ESAT PRACTICE TEST SERIES

ESAT Mock Test 1

Math 1 + Physics + Math 2

Duration: 2 hours (3 x 40 min)

Correct answers, worked shortcuts and diagram review booklet

ThrivingScholars

Worked solutions and answer key

This booklet contains the full student-facing answer key and concise worked solutions for ESAT Mock Test 1. Each question has the correct answer beside it, the correct option highlighted, and a short method underneath.

Math 1

1 G 2 C 3 B 4 B 5 E 6 G 7 D 8 C 9 B 10 C 11 A 12 D
13 B 14 A 15 D 16 C 17 B 18 D 19 C 20 E 21 E 22 C 23 E 24 D
25 E 26 C 27 C

Physics

1 F 2 A 3 D 4 E 5 G 6 C 7 D 8 E 9 D 10 D 11 F 12 B
13 C 14 G 15 D 16 E 17 B 18 C 19 G 20 D 21 E 22 G 23 H 24 C
25 D 26 C 27 B

Math 2

1 A 2 F 3 D 4 A 5 D 6 D 7 F 8 B 9 D 10 D 11 B 12 E
13 E 14 B 15 C 16 A 17 D 18 A 19 C 20 B 21 A 22 A 23 C 24 C
25 D 26 C 27 D

ESAT MOCK TEST 1

Math 1

27 questions · Answers placed beside each question for fast review.

MATH 1

Question 1

Answer **G**

If the 5-digit number $1X2X3$ is divisible by 9, what is X?

- **A** 0
- **B** 1
- **C** 2
- **D** 3
- **E** 4
- **F** 5
- **G** 6

SHORT SOLUTION

The digit sum is $1+X+2+X+3=6+2X$. For divisibility by 9, $6+2X$ must be a multiple of 9. From the choices, $X=6$ gives 18.

MATH 1

Question 2

Answer **C**

If we define $a \star b = a^2 - b^2$, what is $5 \star 4$?

- **A** 1
- **B** 7
- **C** 9
- **D** 16
- **E** 25
- **F** 41

SHORT SOLUTION

Apply the definition directly: $5 \star 4 = 5^2 - 4^2 = 25 - 16 = 9$.

MATH 1

Question 3

Answer **B**

Jack tosses two fair dice. What is the probability that the sum of the two values is no less than 9?

Recall that

$$\text{Probability} = \frac{\text{Number of valid cases}}{\text{Number of all possible cases}}.$$

A $\frac{1}{4}$

B $\frac{5}{18}$

C $\frac{1}{3}$

D $\frac{5}{12}$

E $\frac{2}{3}$

SHORT SOLUTION

For two dice, sums at least 9 are $9, 10, 11, 12$, giving $4+3+2+1=10$ outcomes. So the probability is $10/36 = 5/18$.

MATH 1

Question 4

Answer **B**

How many 4-digit integers $\overline{abcd}$ satisfy that

1 The thousands digit a and the tens digit c are even numbers;

2 The hundreds digit b and the unit digit d are odd numbers.

3 This number is divisible by 5?

A 80

B 100

C 125

D 250

E 500

SHORT SOLUTION

Since the number is divisible by 5, the final digit must be 5 because d is odd. Then $a \in \{2, 4, 6, 8\}$, b has 5 choices, and c has 5 choices, giving $4 \cdot 5 \cdot 5 \cdot 1 = 100$.

MATH 1

Question 5

Answer E

If $\frac{m}{10} = \frac{6}{n} = \frac{2}{5}$, what is $m + n$?

A 10

B 13

C 14

D 16

E 19

SHORT SOLUTION

From $m/10 = 2/5$, $m = 4$. From $6/n = 2/5$, $30 = 2n$, so $n = 15$. Hence $m + n = 19$.

MATH 1

Question 6

Answer G

Nancy has 100 marbles. She gives 20% of them to her friend Pedro. Then Nancy gives 25% of what is left to another friend, Ben. Finally, Nancy gives 10% of what is now left in the bag to her brother Jimmy. What percentage of her original bag of marbles does Nancy have left for herself?

A 20

B 30

C $33\frac{1}{3}$

D 38

E 45

F 50

G 54

H 60

SHORT SOLUTION

Track what remains: after 20% is given away, 80 remain; after giving 25% of 80, 60 remain; after giving 10% of 60, 54 remain.

MATH 1

Question 7

Answer **D**

What is the sum of all positive 2-digit integers that have a remainder of 1 when divided by 9 and a remainder of 2 when divided by 5?

- **A** 29
- **B** 56
- **C** 84
- **D** 119
- **E** 164

SHORT SOLUTION

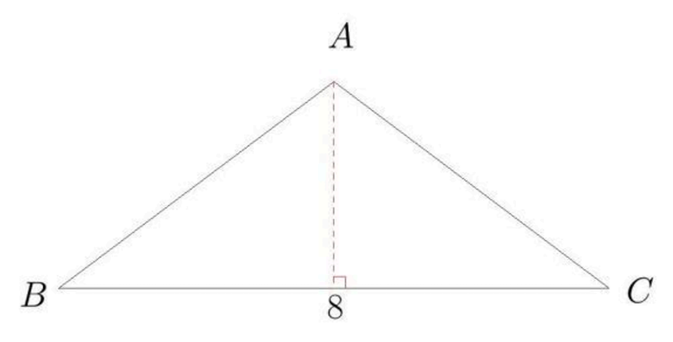
The numbers satisfying both conditions are 37 and 82: each is 1 more than a multiple of 9 and 2 more than a multiple of 5. Their sum is 119.

MATH 1

Question 8

Answer **C**

$\triangle ABC$ is an isosceles triangle. $AB = AC$. If $BC = 8$ and the area of $\triangle ABC$ is 12, what is the perimeter of $\triangle ABC$?



- **A** 12
- **B** 16
- **C** 18
- **D** 19
- **E** 20

SHORT SOLUTION

The height is h where $\frac{1}{2} \cdot 8 \cdot h = 12$, so $h = 3$. The altitude bisects the base, giving half-base 4, so each equal side is $\sqrt{3^2 + 4^2} = 5$. Perimeter $= 8 + 5 + 5 = 18$.

MATH 1

Question 9

Answer **B**

What is the smaller angle between the hour hand and minute hand of a clock at $5{:}50\ \mathrm{pm}$?

A $100°$

B $125°$

C $150°$

D $155°$

E $165°$

SHORT SOLUTION

At $5{:}50$, the minute hand is at $300°$. The hour hand is at $5 \cdot 30 + 50 \cdot 0.5 = 175°$. The smaller angle is $300 - 175 = 125°$.

MATH 1

Question 10

Answer **C**

The average age of 5 people in a room is 40 years. A 36-year-old person leaves the room. What is the average age of the four remaining people?

A 39

B 40

C 41

D 43

E 45

SHORT SOLUTION

The original total age is $5 \cdot 40 = 200$. After the 36-year-old leaves, the total is 164, so the new average is $164/4 = 41$.

MATH 1

Question 11

Answer A

In how many ways can we write 127 as the sum of two prime numbers?

- **A** 0
- **B** 1
- **C** 2
- **D** 3
- **E** 4

SHORT SOLUTION

An odd number written as a sum of two primes must use 2 plus an odd prime. Here $127 = 2 + 125$, but 125 is not prime, so there are no ways.

MATH 1

Question 12

Answer D

If for some positive integer n,

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\cdots\left(1+\frac{1}{n}\right) = 50,$$

what is n?

- **A** 20
- **B** 39
- **C** 65
- **D** 99
- **E** 100

SHORT SOLUTION

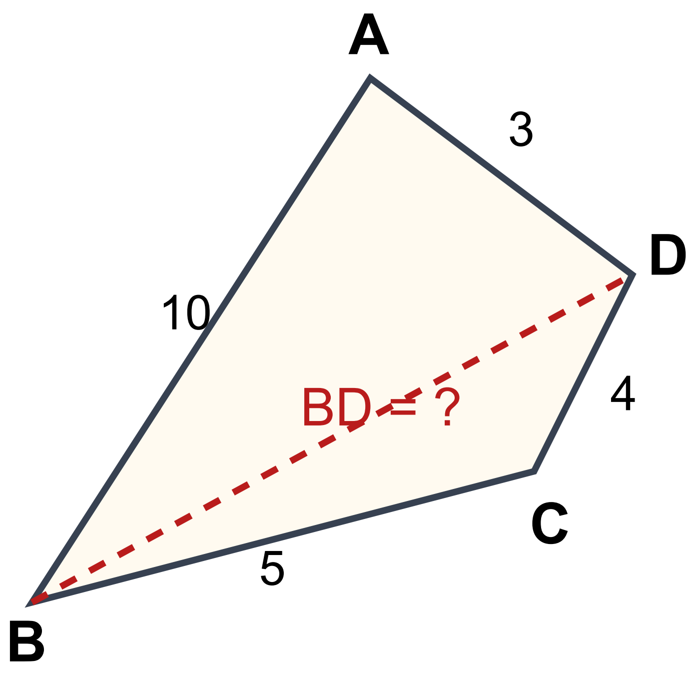
Each factor is $1 + 1/k = (k+1)/k$. The product telescopes: $\frac{3}{2} \cdot \frac{4}{3} \cdots \frac{n+1}{n} = \frac{n+1}{2} = 50$. Thus $n = 99$.

