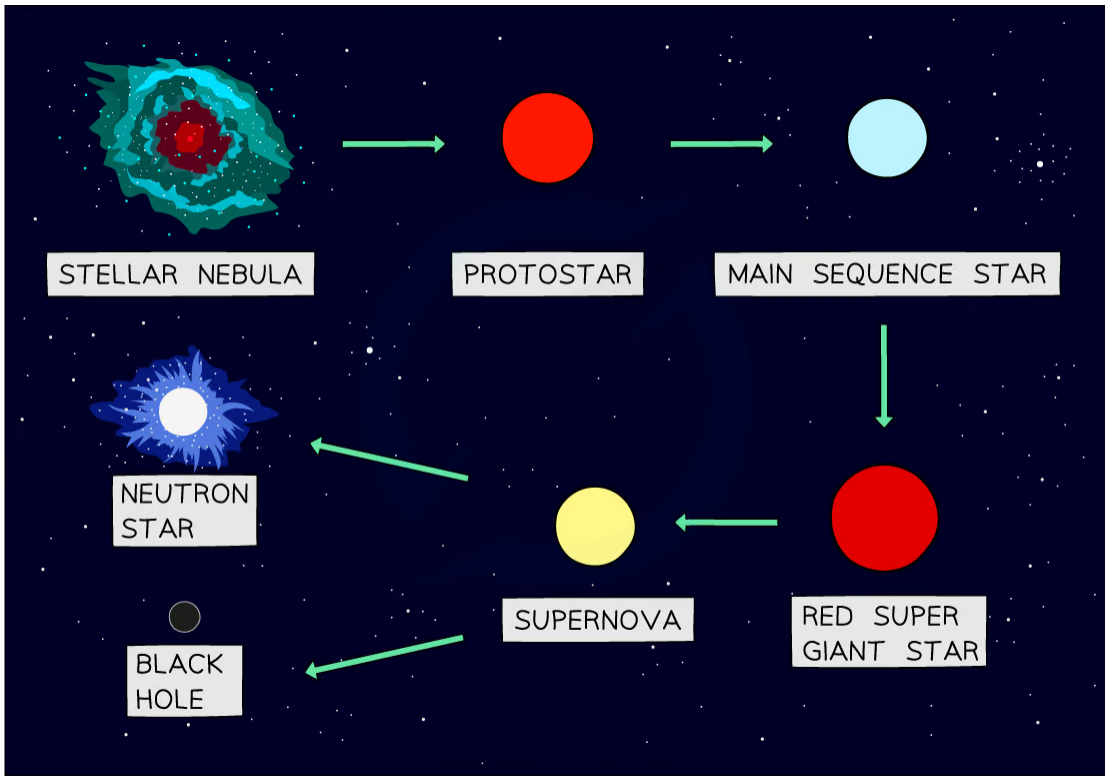
- The star follows the same process as the formation of a red giant
  - The **shell burning** and **core burning** cycle in massive stars goes beyond that of low-mass stars, fusing elements up to **iron**

5. Supernova

- The iron core collapses
- The outer shell is blown out in an explosive **supernova**

6. Neutron Star (or Black Hole)

- After the supernova explosion, the collapsed **neutron core** can remain intact having formed a **neutron star**
  - If the neutron core mass is greater than 3 times the solar mass, the pressure on the core becomes so great that the core collapses and produces a **black hole**



Lifecycle of massive stars

YOUR NOTES

YOUR NOTES

Worked Example

Stars less massive than our Sun will leave the main sequence and become red giants.

Describe and explain the next stages of evolution for such stars.

Answer:

Step 1: Underline the command words

- **Describe** questions require details of the processes occurring
- **Explain** questions require details of how and why those processes occur
  - This question requires **both**

Step 2: Identify the mass of the star

- The stars in the question are less massive than the Sun, therefore it is referring to **low-mass stars**

Step 3: Identify the stage of life cycle being asked about

- The question asks for the **next** stage of evolution after **becoming** a red giant
- It requires an explanation of the processes **during** the red giant phase

Step 4: Plan the answer

- Make a list of the remaining stages in the evolution of a low-mass star
  - Red giant
  - Planetary nebula
  - White dwarf
- Add to the list any important points or key words that need to be included in the answer
- Red giant
  - Fuel runs out
  - Forces no longer balanced
  - Expands and cools
  - Fusion continues in shell
- Planetary nebula
  - Carbon-oxygen core not hot enough for further fusion
  - Outer layers released
- White dwarf
  - Core collapses leaving remnant core

Step 5: Begin writing the answer using words from the question stem

- Low-mass stars will leave the main sequence and become red giants...

Step 6: Use the plan to keep the answer concise and logically sequenced

- Low-mass stars leave the main sequence and become red giants when the hydrogen in the core runs out.
- Reduced energy released by fusion leads to radiation pressure decreasing
  - Radiation pressure and gas pressure no longer balance the gravitational pressure and the core collapses
  - Fusion no longer takes place inside the core
- The outer layers expand and cool to form a red giant
- Temperatures generated by the collapsing core are high enough for fusion to occur in the shell around the core
  - Contraction of the core produces temperatures great enough for the fusion of helium into carbon and oxygen inside the core
  - The carbon-oxygen core is not hot enough for further fusion, so the core collapses
- The outer layers are ejected forming a planetary nebula
- The remnant core remains intact leaving a hot, dense, solid core called a white dwarf

YOUR NOTES

YOUR NOTES

Worked Example

Describe the evolution of a star much more massive than our Sun from its formation to its eventual death.

Answer:

Step 1: Underline the command word

- Underline the command word 'describe'
- A **describe** question does **not** need you to **explain why** the processes happen, but you do need to go into detail about what happens in each stage

Step 2: Identify the mass of the star

- The star in the question is more massive than the Sun, therefore it is referring to **high-mass stars**

Step 3: Identify the stage of life cycle being asked about

- This question is about the whole life cycle from formation to death
- The common first five steps are needed, and then the steps which only apply to massive stars

Step 4: Plan the answer

- Use the white space around the question to plan your answer
- List the stages that a massive star goes through, this will help you form your answer in a logical sequence of events
  - Nebula
  - Protostar
  - Nuclear fusion
  - Main sequence
  - Red super giant
  - Supernova
  - Neutron star/black hole

Step 5: Add to the list any important points or key words that need to be included for each stage

- Nebula – gravitational collapse
- Protostar – heats up and glows
- Nuclear fusion – H to He generates energy
- Main sequence – stable, forces balanced
- Red supergiant – expands and cools
- Supernova – core collapses
- Neutron star/black hole – remnants

Step 6: Begin writing the answer using words from the question stem to begin

- A star more massive than our Sun will form from...

Step 7: Use the plan to keep the answer concise and logically sequenced

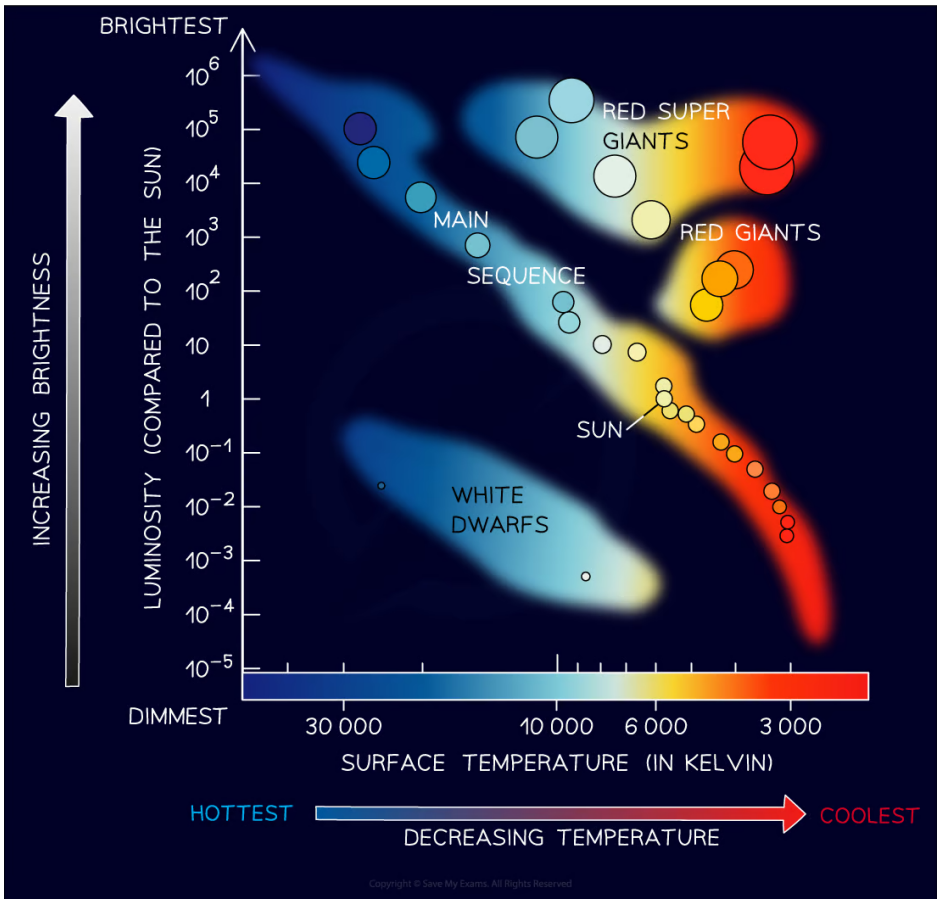
- A star more massive than our Sun will form from clouds of gas and dust called a nebula
- The gravitational collapse of matter increases the temperature of the cloud causing it to glow - this is a protostar
- Nuclear fusion of hydrogen nuclei to helium nuclei generates massive amounts of energy
- The outward radiation pressure and gas pressure balances the inward gravitational pressure and the star become stable entering the main sequence stage
- When the hydrogen runs out, the outer layers of the star expand and cool forming a red supergiant
- Eventually, the core collapses and the star explodes in a supernova
- The remnant core either remains intact forming a neutron star, or the core collapses further resulting in a black hole

YOUR NOTES

YOUR NOTES

The Hertzsprung–Russell (HR) Diagram

- Danish astronomer Ejnar Hertzsprung, and American astronomer Henry Noris Russell, independently plotted the luminosity of different stars against their **temperature**
  - **Luminosity**, relative to the Sun, on the y-axis, goes from dim (at the bottom) to bright (at the top)
  - **Temperature**, in degrees Kelvin, on the x-axis, goes from **hot** (on the left) **to cool** (on the right)



The Hertzsprung-Russell Diagram depicts the luminosity of stars against their temperature

- Hertzsprung and Russel found that the stars clustered in **distinct areas**
- Most stars are clustered in a band called the **main sequence**
  - For main sequence stars, luminosity **increases** with surface temperature
- A smaller number of stars clustered above the main sequence in two areas, **red giants,** and **red supergiants**
  - These stars show an increase in luminosity at **cooler** temperatures
  - The only explanation for this is that these stars are much **larger** than main sequence stars

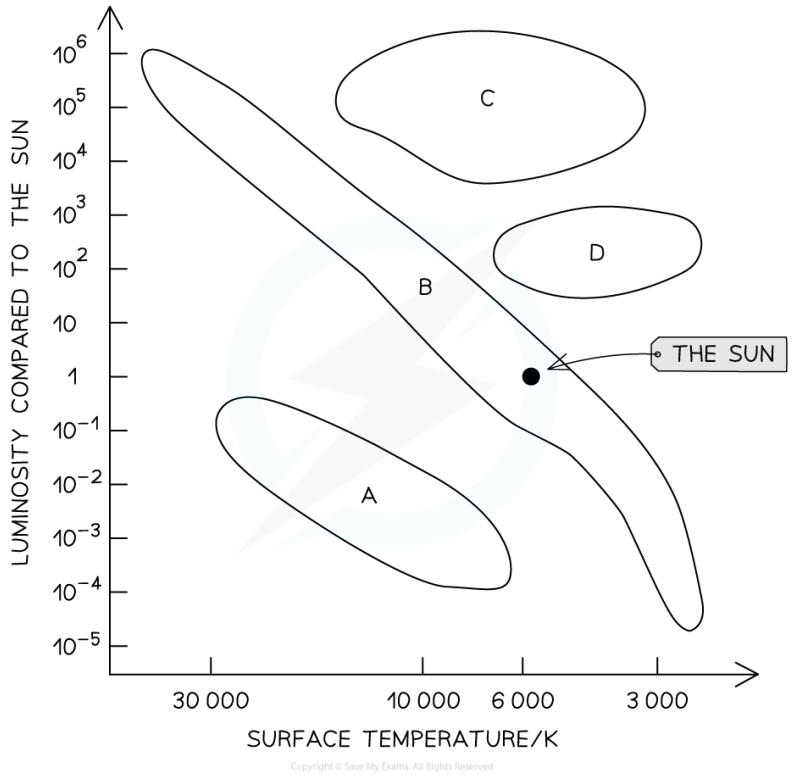
- Below and to the left of the main sequence are the **white dwarf stars**
  - These stars are **hot**, but not very luminous
  - Therefore, they must be much **smaller** than main sequence stars
- The Hertzsprung-Russell Diagram only shows stars that are in **stable phases**
  - Transitory phases happen quickly in relation to the lifetime of a star
  - Black holes cannot be seen since they emit no light

YOUR NOTES

YOUR NOTES

Worked Example

Stars can be classified using the Hertzsprung-Russell (H-R) Diagram.



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(a)
State the types of stars found in areas A, B, C and D
(b)
On the H-R diagram, plot the star with a surface temperature of 20 000 K and a luminosity 10 000 times greater than the Sun and label it Star X.

Answer:

(a)

Step 1: Identify the main sequence on the HR diagram

- The main sequence is the easiest to recognise as it is the long band diagonally central to the diagram where the majority of stars are found
- The main sequence is region **B**

Step 2: Identify the white dwarf region on the HR diagram

- White dwarf stars are hot, but not very luminous
- Identify the area with a **lower luminosity** than the main sequence
- The white dwarf region is area **A**

Step 3: Identify the red giant and red supergiant regions on the HR diagram

YOUR NOTES

- Red giants and super red giants have a greater luminosity than main sequence stars at a lower temperature
- That means that they are bigger than main sequence stars
- The bigger they are, the more luminous they are
- So, the super red giants are **more luminous** than the red giants and will appear **above** them on the graph
- The super red giant region is area **C**
- The red giant region is area **D**

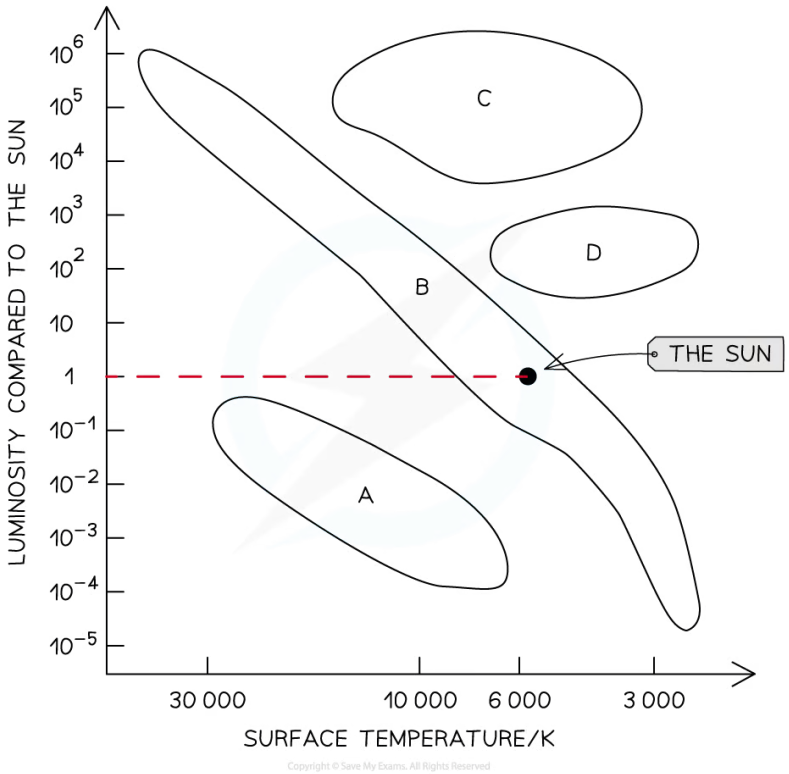
(b)

Step 1: List the known quantities

- Surface temperature of Star **X** = 20 000 K
- Luminosity of Star **X** = 10 000 times that of the Sun

Step 2: Use the graph to find the value for the luminosity of the Sun

- Use a ruler and pencil to draw a line from the position of the sun to the luminosity axis (y-axis)
- The Sun's luminosity on this scale is 1 because the luminosities given are relative to the luminosity of the sun



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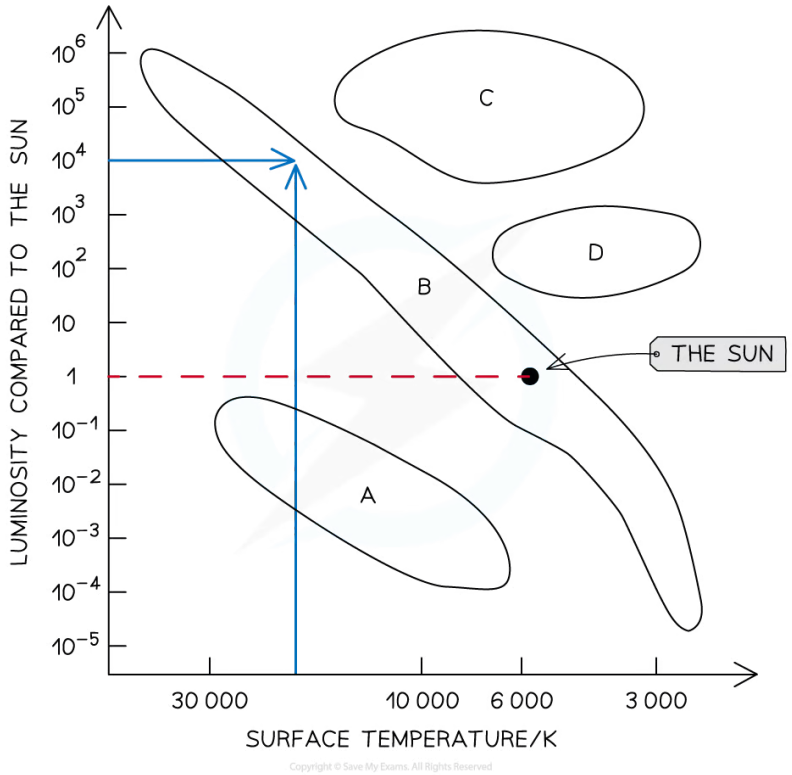
Step 3: Calculate the luminosity of Star X

- Star **X** is 10 000 times that of the Sun
- The luminosity of the Sun is 1

$10\,000 \times 1 = 10\,000$ or 10^4

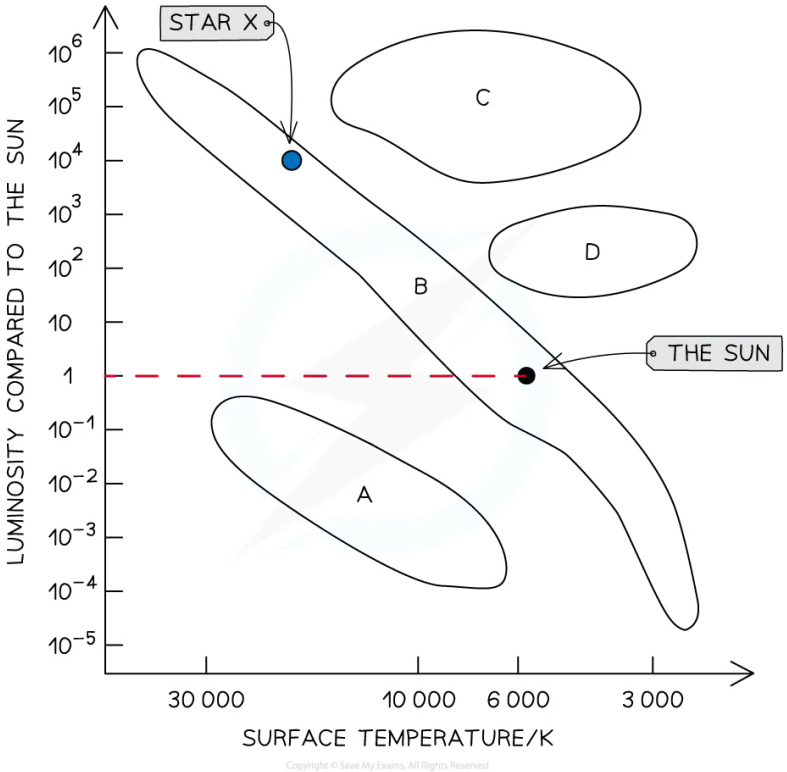
Step 4: Plot the position of Star X on the HR diagram

