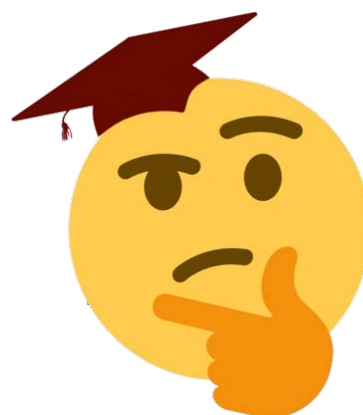


A LEVEL Cambridge Topical Past Papers

FURTHER MECHANICS

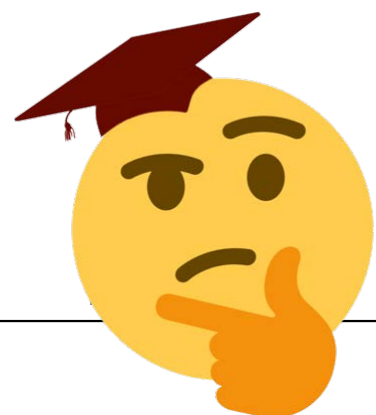
2017 — 2023

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	ANSWERS	Page 245



1 - (9231/21_Summer_2017_Q1) - *Momentum And Impulse*

A bullet of mass 0.08 kg is fired horizontally into a fixed vertical barrier. It enters the barrier horizontally with speed 300 m s^{-1} and emerges horizontally after 0.02 s . There is a constant horizontal resisting force of magnitude 1000 N . Find the speed with which the bullet emerges from the barrier. [3]



2 - (9231/21_Summer_2017_Q3) - Momentum And Impulse

Two uniform small smooth spheres A and B have equal radii and masses $3m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e .

- (i) Find, in terms of u and e , expressions for the velocities of A and B after the collision. [3]

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Sphere B continues to move until it strikes a fixed smooth vertical barrier which is perpendicular to the direction of motion of B . The coefficient of restitution between B and the barrier is $\frac{3}{4}$. When the spheres subsequently collide, A is brought to rest.

- (ii) Find the value of e . [7]

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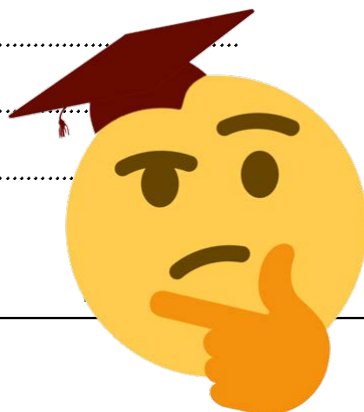
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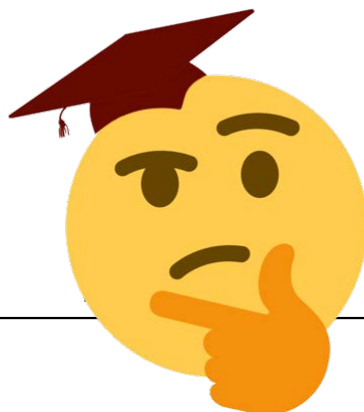


3 - (9231/23_Summer_2017_Q3) - Momentum And Impulse

Two uniform small smooth spheres A and B have equal radii and each has mass m . Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$. Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A . The coefficient of restitution between B and the wall is $\frac{1}{3}$.

(i) Show that the speed of B after its collision with the wall is $\frac{5}{18}u$. [4]

(ii) Find the distance of B from the wall when it collides with A for the second time. [6]

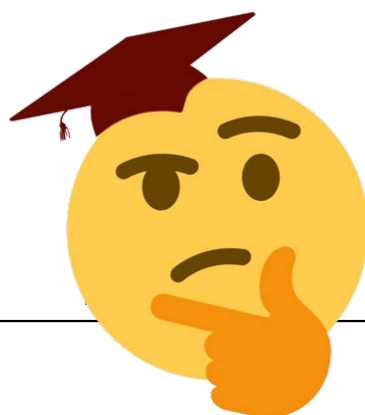


4 - (9231/21_Winter_2017_Q3) - Momentum And Impulse

Three uniform small smooth spheres A , B and C have equal radii and masses m , km and m respectively, where k is a constant. The spheres are moving in the same direction along a straight line on a smooth horizontal surface, with B between A and C . The speeds of A , B and C are $2u$, u and $\frac{4}{3}u$ respectively. The coefficient of restitution between any pair of the spheres is $\frac{1}{2}$. After sphere A has collided with sphere B , sphere B collides with sphere C .

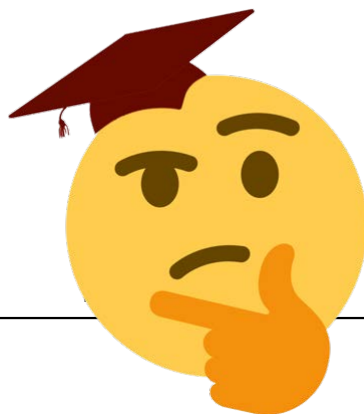
(i) Find an inequality satisfied by k . [5]

(ii) Given that $k = 2$, show that after B has collided with C there are no further collisions between any of the three spheres. [5]



5 - (9231/21_Summer_2018_Q1) - *Momentum And Impulse*

A bullet of mass m kg is fired horizontally into a fixed vertical block of material. It enters the block horizontally with speed 250 m s^{-1} and emerges horizontally with speed 70 m s^{-1} after 0.04 s . The block offers a constant horizontal resisting force of magnitude 450 N . Find the value of m . [3]



6 - (9231/23_Summer_2018_Q2) - Momentum And Impulse

Two uniform small spheres A and B have equal radii and masses $4m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e .

- (i) Show that after the collision A moves with speed $\frac{1}{5}u(4 - e)$ and find the speed of B . [4]

Sphere B continues to move until it collides with a fixed smooth vertical barrier which is perpendicular to the direction of motion of B . The coefficient of restitution between B and the barrier is $\frac{3}{4}e$. After this collision, the speeds of A and B are equal.

- (ii) Find the value of e . [3]

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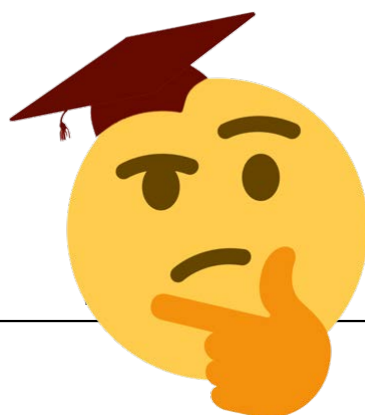
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The spheres A and B now collide directly again.

- (iii) Determine whether sphere B collides with the barrier for a second time. [2]



7 - (9231/21_Summer_2018_Q3) - Momentum And Impulse

Two identical uniform small spheres A and B , each of mass m , are moving towards each other in a straight line on a smooth horizontal surface. Their speeds are u and ku respectively, and they collide directly. The coefficient of restitution between the spheres is e . Sphere B is brought to rest by the collision.