MATH 1

Question 13

Answer **B**

In quadrilateral $ABCD$, $AB = 10$, $BC = 5$, $CD = 4$ and $AD = 3$. If the length of BD is an integer, what is BD?



A 7

B 8

C 9

D 10

E there is not enough information

SHORT SOLUTION

Use the triangle inequality in the two triangles that share BD. For $\triangle ABD$, \(|10-3|

MATH 1

Question 14

Answer **A**

What is the unit digit of 7^{2020}? Here

$$7^n = \underbrace{7 \times 7 \times 7 \times \cdots \times 7}_{n}.$$

(Hint: check unit digits of 7, 7^2, 7^3, $\ldots$ and find the pattern.)

- **A** 1
- **B** 3
- **C** 5
- **D** 7
- **E** 9

SHORT SOLUTION

The units digits of powers of 7 cycle as $7, 9, 3, 1$. Since 2020 is divisible by 4, the units digit is the fourth term of the cycle, 1.

MATH 1

Question 15

Answer **D**

Here x, y, z and u represent four different numbers.

$$\begin{array}{cc} x & y \\ +z & x \\ \hline u & x \end{array} \qquad \begin{array}{cc} x & y \\ -z & x \\ \hline & x \end{array}$$

$x + y + z + u =$

- **A** 12
- **B** 14
- **C** 16
- **D** 18
- **E** 20

SHORT SOLUTION

From the subtraction, $(10x + y) - (10z + x) = x$, so $8x + y = 10z$. From the addition, $(10x + y) + (10z + x) = 10u + x$, so $10x + y + 10z = 10u$. The distinct digit solution is $(x, y, z, u) = (5, 0, 4, 9)$, giving 18.

MATH 1

Question 16

Answer **C**

Billy, a seasonal worker in the town of Cowra, collected 8 buckets of cherries on his first day. Each day after that he increased the number of buckets he picked by 2 buckets per day. According to this pattern, how many buckets will he collect in the first 20 days?

- **A** 308
- **B** 500
- **C** 540
- **D** 580
- **E** 600

SHORT SOLUTION

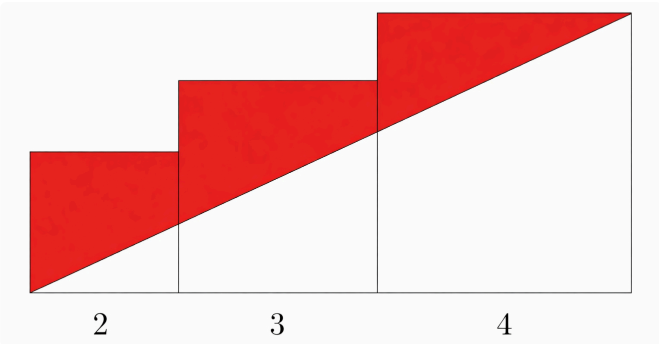
The buckets form an arithmetic sequence: $8, 10, 12, \ldots$. The 20th term is $8 + 19 \cdot 2 = 46$. Sum $= \frac{20}{2}(8 + 46) = 540$.

MATH 1

Question 17

Answer **B**

Three squares are lined up horizontally as shown. Their side lengths are 2, 3 and 4 respectively. A straight line is drawn from the top right corner of the largest square to the bottom left hand corner of the smallest square. What is the area of the shaded region?



A 10

B 11

C 13

D 16

E 18

SHORT SOLUTION

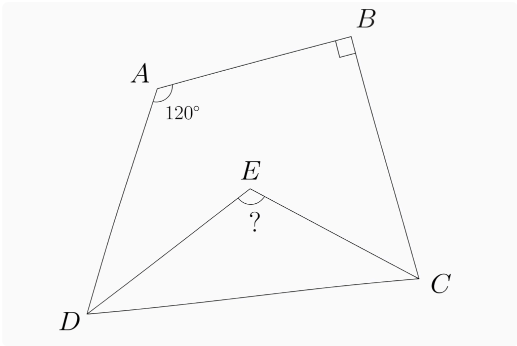
The total area of the three squares is $2^2 + 3^2 + 4^2 = 29$. The line cuts off one large triangle of base 9 and height 4, area 18. Shaded area $= 29 - 18 = 11$.

MATH 1

Question 18

Answer **D**

In quadrilateral $ABCD$, $\angle A = 120°$ and $\angle B = 90°$. ED and EC bisect $\angle ADC$ and $\angle BCD$ respectively. What is $\angle E$?



A 90

B 95

C 100

D 105

E 110

SHORT SOLUTION

In the quadrilateral, $\angle C + \angle D = 360° - 120° - 90° = 150°$. Since EC and ED bisect those angles, triangle DCE has angles at C, D summing to $75°$. Therefore $\angle E = 180° - 75° = 105°$.

MATH 1

Question 19

Answer **C**

In a mathematics contest with 10 problems, a student gains 6 points for a correct answer and loses 1 point for an incorrect answer. If Olivia answered every problem and her score was 32, how many correct answers did she have?

A 4

B 5

C 6

D 7

E 8

SHORT SOLUTION

Let c be the number correct. Then incorrect answers are $10 - c$, so $6c - (10 - c) = 32$. Hence $7c = 42$ and $c = 6$.

MATH 1

Question 20

Answer E

Emily and Owen are going to a library that is 1 mile from their house. They leave home simultaneously. Emily rides her bicycle to the library at a constant speed of 12 miles per hour. Owen walks to the library at a constant speed of 5 miles per hour. How many minutes before Owen does Emily arrive?

A 3

B 4

C 5

D 6

E 7

SHORT SOLUTION

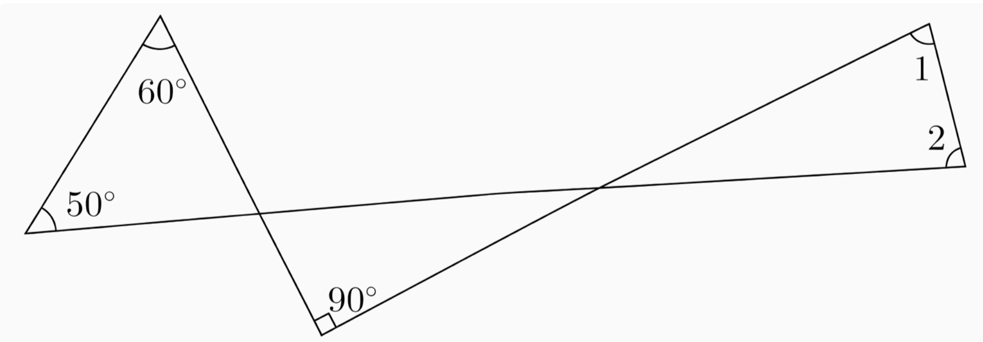
Emily takes $1/12$ hour $= 5$ minutes. Owen takes $1/5$ hour $= 12$ minutes. Emily arrives $12 - 5 = 7$ minutes earlier.

MATH 1

Question 21

Answer E

In the following picture, if $\angle 1 = \angle 2$, what is $\angle 1$?



A $60°$

B $65°$

C $70°$

D $75°$

E $80°$

SHORT SOLUTION

The left triangle gives the angle at the first intersection as $180° - 50° - 60° = 70°$. The next triangle has a right angle, so the angle at the second intersection is $20°$. The final small triangle has angles $x, x, 20°$, so $2x = 160°$, giving $x = 80°$.

MATH 1

Question 22

Answer **C**

Jack, a zoo keeper, is going to give N peaches to a group of monkeys. If he gives 10 peaches to each monkey, two monkeys will receive no peach. Or, the peaches are exactly enough to give 8 to each monkey. What is the sum of digits of N?

