AP® Calculus AB Exam

SECTION I: Multiple Choice **2019**

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CALCULUS AB

SECTION I, Part A

Time—1 hour

Number of questions—30

NO CALCULATOR IS ALLOWED FOR THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. $\displaystyle\int \frac{x^2}{4}\,dx =$

(A) $\dfrac{x}{2} + C$ (B) $\dfrac{x^3}{12} + C$ (C) $\dfrac{x^3}{4} + C$ (D) $\dfrac{3x^3}{4} + C$

(B)	**Correct.** By the power rule for antiderivatives, the antiderivative of x^n is $\dfrac{x^{n+1}}{n+1}$ for $n \neq -1$. Therefore, $\displaystyle\int \frac{x^2}{4}\,dx = \frac{1}{4}\int x^2\,dx = \frac{1}{4}\cdot\frac{x^3}{3} + C = \frac{x^3}{12} + C.$

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2. Which of the following is an equation of the line tangent to the graph of $y = \cos x$ at $x = \dfrac{\pi}{2}$?

(A) $y = x + \dfrac{\pi}{2}$

(B) $y = x - \dfrac{\pi}{2}$

(C) $y = -x + \dfrac{\pi}{2}$

(D) $y = -x - \dfrac{\pi}{2}$

(C)	**Correct.** The slope of the tangent line is the value of the derivative at $x = \dfrac{\pi}{2}$. $\dfrac{dy}{dx} = -\sin x \Rightarrow \left.\dfrac{dy}{dx}\right\|_{x=\frac{\pi}{2}} = -\sin\left(\dfrac{\pi}{2}\right) = -1$ At $x = \dfrac{\pi}{2}$, $y = \cos\left(\dfrac{\pi}{2}\right) = 0.$ An equation of the tangent line at the point $\left(\dfrac{\pi}{2}, 0\right)$ is therefore $y = -1\left(x - \dfrac{\pi}{2}\right) + 0 = -x + \dfrac{\pi}{2}.$

3. $\frac{d}{dx}\left(2(\sin\sqrt{x})^2\right) =$

(A) $4\cos\left(\frac{1}{2\sqrt{x}}\right)$ (B) $4\sin\sqrt{x}\cos\sqrt{x}$ (C) $\frac{2\sin\sqrt{x}}{\sqrt{x}}$ (D) $\frac{2\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

(D)	**Correct**. The chain rule must be used twice for this composition of three functions. $\frac{d}{dx}\left(2(\sin\sqrt{x})^2\right) = 2\cdot 2(\sin\sqrt{x})\cdot\left(\frac{d}{dx}(\sin\sqrt{x})\right)$ $= 2\cdot 2(\sin\sqrt{x})\cdot\left(\cos\sqrt{x}\cdot\frac{d}{dx}(\sqrt{x})\right)$ $= 2\cdot 2(\sin\sqrt{x})\cdot\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$ $= \frac{2\sin\sqrt{x}\cos\sqrt{x}}{\sqrt{x}}$

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4. $\int \frac{x^2+1}{\left(x^3+3x-5\right)^3}\,dx =$

(A) $-\frac{3}{2}\cdot\frac{1}{\left(3x^2+3\right)^2} + C$

(B) $-\frac{1}{6}\cdot\frac{1}{\left(3x^2+3\right)^2} + C$

(C) $-\frac{3}{2}\cdot\frac{1}{\left(x^3+3x-5\right)^2} + C$

(D) $-\frac{1}{6}\cdot\frac{1}{\left(x^3+3x-5\right)^2} + C$

(D)	**Correct**. Starting with the substitution $u = x^3 + 3x - 5$, $u = x^3 + 3x - 5 \Rightarrow \frac{du}{dx} = 3x^2 + 3 = 3\left(x^2+1\right) \Rightarrow dx = \frac{du}{3\left(x^2+1\right)}$. Substituting for $x^3 + 3x - 5$ and for dx gives $\int \frac{x^2+1}{\left(x^3+3x-5\right)^3}\,dx = \int \frac{1}{u^3}\cdot\frac{1}{3}\,du = \frac{1}{3}\cdot\left(-\frac{1}{2u^2}\right) + C = -\frac{1}{6}\cdot\frac{1}{\left(x^3+3x-5\right)^2} + C.$

5. The function f is given by $f(x) = 4x^3 - x^4$. On what intervals is the graph of f concave up?

(A) $(-\infty, 0)$ and $(2, \infty)$

(B) $(-\infty, 3)$

(C) $(0, 2)$ only

(D) $(0, 3)$ only

(C)	**Correct**. The graph of f will be concave up on intervals where $f''(x) > 0$. $f'(x) = 12x^2 - 4x^3$ $f''(x) = 24x - 12x^2 = 12x(2 - x)$ The graph of f'' is a parabola opening downward and with zeros at $x = 0$ and $x = 2$. Therefore, $f''(x) > 0$ on the interval between the two zeros, or $0 < x < 2$.

6. If $x + 3y^{1/3} = y$, what is $\frac{dy}{dx}$ at the point $(2, 8)$?

(A) $\frac{1}{3}$ (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) $\frac{4}{3}$

(D)	**Correct**. The chain rule is the basis for implicit differentiation. $1 + y^{-\frac{2}{3}}\frac{dy}{dx} = \frac{dy}{dx}$ The point $(2, 8)$ is on the curve since $x = 2$ and $y = 8$ satisfy the equation. At this point, $1 + \frac{1}{4}\frac{dy}{dx} = \frac{dy}{dx} \Rightarrow \frac{3}{4}\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{4}{3}$.

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7. $\frac{d}{dx}\left(x^5 - 5^x\right) =$

(A) $\frac{x^6}{6} - \frac{5^x}{\ln 5}$ (B) $5x^4 - 5^x$ (C) $5x^4 - x \cdot 5^{x-1}$ (D) $5x^4 - (\ln 5)5^x$

(D)	**Correct**. The derivative of x^5 is $5x^4$ by the power rule, and the derivative of the exponential function 5^x is $(\ln 5)5^x$. Therefore, $\frac{d}{dx}\left(x^5 - 5^x\right) = 5x^4 - (\ln 5)5^x$.

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8. $\lim_{x \to \infty} \frac{10 - 6x^2}{5 + 3e^x}$ is

(A) -2 (B) 0 (C) 2 (D) nonexistent

(B)	**Correct**. The numerator of $\frac{10 - 6x^2}{5 + 3e^x}$ is a translated power function and the denominator is a translated exponential function. Since the exponential function e^x grows faster than the power function x^2, the relative magnitude of the denominator compared to the numerator will result in this expression converging to 0 as x goes to infinity.

9. Let R be the region bounded by the graphs of $y = 2x$ and $y = 4x - x^2$. What is the area of R ?

(A) $\frac{2}{3}$ (B) $\frac{4}{3}$ (C) $\frac{16}{3}$ (D) $\frac{28}{3}$

(B)	**Correct**. The graphs of $y = 2x$ and $y = 4x - x^2$ intersect when $x = 0$ and $x = 2$. The graph of $y = 4x - x^2$ lies above the graph $y = 2x$ on the interval $0 \le x \le 2$. (One way to see this is to sketch a graph of the parabola and the line, observing that the graph of $y = 4x - x^2$ has a slope of 4 at $x = 0$, while the graph of $y = 2x$ has a slope of 2.) The area of the region bounded by the two graphs is therefore $\int_0^2 \left(4x - x^2 - 2x\right) dx = \int_0^2 \left(2x - x^2\right) dx = \left(x^2 - \frac{x^3}{3}\right)\Big\vert_0^2 = 4 - \frac{8}{3} = \frac{4}{3}$.

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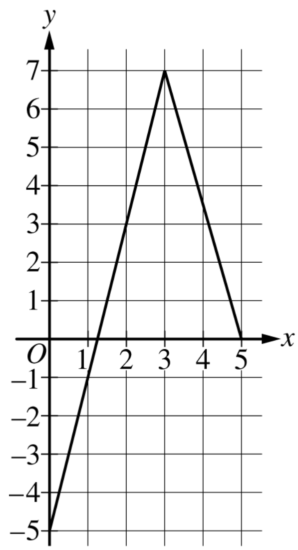

Graph of f

10. The graph of a function f is shown above. If g is the function defined by $g(x) = \dfrac{x^2 + 1}{f(x)}$, what is the value of $g'(2)$?