- (i) Show that $e = \frac{k-1}{k+1}$. [3]

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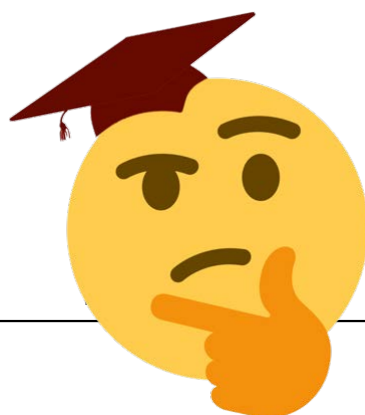
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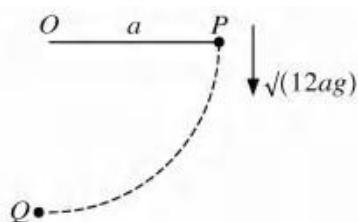
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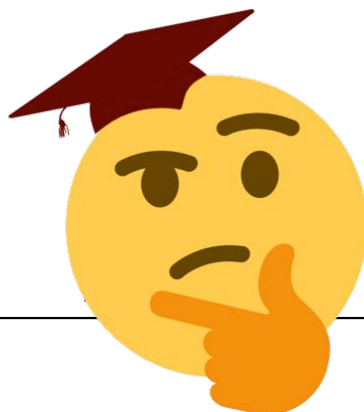
- (ii) Given that 60% of the total initial kinetic energy is lost in the collision, find the values of k and e . [6]



8 - (9231/23_Summer_2018_Q5) - Momentum And Impulse, Circular Motion



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is held with the string taut and horizontal. It is projected downwards with speed $\sqrt{12ag}$. At the lowest point of its motion, P collides directly with a particle Q of mass km which is at rest (see diagram). In the collision, P and Q coalesce. The tension in the string immediately after the collision is half of its value immediately before the collision. Find the possible values of k . [11]



9 - (9231/21_Winter_2018_Q2) - Momentum And Impulse

Two uniform small smooth spheres A and B have equal radii and masses $5m$ and $2m$ respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is moving towards it with speed $2u$. The coefficient of restitution between the spheres is e .

- (i) Show that the speed of B after the collision is $\frac{1}{7}u(1 + 15e)$ and find an expression for the speed of A . [4]

In the collision, the speed of A is halved and its direction of motion is reversed.

- (ii) Find the value of e . [2]

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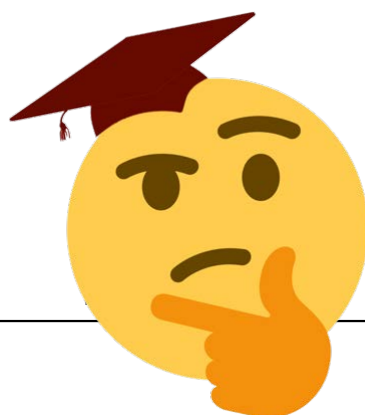
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- (iii) For this collision, find the ratio of the loss of kinetic energy of A to the loss of kinetic energy of B . [3]



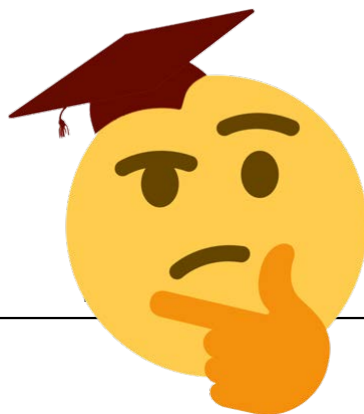
10 – (9231/22_Winter_2018_Q2) – *Momentum And Impulse*

Two uniform small smooth spheres A and B have equal radii and masses $2m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is $\frac{2}{3}$.

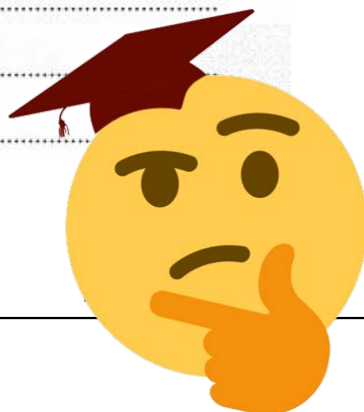
- (i) Find, in terms of u , the speeds of A and B after this collision. [4]

Sphere B is initially at a distance d from a fixed smooth vertical wall which is perpendicular to the direction of motion of A . The coefficient of restitution between B and the wall is $\frac{1}{2}$.

- (ii) Find, in terms of d and u , the time that elapses between the first and second collisions between A and B . [5]



A bullet of mass 0.2 kg is fired into a fixed vertical barrier. It enters the barrier horizontally with speed 250 m s^{-1} and emerges horizontally after a time T seconds with speed 40 m s^{-1} . There is a constant horizontal resisting force of magnitude 1200 N . Find T . [4]

[illegible]

12 - (9231/21_Summer_2019_Q3) - Momentum And Impulse

Three uniform small spheres A , B and C have equal radii and masses $2m$, $4m$ and m respectively. The spheres are moving in a straight line on a smooth horizontal surface, with B between A and C . The coefficient of restitution between each pair of spheres is e . Spheres A and B are moving towards each other with speeds $2u$ and u respectively. The first collision is between A and B .

- (i) Find the velocities of A and B after this collision. [3]

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Sphere C is moving towards B with speed $\frac{4}{3}u$ and now collides with it. As a result of this collision, B is brought to rest.

- (ii) Find the value of e . [4]

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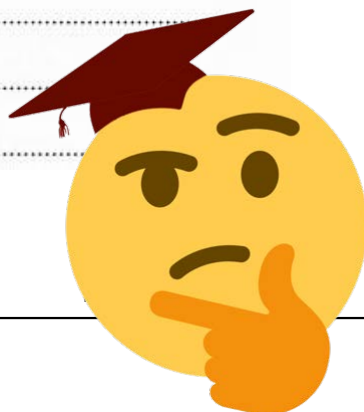
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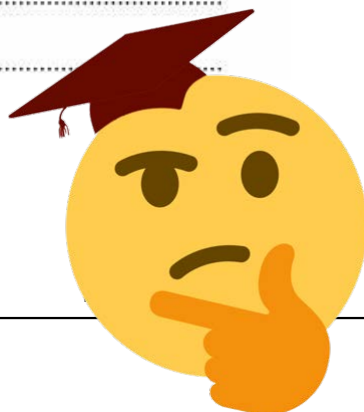
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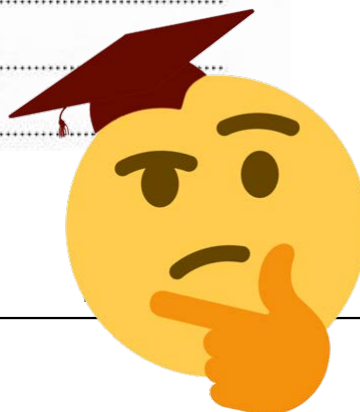


(iii) Find the total kinetic energy lost by the three spheres as a result of the two collisions. [3]



Three uniform small spheres A , B and C have equal radii and masses $3m$, m and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C . The coefficient of restitution between each pair of spheres is e . Sphere A is projected directly towards B with speed u .

- [illegible]



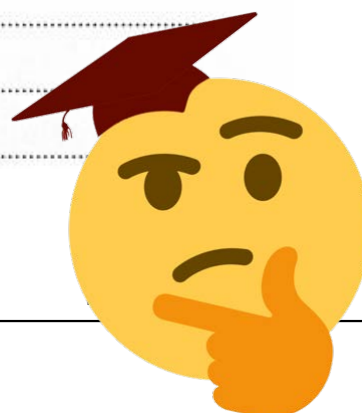
(ii) Given that A and C are moving with equal speeds after these two collisions, find the value of e . [3]



Answer only **one** of the following two alternatives.