- **A** 5
- **B** 6
- **C** 8
- **D** 9
- **E** 10

SHORT SOLUTION

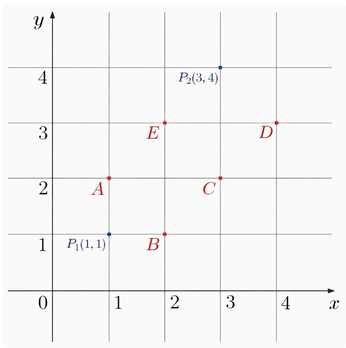
Let there be m monkeys. The two descriptions give $N = 8m$ and $N = 10(m-2)$. Thus $8m = 10m - 20$, so $m = 10$ and $N = 80$. Digit sum $= 8$.

MATH 1

Question 23

Answer **E**

Every point on the plane can be assigned with a coordinate that has two numbers. For example, the coordinate of point P_1 in the following picture is $(1, 1)$ and the coordinate of point P_2 is $(3, 4)$. Which of the following points has the coordinate $(2, 3)$?



A A

B B

C C

D D

E E

SHORT SOLUTION

Read the grid directly: the point at $(2, 3)$ is labelled E.

MATH 1

Question 24

Answer **D**

A pizza-shop offers a basic version of pizza with mozzarella and tomatoes. Two different toppings from the following list must be added: anchovies, artichokes, mushrooms, capers. Moreover, for each pizza three different sizes are available: small, medium, large. How many different types of pizza are available at all?

A 11

B 12

C 15

D 18

E 36

SHORT SOLUTION

Choose 2 toppings from 4: $\binom{4}{2} = 6$. For each, there are 3 sizes, so total pizzas $= 6 \cdot 3 = 18$.

MATH 1

Question 25

Answer **E**

Bag A: 1 Snickers bar + 1 Butterfinger bar.
Bag B: 1 Butterfinger bar + 3 Reese's Peanut Butter Cups.
Bag C: 1 Snickers bar + 2 Reese's Peanut Butter Cups.

If each bag costs 50 cents, what is the price of
1 Snickers bar + 1 Butterfinger bar + 1 Reese's Peanut Butter Cup,
in cents?

- **A** 40
- **B** 45
- **C** 50
- **D** 55
- **E** 60

SHORT SOLUTION

Let the prices be S, B, R. The bags give $S+B=50$, $B+3R=50$, and $S+2R=50$. Solving gives $S=30$, $B=20$, $R=10$, so $S+B+R=60$.

MATH 1

Question 26

Answer **C**

For how many positive integers x is

$$(x-1)(x-4)(x-9)$$

negative?

- **A** 2
- **B** 3
- **C** 4
- **D** 5
- **E** 6

SHORT SOLUTION

The roots are 1, 4 and 9. Testing the sign in the intervals shows the product is negative for $x<1$ and \(4

MATH 1

Question 27

Answer **C**

A gumball machine contains 9 red, 7 white, and 8 blue gumballs. The least number of gumballs a person must buy to be sure of getting four gumballs of the same color is

- **A** 8
- **B** 9
- **C** 10
- **D** 12
- **E** 18

SHORT SOLUTION

The most gumballs possible without having four of one colour is $3+3+3=9$. One more guarantees four of some colour, so the least number is 10.

ESAT MOCK TEST 1

Physics

27 questions · Answers placed beside each question for fast review.

PHYSICS

Question 1

Answer **F**

Which of the following statements are **FALSE**?

A Electromagnetic waves cause things to heat up.

B X-rays and gamma rays can knock electrons out of their orbits.

C Loud sounds can make objects vibrate.

D Wave power can be used to generate electricity.

E Since waves carry energy away, the source of a wave loses energy.

F The amplitude of a wave determines its mass.

SHORT SOLUTION

Waves are not objects with mass. Amplitude affects energy/intensity, not mass, so statement F is false.

PHYSICS

Question 2

Answer **A**

A spacecraft is analysing a newly discovered exoplanet. A rock of unknown mass falls from rest on the planet from a height of $30\,\mathrm{m}$, ignoring air resistance. Given that $g = 5.4\,\mathrm{m\,s^{-2}}$ on the planet, calculate the speed of the rock when it hits the ground and the time it took to fall.

	Speed ($\mathrm{ms^{-1}}$)	Time (s)
A	18	3.3
B	18	3.1
C	12	3.3
D	10	3.7
E	9	2.3
F	1	0.3

SHORT SOLUTION

Use $v^2 = u^2 + 2as$ with $u = 0$: $v^2 = 2(5.4)(30) = 324$, so $v = 18\,\mathrm{m\,s^{-1}}$. Also $s = \frac{1}{2}at^2$, so $30 = 2.7t^2$ and $t \approx 3.3\,\mathrm{s}$.

PHYSICS

Question 3

Answer **D**

A canoe floating on the sea rises and falls 7 times in 49 seconds. The waves pass it at a speed of $5\,\mathrm{m\,s^{-1}}$. How long are the waves?

- **A** $12\,\mathrm{m}$
- **B** $22\,\mathrm{m}$
- **C** $25\,\mathrm{m}$
- **D** $35\,\mathrm{m}$
- **E** $57\,\mathrm{m}$
- **F** $75\,\mathrm{m}$

SHORT SOLUTION

The period is $T = 49/7 = 7\,\mathrm{s}$. Since $v = f\lambda = \lambda/T$, $\lambda = vT = 5 \cdot 7 = 35\,\mathrm{m}$.

PHYSICS

Question 4

Answer **E**

Miss Orrell lifts her $37.5\,\mathrm{kg}$ bike for a distance of $1.3\,\mathrm{m}$ in $5\,\mathrm{s}$. The acceleration of free fall is $10\,\mathrm{m\,s^{-2}}$. What is the average power that she develops?

- **A** $9.8\,\mathrm{W}$
- **B** $12.9\,\mathrm{W}$
- **C** $57.9\,\mathrm{W}$
- **D** $79.5\,\mathrm{W}$
- **E** $97.5\,\mathrm{W}$
- **F** $98.0\,\mathrm{W}$

SHORT SOLUTION

Work done $= mgh = 37.5 \cdot 10 \cdot 1.3 = 487.5\,\mathrm{J}$. Power $= 487.5/5 = 97.5\,\mathrm{W}$.

PHYSICS

Question 5

Answer **G**

A truck accelerates at $5.6\,\mathrm{m\,s^{-2}}$ from rest for 8 seconds. Calculate the final speed and the distance travelled in 8 seconds.

	Final Speed ($\mathrm{ms^{-1}}$)	Distance (m)
A	40.8	119.2
B	40.8	129.6
C	42.8	179.2
D	44.1	139.2
E	44.1	179.7
F	44.2	129.2
G	44.8	179.2
H	44.8	179.7

SHORT SOLUTION

From rest, $v = at = 5.6 \cdot 8 = 44.8\,\mathrm{m\,s^{-1}}$. Distance $s = \frac{1}{2}at^2 = \frac{1}{2} \cdot 5.6 \cdot 64 = 179.2\,\mathrm{m}$.

PHYSICS

Question 6

Answer C

Which of the following statements is true when a sky diver jumps out of a plane?

A The sky diver leaves the plane and will accelerate until the air resistance is greater than their weight.

B The sky diver leaves the plane and will accelerate until the air resistance is less than their weight.

C The sky diver leaves the plane and will accelerate until the air resistance equals their weight.

D The sky diver leaves the plane and will accelerate until the air resistance equals their weight squared.

E The sky diver will travel at a constant velocity after leaving the plane.

SHORT SOLUTION

Initially weight is greater than air resistance, so the skydiver accelerates. As speed rises, air resistance increases until it equals weight; then acceleration becomes zero.

PHYSICS

Question 7

Answer D

A $100\,\mathrm{g}$ apple falls from rest on Isaac's head from a height of $20\,\mathrm{m}$, ignoring air resistance. Calculate the apple's momentum before the point of impact. Take $g = 10\,\mathrm{m\,s^{-2}}$.

A $0.1\,\mathrm{kg\,m\,s^{-1}}$

B $0.2\,\mathrm{kg\,m\,s^{-1}}$

C $1\,\mathrm{kg\,m\,s^{-1}}$

D $2\,\mathrm{kg\,m\,s^{-1}}$

E $10\,\mathrm{kg\,m\,s^{-1}}$

F $20\,\mathrm{kg\,m\,s^{-1}}$

SHORT SOLUTION

The apple mass is $0.1\,\mathrm{kg}$. Its speed before impact is $v = \sqrt{2gh} = \sqrt{2 \cdot 10 \cdot 20} = 20\,\mathrm{m\,s^{-1}}$. Momentum $p = mv = 0.1 \cdot 20 = 2\,\mathrm{kg\,m\,s^{-1}}$.