- Locate the surface temperature of Star **X** at 20 000 K
- Locate the luminosity of Star **X** at 10^4



- Plot the point and label it Star X

YOUR NOTES



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Exam Tip

You need to be able to identify the distinct areas of the Hertzsprung-Russell diagram out of context like in this exam question

Emission & Absorption Spectra in Stars

Types of Spectra

- There are three types of light spectra:
  - Continuous emission spectra
  - Emission line spectra
  - Absorption line spectra

Continuous Spectra

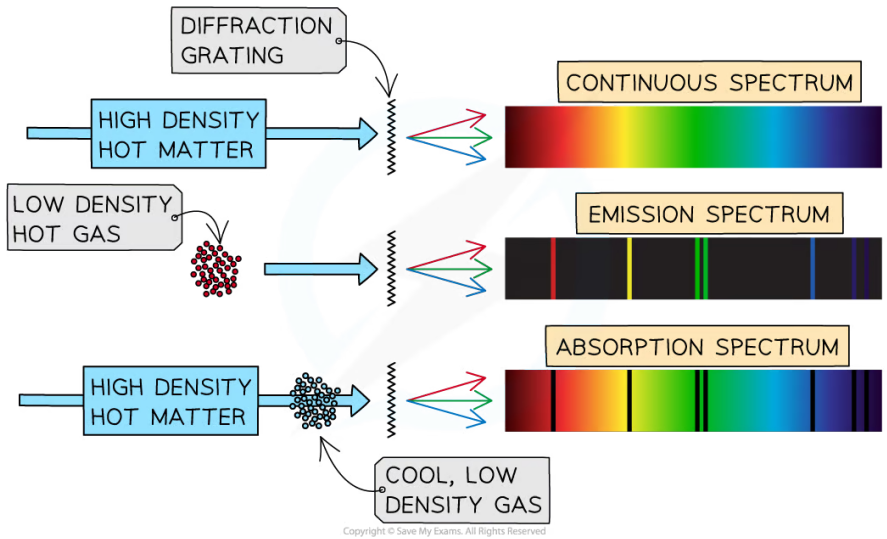
- In a **continuous spectrum**, photons emitted from the core of a star contain all the wavelengths and frequencies of the electromagnetic spectrum
- Continuous spectra are produced from **hot, dense sources**, such as the cores of stars

Emission Spectra

- When an electron transitions from a higher energy level to a lower energy level, this results in the **emission** of a photon
- Each transition corresponds to a different wavelength of light and this corresponds to a line in the spectrum
- The resulting emission spectrum contains a set of discrete wavelengths represented by **coloured lines** on a **black** background
- Emission line spectra are produced by **hot, low-pressure gases**

Absorption Spectra

- An atom can be raised to an excited state by the absorption of a photon
- Absorption spectra are observed when white light passes through a **cool, low-pressure gas**
- Some wavelengths appear to be missing in an absorption spectrum which correspond to the lines in the emission spectra of the same element



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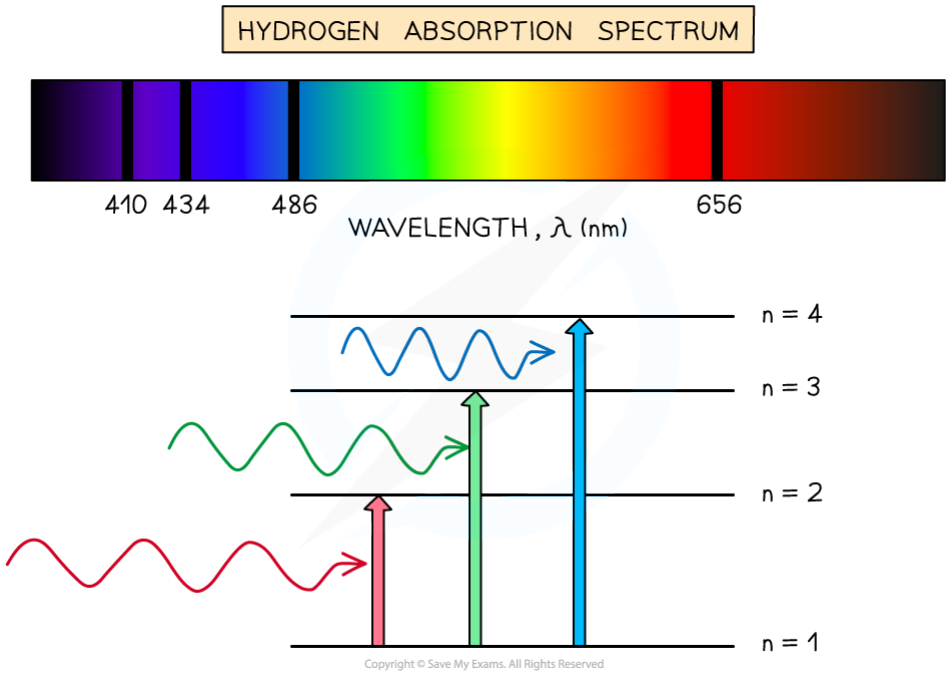
An absorption spectrum is the combination of an emission spectrum on top of a continuous spectrum

YOUR NOTES

Chemical Composition of Stars

- Stellar spectral lines are caused by the interactions between **photons** and the **atoms** present in **gaseous layers** of stars
- Photons produced by fusion reactions in a star's core move towards the layers of gas in the outer atmosphere of the star
  - The photons produced in the core form a **continuous spectrum**
  - Photons are absorbed by the gas atoms, which excite and re-emit other photons of various frequencies in random directions
- The light from a star can be analysed using **spectroscopy**
  - The atmospheres of stars are not hot enough to produce an emission line spectrum
  - Therefore, stars are found to emit an **absorption line spectrum**



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An absorption line will appear in a spectrum if an absorbing material is placed between a source and the observer

- An absorption line spectrum is the equivalent of an emission line spectrum but it is made of **dark lines** on top of a **continuous spectrum**
  - The dark lines represent the frequencies or wavelengths that are absorbed by a medium, such as a gas, when light passes through it
  - The absorption spectral lines represent the **energy** that has been **absorbed by electrons** in the outer atmosphere of the star
- Each gas produces a **unique pattern** of spectral lines due to the specific transition between the element's energy levels
  - The presence of spectral lines in a star's absorption spectrum act as **fingerprints**

YOUR NOTES

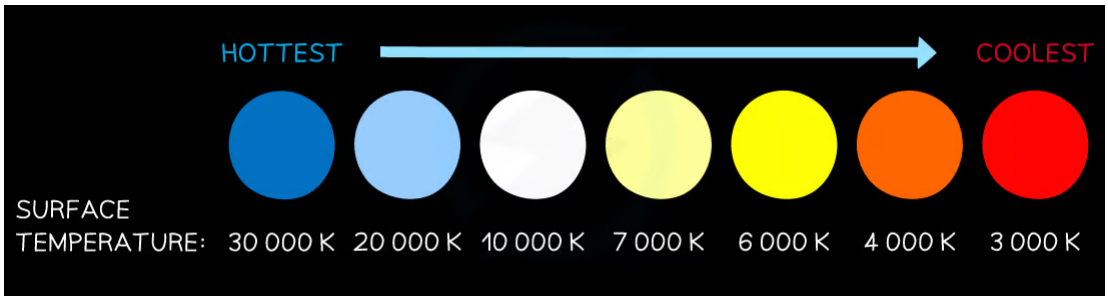
- They can be used to determine the presence of a **certain element** within the star

- The **chemical composition** of a star can be investigated even when extremely distant
  - If the element is present in the star, its characteristic pattern of spectral lines will appear as dark lines in the absorption line spectrum of the star
- The Sun is predominantly made up of **hydrogen** and **helium** gas
  - The chemical composition of the Sun can be verified using the **emission line spectra** of the two gases compared with the **absorption line spectrum** of the Sun

- For example, the hydrogen emission line spectrum includes lines at:
  - 2 nm, 486.3 nm and 656.5 nm
- While helium spectrum includes lines at:
  - 7 nm and 587.7 nm
- The same wavelengths can be seen as dark lines on top of the Sun's continuous spectrum

Surface Temperature of Stars

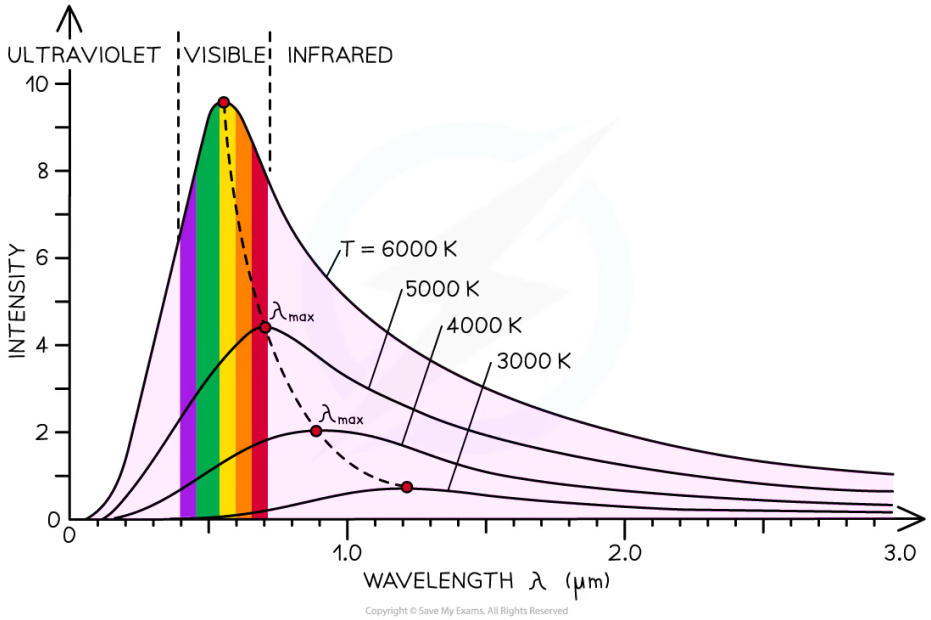
- The spectra of stars consist of a wide distribution of wavelengths
- Each wavelength of radiation has a different intensity
- The peak wavelength refers to the wavelength with the highest intensity



The colour of a star correlates to its temperature

- Stars are the closest approximation to black-body radiators that exist
- Therefore, the colour of a star i.e. its peak emission wavelength, can be attributed to its temperature according to Wien's law, where:
  - A shorter peak wavelength corresponds to a higher temperature at the peak intensity, so **hotter** stars tend to be **white or blue**
  - A longer peak wavelength corresponds to a lower temperature at the peak intensity, so **cooler** stars tend to be **red or yellow**

YOUR NOTES



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The intensity-wavelength graph shows how thermodynamic temperature links to the peak wavelength for four different stars

YOUR NOTES

Worked Example

Explain why:

a)
Hot, dense sources produce continuous spectra
b)
Hot, low pressure gases produce emission spectra
c)
Hot, dense sources observed through cool, low pressure gases produce absorption spectra

Answer:

(a) Hot, dense sources, such as the cores of stars, produce continuous spectra because:

- In a hot, dense material, the atoms or molecules are so close together that they interact with one another
- This leads to a spread of energy states that are not clearly defined
- Therefore, photons of all frequencies are emitted leading to an uninterrupted band of colour

(b) Hot, low pressure gases produce emission line spectra, because:

- Hot gases produce emission line spectra when photons are emitted due to the transition of electrons
  between discrete energy levels in atoms of the gas
- The line spectrum has certain, fixed frequencies related to the differences in energy between the various energy levels of the atoms of the gas
- In a low pressure gas, the atoms or molecules are not close together
- This means the energy levels of the gas atoms or molecules are clearly quantised and well-defined
- Therefore, only photons which correspond to the differences in energy between the energy levels of a bound electron are seen

(c) Hot, dense sources observed through cold gases produce absorption spectra because:

- Atoms of different elements in the cold gas absorb energy emitted from the hot source but only at particular energy values
- These particular energy values correspond to the differences in energy between the energy levels of a bound electron
- This means that particular frequencies of light are absorbed, creating black lines in the continuous emission spectrum

Exam Tip

Given an absorption line spectrum for a specific star, you can be asked to identify a star of similar chemical composition. It is important to pay attention to the spacing between the lines to be able to correctly identify the most similar star to the given one.

YOUR NOTES

Astronomical Unit Conversions

- Astronomical distances are very large and as a result, are usually measured using:
  - Astronomical Units (AU)
  - Light-years (ly)
  - Parsecs (pc)

Astronomical Unit (AU)

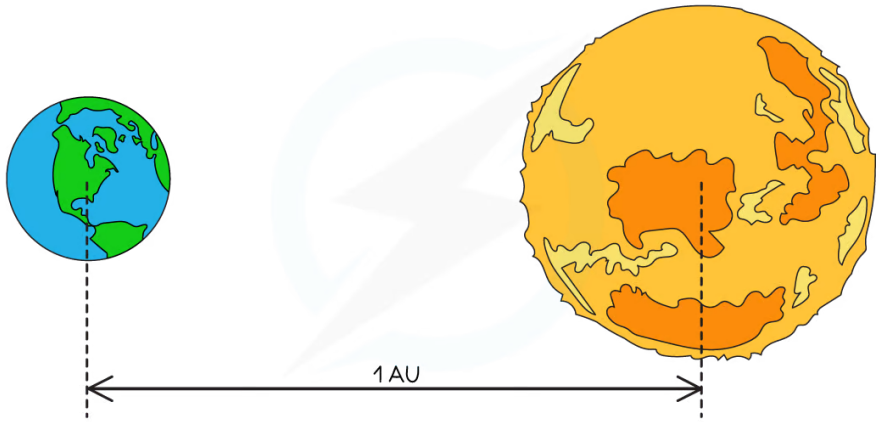
- The astronomical unit (AU) is defined as

The mean distance from the centre of the Earth to the centre of the Sun

- As the Earth's orbit around the Sun is **elliptical** it will be slightly closer to the Sun in January (1.471×10^{11} m) than it is in July (1.521×10^{11} m)
- Calculating the mean of these two values gives:

$$\frac{(1.471 \times 10^{11}) + (1.521 \times 10^{11})}{2} = 1.496 \times 10^{11} \text{ m}$$

- Therefore, 1 astronomical unit = 1.496×10^{11} m ≈ 1.5×10^{11} m
- The astronomical unit is useful for studying distances on the scale of the **solar system**



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Light-year (ly)

- A light-year is defined as:

The distance travelled by light in one year

- This can be calculated using:

Distance = speed × time

- Where:
  - The speed of light is 3×10^{8} m s^{-1}

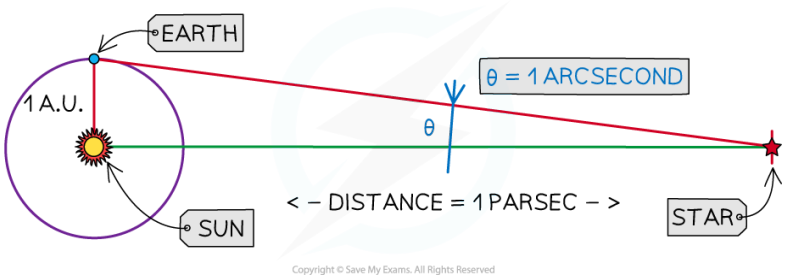
YOUR NOTES

- 1 year = 60 × 60 × 24 × 365 = 3.15×10^7 s
- Hence, the distance travelled by light in one year = $(3 \times 10^8) \times (3.15 \times 10^7) = 9.46 \times 10^{15}$ m
  - Therefore, 1 light-year ≈ 9.5×10^{15} m

Parsec (pc)

- Angles smaller than 1 degree can be measured in arcminutes or arcseconds
  - 1 degree = 60 arcminutes
  - 1 arcminute = 60 arcseconds
  - Therefore, 1 degree = 60 x 60 = 3600 arcseconds
  - 1 arcsecond = 1/3600 degree
- The parsec is defined as

> **A unit of distance that gives a parallax angle of 1 second of an arc (of a degree), using the radius of the Earth's orbit (1 AU) as the baseline of a right-angled triangle**



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- Given that 1 AU = 1.496×10^{11} m, trigonometry can be used to express 1 parsec in metres:

$$\tan\theta = \frac{opp}{adj} = \frac{1\ AU}{1\ pc}$$

$$\tan\left(\frac{1}{3600}\right) = \frac{1\ AU}{1\ pc}$$

$$1\ \text{pc} = \frac{1\ AU}{\tan\left(\frac{1}{3600}\right)} = \frac{1.496 \times 10^{11}}{\tan\left(\frac{1}{3600}\right)} = 3.09 \times 10^{16}\ \text{m}$$

- Therefore, 1 parsec ≈ 3.1×10^{16} m
- The parsec (1 pc = 3.1×10^{16} m) and the light-year (1 ly = 9.5×10^{15} m) are much **greater** in size than the astronomical unit (1 AU = 1.496×10^{11} m)
- This makes them useful when studying **interstellar distances**
  - For example, on the scale of distances between the Earth and stars, or neighbouring galaxies

YOUR NOTES

Worked Example

The closest star to Earth is a triple-star system called Alpha Centauri, which is approximately 4.35 light-years from Earth.

Calculate the distance between the Earth and Alpha Centauri in:

(a)
Astronomical units
(b)
Parsecs

An astronomical unit is 1.496×10^{11} m.

Answer:

(a)

Step 1: List the known quantities

- 1 light-year $\approx 9.5 \times 10^{15}$ m (from data booklet)
- 1 AU = 1.496×10^{11} m
- Distance to Alpha Centauri = 4.35 ly

Step 2: Convert 4.35 light-years into metres

- 4.35 ly = $4.35 \times (9.5 \times 10^{15}) = 4.13 \times 10^{16}$ m

Step 3: Convert from metres into AU

- 4.13×10^{16} m = $\frac{4.13 \times 10^{16}}{1.496 \times 10^{11}} = 2.8 \times 10^{5}$ AU (to 2 s.f)

(b)

Step 1: List the known quantities

- 1 parsec $\approx 3.1 \times 10^{16}$ m (from data booklet)
- 4.35 ly = 4.13×10^{16} m (from part a)

Step 2: Convert from metres into parsecs

- 4.13×10^{16} m = $\frac{4.13 \times 10^{16}}{3.1 \times 10^{16}} = 1.3$ pc (to 2 s.f)

Exam Tip

You do not need to learn all of the conversion factors for astronomical distances, you just need to know how to use them! The following are given in the data booklet:

$$1 \text{ light-year} \approx 9.5 \times 10^{15} \text{ m}$$

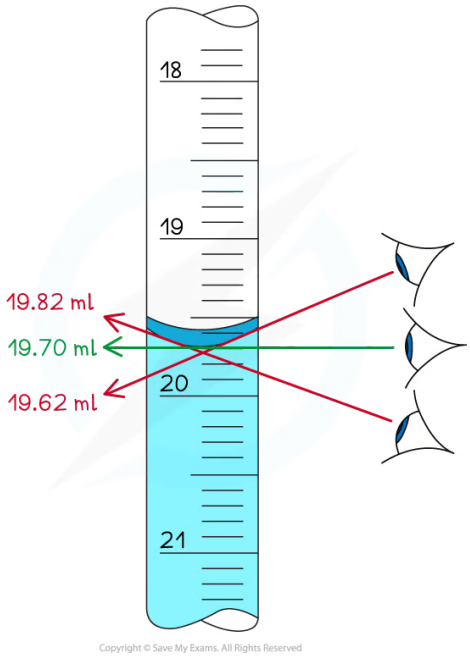
$$1 \text{ parsec} \approx 3.1 \times 10^{16} \text{ m}$$

However, the astronomical unit (AU) is not, so this could be useful to learn by heart!

