ESAT PRACTICE TEST SERIES

ESAT Mock Test 2

Math 1 + Physics + Math 2

- Duration: 2 hours (3 × 40 min)
- All topics

SOLUTION BOOK

ThrivingScholars

ANSWER KEY

ESAT Mock Test 2

Correct answers for Mathematics 1, Physics and Mathematics 2.

Mathematics 1

1	C	2	B	3	C
4	C	5	D	6	B
7	A	8	C	9	C
10	D	11	C	12	A
13	D	14	C	15	D
16	C	17	C	18	C
19	A	20	E	21	D
22	B	23	A	24	D
25	E	26	D	27	C

Physics

1	D	2	G	3	F
4	B	5	A	6	C
7	G	8	D	9	A
10	B	11	H	12	E
13	G	14	A	15	D
16	F	17	B	18	D
19	E	20	F	21	E
22	D	23	C	24	C
25	C	26	A	27	B

Mathematics 2

1	C	2	B	3	C
4	A	5	A	6	C
7	B	8	D	9	C
10	B	11	B	12	B
13	E	14	B	15	C
16	C	17	B	18	B
19	E	20	E	21	G
22	G	23	D	24	A
25	E	26	D	27	C

ESAT MOCK TEST 2

Mathematics 1

27 questions with answers and worked solutions.

M1-01

Answer C

Question 1

If

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - b \cdot c,$$

what is the value of

$$\begin{vmatrix} 3 & 4 \\ 6 & 8 \end{vmatrix}?$$

- **A** -2
- **B** -1
- **C** 0
- **D** 1
- **E** 2

SOLUTION

Use the definition directly: $3 \cdot 8 - 4 \cdot 6 = 24 - 24 = 0$.

M1-02

Answer B

Question 2

The stock price of a company was $\$100$ in 2005. It went up by 20% in 2006. However, due to the financial crisis, the stock price went down by 20% in 2007. What was the stock price at the end of 2007?

- **A** $\$80$
- **B** $\$96$
- **C** $\$98$
- **D** $\$100$
- **E** $\$120$

SOLUTION

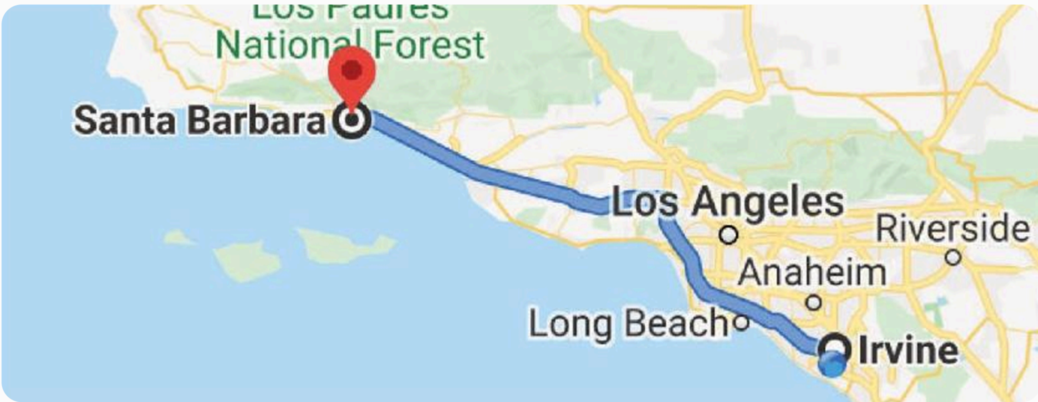
After the 20% increase the price is $100 \times 1.2 = 120$. A 20% fall from that gives $120 \times 0.8 = 96$.

M1-03

Answer C

Question 3

Irvine (CA) and Santa Barbara are approximately 120 miles apart. Austin drove from Irvine to visit his brother Benjamin in Santa Barbara, averaging 60 miles per hour. On his return trip along the same route, due to some heavy traffic on I-5, he averaged only 40 miles per hour. What was the average speed for the round trip, in miles per hour?



Recall that

$$\text{Average speed} = \frac{\text{Total travel distance}}{\text{Total amount of time}}.$$

- **A** 45
- **B** 46
- **C** 48
- **D** 50
- **E** 54

SOLUTION

The outward journey takes $120/60 = 2$ hours and the return journey takes $120/40 = 3$ hours. Total distance is 240 miles in 5 hours, so the average speed is 48 mph.

M1-04 **Answer C**

Question 4

Find the number a such that

$$\frac{7}{8} = \frac{1}{1+\frac{1}{a}}$$

- **A** -7
- **B** 1
- **C** 7
- **D** 8
- **E** 56

SOLUTION

Take reciprocals: $\frac{8}{7} = 1 + \frac{1}{a}$. Hence $\frac{1}{a} = \frac{1}{7}$, so $a = 7$.

M1-05 **Answer D**

Question 5

Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, "I didn't get the lowest score among the three of us," and Bridget adds, "I didn't get the highest score among the three of us." What is the ranking of the three girls from highest to lowest?

- **A** Hannah, Cassie, Bridget
- **B** Hannah, Bridget, Cassie
- **C** Cassie, Bridget, Hannah
- **D** Cassie, Hannah, Bridget
- **E** Bridget, Cassie, Hannah

SOLUTION

Cassie can safely say she is not lowest only if her score is higher than Hannah's known score. Bridget can safely say she is not highest only if her score is lower than Hannah's. Therefore the order is Cassie, Hannah, Bridget.

M1-06

Answer B

Question 6

The following is an arithmetic sequence that has 10 numbers:

$$1,\ a_2,\ a_3,\ a_4,\ \ldots,\ a_{10}.$$

If these 10 numbers add up to 100, what is the second number a_2?

- **A** 2
- **B** 3
- **C** 4
- **D** 5
- **E** 6

SOLUTION

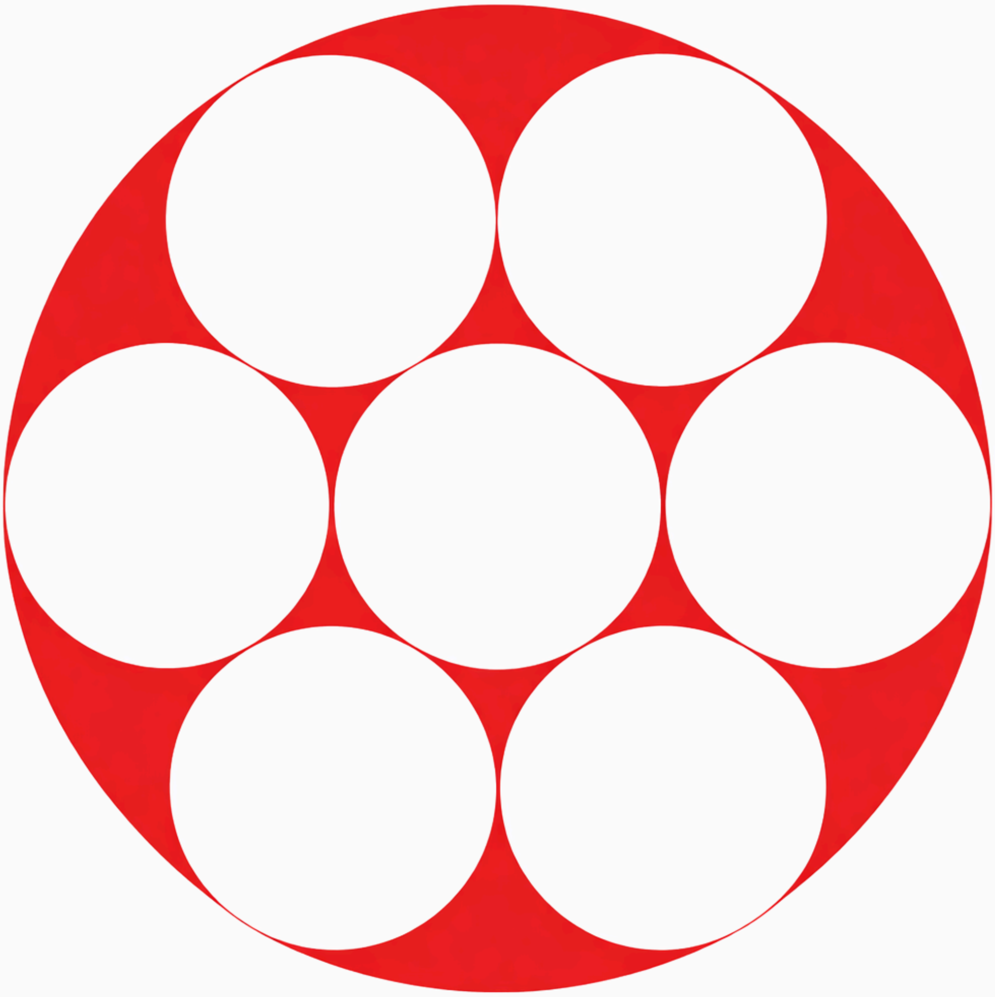
For an arithmetic sequence, $S_{10} = \frac{10}{2}(a_1 + a_{10})$. Thus $100 = 5(1 + a_{10})$, so $a_{10} = 19$. The common difference is $(19 - 1)/9 = 2$, hence $a_2 = 3$.

M1-07

Answer **A**

Question 7

Seven small circles with radius 1 are circumscribed by a large circle. Adjacent circles are tangent (i.e. they touch at only one point). Six small circles are externally tangent to the center one and internally tangent to the big one. What fraction of the large circle's area is shaded?



A $\frac{2}{9}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{1}{2}$

E $\frac{5}{9}$

SOLUTION

The large circle has radius 3: from the centre of the middle small circle to the outside is $1+1+1$. The shaded area is $9\pi - 7\pi = 2\pi$, so the fraction shaded is $\frac{2\pi}{9\pi} = \frac{2}{9}$.

M1-08 **Answer C**

Question 8

x, y and z are three numbers satisfying that

$$\begin{cases} x + 2y + 3z = 25 \\ 3x + 2y + z = 35 \end{cases}$$

What is $x + y + z$?

- **A** 10
- **B** 12
- **C** 15
- **D** 16
- **E** There are more than one possible values

SOLUTION

Add the two equations: $(x + 2y + 3z) + (3x + 2y + z) = 60$. Hence $4(x + y + z) = 60$, so $x + y + z = 15$.

M1-09 **Answer C**

Question 9

A bag contains 5 balls (three red and 2 blue). Leo picks two balls at random. What is the probability that he gets two balls of the same color?