PHYSICS

Question 8

Answer **E**

Which of the following do all electromagnetic waves have in common?

1. They can travel through a vacuum.
2. They can be reflected.
3. They are the same length.
4. They have the same amount of energy.
5. They can be polarised.

- **A** 1, 2 and 3 only
- **B** 1, 2, 3 and 4 only
- **C** 4 and 5 only
- **D** 3 and 4 only
- **E** 1, 2 and 5 only
- **F** 1 and 5 only

SHORT SOLUTION

All electromagnetic waves can travel through vacuum, reflect, and be polarised. They do not all have the same wavelength or energy.

PHYSICS

Question 9

Answer **D**

A battery with an internal resistance of $0.8\,\Omega$ and e.m.f of $36\,\mathrm{V}$ is used to power a drill with resistance $1\,\Omega$. What is the current in the circuit when the drill is connected to the power supply?

- **A** $5\,\mathrm{A}$
- **B** $10\,\mathrm{A}$
- **C** $15\,\mathrm{A}$
- **D** $20\,\mathrm{A}$
- **E** $25\,\mathrm{A}$
- **F** $30\,\mathrm{A}$

SHORT SOLUTION

The internal and drill resistances are in series, so $R_{\text{total}} = 0.8 + 1 = 1.8\,\Omega$. Current $I = V/R = 36/1.8 = 20\,\mathrm{A}$.

PHYSICS

Question 10

Answer **D**

Officer Bailey throws a $20\,\mathrm{g}$ dart at a speed of $100\,\mathrm{m\,s^{-1}}$. It strikes the dartboard and is brought to rest in 10 milliseconds. Calculate the average force exerted on the dart by the dartboard.

- **A** $0.2\,\mathrm{N}$
- **B** $2\,\mathrm{N}$
- **C** $20\,\mathrm{N}$
- **D** $200\,\mathrm{N}$
- **E** $2000\,\mathrm{N}$
- **F** $20000\,\mathrm{N}$

SHORT SOLUTION

Mass $= 0.020\,\mathrm{kg}$, change in velocity $= 100\,\mathrm{m\,s^{-1}}$, and time $= 0.010\,\mathrm{s}$. Average force $F = \Delta p/\Delta t = 0.020 \cdot 100/0.010 = 200\,\mathrm{N}$.

PHYSICS

Question 11

Answer **F**

Professor Huang lifts a $50\,\mathrm{kg}$ bag through a distance of $0.7\,\mathrm{m}$ in $3\,\mathrm{s}$. What average power does she develop to 3 significant figures? Take $g = 10\,\mathrm{m\,s^{-2}}$.

- **A** $112\,\mathrm{W}$
- **B** $113\,\mathrm{W}$
- **C** $114\,\mathrm{W}$
- **D** $115\,\mathrm{W}$
- **E** $116\,\mathrm{W}$
- **F** $117\,\mathrm{W}$

SHORT SOLUTION

Work done $= mgh = 50 \cdot 10 \cdot 0.7 = 350\,\mathrm{J}$. Power $= 350/3 = 116.7\,\mathrm{W}$, which is $117\,\mathrm{W}$ to 3 significant figures.

PHYSICS

Question 12

Answer **B**

An electric scooter is travelling at a speed of $30\,\mathrm{m\,s^{-1}}$. The motor provides a driving force of $300\,\mathrm{N}$ in the direction of motion. Assuming all electrical power is converted into mechanical power, and given that the motor runs at $200\,\mathrm{V}$, calculate the current in the scooter.

- **A** $4.5\,\mathrm{A}$
- **B** $45\,\mathrm{A}$
- **C** $450\,\mathrm{A}$
- **D** $4500\,\mathrm{A}$
- **E** $45000\,\mathrm{A}$
- **F** More information needed.

SHORT SOLUTION

Use the motor driving force for power: $P = Fv = 300 \cdot 30 = 9000\,\mathrm{W}$. Since $P = VI$, $I = 9000/200 = 45\,\mathrm{A}$.

PHYSICS

Question 13

Answer **C**

Which of the following statements about the physical definition of work are correct?

1. $\text{Work done} = \frac{\text{Force}}{\text{distance}}$
2. The unit of work is equivalent to $\mathrm{kg\,m\,s^{-2}}$.
3. Work is defined as a force causing displacement of the body upon which it acts.

- **A** Only 1
- **B** Only 2
- **C** Only 3
- **D** 1 and 2
- **E** 2 and 3
- **F** 1 and 3

SHORT SOLUTION

Work is force times displacement in the direction of the force, not force divided by distance. Its unit is the joule, $\mathrm{kg\,m^2s^{-2}}$, not $\mathrm{kg\,m\,s^{-2}}$. Only statement 3 is correct.

PHYSICS

Question 14

Answer **G**

Which of the following statements about kinetic energy are correct?

1. It is defined as $E_k = \frac{mv^2}{2}$.
2. The unit of kinetic energy is equivalent to $\mathrm{Pa} \times \mathrm{m}^3$.
3. Kinetic energy is equal to the amount of energy needed to decelerate the body in question from its current speed.

- **A** Only 1
- **B** Only 2
- **C** Only 3
- **D** 1 and 2
- **E** 2 and 3
- **F** 1 and 3
- **G** 1, 2 and 3

SHORT SOLUTION

$E_k = \frac{1}{2}mv^2$. Also $\mathrm{Pa} \cdot \mathrm{m}^3 = (\mathrm{N}/\mathrm{m}^2)\mathrm{m}^3 = \mathrm{N\ m} = \mathrm{J}$. Kinetic energy is the work needed to bring the body to rest, so all three are correct.

PHYSICS

Question 15

Answer **D**

In relation to radiation, which of the following statements is **FALSE**?

- **A** Radiation is the emission of energy in the form of waves or particles.
- **B** Radiation can be either ionizing or non-ionizing.
- **C** Gamma radiation has very high energy.
- **D** Alpha radiation is of higher energy than beta radiation.
- **E** X-rays are an example of wave radiation.

SHORT SOLUTION

Radiation can be waves or particles, and gamma rays and X-rays are high-energy electromagnetic radiation. The statement that alpha radiation is always of higher energy than beta radiation is not a reliable general definition, so D is false.

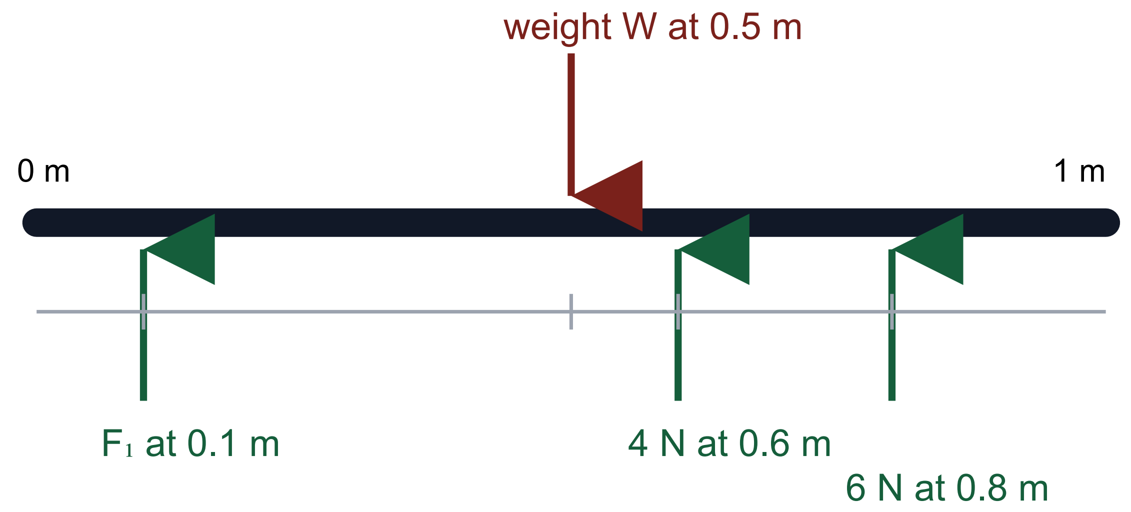
PHYSICS

Question 16

Answer **E**

A uniform rod of length $1\,\mathrm{m}$ is balanced on 3 supports so that it is in equilibrium. The position of the first support is at $x_1 = 0.1\,\mathrm{m}$, where x is the distance along the rod. The second support is at $x_2 = 0.6\,\mathrm{m}$ and applies a force, F_2, of $4\,\mathrm{N}$ to the rod. The third support is at $x_3 = 0.8\,\mathrm{m}$ and applies a force, F_3, of $6\,\mathrm{N}$ to the rod.