Parallax Calculations

YOUR NOTES

- The **principle of parallax** is based on how the position of an object appears to change depending on where it is observed from
- When **observing** the volume of liquid in a measuring cylinder the parallax principle will result in the observer obtaining different values based on where they viewed the bottom of the meniscus from



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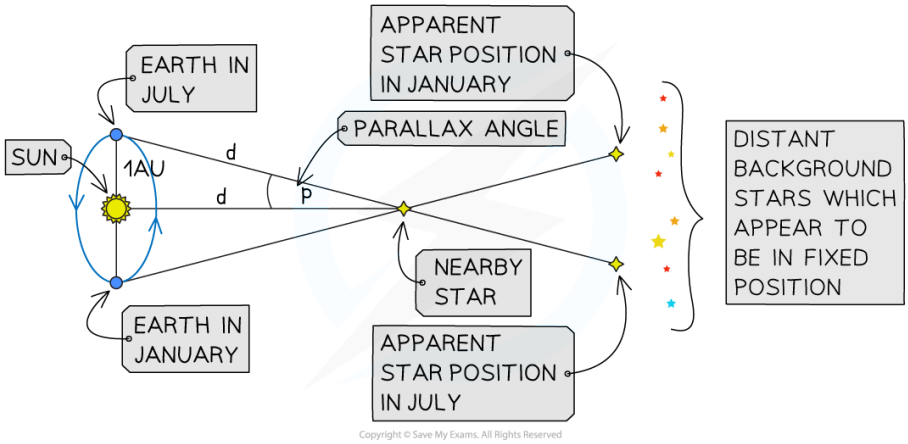
Parallax error describes the false readings that can be made by taking measurements from different angles

- **Stellar parallax** can be used to measure the distance to **nearby stars**
- Stellar Parallax is defined as:

  > **The apparent shifting in position of a nearby star against a background of distant stars when viewed from different positions of the Earth, during the Earth's orbit about the Sun**

- It involves observing how the position of a nearby star changes over a period of time against a fixed background of distant stars
  - To an **observer,** the position of distant stars does not change with time
- If a **nearby star** is viewed from the Earth at 6 months intervals (e.g. once in January and once again in July), the Earth will be at a **different position** in its orbit around the Sun
  - The nearby star will appear in a different position compared to the backdrop of distant stars
  - The distant stats will appear to not have moved
- This **apparent movement** of the nearby star is called the stellar parallax

YOUR NOTES



- Applying trigonometry to the parallax equation:

$$\tan p = \frac{1\ AU}{d}$$

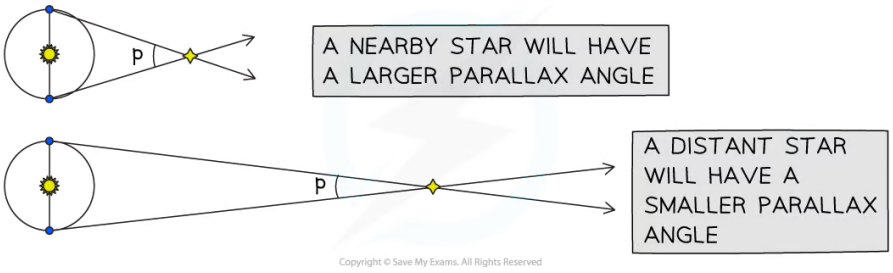
- Where:
  - 1 AU = radius of the Earth's orbit around the sun
  - p = parallax angle from Earth to the nearby star
  - d = distance to the nearby star
- For small angles, expressed in radians, $\tan p \approx p$, therefore:

$$p = \frac{1\ AU}{d}$$

- If the distance to the nearby star is to be measured in parsec, then it can be shown that the relationship between the distance to a star from Earth and the angle of stellar parallax is given by

$$p = \frac{1}{d}$$

- Where:
  - p = parallax (")
  - d = the distance to the nearby star (pc)
- This equation is accurate for distances of up to 100 pc
- For distances larger than 100 pc, the angles involved are so small they are too difficult to measure accurately



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YOUR NOTES

YOUR NOTES

Worked Example

The nearest star to Earth, Proxima Centauri, has a parallax of 0.768 seconds of arc.

Calculate the distance of Proxima Centauri from Earth

(a) in parsecs

(b) in light-years

Answer:

(a)

Step 1: List the known quantities

- Parallax, $p = 0.768''$

Step 2: State the parallax equation

$$p = \frac{1}{d}$$

Step 3: Rearrange and calculate the distance d

$$d = \frac{1}{p} = \frac{1}{0.768} = 1.30\ pc$$

(b)

Step 1: State the conversion between parsecs and metres

- From the data booklet:

$$1\ \text{parsec} \approx 3.1 \times 10^{16}\ \text{m}$$

Step 2: Convert 1.30 pc to m

$$1.30\ \text{pc} = 1.30 \times (3.1 \times 10^{16}) = 4.03 \times 10^{16}\ \text{m}$$

Step 3: State the conversion between light-years and metres

- From the data booklet

$$1\ \text{light-year} \approx 9.5 \times 10^{15}\ \text{m}$$

Step 4: Convert 4.03×10^{16} m into light-years

$$\frac{4.03 \times 10^{16}}{9.5 \times 10^{15}} = 4.2\ \text{ly (to 2 s.f)}$$

Exam Tip

Make sure you know the units for arc seconds (") and arc minutes (')

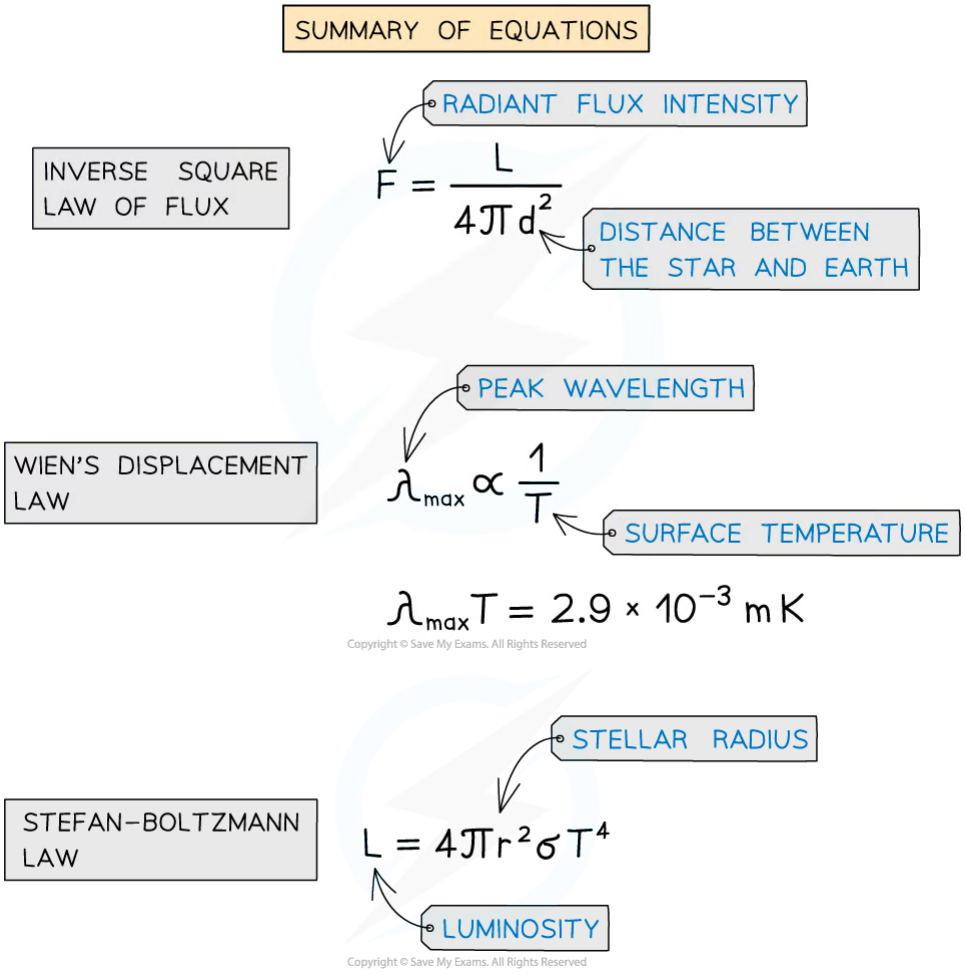
- 1 arcminute is denoted by 1'
- 1 arcsecond is denoted by 1"

YOUR NOTES

YOUR NOTES

Determination of Stellar Radii

- The radius of a star can be estimated by combining Wien's displacement law and the Stefan–Boltzmann law
- The procedure for this is as follows:
  - Using Wien's displacement law to find the surface temperature of the star
  - Using the inverse square law of flux equation to find the luminosity of the star (if given the radiant flux and stellar distance)
  - Then, using the Stefan-Boltzmann law, the stellar radius can be obtained



YOUR NOTES

Worked Example

Betelguese is our nearest red giant star. It has a luminosity of 4.49×10^{31} W and emits radiation with a peak wavelength of 850 nm.

Calculate the ratio of the radius of Betelgeuse r_B to the radius of the Sun r_S.

Radius of the Sun, $r_s = 6.95 \times 10^8$ m.

Answer:

Step 1: List the known quantities

- Luminosity of Betelgeuse, $L = 4.49 \times 10^{31}$ W
- Peak wavelength of Betelgeuse, $\lambda_{max} = 850$ nm $= 850 \times 10^{-9}$ m
- Radius of the Sun, $r_s = 6.95 \times 10^8$ m

Step 2: Write down Wien's displacement law

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{ m K}$$

Step 3: Rearrange Wien's displacement law to find the surface temperature of Betelguese

$$T = \frac{2.9 \times 10^{-3}}{\lambda_{max}} = \frac{2.9 \times 10^{-3}}{850 \times 10^{-9}} = 3410 \text{ K}$$

Step 4: Write down the Stefan-Boltzmann law

$$L = 4\pi r^2 \sigma T^4$$

Step 5: Rearrange for r and calculate the stellar radius of Betelguese

$$r_B = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{(4.49 \times 10^{31})}{4\pi \times (5.67 \times 10^{-8}) \times (3410)^4}} = 6.83 \times 10^{11} \text{ m}$$

Step 6: Calculate the ratio r_B / r_s

$$\frac{r_B}{r_s} = \frac{6.83 \times 10^{11}}{6.95 \times 10^8} = 983$$

- Therefore, the radius of Betelguese is about 1000 times larger than the Sun's radius

IB Physics HL

YOUR NOTES

Measurements in Physics

CONTENTS

Fundamental & Derived Units in IB Physics
Using Scientific Notation in Physics
Using Dimensional Analysis
Measurement Techniques in IB Physics

YOUR NOTES

SI Units & Prefixes

- There is a seemingly endless number of units in Physics
- These can all be reduced to seven base units from which every other unit can be derived
- These seven units are referred to as the SI Base Units; making up the system of measurement officially used in almost every country around the world

SI Base Quantities Table

Quantity	Si Base Unit	Symbol
Mass	Kilogram	kg
Length	Metre	m
Time	Second	s
Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol

Six SI quantities are shown. The seventh quantity, the candela, measures luminous intensity and is not covered in IB Physics. You may meet it later if you continue with Physics at university.

Derived Units

YOUR NOTES

- Derived units are derived from the seven SI Base units
- The base units of physical quantities such as:
  - Newtons, **N**
  - Joules, **J**
  - Pascals, **Pa,** can be deduced

- To deduce the base units, it is necessary to use the definition of the quantity
- The Newton (N), the unit of force, is defined by the equation:
  - Force = mass × acceleration
  - $N = kg \times m\,s^{-2} = kg\,m\,s^{-2}$
  - Therefore, the Newton (N) in SI base units is $\mathbf{kg\,m\,s^{-2}}$

- The Joule (J), the unit of energy, is defined by the equation:
  - Energy = ½ × mass × velocity^2
  - $J = kg \times (m\,s^{-1})^2 = kg\,m^2\,s^{-2}$
  - Therefore, the Joule (J) in SI base units is $\mathbf{kg\,m^2\,s^{-2}}$

- The Pascal (Pa), the unit of pressure, is defined by the equation:
  - Pressure = force ÷ area
  - $Pa = N \div m^2 = (kg\,m\,s^{-2}) \div m^2 = kg\,m^{-1}\,s^{-2}$
  - Therefore, the Pascal (Pa) in SI base units is $\mathbf{kg\,m^{-1}\,s^{-2}}$

YOUR NOTES

Orders of Magnitude

- When a number is expressed in an **order of 10**, this is an **order of magnitude**
  - For example, if a number is described as 3×10^8 then that number is actually 3 x 100 000 000
  - The **order of magnitude** of 3×10^8 is just $\mathbf{10^8}$
- When the number is greater than 5, round up to the next order of magnitude
  - For example, the **order of magnitude** of 6×10^8 is $\mathbf{10^9}$
- A quantity is an order of magnitude **larger** than another quantity if it is about **ten times larger**
- Similarly, **two orders of magnitude** would be **100 times larger,** or 10^2
  - In physics, orders of magnitude can be very large or very small
- When estimating values, it's best to give the **estimate** of an order of magnitude to **the nearest power of 10**
  - For example, the diameter of the Milky Way is approximately 1 000 000 000 000 000 000 000 m
- It is inconvenient to write this many zeros, so it's best to use **scientific notation** as follows:

$$1\,000\,000\,000\,000\,000\,000\,000 = 1 \times 10^{21}\ \text{m}$$

- The order of magnitude is 10^{21}
- Orders of magnitude make it easier to compare the relative sizes of objects
  - For example, a quantity with an order of magnitude of 10^6 is 10 000 times larger than a quantity with a magnitude of 10^2

Approximation & Estimation

- To estimate is to obtain an approximate value
  - For **very** large or small quantities, using **orders of magnitudes** to estimate calculations is a valid approach
- Estimation is typically done to the nearest order of magnitude

Estimating Physical Quantities Table

Object of Interest	Approximate Length (m)	Order of Magnitude (m)
distance to the edge of the observable Universe	4.40×10^{26}	10^{26}
distance from Earth to Neptune	4.5×10^{12}	10^{12}
distance from London to Cape Town	9.7×10^6	10^7
length of a human	1.7	10^0

length of an ant	9×10^{-4}	10^{-3}
length of a bacteria cell	2×10^{-6}	10^{-6}

YOUR NOTES

Worked Example

Estimate the order of magnitude of the following:

(a) The temperature of an oven (in Kelvin)

(b) The volume of the Earth (in m^3)

(c) The number of seconds in a person's life if they live to be 95 years old

Answer:

(a) The temperature of an oven

Step 1: Identify the approximate temperature of an oven

- A conventional oven works at ~200 °C which is 473 K

Step 2: Identify the order of magnitude

- Since this could be written as 4.73×10^2 K
- The order of magnitude is $\sim 10^2$

(b) The volume of the Earth

Step 1: Identify the approximate radius of the Earth

- The radius of the Earth is $\sim 6.4 \times 10^6$ m

Step 2: Use the radius to calculate the volume

- The volume of a sphere is equal to:

$$V = 4/3\,\pi r^3$$

$$V = 4/3 \times \pi \times (6.4 \times 10^6)^3$$

$$V = 1.1 \times 10^{21}\ m^3$$

Step 3: Identify the order of magnitude

- The order of magnitude is $\sim 10^{21}$

(c) The number of seconds in 95 years

Step 1: Find the number of seconds in a single year

1 year = 365 days with 24 hours each with 60 minutes with 60 seconds

365 × 24 × 60 × 60 = 31 536 000 seconds in a year

Step 2: Find the number of seconds in 95 years

95 × 31 536 000 = 283 824 000 seconds

- This is approximately 2.84×10^8 seconds

- Therefore the order of magnitude is $\sim 10^8$

YOUR NOTES

Scientific Notation

- In physics, **measured quantities** cover a large range from the very large to the very small
- Scientific notation is a form that is based on powers of 10
- The scientific form must have **one digit** in front of the decimal place
  - Any remaining digits remain behind the decimal place
  - The magnitude of the value comes from multiplying by 10^n where n is called 'the power'
  - This power is positive when representing large numbers or negative when representing small numbers

Worked Example

Express 4 600 000 in scientific notation.

Answer:

Step 1: Write the convention for scientific notation

- To convert into scientific notation, only one digit may remain in front of the decimal point
  - Therefore, the scientific notation must be 4.6×10^n
- The value of *n* is determined by the number of decimal places that must be moved to return to the original number (i.e. 4 600 000)

Step 2: Identify the number of digits after the 4

- In this case, that number is +6

Step 3: Write the final answer in scientific notation

- The solution is: $\mathbf{4.6 \times 10^6}$

Metric Multipliers

- When dealing with magnitudes of 10, there are **metric names** for many common quantities
- These are known as metric multipliers and they change the **size** of the **quantity** they are applied to
  - They are represented by prefixes that go in front of the measurement
- Some common examples that are well-known include
  - **kilo**metres, **k**m ($\times 10^3$)
  - **centi**metres, **c**m ($\times 10^{-2}$)
  - **milli**grams, **m**g ($\times 10^{-3}$)
- Metric multipliers are represented by a single letter symbol such as centi- (c) or Giga- (G)
  - These letters go in front of the quantity of interest

- For example, centimetres (cm) or Gigawatts (GW)

YOUR NOTES

Common Metric Multipliers Table

Prefix	Abbreviation	Value
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
hecto	h	10^{2}
deca	da	10^{1}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

YOUR NOTES

Worked Example

What is 3.6 Mm + 2700 km, in m?

Answer:

Step 1: Check which metric multipliers are in this problem

- M represents **Mega-** which is **× 10^6** (not milli- which is small m!)
- k represents **kilo-** which is a multiplier of **× 10^3**

Step 2: Apply these multipliers to get both quantities to be metres

$$3.6 \times 10^6 \text{ m} + 2.7 \times 10^6 \text{ m}$$

Step 3: Write the final answer in units of metres

$$6.3 \times 10^6 \text{ m}$$

Significant Figures

- Significant figures are the digits that accurately represent a given quantity
- Significant figures describe the precision with which a quantity is known
  - If a quantity has **more significant figures** then **more precise** information is known about that quantity

Rules for Significant Figures

- Not all digits that a number may show are significant
- In order to know how many digits in a quantity are significant, these rules can be followed
  - **Rule 1:** In an integer, all digits count as significant if the last digit is non-zero
    - **Example:** 702 has 3 significant figures
  - **Rule 2:** Zeros at the end of an integer do not count as significant
    - **Example:** 705,000 has 3 significant figures
  - **Rule 3:** Zeros in front of an integer do not count as significant
    - **Example:** 0.002309 has 4 significant figures
  - **Rule 4:** Zeros at the end of a number less than zero count as significant, but those in front do not.
    - **Example:** 0.0020300 has 5 significant figures
  - **Rule 5:** Zeros after a decimal point are also significant figures.
    - **Example:** 70.0 has 3 significant figures
- Combinations of numbers must always be to the smallest number significant figures

YOUR NOTES

Worked Example

What is the solution to this problem to the correct number of significant figures: 18 × 384?

Answer:

Step 1: Identify the smallest number of significant figures

- 18 has only 2 significant figures, while 384 has 3 significant figures
- Therefore, the final answer should be to 2 significant figures

Step 2: Do the calculation with the maximum number of digits

$$18 \times 384 = 6912$$

Step 3: Round to the final answer to 2 significant figures

$$6.9 \times 10^3$$

Exam Tip

When studying IB DP Physics, it is recommended to state your answer on a single line explicitly (if possible) with all necessary details to ensure the examiners can mark correctly and for best practice.

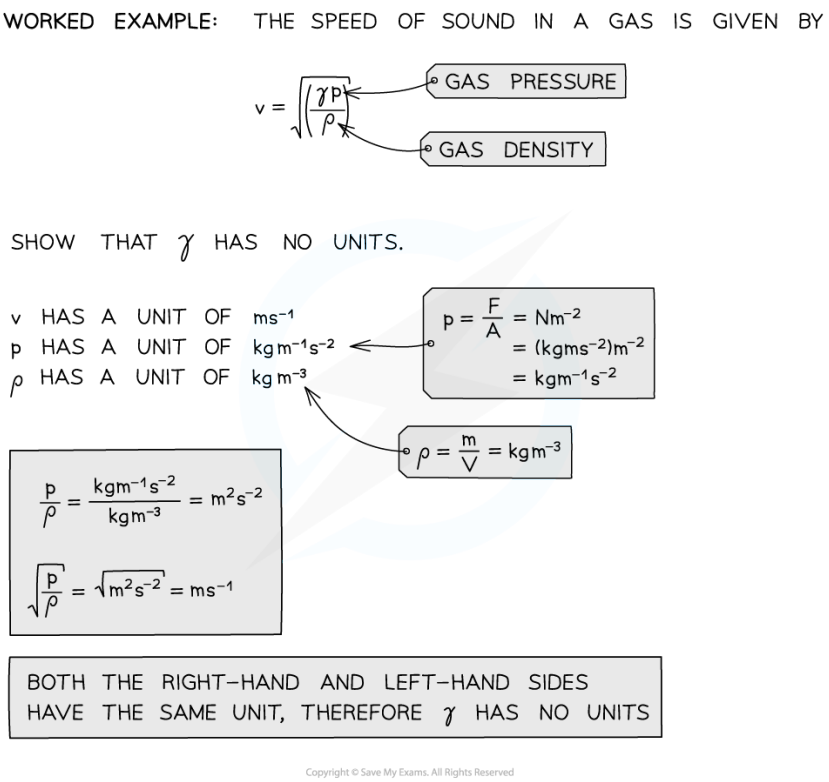
You are expected to know metric multipliers for your exams. Make sure you become familiar with them in order to avoid any mistakes.

