AMC 10 DIAGNOSTIC SERIES

AMC 10 Diagnostic Test

25 Questions + Fully Worked Solutions

Recommended test time: 75 minutes

Five-option multiple choice · No calculator

ThrivingScholars

Answer key and diagnostic map

This booklet contains all 25 questions, embedded diagrams, the verified answer key, and a complete worked solution beneath every problem.

1 C · 2 D · 3 B · 4 C · 5 B · 6 D · 7 D · 8 C · 9 E · 10 B · 11 C · 12 A · 13 C
14 E · 15 A · 16 D · 17 B · 18 A · 19 E · 20 C · 21 C · 22 D · 23 E · 24 E · 25 C

Band	Questions	Main skills sampled
Foundation	1-6	Arithmetic, algebraic modelling, counting and basic geometry
Core AMC 10	7-12	Quadratics, polygons, probability, solid geometry and congruences
Intermediate	13-17	Similarity, recurrences, tangential geometry and factorial valuation
Advanced	18-21	Perfect squares, circle geometry, valuations and coordinate geometry
Stretch	22-25	Graph reasoning, sequences, floor functions and regular graphs

Editorial corrections applied: Question 2 has unambiguous cutting language; Question 17 explicitly states positive integer side lengths; and Question 24 uses the standard floor-function definition.

AMC 10 DIAGNOSTIC TEST

Questions and worked solutions

Correct options are highlighted for review.

ALGEBRA

Question 1

Answer C

Airline A charges 8 cents per mile with an initial fee of $\$50$. Airline B charges 10 cents per mile with an initial fee of $\$40$. How many miles must a flight be if it costs the same for both airlines?

- **A** 50
- **B** 100
- **C** 500
- **D** 1000
- **E** 5000

FULLY WORKED SOLUTION

Let m be the number of miles. The two costs are $$50+0.08m \quad\text{and}\quad 40+0.10m.$$ Set them equal: $$50+0.08m=40+0.10m.$$ Hence $10=0.02m$, so $$m=500.$$ Therefore the required distance is $\boxed{500}$ miles.

ARITHMETIC / SCHEDULING

Question 2

Answer D

A woodworker has a \(20\)-foot board and, beginning on Monday, cuts off one \(2\)-foot piece at the end of each working day. The woodworker does not work on Saturdays or Sundays. The process stops when the remaining board is itself a \(2\)-foot piece. On what day is the final cut made?

- **A** Monday
- **B** Tuesday
- **C** Wednesday
- **D** Thursday
- **E** Friday

FULLY WORKED SOLUTION

The board is to become ten \(2\)-foot pieces. Starting with one board, producing ten pieces requires only nine cuts, because the last \(2\)-foot section is left uncut. The first five cuts occur Monday through Friday. After the weekend, cuts \(6,7,8,9\) occur on Monday, Tuesday, Wednesday, and Thursday. Therefore the final cut is made on \(\boxed{\text{Thursday}}\).

RATES

Question 3

Answer B

Larry can make one batch of \(3\) cookies every \(8\) minutes. Mary can make one batch of \(5\) cookies every \(12\) minutes. They both start making cookies at 3:00 PM. At what time will they have made a combined total of \(60\) cookies?

- **A** 4:12 PM
- **B** 4:20 PM
- **C** 4:24 PM
- **D** 4:32 PM
- **E** 4:36 PM

FULLY WORKED SOLUTION

After \72\)*minutes*, *Larryhascompleted*\(9\)*batchesandMaryhascompleted*\(6\) : \[9(3+65=27+30=57. \] At \ 80\)*minutes*, *Larrycompleteshistenthbatch whileMarystillhas*\(30\)*cookies* : \[10(3+65=60. \] Eighty minutes after 3:00 PM is \(\boxed{\text{4:20 PM}}\).

GEOMETRY / AREA

Question 4

Answer C

Triangle \(ABC\) has area \(60\) and \(BC=9\). Point \(D\) lies on line \(BC\) with \(CD=6\). What is the positive difference between the two possible areas of triangle \(ABD\)?