(A) $-\dfrac{8}{9}$ (B) $\dfrac{1}{9}$ (C) 1 (D) $\dfrac{32}{9}$

(A)	**Correct**. The derivative of g is found using the quotient rule. $g'(x) = \dfrac{2xf(x) - f'(x)\left(x^2 + 1\right)}{(f(x))^2}$ The graph of f is used to determine that $f(2) = 3$ and $f'(2) = \dfrac{7-3}{3-2} = 4$. Then $g'(2) = \dfrac{4f(2) - f'(2)(5)}{(f(2))^2} = \dfrac{(4)(3) - (4)(5)}{9} = -\dfrac{8}{9}$.

11. $\int_{-1}^{1} \frac{x^2 - x}{x}\, dx$ is

(A) -2 (B) 0 (C) $\frac{4}{3}$ (D) nonexistent

(A)	**Correct**. The function $f(x) = \frac{x^2 - x}{x}$ has a removable discontinuity at $x = 0$, since $f(0)$ is undefined but $\lim_{x\to 0} \frac{x^2 - x}{x} = \lim_{x\to 0} (x - 1) = -1$. The definition of the definite integral can be extended to functions with removable discontinuities. If $g(x) = x - 1$, then $g(x) = f(x)$ for all x except $x = 0$, and therefore $\int_{-1}^{1} f(x)\, dx = \int_{-1}^{1} g(x)\, dx = \int_{-1}^{1} (x-1)\, dx = \left(\frac{1}{2}x^2 - x\right)\Big\vert_{-1}^{1} = \left(\frac{1}{2} - 1\right) - \left(\frac{1}{2} + 1\right) = -2.$

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x	0	4	8	12	16
$f(x)$	8	0	2	10	1

12. The table above gives selected values for the differentiable function *f*. In which of the following intervals must there be a number c such that $f'(c) = 2$?

(A) $(0, 4)$ (B) $(4, 8)$ (C) $(8, 12)$ (D) $(12, 16)$

(C)	**Correct**. The function f is continuous on the closed interval $[8, 12]$ and differentiable on the open interval $(8, 12)$. By the Mean Value Theorem, there is a number c in the interval $(8, 12)$ such that $f'(c) = \frac{f(12) - f(8)}{12 - 8} = \frac{10 - 2}{4} = 2.$

13. $\lim_{x \to 0} \frac{\sin x}{e^x - 1}$ is

(A) 1 (B) $\frac{1}{e}$ (C) 0 (D) nonexistent

(A)	**Correct**. Since $\lim_{x \to 0} \sin x = 0$ and $\lim_{x \to 0} \left(e^x - 1\right) = 0$, the indeterminate limit can be evaluated using L'Hospital's Rule, as follows. $\lim_{x \to 0} \frac{\sin x}{e^x - 1} = \lim_{x \to 0} \frac{\cos x}{e^x} = \frac{\cos 0}{e^0} = \frac{1}{1} = 1$

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14. A particle moves along a straight line so that at time $t \geq 0$ its acceleration is given by $a(t) = 12t$. At time $t = 0$, the velocity of the particle is 2 and the position of the particle is 5. Which of the following is an expression for the position of the particle at time $t \geq 0$?

(A) $6t^2 + 5$

(B) $6t^3 + 2t + 5$

(C) $2t^3 + 5$

(D) $2t^3 + 2t + 5$

(D)	**Correct**. Since the acceleration is given, the position can be found using antidifferentiation and the values of the velocity and position at time $t = 0$. $a(t) = 12t \Rightarrow v(t) = 6t^2 + C_1$; $v(0) = 2 \Rightarrow 2 = 0 + C_1 \Rightarrow C_1 = 2$ $v(t) = 6t^2 + 2 \Rightarrow s(t) = 2t^3 + 2t + C_2$; $s(0) = 5 \Rightarrow 5 = 0 + 0 + C_2 \Rightarrow C_2 = 5$ The position of the particle is $s(t) = 2t^3 + 2t + 5$ for $t \geq 0$.

$$f(x) = \begin{cases} -x^2 + 3 & \text{if } x \le 5 \\ -10x + 28 & \text{if } x > 5 \end{cases}$$

15. Let f be the function defined above. Which of the following statements about f is true?

(A) f is continuous and differentiable at $x = 5$.

(B) f is continuous but not differentiable at $x = 5$.

(C) f is differentiable but not continuous at $x = 5$.

(D) f is defined but neither continuous nor differentiable at $x = 5$.

<table>
<tr>
<td>(A)</td>
<td>Correct. This statement is true.
$\lim\limits_{x\to 5^-} f(x) = \lim\limits_{x\to 5^-} \left(-x^2 + 3\right) = -25 + 3 = -22$
$\lim\limits_{x\to 5^+} f(x) = \lim\limits_{x\to 5^+} (-10x + 28) = -50 + 28 = -22$
Therefore, $\lim\limits_{x\to 5} f(x)$ exists and $\lim\limits_{x\to 5} f(x) = -22 = f(5)$, so f is continuous at $x = 5$.
$\lim\limits_{h\to 0^-} \dfrac{f(5+h) - f(5)}{h} = \lim\limits_{h\to 0^-} \dfrac{-(5+h)^2 + 3 - (-22)}{h} = \lim\limits_{h\to 0^-} \dfrac{-10h - h^2}{h} = -10$
$\lim\limits_{h\to 0^+} \dfrac{f(5+h) - f(5)}{h} = \lim\limits_{h\to 0^+} \dfrac{-10(5+h) + 28 - (-22)}{h} = \lim\limits_{h\to 0^+} \dfrac{-10h}{h} = -10$
Therefore, f is also differentiable at $x = 5$ and $f'(5) = -10$. An alternative way to see that the piecewise-defined function f is differentiable at $x = 5$ is to observe that $f'(x) = \begin{cases} -2x & \text{for } x < 5 \\ -10 & \text{for } x > 5. \end{cases}$ Since f is continuous at $x = 5$ and the derivatives $-2x$ and -10 are equal at $x = 5$, f is differentiable at $x = 5$.</td>
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16. If $\frac{dy}{dx} = 2 - y$, and if $y = 1$ when $x = 1$, then $y =$

(A) $2 - e^{x-1}$ (B) $2 - e^{1-x}$ (C) $2 - e^{-x}$ (D) $2 + e^{-x}$

(B)	**Correct**. The differential equation can be solved using separation of variables and the initial condition to determine the appropriate value for the arbitrary constant. $\frac{dy}{dx} = 2 - y \Rightarrow \frac{dy}{2-y} = dx$ $\int \frac{1}{2-y}\, dy = \int dx \Rightarrow -\ln\lvert 2-y\rvert = x + C$ $-\ln\lvert 1\rvert = 1 + C \Rightarrow C = -1$ $-\ln\lvert 2-y\rvert = x - 1 \Rightarrow \ln\lvert 2-y\rvert = -x + 1 \Rightarrow \lvert 2-y\rvert = e^{1-x}$ Since $2 - y > 0$ at the initial value $y = 1$, the solution to the differential equation is $2 - y = e^{1-x}$, or $y = 2 - e^{1-x}$.

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17. Let g be the function given by $g(x) = \int_3^x \left(t^2 - 5t - 14\right) dt$. What is the x-coordinate of the point of inflection of the graph of g ?

(A) -2 (B) $\frac{5}{2}$ (C) 3 (D) 7

(B)	**Correct**. To find the point of inflection of the graph of g, determine where g'' changes sign. $g'(x) = x^2 - 5x - 14$ $g''(x) = 2x - 5$ Then $g''(x) = 0$ at $x = \frac{5}{2}$. Since $g''(x) < 0$ for $x < \frac{5}{2}$ and $g''(x) > 0$ for $x > \frac{5}{2}$, the graph of g changes concavity at $x = \frac{5}{2}$ and therefore, the graph of g has a point of inflection at $x = \frac{5}{2}$.

18. $\frac{d}{dx}\left(x^3\sec(2x)\right) =$

(A) $6x^2\sec(2x)\tan(2x)$

(B) $2x^3\tan^2(2x) + 3x^2\sec(2x)$

(C) $x^3\sec(2x)\tan(2x) + 3x^2\sec(2x)$

(D) $2x^3\sec(2x)\tan(2x) + 3x^2\sec(2x)$

(D)	**Correct**. A combination of the product rule and the chain rule is used to compute the derivative. $\frac{d}{dx}\left(x^3\sec(2x)\right) = x^3 \cdot \frac{d}{dx}(\sec(2x)) + 3x^2 \cdot \sec(2x)$ $= x^3 \cdot (\sec(2x)\tan(2x) \cdot 2) + 3x^2\sec(2x)$ $= 2x^3\sec(2x)\tan(2x) + 3x^2\sec(2x)$

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19. A particle moves along the y-axis so that at time $t \geq 0$ its position is given by $y(t) = t^3 - 4t^2 + 4t + 3$. Which of the following statements describes the motion of the particle at time $t = 1$?