What is the force applied by the first support, F_1?



A $4.5\,\mathrm{N}$	**B** $5.0\,\mathrm{N}$
C $5.3\,\mathrm{N}$	**D** $6.7\,\mathrm{N}$
E $5.5\,\mathrm{N}$	**F** $11.0\,\mathrm{N}$
G $10.5\,\mathrm{N}$	**H** $7.3\,\mathrm{N}$

SHORT SOLUTION

Let the rod weight be W. Force balance gives $W = F_1 + 4 + 6 = F_1 + 10$. Taking moments about the left end: $0.1F_1 + 0.6(4) + 0.8(6) = 0.5W$. Hence $0.1F_1 + 7.2 = 0.5(F_1 + 10)$, so $F_1 = 5.5\,\mathrm{N}$.

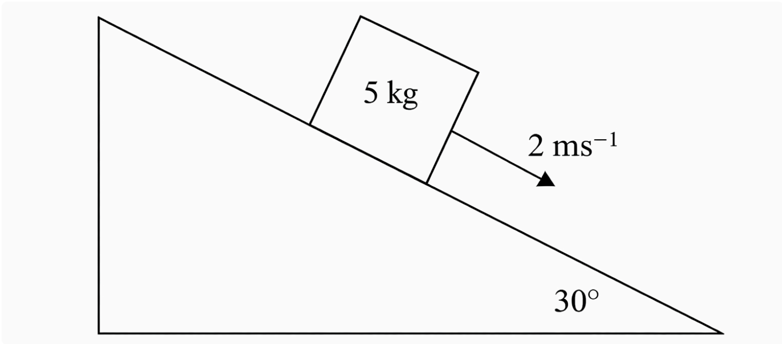
PHYSICS

Question 17

Answer **B**

A block of mass $5\,\mathrm{kg}$ slides down a rough slope angled at $30°$ to the horizontal. The block moves at a constant velocity of $2\,\mathrm{m\,s^{-1}}$. (Take $g = 10\,\mathrm{m\,s^{-2}}$.)

What is the frictional force on the block?



A $14\,\mathrm{N}$

B $25\,\mathrm{N}$

C $\frac{50}{\sqrt{2}}\,\mathrm{N}$

D $50\sqrt{2}\,\mathrm{N}$

E $25\sqrt{3}\,\mathrm{N}$

F $55\,\mathrm{N}$

G $50\sqrt{3}\,\mathrm{N}$

H $50\,\mathrm{N}$

SHORT SOLUTION

Constant velocity means zero resultant force along the slope. The downslope component of weight is $mg\sin 30° = 5 \cdot 10 \cdot \frac{1}{2} = 25\,\mathrm{N}$, so friction is $25\,\mathrm{N}$ upslope.

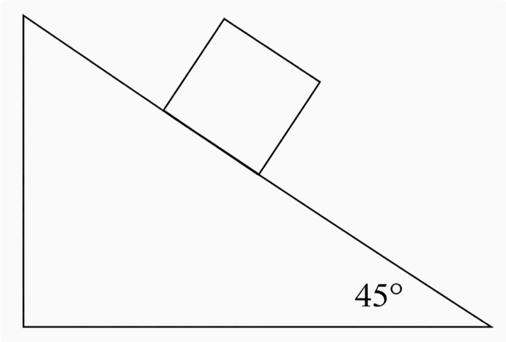
PHYSICS

Question 18

Answer **C**

A block slides down a smooth slope angled at 45° to the horizontal. The block is initially stationary. (Take $g = 10\,\mathrm{m\,s^{-2}}$.)

What is the velocity of the block after $10\,\mathrm{s}$?



A $50\,\mathrm{m\,s^{-1}}$

B $40\,\mathrm{m\,s^{-1}}$

C $\frac{100}{\sqrt{2}}\,\mathrm{m\,s^{-1}}$

D $\frac{200}{\sqrt{3}}\,\mathrm{m\,s^{-1}}$

E $\frac{100}{\sqrt{3}}\,\mathrm{m\,s^{-1}}$

F $75\,\mathrm{m\,s^{-1}}$

G $\frac{200}{3}\,\mathrm{m\,s^{-1}}$

H $\frac{200}{\sqrt{2}}\,\mathrm{m\,s^{-1}}$

SHORT SOLUTION

On a smooth slope, acceleration down the plane is $g\sin 45^\circ = 10/\sqrt{2}$. Starting from rest for 10 seconds gives $v = at = 100/\sqrt{2}\,\mathrm{m\,s^{-1}}$.

PHYSICS

Question 19

Answer **G**

An object of mass $10\,\mathrm{kg}$ falls from a great height. If the force due to air resistance can be modelled as $F_a = 0.5u\,\mathrm{N}$ where u is the velocity of the free-falling object, what is the terminal velocity of the object? (Take $g = 10\,\mathrm{N\,kg^{-1}}$.)

- **A** $700\,\mathrm{m\,s^{-1}}$
- **B** $150\,\mathrm{m\,s^{-1}}$
- **C** $50\,\mathrm{m\,s^{-1}}$
- **D** $100\,\mathrm{m\,s^{-1}}$
- **E** $800\,\mathrm{m\,s^{-1}}$
- **F** $400\,\mathrm{m\,s^{-1}}$
- **G** $200\,\mathrm{m\,s^{-1}}$
- **H** $500\,\mathrm{m\,s^{-1}}$

SHORT SOLUTION

At terminal velocity, air resistance equals weight: $0.5u = mg = 10 \cdot 10 = 100$. Therefore $u = 200\,\mathrm{m\,s^{-1}}$.

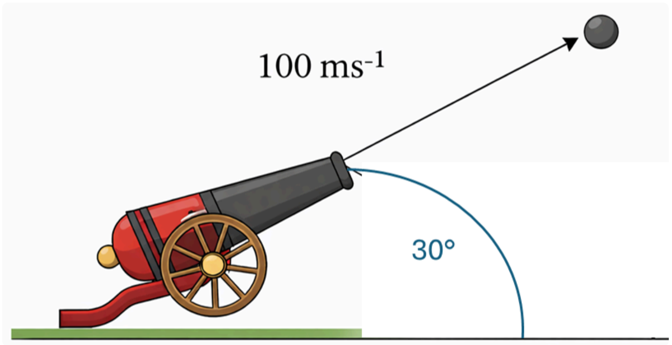
PHYSICS

Question 20

Answer **D**

A cannonball is fired from ground level at $30°$ to the horizontal with an initial velocity of $100\,\mathrm{m\,s^{-1}}$. The ground is flat and air resistance is negligible. (Take $g = 10\,\mathrm{N\,kg^{-1}}$.)

How far does the cannonball travel horizontally before it hits the ground?



- **A** $250\,\mathrm{m}$
- **B** $250\sqrt{2}\,\mathrm{m}$
- **C** $300\,\mathrm{m}$
- **D** $500\sqrt{3}\,\mathrm{m}$
- **E** $500\,\mathrm{m}$
- **F** $\frac{500}{\sqrt{2}}\,\mathrm{m}$
- **G** $600\sqrt{3}\,\mathrm{m}$
- **H** $600\sqrt{2}\,\mathrm{m}$

SHORT SOLUTION

Use projectile range $R = \frac{u^2 \sin 2\theta}{g}$. Thus $R = \frac{100^2 \sin 60°}{10} = 1000 \cdot \frac{\sqrt{3}}{2} = 500\sqrt{3}\,\mathrm{m}$.