YOUR NOTES

Using Dimensional Analysis

- An important skill is to be able to check the homogeneity of physical equations using the SI base units
  - This is also known as **dimensional analysis**
- The units on either side of the equation should be the same
- To check the homogeneity of physical equations:
  - Check the units on both sides of an equation
  - Determine if they are equal
  - If they do not match, the equation will need to be adjusted

Worked Example



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Measurement Techniques

- Common instruments used in Physics are:
  - **Metre rules** - to measure distance and length
  - **Thermometers** - to measure temperature
  - **Measuring cylinders** - to measure to volume of liquid or the volume of displaced liquid
  - **Balances** - to measure mass
  - **Newtonmeters** - to measure force
  - **Protractors** - to measure angles
  - **Stopwatches** - to measure time
  - **Ammeters** - to measure current
  - **Voltmeters** - to measure potential difference
  - **Sound meter** - to measure the intensity of sound
  - **Light meter** - to measure the intensity of light
- More complicated instruments such as the **micrometer** screw gauge and **Vernier calipers** can be used to measure thicknesses, diameters and lengths to a greater degree of accuracy

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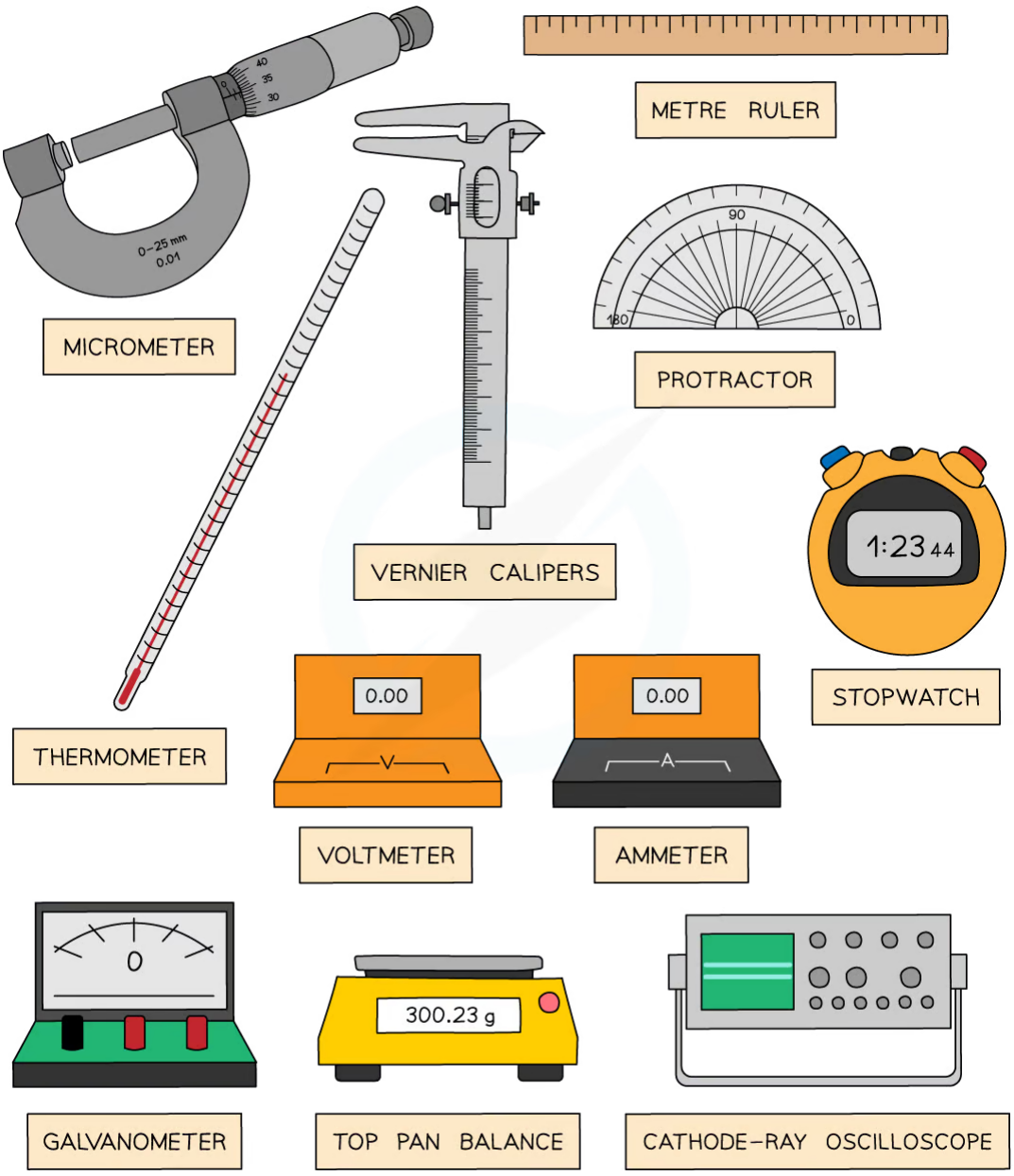

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- When using measuring instruments like these you need to ensure that you are fully aware of what each division on a scale represents
  - This is known as the **resolution**
- The resolution is the smallest change in the physical quantity being measured that results in a change in the reading given by the measuring instrument
- The smaller the change that can be measured by the instrument, the greater the degree of resolution
- For example, a standard mercury thermometer has a resolution of 1°C whereas a typical digital thermometer will have a resolution of 0.1°C
  - The digital thermometer has a higher resolution than the mercury thermometer

Measuring Instruments Table

Quantity	Instrument	Typical Resolution
Length	Meter Rule	1 mm
Thickness or length	Vernier Calipers	0.05 mm
Thickness or length	Micrometer	0.001 mm
Mass	Top-Pan Balance	0.01 g
Angle	Protractor	1°
Time	Stopwatch	0.01 s
Temperature	Thermometer	1 °C
Potential Difference	Voltmeter	1 mV - 0.1 V
Current	Ammeter	1 mA - 0.1 A

YOUR NOTES

YOUR NOTES

Controlling Variables

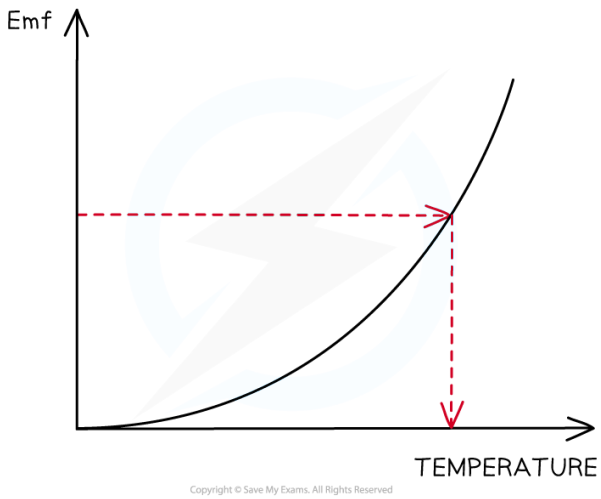
- For an experiment to be **valid**, it is essential that any variable that may affect the outcome of an experiment is controlled
- Some of the key practical skills that are required to do so are as follows:
  - Calibration of measuring apparatus
  - Keeping certain environmental conditions constant
  - Insulation against heat loss or gain
  - Reduction of friction
  - Reduction of electrical resistance
  - Taking background radiation into account

Calibration of Measuring Apparatus

- Calibration is a comparison between a **known** measurement and the measurement you achieve using the instrument
- This checks the accuracy of the instrument, especially for higher readings
  - For example, checking whether a meter (e.g., voltmeter, micrometer, ammeter) reads **zero** before measurements are made
  - This helps to avoid zero error

Calibrating sensors

- Calibration curves are used to **convert measurements** made on one measurement scale to another measurement scale
- These are useful in experiments when the instruments used have outputs which are not proportional to the value they are measuring
  - For example, e.m.f and temperature (thermocouple) or resistance against temperature (thermistor)
- The calibration curve for a thermocouple, in which the e.m.f varies with temperature, is shown below:



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A curve of voltage against temperature can be used as a temperature sensor

Maintaining Constant Conditions

YOUR NOTES

- In an experiment, a variable is any factor that could change or be changed
- There are different types of variables within an experiment
  - The **independent variable**: the only variable that should be changed throughout an experiment
  - The **dependent variable**: the variable that is measured to determine the outcome of an experiment (the results)
  - The **controlled variables**: any other variables that may affect the results of the experiment that need to be controlled or monitored
- It is essential that any variable that may affect the outcome of an experiment is controlled in order for the results to be **valid** and to have a **fair test**
  - A fair test is one in which **only** the independent variable has been allowed to affect the dependent variable

Controlling Heat Losses & Gains

- Energy transfers by heating due to **conduction** are one of the most common sources of dissipated energy
- To reduce energy transfers by **conduction**, materials with a **low** thermal conductivity should be used
  - Materials with low thermal conduction are called **insulators**
- **Insulation** reduces energy transfers from both **conduction** and **convection**
- The effectiveness of an insulator is dependent upon:

1. The thermal conductivity of the material

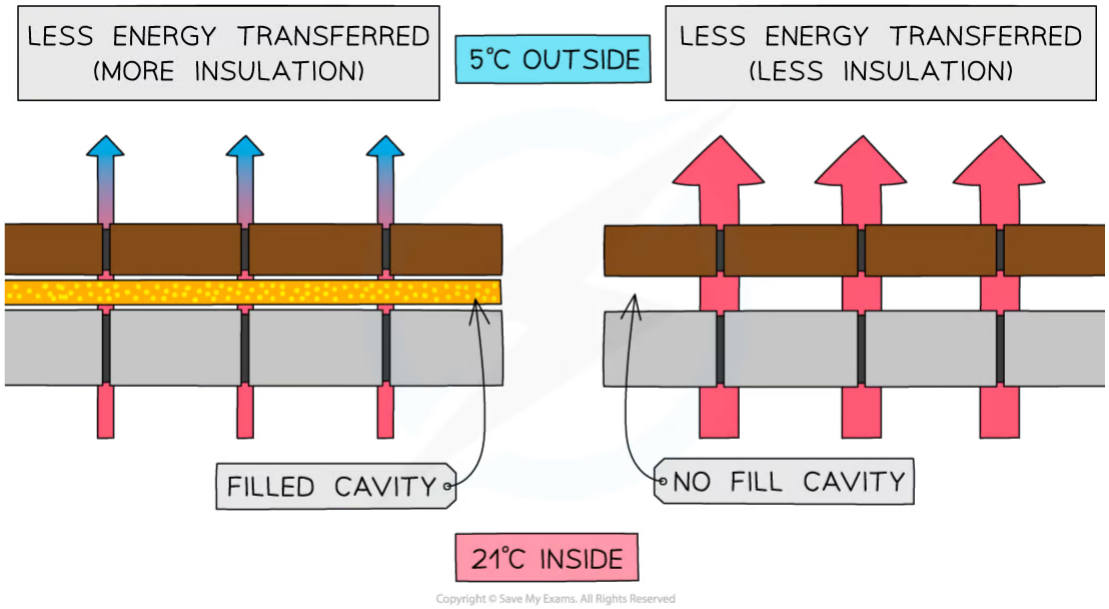
- The lower the conductivity, the lower the amount of heat loss

2. The density of the material

- The more dense the insulator, the more conduction can occur
- In a denser material, the particles are closer together so they can transfer energy to one another more easily

3. The thickness of the material

- The thicker the material, the lower the amount of heat loss



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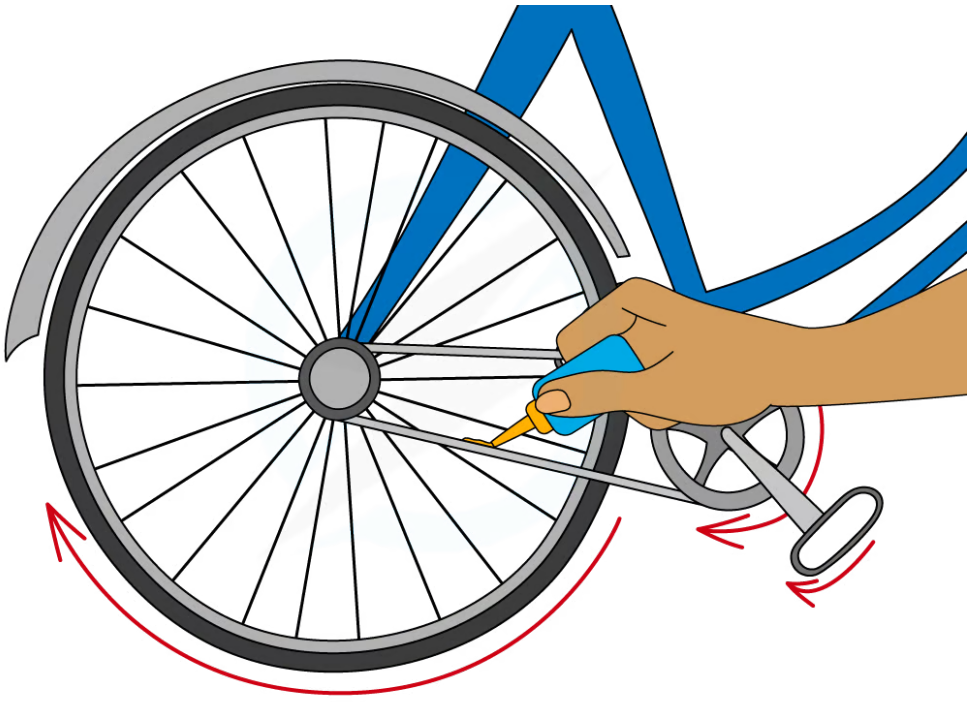
Less energy is transferred by conduction and convection if the cavity is insulated

Reducing Friction

- In a mechanical system, there is often **friction** between the moving parts of the machinery
- This results in unwanted energy transfers **by heating** the machinery and the surroundings
- **Friction** can be reduced by:
  - Adding bearings to prevent components from directly rubbing together
  - Lubricating parts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Lubricating parts of a bicycle to reduce friction

YOUR NOTES

Reducing Electrical Resistance

- In electric circuits, there is **resistance** as current flows through the wires and components
- This results in unwanted energy transfers **by heating** to the wires, components and the surroundings
- **Resistance** can be reduced by:
  - Using components with lower resistance
  - Reducing the current

Adjusting for Background Radiation

- Although most background radiation is natural, a small amount of it comes from artificial sources, such as **medical procedures** (including X-rays)
- Levels of background radiation can vary significantly from place to place
- When conducting experiments to measure the radiation coming from radioactive sources, the background radiation must be taken into account, to do this:
  - Place a **Geiger-Muller tube** away from any radioactive sources and measure the **background count**
  - Carry out the experiment with the radioactive source
  - Subtract the background count from each reading to obtain the count rate from the source only

YOUR NOTES

Worked Example

A student measures the background radiation count in a laboratory and obtains the following readings:

Count rate/ counts min^{-1}	69	68	70	71	69	72

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The student is trying to verify the inverse square law of gamma radiation on a sample of Radium-226. He collects the following data:

Distance / cm		10	20	30	40	50	60	70	80	90
Count rate/ counts min^{-1}	1	586	202	123	100	89	87	79	78	76
	2	569	193	136	102	94	85	83	77	74
	3	591	199	122	104	90	80	81	79	78

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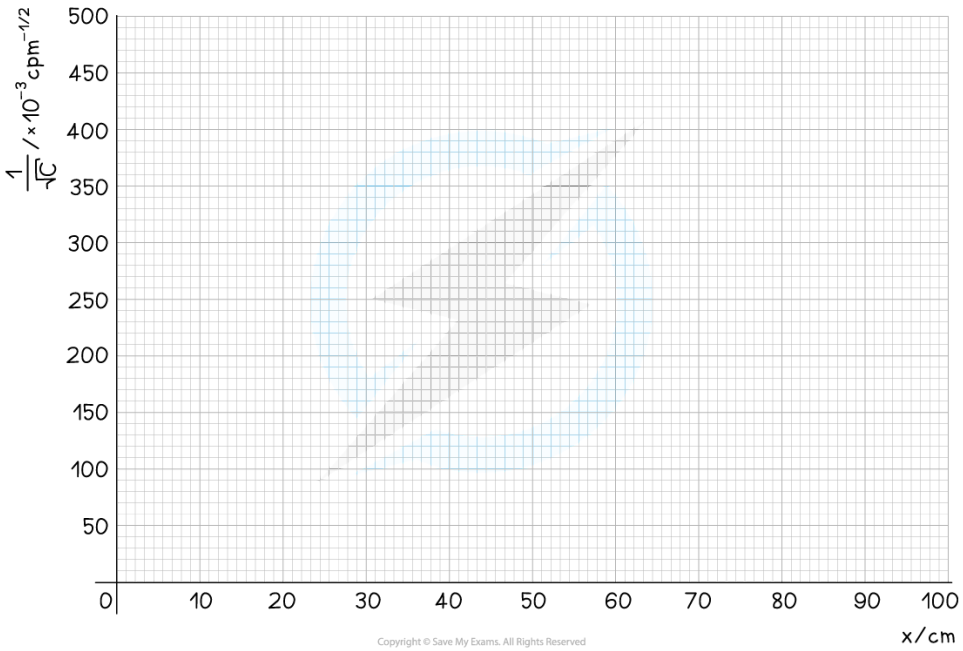
Use this data to determine if the student's data follows an inverse square law.