(A) The particle is moving down the y-axis with decreasing velocity.

(B) The particle is moving down the y-axis with increasing velocity.

(C) The particle is moving up the y-axis with decreasing velocity.

(D) The particle is moving up the y-axis with increasing velocity.

(A)	**Correct**. The velocity of the particle is $v(t) = 3t^2 - 8t + 4$. At time $t = 1$, $v(1) = -1$. Since the velocity is negative, the particle is moving down the y-axis. The rate of change of the velocity is $v'(t) = 6t - 8$. At time $t = 1$, $v'(1) = -2$. Since this is negative, the particle is moving with decreasing velocity at time $t = 1$.

20. If $\int_1^4 f(x)\,dx = 8$ and $\int_1^4 g(x)\,dx = -2$, which of the following cannot be determined from the information given?

(A) $\int_4^1 g(x)\,dx$

(B) $\int_1^4 3f(x)\,dx$

(C) $\int_1^4 3f(x)g(x)\,dx$

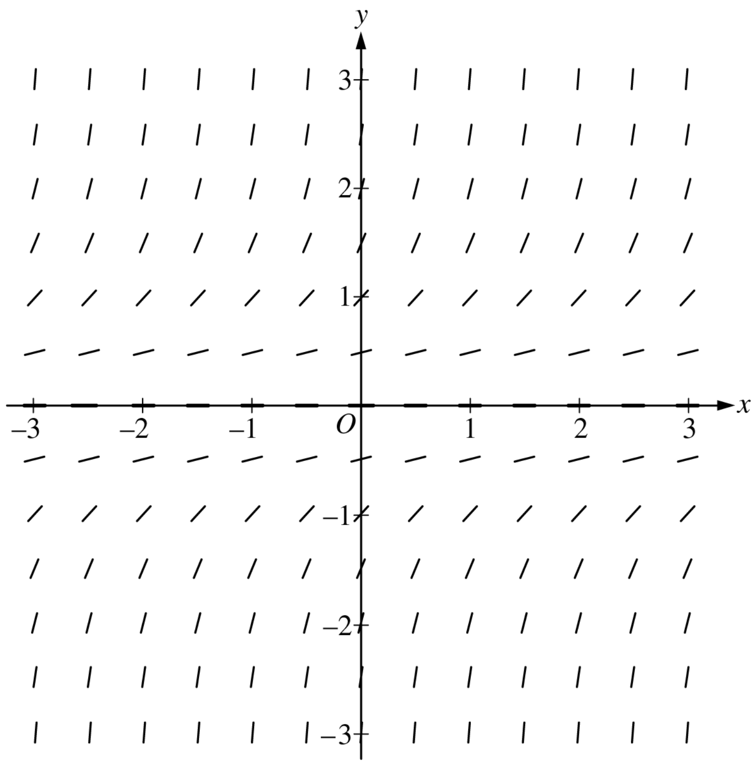
(D) $\int_1^4 (3f(x) + g(x))\,dx$

(C)	**Correct**. It is not true in general that $\int_1^4 3f(x)g(x)\,dx = \int_1^4 3f(x)\,dx \cdot \int_1^4 g(x)\,dx$, so the individual values of $\int_1^4 f(x)\,dx$ and $\int_1^4 g(x)\,dx$ cannot be used to determine the value of $\int_1^4 3f(x)g(x)\,dx$. For example, if $f(x) = \frac{8}{3}$ and $g(x) = -\frac{2}{3}$, then $\int_1^4 f(x)\,dx = 8$, $\int_1^4 g(x)\,dx = -2$, and $\int_1^4 3f(x)g(x)\,dx = \int_1^4 -\frac{16}{3}\,dx = -16$. However, if $f(x) = \frac{16}{9}(x-1)$ and $g(x) = -\frac{4}{9}(x-1)$, then $\int_1^4 f(x)\,dx = 8$ and $\int_1^4 g(x)\,dx = -2$ as before, but now $\int_1^4 3f(x)g(x)\,dx = \int_1^4 -\frac{64}{27}(x-1)^2\,dx = -\frac{64}{3}$.

21. For any real number x, $\lim_{h \to 0} \frac{\sin(2(x+h)) - \sin(2x)}{h} =$

(A) 0 (B) 1 (C) $\cos(2x)$ (D) $2\cos(2x)$

(D)	**Correct**. The limit of this difference quotient is of the form $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, where $f(x) = \sin(2x)$. This is one way to express the derivative of f. By the chain rule, $f'(x) = \cos(2x) \cdot 2$.



22. Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = |x + y|$ (B) $\frac{dy}{dx} = x^3$ (C) $\frac{dy}{dx} = y^3$ (D) $\frac{dy}{dx} = y^2$

(D)	**Correct**. The segments in the slope field suggest that (1) the slopes depend only on the variable y, (2) the slopes are nonnegative, and (3) the slopes are zero when $y = 0$. The differential equation $\frac{dy}{dx} = y^2$ satisfies all three conditions. The differential equation $\frac{dy}{dx} = \lvert x + y \rvert$ does not satisfy conditions (1) and (3). The differential equation $\frac{dy}{dx} = x^3$ does not satisfy any of the conditions. The differential equation $\frac{dy}{dx} = y^3$ does not satisfy condition (2) when $y < 0$.

23. The base of a solid is the region in the first quadrant bounded by the y-axis, the x-axis, the graph of $y = e^x$, and the vertical line $x = 1$. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

(A) $e - 1$

(B) $\frac{1}{2}e^2 - \frac{1}{2}$

(C) $e^2 - 1$

(D) $2e^2 - 2$

(B)	**Correct**. The area of a square of side length s is s^2. A typical cross section of the solid is a square with side from the x-axis to the graph of $y = e^x$. The length of the side of the square is therefore $s = e^x$, so the area of the square is $\left(e^x\right)^2 = e^{2x}$. The volume of the solid is found using the definite integral of the cross-sectional area. $\int_0^1 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big\|_0^1 = \frac{1}{2}e^2 - \frac{1}{2}$

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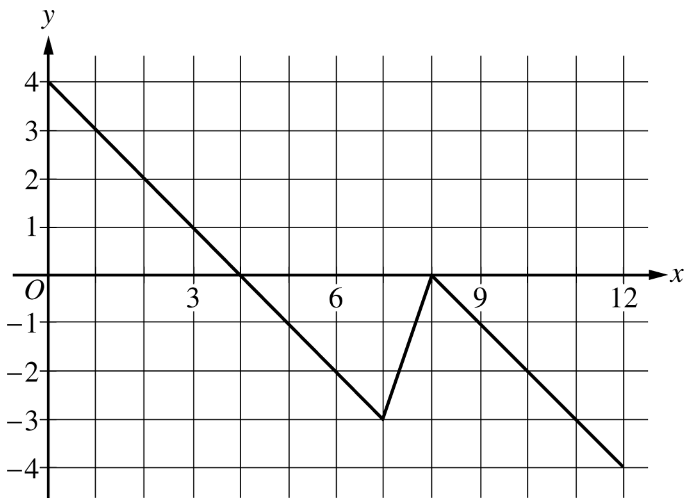

Graph of f

24. The graph of the piecewise linear function f is shown above. What is the value of $\int_0^{12} f'(x)\,dx$?

(A) -8 (B) -6 (C) 0 (D) 22

(A)	**Correct**. By the Fundamental Theorem of Calculus, $\int_0^{12} f'(x)\,dx = f(12) - f(0) = (-4) - 4 = -8,$ where values of f at $x = 0$ and $x = 12$ are obtained from the graph.

25. The equation $y = 2e^{6x} - 5$ is a particular solution to which of the following differential equations?

(A) $y' - 6y - 30 = 0$

(B) $2y' - 12y + 5 = 0$

(C) $y'' - 5y' - 6y = 0$

(D) $y'' - 2y' + y + 5 = 0$

(A)	**Correct**. One way to verify that a function is a solution to a differential equation is to check that the function and its derivatives satisfy the differential equation. The differential equation in this option involves y and y'. The correct derivative must be computed and the algebra correctly done to verify that the differential equation is satisfied. $y' = 12e^{6x}$ $y' - 6y - 30 = 12e^{6x} - 6\left(2e^{6x} - 5\right) - 30 = 12e^{6x} - 12e^{6x} + 30 - 30 = 0$

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26. What is the value of x at which the minimum value of $y = 3x^{4/3} - 2x$ occurs on the closed interval $[0, 1]$?