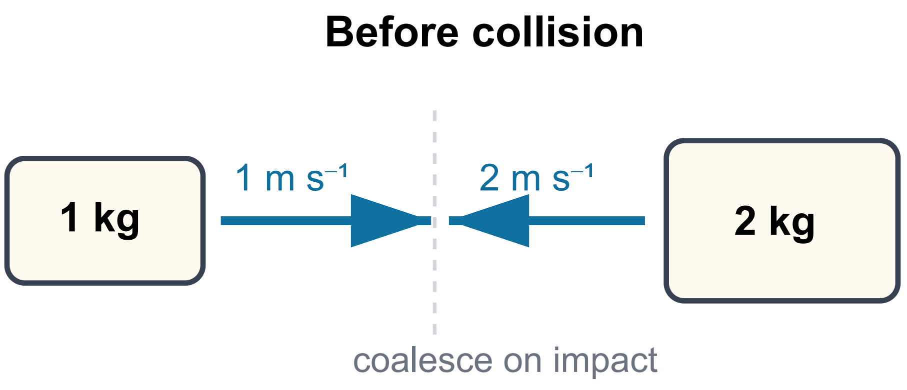
PHYSICS

Question 21

Answer **E**

Two blocks, one of mass $1\,\mathrm{kg}$ and one of mass $2\,\mathrm{kg}$, move directly towards each other and collide inelastically. Before collision, their velocities have magnitudes of $1\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively.

Assuming the two blocks coalesce on impact, what fraction of the kinetic energy is lost (e.g. to heat)?



A 1

B 0

C $\frac{1}{3}$

D $\frac{1}{2}$

E $\frac{2}{3}$

F $\frac{1}{4}$

G $\frac{3}{4}$

H $\frac{2}{5}$

SHORT SOLUTION

Initial momentum is $1(1) - 2(2) = -3$, so after coalescing, $v = -3/(1+2) = -1\,\mathrm{m\,s^{-1}}$. Initial KE is $\frac{1}{2}(1)(1^2) + \frac{1}{2}(2)(2^2) = 4.5$. Final KE is $\frac{1}{2}(3)(1^2) = 1.5$. Fraction lost $= (4.5 - 1.5)/4.5 = 2/3$.

PHYSICS

Question 22

Answer **G**

An object of mass $20\,\mathrm{kg}$ falls $100\,\mathrm{m}$ and experiences an average resistive force of $40\,\mathrm{N}$ due to air resistance. If the object is initially at rest, what is its final velocity? (Take $g = 10\,\mathrm{N\,kg^{-1}}$.)

- **A** $60\,\mathrm{m\,s^{-1}}$
- **B** $420\,\mathrm{m\,s^{-1}}$
- **C** $360\,\mathrm{m\,s^{-1}}$
- **D** $200\,\mathrm{m\,s^{-1}}$
- **E** $80\,\mathrm{m\,s^{-1}}$
- **F** $500\,\mathrm{m\,s^{-1}}$
- **G** $40\,\mathrm{m\,s^{-1}}$
- **H** $800\,\mathrm{m\,s^{-1}}$

SHORT SOLUTION

Energy lost to air resistance is $40 \cdot 100 = 4000\,\mathrm{J}$. Gravitational energy lost is $mgh = 20 \cdot 10 \cdot 100 = 20000\,\mathrm{J}$. Final KE $= 16000\,\mathrm{J} = \frac{1}{2}(20)v^2$, so $v = 40\,\mathrm{m\,s^{-1}}$.

PHYSICS

Question 23

Answer **H**

A velocity profile is given by

$$v = \frac{(5t+3)^2}{t^{\frac{1}{2}}}$$

(SI units). Find the total distance travelled from $t = 1$ to $t = 4\,\mathrm{s}$.

- **A** $490\,\mathrm{m}$
- **B** $955\,\mathrm{m}$
- **C** $1082\,\mathrm{m}$
- **D** $683\,\mathrm{m}$
- **E** $575\,\mathrm{m}$
- **F** $23\,\mathrm{m}$
- **G** $1820\,\mathrm{m}$
- **H** $468\,\mathrm{m}$

SHORT SOLUTION

Rewrite $v = (5t+3)^2 t^{-1/2} = 25t^{3/2} + 30t^{1/2} + 9t^{-1/2}$. Integrating gives $10t^{5/2} + 20t^{3/2} + 18t^{1/2}$. From 1 to 4: $(320 + 160 + 36) - (10 + 20 + 18) = 468\,\mathrm{m}$.

PHYSICS

Question 24

Answer **C**

A spring is used to lift $20\,\mathrm{cm}^3$ of loosely packed sand in a small bucket. The spring constant, k, is $4\,\mathrm{N\,m^{-1}}$ and the sand causes the spring to extend by $10\,\mathrm{cm}$. What is the density of the sand at this packing fraction? (Take $g = 10\,\mathrm{N\,kg^{-1}}$.)

- **A** $3200\,\mathrm{kg\,m^{-3}}$
- **B** $32000\,\mathrm{kg\,m^{-3}}$
- **C** $2000\,\mathrm{kg\,m^{-3}}$
- **D** $1000\,\mathrm{kg\,m^{-3}}$
- **E** $20000\,\mathrm{kg\,m^{-3}}$
- **F** $2500\,\mathrm{kg\,m^{-3}}$
- **G** $4500\,\mathrm{kg\,m^{-3}}$
- **H** $800\,\mathrm{kg\,m^{-3}}$

SHORT SOLUTION

The spring force is $F = kx = 4 \cdot 0.10 = 0.4\,\mathrm{N}$, so mass $m = F/g = 0.04\,\mathrm{kg}$. Volume $20\,\mathrm{cm}^3 = 2.0 \times 10^{-5}\,\mathrm{m}^3$. Density $= 0.04/(2.0 \times 10^{-5}) = 2000\,\mathrm{kg\,m^{-3}}$.

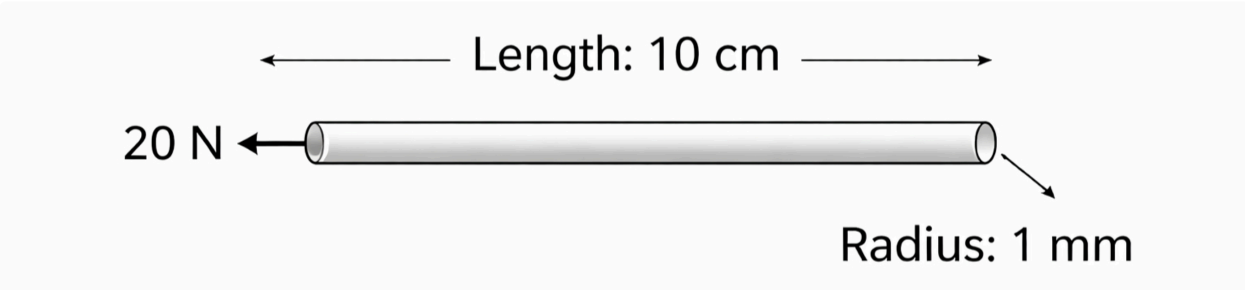
PHYSICS

Question 25

Answer **D**

A cylindrical wire of radius $1\,\mathrm{mm}$, and length $10\,\mathrm{cm}$ is extended by applying a tension of $20\,\mathrm{N}$ to it, resulting in an elastic extension of $2\,\mathrm{mm}$.

What is the Young's modulus of the material?



- **A** $2\pi\,\mathrm{GPa}$
- **B** $\frac{2}{\pi}\,\mathrm{GPa}$
- **C** $\pi\,\mathrm{GPa}$
- **D** $\frac{1}{\pi}\,\mathrm{GPa}$
- **E** $20\pi\,\mathrm{MPa}$
- **F** $\frac{20}{\pi}\,\mathrm{MPa}$
- **G** $10\pi\,\mathrm{GPa}$
- **H** $\frac{10}{\pi}\,\mathrm{GPa}$

SHORT SOLUTION

Young's modulus $E = \frac{FL}{A\Delta L}$. With $F = 20$, $L = 0.10$, $A = \pi(10^{-3})^2$, and $\Delta L = 2 \times 10^{-3}$, $E = \frac{1}{\pi} \times 10^9\,\mathrm{Pa} = \frac{1}{\pi}\,\mathrm{GPa}$.

PHYSICS

Question 26

Answer **C**

A uranium-235 nucleus can undergo fission to produce two smaller nuclei. Which of the diagrams, if any, could represent this process?

Diagram 1

n + U-235 → Xe-137 + Sr-96 + 2n

Diagram 2

n + U-235 → Xe-137 + Sr-96 + 3n

Diagram 3

n + U-235 → Xe-140 + Sr-96 + 3n

- **A** none of them
- **B** 1 only
- **C** 2 only
- **D** 3 only
- **E** 1 and 2 only
- **F** 1 and 3 only

SHORT SOLUTION

Check conservation of nucleon number and proton number. Diagram 2 conserves both: $1 + 235 = 137 + 96 + 3$ and $92 = 54 + 38$. Diagrams 1 and 3 do not conserve mass number as drawn.