- **A** \(20\)
- **B** \(60\)
- **C** \(80\)
- **D** \(100\)
- **E** \(120\)

FULLY WORKED SOLUTION

Let \h\)bethealtitudefrom\(A\)toline\(BC\). Since\[$\frac{1}{2}$(9h=60, \] we get \
h = $\frac{40}{3}$\). Therearetwopossiblepositionsof\(D\) : −If\(D\)liesbetween\(B\)and\(C\), then\(BD = 9 − 6 = 3\), so\[[AB
\leftMissing or unrecognized delimiter for \right=20. \] - If \
D\)liesbeyond\(C\), then\(BD = 9 + 6 = 15\), so\[[ABD] = $\frac{1}{2}$(15\leftMissing or unrecognized delimiter for \right=100. \]
The positive difference is \(100-20=\boxed{80}\).

NUMBER THEORY / REMAINDERS

Question 5

Answer B

When a number is divided by \(8\), it has a certain quotient and remainder \(5\). When the same number is divided by \(5\), its quotient is doubled and its remainder is \(3\). What is the number?

- **A** \(8\)
- **B** \(13\)
- **C** \(21\)
- **D** \(40\)
- **E** \(53\)

FULLY WORKED SOLUTION

Let \
q\)bethequotientupondivisionby\(8\). Then\[N = 8q + 5.\]Thequotientupondivisionby\(5\)is\(2q\), so\[N = 5(2q
+3=10q+3. \] Equating the two expressions gives \[8q+5=10q+3 \implies q=1. \] Thus \N = 8(1+5=\boxed{13}\).

COUNTING

Question 6

Answer D

How many four-digit numbers consist of only odd digits or only even digits?

A \(500\)

B \(625\)

C \(1000\)

D \(1125\)

E \(1250\)

FULLY WORKED SOLUTION

For an all-odd number, each of the four digits has \ 5\)*choices* : \[$5^4 = 625.$\]*For all* − *even four* − *digit number*, *the first digit has*\(4\)*choices*\(($2, 4, 6, 8$\), while each remaining digit has \5\)*choices*\(($0, 2, 4, 6, 8$\): \[4\cdot5^3=500. \] The cases are disjoint, so the total is \[625+500=\boxed{1125}. \]

ALGEBRA / QUADRATICS

Question 7

Answer D

What is the sum of all integers \(n\) such that the equation \[x^2-2nx+n^2-4=0 \] has both solutions strictly between \(-3\) and \(5\)?

A \(-3\)

B \(-1\)

C \(1\)

D \(3\)

E \(5\)

FULLY WORKED SOLUTION

Rewrite the equation as \[$x-n$^2=4. \] Its roots are \(n-2\) and \(n+2\). Both must lie strictly between \(-3\) and \(5\): \[n-2>-3 \implies n>-1, \] \[n+2<5 \implies n<3. \] Thus \(n\in\{0,1,2\}\), whose sum is \(\boxed{3}\).

GEOMETRY / POLYGONS

Question 8

Answer C

A regular \(n\)-gon has \(2015\) diagonals. What is the value of \(n\)?

A \(55\)

B \(58\)

C \(65\)

D \(68\)

E \(70\)

FULLY WORKED SOLUTION

An \
$n\backslash) - gonhas\backslash[\frac{n(n-3)}{2}\backslash]diagonals. Hence\backslash[\frac{n(n-3)}{2} = 2015, \backslash]so\backslash[n^2 - 3n - 4030 = 0.\backslash]Thediscriminantis\backslash[9 + 4(4030$
=16129=127^2. \] Therefore \[n=\frac{3+127}{2}=\boxed{65}. \]

PROBABILITY

Question 9

Answer E

A bag contains \(3\) red balls, \(4\) blue balls, and \(5\) green balls. Three balls are chosen at random without replacement. What is the probability that all three balls have different colours?

A \(\frac1{22}\)

B \(\frac1{11}\)

C \(\frac{12}{65}\)

D \(\frac5{26}\)

E \(\frac3{11}\)

FULLY WORKED SOLUTION

There are \(12\) balls, so the number of three-ball selections is \[\binom{12}{3}=220. \] To obtain one ball of each colour, choose one red, one blue, and one green: \[3\cdot4\cdot5=60. \] Thus the probability is \[\frac{60}{220}=\boxed{\frac3{11}}. \]

NUMBER THEORY / DIFFERENCE OF SQUARES

Question 10

Answer B

If \(x\) and \(y\) are positive integers such that \[x^2=31+y^2, \] what is the value of \(x^2+y^2\)?