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{2}$ (D) 1

(B)	**Correct**. The absolute minimum will occur at a critical point or one of the endpoints. $y' = 4x^{\frac{1}{3}} - 2 = 0 \Rightarrow x^{\frac{1}{3}} = \frac{1}{2} \Rightarrow x = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$ The candidates are $x = 0,\ x = \frac{1}{8},$ and $x = 1.$ When $x = 0,\ y = 0.$ When $x = \frac{1}{8},\ y = 3\left(\frac{1}{8}\right)^{\frac{4}{3}} - 2\left(\frac{1}{8}\right) = \frac{3}{16} - \frac{1}{4} = -\frac{1}{16}.$ When $x = 1,\ y = 1.$ The absolute minimum is therefore at $x = \frac{1}{8}.$ Alternatively, since $x = \frac{1}{8}$ is the only critical point and the Second Derivative Test shows that it is the location of a local minimum, it must also be the location of the absolute minimum on the interval $[0, 1]$.

27. At time $t = 0$, a storage tank is empty and begins filling with water. For $t > 0$ hours, the depth of the water in the tank is increasing at a rate of $W(t)$ feet per hour. Which of the following is the best interpretation of the statement $W'(2) > 3$?

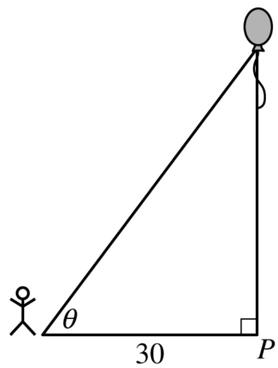
(A) Two hours after the tank begins filling with water, the depth of the water is increasing at a rate greater than 3 feet per hour.

(B) Over the first two hours after the tank begins filling with water, the depth of the water is always increasing at a rate greater than 3 feet per hour.

(C) Two hours after the tank begins filling with water, the rate at which the depth of the water is rising is increasing at a rate greater than 3 feet per hour per hour.

(D) Over the first two hours after the tank begins filling with water, the rate at which the depth of the water is rising is always increasing at a rate greater than 3 feet per hour per hour.

(C)	**Correct**. In the expression $W'(2)$, the 2 represents the value of the independent variable and is therefore the number of hours since the tank began filling with water. $W'(2)$, being the value of a derivative, is the rate of change of W, that is, the rate of change of the rate at which the depth of the water is rising; in this case, 2 hours after the tank begins filling with water. The units for the derivative would be the units of W per unit of time; thus, feet per hour per hour. The statement says that at time 2 hours after the tank begins filling with water, the rate at which the depth of the water is rising, $W(t)$, is increasing at a rate that is greater than 3 feet per hour per hour.



28. A person stands 30 feet from point P and watches a balloon rise vertically from the point, as shown in the figure above. The balloon is rising at a constant rate of 2 feet per second. What is the rate of change, in radians per second, of angle θ at the instant when the balloon is 40 feet above point P ?

(A) $\frac{3}{100}$ (B) $\frac{3}{125}$ (C) $\frac{1}{12}$ (D) $\frac{5}{27}$

(B)	**Correct**. If $h(t)$ is the height of the balloon above the point P at time t, then $\tan\theta = \frac{h}{30}$. Using implicit differentiation with respect to t shows that $\sec^2\theta\frac{d\theta}{dt} = \frac{1}{30}\frac{dh}{dt}$. At the instant when $h = 40$, the distance from the person to the balloon is $\sqrt{30^2 + 40^2} = \sqrt{2500} = 50$. At this instant, $\sec\theta = \frac{50}{30} = \frac{5}{3}$ and therefore $\left(\frac{5}{3}\right)^2\frac{d\theta}{dt} = \frac{1}{30}\cdot 2 = \frac{1}{15} \Rightarrow \frac{d\theta}{dt} = \frac{1}{15}\cdot\frac{9}{25} = \frac{3}{125}$.

29. How many vertical asymptotes does the graph of $y = \frac{x-2}{x^4 - 16}$ have?

(A) One (B) Two (C) Three (D) Four

(A)	**Correct**. Since $\frac{x-2}{x^4-16} = \frac{x-2}{(x^2-4)(x^2+4)} = \frac{x-2}{(x-2)(x+2)(x^2+4)} = \frac{1}{(x+2)(x^2+4)}$ for $x \neq 2$, the graph has only one vertical asymptote, which is at $x = -2$. The graph has a removable discontinuity at $x = 2$.

GO ON TO THE NEXT PAGE.

30. For what value of b does the integral $\int_1^b x^2\,dx$ equal $\lim_{n\to\infty} \sum_{k=1}^{n} \left(1 + \frac{2k}{n}\right)^2 \frac{2}{n}$?

(A) $b = 2$ only

(B) $b = 3$ only

(C) b could be any real number.

(D) There is no such value of b.

(B)	**Correct**. The sum can be interpreted as a right Riemann sum in the form $\sum_{k=1}^{n} f(1 + k\Delta x)\Delta x,$ where $f(x) = x^2$ and $\Delta x = \frac{2}{n}.$ The value of Δx corresponds to an interval of length $2.$ The sum starts with the right endpoint $1 + \Delta x$ and ends with the right endpoint $1 + n\Delta x = 1 + 2 = 3,$ so the Riemann sum is over the interval $[1, 3].$ The limit of the Riemann sum is the definite integral $\int_1^3 f(x)\,dx.$ There could not be another value of b for which $\int_1^b x^2\,dx$ has the same value as $\int_1^3 x^2\,dx$ since $I(b) = \int_1^b x^2\,dx$ is a strictly increasing function of $b.$ Therefore, $b = 3$ is the only choice.

CALCULUS AB

SECTION I, Part B

Time—45 minutes

Number of questions—15

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in this exam booklet. Do not spend too much time on any one problem.

BE SURE YOU FILL IN THE CIRCLES ON THE ANSWER SHEET THAT CORRESPOND TO QUESTIONS NUMBERED 1–15.

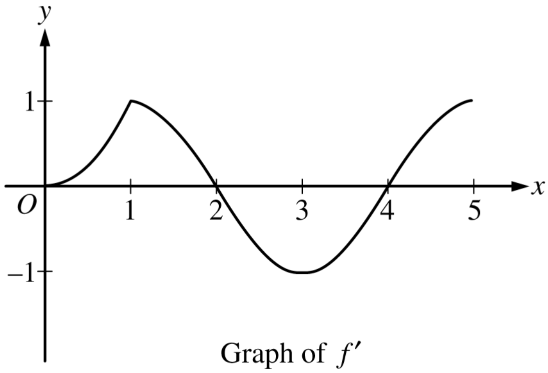
YOU MAY NOT RETURN TO QUESTIONS NUMBERED 1–30.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f'

1. The function f is continuous on the closed interval [0,5]. The graph of f', the derivative of f, is shown above. On which of the following intervals is f increasing?

(A) [0,1] and [2,4]

(B) [0,1] and [3,5]

(C) [0,1] and [4,5] only

(D) [0,2] and [4,5]

(D)	**Correct**. The function f is increasing on closed intervals where f' is positive on the corresponding open intervals. The graph indicates that $f'(x) > 0$ on the intervals $(0, 2)$ and $(4, 5)$, so f is increasing on the intervals $[0, 2]$ and $[4, 5]$.

2. The height of an object at time $t \geq 1$ is given by $h(t) = t^2 - \frac{16}{t} + 15$. What is the velocity of the object at time $t = 3$?

(A) 0.815 (B) 7.778 (C) 18.667 (D) 21.089

(B)	**Correct**. The velocity is the derivative of the height. Using the calculator, $v(3) = h'(3) = 7.778$.

---

3. The function g is differentiable and satisfies $g(-1) = 4$ and $g'(-1) = 2$. What is the approximation of $g(-1.2)$ using the line tangent to the graph of g at $x = -1$?

(A) 3.6 (B) 3.8 (C) 4.2 (D) 4.4

(A)	**Correct.** An equation of the line tangent to the graph of g at $x = a$ is $y = g(a) + g'(a)(x - a)$. In this question, $a = -1$. The value of y when $x = -1.2$ would be an approximation to $g(-1.2)$. $g(-1.2) \approx g(-1) + g'(-1)(-1.2 - (-1)) = 4 + 2(-0.2) = 3.6$

4. Tara's heart rate during a workout is modeled by the differentiable function h, where $h(t)$ is measured in beats per minute and t is measured in minutes from the start of the workout. Which of the following expressions gives Tara's average heart rate from $t = 30$ to $t = 60$?

(A) $\int_{30}^{60} h(t)\,dt$

(B) $\frac{1}{30}\int_{30}^{60} h(t)\,dt$

(C) $\frac{1}{30}\int_{30}^{60} h'(t)\,dt$

(D) $\frac{h'(30) + h'(60)}{2}$

(B)	**Correct.** The average value of a function f over an interval $[a, b]$ is $\frac{1}{b-a}\int_{a}^{b} f(x)\,dx$. Tara's average heart rate from $t = 30$ to $t = 60$ is the average value of the function h over the interval $[30, 60]$ and would therefore be given by the expression $\frac{1}{60-30}\int_{30}^{60} h(t)\,dt$.