Recall that

$$\text{Probability} = \frac{\text{Number of valid cases}}{\text{Number of all possible cases}}.$$

- **A** $\frac{1}{5}$
- **B** $\frac{3}{10}$
- **C** $\frac{2}{5}$
- **D** $\frac{1}{2}$
- **E** $\frac{3}{5}$

SOLUTION

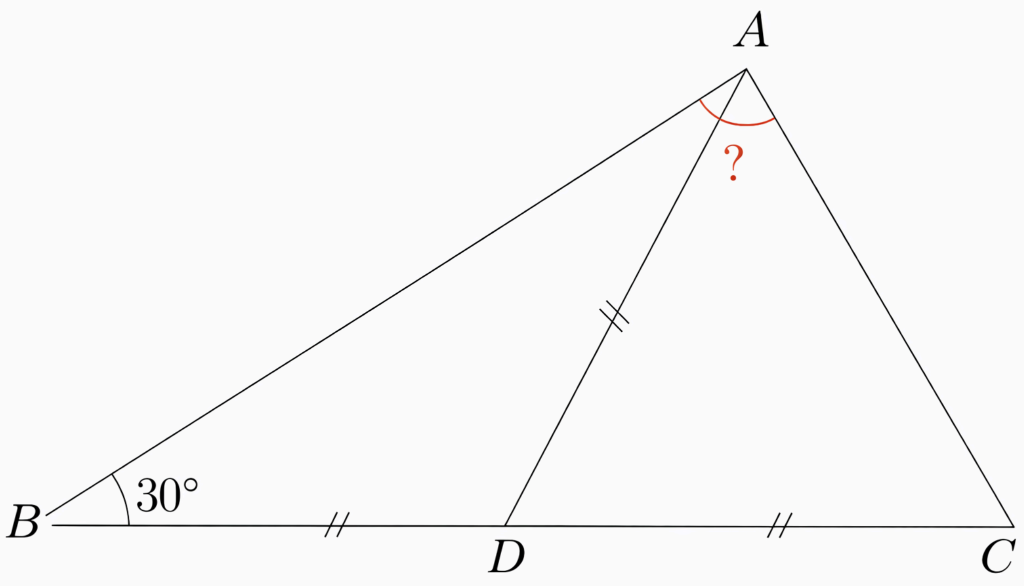
There are $\binom{5}{2} = 10$ equally likely pairs. Same colour pairs are $\binom{3}{2} + \binom{2}{2} = 3 + 1 = 4$. The probability is $4/10 = 2/5$.

M1-10

Answer D

Question 10

In $\triangle ABC$, D is the midpoint on BC. If $AD = BD = CD$ and $\angle B = 30^\circ$, what is $\angle BAC$?



A 60°

B 80°

C 86°

D 90°

E 95°

SOLUTION

Since $AD = BD$, triangle ABD has angles $30^\circ, 30^\circ, 120^\circ$. Therefore $\angle ADC = 60^\circ$. With $AD = CD$, triangle ADC is equilateral, so $\angle DAC = 60^\circ$. Thus $\angle BAC = 30^\circ + 60^\circ = 90^\circ$.

M1-11

Answer C

Question 11

How many 2-digit positive integers are divisible by 3 or 5 or both?

A 18

B 30

C 42

D 48

E 50

SOLUTION

Two-digit multiples of 3: 30. Multiples of 5: 18. Multiples of both 3 and 5, i.e. 15: 6. By inclusion-exclusion, the count is $30 + 18 - 6 = 42$.

M1-12

Answer A

Question 12

If a is the smallest two-digit number such that

$$\operatorname{lcm}(a, 12) = 60,$$

how many different divisors does a have? Recall that "lcm" stands for the *least common multiple*.

A 4

B 6

C 8

D 10

E 12

SOLUTION

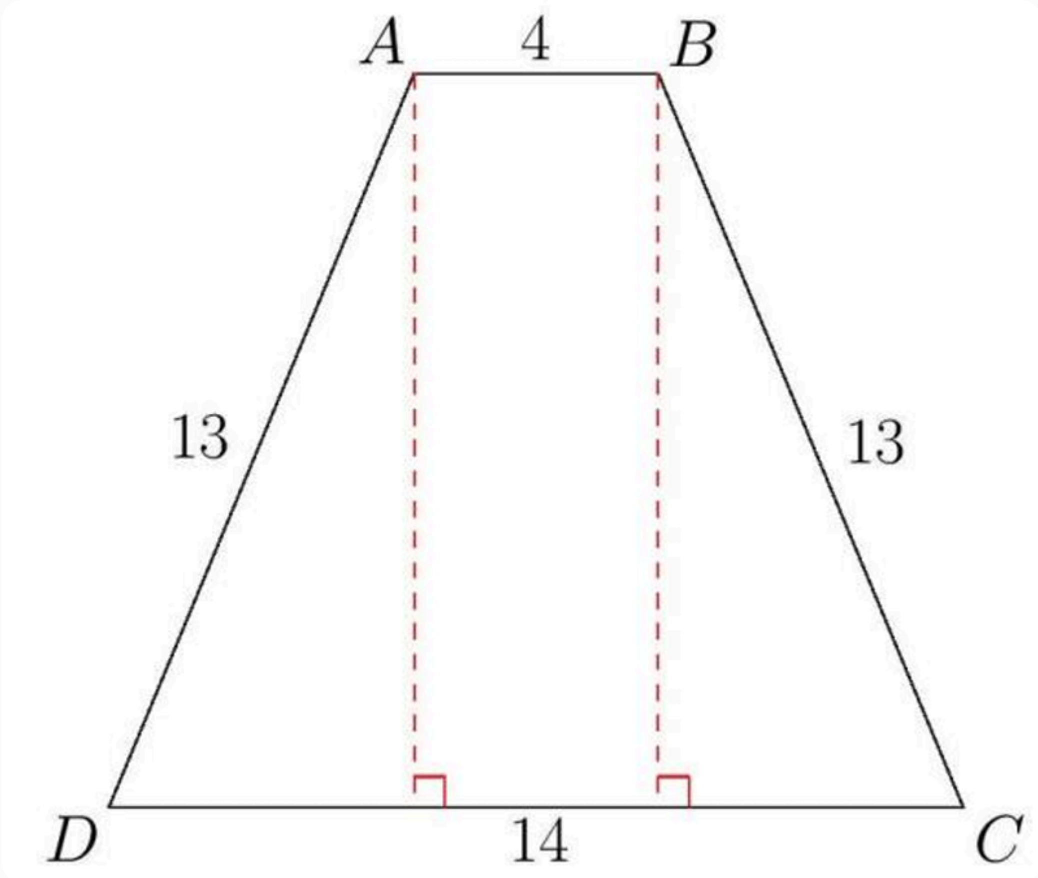
Since $\operatorname{lcm}(a, 12) = 60$, the smallest two-digit possible value is $a = 10$, because $\operatorname{lcm}(10, 12) = 60$. The divisors of 10 are $1, 2, 5, 10$, so there are 4.

M1-13

Answer **D**

Question 13

$ABCD$ is an isosceles trapezoid. AB is parallel with CD. If $AB = 4$, $BC = AD = 13$ and $CD = 14$, what is the area of the trapezoid?



A 48

B 78

C 90

D 108

E 110

SOLUTION

The difference between the bases is $14 - 4 = 10$, so each side overhang is 5. The height is $\sqrt{13^2 - 5^2} = 12$. Area $= \frac{1}{2}(4 + 14) \cdot 12 = 108$.

M1-14

Answer C

Question 14

A water tank weighs 190 kg when it is full of water and 120 kg when half full. How much does it weigh, in kilograms, when empty?

- A 30
- B 40
- C 50
- D 60
- E 70

SOLUTION

The difference between full and half-full is 70 kg, which is half the water. Half-full weight 120 kg therefore equals empty tank plus 70 kg, so the empty tank weighs 50 kg.

M1-15

Answer D

Question 15

If it is 8 am now, what time will it be 2020 hours later?

- A 7 am
- B 8 am
- C 11 am
- D 12 pm
- E 7 pm

SOLUTION

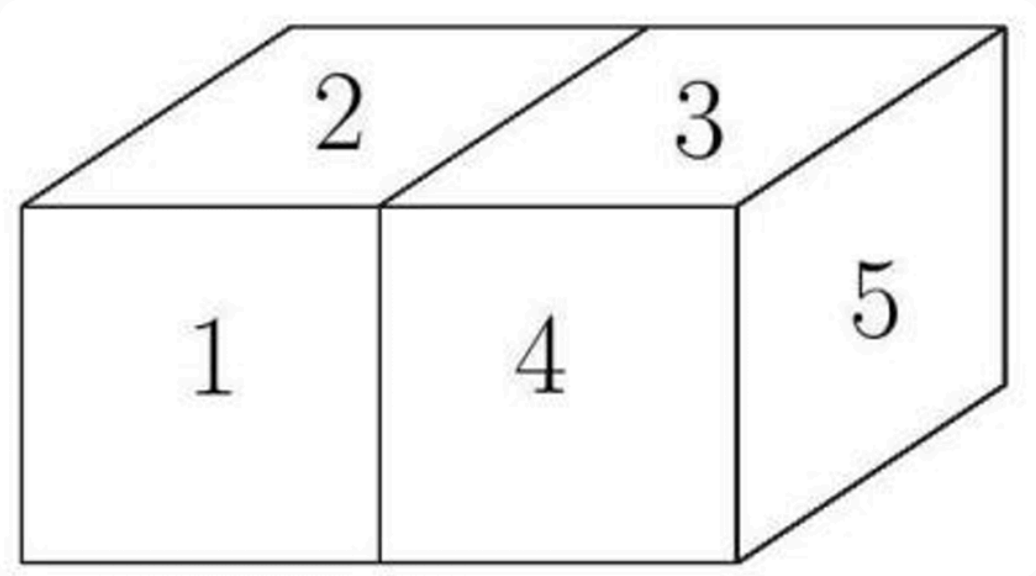
$2020 \equiv 4 \pmod{24}$. Four hours after 8 am is 12 pm.

M1-16

Answer C

Question 16

Two cubes with numbers on their faces are put next to each other as follows. For each cube, the sum of numbers on every two opposite faces is 15. What is the sum of numbers on those 7 faces that are hidden in the picture?



A 60

B 65

C 75

D 76

E 80

SOLUTION

Each cube has 3 opposite pairs, each summing to 15, so one cube totals 45 and two cubes total 90. The visible faces sum to $1 + 2 + 3 + 4 + 5 = 15$. Hidden faces sum to $90 - 15 = 75$.

M1-17

Answer C

Question 17

A positive integer is called 13-special if the quotient and remainder are the same when this integer is divided by 13. How many positive 13-special integers are between 1 and 100 (inclusive)?

