CollegeBoard
Advanced Placement Program

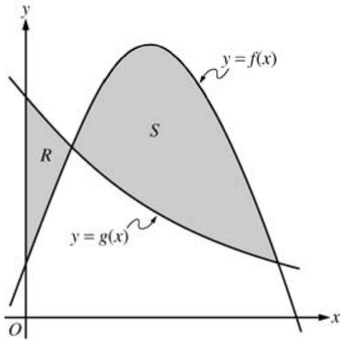
AP® Calculus BC
2005 Scoring Guidelines

The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 4,700 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three and a half million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

Visit the College Board on the Web: www.collegeboard.com.
AP Central is the official online home for the AP Program and Pre-AP: apcentral.collegeboard.com.

Question 1



Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Find the area of S.

(c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.

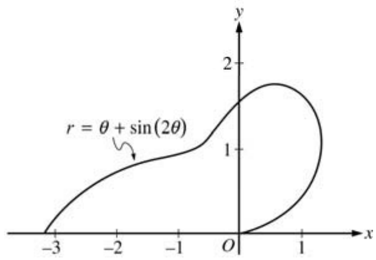
$f(x) = g(x)$ when $\frac{1}{4} + \sin(\pi x) = 4^{-x}$.

f and g intersect when $x = 0.178218$ and when $x = 1$.
Let $a = 0.178218$.

Solution	Points
(a) $\int_0^a (g(x) - f(x))\, dx = 0.064$ or 0.065	3 : { 1 : limits; 1 : integrand; 1 : answer }
(b) $\int_a^1 (f(x) - g(x))\, dx = 0.410$	3 : { 1 : limits; 1 : integrand; 1 : answer }
(c) $\pi\int_a^1 \left((f(x)+1)^2 - (g(x)+1)^2\right) dx = 4.558$ or 4.559	3 : { 2 : integrand; 1 : limits, constant, and answer }

Question 2

The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.



(a) Find the area bounded by the curve and the x-axis.

(b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

(c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r? What does this fact say about the curve?

(d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

(a) Area $= \frac{1}{2}\int_0^{\pi} r^2\, d\theta$

$= \frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2\, d\theta = 4.382$

3 : { 1 : limits and constant; 1 : integrand; 1 : answer }

(b) $-2 = r\cos(\theta) = (\theta + \sin(2\theta))\cos(\theta)$

$\theta = 2.786$

2 : { 1 : equation; 1 : answer }

(c) Since $\frac{dr}{d\theta} < 0$ for $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, r is decreasing on this interval. This means the curve is getting closer to the origin.

2 : { 1 : information about r; 1 : information about the curve }

(d) The only value in $\left[0, \frac{\pi}{2}\right]$ where $\frac{dr}{d\theta} = 0$ is $\theta = \frac{\pi}{3}$.

θ	r
0	0
$\frac{\pi}{3}$	1.913
$\frac{\pi}{2}$	1.571

The greatest distance occurs when $\theta = \frac{\pi}{3}$.

2 : { 1 : $\theta = \frac{\pi}{3}$ or 1.047; 1 : answer with justification }

AP® CALCULUS BC
2005 SCORING GUIDELINES

Question 3

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

(a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

(b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

(c) Find $\int_0^8 T'(x)\,dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)\,dx$ in terms of the temperature of the wire.

(d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.

(a) $\dfrac{T(8)-T(6)}{8-6} = \dfrac{55-62}{2} = -\dfrac{7}{2}\,°\text{C/cm}$

1 : answer

(b) $\dfrac{1}{8}\int_0^8 T(x)\,dx$

Trapezoidal approximation for $\int_0^8 T(x)\,dx$:

$$A = \frac{100+93}{2}\cdot 1 + \frac{93+70}{2}\cdot 4 + \frac{70+62}{2}\cdot 1 + \frac{62+55}{2}\cdot 2$$

Average temperature $\approx \dfrac{1}{8}A = 75.6875°\text{C}$

3 : { 1 : $\dfrac{1}{8}\int_0^8 T(x)\,dx$; 1 : trapezoidal sum; 1 : answer }

(c) $\int_0^8 T'(x)\,dx = T(8) - T(0) = 55 - 100 = -45°\text{C}$

The temperature drops 45°C from the heated end of the wire to the other end of the wire.

2 : { 1 : value; 1 : meaning }

(d) Average rate of change of temperature on $[1, 5]$ is $\dfrac{70-93}{5-1} = -5.75$.