---

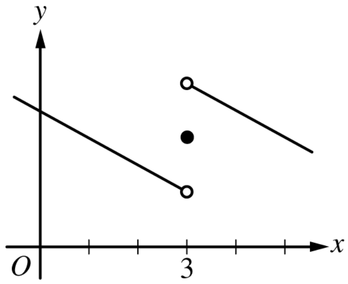
5. Let g be the function with first derivative $g'(x) = \sqrt{x^3 + x}$ for $x > 0$. If $g(2) = -7$, what is the value of $g(5)$?

(A) 4.402 (B) 11.402 (C) 13.899 (D) 20.899

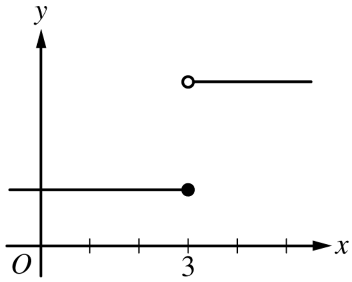
(C)	**Correct.** By the Fundamental Theorem of Calculus, $g(5) - g(2) = \int_{2}^{5} g'(x)\,dx$. Therefore, $g(5) = g(2) + \int_{2}^{5} \sqrt{x^3 + x}\,dx = -7 + \int_{2}^{5} \sqrt{x^3 + x}\,dx = 13.899$, where the evaluation of the definite integral is done with the calculator.

6. If f is a function that has a removable discontinuity at $x = 3$, which of the following could be the graph of f ?

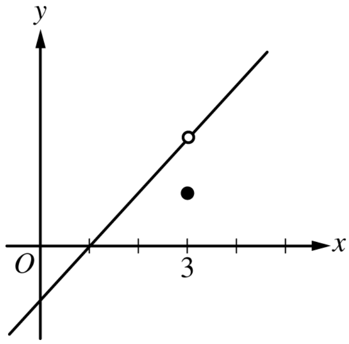
(A)



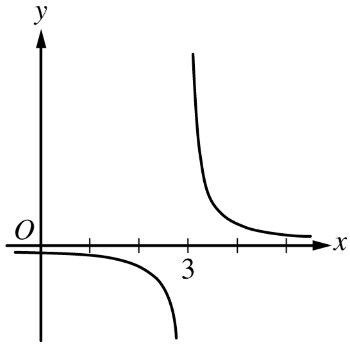
(B)



(C)



(D)



(C)	**Correct**. A removable discontinuity occurs at $x = c$ if $\lim_{x\to c} f(x)$ exists, but $f(c)$ does not exist or is not equal to the value of the limit. This graph could be the graph of f since $\lim_{x\to 3} f(x)$ exists but is not equal to $f(3)$.

7. Let f be a continuous function such that $\int_0^{17} f(x)\,dx = 8$, $\int_{17}^{20} f(x)\,dx = -3$, and $\int_{13}^{20} f(x)\,dx = 7$. What is the value of $\int_0^{13} f(x)\,dx$?

(A) -2 (B) 4 (C) 12 (D) 18

(A)	**Correct**. Using the property of definite integrals over adjacent intervals, $\int_0^{20} f(x)\,dx = \int_0^{17} f(x)\,dx + \int_{17}^{20} f(x)\,dx = 8 + (-3) = 5.$ Another application of the same property gives $\int_0^{20} f(x)\,dx = \int_0^{13} f(x)\,dx + \int_{13}^{20} f(x)\,dx \Rightarrow \int_0^{13} f(x)\,dx = \int_0^{20} f(x)\,dx - \int_{13}^{20} f(x)\,dx.$ Therefore, $\int_0^{13} f(x)\,dx = \int_0^{20} f(x)\,dx - \int_{13}^{20} f(x)\,dx = 5 - 7 = -2.$

$$f(x) = \begin{cases} k^3 + x & \text{for } x < 3 \\ \dfrac{16}{k^2 - x} & \text{for } x \geq 3 \end{cases}$$

8. Let f be the function defined above, where k is a positive constant. For what value of k, if any, is f continuous?

(A) 2.081 (B) 2.646 (C) 8.550 (D) There is no such value of k.

(A)	**Correct**. The limit at $x = 3$ exists if the left-hand and right-hand limits are equal. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) \Rightarrow k^3 + 3 = \dfrac{16}{k^2 - 3}$ The solution to this equation for $k > 0$ is $k = 2.081$. With this value of k, $\lim_{x\to 3} f(x)$ exists and is equal to $f(3)$. Therefore, f is continuous at $x = 3$.

x	$f(x)$
–5	–9
0	1
2	5

9. The table above gives values of a continuous function f at selected values of x. Based on the information in the table, which of the following statements must be true?

(A) f has at most one zero.

(B) f has a relative maximum at $x = 2$.

(C) There exists a value c, where $-5 < c < 2$, such that $f(c) = 4$.

(D) There exists a value c, where $-5 < c < 2$, such that $f'(c) = 2$.

(C)	**Correct**. Since f is continuous on the closed interval $[-5, 2]$ and $f(-5) < 4 < f(2)$, then by the Intermediate Value Theorem there must be a value c in the open interval $(-5, 2)$ such that $f(c) = 4$.

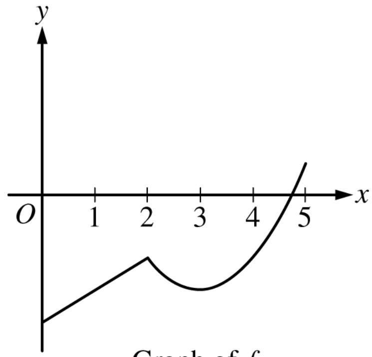
---

10. The function f is an antiderivative of the function g defined by $g(x) = 3 - \sqrt{x^2 + x + 4\cos x}$. Which of the following is the x-coordinate of the location of a local maximum for the graph of $y = f(x)$?

(A) -3.961 (B) -2.161 (C) 1.494 (D) 3.140

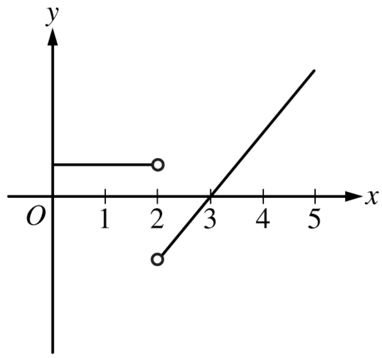
(D)	**Correct**. A local maximum for the graph of $y = f(x)$ occurs at a value of x where $f' = g$ changes from positive to negative. The graph of $y = g(x)$ crosses the x-axis from positive to negative at $x = 3.140$.

11. The graph of $y = f(x)$ on the interval $0 < x < 5$ is shown above. Which of the following could be the graph of $y = f(x)$?

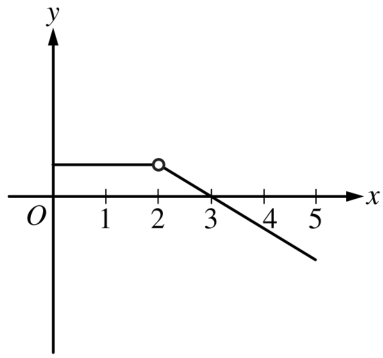


Graph of f

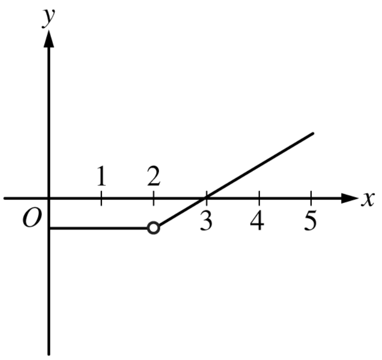
(A)



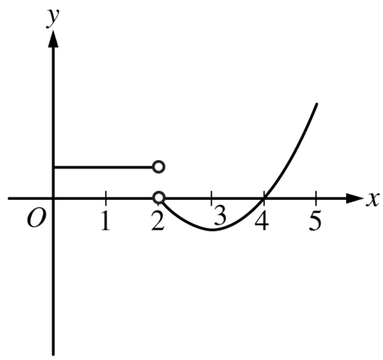
(B)



(C)



(D)



(A)	**Correct.** The graph of f indicates that f is increasing from $x = 0$ to $x = 2$, then decreasing from $x = 2$ to $x = 3$, and then increasing from $x = 3$ to $x = 5$. Therefore, the graph of f' should be positive from $x = 0$ to $x = 2$, negative from $x = 2$ to $x = 3$, and positive from $x = 3$ to $x = 5$. This graph is the only one that has this behavior, so it could be the graph of f'. Some other features of the graph of f support this conclusion. Since f is not differentiable at $x = 2$, the graph of f' should not be defined at $x = 2$. Since f has a local minimum at $x = 3$ and is differentiable there, $f'(3)$ should equal 0. This graph is consistent with those observations.