- A 5
- B 6
- C 7
- D 10
- E 12

SOLUTION

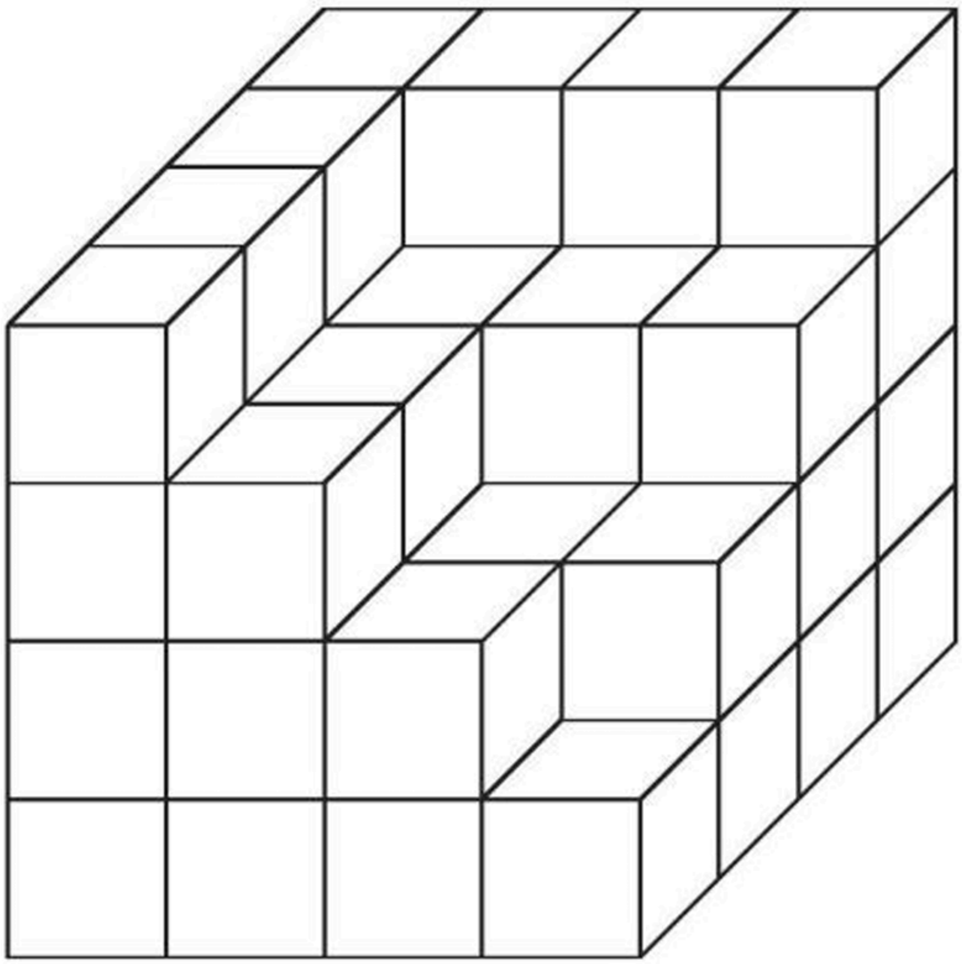
If quotient and remainder are equal to q, then the number is $13q + q = 14q$. Since the remainder is less than 13, $q = 1, 2, \ldots, 12$. Also $14q \leq 100$, so $q \leq 7$. There are 7 such integers.

M1-18

Answer C

Question 18

The following three-dimensional figure is made of small cubes with equal side length. How many small cubes are there?



A 48

B 49

C 50

D 51

E 52

SOLUTION

Count by horizontal layers from bottom to top: $16+14+12+8=50$ cubes.

M1-19

Answer A

Question 19

There are 7 cards with seven different numbers 1–7 written on them. Austen, Sam and Stephanie each takes 2 cards.

Austen said: "Numbers on my two cards add up to 7."

Sam said: "The difference between my two numbers is 1."

Stephanie said: "The product of my two numbers is 12."

What is the number on the card that is left?

A 1

B 2

C 3

D 4

E 7

SOLUTION

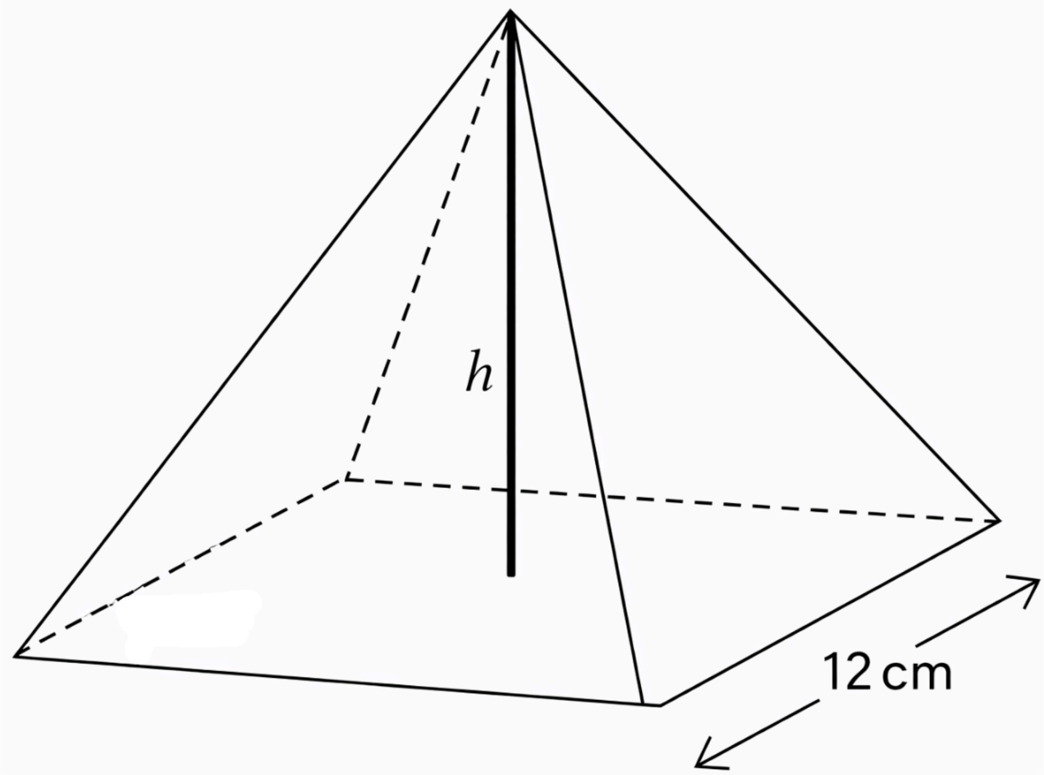
Stephanie's product must be $(3, 4)$ or $(2, 6)$. Testing with Austen's sum 7 and Sam's difference 1 gives the only consistent split: Austen $(2, 5)$, Stephanie $(3, 4)$, Sam $(6, 7)$. The card left is 1.

M1-20

Answer **E**

Question 20

A solid pyramid has a square base of side length $12\ \mathrm{cm}$ and a vertical height of $h\ \mathrm{cm}$. The volume of the pyramid, in cm^3, is equal to the total surface area of the pyramid, in cm^2. What is the value of h?



$$\text{Volume of pyramid} = \frac{1}{3} \times \text{area of base} \times \text{vertical height.}$$

A $\frac{72}{35}$

B $2\sqrt{3}$

C 6

D $\frac{144}{23}$

E 8

F $2\sqrt{21}$

SOLUTION

The slant height of a face is $\sqrt{h^2+6^2}$. Surface area is $144+24\sqrt{h^2+36}$, while volume is $48h$. So $48h = 144 + 24\sqrt{h^2+36}$, giving $2h = 6 + \sqrt{h^2+36}$. Squaring gives $3h^2 - 24h = 0$, hence $h = 8$.

M1-21

Answer **D**

Question 21

The average score of students in team A and team B altogether is 18 in an AMC 8 contest. Team A has 20 students with an average score of 14. If the average score of students in class B is 23, how many students are in team B?

- **A** 12
- **B** 14
- **C** 15
- **D** 16
- **E** 18

SOLUTION

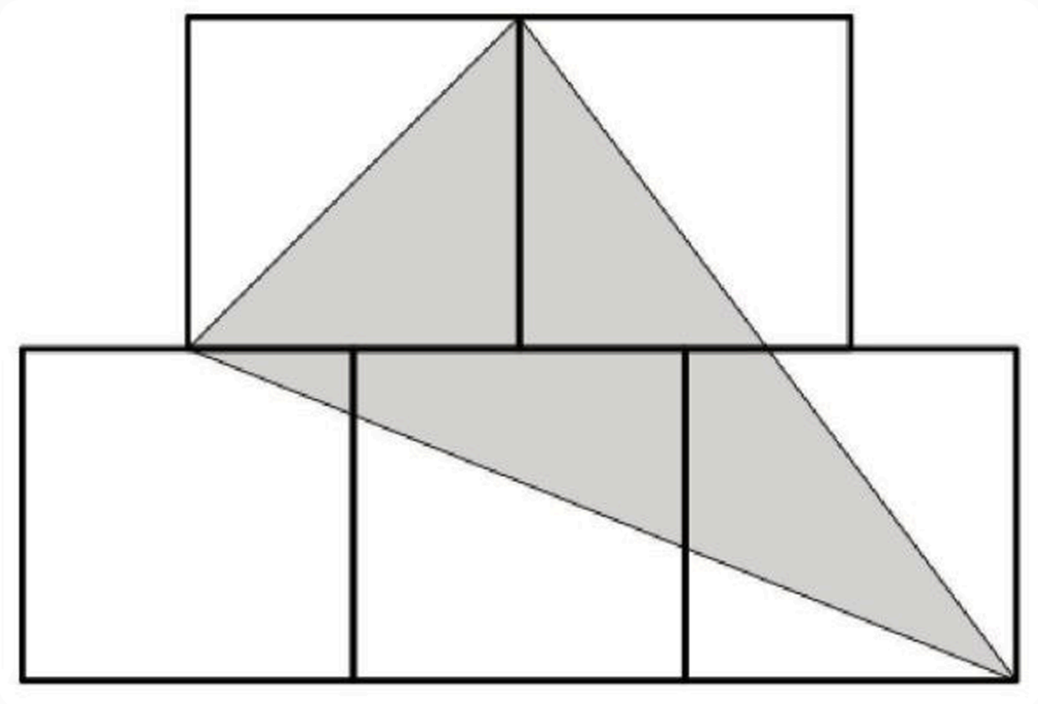
Let team B have n students. Then $\frac{20\cdot 14+23n}{20+n}=18$. Hence $280+23n=360+18n$, so $n=16$.

M1-22

Answer **B**

Question 22

The diagram shows 5 identical squares of side length 2 arranged symmetrically. What fraction of the diagram's area is shaded?



A $\frac{3}{5}$

B $\frac{7}{20}$

C $\frac{7}{10}$

D $\frac{3}{4}$

E $\frac{4}{5}$

SOLUTION

The five squares have total area $5 \cdot 2^2 = 20$. The shaded region is the triangle with vertices at the left meeting point, the top middle point and the bottom-right corner, with area 7. Therefore the shaded fraction is $7/20$.