A particle P , of mass m , is able to move in a vertical circle on the smooth inner surface of a sphere with centre O and radius a . Points A and B are on the inner surface of the sphere and AOB is a horizontal diameter. Initially, P is projected vertically downwards with speed $\sqrt{\frac{21}{2}ag}$ from A and begins to move in a vertical circle. At the lowest point of its path, vertically below O , the particle P collides with a stationary particle Q , of mass $4m$, and rebounds. The speed acquired by Q , as a result of the collision, is just sufficient for it to reach the point B .

- (i) Find the speed of P and the speed of Q immediately after their collision. [7]

[illegible]

(ii) Find $\cos \theta$.

[5]

[illegible]

OR

A farmer grows two different types of cherries, Type *A* and Type *B*. He assumes that the masses of each type are normally distributed. He chooses a random sample of 8 cherries of Type *A*. He finds that the sample mean mass is 15.1 g and that a 95% confidence interval for the population mean mass, μ g, is $13.5 \leq \mu \leq 16.7$.

(i) Find an unbiased estimate for the population variance of the masses of cherries of Type *A*. [3]

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The farmer now chooses a random sample of 6 cherries of Type *B* and records their masses as follows.

12.2 13.3 16.4 14.0 13.9 15.4

(ii) Test at the 5% significance level whether the mean mass of cherries of Type *B* is less than the mean mass of cherries of Type *A*. You should assume that the population variances for the two types of cherry are equal. [9]

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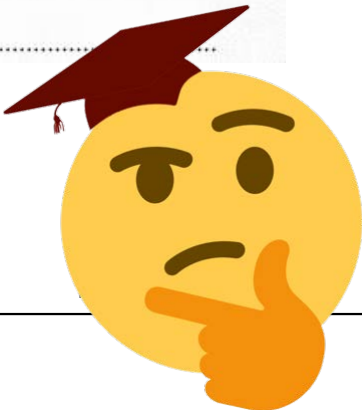
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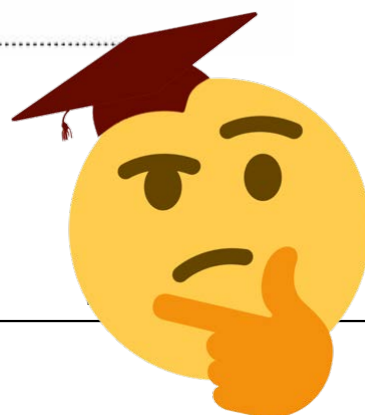
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15 - (9231/21_Winter_2019_Q3) - Momentum And Impulse

Three uniform small spheres A , B and C have equal radii and masses $5m$, $5m$ and $3m$ respectively. The spheres are at rest on a smooth horizontal surface, in a straight line, with B between A and C . The coefficient of restitution between each pair of spheres is e . Sphere A is projected directly towards B with speed u .

- (i) Show that the speed of A after its collision with B is $\frac{1}{2}u(1 - e)$ and find the speed of B . [3]

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Sphere B now collides with sphere C . Subsequently there are no further collisions between any of the spheres.

- (ii) Find the set of possible values of e . [6]

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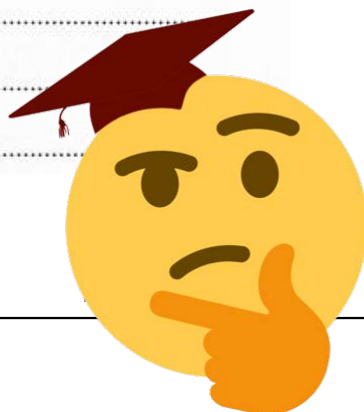
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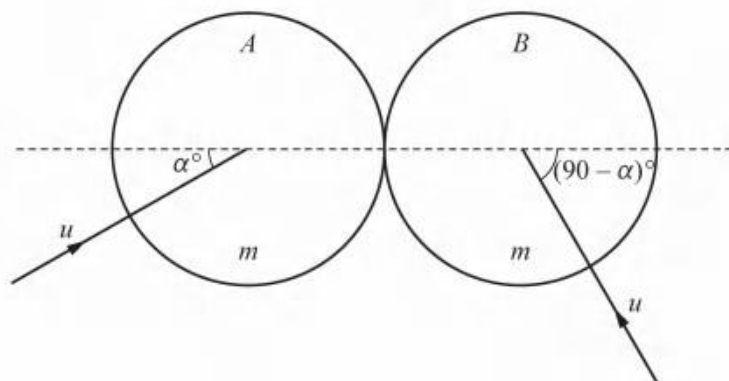
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16 – (9231/33_Summer_2020_Q5) – Momentum And Impulse

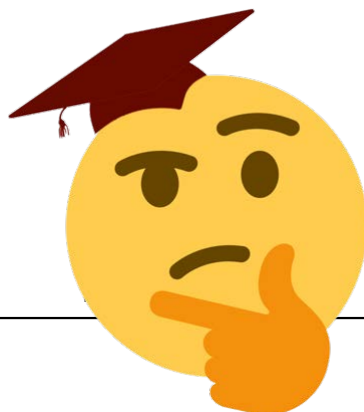


Two uniform smooth spheres A and B of equal radii each have mass m . The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A 's direction of motion makes an angle of α° with the line of centres, and B 's direction of motion is perpendicular to that of A (see diagram). The coefficient of restitution between the spheres is e .

Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan \alpha = \frac{1+e}{1-e}$. [4]

(b) Given that $\tan \alpha = 2$, find the speed of A after the collision. [4]



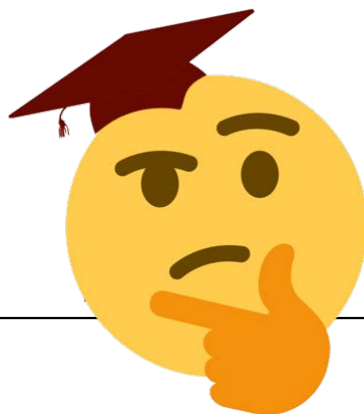
17 – (9231/31_Summer_2020_Q6) – Momentum And Impulse

A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α with the barrier. The coefficient of restitution between P and the barrier is e . As a result of the impact, the direction of motion of P is turned through 90° .

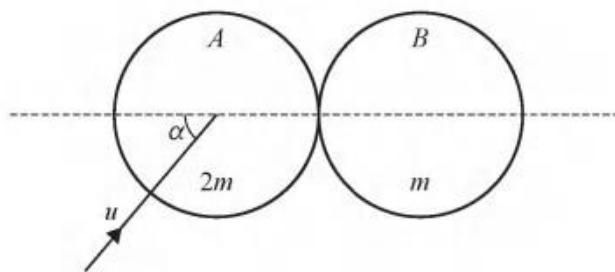
- (a) Show that $\tan^2 \alpha = \frac{1}{e}$. [3]

The particle P loses two-thirds of its kinetic energy in the impact.

- (b) Find the value of α and the value of e . [5]



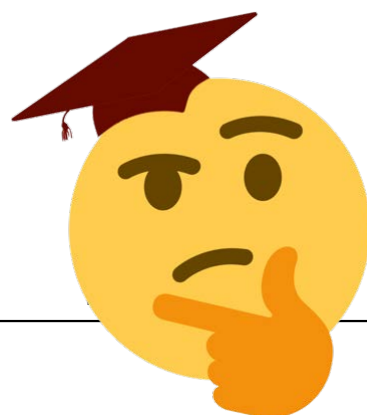
18 - (9231/32_Winter_2020_Q2) - Momentum And Impulse



Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B . Immediately before the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan \alpha = \frac{4}{3}$ (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

Find the speed of A after the collision.