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Answer:

Step 1: Determine a mean value of background radiation

$$\text{Count rate} = \frac{69 + 68 + 70 + 71 + 69 + 72}{6} = 69.8 = 70$$

YOUR NOTES

- The background radiation must be subtracted from each count rate reading to determine the corrected count rate, C

Step 2: Compare the inverse square law to the equation of a straight line

- According to the inverse square law, the intensity, *I*, of the γ radiation from a point source depends on the distance, *x*, from the source

$$I \propto \frac{1}{x^2}$$

- Intensity is proportional to the corrected count rate, C, so

$$C \propto \frac{1}{x^2} \rightarrow \frac{1}{C} \propto x^2 \rightarrow \frac{1}{\sqrt{C}} \propto x \rightarrow \frac{1}{\sqrt{C}} = kx$$

- The graph provided is of the form $1/C^{-1/2}$ against *x*
- Comparing this to the equation of a straight line, y = mx
  - $y = 1/C^{-1/2}$ (counts $min^{-1/2}$)
  - x = x (m)
  - Gradient = constant, k
- If it is a straight line graph through the origin, this shows they are directly proportional, and the inverse square relationship is confirmed

Step 3: Calculate C (corrected average count rate) and $C^{-1/2}$

Distance x / cm	10	20	30	40	50	60	70	80	90
Average count rate/ counts min^{-1}	582	198	127	102	91	84	81	78	76
Corrected average count rate C/ counts min^{-1}	512	128	57	32	21	14	11	8	6
$\frac{1}{\sqrt{C}}$ / $cpm^{-1/2}$	0.044	0.088	0.132	0.177	0.218	0.267	0.302	0.354	0.408

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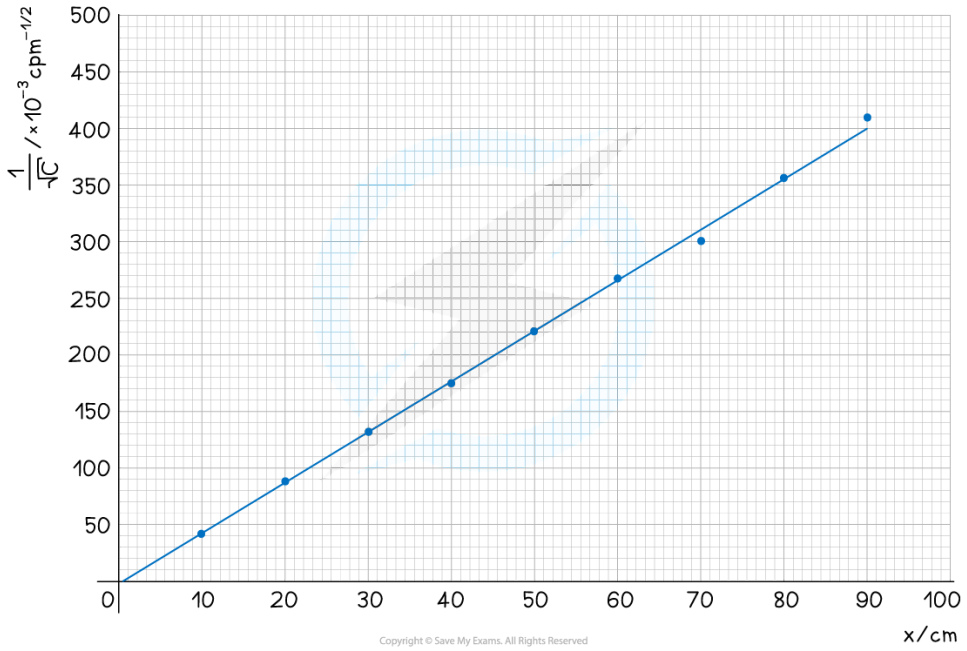
Step 4: Plot a graph of $C^{-1/2}$ against x and draw a line of best fit



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- The graph shows $C^{-1/2}$ is directly proportional to x, therefore, the data follows an inverse square law

YOUR NOTES

IB Physics HL

YOUR NOTES

Processing Uncertainties

CONTENTS

Random & Systematic Errors
Calculating Uncertainties
Determining Uncertainties from Graphs

YOUR NOTES

Random & Systematic Errors

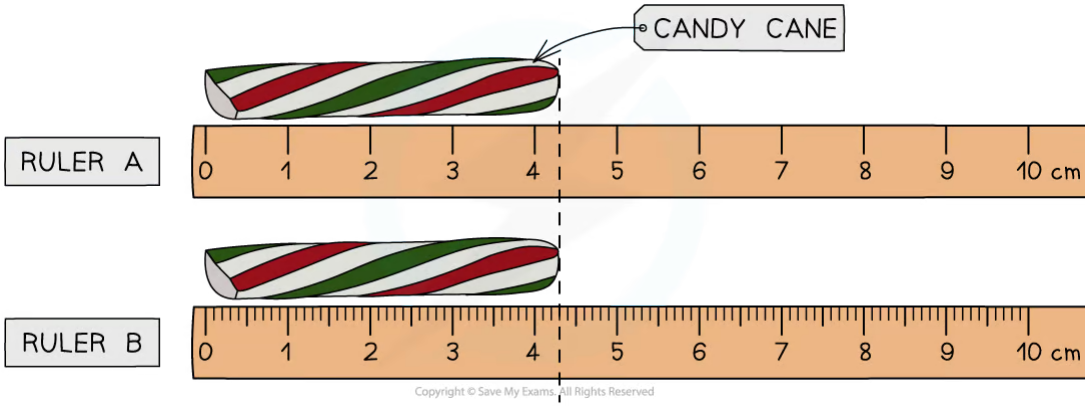
- Measurements of quantities are made with the aim of finding the true value of that quantity
  - In reality, it is impossible to obtain the true value of any quantity as there will always be a degree of **uncertainty**
- The uncertainty is an estimate of the difference between a **measurement reading** and the **true value**
- The two types of **measurement errors** that lead to uncertainty are:
  - Random errors
  - Systematic errors

Random Errors

- Random errors cause unpredictable fluctuations in an instrument's readings as a result of uncontrollable factors, such as environmental conditions
- This affects the precision of the measurements taken, causing a wider spread of results about the mean value
- To **reduce** random error:
  - **Repeat** measurements several times and calculate an average from them

Reading Errors

- When measuring a quantity using an **analogue** device such as a ruler, the uncertainty in that measured quantity is **±0.5 the smallest measuring interval**
- When measuring a quantity using a **digital** device such as a digital scale or stopwatch, the uncertainty in that measured quantity is **±1 the smallest measuring interval**
- To **reduce** reading errors:
  - Use a **more precise** device with **smaller measuring intervals** and therefore less uncertainty



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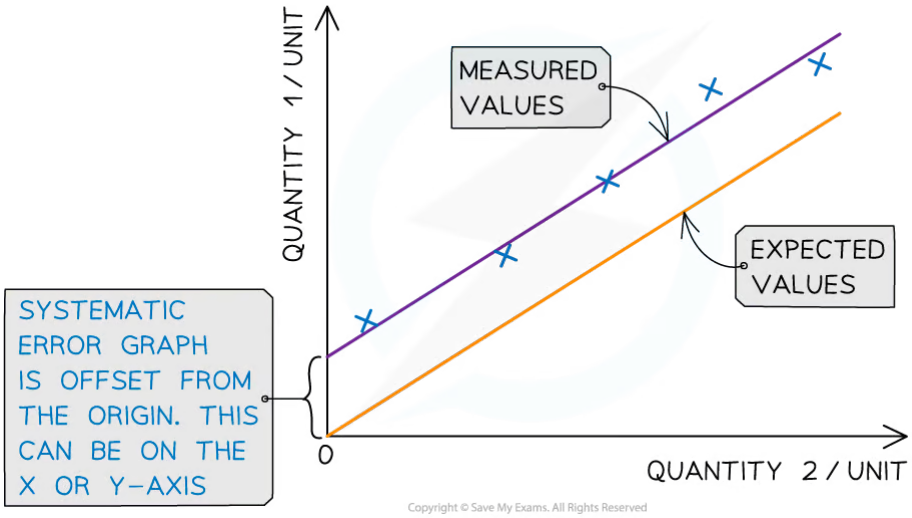
Both rulers measure the same candy cane, yet Ruler B is more precise than Ruler A due to a smaller interval size

Systematic Errors

YOUR NOTES

- Systematic errors arise from the use of faulty instruments or from flaws in the experimental method
- This type of error is repeated consistently every time the instrument is used or the method is followed, which affects the accuracy of all readings obtained
- To **reduce** systematic errors:
  - Instruments should be **recalibrated**, or different instruments should be used
  - Corrections or adjustments should be made to the technique



Systematic errors on graphs are shown by the offset of the line from the origin

Zero Errors

- This is a type of systematic error which occurs when an instrument gives a reading when the **true reading is zero**
  - For example, a top-ban balance that starts at 2 g instead of 0 g
- To **account for** zero errors
  - Take the **difference** of the **offset** from each value
  - For example, if a scale starts at 2 g instead of 0 g, a measurement of 50 g would actually be 50 – 2 = 48 g
  - The offset could be positive or negative

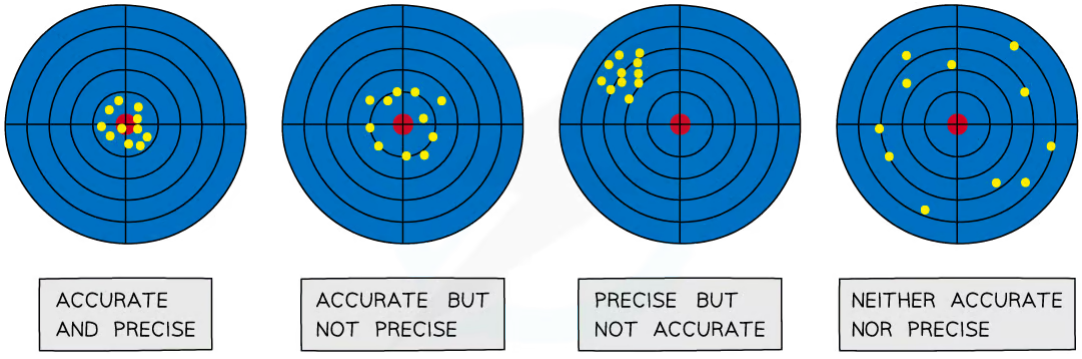
Precision

- Precise measurements are ones in which there is **very little spread** about the **mean** value, in other words, **how close** the measured values are **to each other**
- If a measurement is repeated several times, it can be described as **precise** when the values are **very similar to, or the same** as, each other
  - Another way to describe this concept is if the **random uncertainty** of a measurement is **small**, then that measurement can be said to be **precise**
- The precision of a measurement is reflected in the values recorded – measurements to a greater number of decimal places are said to be more **precise** than those to a whole number

YOUR NOTES

Accuracy

- A measurement is considered **accurate** if it is close to the true value
  - Another way to describe this concept is if the **systematic error** of a measurement is **small**, then that measurement can be said to be **accurate**
- The accuracy can be **increased by repeating measurements** and finding a mean of the results
- Repeating measurements also helps to identify anomalies that can be omitted from the final results



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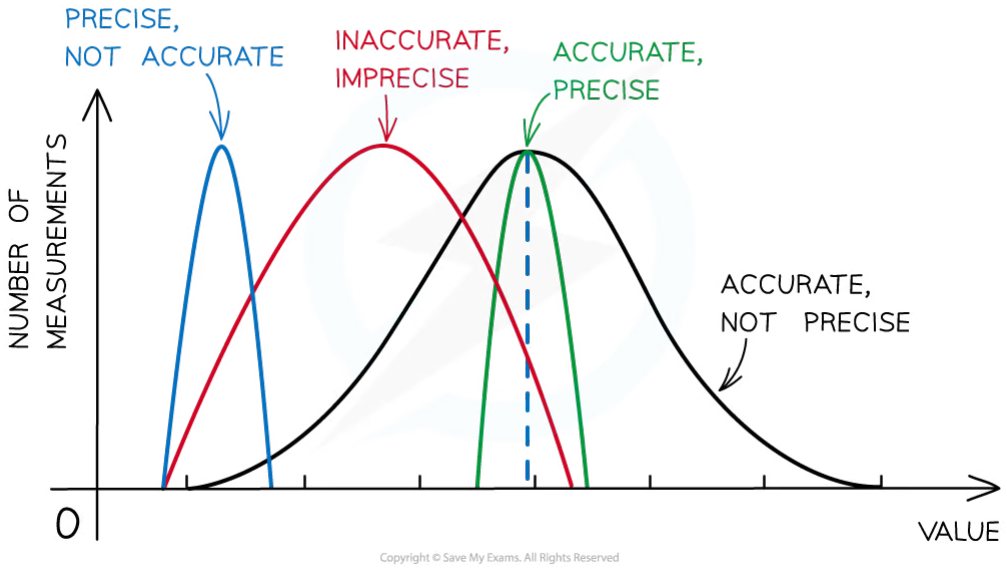
The difference between precise and accurate results



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Representing precision and accuracy on a graph

Reliability

- Reliability is defined as

A measure of the ability of an experimental procedure to produce the expected results when using the same method and equipment

- A reliable **experiment** is one which produces consistent results when repeated many times
- Similarly, a reliable **measurement** is one which can be reproduced consistently when measured repeatedly
- When thinking about the reliability of an experiment, a **good question** to ask is
  - Would similar conclusions be reached if someone repeated this experiment?

Validity

- The validity of an experiment relates to the experimental method and the appropriate choice of variables
- Validity is defined as

A measure of the suitability of an experimental procedure to measure what it is intended to measure

- It is essential that any variables that may affect the outcome of an experiment are **identified** and **controlled** in order for the results to be **valid**
- For example, when using Charles' law to determine absolute zero, **pressure** must be kept constant
- When thinking about the validity of an experiment, a **good question** to ask is
  - How relevant is this experiment to my original research question?

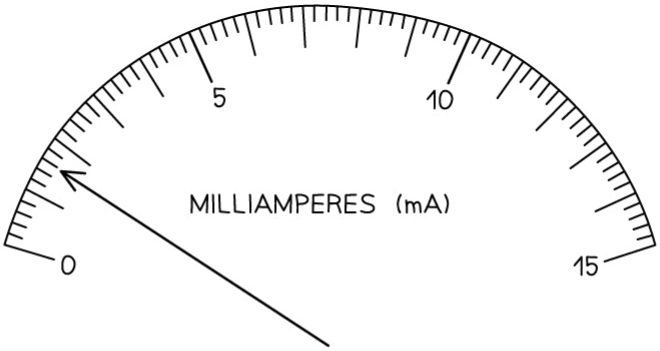
YOUR NOTES

Calculating Uncertainties

- There is always a degree of uncertainty when measurements are taken; the uncertainty can be thought of as the difference between the **actual** reading taken (caused by the equipment or techniques used) and the **true value**
- Uncertainties are **not** the same as errors
  - Errors can be thought of as issues with equipment or methodology that cause a reading to be different from the true value
  - The uncertainty is a range of values around a measurement within which the true value is expected to lie, and is an **estimate**
- For example, if the true value of the mass of a box is 950 g, but a systematic error with a balance gives an actual reading of 952 g, the uncertainty is ±2 g
- These uncertainties can be represented in a number of ways:
  - **Absolute Uncertainty:** where uncertainty is given as a fixed quantity
  - **Fractional Uncertainty:** where uncertainty is given as a fraction of the measurement
  - **Percentage Uncertainty:** where uncertainty is given as a percentage of the measurement

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

- To find uncertainties in different situations:
  - **The uncertainty in a reading:** ± half the smallest division
  - **The uncertainty in a measurement:** at least ±1 smallest division
  - **The uncertainty in repeated data:** half the range i.e. ± ½ (largest - smallest value)
  - **The uncertainty in digital readings:** ± the last significant digit unless otherwise quoted
  - **The uncertainty in the natural log of a value:** absolute uncertainty in ln(x) = $\frac{\text{uncertainty in } x}{x}$



SMALLEST DIVISION = 0.2 mA

READING (I) = 1.6 mA

$$\text{ABSOLUTE UNCERTAINTY } (\Delta I) = \frac{1}{2} \times 0.2 \text{ mA} = 0.1 \text{ mA}$$

$$I = 1.6 \pm 0.1 \text{ mA}$$

$$\text{FRACTIONAL UNCERTAINTY} = \frac{\text{UNCERTAINTY}}{\text{VALUE}} = \frac{0.1}{1.6} = \frac{1}{16}$$

$$I = 1.6 \pm \frac{1}{16} \text{ mA}$$

$$\text{PERCENTAGE UNCERTAINTY } (\%) = \frac{\text{UNCERTAINTY}}{\text{VALUE}} \times 100 = \frac{0.1}{1.6} \times 100 = 6.2\%$$

$$I = 1.6 \pm 6.2\% \text{ mA}$$

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How to calculate absolute, fractional and percentage uncertainty

- Always make sure your absolute or percentage uncertainty is to the same number of **significant figures** as the reading

Combining Uncertainties

- When combining uncertainties, the rules are as follows:

Operation	Example	Propagation Rule
Addition & Subtraction	$y = a \pm b$	$\Delta y = \Delta a + \Delta b$ The sum of the absolute uncertainties

YOUR NOTES

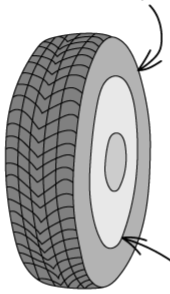
Multiplication & Division	$y = a \times b$ or $y = \frac{a}{b}$	$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$ The sum of the fractional uncertainties
Power	$y = a^{\pm n}$	$\frac{\Delta y}{y} = n\left(\frac{\Delta a}{a}\right)$ The magnitude of n times the fractional uncertainty

Adding / Subtracting Data

- **Add** together the absolute uncertainties

ADDING / SUBTRACTING DATA

DIAMETER OF TYRE (d_1) = 55.0 ± 0.5 cm



DIAMETER OF INNER TYRE (d_2) = 21.0 ± 0.7cm

DIFFERENCE IN DIAMETERS ($d_1 - d_2$) = 55.0 − 21.0 = 34.0 cm

UNCERTAINTY IN DIFFERENCE = ±(0.5 + 0.7) = ± 1.2 cm

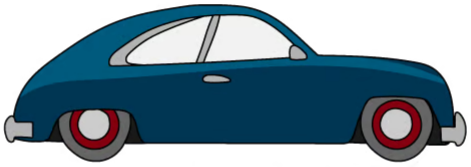
$d_1 - d_2$ = 34.0 ± 1.2 cm

Multiplying / Dividing Data

- **Add** the percentage or fractional uncertainties

YOUR NOTES

MULTIPLYING / DIVIDING DATA



DISTANCE = 50.0 ± 0.1 m

TIME = 5.00 ± 0.05 s

$$\text{SPEED } (v) = \frac{\text{DISTANCE } (s)}{\text{TIME } (t)}$$

$$V = \frac{50.0}{5.0} = 10.0 \text{ ms}^{-1}$$

$$\frac{\Delta v}{v} = \frac{\Delta s}{s} + \frac{\Delta t}{t} = \frac{0.1}{50.0} + \frac{0.05}{5.00} = 0.002 + 0.01 = 0.012$$

$$\text{ABSOLUTE UNCERTAINTY } (\Delta v) = 10.0 \times 0.012 = \pm 0.12 \text{ ms}^{-1}$$

$$v = 10.0 \pm 0.12 \text{ ms}^{-1}$$

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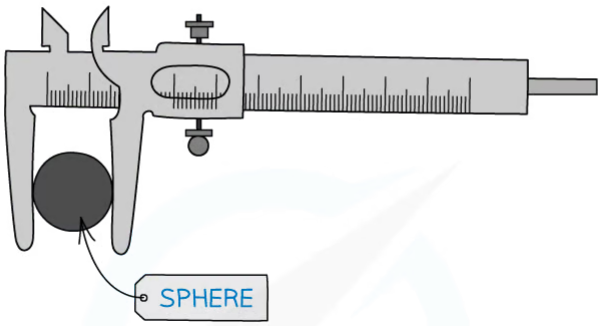
Raising to a Power

- **Multiply** the percentage uncertainty by the power

YOUR NOTES

RAISING TO A POWER



$$V = \frac{4}{3}\pi r^3$$

$$r = 2.50 \pm 0.02 \text{ cm}$$

$$V = \frac{4}{3}\pi (2.50)^3 = 65.5 \text{ cm}^3$$

$$\frac{\Delta V}{V} = 3 \times \frac{\Delta r}{r} = 3 \times \frac{0.02}{2.50} = 0.024$$

ABSOLUTELY UNCERTAINTY $(\Delta V) = 65.5 \times 0.024 = 1.57 \text{cm}^3$

PERCENTAGE UNCERTAINTY $(\%\Delta V) = 100 \times 0.024 = 2.4\%$

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Exam Tip

Remember:

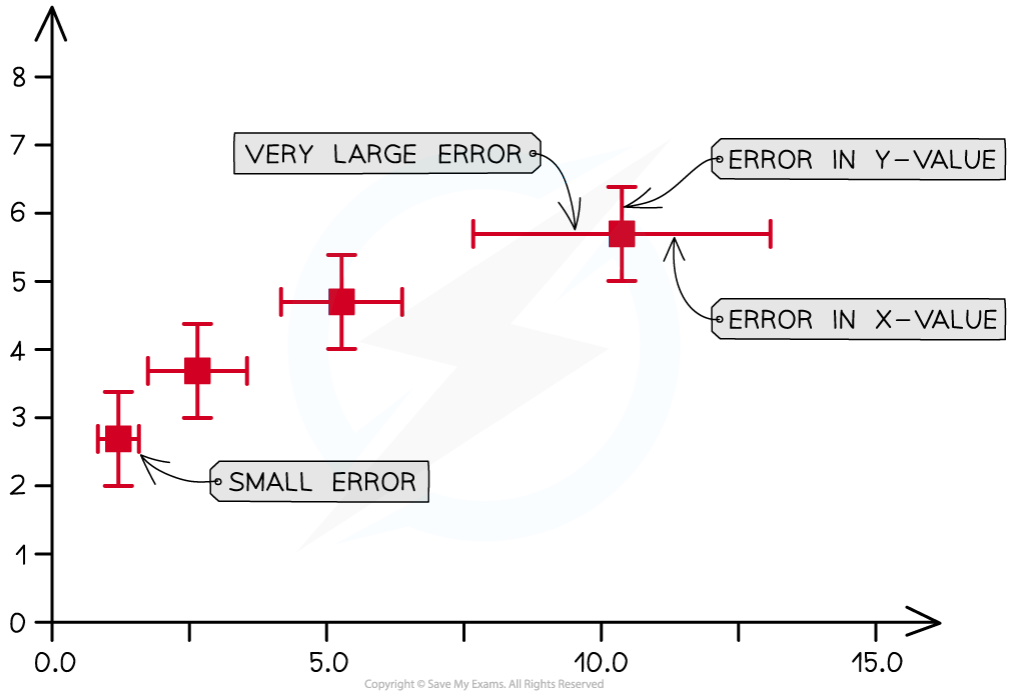
- Absolute uncertainties (denoted by Δ) have the same units as the quantity
- Percentage uncertainties have no units
- The uncertainty in constants, such as π, is taken to be zero

Uncertainties in trigonometric and logarithmic functions will not be tested in the exam, so just remember these rules and you'll be fine!