M1-23

Answer **A**

Question 23

How many 7-digit palindromes (numbers that read the same backward as forward) can be formed using the digits $2, 2, 3, 3, 5, 5, 5$? For example, one such number is 2355532.

- **A** 6
- **B** 12
- **C** 24
- **D** 36
- **E** 48

SOLUTION

A 7-digit palindrome is determined by its first three digits and middle digit. The only digit with odd multiplicity is 5, so the middle digit is 5. The pairs $2, 3, 5$ can be arranged in $3! = 6$ ways.

M1-24

Answer **D**

Question 24

A standard, six-sided die is rolled three times. What is the probability that the three rolls are either all the same or all different?

- **A** $\frac{1}{6}$
- **B** $\frac{3}{8}$
- **C** $\frac{5}{12}$
- **D** $\frac{7}{12}$
- **E** $\frac{11}{36}$

SOLUTION

Total outcomes are $6^3 = 216$. All same: 6 outcomes. All different: $6 \cdot 5 \cdot 4 = 120$. Probability $= (6 + 120)/216 = 126/216 = 7/12$.

M1-25

Answer E

Question 25

What is the sum of the first digit and last digit of the smallest positive integer whose digits add to 2019?

A 4

B 6

C 8

D 9

E 12

SOLUTION

To minimise the integer, use the fewest digits: $\lceil 2019/9 \rceil = 225$. The first digit must be 3, followed by 224 nines. The first plus last digit is $3 + 9 = 12$.

M1-26

Answer D

Question 26

What is the minimum positive integer n such that the following statement is correct?

"Randomly choose n people. We can ALWAYS find at least three of them who were born in the same month."

A 13

B 20

C 24

D 25

E 36

SOLUTION

With 12 months, it is possible to choose 24 people with only two in each month. The next person guarantees three people in some month, so $n = 25$.

M1-27

Answer C

Question 27

2020 beads (black and white) are put on a string as follows. The first bead on the left is white. How many black beads are on the string?

A 1340

B 1344

C 1346

D 1350

E 1352

SOLUTION

The pattern repeats as $\text{white, black, black}$. In $2020 = 3 \cdot 673 + 1$ beads there are 673 full groups, giving $2 \cdot 673 = 1346$ black beads, plus one extra white.

ESAT MOCK TEST 2

Physics

27 questions with answers and worked solutions.

P-01

Answer D

Question 1

A $1000\,\mathrm{kg}$ car accelerates from rest at $5\,\mathrm{m\,s^{-2}}$ for $10\,\mathrm{s}$. Then, a braking force is applied to bring it to rest within $20\,\mathrm{s}$. What distance has the car travelled?

- A $125\,\mathrm{m}$
- B $250\,\mathrm{m}$
- C $650\,\mathrm{m}$
- D $750\,\mathrm{m}$
- E $1200\,\mathrm{m}$
- F More information needed

SOLUTION

First phase: $v = at = 5 \cdot 10 = 50\,\mathrm{m\,s^{-1}}$, distance $= \frac{1}{2}at^2 = 250$ m. Braking phase has average speed $(50+0)/2 = 25\,\mathrm{m\,s^{-1}}$ for 20 s, so distance 500 m. Total 750 m.

P-02

Answer G

Question 2

An electric heater is connected to $120\,\mathrm{V}$ mains by a copper wire that has a resistance of $8\,\Omega$. What is the power of the heater?

- A $90\,\mathrm{W}$
- B $180\,\mathrm{W}$
- C $900\,\mathrm{W}$
- D $1800\,\mathrm{W}$
- E $9000\,\mathrm{W}$
- F $18000\,\mathrm{W}$
- G More information needed

SOLUTION

The resistance of the heater itself is not given. The resistance of the connecting wire alone is not enough to determine the heater power, so more information is needed.

P-03

Answer F

Question 3

In a particle accelerator, electrons are accelerated through a potential difference of $40\,\mathrm{MV}$ and emerge with an energy of $40\,\mathrm{MeV}$ $\left(1\,\mathrm{MeV} = 1.60 \times 10^{-13}\,\mathrm{J}\right)$. Each pulse contains 5000 electrons. The current is zero between pulses. Assuming that the electrons have zero energy prior to being accelerated, what is the power delivered by the electron beam?

A $1\,\mathrm{kW}$

B $10\,\mathrm{kW}$

C $100\,\mathrm{kW}$

D $1000\,\mathrm{kW}$

E $10000\,\mathrm{kW}$

F More information needed

SOLUTION

Each pulse energy can be found, but power requires energy per unit time. The pulse rate or time interval between pulses is not given, so more information is needed.

P-04

Answer B

Question 4

Which of the following statements is true?

A When an object is in equilibrium with its surroundings, there is no energy transferred to or from the object and so its temperature remains constant.

B When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate and so its temperature remains constant.

C Radiation is faster than convection but slower than conduction.

D Radiation is faster than conduction but slower than convection.

E None of the above.

SOLUTION

At thermal equilibrium, the object still emits and absorbs radiation, but the two rates are equal. That is why its temperature remains constant.

P-05

Answer A

Question 5

A $6\,\mathrm{kg}$ block is pulled from rest along a horizontal frictionless surface by a constant horizontal force of $12\,\mathrm{N}$. Calculate the speed of the block after it has moved $300\,\mathrm{cm}$.

A $2\sqrt{3}\,\mathrm{m\,s^{-1}}$

B $4\sqrt{3}\,\mathrm{m\,s^{-1}}$

C $3\sqrt{2}\,\mathrm{m\,s^{-1}}$

D $12\,\mathrm{m\,s^{-1}}$

E $\sqrt{\frac{3}{2}}\,\mathrm{m\,s^{-1}}$

SOLUTION

The acceleration is $a = F/m = 12/6 = 2\,\mathrm{m\,s^{-2}}$. The distance is $300\ \mathrm{cm} = 3\ \mathrm{m}$. Using $v^2 = u^2 + 2as$, $v^2 = 12$, so $v = 2\sqrt{3}\,\mathrm{m\,s^{-1}}$.

P-06

Answer C

Question 6

A $100\,\mathrm{V}$ heater heats 1.5 litres of pure water from $10°\mathrm{C}$ to $50°\mathrm{C}$ in 50 minutes. Given that $1\,\mathrm{kg}$ of pure water requires $4000\,\mathrm{J}$ to raise its temperature by $1°\mathrm{C}$, calculate the resistance of the heater.

A $12.5\,\Omega$

B $25\,\Omega$

C $125\,\Omega$

D $250\,\Omega$

E $500\,\Omega$

F $850\,\Omega$

SOLUTION

The water has mass about 1.5 kg. Energy needed is $mc\Delta T = 1.5 \cdot 4000 \cdot 40 = 240000$ J. Time is 3000 s, so power is 80 W. Since $P = V^2/R$, $R = 100^2/80 = 125\,\Omega$.

P-07

Answer G

Question 7

Which of the following statements are true?

1. Nuclear fission is the basis of nuclear energy. 2. Following fission, the resulting atoms are a different element to the original one. 3. Nuclear fission often results in the production of free neutrons and photons.

A Only 1	B Only 2
C Only 3	D 1 and 2
E 2 and 3	F 1 and 3
G 1, 2 and 3	H None of the statements are true

SOLUTION

Fission is used in nuclear power, fission products are different nuclei, and neutrons and photons are commonly released. Therefore all three statements are true.

P-08

Answer D

Question 8

Which of the following statements are true? Assume $g = 10\,\mathrm{m\,s^{-2}}$.

1. Gravitational potential energy is defined as $\Delta E_p = m \times g \times \Delta h$. 2. Gravitational potential energy is a measure of the work done against gravity. 3. A reservoir situated $1\,\mathrm{km}$ above ground level with 10^6 litres of water has a potential energy of 1 Giga Joule.

- **A** Only 1
- **B** Only 2
- **C** Only 3
- **D** 1 and 2
- **E** 2 and 3
- **F** 1 and 3
- **G** 1, 2 and 3
- **H** None of the statements are true

SOLUTION

Statements 1 and 2 are true. For statement 3, 10^6 litres of water has mass 10^6 kg, so $mgh = 10^6 \cdot 10 \cdot 1000 = 10^{10}$ J, which is 10 GJ, not 1 GJ.

P-09

Answer **A**

Question 9

A stone is thrown horizontally from the top of a cliff at $10\,\mathrm{m\,s^{-1}}$. The stone falls $50\,\mathrm{m}$ before it hits the ground. Take g as $10\,\mathrm{N\,kg^{-1}}$ and neglect air resistance. What is the speed of the stone when it hits the ground?

- **A** $10\sqrt{11}\,\mathrm{m\,s^{-1}}$
- **B** $11\sqrt{10}\,\mathrm{m\,s^{-1}}$
- **C** $50\sqrt{10}\,\mathrm{m\,s^{-1}}$
- **D** $10\sqrt{5}\,\mathrm{m\,s^{-1}}$
- **E** $15\sqrt{11}\,\mathrm{m\,s^{-1}}$
- **F** $50\sqrt{5}\,\mathrm{m\,s^{-1}}$
- **G** $5\sqrt{10}\,\mathrm{m\,s^{-1}}$
- **H** $11\sqrt{5}\,\mathrm{m\,s^{-1}}$

SOLUTION

The horizontal velocity stays $10\,\mathrm{m\,s^{-1}}$. Vertically, $v_y^2 = 2gh = 2 \cdot 10 \cdot 50 = 1000$, so $v_y = 10\sqrt{10}$. Resultant speed $= \sqrt{10^2 + (10\sqrt{10})^2} = 10\sqrt{11}$.

P-10

Answer **B**

Question 10

A sailing boat has a mass of $200\,\mathrm{kg}$. When the boat travels at $30\,\mathrm{m\,s^{-1}}$ the resistive force exerted on the hull by the water is $300\,\mathrm{N}$. If the pressure exerted on the sail by the wind is $40\,\mathrm{N\,m^{-2}}$, and the boat is still accelerating at $1\,\mathrm{m\,s^{-2}}$ when it travels at $30\,\mathrm{m\,s^{-1}}$, how big is the sail?

- **A** $10\,\mathrm{m^2}$
- **B** $\frac{25}{2}\,\mathrm{m^2}$
- **C** $\frac{10}{4}\,\mathrm{m^2}$
- **D** $12\,\mathrm{m^2}$
- **E** $24\,\mathrm{m^2}$
- **F** $\frac{25}{4}\,\mathrm{m^2}$
- **G** $\frac{15}{2}\,\mathrm{m^2}$
- **H** $\frac{16}{3}\,\mathrm{m^2}$

SOLUTION

The net force needed for acceleration is $ma = 200 \cdot 1 = 200$ N. The sail force must overcome 300 N resistance as well, so it is 500 N. Area $= F/P = 500/40 = 25/2\,\mathrm{m^2}$.

