

Week 2: Pre-AMC Level

Problem 1 If $\frac{a}{c} \bigg| \frac{b}{d} = a \cdot d - b \cdot c$, what is the value of $\frac{3}{6} \bigg| \frac{4}{8}$?

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

Problem 2 The stock price of a company was \$100 in 2005. It went up by 20% in 2006. However, due to the financial crisis, the stock price went down by 20% in 2007. What was the stock price at the end of 2007?

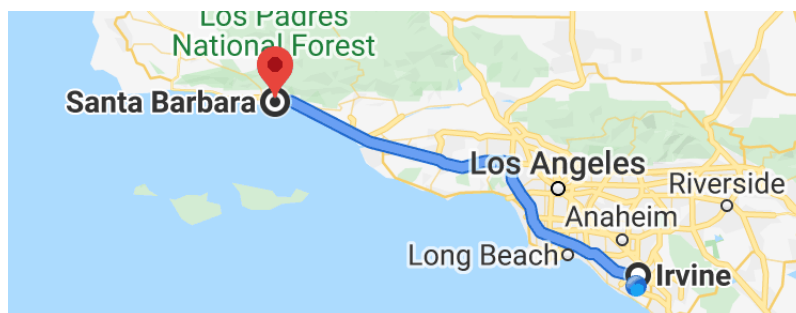
- (A) \$80 (B) \$96 (C) \$98 (D) \$100 (E) \$120

Problem 3 Irvine (CA) and Santa Barbara are approximate 120 miles apart. Austin drove from Irvine to visit his brother Benjamin in Santa Barbara, averaging 60 miles per hour. On his return trip along the same route, due to some heavy traffic on I-5, he averaged only 40 miles per hour. What was the average speed for the round trip, in miles per hour?

(Recall that

$$\text{Average speed} = \frac{\text{Total travel distance}}{\text{Total amount of time}}.$$

)



- (A) 45 (B) 46 (C) 48 (D) 50 (E) 54

Problem 4 Find the number a such that

$$\frac{7}{8} = \frac{1}{1 + \frac{1}{a}}.$$

Problem 5 [AMC8 Problem] Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, 'I didn't get the lowest score in our class,' and Bridget adds, 'I didn't get the highest score.' What is the ranking of the three girls from highest to lowest?

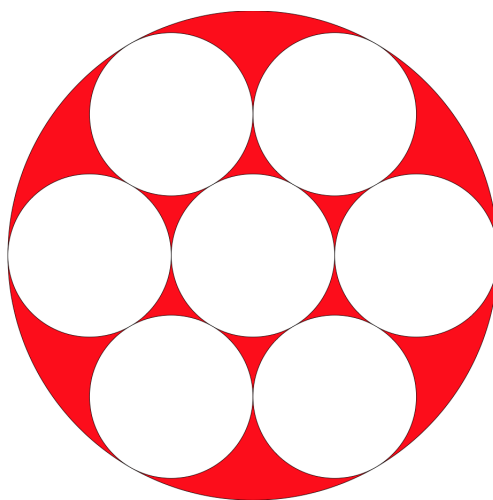
- (A) Hannah, Cassie, Bridget (B) Hannah, Bridget, Cassie
(C) Cassie, Bridget, Hannah (D) Cassie, Hannah, Bridget
(E) Bridget, Cassie, Hannah

Problem 6 The following is an arithmetic sequence that has 10 numbers

$$1, a_2, a_3, a_4, \dots, a_{10}.$$

If these 10 numbers add up to 100, what is the second number a_2 ?

Problem 7 Seven small circles with radius 1 are circumscribed by a large circle. Adjacent circles are tangent (i.e, they touch at only one point). Six small circles are externally tangent to center one and internally tangent to the big one. What fraction of the large circle's area is shaded?



- (A) $\frac{2}{9}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{5}{9}$

Problem 8 x , y and z are three numbers satisfying that

$$\begin{cases} x + 2y + 3z = 25 \\ 3x + 2y + z = 35. \end{cases}$$

What is $x + y + z$?

- (A) 10 (B) 12 (C) 15 (D) 16
(E) There are more than one possible values

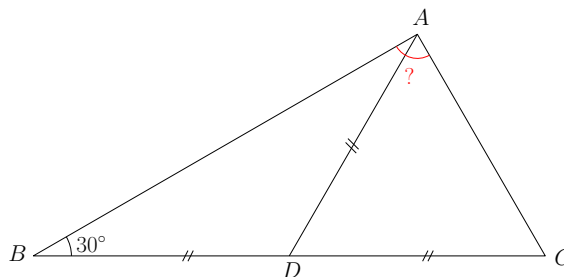
Problem 9 A bag contains 5 balls (three red and 2 blue). Leo picks two balls at random. What is the probability that he gets two balls of the same color?