YOUR NOTES

Determining Uncertainties from Graphs

- The uncertainty in a measurement can be shown on a graph as an **error bar**
- This bar is drawn above and below the point (or from side to side) and shows the **uncertainty** in that measurement
- Error bars are plotted on graphs to show the **absolute uncertainty** of values plotted



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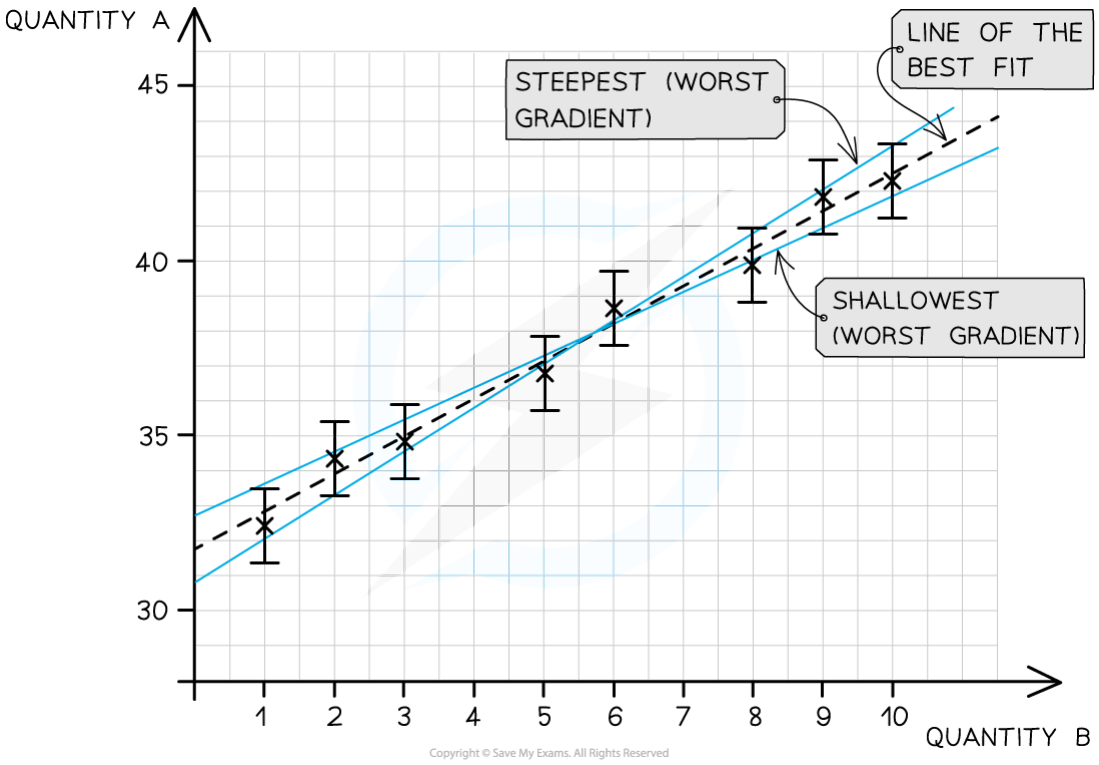
Representing error bars on a graph

- To calculate the **uncertainty in a gradient**, two lines of best fit should be drawn on the graph:
  - The 'best' line of best fit, which passes as **close** to the points **as possible**
  - The 'worst' line of best fit, either the **steepest possible** or the **shallowest possible** line which fits within all the error bars



The line of best fit passes as close as possible to all the points. The steepest and shallowest lines are known as the worst fit

- The percentage uncertainty in the **gradient** can be found using the magnitude of the 'best' and 'worst' gradients:

$$\text{percentage uncertainty} = \frac{best\ gradient\ -\ worst\ gradient}{best\ gradient} \times 100\%$$

- Either the steepest or shallowest line of best fit may have the 'worst' gradient on a case-by-case basis.
  - The 'worst' gradient will be the one with the **greatest difference** in magnitude from the 'best' line of best fit.
  - The equation **above** is for the case where the 'worst' gradient is the **shallowest**.
  - If the 'worst' gradient is the **steepest**, then the 'worst' gradient should be **subtracted** from the 'best' gradient and **then** divided by the best gradient and multiplied by 100
- Alternatively, the **average** of the two maximum and minimum lines can be used to calculate the percentage uncertainty:

$$\text{percentage uncertainty} = \frac{max.\ gradient\ -\ min.\ gradient}{2} \times 100\%$$

- The percentage uncertainty in the **y-intercept** can be found using:

$$\text{percentage uncertainty} = \frac{best\ y\ intercept\ -\ worst\ y\ intercept}{best\ y\ intercept} \times 100\%$$

$$\text{percentage uncertainty} = \frac{max.\ y\ intercept\ -\ min.\ y\ intercept}{2} \times 100\%$$

YOUR NOTES

Percentage Difference

- The percentage difference gives an indication of how close the **experimental value** achieved from an experiment is to the **accepted value**
  - It is **not** a percentage uncertainty
- The percentage difference is defined by the equation:

$$\text{percentage difference} = \frac{experimental\ value\ -\ accepted\ value}{accepted\ value} \times 100\%$$

- The experimental value is sometimes referred to as the 'measured' value
- The accepted value is sometimes referred to as the 'true' value
  - This may be labelled on a component such as the capacitance of a capacitor or the resistance of a resistor
  - Or, from a reputable source such as a peer-reviewed data booklet
- For example, the acceleration due to gravity g is known to be 9.81 m s^{-2}. This is its **accepted value**
  - From an experiment, the value of g may be found to be 10.35 m s^{-2}
  - Its **percentage difference** would therefore be 5.5 %
- The **smaller** the percentage difference, the more **accurate** the results of the experiment

YOUR NOTES

Worked Example

On the axes provided, plot the graph for the following data and draw error bars and lines of best and worst fit.

Force / N	10	20	30	40	50	60	70	80
Extension / mm	8.5 ± 1	11 ± 0.5	15 ± 1	15 ± 2	20 ± 1.5	19.5 ± 2	22 ± 0.5	26 ± 1

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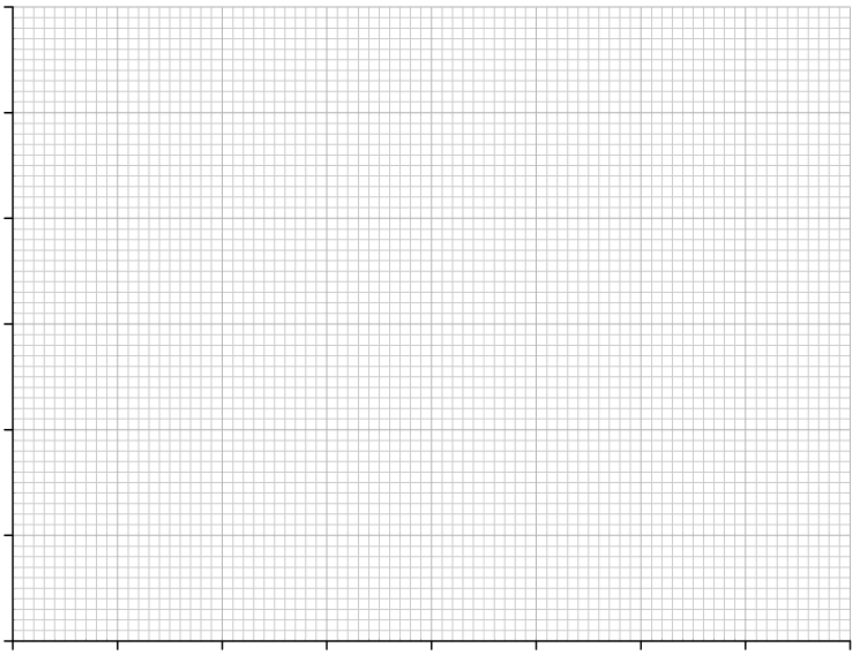
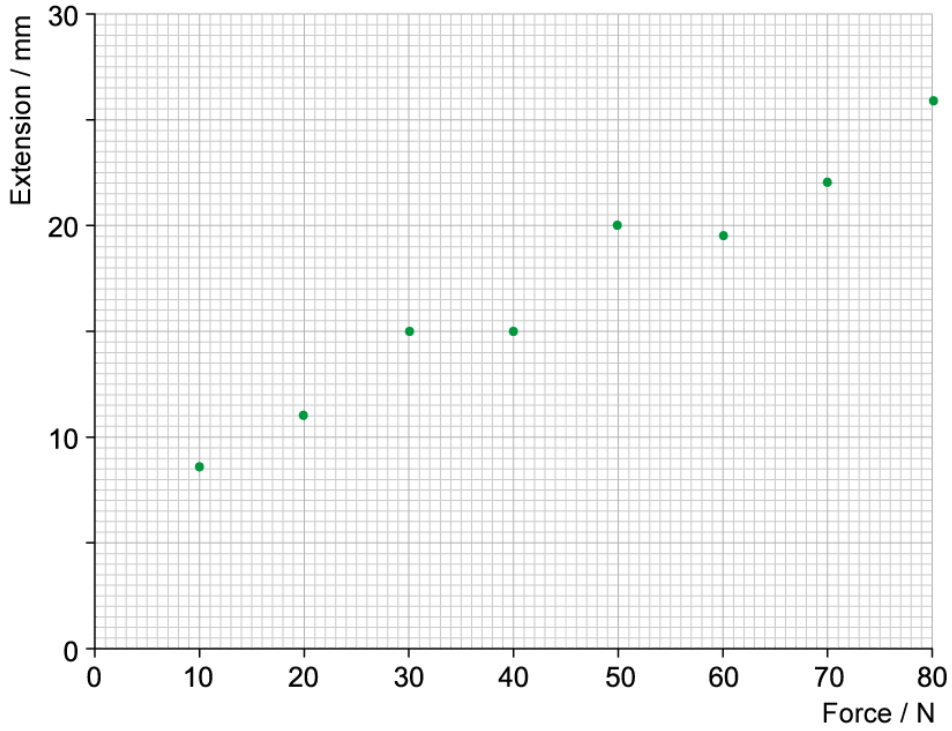
Find the percentage uncertainty in the gradient from your graph.



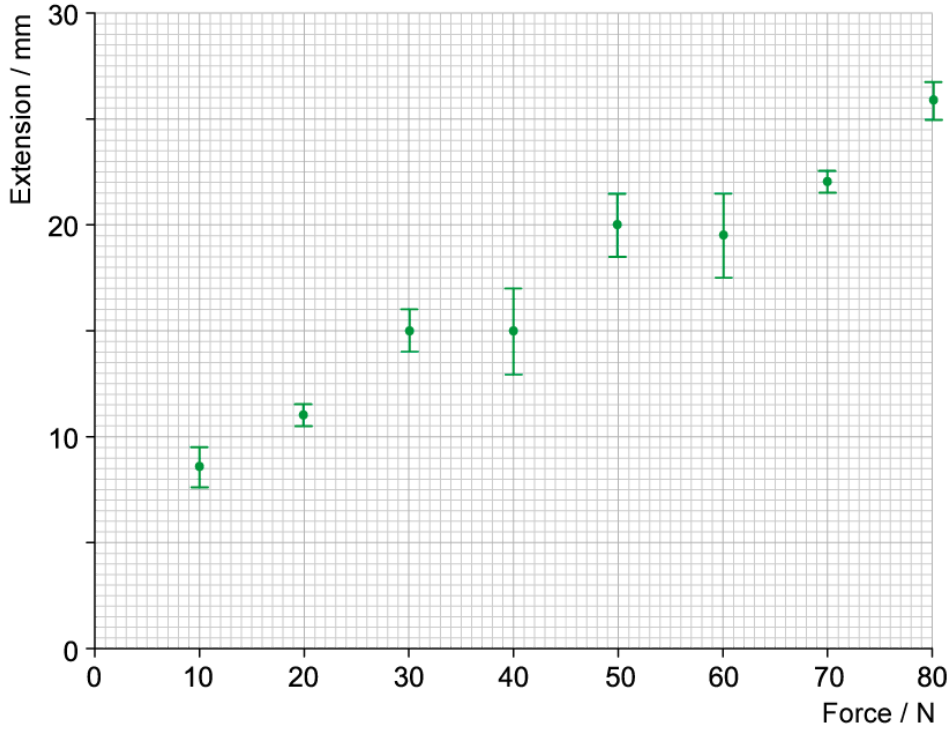
Answer:

Step 1: Draw sensible scales on the axes and plot the data

YOUR NOTES

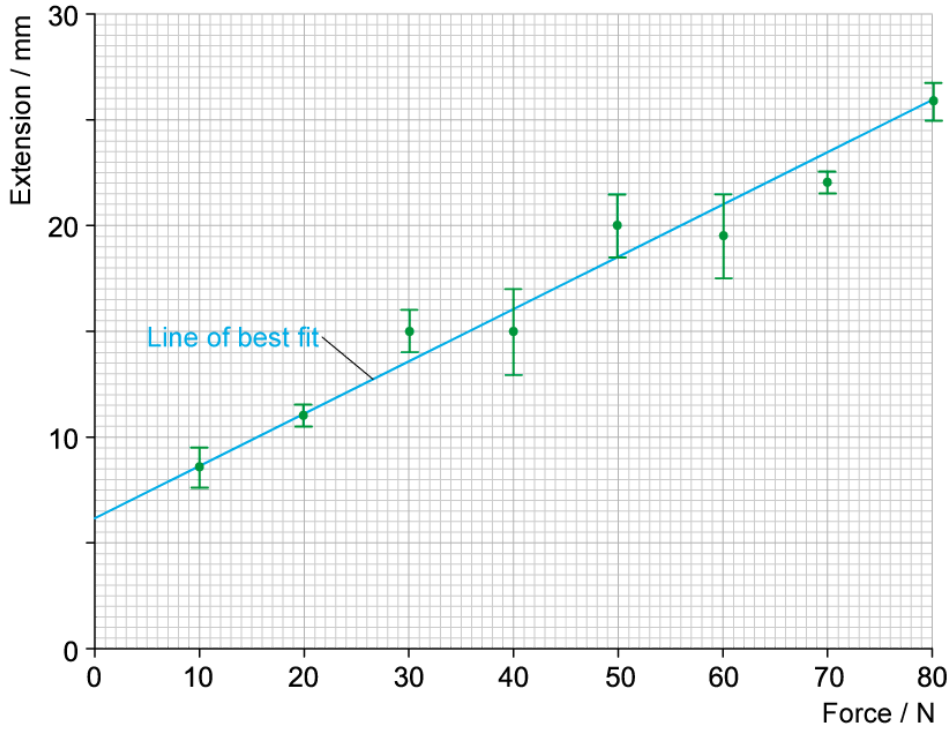


Step 2: Draw the errors bars for each point

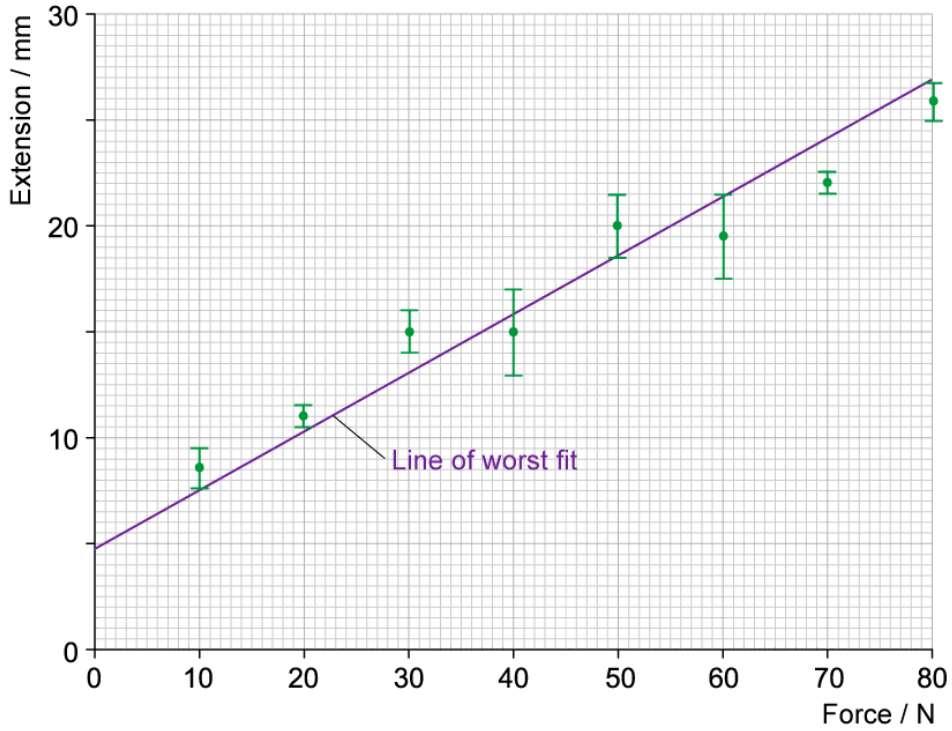


Step 3: Draw the line of best fit

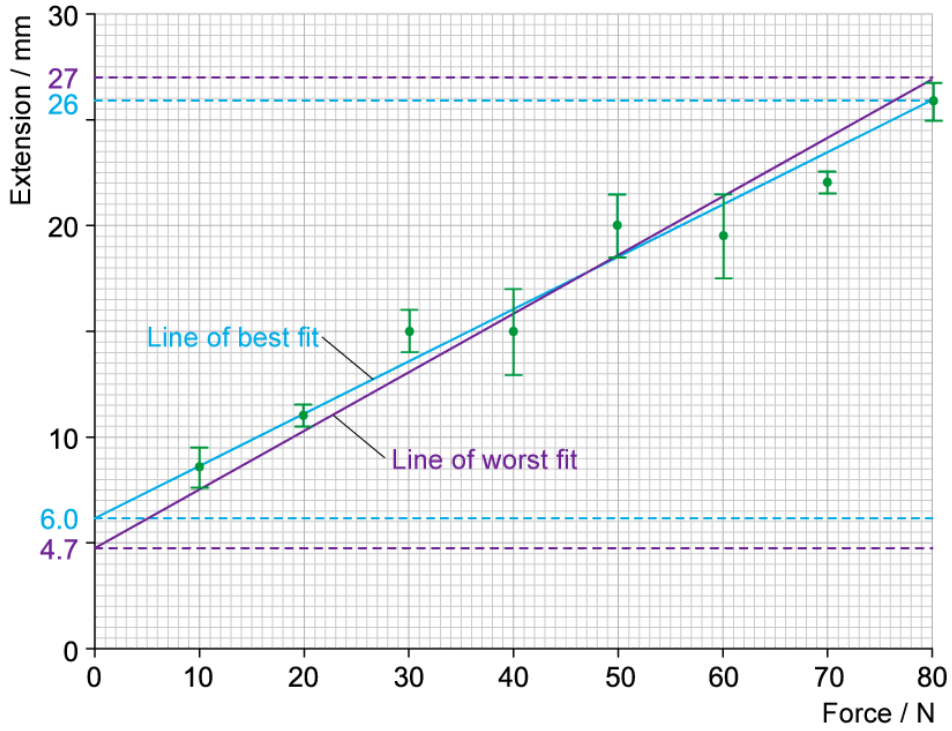
YOUR NOTES



Step 4: Draw the line of worst fit



Step 5: Work out the gradient of each line and calculate the percentage uncertainty



- best gradient = $\frac{\Delta y}{\Delta x} = \frac{26 - 6}{80 - 0} = 0.25$
- worst gradient = $\frac{\Delta y}{\Delta x} = \frac{27 - 4.7}{80 - 0} = 0.28$
- % uncertainty = $\frac{0.28 - 0.25}{0.25} \times 100\% = 12\%$

Exam Tip

A common misconception is that error bars need to all be the same size. In physics, this is not the case and each data point can have different error bar sizes as they have different uncertainties.