Average rate of change of temperature on $[5, 6]$ is $\dfrac{62-70}{6-5} = -8$.

No. By the MVT, $T'(c_1) = -5.75$ for some c_1 in the interval $(1, 5)$ and $T'(c_2) = -8$ for some c_2 in the interval $(5, 6)$. It follows that T' must decrease somewhere in the interval (c_1, c_2). Therefore T'' is not positive for every x in $[0, 8]$.

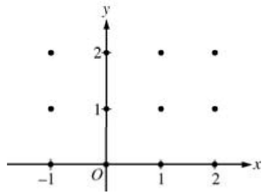
2 : { 1 : two slopes of secant lines; 1 : answer with explanation }

Units of °C/cm in (a), and °C in (b) and (c)

1 : units in (a), (b), and (c)

Question 4

Consider the differential equation $\frac{dy}{dx} = 2x - y.$



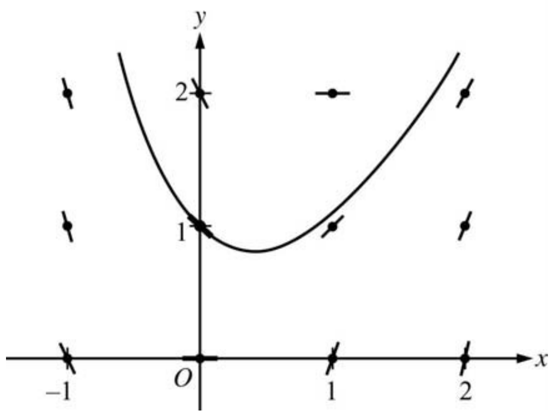
(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and sketch the solution curve that passes through the point $(0, 1)$. (**Note: Use the axes provided in the pink test booklet.**)

(b) The solution curve that passes through the point $(0, 1)$ has a local minimum at $x = \ln\left(\frac{3}{2}\right)$. What is the y-coordinate of this local minimum?

(c) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.4)$. Show the work that leads to your answer.

(d) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine whether the approximation found in part (c) is less than or greater than $f(-0.4)$. Explain your reasoning.

(a)



$3: \begin{cases} 1: \text{zero slopes} \\ 1: \text{nonzero slopes} \\ 1: \text{curve through } (0, 1) \end{cases}$

(b) $\frac{dy}{dx} = 0$ when $2x = y$

The y-coordinate is $2\ln\left(\frac{3}{2}\right)$.

$2: \begin{cases} 1: \text{sets } \frac{dy}{dx} = 0 \\ 1: \text{answer} \end{cases}$

(c)
$$f(-0.2) \approx f(0) + f'(0)(-0.2) = 1 + (-1)(-0.2) = 1.2$$
$$f(-0.4) \approx f(-0.2) + f'(-0.2)(-0.2) \approx 1.2 + (-1.6)(-0.2) = 1.52$$

$2: \begin{cases} 1: \text{Euler's method with two steps} \\ 1: \text{Euler approximation to } f(-0.4) \end{cases}$

(d) $\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - 2x + y$

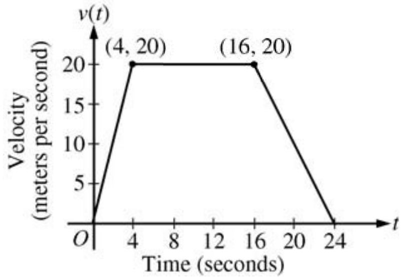
$\frac{d^2y}{dx^2}$ is positive in quadrant II because $x < 0$ and $y > 0$.

$1.52 < f(-0.4)$ since all solution curves in quadrant II are concave up.

$2: \begin{cases} 1: \frac{d^2y}{dx^2} \\ 1: \text{answer with reason} \end{cases}$

Question 5

A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.



(a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.

(b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.

(c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.

(d) Find the average rate of change of v over the interval $8 \le t \le 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

(a) $\int_0^{24} v(t)\,dt = \frac{1}{2}(4)(20) + (12)(20) + \frac{1}{2}(8)(20) = 360$

The car travels 360 meters in these 24 seconds.