P-11

Answer **H**

Question 11

A watch, when submerged in water, can withstand an average force of $10\,\mathrm{kN}$ on its surface. The total surface area of the watch is $50\,\mathrm{cm}^2$. Given that atmospheric pressure is $1\,\mathrm{bar} = 10^5\,\mathrm{Pa}$, the density of water is $1000\,\mathrm{kg\,m^{-3}}$, and g is $10\,\mathrm{N\,kg^{-1}}$, what is the deepest the watch can be taken underwater?

A $120\,\mathrm{m}$	B $30\,\mathrm{m}$
C $470\,\mathrm{m}$	D $200\,\mathrm{m}$
E $530\,\mathrm{m}$	F $500\,\mathrm{m}$
G $50\,\mathrm{m}$	H $190\,\mathrm{m}$

SOLUTION

The maximum pressure is $F/A = 10000/(50 \times 10^{-4}) = 2.0 \times 10^6$ Pa. Underwater pressure is $10^5 + \rho g h$. Thus $h = (2.0 \times 10^6 - 10^5)/(1000 \cdot 10) = 190$ m.

P-12

Answer **E**

Question 12

A plane of mass $10{,}000\,\mathrm{kg}$ coming into land hits the runway travelling at $100\,\mathrm{m\,s^{-1}}$. It decelerates uniformly over a distance of $500\,\mathrm{m}$, until it reaches a taxi velocity of $10\,\mathrm{m\,s^{-1}}$. What is the resultant force on the aircraft?

A $1.00\,\mathrm{kN}$	B $1.80\,\mathrm{kN}$
C $9.09\,\mathrm{kN}$	D $10.0\,\mathrm{kN}$
E $99.0\,\mathrm{kN}$	F $100\,\mathrm{kN}$
G $101\,\mathrm{kN}$	H $200\,\mathrm{kN}$

SOLUTION

Use $v^2 = u^2 + 2as$: $10^2 = 100^2 + 2a(500)$, so $a = -9.9\,\mathrm{m\,s^{-2}}$. The resultant force magnitude is $10000 \cdot 9.9 = 99000$ N, or 99.0 kN.

P-13

Answer G

Question 13

Two cups in different buildings are connected by a wire, which is $1000\,\mathrm{m}$ long. If one person sings a note with a frequency of $250\,\mathrm{Hz}$ into the cup, a $0.2\,\mathrm{s}$ delay is recorded until it is first heard in the other cup. What is the wavelength of the sound wave travelling through the wire?

- **A** $0.20\,\mathrm{m}$
- **B** $0.80\,\mathrm{m}$
- **C** $1.0\,\mathrm{m}$
- **D** $4.0\,\mathrm{m}$
- **E** $5.0\,\mathrm{m}$
- **F** $10\,\mathrm{m}$
- **G** $20\,\mathrm{m}$
- **H** $200\,\mathrm{m}$

SOLUTION

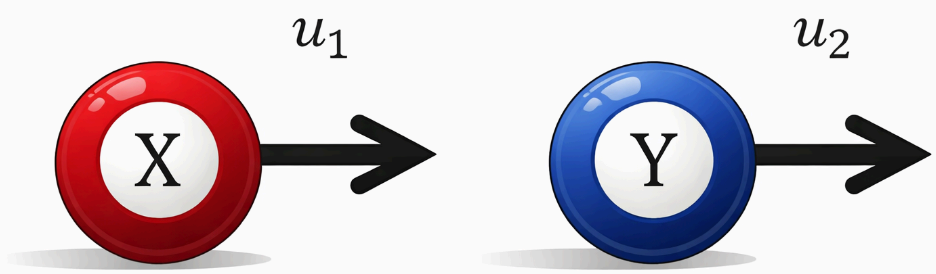
The wave speed in the wire is $1000/0.2 = 5000\,\mathrm{m\,s^{-1}}$. Since $v = f\lambda$, $\lambda = 5000/250 = 20$ m.

P-14

Answer A

Question 14

Two identical snooker balls (X and Y) of mass $4\,\mathrm{kg}$, with velocities u_1 and u_2 respectively (in the positive x direction, $u_1 > u_2$) collide elastically. After the collision, ball X comes to rest, while ball Y moves off with velocity v. If $u_1 = 1\,\mathrm{m\,s^{-1}}$, what is the value of u_2?



- **A** $0.000\,\mathrm{m\,s^{-1}}$
- **B** $0.100\,\mathrm{m\,s^{-1}}$
- **C** $0.125\,\mathrm{m\,s^{-1}}$
- **D** $0.250\,\mathrm{m\,s^{-1}}$
- **E** $0.375\,\mathrm{m\,s^{-1}}$
- **F** $0.500\,\mathrm{m\,s^{-1}}$
- **G** $1.00\,\mathrm{m\,s^{-1}}$
- **H** $2.00\,\mathrm{m\,s^{-1}}$

SOLUTION

For equal masses in a 1D elastic collision, the velocities are exchanged. Since ball X comes to rest, ball Y must initially have been at rest, so $u_2 = 0$.

P-15

Answer **D**

Question 15

A car of mass $500\,\mathrm{kg}$ is climbing a slope inclined at 30° to the horizontal. The car is travelling at a speed of $20\,\mathrm{m\,s^{-1}}$, when the engine breaks down, so that the only forces acting on the car are gravity and a constant friction force of $7500\,\mathrm{N}$. The car travels a distance x parallel to the slope before coming to rest instantaneously. How far is x?

- **A** $1.0\,\mathrm{m}$
- **B** $2.0\,\mathrm{m}$
- **C** $5.0\,\mathrm{m}$
- **D** $10\,\mathrm{m}$
- **E** $13\,\mathrm{m}$
- **F** $100\,\mathrm{m}$
- **G** $130\,\mathrm{m}$
- **H** $200\,\mathrm{m}$

SOLUTION

The initial kinetic energy is $\frac{1}{2} \cdot 500 \cdot 20^2 = 100000$ J. The opposing force along the slope is $500 \cdot 10 \sin 30^\circ + 7500 = 2500 + 7500 = 10000$ N. Distance $= 100000/10000 = 10$ m.

P-16

Answer F

Question 16

A hydroelectric dam stores energy by pumping water from a lower reservoir to a higher reservoir using an electric pump (supplied with $100\,\mathrm{V}$ and $10\,\mathrm{A}$). This power can then be harnessed again by allowing the water to fall down to the lower reservoir, driving a turbine, generating electricity. It takes $1000\,\mathrm{s}$ for the pump to lift $1000\,\mathrm{kg}$ of water to a height of $75\,\mathrm{m}$ above the reservoir. This mass of water is then used to drive a turbine, which converts the gravitational potential energy of the water into electrical energy with a 33.3% efficiency. What percentage of the original electrical input energy is lost over the full storage-and-generation cycle?

- A 0%
- B 25%
- C 33.3%
- D 50%
- E 66.7%
- F 75%
- G 80%

SOLUTION

Input energy is $VIt = 100 \cdot 10 \cdot 1000 = 1.0 \times 10^6$ J. Stored GPE is $mgh = 1000 \cdot 10 \cdot 75 = 7.5 \times 10^5$ J. The turbine returns only 33.3% of this, about 2.5×10^5 J, so 75% of the input energy is lost over the storage-and-return process.

P-17

Answer **B**

Question 17

An object falling through the air in a straight-line experiences quadratic air resistance, such that the acceleration, a, of the object as a function of its velocity, v, is given by:

$$a(v) = g - bv^2$$

Where b is a non-negative constant and g is acceleration due to gravity. Which of the following is an expression for the terminal velocity, v_0, of the object?

- **A** $g - b$
- **B** $\sqrt{\frac{g}{b}}$
- **C** $\frac{g}{b}$
- **D** $g - \sqrt{b}$
- **E** $b - \sqrt{g}$
- **F** $\sqrt{\frac{b}{g}}$
- **G** $-2bv$
- **H** $gv - \frac{1}{3}bv^3$

SOLUTION

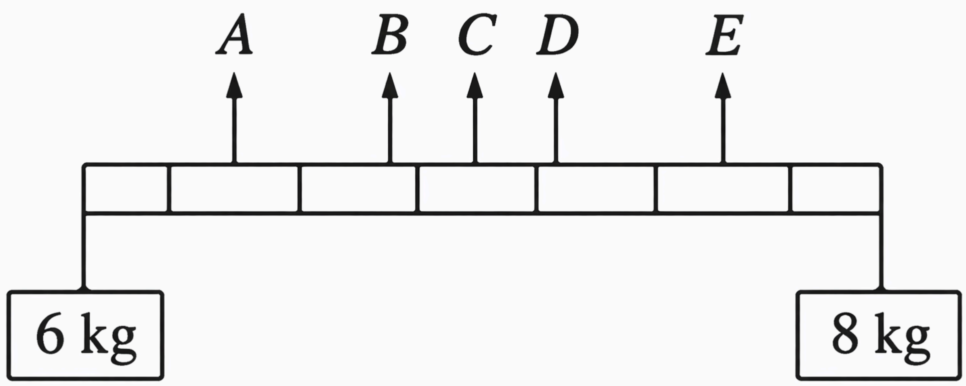
Terminal velocity occurs when acceleration is zero: $0 = g - bv_0^2$. Therefore $v_0 = \sqrt{g/b}$.

P-18

Answer D

Question 18

Two objects, of masses $6\ \mathrm{kg}$ and $8\ \mathrm{kg}$, are hung from the ends of a stick that is $70\ \mathrm{cm}$ long and has marks every $10\ \mathrm{cm}$, as shown in the diagram. If the mass of the stick is negligible, at which of the indicated points should a cord be attached if the stick is to remain horizontal when suspended from the cord?



- A A
- B B
- C C
- D D
- E E

SOLUTION

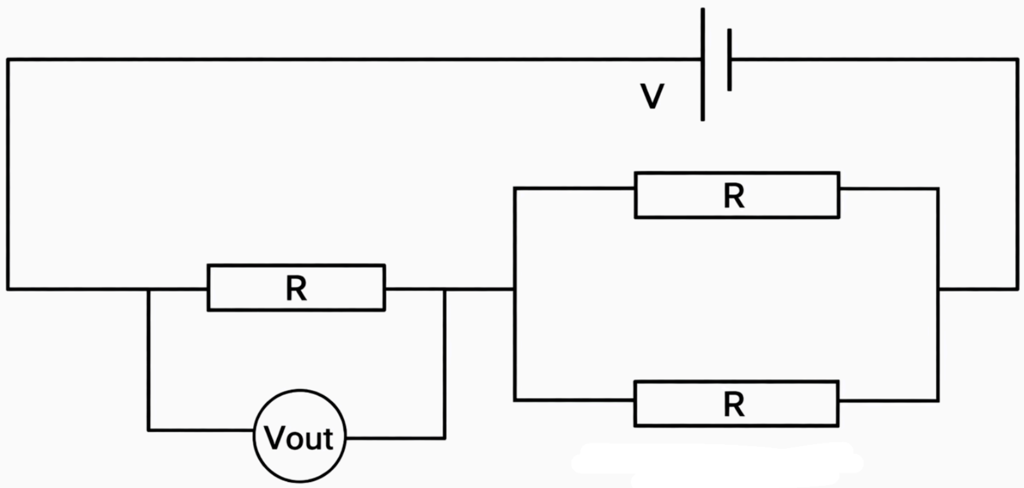
Balance moments about the support. If the support is x cm from the 6 kg mass, then $6x = 8(70 - x)$, giving $x = 40$ cm. The 40 cm mark is point D.