[5]



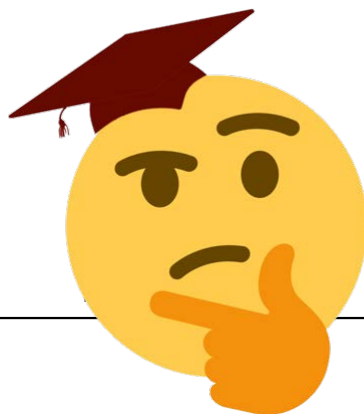
19 – (9231/32_Winter_2020_Q4) – *Momentum And Impulse*

A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r . The angle between the vertical and the normal reaction of the surface on P is θ .

(a) Show that $\cos \theta = \frac{g}{\omega^2 r}$. [3]

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

(b) Find x in terms of r . [4]



20 – (9231/31_Winter_2020_Q6) – Momentum And Impulse

Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B . The coefficient of restitution between the spheres is e .

- (a) Find, in terms of u and e , the velocities of A and B after the collision. [3]

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Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B , where $\tan \theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B .

- (b) Find the possible values of e . [7]

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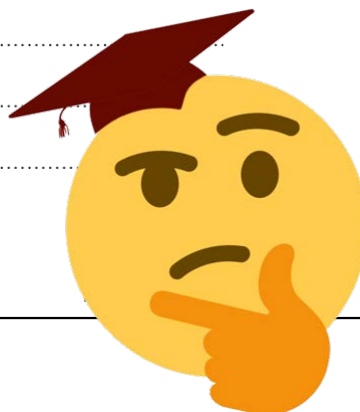
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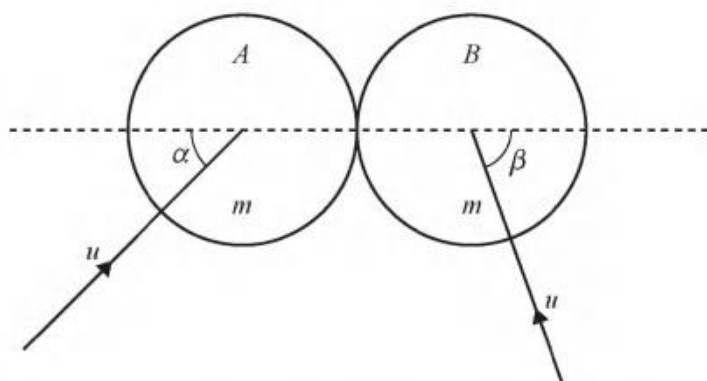
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21 – (9231/31_Summer_2021_Q6) – Momentum And Impulse

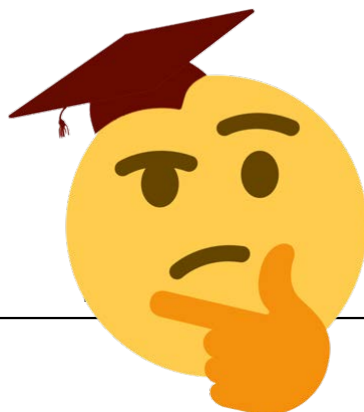


Two uniform smooth spheres A and B of equal radii each have mass m . The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision, A 's direction of motion makes an angle α with the line of centres, and B 's direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$ and $2\cos\beta = \cos\alpha$.

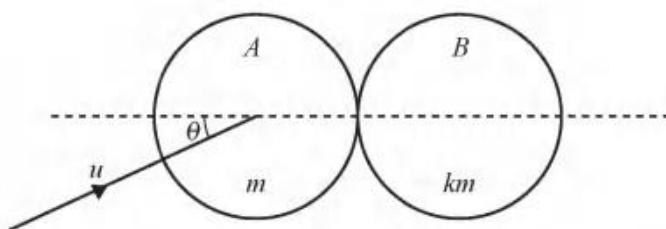
- (a) Show that the direction of motion of A after the collision is perpendicular to the line of centres. [4]

The total kinetic energy of the spheres after the collision is $\frac{3}{4}mu^2$.

- (b) Find the value of α . [4]



22 – (9231/33_Summer_2021_Q6) – Momentum And Impulse

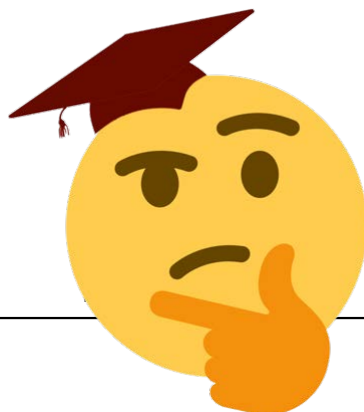


Two uniform smooth spheres A and B of equal radii have masses m and km respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides with sphere B which is at rest. Immediately before the collision, A 's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

- (a) Show that the speed of B after the collision is $\frac{4u \cos \theta}{3(1+k)}$. [3]

70% of the total kinetic energy of the spheres is lost as a result of the collision.

- (b) Given that $\tan \theta = \frac{1}{3}$, find the value of k . [6]



23 – (9231/32_Winter_2021_Q1) – *Momentum And Impulse*

A particle is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane. The particle moves freely under gravity.

- (a) Write down the horizontal and vertical components of the velocity of the particle at time T after projection. [2]

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At time T after projection, the direction of motion of the particle is perpendicular to the direction of projection.

- (b) Express T in terms of u , g and α . [2]

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- (c) Deduce that $T > \frac{u}{g}$. [1]

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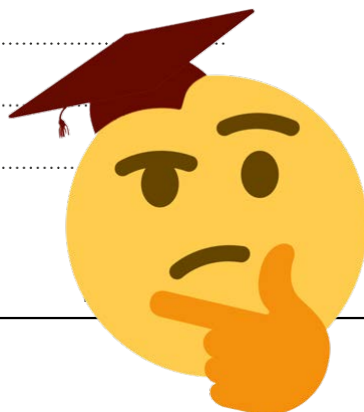
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24 – (9231/31_Winter_2021_Q2) – *Momentum And Impulse*

A particle P of mass m kg moves along a horizontal straight line with acceleration a ms⁻² given by

$$a = \frac{v(1-2t^2)}{t},$$

where v ms⁻¹ is the velocity of P at time t s.