$2: \begin{cases} 1: \text{value} \\ 1: \text{meaning with units} \end{cases}$

(b) $v'(4)$ does not exist because

$$\lim_{t \to 4^-} \left(\frac{v(t) - v(4)}{t - 4} \right) = 5 \ne 0 = \lim_{t \to 4^+} \left(\frac{v(t) - v(4)}{t - 4} \right).$$

$$v'(20) = \frac{20 - 0}{16 - 24} = -\frac{5}{2} \text{ m/sec}^2$$

$3: \begin{cases} 1: v'(4) \text{ does not exist, with explanation} \\ 1: v'(20) \\ 1: \text{units} \end{cases}$

(c)

$$a(t) = \begin{cases} 5 & \text{if } 0 < t < 4 \\ 0 & \text{if } 4 < t < 16 \\ -\frac{5}{2} & \text{if } 16 < t < 24 \end{cases}$$

$a(t)$ does not exist at $t = 4$ and $t = 16$.

$2: \begin{cases} 1: \text{finds the values } 5, 0, -\frac{5}{2} \\ 1: \text{identifies constants with correct intervals} \end{cases}$

(d) The average rate of change of v on $[8, 20]$ is

$$\frac{v(20) - v(8)}{20 - 8} = -\frac{5}{6} \text{ m/sec}^2.$$

No, the Mean Value Theorem does not apply to v on $[8, 20]$ because v is not differentiable at $t = 16$.

$2: \begin{cases} 1: \text{average rate of change of } v \text{ on } [8, 20] \\ 1: \text{answer with explanation} \end{cases}$

Question 6

Let f be a function with derivatives of all orders and for which $f(2) = 7$. When n is odd, the nth derivative of f at $x = 2$ is 0. When n is even and $n \geq 2$, the nth derivative of f at $x = 2$ is given by $f^{(n)}(2) = \dfrac{(n-1)!}{3^n}$.

(a) Write the sixth-degree Taylor polynomial for f about $x = 2$.

(b) In the Taylor series for f about $x = 2$, what is the coefficient of $(x-2)^{2n}$ for $n \geq 1$?

(c) Find the interval of convergence of the Taylor series for f about $x = 2$. Show the work that leads to your answer.

(a) $$P_6(x) = 7 + \frac{1!}{3^2}\cdot\frac{1}{2!}(x-2)^2 + \frac{3!}{3^4}\cdot\frac{1}{4!}(x-2)^4 + \frac{5!}{3^6}\cdot\frac{1}{6!}(x-2)^6$$

3 : { 1 : polynomial about $x = 2$
2 : $P_6(x)$
$\langle -1 \rangle$ each incorrect term
$\langle -1 \rangle$ max for all extra terms, $+\cdots$, misuse of equality }

(b) $$\frac{(2n-1)!}{3^{2n}}\cdot\frac{1}{(2n)!} = \frac{1}{3^{2n}(2n)}$$

1 : coefficient

(c) The Taylor series for f about $x = 2$ is

$$f(x) = 7 + \sum_{n=1}^{\infty}\frac{1}{2n\cdot 3^{2n}}(x-2)^{2n}.$$

$$L = \lim_{n\to\infty}\left|\frac{\frac{1}{2(n+1)}\cdot\frac{1}{3^{2(n+1)}}(x-2)^{2(n+1)}}{\frac{1}{2n}\cdot\frac{1}{3^{2n}}(x-2)^{2n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{2n}{2(n+1)}\cdot\frac{3^{2n}}{3^2 3^{2n}}(x-2)^2\right| = \frac{(x-2)^2}{9}$$

$L < 1$ when $|x-2| < 3$.

Thus, the series converges when $-1 < x < 5$.

When $x = 5$, the series is $7 + \sum_{n=1}^{\infty}\dfrac{3^{2n}}{2n\cdot 3^{2n}} = 7 + \dfrac{1}{2}\sum_{n=1}^{\infty}\dfrac{1}{n}$,

which diverges, because $\sum_{n=1}^{\infty}\dfrac{1}{n}$, the harmonic series, diverges.

When $x = -1$, the series is $7 + \sum_{n=1}^{\infty}\dfrac{(-3)^{2n}}{2n\cdot 3^{2n}} = 7 + \dfrac{1}{2}\sum_{n=1}^{\infty}\dfrac{1}{n}$,

which diverges, because $\sum_{n=1}^{\infty}\dfrac{1}{n}$, the harmonic series, diverges.

The interval of convergence is $(-1, 5)$.

5 : { 1 : sets up ratio
1 : computes limit of ratio
1 : identifies interior of interval of convergence
1 : considers both endpoints
1 : analysis/conclusion for both endpoints }