- **A** \(31\)
- **B** \(481\)
- **C** \(705\)
- **D** \(736\)
- **E** \(961\)

FULLY WORKED SOLUTION

We have \[x^2-y^2=31, \] so \[$x-yx+y$=31. \] Because \(31\) is prime and \(x,y>0\), \[x-y=1,\qquad x+y=31. \] Hence \(x=16\) and \(y=15\). Therefore \[x^2+y^2=16^2+15^2=256+225=\boxed{481}. \]

GEOMETRY / SOLIDS

Question 11

Answer C

The surface area of a sphere is twice the surface area of a cube. What is the ratio of the volume of the sphere to the volume of the cube?

- **A** \(\frac{2\sqrt{3\pi}}{\pi}\)
- **B** \(2\)
- **C** \(\frac{4\sqrt{3\pi}}{\pi}\)
- **D** \(4\)
- **E** \(2\pi\)

FULLY WORKED SOLUTION

Let the sphere have radius \r\)$andthecubehavesidelength$\(s\)$.Theconditiongives$\[$4\pi r^2 = 2(6s^2$=12s^2, \] so \[\frac{r}{s}=\sqrt{\frac3\pi}. \] The volume ratio is \[\frac{\frac43\pi r^3}{s^3} =\frac43\pi\left Missing or unrecognized delimiter for \right ^{3/2} =\boxed{\frac{4\sqrt{3\pi}}{\pi}}. \]

NUMBER THEORY / CONGRUENCES

Question 12

Answer **A**

When a teacher divided a class into equal groups of 3, there were 2 students left over. When the class was divided into equal groups of 5, there were 4 students left over. When it was divided into equal groups of 4, there were no students left over. What is the least possible number of students?

- **A** 44
- **B** 56
- **C** 68
- **D** 84
- **E** 104

FULLY WORKED SOLUTION

Let the class size be N. The first two conditions are $$N\equiv -1\pmod3,\qquad N\equiv -1\pmod5.$$ Thus $$N\equiv -1\pmod{15}.$$ The positive numbers in this class are $14,29,44,59,\ldots$. The first one divisible by 4 is $\boxed{44}$.

GEOMETRY / SIMILARITY

Question 13

Answer C

In square \(ABCD\), point \(E\) lies on segment \(AD\), and segments \(BD\) and \(CE\) intersect at \(F\). The line \(CE\) is extended past \(E\) to meet the extension of \(AB\) at \(G\). If \(EF=9\) and \(CF=15\), what is \(EG\)?

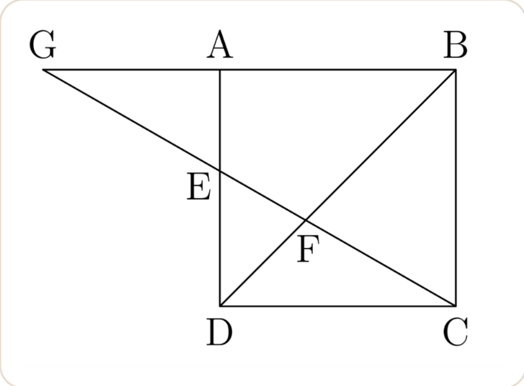

Diagram for Question 13

- **A** \(6\)
- **B** \(12\)
- **C** \(16\)
- **D** \(20\)
- **E** \(24\)

FULLY WORKED SOLUTION

Since \
$CF = 15$\)and\($EF = 9$\), \[$CE = 24,$ \]and\(F\)divides\(CE\)intheratio\[$CF : FE = 15 : 9 = 5 : 3.$\]Put\($D = (0,0$
\), \$C = (s, 0$\), \$A = (0, s$\), and \$E = (0, e$\). Since \Missing or unrecognized delimiter for \right. \] The diagonal \
BD\)hasequation\($y = x$\), so\[$\frac{3s}{8} = \frac{5e}{8} \implies \frac{s}{e} = \frac{5}{3}.$ \]Alongline\(CEG\), reachingheight\(s\)insteadofheight\(e\)sca
=40. \] Therefore \[EG=CG-CE=40-24=\boxed{16}. \]

COMBINATORICS / RECURRENCE

Question 14

Answer **E**

There is an ample supply of identical \(1\times1\) tiles and identical \(2\times2\) tiles. Using these two types of tiles, how many ways are there to tile a \(3\times8\) grid?