- (a) Find an expression for v in terms of t and an arbitrary constant. [3]

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- (b) Given that $a = 5$ when $t = 1$, find an expression, in terms of m and t , for the horizontal force acting on P at time t . [3]

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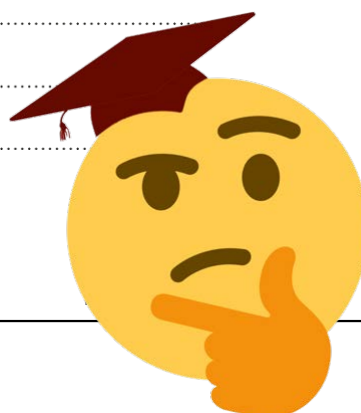
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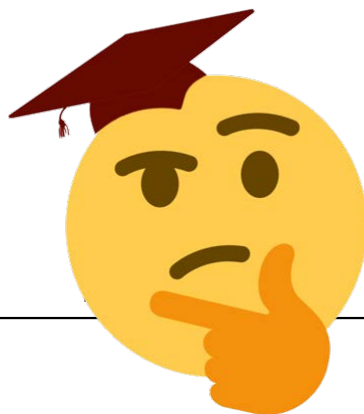


25 – (9231/31_Winter_2021_Q5) – Momentum And Impulse

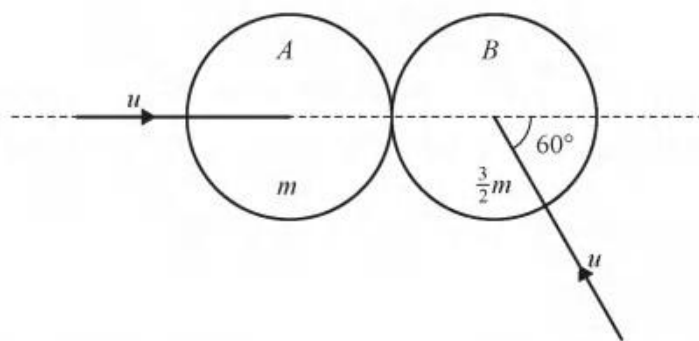
A particle P is projected from a point O on a horizontal plane and moves freely under gravity. Its initial speed is $u \text{ ms}^{-1}$ and its angle of projection is $\sin^{-1}(\frac{4}{5})$ above the horizontal. At time 8 s after projection, P is at the point A . At time 32 s after projection, P is at the point B . The direction of motion of P at B is perpendicular to its direction of motion at A .

Find the value of u .

[7]

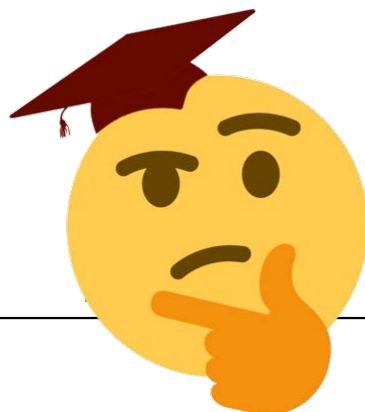


26 – (9231/32_Winter_2021_Q5) – Momentum And Impulse



Two uniform smooth spheres A and B of equal radii have masses m and $\frac{3}{2}m$ respectively. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A 's direction of motion is along the line of centres, and B 's direction of motion makes an angle of 60° with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$.

- (a) Find the angle through which the direction of motion of B is deflected by the collision. [6]
- (b) Find the loss in the total kinetic energy of the system as a result of the collision. [3]



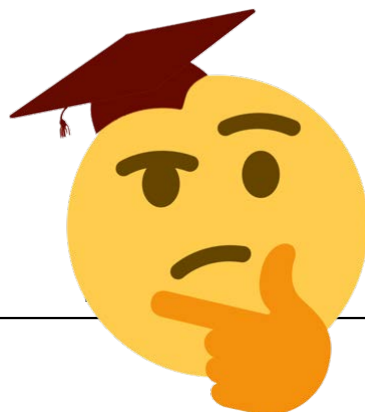
27 – (9231/32_Winter_2021_Q6) – Momentum And Impulse

A particle P of mass 2 kg moves along a horizontal straight line. The point O is a fixed point on this line. At time $t\text{ s}$ the velocity of P is $v\text{ ms}^{-1}$ and the displacement of P from O is $x\text{ m}$.

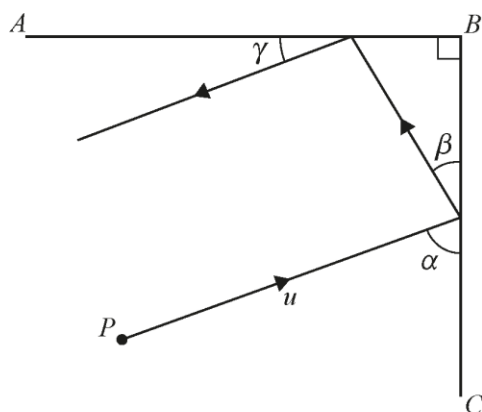
A force of magnitude $\left(8x - \frac{128}{x^3}\right)\text{ N}$ acts on P in the direction OP . When $t = 0$, $x = 8$ and $v = -15$.

(a) Show that $v = -\frac{2}{x}(x^2 - 4)$. [5]

(b) Find an expression for x in terms of t . [4]



28 – (9231/31_Winter_2021_Q7) – Momentum And Impulse



The smooth vertical walls AB and CB are at right angles to each other. A particle P is moving with speed u on a smooth horizontal floor and strikes the wall CB at an angle α . It rebounds at an angle β to the wall CB . The particle then strikes the wall AB and rebounds at an angle γ to that wall (see diagram). The coefficient of restitution between each wall and P is e .

- (a) Show that $\tan \beta = e \tan \alpha$. [3]

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- (b) Express γ in terms of α and explain what this result means about the final direction of motion of P . [4]

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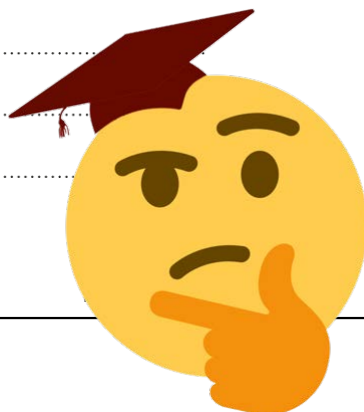
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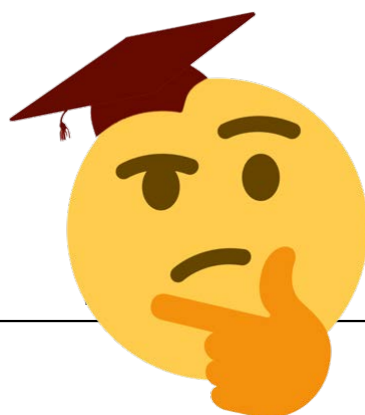
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As a result of the two impacts the particle loses $\frac{8}{9}$ of its initial kinetic energy.

(c) Given that $\alpha + \beta = 90^\circ$, find the value of e and the value of $\tan \alpha$.

[4]



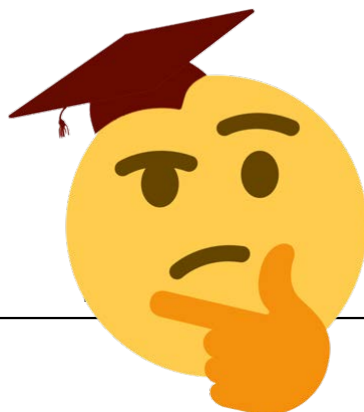
29 – (9231/31_Summer_2022_Q6) – Momentum And Impulse

Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are on a horizontal surface. Sphere A is travelling with speed u towards sphere B which is at rest. The spheres collide. Immediately before the collision, the direction of motion of A makes an angle α with the line of centres. The coefficient of restitution between the spheres is $\frac{1}{2}$.