P-19

Answer E

Question 19

In the below circuit, a cell with an e.m.f of $V = 1\,\mathrm{V}$ drives a current through three identical resistors R. What is the potential difference across the voltmeter marked V_{out}?



A $\frac{1}{6}V$

B $\frac{1}{4}V$

C $\frac{1}{3}V$

D $\frac{1}{2}V$

E $\frac{2}{3}V$

F $\frac{3}{4}V$

SOLUTION

The two right-hand resistors are in parallel, so their equivalent resistance is $R/2$. This is in series with the left resistor R, total $3R/2$. The voltmeter is across the left resistor, so its share of the voltage is $R/(3R/2) = 2/3$.

P-20

Answer F

Question 20

A particle's velocity, v, as a function of time, t, is given by the function:

$$v(t) = t^5 - 5t^3 + 4t$$

In the entire range of t $(-\infty < t < \infty)$, how many times does the particle instantaneously come to rest?

- A 0
- B 1
- C 2
- D 3
- E 4
- F 5
- G 6

SOLUTION

Set $v(t) = 0$: $t^5 - 5t^3 + 4t = t(t^2 - 1)(t^2 - 4)$. The roots are $-2, -1, 0, 1, 2$, so the particle is instantaneously at rest 5 times.

P-21

Answer E

Question 21

Velocity, v, is the rate of change of displacement, s. For the same particle as in Question 20 (with the same velocity function), what is the distance travelled by the particle between $t = -1\,\mathrm{s}$ and $t = 1\,\mathrm{s}$?

- A $0\,\mathrm{m}$
- B $\frac{1}{12}\,\mathrm{m}$
- C $\frac{1}{8}\,\mathrm{m}$
- D $\frac{11}{12}\,\mathrm{m}$
- E $\frac{11}{6}\,\mathrm{m}$
- F $\frac{13}{6}\,\mathrm{m}$
- G $4\,\mathrm{m}$

SOLUTION

On $(0, 1)$, $v(t) > 0$, and the velocity is an odd function. Distance from -1 to 1 is $2\int_0^1 (t^5 - 5t^3 + 4t)\,dt = 2\left(\frac{1}{6} - \frac{5}{4} + 2\right) = \frac{11}{6}$ m.

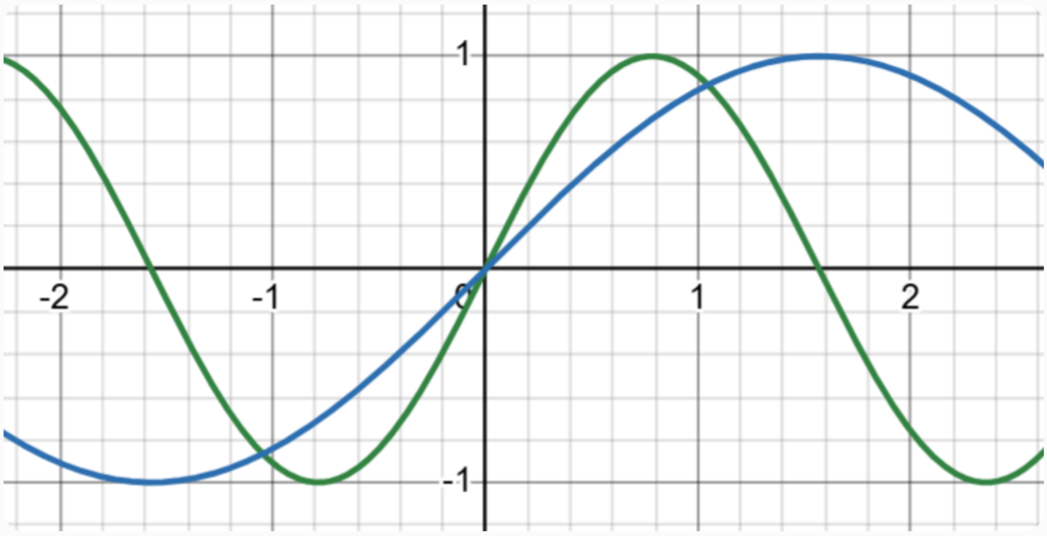
P-22

Answer D

Question 22

An oscilloscope measures an electrical signal from an alternating current power supply. The voltage, V, measured by the oscilloscope has the form $V = V_0 \sin(\omega t + \phi)$, where V_0, ω and ϕ are all positive constants.

If the blue curve is the original signal, which of the following operations would give rise to the signal shown by the green curve?



- **A** Halve ϕ
- **B** Double ϕ
- **C** Halve ω
- **D** Double ω
- **E** Halve V_0
- **F** Double V_0
- **G** Increase ϕ by $180°$

SOLUTION

The green curve has the same amplitude and passes through the origin in the same direction, but it completes oscillations more quickly. That means the angular frequency ω has been doubled.

P-23

Answer **C**

Question 23

An object of mass $0.1\,\mathrm{kg}$ sits on a rough plane, inclined at θ to the horizontal. If the coefficient of friction of the plane is $\dfrac{1}{\sqrt{3}}$, what is the maximum value of θ (in degrees) without the object slipping?

- **A** 0°
- **B** 15°
- **C** 30°
- **D** 45°
- **E** 60°
- **F** 75°

SOLUTION

At the limiting angle for no slipping, $\tan\theta = \mu$. Since $\mu = 1/\sqrt{3}$, $\theta = 30^\circ$.

P-24

Answer C

Question 24

The Tsiolkovsky rocket equation (below) is used to calculate the change in velocity, Δv, experienced by a spacecraft during a manoeuvre in terms of its exhaust velocity, u, and the ratio of its initial mass, m_0, and final mass, m_f.

$$\Delta v = u \log_e\left(\frac{m_0}{m_f}\right)$$

In order to achieve place a payload of mass $1\,\mathrm{kg}$ in a circular orbit, a Δv of $5000\,\mathrm{m\,s^{-1}}$ must be achieved. If using a rocket with an exhaust velocity of $2500\,\mathrm{m\,s^{-1}}$, what is the minimum mass of fuel that can be used in order to place the payload in circular orbit?

- **A** $e^2\,\mathrm{kg}$
- **B** $e^{\frac{1}{2}}\,\mathrm{kg}$
- **C** $e^2 - 1\,\mathrm{kg}$
- **D** $e^{\frac{1}{2}} - 1\,\mathrm{kg}$
- **E** $e^2 + 1\,\mathrm{kg}$
- **F** $\log_e 2\,\mathrm{kg}$
- **G** $\log_e 2 - 1\,\mathrm{kg}$
- **H** $\log_e 2 + 1\,\mathrm{kg}$

SOLUTION

The rocket equation gives $5000 = 2500\ln(m_0/m_f)$, so $m_0/m_f = e^2$. With final payload mass $m_f = 1$ kg, initial mass is e^2 kg, so fuel mass is $e^2 - 1$ kg.

P-25

Answer C

Question 25

Stationary waves can be set up in tube with one closed end and one open end, such that there is a node at the closed end and an antinode at the open end. What is the second lowest frequency note that can be produced by a tube which is $0.6\,\mathrm{m}$ long, given that the speed of sound is $340\,\mathrm{m\,s^{-1}}$?

- A $283\,\mathrm{Hz}$
- B $378\,\mathrm{Hz}$
- C $425\,\mathrm{Hz}$
- D $567\,\mathrm{Hz}$
- E $850\,\mathrm{Hz}$
- F $1700\,\mathrm{Hz}$

SOLUTION

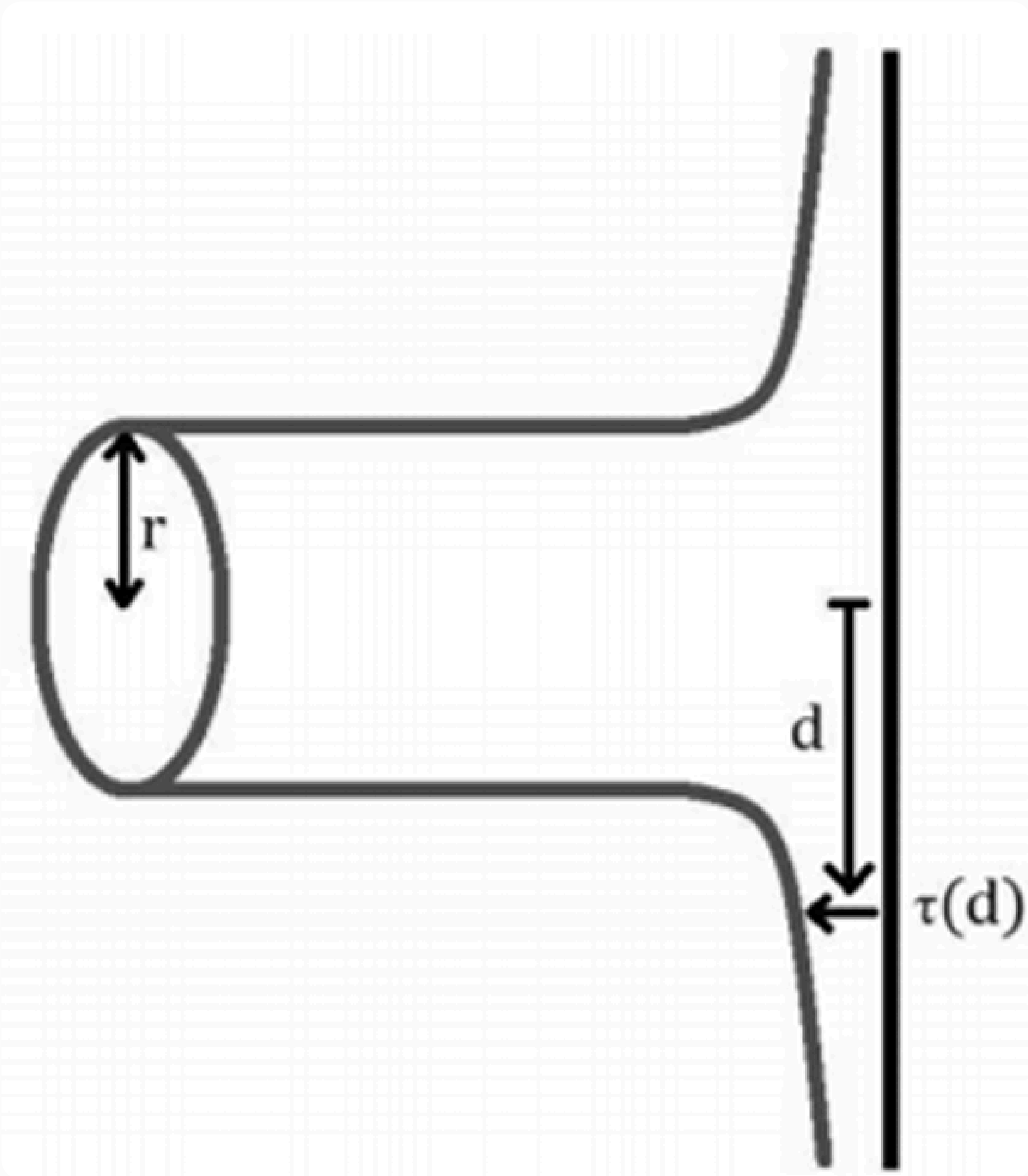
For a tube closed at one end and open at the other, resonances are $f_n = (2n-1)v/(4L)$. The second lowest frequency is $3v/(4L) = 3 \cdot 340/(4 \cdot 0.6) = 425$ Hz.