- **A** \(34\)
- **B** \(66\)
- **C** \(96\)
- **D** \(130\)
- **E** \(171\)

FULLY WORKED SOLUTION

Let \
T_n\)$bethenumberoftilingsofa$\($3 \times n$\)$grid. Attheleftedge, either: 1.thefirstcolumnisfilledentirelywith$\($1 \times 1$\)$tiles, le$
=\boxed{171}. \]

GEOMETRY / TANGENTIAL QUADRILATERALS

Question 15

Answer **A**

An isosceles trapezoid has parallel sides of lengths \(15\) and \(60\). A circle is inscribed in the trapezoid, touching all four sides. What is the radius of the circle?

- **A** \(15\)
- **B** \(25\)
- **C** \(30\)
- **D** \(45\)
- **E** \(50\)

FULLY WORKED SOLUTION

Let each leg have length \
L\). $Inanytangentialquadrilateral, thesumsofoppositesidelengthsareequal, so$\[$15 + 60 = 2L \implies L = 37.5.$ \]$Thebasedi$
$37.5 + 22.5$=15\cdot60=900, \] so \(h=30\). The height is the diameter of the incircle, hence \[r=\frac{30}{2}=\boxed{15}. \]

NUMBER THEORY / FACTORIALS

Question 16

Answer D

There exist three-digit integers \n\)suchthat\((n − 1!\) ends in \(A\) consecutive zeroes while \(n!\) ends in \(A+3\) consecutive zeroes. What is the sum of the digits of the largest possible value of \(n-1\)?

- **A** \(7\)
- **B** \(8\)
- **C** \(13\)
- **D** \(19\)
- **E** \(20\)

FULLY WORKED SOLUTION

The increase in the number of trailing zeroes from \(n − 1!\) to \ n!\)equalsthenumberoffactorsof\(5\)in\(n\).Weneed\[v_5(n=3. \] Thus \(n\) must be divisible by \(5^3=125\), but not by \(5^4=625\). The largest three-digit multiple satisfying this is \[n=875=7\cdot5^3. \] Hence \(n-1=874\), and the digit sum is \[8+7+4=\boxed{19}. \]

NUMBER THEORY / FACTORING

Question 17

Answer B

A rectangle with positive integer side lengths has area \(A\text{ cm}^2\) and perimeter \(P\text{ cm}\). Which of the following numbers cannot equal \(A+P\)?

- **A** \(100\)
- **B** \(102\)
- **C** \(104\)
- **D** \(106\)
- **E** \(108\)

FULLY WORKED SOLUTION

Let the positive integer side lengths be \ x\)and\(y\).Then\[A + P = xy + 2x + 2y. \]Adding\(4\)gives\[A + P + 4 = (x + 2y + 2. \] Both factors must be integers at least \(3\). Check the choices after adding \(4\): \[104=8\cdot13,\quad 106=2\cdot53,\quad 108=6\cdot18, \] \[110=5\cdot22,\quad 112=7\cdot16. \] Only \(106\) cannot be written as a product of two integers both at least \(3\). Therefore \[A+P=102 \] is impossible, so the answer is \(\boxed{102}\).

NUMBER THEORY / PERFECT SQUARES

Question 18

Answer A

For how many positive integers \(n\) is \[n^{2015}-n \] the square of an integer?

- **A** \(1\)
- **B** \(2\)
- **C** \(3\)
- **D** \(4\)
- **E** \(5\)

FULLY WORKED SOLUTION

Factor: \[n^{2015}-n=n$n^{2014}-1$. \] Also \[\gcd$n, n^{2014}-1$=1. \] If their product is a square, each relatively prime factor must be a square. Write \[n=a^2,\qquad n^{2014}-1=b^2. \] Then \[a^{4028}-b^2=1, \] so \[$a^{2014}-ba^{2014}+b$=1. \] Both factors are nonnegative integers, so both must equal \(1\). This gives \(a=1\), hence \(n=1\). Indeed, \[1^{2015}-1=0=0^2. \] Therefore there is exactly \(\boxed{1}\) positive integer.

GEOMETRY / CIRCLES

Question 19

Answer E

Points \(A,B,C,D\) lie on a circle. Chords \(AC\) and \(BD\) are perpendicular and intersect at \(E\) inside the circle. If \(AC=27\), \(AD=5\), and \(AE=3\), what is the distance from \(E\) to the centre of the circle?