12. At time t, $0 < t < 2$, the velocity of a particle moving along the x-axis is given by $v(t) = t\sin\left(t^3\right)$. Let $t = b$ be the time at which the particle changes direction from moving left to moving right. What is the total distance traveled by the particle during the time interval $0 < t < b$?

(A) 0.212 (B) 0.612 (C) 1.011 (D) 1.208

(C)	**Correct**. The graph of the velocity over the interval $(0, 2)$ shows that the velocity changes from positive to negative, then back to positive. The time at which the particle changes direction from moving left to moving right, therefore, is the second zero of $v(t)$, where the velocity changes from negative to positive. This zero is at $t = b = 1.84527.$ Store this value in the calculator, and use the stored value for the limit of integration in order to ensure accuracy. The total distance traveled by the particle during the time interval $0 < t < b$ is $\int_0^b \lvert v(t)\rvert\, dt = \int_0^{1.84527} \left\lvert t\sin\left(t^3\right)\right\rvert\, dt = 1.011.$

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13. Let f be the function defined by $f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{1}{2}x^2 - \frac{1}{2}x$. For how many values of x in the open interval $(0, 1.565)$ is the instantaneous rate of change of f equal to the average rate of change of f on the closed interval $[0, 1.565]$?

(A) Zero (B) One (C) Three (D) Four

(C)	**Correct**. The average rate of change of f on the closed interval $[0, 1.565]$ is $\frac{f(1.565) - f(0)}{1.565 - 0} = -0.39206.$ The instantaneous rate of change of f is the derivative, $f'(x) = x^3 - 2x^2 + x - \frac{1}{2}.$ The graph of f', produced using the calculator, intersects the horizontal line $y = -0.39206$ three times in the open interval $(0, 1.565)$.

14. Let g be a twice-differentiable function with $g'(x) > 0$ and $g''(x) > 0$ for all real numbers x, such that $g(3) = 12$ and $g(5) = 18$. Which of 20, 21, and 22 are possible values for $g(6)$?

(A) 21 only (B) 22 only (C) 20 and 21 only (D) 21 and 22 only

(B)	**Correct**. The graph of g is increasing because $g'(x) > 0$ and concave up because $g''(x) > 0$. The secant line through the points $(3, 12)$ and $(5, 18)$ is $y = 3(x - 3) + 12$. Because the graph of g is increasing and concave up, the graph will lie above the secant line for $x > 5$. In particular, the value of $g(6)$ is strictly greater than the value of y on the secant line at $x = 6$, that is, $g(6) > 3(6 - 3) + 12 = 21$. Therefore, 22 is the only possible value for $g(6)$.

x	2	3	4
$f(x)$	1	2	6
$f'(x)$	4	5	3

15. The table above gives values of the differentiable function f and its derivative at selected values of x. If g is the inverse function of f, which of the following is an equation of the line tangent to the graph of g at the point where $x = 2$?

(A) $y = -\frac{1}{5}(x-2)+3$

(B) $y = -\frac{1}{4}(x-2)+1$

(C) $y = \frac{1}{5}(x-2)+3$

(D) $y = 4(x-2)+1$

(C)	**Correct**. Since $f(g(x)) = x,$ the chain rule can be used to determine that $f'(g(x))g'(x) = 1.$ Substituting $x = 2$ gives $1 = f'(g(2))g'(2) = f'(3)g'(2) \Rightarrow g'(2) = \frac{1}{f'(3)} = \frac{1}{5}.$ Since $g(2) = f^{-1}(2) = 3,$ an equation of the line tangent to the graph of g at $x = 2$ is therefore $y = \frac{1}{5}(x-2)+3.$

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of questions—2

A GRAPHING CALCULATOR IS REQUIRED FOR THESE QUESTIONS.

t (hours)	2	5	9	11	12
$L(t)$ (cars per hour)	15	40	24	68	18

1. The rate at which cars enter a parking lot is modeled by $E(t) = 30 + 5(t-2)(t-5)e^{-0.2t}$. The rate at which cars leave the parking lot is modeled by the differentiable function L. Selected values of $L(t)$ are given in the table above. Both $E(t)$ and $L(t)$ are measured in cars per hour, and time t is measured in hours after 5 A.M. ($t = 0$). Both functions are defined for $0 \le t \le 12$.

(a) What is the rate of change of $E(t)$ at time $t = 7$? Indicate units of measure.

(a) $E'(7) = 6.164924$

The rate of change of $E(t)$ at time $t = 7$ is 6.165 (or 6.164) cars per hour per hour.

1 : answer with units

(b) How many cars enter the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole number.

(b) $\int_0^{12} E(t)\, dt = 520.070489$

To the nearest whole number, 520 cars enter the parking lot from time $t = 0$ to time $t = 12$.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Continue question 1 on page 5.

(c) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate $\int_2^{12} L(t)\,dt$. Using correct units, explain the meaning of $\int_2^{12} L(t)\,dt$ in the context of this problem.

(c) $$\int_2^{12} L(t)\,dt \approx (5-2)\cdot\frac{L(2)+L(5)}{2} + (9-5)\cdot\frac{L(5)+L(9)}{2} + (11-9)\cdot\frac{L(9)+L(11)}{2} + (12-11)\cdot\frac{L(11)+L(12)}{2}$$
$$= 3\cdot\frac{15+40}{2} + 4\cdot\frac{40+24}{2} + 2\cdot\frac{24+68}{2} + 1\cdot\frac{68+18}{2}$$
$$= 345.5$$

$\int_2^{12} L(t)\,dt$ is the number of cars that leave the parking lot in the 10 hours between 7 A.M. ($t = 2$) and 5 P.M. ($t = 12$).

3 : { 1 : trapezoidal sum; 1 : approximation; 1 : explanation }

(d) For $0 \le t < 6$, 5 dollars are collected from each car entering the parking lot. For $6 \le t \le 12$, 8 dollars are collected from each car entering the parking lot. How many dollars are collected from the cars entering the parking lot from time $t = 0$ to time $t = 12$? Give your answer to the nearest whole dollar.

(d) $$5\int_0^6 E(t)\,dt + 8\int_6^{12} E(t)\,dt = 3530.1396$$

To the nearest dollar, 3530 dollars are collected from time $t = 0$ to time $t = 12$.

3 : { 1 : integrand; 1 : limits and constants; 1 : answer }

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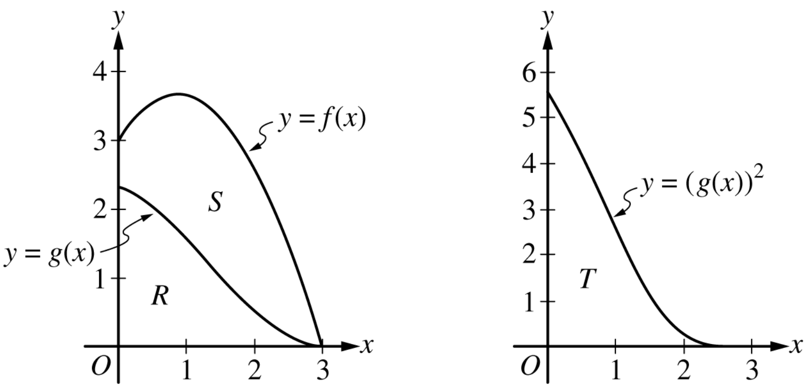


2. The function f is defined by $f(x) = 3(1+x)^{0.5}\cos\left(\frac{\pi x}{6}\right)$ for $0 \le x \le 3$. The function g is continuous and decreasing for $0 \le x \le 3$ with $g(3) = 0$.

The figure above on the left shows the graphs of f and g and the regions R and S. R is the region bounded by the graph of g and the x- and y-axes. Region R has area 3.24125. S is the region bounded by the y-axis and the graphs of f and g.

The figure above on the right shows the graph of $y = (g(x))^2$ and the region T. T is the region bounded by the graph of $y = (g(x))^2$ and the x- and y-axes. Region T has area 5.32021.

(a) Find the area of region S.

(a) $\int_0^3 (f(x) - g(x))\,dx = \int_0^3 f(x)\,dx - \int_0^3 g(x)\,dx$

$= \int_0^3 f(x)\,dx - 3.24125 = 4.919585$

The area of region S is 4.920 (or 4.919).