P-26

Answer **A**

Question 26

A cylindrical jet of water, with a radius r and velocity v, strikes a wall and spreads out into a plane. The velocity and density of the water remain constant. Which of the following is an expression for the thickness of the plane, τ, as a function of the distance from the point of impact of the jet, d?



Mass is conserved. Consider the mass coming in and out in a certain amount of time.

A $\frac{r^2}{2d}$

B $\frac{v^2}{rd}$

C dr^2

D $\frac{d}{r^2}$

E $\frac{2d}{v}$

F $\frac{2r}{dv}$

SOLUTION

Conserve volume flow. Incoming flow is $\pi r^2 v$. At distance d, outgoing sheet flow is circumference $2\pi d$ times thickness τ times speed v, so $2\pi d\tau v = \pi r^2 v$. Hence $\tau = \frac{r^2}{2d}$.

P-27

Answer **B**

Question 27

A cannon on top of a $20\,\mathrm{m}$ castle wall is fired horizontally at $100\,\mathrm{m\,s^{-1}}$. How far away from the foot of the castle wall does the cannon ball land (you may ignore the effect of air resistance)?

- A $100\,\mathrm{m}$
- B $200\,\mathrm{m}$
- C $300\,\mathrm{m}$
- D $400\,\mathrm{m}$
- E $500\,\mathrm{m}$
- F $600\,\mathrm{m}$
- G $700\,\mathrm{m}$
- H $800\,\mathrm{m}$

SOLUTION

The fall time from height 20 m is given by $20 = \frac{1}{2}gt^2 = 5t^2$, so $t = 2$ s. Horizontal distance is $100 \cdot 2 = 200$ m.

ESAT MOCK TEST 2

Mathematics 2

27 questions with answers and worked solutions.

M2-01

Answer C

Question 1

Which of the following graphs do not intersect? 1. $y = x$ 2. $y = x^2$ 3. $y = 1 - x^2$ 4. $y = 2$

- A 1 and 2
- B 2 and 3
- C 3 and 4
- D 1 and 3
- E 1 and 4
- F 2 and 4

SOLUTION

Graph 3 and graph 4 do not intersect because $1 - x^2 = 2$ gives $x^2 = -1$, which has no real solution.

M2-02

Answer B

Question 2

Calculate the product of 897,653 and 0.009764.

- A 87646.8
- B 8764.68
- C 876.468
- D 87.6468
- E 8.76468
- F 0.876468

SOLUTION

$897653 \times 0.009764 = 8764.683892$, so the matching option is 8764.68.

M2-03

Answer C

Question 3

Solve for x:

$$\frac{7x+3}{10}+\frac{3x+1}{7}=14$$

A $\frac{929}{51}$

B $\frac{949}{47}$

C $\frac{949}{79}$

D $\frac{980}{79}$

SOLUTION

Multiply by 70: $7(7x+3)+10(3x+1)=980$. Hence $49x+21+30x+10=980$, so $79x=949$ and $x=949/79$.

M2-04

Answer A

Question 4

What is the area of an equilateral triangle with side length x?

A $\frac{x^2\sqrt{3}}{4}$

B $\frac{x\sqrt{3}}{4}$

C $\frac{x^2}{2}$

D $\frac{x}{2}$

E x^2

F x

SOLUTION

An equilateral triangle of side x has height $x\sqrt{3}/2$. Area $=\frac{1}{2}x\cdot\frac{x\sqrt{3}}{2}=\frac{x^2\sqrt{3}}{4}$.

M2-05

Answer **A**

Question 5

Simplify

$$3-\frac{7x\left(25x^2-1\right)}{49x^2\left(5x+1\right)}$$

- **A** $3-\frac{5x-1}{7x}$
- **B** $3-\frac{5x+1}{7x}$
- **C** $3+\frac{5x-1}{7x}$
- **D** $3+\frac{5x+1}{7x}$
- **E** $3-\frac{5x^2}{49}$
- **F** $3+\frac{5x^2}{49}$

SOLUTION

Factor $25x^2-1=(5x-1)(5x+1)$. Then $\frac{7x(25x^2-1)}{49x^2(5x+1)}=\frac{5x-1}{7x}$. So the expression is $3-\frac{5x-1}{7x}$.

M2-06

Answer **C**

Question 6

Solve the equation $x^2-10x-100=0$.

- **A** $-5\pm 5\sqrt{5}$
- **B** $-5\pm\sqrt{5}$
- **C** $5\pm 5\sqrt{5}$
- **D** $5\pm\sqrt{5}$
- **E** $5\pm 5\sqrt{125}$
- **F** $-5\pm\sqrt{125}$

SOLUTION

Using the quadratic formula, $x=\frac{10\pm\sqrt{100+400}}{2}=\frac{10\pm10\sqrt{5}}{2}=5\pm5\sqrt{5}$.

M2-07

Answer **B**

Question 7

Rearrange $x^2 - 4x + 7 = y^3 + 2$ to make x the subject.

- **A** $x = 2 \pm \sqrt{y^3 + 1}$
- **B** $x = 2 \pm \sqrt{y^3 - 1}$
- **C** $x = -2 \pm \sqrt{y^3 - 1}$
- **D** $x = -2 \pm \sqrt{y^3 + 1}$
- **E** x cannot be made the subject for this equation.

SOLUTION

Complete the square: $x^2 - 4x + 7 = (x - 2)^2 + 3$. Thus $(x - 2)^2 = y^3 - 1$, giving $x = 2 \pm \sqrt{y^3 - 1}$.

M2-08

Answer **D**

Question 8

Rearrange $3x + 2 = \sqrt{7x^2 + 2x + y}$ to make y the subject.

- **A** $y = 4x^2 + 8x + 2$
- **B** $y = 4x^2 + 8x + 4$
- **C** $y = 2x^2 + 10x + 2$
- **D** $y = 2x^2 + 10x + 4$
- **E** $y = x^2 + 10x + 2$
- **F** $y = x^2 + 10x + 4$

SOLUTION

Square both sides: $(3x + 2)^2 = 7x^2 + 2x + y$. Therefore $9x^2 + 12x + 4 = 7x^2 + 2x + y$, so $y = 2x^2 + 10x + 4$.

M2-09

Answer C

Question 9

Evaluate:

$$5\left[5\left(6^2-5\times 3\right)+400^{1/2}\right]^{1/3}+7$$

A 0

B 25

C 32

D 49

E 56

F 200

SOLUTION

Inside the cube root: $6^2-5\cdot 3=21$, so $5(21)+20=125$. Hence the expression is $5\cdot 5+7=32$.

M2-10

Answer B

Question 10

What is the area of a regular hexagon with side length 1?

A $3\sqrt{3}$

B $\frac{3\sqrt{3}}{2}$

C $\sqrt{3}$

D $\frac{\sqrt{3}}{2}$

E 6

F More information needed

SOLUTION

A regular hexagon of side 1 is made of 6 equilateral triangles of side 1. Its area is $6\cdot\frac{\sqrt{3}}{4}=\frac{3\sqrt{3}}{2}$.

M2-11

Answer **B**

Question 11

Dexter moves into a new rectangular room that is 19 metres longer than it is wide, and its total area is 780 square metres. What are the room's dimensions?

- **A** Width = $20\,\mathrm{m}$; Length = $-39\,\mathrm{m}$
- **B** Width = $20\,\mathrm{m}$; Length = $39\,\mathrm{m}$
- **C** Width = $39\,\mathrm{m}$; Length = $20\,\mathrm{m}$
- **D** Width = $-39\,\mathrm{m}$; Length = $20\,\mathrm{m}$
- **E** Width = $-20\,\mathrm{m}$; Length = $39\,\mathrm{m}$

SOLUTION

Let the width be w. Then length is $w+19$, so $w(w+19)=780$. Since $20\cdot 39=780$, the dimensions are width 20 m and length 39 m.

M2-12

Answer **B**

Question 12

Tom uses 34 meters of fencing to enclose his rectangular lot. He measured the diagonals to be 13 metres long. What is the length and width of the lot?

- **A** $3\,\mathrm{m}$ by $4\,\mathrm{m}$
- **B** $5\,\mathrm{m}$ by $12\,\mathrm{m}$
- **C** $6\,\mathrm{m}$ by $12\,\mathrm{m}$
- **D** $8\,\mathrm{m}$ by $15\,\mathrm{m}$
- **E** $9\,\mathrm{m}$ by $15\,\mathrm{m}$
- **F** $10\,\mathrm{m}$ by $10\,\mathrm{m}$

SOLUTION

Perimeter gives $L+W=17$. The diagonal gives $L^2+W^2=13^2=169$. The pair $(5,12)$ satisfies both conditions, so the lot is 5 m by 12 m.

M2-13

Answer E

Question 13

Solve

$$\frac{3x-5}{2}+\frac{x+5}{4}=x+1.$$

A 1

B 1.5

C 3.5

D 4.5

E None of the above

SOLUTION

Multiply by 4: $2(3x-5)+(x+5)=4x+4$. Thus $6x-10+x+5=4x+4$, so $3x=9$ and $x=3$. Since 3 is not listed, the correct option is "None of the above".