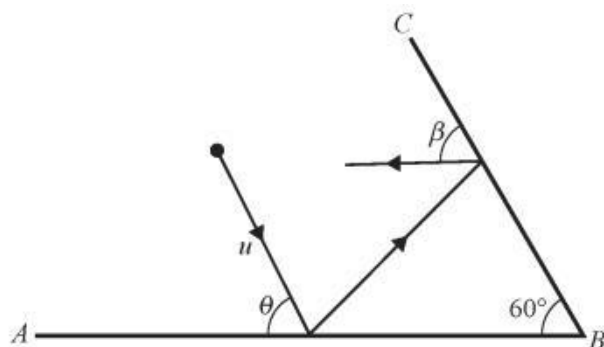
- (a) Show that the speed of B after the collision is $\frac{3u \cos \alpha}{2(1+k)}$ and find also an expression for the speed of A along the line of centres after the collision, in terms of k , u and α . [4]

After the collision, the kinetic energy of A is equal to the kinetic energy of B .

- (b) Given that $\tan \alpha = \frac{2}{3}$, find the possible values of k . [5]



30 – (9231/33_Summer_2022_Q6) – Momentum And Impulse

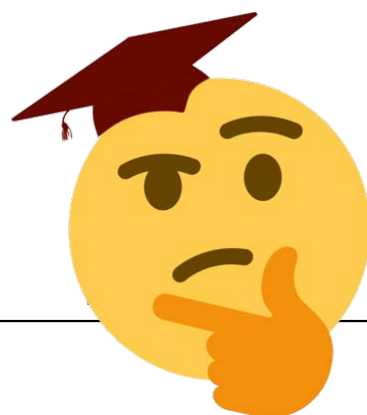


AB and BC are two fixed smooth vertical barriers on a smooth horizontal surface, with angle $ABC = 60^\circ$. A particle of mass m is moving with speed u on the surface. The particle strikes AB at an angle θ with AB . It then strikes BC and rebounds at an angle β with BC (see diagram). The coefficient of restitution between the particle and each barrier is e and $\tan \theta = 2$.

The kinetic energy of the particle after the first collision is 40% of its kinetic energy before the first collision.

(a) Find the value of e . [4]

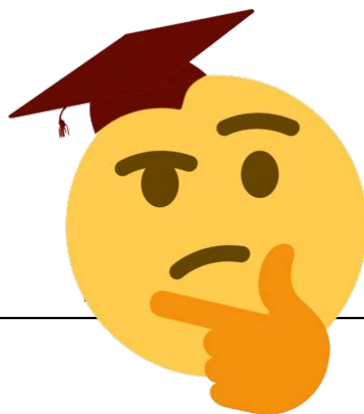
(b) Find the size of angle β . [4]



31 – (9231/31_Summer_2022_Q7) – *Momentum And Impulse*

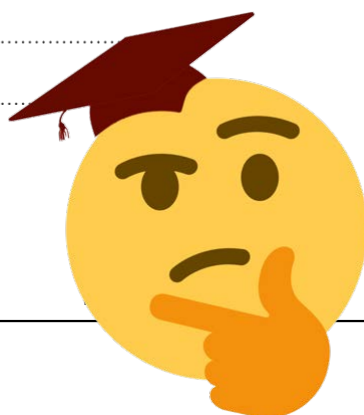
Particles P and Q are projected in the same vertical plane from a point O at the top of a cliff. The height of the cliff exceeds 50 m. Both particles move freely under gravity. Particle P is projected with speed $\frac{35}{2} \text{ m s}^{-1}$ at an angle α above the horizontal, where $\tan \alpha = \frac{4}{3}$. Particle Q is projected with speed $u \text{ m s}^{-1}$ at an angle β above the horizontal, where $\tan \beta = \frac{1}{2}$. Particle Q is projected one second after the projection of particle P . The particles collide T s after the projection of particle Q .

- (a) Write down expressions, in terms of T , for the horizontal displacements of P and Q from O when they collide and hence show that $4uT = 21\sqrt{5}(T+1)$. [4]



[4]

[3]

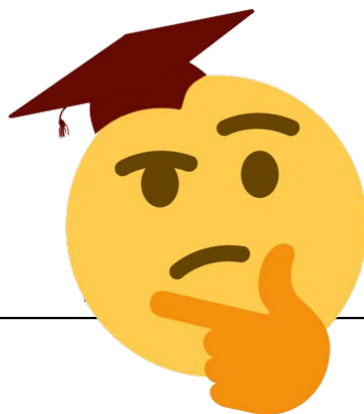


32 - (9231/32_Winter_2022_Q3) - Momentum And Impulse

One end of a light elastic string, of natural length a and modulus of elasticity $\frac{16}{3}Mg$, is attached to a fixed point O . A particle P of mass $4M$ is attached to the other end of the string and hangs vertically in equilibrium. Another particle of mass $2M$ is attached to P and the combined particle is then released from rest. The speed of the combined particle when it has descended a distance $\frac{1}{4}a$ is v .

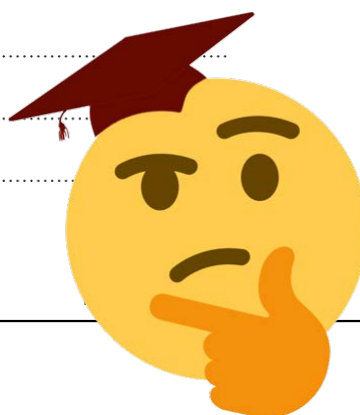
Find an expression for v in terms of g and a .

[6]



A particle of mass 0.5 kg moves along a horizontal straight line. Its velocity is $v \text{ ms}^{-1}$ at time $t \text{ s}$. The forces acting on the particle are a driving force of magnitude 50 N and a resistance of magnitude $2v^2 \text{ N}$. The initial velocity of the particle is 3 ms^{-1} .

(a) Find an expression for v in terms of t . [7]



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(b) Deduce the limiting value of v . [1]

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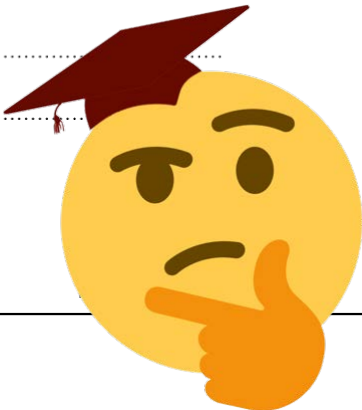
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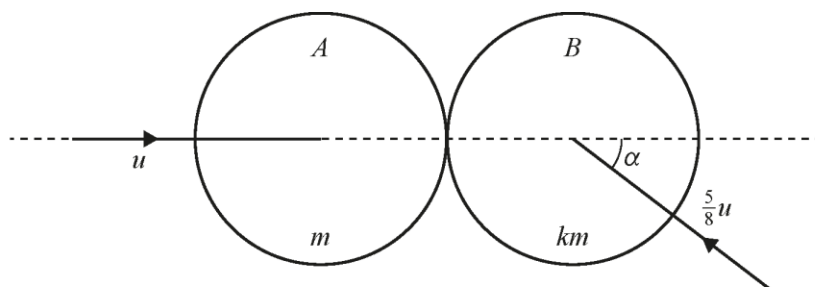
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34 - (9231/31_Winter_2022_Q6) - Momentum And Impulse

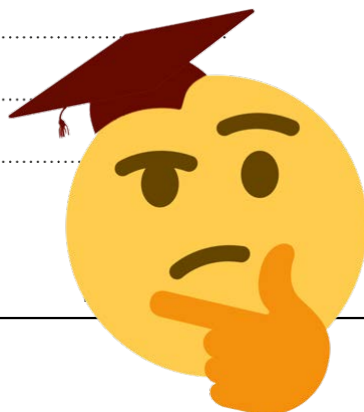


Two uniform smooth spheres A and B of equal radii have masses m and km respectively. The two spheres are moving on a horizontal surface with speeds u and $\frac{5}{8}u$ respectively. Immediately before the spheres collide, A is travelling along the line of centres, and B 's direction of motion makes an angle α with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{2}{3}$ and $\tan \alpha = \frac{3}{4}$.