3 : { 1 : definite integral of f
1 : uses area of R
1 : answer }

(b) Find the volume of the solid generated when region S is revolved about the horizontal line $y = -3$.

(b) $\pi\int_0^3\left((f(x)+3)^2 - (g(x)+3)^2\right)dx$

$$= \pi\left(\int_0^3 (f(x)+3)^2\,dx - \int_0^3\left((g(x))^2 + 6g(x) + 9\right)dx\right)$$

$$= \pi\left(\int_0^3 (f(x)+3)^2\,dx - \int_0^3 (g(x))^2\,dx - 6\int_0^3 g(x)\,dx - \int_0^3 9\,dx\right)$$

$$= \pi\left(\int_0^3 (f(x)+3)^2\,dx - 5.32021 - 6\cdot 3.24125 - 9\cdot 3\right)$$

$$= 156.263709$$

The volume of the solid is 156.264 (or 156.263).

4 :
- 1 : form of integrand
- 1 : integrand
- 1 : uses areas of R and T
- 1 : limits, constant, and answer

(c) Region S is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 7 times the length of its base in region S. Write, but do not evaluate, an integral expression for the volume of this solid.

(c) Volume $= \int_0^3 7(f(x) - g(x))^2\,dx$

2 :
- 1 : integrand
- 1 : expression

CALCULUS AB

SECTION II, Part B

Time—1 hour

Number of questions—4

NO CALCULATOR IS ALLOWED FOR THESE QUESTIONS.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

$$f(x) = \begin{cases} \sqrt{9 - x^2} & \text{for } -3 \le x \le 0 \\ -x + 3\cos\left(\dfrac{\pi x}{2}\right) & \text{for } 0 < x \le 4 \end{cases}$$

3. Let f be the function defined above.

(a) Find the average rate of change of f on the interval $-3 \le x \le 4$.

(a) Average rate of change $= \dfrac{f(4) - f(-3)}{4 - (-3)} = \dfrac{-1 - 0}{7} = -\dfrac{1}{7}$

1 : answer

(b) Write an equation for the line tangent to the graph of f at $x = 3$.

(b) $f(3) = -3 + 3\cos\left(\dfrac{3\pi}{2}\right) = -3$

For $0 < x < 4$, $f'(x) = -1 + \left(-3\sin\left(\dfrac{\pi x}{2}\right)\right) \cdot \dfrac{\pi}{2}$

$f'(3) = -1 + \left(-3\sin\left(\dfrac{3\pi}{2}\right)\right) \cdot \dfrac{\pi}{2} = -1 + \dfrac{3\pi}{2}$

An equation for the tangent line is $y = -3 + \left(-1 + \dfrac{3\pi}{2}\right)(x - 3)$.

$2 : \begin{cases} 1 : f'(3) \\ 1 : \text{equation} \end{cases}$

Continue question 3 on page 15.

NO CALCULATOR ALLOWED

(c) Find the average value of f on the interval $-3 \le x \le 4$.

(c) The average value of f on the interval $-3 \le x \le 4$ is $\frac{1}{4-(-3)}\int_{-3}^{4} f(x)\,dx$.

$$\int_{-3}^{4} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_{0}^{4} f(x)\,dx$$

$$\int_{-3}^{0} f(x)\,dx = \int_{-3}^{0} \sqrt{9-x^2}\,dx = \frac{9\pi}{4}$$

$$\int_{0}^{4} f(x)\,dx = \int_{0}^{4} \left(-x + 3\cos\left(\frac{\pi x}{2}\right)\right) dx = \left[-\frac{1}{2}x^2 + \frac{6}{\pi}\sin\left(\frac{\pi x}{2}\right)\right]_0^4 = -8$$

$$\frac{1}{4-(-3)}\int_{-3}^{4} f(x)\,dx = \frac{1}{7}\left(\frac{9\pi}{4} - 8\right)$$

4 :
- 1 : integrals of f over $-3 \le x \le 0$ and $0 \le x \le 4$
- 1 : value of $\int_{-3}^{0} \sqrt{9-x^2}\,dx$
- 1 : antiderivative of $-x + 3\cos\left(\frac{\pi x}{2}\right)$
- 1 : answer

(d) Must there be a value of x at which $f(x)$ attains an absolute maximum on the closed interval $-3 \le x \le 4$? Justify your answer.

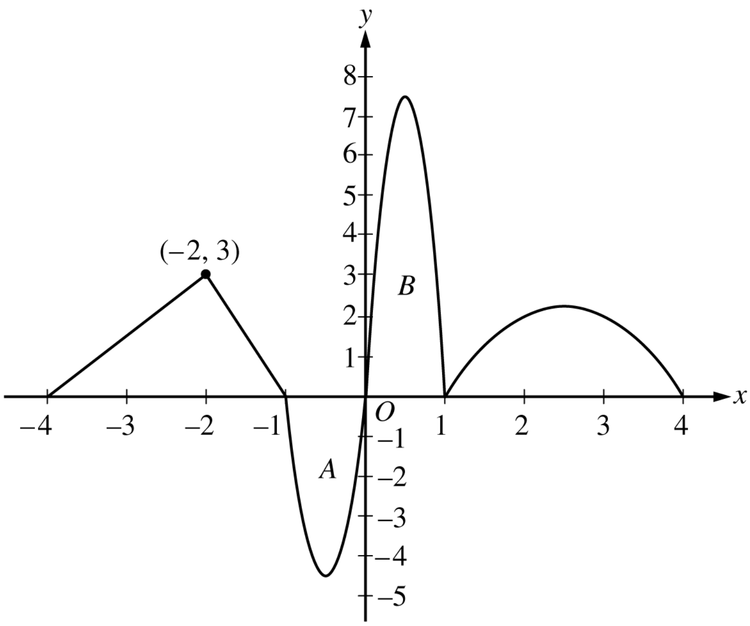
(d) $\lim_{x\to 0^-} f(x) = f(0) = 3$ and $\lim_{x\to 0^+} f(x) = 3$, so f is continuous at $x = 0$.

Because f is continuous on $[-3, 4]$, the Extreme Value Theorem guarantees that f attains an absolute maximum on $[-3, 4]$.

2 :
- 1 : continuity at $x = 0$
- 1 : answer with justification

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED



Graph of f

4. The continuous function f is defined for $-4 \le x \le 4$. The graph of f, shown above, consists of two line segments and portions of three parabolas. The graph has horizontal tangents at $x = -\frac{1}{2}$, $x = \frac{1}{2}$, and $x = \frac{5}{2}$. It is known that $f(x) = -x^2 + 5x - 4$ for $1 \le x \le 4$. The areas of regions A and B bounded by the graph of f and the x-axis are 3 and 5, respectively. Let g be the function defined by $g(x) = \int_{-4}^{x} f(t)\,dt$.

(a) Find $g(0)$ and $g(4)$.

(a) $g(0) = \int_{-4}^{0} f(t)\,dt = \frac{9}{2} - 3 = \frac{3}{2}$

$$g(4) = \int_{-4}^{4} f(t)\,dt$$
$$= \int_{-4}^{0} f(t)\,dt + \int_{0}^{1} f(t)\,dt + \int_{1}^{4} f(t)\,dt$$
$$= \frac{3}{2} + 5 + \int_{1}^{4} \left(-t^2 + 5t - 4\right) dt$$
$$= \frac{3}{2} + 5 + \left[-\frac{1}{3}t^3 + \frac{5}{2}t^2 - 4t\right]_1^4$$
$$= \frac{3}{2} + 5 + \left[\left(-\frac{1}{3}\cdot 4^3 + \frac{5}{2}\cdot 4^2 - 4\cdot 4\right) - \left(-\frac{1}{3}\cdot 1^3 + \frac{5}{2}\cdot 1^2 - 4\cdot 1\right)\right]$$
$$= \frac{3}{2} + 5 + \left(\frac{8}{3} - \left(-\frac{11}{6}\right)\right) = 11$$

$$4 : \begin{cases} 1 : g(0) \\ 1 : \text{integral of } f \text{ over } 1 \le t \le 4 \\ 1 : \text{antiderivative} \\ 1 : g(4) \end{cases}$$

Continue question 4 on page 17.

NO CALCULATOR ALLOWED

(b) Find the absolute minimum value of g on the closed interval $[-4, 4]$. Justify your answer.

(b) $g'(x) = f(x)$ is negative for $-1 < x < 0$, and nonnegative elsewhere. Thus, the absolute minimum value of g on $[-4, 4]$ can only occur at $x = -4$ or $x = 0$.