IB Physics HL

YOUR NOTES

Scalars & Vectors

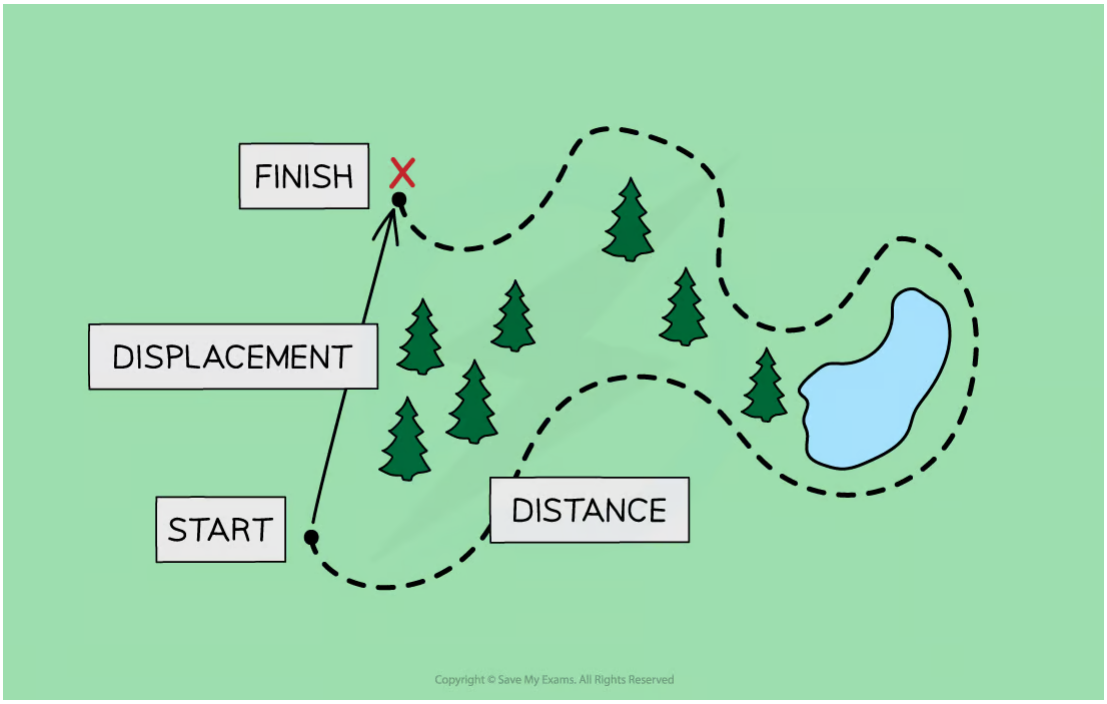
CONTENTS

Scalar & Vector Quantities
Combining & Resolving Vectors
Scale Diagrams

YOUR NOTES

Scalar & Vector Quantities

- A **scalar** is a quantity which **only** has a magnitude (size)
- A **vector** is a quantity which has **both** a **magnitude** and a **direction**
- For example, if a person goes on a hike in the woods to a location which is a couple of miles from their starting point
  - As the crow flies, their **displacement** will only be a few miles but the **distance** they walked will be much longer



Displacement is a vector while distance is a scalar quantity

- **Distance** is a **scalar** quantity
  - This is because it describes how an object has travelled overall, but not the direction it has travelled in
- **Displacement** is a **vector** quantity
  - This is because it describes how far an object is from where it started and in what direction
- Some common scalar and vector quantities are shown in the table below:

Scalars and Vectors Table

YOUR NOTES

SCALARS	VECTORS
DISTANCE	DISPLACEMENT
SPEED	VELOCITY
MASS	ACCELERATION
TIME	FORCE
ENERGY	MOMENTUM
VOLUME	
DENSITY	
PRESSURE	
ELECTRIC CHARGE	
TEMPERATURE	

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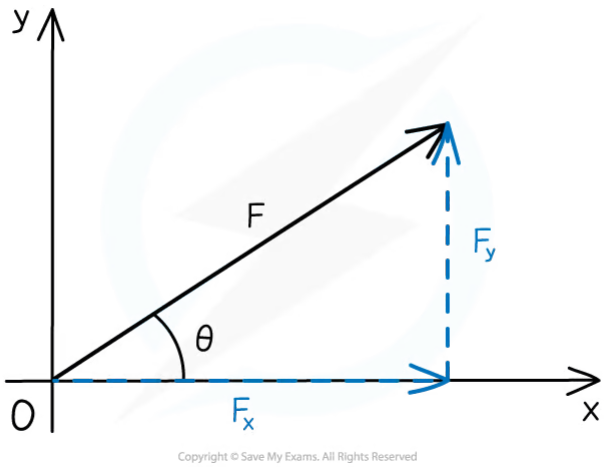
Representing Vectors

- **Vectors** are represented by an arrow
  - The **arrowhead** indicates the **direction** of the vector
  - The **length** of the arrow represents the **magnitude**



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The force vector F has both a direction and a magnitude

- **Component** vectors are sometimes drawn with a dotted line and a **subscript** indicating horizontal or vertical

- For example, F_x is the horizontal component and F_y is the vertical component of the force F

Exam Tip

Do you have trouble figuring out if a quantity is a vector or a scalar? Just think - can this quantity have a minus sign? For example - can you have negative energy? No. Can you have negative displacement? Yes!

YOUR NOTES

Combining & Resolving Vectors

- Vectors can be changed in a variety of ways, such as
  - Combining through vector **addition** or **subtraction**
  - Combining through vector **multiplication**
  - Resolving into **components** through trigonometry

Combining Vectors

- Vectors can be combined by **adding** or **subtracting** them to produce the **resultant vector**
  - The **resultant vector** is sometimes known as the 'net' vector (e.g. the net force)
- There are two methods that can be used to combine vectors: the **triangle method** and the **parallelogram method**

Triangle method

- To combine vectors using the triangle method:
  - **Step 1:** link the vectors head-to-tail
  - **Step 2:** the resultant vector is formed by connecting the tail of the first vector to the head of the second vector

Parallelogram method

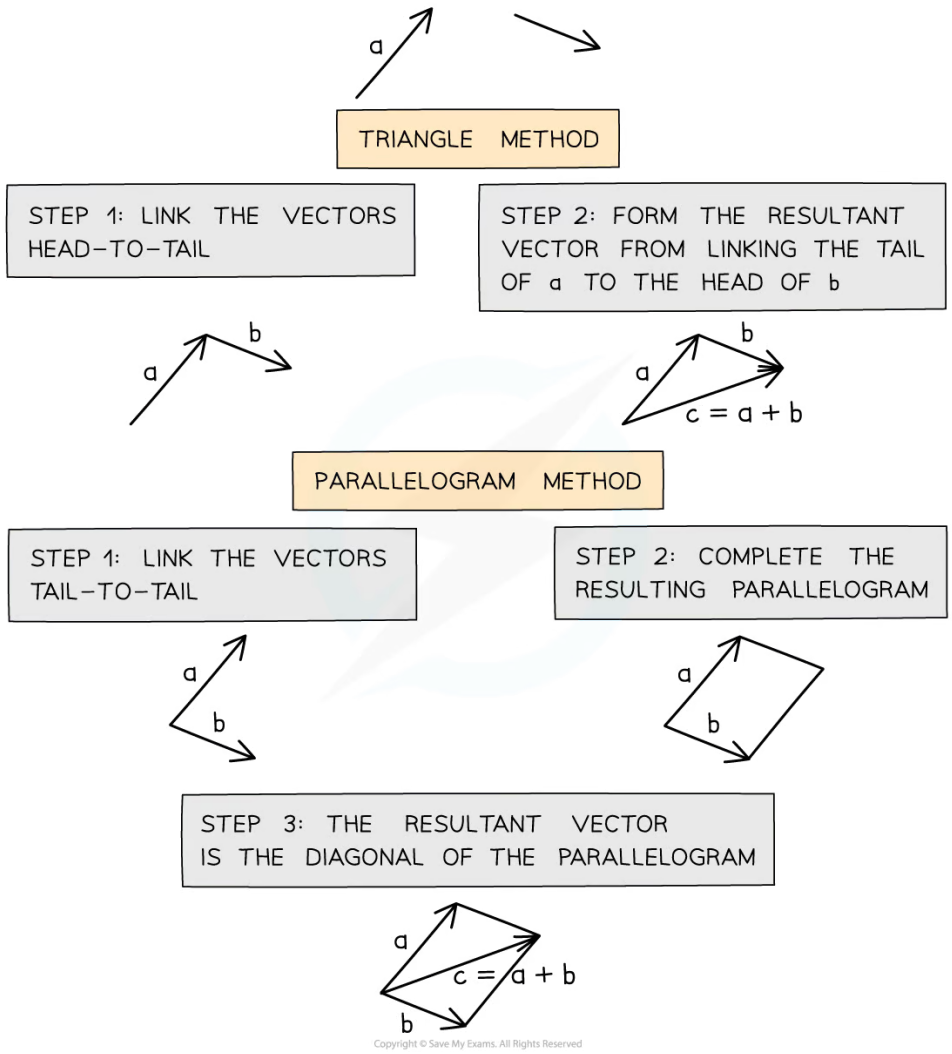
- To combine vectors using the parallelogram method:
  - **Step 1:** link the vectors tail-to-tail
  - **Step 2:** complete the resulting parallelogram
  - **Step 3:** the resultant vector is the diagonal of the parallelogram

YOUR NOTES

Worked Example

Draw the vector c = a + b.

Answer:



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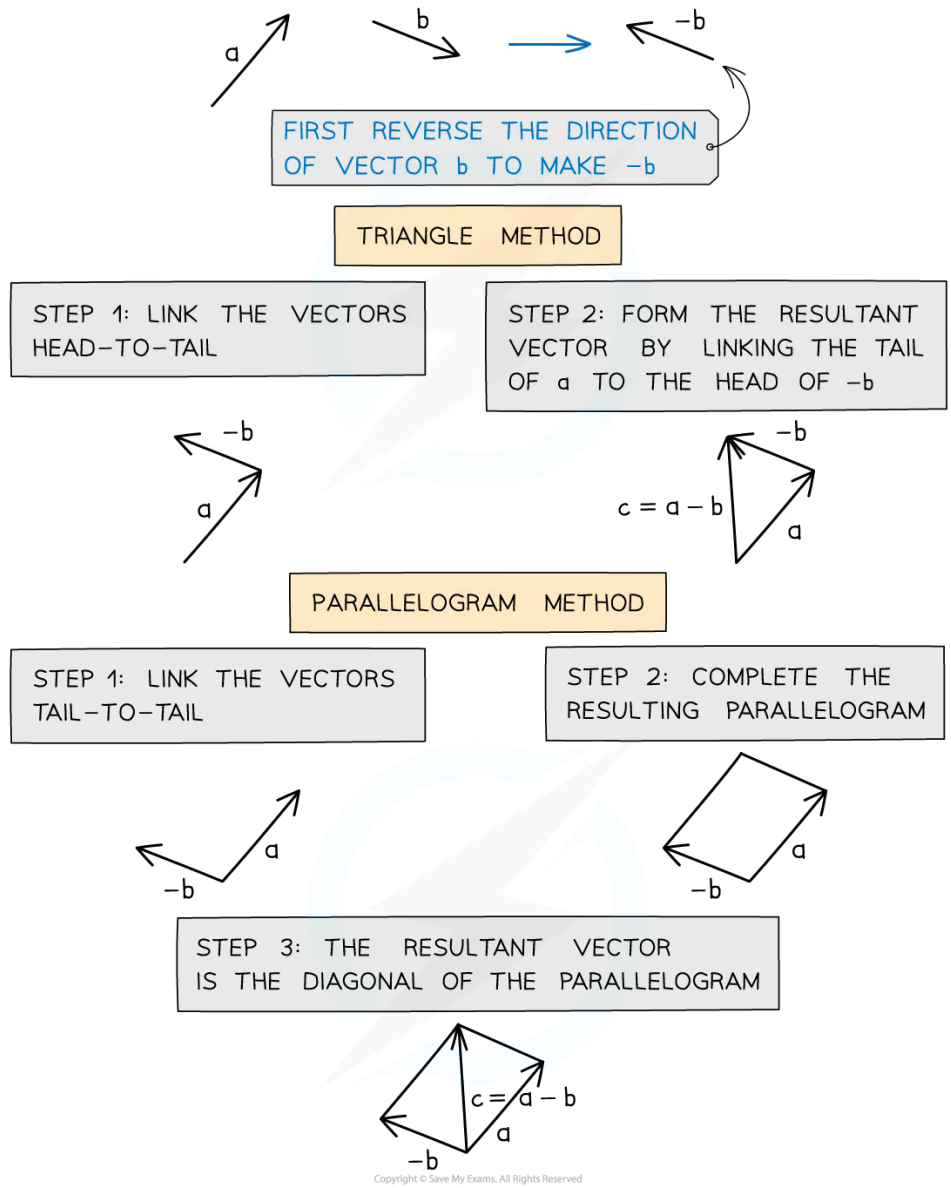
YOUR NOTES

Worked Example

Draw the vector c = a – b.

Answer:



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Vector Multiplication

- The product of a scalar and a vector is **always** a vector
- For example, consider the scalar quantity **mass** m and the vector quantity **acceleration** $\vec{a}$
- The product of mass m and acceleration $\vec{a}$ gives rise to a vector quantity **force** $\vec{F}$

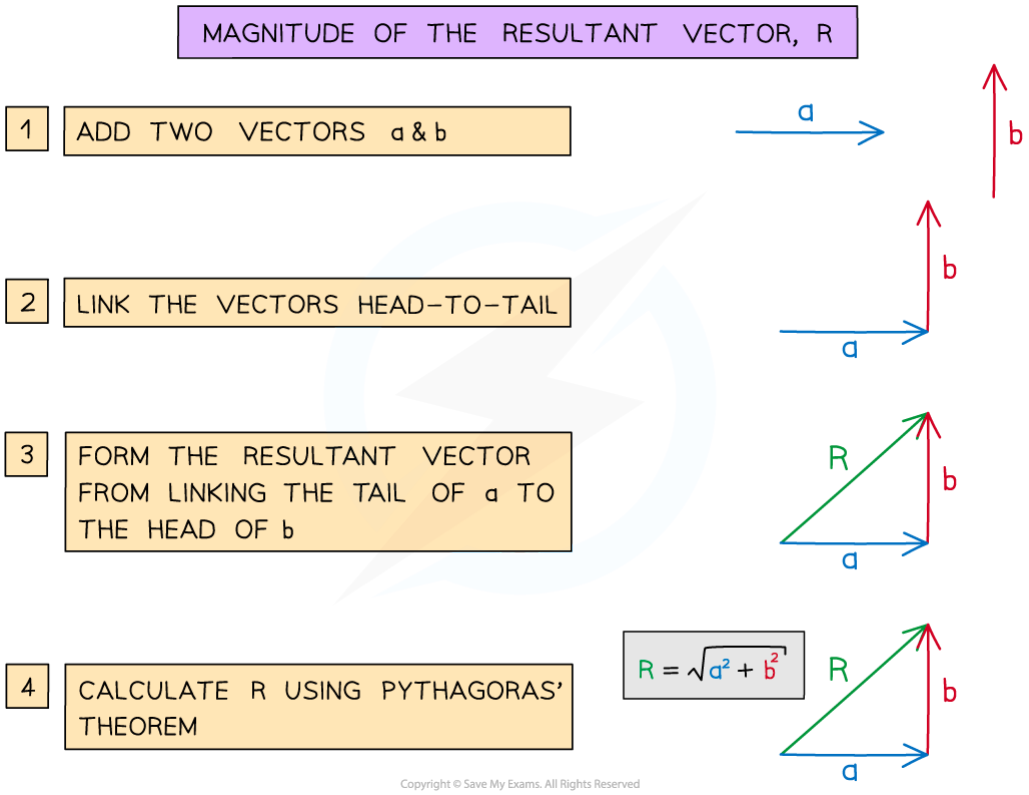
$$\vec{F} = m \times \vec{a}$$

- For another example, consider the scalar quantity **mass** m and the vector quantity **velocity** $\vec{v}$
- The product of mass m and velocity $\vec{v}$ gives rise to a vector quantity **momentum** $\vec{p}$

$$\vec{p} = m \times \vec{v}$$

Resolving Vectors

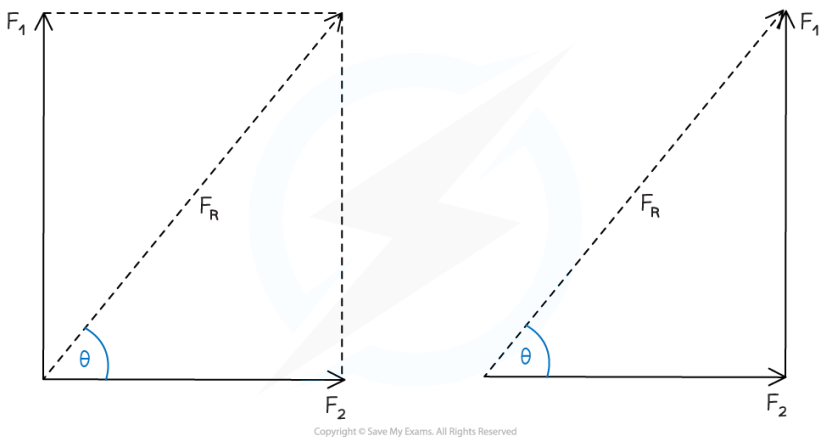
- Two vectors can be represented by a single **resultant vector**
  - Resolving a vector is the opposite of adding vectors
- A single resultant vector can be resolved
  - This means it can be represented by **two** vectors, which in combination have the same effect as the original one



The magnitude of the resultant vector is found by using Pythagoras' Theorem

- When a single resultant vector is broken down into its **parts**, those parts are called **components**
- For example, a force vector of magnitude F_R and an angle of θ to the horizontal is shown below

YOUR NOTES



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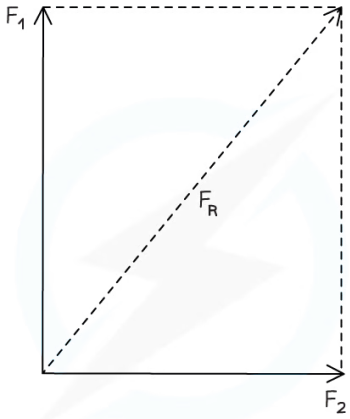
Resolving two force vectors F_1 and F_2 into a resultant force vector F_R

- It is possible to **resolve** this vector into its **horizontal** and **vertical** components using trigonometry



RESULTANT FORCE = EFFECT OF FORCE 1 + EFFECT OF FORCE 2

$$F_R^2 = F_1^2 + F_2^2$$

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The resultant force F_R can be split into its horizontal and vertical components

- The direction of the resultant vector is found from the angle it makes with the horizontal or vertical
  - The question should imply which angle it is referring to (i.e. calculate the angle from the x-axis)
- Calculating the angle of this resultant vector from the horizontal or vertical can be done using **trigonometry**
  - Either the sine, cosine or tangent formula can be used depending on which vector magnitudes are calculated
- For the **horizontal** component, $F_x = F \cos \theta$
- For the **vertical** component, $F_y = F \sin \theta$

YOUR NOTES

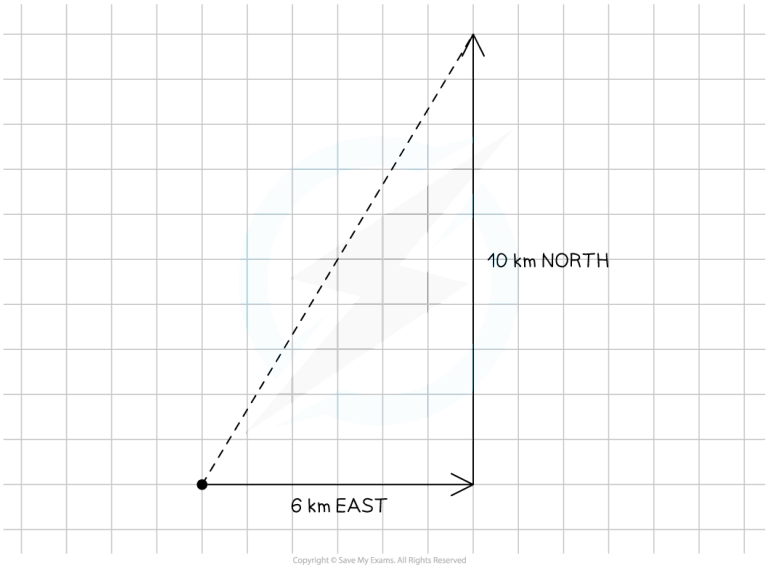
Worked Example

A hiker walks a distance of 6 km due east and 10 km due north.

Calculate the magnitude of their displacement and its direction from the horizontal.