PHYSICS

Question 27

Answer **B**

A wave travels at $30\,\mathrm{m}$ in 10 seconds and has a frequency, f, of $2\,\mathrm{Hz}$. How many complete wave cycles occur if the wave travels $120\,\mathrm{m}$?

- **A** 100
- **B** 80
- **C** 50
- **D** 40
- **E** 120
- **F** 20
- **G** 30
- **H** 240

SHORT SOLUTION

The wave speed is $30/10 = 3\,\mathrm{m\,s^{-1}}$. Wavelength $\lambda = v/f = 3/2 = 1.5\,\mathrm{m}$. Over $120\,\mathrm{m}$, the number of cycles is $120/1.5 = 80$.

ESAT MOCK TEST 1

Math 2

27 questions · Answers placed beside each question for fast review.

MATH 2

Question 1

Answer A

Rearrange

$$\frac{7x+10}{9x+5} = 3y^2 + 2$$

to make x the subject.

A $\frac{15y^2}{7-9(3y^2+2)}$

B $\frac{15y^2}{7+9(3y^2+2)}$

C $-\frac{15y^2}{7-9(3y^2+2)}$

D $-\frac{15y^2}{7+9(3y^2+2)}$

E $-\frac{5y^2}{7+9(3y^2+2)}$

F $\frac{5y^2}{7+9(3y^2+2)}$

SHORT SOLUTION

Let $A = 3y^2 + 2$. Then $7x + 10 = A(9x + 5)$, so $x(7 - 9A) = 5A - 10 = 15y^2$. Therefore $x = \frac{15y^2}{7-9(3y^2+2)}$.

MATH 2

Question 2

Answer **F**

Simplify

$$3x\left(\frac{3x^7}{\frac{1}{x^3}}\right)^3$$

A $9x^{20}$

B $27x^{20}$

C $87x^{20}$

D $9x^{21}$

E $27x^{21}$

F $81x^{31}$

SHORT SOLUTION

Dividing by $1/x^3$ is the same as multiplying by x^3, so the bracket becomes $3x^{10}$. Cubing gives $27x^{30}$, and multiplying by $3x$ gives $81x^{31}$.

MATH 2

Question 3

Answer **D**

For $x \geq 0$, simplify

$$2x\left[(2x)^7\right]^{\frac{1}{14}}$$

A $2x\sqrt{2x^4}$

B $2x\sqrt{2x^3}$

C $2\sqrt{2x^4}$

D $2\sqrt{2x^3}$

E $8x^3$

F $8x$

SHORT SOLUTION

$[(2x)^7]^{1/14} = (2x)^{1/2} = \sqrt{2x}$. Then $2x\sqrt{2x} = 2\sqrt{2x^3}$ for $x \geq 0$.

MATH 2

Question 4

Answer **A**

What is the circumference of a circle with an area of 10π?

- **A** $2\pi\sqrt{10}$
- **B** $\pi\sqrt{10}$
- **C** 10π
- **D** 20π
- **E** $\sqrt{10}$
- **F** More information needed.

SHORT SOLUTION

Area $= \pi r^2 = 10\pi$, so $r = \sqrt{10}$. Circumference $= 2\pi r = 2\pi\sqrt{10}$.

MATH 2

Question 5

Answer **D**

If $a \cdot b = (ab) + (a + b)$, then calculate the value of $(3 \cdot 4) \cdot 5$.

- **A** 19
- **B** 54
- **C** 100
- **D** 119
- **E** 132

SHORT SOLUTION

First, $3 \cdot 4 = 3 \cdot 4 + (3 + 4) = 12 + 7 = 19$. Then $19 \cdot 5 = 19 \cdot 5 + (19 + 5) = 95 + 24 = 119$.

MATH 2

Question 6

Answer **D**

If $a \cdot b = \frac{a^b}{a}$, calculate $(2 \cdot 3) \cdot 2$.

- **A** $\frac{16}{3}$
- **B** 1
- **C** 2
- **D** 4
- **E** 8

SHORT SOLUTION

$2 \cdot 3 = 2^3/2 = 4$. Then $4 \cdot 2 = 4^2/4 = 4$.

MATH 2

Question 7

Answer **F**

Solve $x^2 + 3x - 5 = 0$.

- **A** $x = -\frac{3}{2} \pm \frac{\sqrt{11}}{2}$
- **B** $x = \frac{3}{2} \pm \frac{\sqrt{11}}{2}$
- **C** $x = -\frac{3}{2} \pm \frac{\sqrt{11}}{4}$
- **D** $x = \frac{3}{2} \pm \frac{\sqrt{11}}{4}$
- **E** $x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$
- **F** $x = -\frac{3}{2} \pm \frac{\sqrt{29}}{2}$

SHORT SOLUTION

Using the quadratic formula, $x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-5)}}{2} = \frac{-3 \pm \sqrt{29}}{2}$.

MATH 2

Question 8

Answer **B**

How many times do the curves $y = x^3$ and $y = x^2 + 4x + 14$ intersect?

- A 0
- B 1
- C 2
- D 3
- E 4

SHORT SOLUTION

Intersections satisfy $x^3 = x^2 + 4x + 14$, or $x^3 - x^2 - 4x - 14 = 0$. This cubic has one real root and two complex roots, so the curves intersect once.

MATH 2

Question 9

Answer **D**

Solve for x, y, and z.

1. $x + y - z = -1$
2. $2x - 2y + 3z = 8$
3. $2x - y + 2z = 9$

	x	y	z
A	2	-15	-14
B	15	2	14
C	14	15	-2
D	-2	15	14
E	2	-15	14
F	No solutions possible		

SHORT SOLUTION

Substitute option D: $x = -2, y = 15, z = 14$. Then $-2 + 15 - 14 = -1$, $-4 - 30 + 42 = 8$, and $-4 - 15 + 28 = 9$. It satisfies all three equations.

MATH 2

Question 10

Answer **D**

Fully factorise: $3a^3 - 30a^2 + 75a$

- **A** $3a(a-3)^3$
- **B** $a(3a-5)^2$
- **C** $3a(a^2-10a+25)$
- **D** $3a(a-5)^2$
- **E** $3a(a+5)^2$

SHORT SOLUTION

Factor out $3a$: $3a^3 - 30a^2 + 75a = 3a(a^2-10a+25) = 3a(a-5)^2$.

MATH 2

Question 11

Answer **B**

Solve for x and y:

$4x + 3y = 48$
$3x + 2y = 34$

	x	y
A	8	6
B	6	8
C	3	4
D	4	3
E	30	12
F	12	30
G	No solutions possible	

SHORT SOLUTION

Eliminate y: double the first equation gives $8x + 6y = 96$; triple the second gives $9x + 6y = 102$. Subtract to get $x = 6$, then $3(6) + 2y = 34$ gives $y = 8$.

MATH 2

Question 12

Answer **E**

Evaluate:

$$-\frac{\left(5^2-4\times 7\right)^2}{-6^2+2\times 7}$$

A $-\frac{3}{50}$

B $\frac{11}{22}$

C $-\frac{3}{22}$

D $\frac{9}{50}$

E $\frac{9}{22}$

F 0

SHORT SOLUTION

The numerator is $-(25-28)^2=-9$. The denominator is $-6^2+14=-36+14=-22$. Hence the value is $(-9)/(-22)=9/22$.

MATH 2

Question 13

Answer **E**

All license plates are 6 characters long. The first 3 characters consist of letters and the next 3 characters of numbers. Letters and digits may be repeated. How many unique license plates are possible?

A 676,000

B 6,760,000

C 67,600,000

D 1,757,600

E 17,576,000

F 175,760,000

SHORT SOLUTION

There are 26^3 ways to choose the letters and 10^3 ways to choose the digits. Total $26^3\cdot 10^3=17,576,000$.

MATH 2

Question 14

Answer **B**

How many solutions are there for:

$$2\left(2(x^2-3x)\right)=-9\,?$$

A 0

B 1

C 2

D 3

E Infinite solutions.

SHORT SOLUTION

The equation becomes $4x^2-12x=-9$, or $4x^2-12x+9=0$. The discriminant is $(-12)^2-4(4)(9)=0$, so there is exactly one solution.