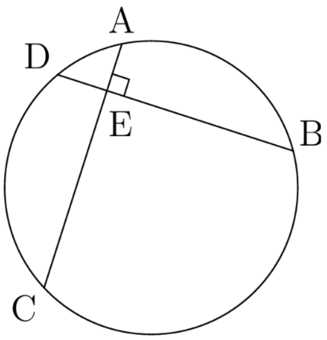

Diagram for Question 19

A \(7\)

B \(\frac{7\sqrt5}{2}\)

C \(\frac{21}{2}\)

D \(\frac{7\sqrt{10}}2\)

E \(\frac{7\sqrt{13}}2\)

FULLY WORKED SOLUTION

Place \$E = (0, 0$\), put \
$AC\backslash)onthe\backslash(x\backslash) - axis, andput\backslash(BD\backslash)onthe\backslash(y\backslash) - axis. Since\backslash(AE = 3\backslash)and\backslash(AC = 27\backslash), take\backslash[A = (-3, 0$
,\qquad C=$24, 0$. \] Let \$D = (0, -d$\). From \$AD = 5\backslash), \backslash[3^2 + d^2 = 5^2 \implies d = 4, \backslash]so\backslash(D = (0, -4$\). By the intersecting-chords theorem, \[AE\cdot EC=BE\cdot ED, \] so \[3$24$=BE4\implies BE=18. \] Thus \$B = (0, 18$\). The perpendicular bisector of \
Missing or unrecognized delimiter for \right. \] Hence \[EO=\sqrt{\left Missing or unrecognized delimiter for \right ^2+7^2}
=\frac12\sqrt{441+196} =\boxed{\frac{7\sqrt{13}}2}. \]

COMBINATORICS / VALUATIONS

Question 20

Answer C

Santa has \(64\) distinct gifts to give to \(40\) different elves. Let \(N\) be the number of distributions for which the numbers of gifts received by any two elves differ by at most \(1\). When \(N\) is written in base \(4\), how many consecutive zeroes appear at the end?

- **A** \(9\)
- **B** \(19\)
- **C** \(20\)
- **D** \(21\)
- **E** \(32\)

FULLY WORKED SOLUTION

Every elf must receive either \$1\)or\(2\)gifts. If\(x\)elvesreceive\(2\), then\[2x+(40-x$=64\implies x=24. \] Choose those \$24\)elvesanddistributethedistinctgifts: \[N=\binom{40}{24}\frac{64!}{(2!)^{24}}. \]Weneed\(v_2(N$\). By Legendre's formula, \[v_2$64!$ =32+16+8+4+2+1=63. \] Also \[v_2\binom{40}{24}=v_2$40!$-v_2$24!$-v_2$16!$=38-22-15=1. \] Therefore \[v_2N=1+63-24=40. \] Each terminal base-\(4\) zero requires a factor \(4=2^2\), so the number of zeroes is \[\left\lfloor\frac{40}{2}\right\rfloor=\boxed{20}. \]

GEOMETRY / COORDINATES

Question 21

Answer C

Equiangular hexagon \(ABCDEF\) has side lengths \[AB=1,\qquad BC=EF=2,\qquad CD=3. \] What is the area of \(ABCDEF\)?

- **A** \(6\sqrt2\)
- **B** \(\frac{13\sqrt2}{2}\)
- **C** \(\frac{11\sqrt3}{2}\)
- **D** \(6\sqrt3\)
- **E** \(12\)

FULLY WORKED SOLUTION

Every interior angle is \
$120^\circ\backslash), sothesidedirectionsmaybetakenas\backslash[0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ. \backslash]Let\backslash(DE=x\backslash)and\backslash(FA=y\backslash). Resolvingthesi$
\), convenient coordinates are \

$$B=(1,0),\quad C=(2,\sqrt3),\backslash]\backslash[D=\left(\frac12,\frac{5\sqrt3}{2}\right),\quad E=\left(-\frac12,\frac{5\sqrt3}{2}\right),\backslash]\backslash[F=\left(-\frac32,\frac{3\sqrt3}{2}\right).\backslash]Applyingtheshoelaceformu$$

=\boxed{\frac{11\sqrt3}{2}}. \]

COMBINATORICS / GRAPH REASONING

Question 22

Answer D

At a large party there are 2015 married couples. Each person knows their own spouse. One man asks every other person how many people at the party they do not know, and receives a different integer from every person. How many people does his wife know? Assume that knowing is symmetric and that no one counts themself.