Answer:

Step 1: Draw a vector diagram



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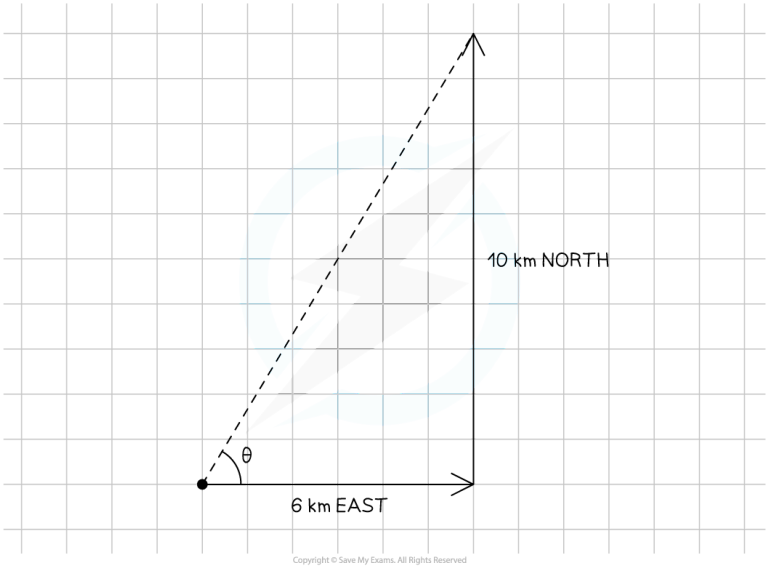
Step 2: Calculate the magnitude of the resultant vector using Pythagoras' Theorem

$$\text{Resultant vector} = \sqrt{6^2 + 10^2}$$

$$\text{Resultant vector} = \sqrt{136}$$

Resultant vector = **11.66**

Step 3: Calculate the direction of the resultant vector using trigonometry



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$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{10}{6}$$

$$\theta = \tan^{-1}\left(\frac{10}{6}\right) = 59°$$

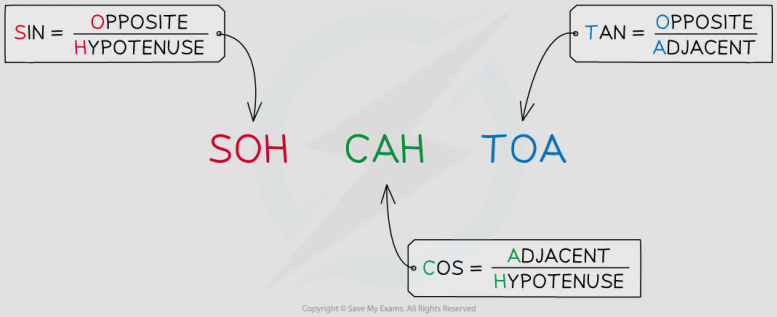
Step 4: State the final answer complete with direction

- Vector magnitude: 12 km
- Direction: 59° east and upwards from the horizontal

YOUR NOTES

Exam Tip

Make sure you are confident using trigonometry as it is used a lot in vector calculations!



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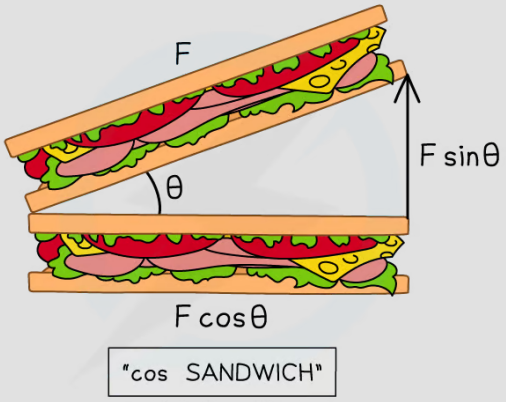
If you're unsure as to which component of the force is cos θ or sin θ, just remember that the cos θ is always the adjacent side of the right-angled triangle AKA, making a 'cos sandwich'



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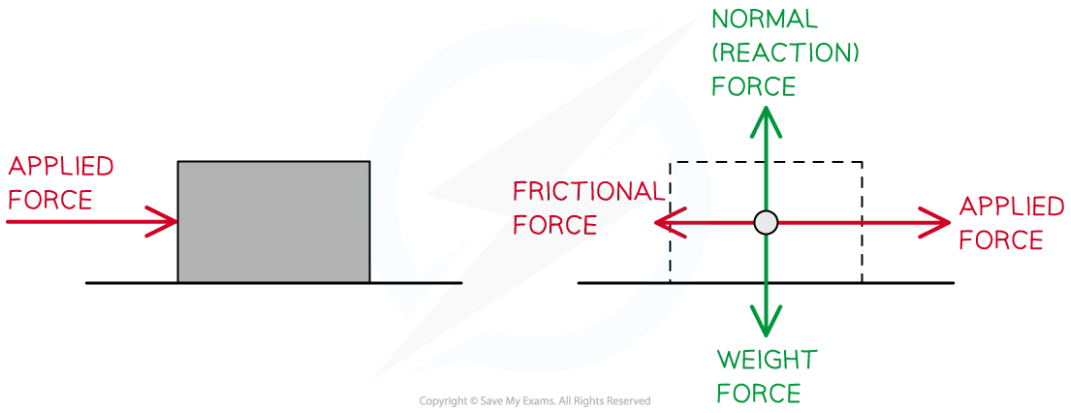
Force as a Vector

YOUR NOTES

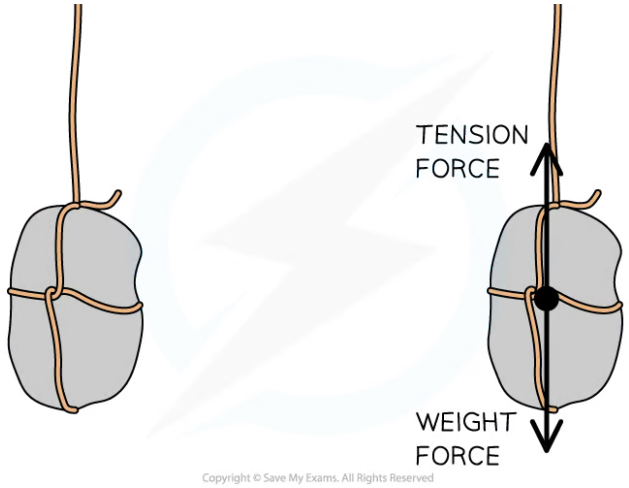
- In physics, vectors appear in many different topic areas
  - Specifically, vectors are often **combined** and **resolved** to solve problems when considering motion, forces, and momentum

- Forces vector diagrams are often represented by free-body force diagrams
- The rules for drawing a free-body diagram are the following:
  - **Rule 1:** Draw a point in the centre of mass of the body
  - **Rule 2:** Draw the body free from contact with any other object
  - **Rule 3:** Draw the forces acting on that body using vectors with length in proportion to its magnitude
  - **Rule 4:** Draw the tail of the vector from the centre of mass and use the tip to indicate the direction



Point particle representation of the forces acting on a moving object on a rough horizontal surface

- The below example shows the forces acting on an object suspended from a stationary rope

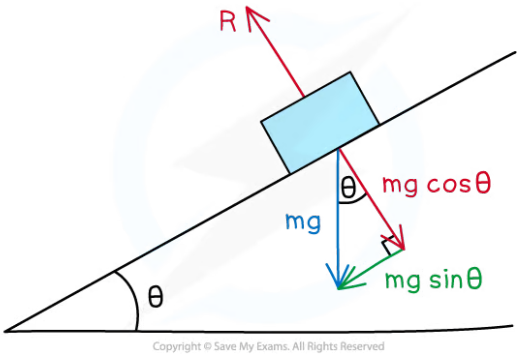


Free-body diagram of an object suspended from a stationary rope

YOUR NOTES

Forces on an Inclined Plane

- A common scenario is an object on an inclined plane
- An inclined plane, or a slope, is a flat surface tilted at an angle, θ



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The weight vector of an object on an inclined plane can be split into its components parallel and perpendicular to the slope

- Inclined slope problems can be simplified by considering the components of the forces as **parallel** or **perpendicular** to the slope
- The **weight** ($W = mg$) of the object is always directed **vertically downwards**
- On the inclined slope, weight can be split into the following components:

Perpendicular to the slope: $\swarrow W = mg\cos\theta$

Parallel to the slope: $\searrow W = mg\sin\theta$

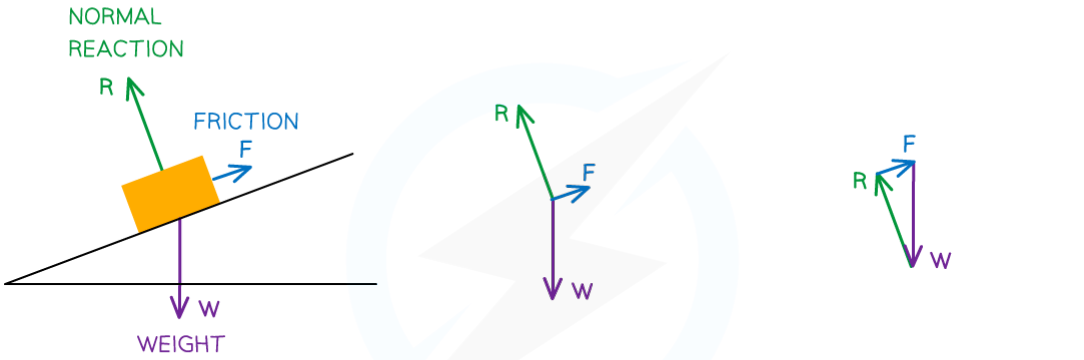
- The **normal** (or reaction) force R is always **vertically upwards**, or perpendicular to the surface
- If there is **no friction**, the parallel component of weight, $mg\sin\theta$, causes the object to move down the slope
- If the object is **not moving** perpendicular to the slope, the normal force is $R = mg\cos\theta$

Equilibrium

- Coplanar forces can be represented by vector triangles
- Forces are in equilibrium if an object is either
  - At rest
  - Moving at **constant** velocity
- In equilibrium, coplanar forces are represented by **closed** vector triangles
  - The vectors, when joined together, form a closed path
- The most common forces on objects are
  - Weight
  - Normal reaction force
  - Tension (from cords and strings)
  - Friction
- The forces on a body in equilibrium are demonstrated below:

YOUR NOTES

A VEHICLE IS AT REST ON A SLOPE AND HAS THREE FORCES ACTING ON IT TO KEEP IT IN EQUILIBRIUM



STEP 1:
DRAW ALL THE FORCES ON THE FREE-BODY DIAGRAM

STEP 2:
REMOVE THE OBJECT AND PUT ALL THE FORCES COMING FROM A SINGLE POINT

STEP 3:
REARRANGE THE FORCES INTO A CLOSED VECTOR TRIANGLE.
KEEP THE SAME LENGTH AND DIRECTION

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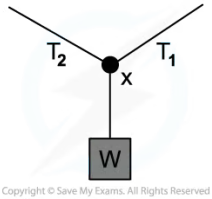
Three forces on an object in equilibrium form a closed vector triangle

YOUR NOTES

Worked Example

A weight hangs in equilibrium from a cable at point **X**. The tensions in the cables are T_1 and T_2 as shown.



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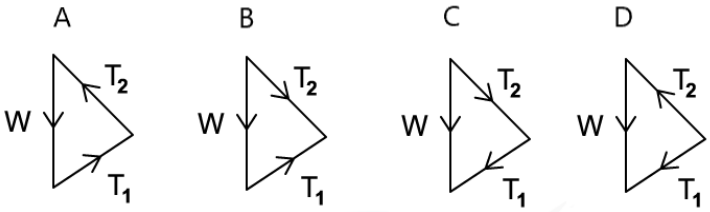
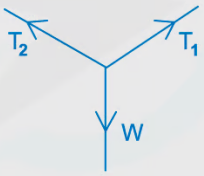
Which diagram correctly represents the forces acting at point **X**?



ANSWER: A

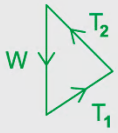
STEP 1

IDENTIFY THE DIRECTION OF ALL THE FORCES



STEP 2

ARRANGE THESE INTO A VECTOR TRIANGLE KEEPING THE SAME MAGNITUDE AND DIRECTIONS



STEP 3

ENSURE THE DIRECTION OF THE VECTORS FORM A CLOSED PATH

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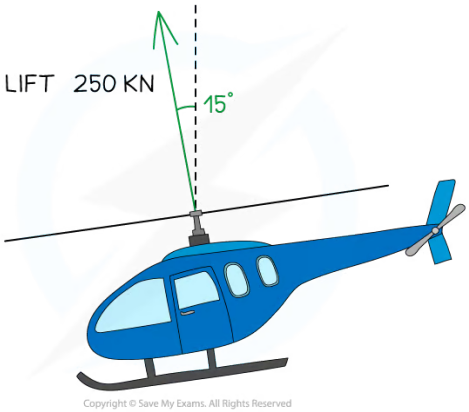
YOUR NOTES

Worked Example

A helicopter provides a lift of 250 kN when the blades are tilted at 15° from the vertical.



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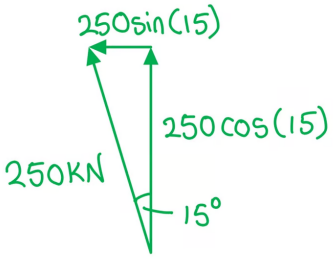
Calculate the horizontal and vertical components of the lift force.

Answer:

Step 1: Draw a vector triangle of the resolved forces



Step 2: Calculate the vertical component of the lift force

Vertical component of force = 250 × cos(15) = 242 kN

Step 3: Calculate the horizontal component of the lift force

Horizontal component of force = 250 × sin(15) = 64.7 kN

Exam Tip

When labelling force vectors, it is important to use conventional and appropriate naming or symbols such as:

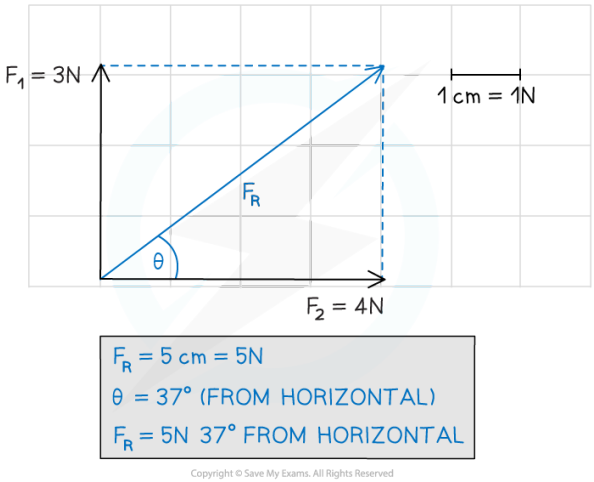
- ***w*** or weight force or ***mg***
- ***N*** or ***R*** for normal reaction force (depending on your local context either of these could be acceptable)

Using unexpected notation can lead to losing marks so try to be consistent with expected conventions.

YOUR NOTES

Scale Diagrams

- There are two methods that can be used to combine or resolve vectors
  - **Calculation** – if the vectors are perpendicular
  - **Scale drawing** – if the vectors are not perpendicular
- Calculating vectors using a scale drawing involves drawings the lengths and angles of the vectors accurately using a sharp **pencil**, **ruler** and **protractor**



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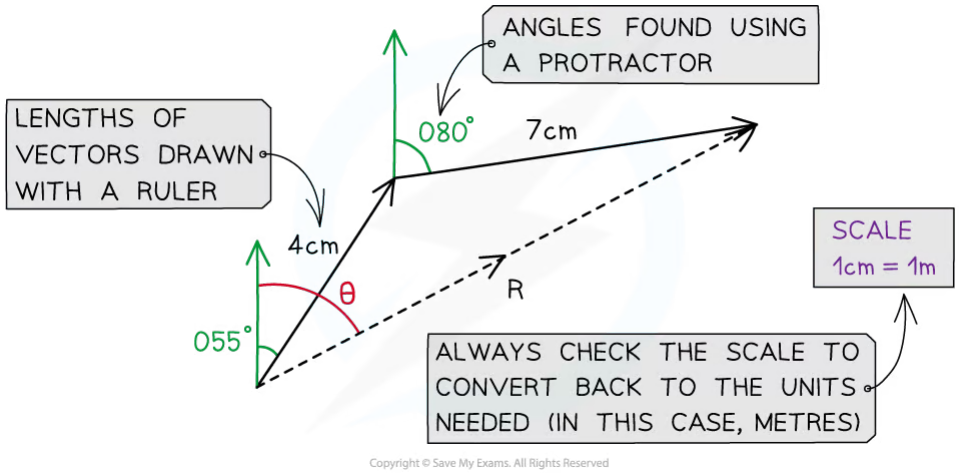
Vectors can be determined using scale diagrams

- When two vectors are **not** at right angles, the resultant vector can be calculated using a scale drawing
  - **Step 1:** Link the vectors head-to-tail if they aren't already
  - **Step 2:** Draw the resultant vector using the triangle or parallelogram method
  - **Step 3:** Measure the length of the resultant vector using a ruler
  - **Step 4:** Measure the angle of the resultant vector (from North if it is a bearing) using a protractor



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A scale drawing of two vector additions. The magnitude of resultant vector R is found using a rule and its direction is found using a protractor

- Note that with scale drawings, a scale may be given for the diagram such as 1 cm = 1 km since only limited lengths can be measured using a ruler
- The final answer is always converted back to the units needed in the diagram
    - Eg. For a scale of 1 cm = 2 km, a resultant vector with a length of 5 cm measured on your ruler is actually 10 km in the scenario

YOUR NOTES

YOUR NOTES

Worked Example

A hiker walks a distance of 6 km due east and 10 km due north.

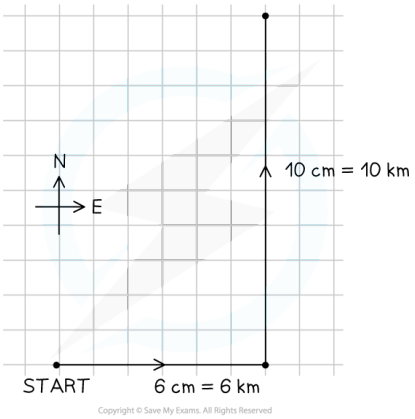
By making a scale drawing of their route, find the magnitude of their displacement and its direction from the horizontal.

Answer:

Step 1: Choose a sensible scale

- The distances are 6 and 10 km, so a scale of 1 cm = 1 km will fit easily on the page, but be large enough for an accurate scale drawing

Step 2: Draw the two components using a ruler and make the measurements accurate to 1 mm



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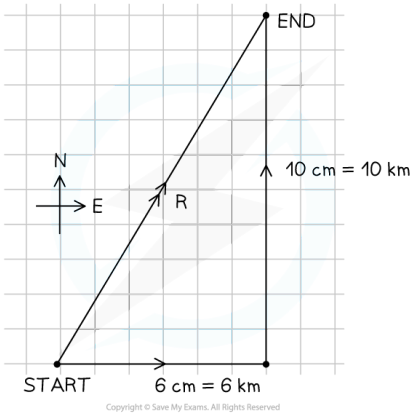
Step 3: Add the resultant vector, remembering the start and finish points of the journey



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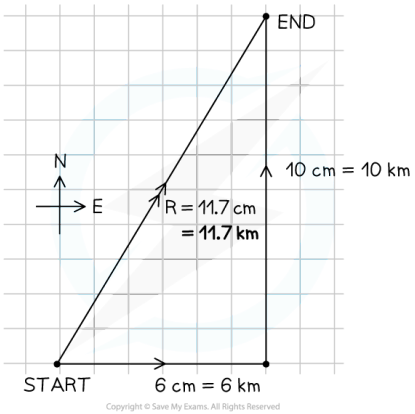
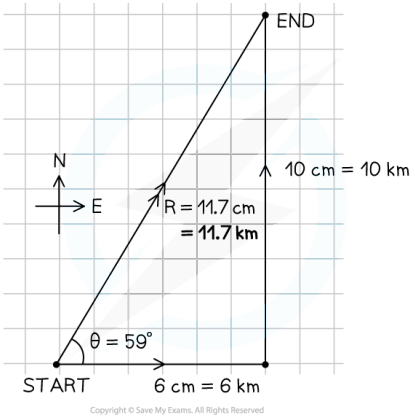
Step 4: Carefully measure the length of the resultant and convert using the scale



Step 5: Measure the angle between the vector and the horizontal line



Step 6: Write the complete answer, giving both magnitude and direction

- Magnitude: $R = 11.7$ km
- Direction: $\theta = 59°$

Exam Tip

It should be noted that some of the examples used on this page demonstrate the use of scale diagrams where the vectors are placed at right angles - it would be quicker to determine the resultant force of these via calculation as simple trigonometry can be used

Scale diagram questions will typically involve vector triangles that do not contain a right angle