MATH 2

Question 15

Answer **C**

Evaluate:

$$\left(x^{\frac{1}{2}}y^{-3}\right)^{\frac{1}{2}}$$

A $\frac{x^{\frac{1}{2}}}{y}$

B $\frac{x}{y^{\frac{3}{2}}}$

C $\frac{x^{\frac{1}{4}}}{y^{\frac{3}{2}}}$

D $\frac{y^{\frac{1}{4}}}{x^{\frac{3}{2}}}$

SHORT SOLUTION

Use index laws: $(x^{1/2}y^{-3})^{1/2}=x^{1/4}y^{-3/2}=\frac{x^{1/4}}{y^{3/2}}$.

MATH 2

Question 16

Answer **A**

Bryan earned a total of £ 1,240 last week from renting out three flats. From this, he had to pay 10% of the rent from the 1-bedroom flat for repairs, 20% of the rent from the 2-bedroom flat for repairs, and 30% from the 3-bedroom flat for repairs.

The 3-bedroom flat costs twice as much as the 1-bedroom flat. Given that the total repair bill was £ 276, calculate the rent for each apartment.

	1 Bedroom	2 Bedrooms	3 Bedrooms
A	280	400	560
B	140	200	280
C	420	600	840
D	250	300	500

SHORT SOLUTION

Let the 1-bedroom rent be a, the 2-bedroom rent be b, and the 3-bedroom rent be $2a$. Then $3a + b = 1240$, and repairs give $0.1a + 0.2b + 0.3(2a) = 276$. Solving gives $a = 280$, $b = 400$, and $2a = 560$.

MATH 2

Question 17

Answer **D**

At a Pizza Parlour, you can order single, double or triple cheese in the crust. You also have the option to include ham, olives, pepperoni, bell pepper, meat balls, tomato slices, and pineapples. How many different types of pizza are available at the Pizza Parlour?

A 10

B 96

C 192

D 384

E 768

F None of the above

SHORT SOLUTION

There are 3 cheese choices. Each of the 7 toppings can be included or not included, giving 2^7 topping combinations. Total $= 3 \cdot 2^7 = 384$.

MATH 2

Question 18

Answer **A**

Solve the simultaneous equations $x^2 + y^2 = 1$ and $x + y = \sqrt{2}$, for $x, y > 0$.

A $(x, y) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

B $(x, y) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

C $(x, y) = \left(\sqrt{2} - 1, 1\right)$

D $(x, y) = \left(\sqrt{2}, \frac{1}{2}\right)$

SHORT SOLUTION

Square $x + y = \sqrt{2}$: $x^2 + y^2 + 2xy = 2$. Since $x^2 + y^2 = 1$, $2xy = 1$. With positive x, y and sum $\sqrt{2}$, this gives $x = y = \sqrt{2}/2$.

MATH 2

Question 19

Answer **C**

Which of the following statements is **FALSE**?

A Congruent objects always have the same dimensions and shape.

B Congruent objects can be mirror images of each other.

C Congruent objects do not always have the same angles.

D Congruent objects can be rotations of each other.

E Two triangles are congruent if they have two sides and the included angle of the same magnitude.

SHORT SOLUTION

Congruent shapes have the same side lengths and the same angles. They may be translated, rotated or reflected, but not enlarged or shrunk. Therefore statement C is false.

MATH 2

Question 20

Answer **B**

Solve the inequality $x^2 \geq 6 - x$.

A $x \leq -3$ and $x \leq 2$

B $x \leq -3$ and $x \geq 2$

C $x \geq -3$ and $x \leq 2$

D $x \geq -3$ and $x \geq 2$

E $x \geq 2$ only

F $x \geq -3$ only

SHORT SOLUTION

Move all terms to one side: $x^2 + x - 6 \geq 0$. Factor: $(x+3)(x-2) \geq 0$. The upward-opening quadratic is non-negative outside the roots, so $x \leq -3$ or $x \geq 2$.

MATH 2

Question 21

Answer **A**

The hypotenuse of an isosceles right-angled triangle is x cm. What is the area of the triangle in terms of x?

A $\frac{x^2}{4}$

B $\frac{x}{4}$

C $\frac{3x^2}{4}$

D $\frac{x^2}{10}$

SHORT SOLUTION

Let each equal leg be a. Then $a^2 + a^2 = x^2$, so $a^2 = x^2/2$. Area $= \frac{1}{2}a^2 = \frac{x^2}{4}$.

MATH 2

Question 22

Answer A

Mr Heard derives a formula:

$$Q = \frac{(X+Y)^2 A}{3B}$$

He doubles the values of X and Y, halves the value of A and triples the value of B. What happens to the value of Q?

- **A** Decreases by $\frac{1}{3}$
- **B** Increases by $\frac{1}{3}$
- **C** Decreases by $\frac{2}{3}$
- **D** Increases by $\frac{2}{3}$
- **E** Increases by $\frac{4}{3}$
- **F** Decreases by $\frac{4}{3}$

SHORT SOLUTION

Doubling both X and Y makes $(X+Y)^2$ four times larger. Halving A makes the factor 2, and tripling B divides by 3. New $Q = (2/3)$ of old Q, so it decreases by $1/3$.

MATH 2

Question 23

Answer C

Consider the graphs $y = x^2 - 2x + 3$, and $y = x^2 - 6x - 10$. Which of the following is true?

- **A** Both equations intersect the x-axis.
- **B** Neither equation intersects the x-axis.
- **C** The first equation does not intersect the x-axis; the second equation intersects the x-axis.
- **D** The first equation intersects the x-axis; the second equation does not intersect the x-axis.

SHORT SOLUTION

Use the discriminant. For $x^2 - 2x + 3$, $b^2 - 4ac = 4 - 12 = -8 < 0$, so no x-intercepts. For $x^2 - 6x - 10$, $36 + 40 = 76 > 0$, so it intersects the x-axis.

MATH 2

Question 24

Answer **C**

Evaluate

$$\log_2\left(\frac{5}{4}\right)+\log_2\left(\frac{6}{5}\right)+\log_2\left(\frac{7}{6}\right)+\cdots+\log_2\left(\frac{64}{63}\right)$$

A -2

B 3

C 4

D 6

SHORT SOLUTION

The logarithms telescope: $\log_2(5/4)+\log_2(6/5)+\cdots+\log_2(64/63)=\log_2(64/4)=\log_2(16)=4$.

MATH 2

Question 25

Answer **D**

Given that $7\cos\theta-3\tan\theta\sin\theta=1$, which one of the following is true?

A $\cos\theta=-\frac{3}{5}$ or $\cos\theta=-\frac{1}{2}$

B $\cos\theta=-\frac{3}{5}$ or $\cos\theta=\frac{1}{2}$

C $\cos\theta=\frac{3}{5}$ or $\cos\theta=\frac{1}{2}$

D $\cos\theta=\frac{3}{5}$ or $\cos\theta=-\frac{1}{2}$

SHORT SOLUTION

Let $c=\cos\theta$. Since $\tan\theta\sin\theta=\sin^2\theta/\cos\theta=(1-c^2)/c$, multiply the equation by c: $7c^2-3(1-c^2)=c$. Thus $10c^2-c-3=0=(5c-3)(2c+1)$, so $c=3/5$ or $c=-1/2$.

MATH 2

Question 26

Answer **C**

k is the smallest positive value of x which is a solution to both the equations $2\sin x + 1 = 0$ and $2\cos 2x = 1$.

How many values of x in the range $0 \leq x \leq k$ are solutions to at least one of these equations?

- **A** 0
- **B** 2
- **C** 3
- **D** 4
- **E** 8

SHORT SOLUTION

$2\sin x + 1 = 0$ gives $\sin x = -1/2$. The first positive common solution with $2\cos 2x = 1$ is $k = 7\pi/6$. In $[0, k]$, the solutions to at least one equation are $\pi/6$, $5\pi/6$, and $7\pi/6$: three values.

MATH 2

Question 27

Answer **D**

How many solutions of the equation $2\sin^3\theta = \sin\theta$ lie in the interval $-\frac{\pi}{2} \leq \theta \leq \pi$?

- **A** 2
- **B** 3
- **C** 4
- **D** 5
- **E** 6
- **F** 7

SHORT SOLUTION

Factor: $2\sin^3\theta = \sin\theta \Rightarrow \sin\theta(2\sin^2\theta - 1) = 0$. In the interval, $\sin\theta = 0$ gives $0, \pi$; $\sin\theta = \sqrt{2}/2$ gives $\pi/4, 3\pi/4$; $\sin\theta = -\sqrt{2}/2$ gives $-\pi/4$. Total 5 solutions.