- **A** 1
- **B** 2
- **C** 2014
- **D** 2015
- **E** 4029

FULLY WORKED SOLUTION

There are 4030 people, and the man asks the other 4029. Because everyone knows at least their spouse, the possible numbers of unknown people are $$0,1,2,\ldots,4028.$$ All answers are different, so every one of these values occurs. The person who knows everyone, answer 0, must be married to the person who knows only one person, answer 4028. Removing that couple, the same argument pairs answers $$1\text{ with }4027,\quad 2\text{ with }4026,\quad\ldots$$ Thus spouses' answers sum to 4028. The unique unpaired response is $$\frac{4028}{2}=2014,$$ and it must belong to the questioner's wife. She therefore does not know 2014 of the other 4029 people, so she knows $$4029-2014=\boxed{2015}$$ people.

ALGEBRA / SEQUENCES

Question 23

Answer E

Brian, Chris, and Paul compare their scores on two tests. Their math scores, in that order, form an arithmetic progression, while their history scores, in the same order, form a geometric progression. Their respective combined scores are $168,169,172$, and the sum of the three math scores is 255. If Paul scored higher in math than in history, what is the range of the three math scores?

- **A** 11
- **B** 15
- **C** 22
- **D** 26
- **E** 30

FULLY WORKED SOLUTION

Write the math scores as $$85-d,\quad 85,\quad 85+d,$$ because their sum is 255. The corresponding history scores are $$83+d,\quad 84,\quad 87-d.$$ These form a geometric progression, so $$84^2=(83+d$$ $87-d$. Expanding gives $$7056=7221+4d-d^2,$$ so $$d^2-4d-165=0.$$ Thus $$d=15\quad\text{or}\quad d=-11.$$ Paul's math score exceeds his history score: $$85+d>87-d\implies d>1,$$ so $d=15$. The math scores are $70,85,100$, and their range is $$100-70=\boxed{30}.$$

NUMBER THEORY / FLOOR FUNCTION

Question 24

Answer E

Let \(\lfloor x\rfloor\) denote the greatest integer less than or equal to \(x\). How many distinct values occur in \[\left\{ \left\lfloor\frac{1^2}{2015}\right\rfloor, \left\lfloor\frac{2^2}{2015}\right\rfloor, \ldots, \left\lfloor\frac{2015^2}{2015}\right\rfloor \right\}? \]

A \(503\)

B \(1008\)

C \(1009\)

D \(1511\)

E \(1512\)

FULLY WORKED SOLUTION

Define \

$a_k = \left\lfloor \frac{k^2}{2015} \right\rfloor.$ \]$Thissequenceisnondecreasing. For$\($k \geq 1007$\), \[$(k+1)^2 - k^2 = 2k+1 \geq 2015,$ \]$soeverysubsequentter$

\) contains a multiple of \(2015\). Across \(k=1,\ldots,1006\), these intervals combine to \((1,1007^2]\). The number of multiples of \(2015\) in this interval is \[\left\lfloor\frac{1007^2}{2015}\right\rfloor=503. \] Hence among the first \(1006\) transitions, \(503\) are increases and \(503\) are repetitions. Since there are \(2015\) terms total, the number of distinct values is \[2015-503=\boxed{1512}. \]

COMBINATORICS / GRAPHS

Question 25

Answer C

There are \(12k\) mathematicians at a conference. Each mathematician shakes hands with \(3k+6\) others. There is a constant \(c\) such that every pair of mathematicians has exactly \(c\) common handshake partners. What is \(k\)?

A \(1\)

B \(2\)

C \(3\)

D \(4\)

E \(5\)

FULLY WORKED SOLUTION

Model the situation as a graph. Let \[v=12k,\qquad r=3k+6. \] Fix a mathematician \ A\). $Thereare$\($r(r-1$\) two-step paths \ $A \to B \to C$\)$with$\($C \neq A$\) : $chooseoneof$\(A\)$'s$\(r\)$neighboursandthenoneofthatneighbour'sother$\($r-1$\)$neighbou$ $=cv-1.$ \] Substituting gives \[$3k+63k+5$=c$12k-1.$ \] Because the stated configuration exists, \(c\) must be an integer. Testing the five choices: \[k=1:\ \frac{72}{11},\quad k=2:\ \frac{132}{23},\quad k=3:\ \frac{210}{35}=6, \] \[k=4:\ \frac{306}{47},\quad k=5:\ \frac{420}{59}. \] Only \(k=3\) works, so \(\boxed{k=3}\).