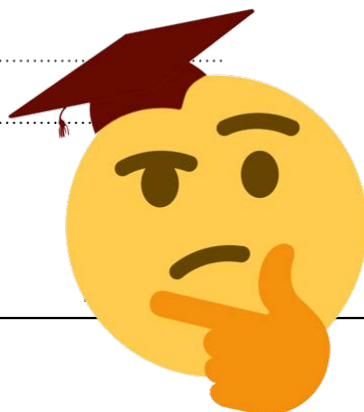
After the collision, the direction of motion of B is perpendicular to the line of centres.

(a) Find the value of k .

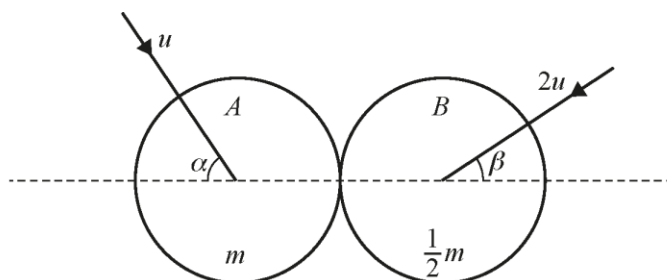
[4]



[4]

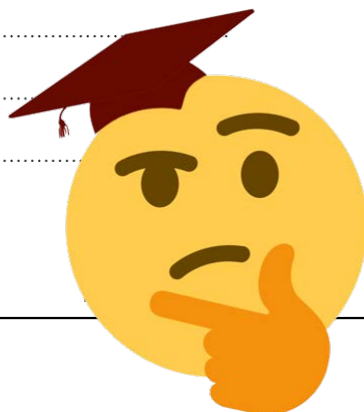


35 - (9231/32_Winter_2022_Q7) - Momentum And Impulse

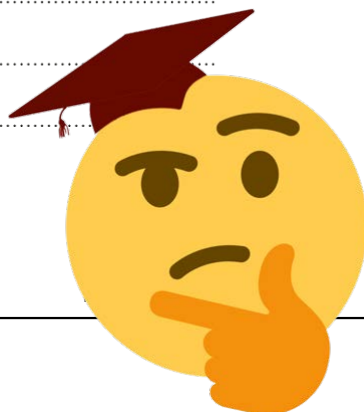


Two uniform smooth spheres A and B of equal radii have masses m and $\frac{1}{2}m$ respectively. The two spheres are moving on a horizontal surface when they collide. Immediately before the collision, sphere A is travelling with speed u and its direction of motion makes an angle α with the line of centres. Sphere B is travelling with speed $2u$ and its direction of motion makes an angle β with the line of centres (see diagram). The coefficient of restitution between the spheres is $\frac{5}{8}$ and $\alpha + \beta = 90^\circ$.

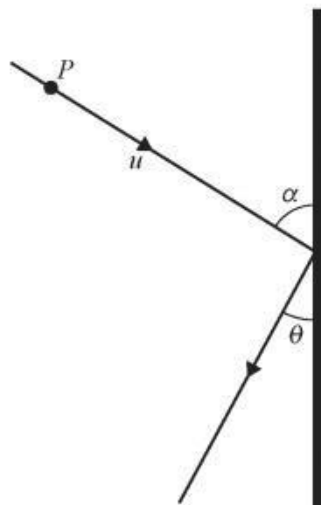
- (a) Find the component of the velocity of B parallel to the line of centres after the collision, giving your answer in terms of u and α . [4]



(b) Find the value of $\tan \alpha$. [5]



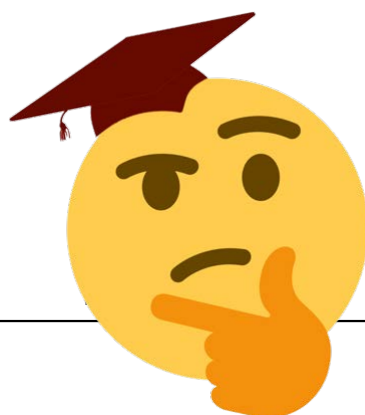
36 – (9231/31_Summer_2023_Q2) – Momentum And Impulse



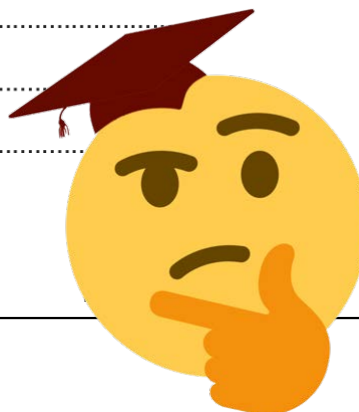
A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. It collides at an angle α with a fixed smooth vertical barrier. After the collision, P moves at an angle θ with the barrier, where $\tan \theta = \frac{1}{2}$ (see diagram). The coefficient of restitution between P and the barrier is e . The particle P loses 20% of its kinetic energy as a result of the collision.

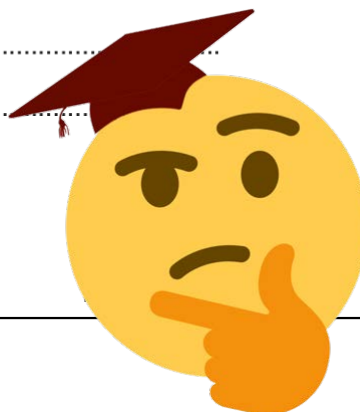
Find the value of e .

[5]



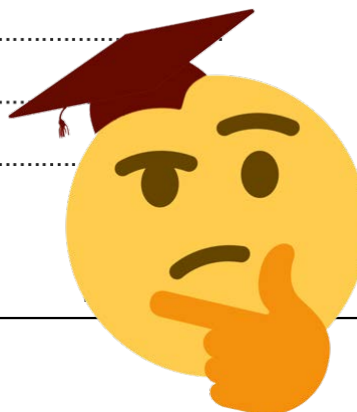
(a) Find the value of e . [5]

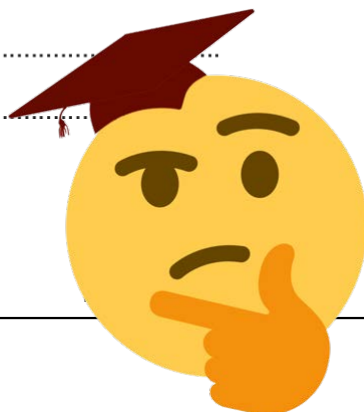
This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small, stylized illustration of a red graduation cap (mortarboard) with a tassel, resting on a yellow cloud-like shape. The rest of the page is empty space for writing.