M2-14

Answer B

Question 14

Differentiate:

$$y=x^2\ln x.$$

A $2x\ln x$

B $2x\ln x+x$

C $x\ln x+2x$

D $x^2\cdot\frac{1}{x}$

E $2x^2\ln x$

SOLUTION

Use the product rule: $\frac{d}{dx}(x^2\ln x)=2x\ln x+x^2\cdot\frac{1}{x}=2x\ln x+x$.

M2-15

Answer C

Question 15

Find the range of $y = \sin x + 2$.

A $[-1, 1]$

B $[0, 2]$

C $[1, 3]$

D $[2, 4]$

E $(-\infty, \infty)$

SOLUTION

Since $-1 \le \sin x \le 1$, adding 2 gives $1 \le y \le 3$.

M2-16

Answer C

Question 16

On the interval $1 \le x \le e$, what is the area of the region between the curves $y = \ln x$ and $y = 1 - x$?

A $\frac{e^2-1}{2}$

B $\frac{e^2-2e+1}{2}$

C $\frac{e^2-2e+3}{2}$

D $e - \frac{1}{e}$

E $e^2 - e + 1$

SOLUTION

On $[1, e]$, $\ln x \ge 1 - x$. The area is $\int_1^e (\ln x - 1 + x)\, dx$. This equals $\left[x \ln x - 2x + \frac{x^2}{2}\right]_1^e = \frac{e^2-2e+3}{2}$.

M2-17

Answer B

Question 17

For $x > 0$, define $f(x) = x + \frac{1}{x} + \ln x$. The minimum value of $f(x)$ is

A $2 + \ln\left(\frac{1}{2}\right)$

B $\sqrt{5} + \ln\left(\frac{\sqrt{5}-1}{2}\right)$

C $\sqrt{5} + \ln\left(\frac{\sqrt{5}+1}{2}\right)$

D $2\sqrt{5} + \ln\left(\frac{\sqrt{5}-1}{2}\right)$

E $\sqrt{5} - \ln\left(\frac{\sqrt{5}-1}{2}\right)$

SOLUTION

Differentiate: $f'(x) = 1 - \frac{1}{x^2} + \frac{1}{x}$. Setting this to zero gives $x^2 + x - 1 = 0$, so $x = \frac{\sqrt{5}-1}{2}$. At this point $x + 1/x = \sqrt{5}$, so the minimum is $\sqrt{5} + \ln\left(\frac{\sqrt{5}-1}{2}\right)$.

M2-18

Answer B

Question 18

For $0 < x < \frac{\pi}{2}$, the maximum value of

$$\ln(\sin x) + 2\ln(\cos x)$$

is

A $\ln\left(\frac{2}{3}\right)$

B $\ln\left(\frac{2}{3\sqrt{3}}\right)$

C $\ln\left(\frac{1}{3}\right)$

D $\ln\left(\frac{4}{9}\right)$

E $\ln\left(\frac{1}{3\sqrt{3}}\right)$

SOLUTION

Maximise $\ln(\sin x) + 2\ln(\cos x) = \ln(\sin x \cos^2 x)$. Let $u = \sin x$, so maximise $u(1-u^2)$. Its derivative is $1 - 3u^2$, so $u = 1/\sqrt{3}$. The maximum product is $\frac{2}{3\sqrt{3}}$, giving $\ln\left(\frac{2}{3\sqrt{3}}\right)$.

M2-19 **Answer E**

Question 19

Given that $y = (2+3x)^6$, what is the coefficient of x^3 in $\frac{dy}{dx}$?

- **A** 240
- **B** 4320
- **C** 4860
- **D** 12960
- **E** 19440

SOLUTION

$\frac{dy}{dx} = 18(2+3x)^5$. The coefficient of x^3 is $18\binom{5}{3}2^2 3^3 = 18 \cdot 10 \cdot 4 \cdot 27 = 19440$.

M2-20 **Answer E**

Question 20

Given that

$$\int_0^2 x^m \, dx = \frac{16\sqrt{2}}{7}$$

and

$$\int_0^2 x^{m+1} \, dx = \frac{32\sqrt{2}}{9},$$

what is the value of m?

- **A** $-\frac{11}{2}$
- **B** $-\frac{9}{2}$
- **C** $-\frac{22}{29}$
- **D** $\frac{7}{22}$
- **E** $\frac{5}{2}$
- **F** $\frac{7}{2}$

SOLUTION

Divide the two given integrals: $\frac{\int_0^2 x^{m+1} dx}{\int_0^2 x^m dx} = \frac{32\sqrt{2}/9}{16\sqrt{2}/7} = \frac{14}{9}$. But the ratio also equals $\frac{2(m+1)}{m+2}$. So $\frac{2(m+1)}{m+2} = \frac{14}{9}$, giving $m = \frac{5}{2}$.

M2-21

Answer G

Question 21

The complete set of values of a for which the equation $3x^2 = (a+2)x - 3$ has two real distinct roots is

A $\;$ no values of a

B $\;$ $-4\sqrt{2} < a < 4\sqrt{2}$

C $\;$ $a < -4\sqrt{2}$ or $a > 4\sqrt{2}$

D $\;$ $-4 < a < 8$

E $\;$ $a < -4$ or $a > 8$

F $\;$ $-8 < a < 4$

G $\;$ $a < -8$ or $a > 4$

H $\;$ all values of a

SOLUTION

Rewrite as $3x^2 - (a+2)x + 3 = 0$. Two distinct real roots require $(a+2)^2 - 36 > 0$, so $|a+2| > 6$. Hence $a < -8$ or $a > 4$.

M2-22

Answer G

Question 22

$f(x) = x^3 - a^2 x$ where a is a positive constant.

Find the complete set of values of x for which $f(x)$ is an increasing function.

A $\;$ $x \leq -a$, $x \geq a$

B $\;$ $-a \leq x \leq a$

C $\;$ $x \leq -a$, $0 \leq x \leq a$

D $\;$ $-a \leq x \leq 0$, $x \geq a$

E $\;$ $x \leq -\frac{a}{3}$, $x \geq \frac{a}{3}$

F $\;$ $-\frac{a}{3} \leq x \leq \frac{a}{3}$

G $\;$ $x \leq -\frac{a}{\sqrt{3}}$, $x \geq \frac{a}{\sqrt{3}}$

H $\;$ $-\frac{a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$

SOLUTION

$f'(x) = 3x^2 - a^2$. The function is increasing where $3x^2 - a^2 \geq 0$, so $|x| \geq \frac{a}{\sqrt{3}}$. Thus $x \leq -\frac{a}{\sqrt{3}}$ or $x \geq \frac{a}{\sqrt{3}}$.

M2-23

Answer D

Question 23

What is the coefficient of x^3 in the expansion of $(1-2x)^5(1+2x)^5$?

- **A** -6400
- **B** -640
- **C** -80
- **D** 0
- **E** 80
- **F** 800
- **G** 960

SOLUTION

$(1-2x)^5(1+2x)^5 = ((1-2x)(1+2x))^5 = (1-4x^2)^5$. This contains only even powers of x, so the coefficient of x^3 is 0.

M2-24

Answer A

Question 24

What is the equation of the line that is normal to the following function at $x = 4$?

$$y = f(x) = x^3 - 3x^{\frac{3}{2}} + 5x - 2$$

- **A** $y = -\frac{1}{44}x + \frac{639}{11}$
- **B** $y = -\frac{1}{8}x + \frac{21}{2}$
- **C** $y = -\frac{1}{8}x + \frac{7}{2}$
- **D** $y = 8x + \frac{2}{3}$
- **E** $y = 8x - \frac{16}{3}$
- **F** $y = -\frac{1}{5}x + \frac{3}{2}$
- **G** $y = -\frac{1}{5}x + \frac{7}{2}$
- **H** $y = 5x - \frac{16}{3}$

SOLUTION

For the function, $f(4) = 4^3 - 3 \cdot 4^{3/2} + 5 \cdot 4 - 2 = 58$. Also $f'(x) = 3x^2 - \frac{9}{2}\sqrt{x} + 5$, so $f'(4) = 44$. The normal gradient is $-1/44$, giving $y - 58 = -\frac{1}{44}(x-4)$, i.e. $y = -\frac{x}{44} + \frac{639}{11}$.

M2-25

Answer **E**

Question 25

The curve $y = x^2$ is translated by the vector $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and then reflected in the line $y = -1$.

Which one of the following is an equation of the resulting curve?

- **A** $y = -3 - (x-4)^2$
- **B** $y = -3 + (x+4)^2$
- **C** $y = 3 - (x+4)^2$
- **D** $y = 3 + (x-4)^2$
- **E** $y = -5 - (x-4)^2$
- **F** $y = -5 + (x+4)^2$
- **G** $y = 5 - (x+4)^2$
- **H** $y = 5 + (x-4)^2$

SOLUTION

After translating $y = x^2$ by $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$, the curve is $y = (x-4)^2 + 3$. Reflection in $y = -1$ sends y to $-2 - y$, so the result is $y = -5 - (x-4)^2$.

M2-26

Answer **D**

Question 26

The graph of the function $y = x^3 + px^2 + qx + 6$, where p and q are real constants, has a local maximum when $x = 2$ and a local minimum when $x = 4$.

What are the values of p and q?

- **A** $p = -3$ and $q = -8$
- **B** $p = -3$ and $q = 8$
- **C** $p = 3$ and $q = -8$
- **D** $p = -9$ and $q = 24$
- **E** $p = 9$ and $q = 24$
- **F** $p = 9$ and $q = -24$

SOLUTION

At stationary points, $y' = 3x^2 + 2px + q = 0$. Since the stationary points are at $x = 2$ and $x = 4$, $y' = 3(x-2)(x-4) = 3x^2 - 18x + 24$. Thus $2p = -18$ and $q = 24$, so $p = -9$.

M2-27

Answer C

Question 27

A square $PQRS$ is drawn above the x-axis with the side PQ on the x-axis.

P is the point $(-5, 0)$ and Q is the point $(1, 0)$.

A circle is drawn inside the square with diameter equal in length to the side of the square.

Which one of the following is an equation of the circle?

A $x^2 + y^2 - 4x + 6y + 4 = 0$

B $x^2 + y^2 - 4x + 6y + 9 = 0$

C $x^2 + y^2 + 4x - 6y + 4 = 0$

D $x^2 + y^2 + 4x - 6y + 9 = 0$

E $x^2 + y^2 - 6x - 4y + 9 = 0$

F $x^2 + y^2 - 6x + 4y + 4 = 0$

G $x^2 + y^2 + 6x - 4y + 4 = 0$

H $x^2 + y^2 + 6x + 4y + 9 = 0$

SOLUTION

The square has side length 6. The circle is centred halfway across the square and 3 units above the x-axis, so its centre is $(-2, 3)$ and radius is 3. Hence $(x + 2)^2 + (y - 3)^2 = 9$, which expands to $x^2 + y^2 + 4x - 6y + 4 = 0$.