Recall that

$$\text{Probability} = \frac{\text{Number of valid cases}}{\text{Number of all possible cases}}.$$

- (A) $\frac{1}{5}$ (B) $\frac{3}{10}$ (C) $\frac{2}{5}$ (D) $\frac{1}{2}$ (E) $\frac{3}{5}$

Problem 10 In $\triangle ABC$, D is the midpoint on BC . If $AD = BD = CD$ and $\angle B = 30^\circ$, what is $\angle BAC$?



- (A) 60° (B) 80° (C) 86° (D) 90° (E) 95°

Problem 11 How many 2-digit positive integers are divisible by 3 or 5 or both?

- (A) 18 (B) 30 (C) 42 (D) 48 (E) 50

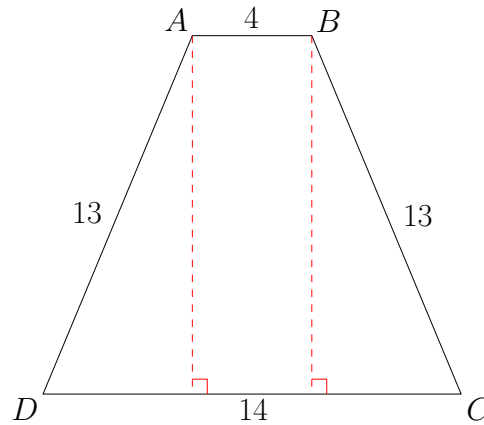
Problem 12 If a is the smallest two-digit number such that

$$\text{lcm}(a, 12) = 60,$$

how many different divisors does a have? Recall that “ lcm ” stands for the *least common multiple*.

- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12

Problem 13 $ABCD$ is an isosceles trapezoid. AB is parallel with CD . If $AB = 4$, $BC = AD = 13$ and $CD = 14$, what is the area of the trapezoid?



- (A) 48 (B) 78 (C) 90 (D) 108 (E) 110

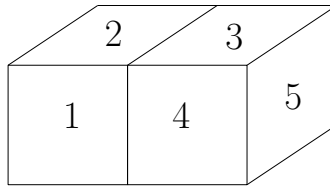
Problem 14 A water tank weighs 190 kg when it is full of water and 120 kg when half full. How much does it weigh, in kilograms, when empty?

- (A) 30 (B) 40 (C) 50 (D) 60 (E) 70

Problem 15 If it is 8 am now, what time will it be 2020 hours later?

- (A) 7am (B) 8am (C) 11am (D) 12pm (E) 7pm

Problem 16 Two cubes with numbers on their faces are put next to each other as follows. For each cube, the sum of numbers on every two opposite faces is 15, what is the sum of numbers on those 7 faces that are hidden in the following picture? (Chinese Hope Cup Math Competition)

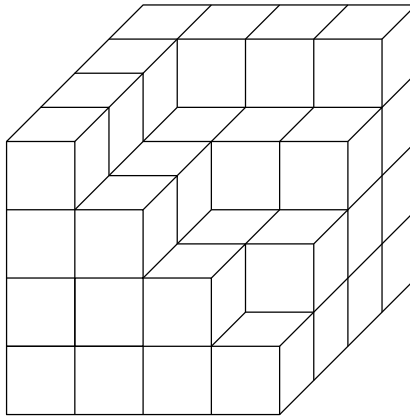


- (A) 60 (B) 65 (C) 75 (D) 76
(E) 80

Problem 17 A positive integer is called 13-special if the quotient and remainder are the same when this integer is divided by 13. How many positive 13-special integers are between 1 and 100 (inclusive)?

- (A) 5 (B) 6 (C) 7 (D) 10 (E) 12

Problem 18 The following three-dimensional figure is made of small cubes with equal side length. How many small cubes are there?



- (A) 48 (B) 49 (C) 50 (D) 51 (E) 52

Problem 19 There are 7 cards with seven different numbers 1-7 written on them. Austen, Sam and Stephanie each takes 2 cards. (Chinese Hope Cup Math Competition)

Austen said: “Numbers on my two cars add up to 7”.

Sam said: "The difference between my two numbers is 1".

Stephanie said: "The product of my two numbers is 12."

What is the number on the card that is left?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 7

Problem 20 Two couples sit around a table. If each person needs to sit next to his/her spouse, in how many different ways can we sit them?



(Hint: use multiplication principle. Note that husband and wife could switch positions.)

- (A) 4 (B) 8 (C) 12 (D) 16 (E) 18

Answers

1. C
2. B (Hint: Up and down percentages are based on the previous stock price instead of the stock price at the very beginning.)
3. C 4. 7 5. D
6. 3 (Hint: According to Gauss's trick, what is the relation between 100 and $1 + a_{10}$?)
7. A 8. C 9. C 10. D
11. C 12. A
13. D (Hint: Draw the height of the trapezoid (the red line in the picture) and employ Pythagorean theorem to find the height. The use "cut and pastes" to find the area.)
14. C 15. D
16. C 17. C 18. C 19. A
20. D (Hint: Use multiplication principle. Note that husband and wife could switch positions.)