$g(-4) = 0$

$g(0) = \frac{3}{2}$

The absolute minimum value of g on $[-4, 4]$ is $g(-4) = 0$.

3 :
- 1 : $g'(x) = f(x)$
- 1 : identifies $x = -4$ and $x = 0$ as candidates
- 1 : answer with justification

(c) Find all intervals on which the graph of g is concave down. Give a reason for your answer.

(c) The graph of g is concave down on the intervals $-2 < x < -\frac{1}{2}$, $\frac{1}{2} < x < 1$, and $\frac{5}{2} < x < 4$ because $g'(x) = f(x)$ is decreasing on these intervals.

2 :
- 1 : intervals
- 1 : reason

NO CALCULATOR ALLOWED

t (hours)	0	1	2	3	4
$B(t)$ (miles per hour)	1	8	1.5	–5	11

5. Brandon and Chloe ride their bikes for 4 hours along a flat, straight road. Brandon's velocity, in miles per hour, at time t hours is given by a differentiable function B for $0 \le t \le 4$. Values of $B(t)$ for selected times t are given in the table above. Chloe's velocity, in miles per hour, at time t hours is given by the piecewise function C defined by

$$C(t) = \begin{cases} te^{4-t^2} & \text{for } 0 \le t \le 2 \\ 12 - 3t - t^2 & \text{for } 2 < t \le 4. \end{cases}$$

(a) How many miles did Chloe travel from time $t = 0$ to time $t = 2$?

(a) $\displaystyle\int_0^2 te^{4-t^2}\,dt = -\frac{1}{2}e^{4-t^2}\Big|_{t=0}^{t=2} = -\frac{1}{2} + \frac{1}{2}e^4$

Chloe traveled $-\frac{1}{2} + \frac{1}{2}e^4$ miles from time $t = 0$ to time $t = 2$.

3 : { 1 : integral; 1 : antiderivative; 1 : answer }

(b) At time $t = 3$, is Chloe's speed increasing or decreasing? Give a reason for your answer.

(b) $C(3) = -6 < 0$
$C'(3) = -9 < 0$

Chloe's speed is increasing at time $t = 3$ because her velocity and acceleration have the same sign.

2 : { 1 : $C(3) < 0$ and $C'(3) < 0$; 1 : answer with reason }

(c) B is differentiable $\Rightarrow$ B is continuous on $[0, 4]$.

Continue question 5 on page 19.

NO CALCULATOR ALLOWED

(c) Is there a time t, for $0 \le t \le 4$, at which Brandon's acceleration is equal to 2.5 miles per hour per hour? Justify your answer.

c) B is differentiable $\Rightarrow B$ is continuous on $[0, 4]$.

$$\frac{B(4) - B(0)}{4 - 0} = \frac{11 - 1}{4 - 0} = 2.5$$

By the Mean Value Theorem, there is a time t, for $0 < t < 4$, such that $B'(t) = 2.5$ miles per hour per hour.

$$2 : \begin{cases} 1 : \dfrac{B(4) - B(0)}{4 - 0} \\ 1 : \text{answer with justification} \end{cases}$$

(d) Is there a time t, for $0 \le t \le 2$, at which Brandon's velocity is equal to Chloe's velocity? Justify your answer.

(d) B and C are continuous on $[0, 2]$, therefore $B - C$ is continuous on $[0, 2]$.

$B(0) - C(0) = 1 - 0 > 0$
$B(2) - C(2) = 1.5 - 2 < 0$

By the Intermediate Value Theorem, there is a time t, for $0 < t < 2$, such that $B(t) - C(t) = 0$, or $B(t) = C(t)$.

$$2 : \begin{cases} 1 : \text{considers } B(t) - C(t) \\ 1 : \text{answer with justification} \end{cases}$$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

6. Consider the curve defined by $2x^2 + 3y^2 - 4xy = 36$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{2y - 2x}{3y - 2x}$.

(a) $\dfrac{d}{dx}\left(2x^2 + 3y^2 - 4xy\right) = \dfrac{d}{dx}(36) \Rightarrow 4x + 6y\dfrac{dy}{dx} - 4y - 4x\dfrac{dy}{dx} = 0$

$\Rightarrow (6y - 4x)\dfrac{dy}{dx} = 4y - 4x \Rightarrow \dfrac{dy}{dx} = \dfrac{4y - 4x}{6y - 4x} = \dfrac{2y - 2x}{3y - 2x}$

2 : { 1 : implicit differentiation; 1 : verification }

(b) Find the slope of the line tangent to the curve at each point on the curve where $x = 6$.

(b) $x = 6 \Rightarrow 2 \cdot 6^2 + 3y^2 - 4 \cdot 6 \cdot y = 36$

$\Rightarrow y^2 - 8y + 12 = 0 \Rightarrow y = 2$ or $y = 6$

$\left.\dfrac{dy}{dx}\right|_{(x,\, y)=(6,\, 2)} = \dfrac{4 - 12}{6 - 12} = \dfrac{4}{3}$

The slope of the line tangent to the curve at $(6, 2)$ is $\dfrac{4}{3}$.

$\left.\dfrac{dy}{dx}\right|_{(x,\, y)=(6,\, 6)} = \dfrac{12 - 12}{18 - 12} = 0$

The slope of the line tangent to the curve at $(6, 6)$ is 0.

2 : { 1 : slope at $(6, 2)$; 1 : slope at $(6, 6)$ }

Continue question 6 on page 21.

NO CALCULATOR ALLOWED

(c) Find the positive value of x at which the curve has a vertical tangent line. Show the work that leads to your answer.

(c) The curve has vertical tangent lines where $3y - 2x = 0$ and $2y - 2x \neq 0$.

$3y - 2x = 0 \Rightarrow y = \frac{2}{3}x \Rightarrow 2x^2 + 3\left(\frac{2}{3}x\right)^2 - 4x \cdot \frac{2}{3}x = 36$

$\Rightarrow \left(2 + \frac{4}{3} - \frac{8}{3}\right)x^2 = \frac{2}{3}x^2 = 36 \Rightarrow x^2 = 54$

Because x is positive, $x = 3\sqrt{6}$.

$2 : \begin{cases} 1 : \text{sets } 3y - 2x = 0 \\ 1 : \text{answer} \end{cases}$

(d) Let x and y be functions of time t that are related by the equation $2x^2 + 3y^2 - 4xy = 36$. At time $t = 1$, the value of x is 2, the value of y is -2, and the value of $\frac{dy}{dt}$ is 4. Find the value of $\frac{dx}{dt}$ at time $t = 1$.

(d) $\frac{d}{dt}\left(2x^2 + 3y^2 - 4xy\right) = \frac{d}{dt}(36)$

$\Rightarrow 4x\frac{dx}{dt} + 6y\frac{dy}{dt} - 4\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right) = 0$

$\Rightarrow (4x - 4y)\frac{dx}{dt} + (6y - 4x)\frac{dy}{dt} = 0$

$(4 \cdot 2 - 4 \cdot (-2))\left.\frac{dx}{dt}\right|_{t=1} + (6 \cdot (-2) - 4 \cdot 2) \cdot 4 = 0 \Rightarrow \left.\frac{dx}{dt}\right|_{t=1} = 5$

— OR —

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{2y - 2x}{3y - 2x} \cdot \frac{dx}{dt}$

$4 = \frac{2 \cdot (-2) - 2 \cdot 2}{3 \cdot (-2) - 2 \cdot 2} \cdot \left.\frac{dx}{dt}\right|_{t=1} = \frac{4}{5} \cdot \left.\frac{dx}{dt}\right|_{t=1} \Rightarrow \left.\frac{dx}{dt}\right|_{t=1} = 5$

$3 : \begin{cases} 1 : \text{chain rules or product rule} \\ 1 : \text{derivative with respect to } t \\ 1 : \text{answer} \end{cases}$

Answer Key for AP Calculus AB
Practice Exam, Section I

Question 1: B
Question 2: C
Question 3: D
Question 4: D
Question 5: C
Question 6: D
Question 7: D
Question 8: B
Question 9: B
Question 10: A
Question 11: A
Question 12: C
Question 13: A
Question 14: D
Question 15: A
Question 16: B
Question 17: B
Question 18: D
Question 19: A
Question 20: C
Question 21: D
Question 22: D
Question 23: B
Question 24: A
Question 25: A
Question 26: B
Question 27: C
Question 28: B
Question 29: A
Question 30: B

Question 76: D
Question 77: B
Question 78: A
Question 79: B
Question 80: C
Question 81: C
Question 82: A
Question 83: A
Question 84: C
Question 85: D
Question 86: A
Question 87: C
Question 88: C
Question 89: B
Question 90: C