[illegible]

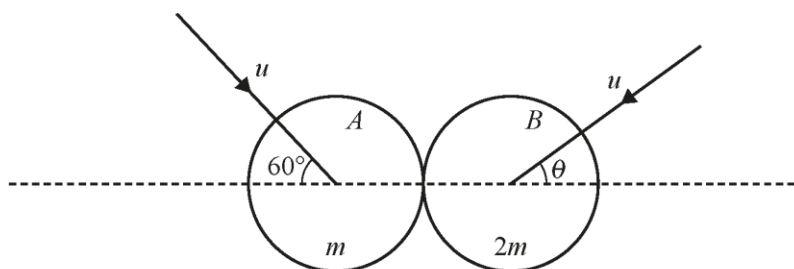
The points O and P are on a horizontal plane, a distance 8 m apart. A ball is thrown from O with speed $u \text{ ms}^{-1}$ at an angle θ above the horizontal, where $\tan \theta = \frac{4}{3}$. At the same instant, a model aircraft is launched with speed 5 ms^{-1} parallel to the horizontal plane from a point 4 m vertically above P . The model aircraft moves in the same vertical plane as the ball and in the same horizontal direction as the ball. The model aircraft moves horizontally with a constant speed of 5 ms^{-1} . After T s, the ball and the model aircraft collide.

- [illegible]



[illegible]

39 – (9231/31_Winter_2023_Q1) – Momentum And Impulse



Two uniform smooth spheres A and B of equal radii have masses m and $2m$ respectively. The two spheres are moving with equal speeds u on a smooth horizontal surface when they collide. Immediately before the collision, A 's direction of motion makes an angle of 60° with the line of centres, and B 's direction of motion makes an angle θ with the line of centres (see diagram). The coefficient of restitution between the spheres is e .

After the collision, the component of the velocity of A along the line of centres is v and B moves perpendicular to the line of centres. Sphere A now has twice as much kinetic energy as sphere B .

- (a) Show that $v = \frac{1}{2}u(4 \cos \theta - 1)$. [1]

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- (b) Find the value of $\cos \theta$. [4]

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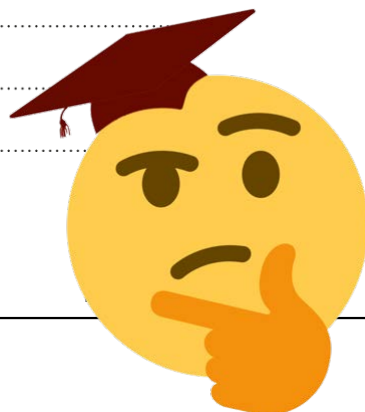
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(c) Find the value of e .

[2]

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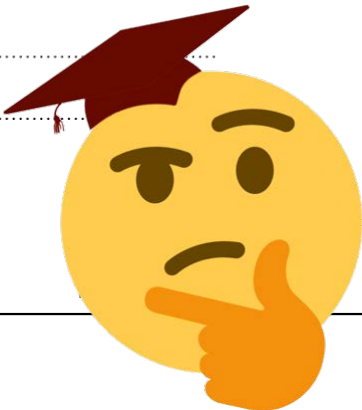
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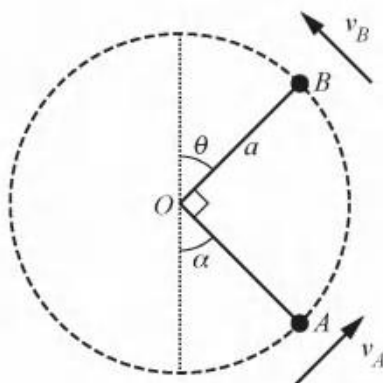
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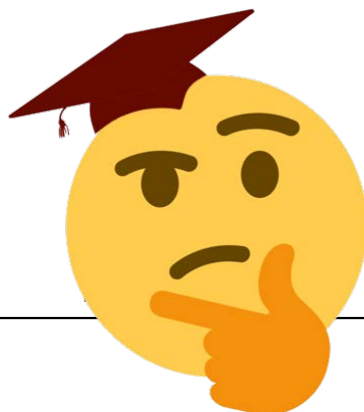


40 – (9231/32_Winter_2023_Q5) – Momentum And Impulse

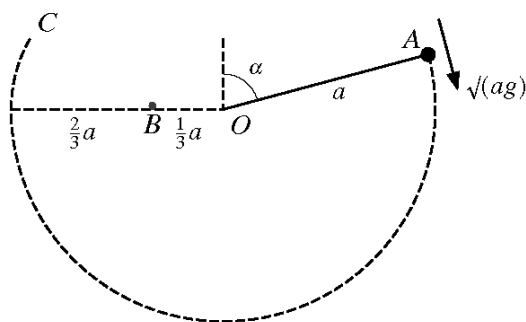


A bead of mass m moves on a smooth circular wire, with centre O and radius a , in a vertical plane. The bead has speed v_A when it is at the point A where OA makes an angle α with the downward vertical through O , and $\cos \alpha = \frac{3}{5}$. Subsequently the bead has speed v_B at the point B , where OB makes an angle θ with the upward vertical through O . Angle AOB is a right angle (see diagram). The reaction of the wire on the bead at B is in the direction OB and has magnitude equal to $\frac{1}{6}$ of the magnitude of the reaction when the bead is at A .

- (a) Find, in terms of m and g , the magnitude of the reaction at B . [6]
- (b) Given that $v_A = \sqrt{kag}$, find the value of k . [2]



1 - (9231/21_Summer_2017_Q5) - Circular Motion



A particle of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The point A is such that $OA = a$ and OA makes an angle α with the upward vertical through O . The particle is held at A and then projected downwards with speed $\sqrt{ag}$ so that it begins to move in a vertical circle with centre O . There is a small smooth peg at the point B which is at the same horizontal level as O and at a distance $\frac{1}{3}a$ from O on the opposite side of O to A (see diagram).

- (i) Show that, when the string first makes contact with the peg, the speed of the particle is $\sqrt{ag(1 + 2 \cos \alpha)}$. [2]

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The particle now begins to move in a vertical circle with centre B . When the particle is at the point C where angle $CBO = 150^\circ$, the tension in the string is the same as it was when the particle was at the point A .

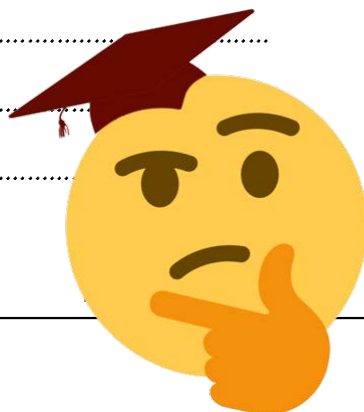
- (ii) Find the value of $\cos \alpha$. [10]

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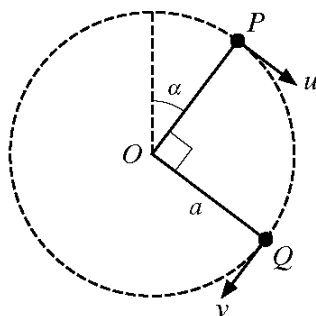
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2 - (9231/23_Summer_2017_Q5) - Circular Motion



A particle of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is moving in complete vertical circles with the string taut. When the particle is at the point P , where OP makes an angle α with the upward vertical through O , its speed is u . When the particle is at the point Q , where angle $QOP = 90^\circ$, its speed is v (see diagram). It is given that $\cos \alpha = \frac{4}{5}$.

- (i) Show that $v^2 = u^2 + \frac{14}{5}ag$. [2]

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The tension in the string when the particle is at Q is twice the tension in the string when the particle is at P .

- (ii) Obtain another equation relating u^2 , v^2 , a and g , and hence find u in terms of a and g . [5]

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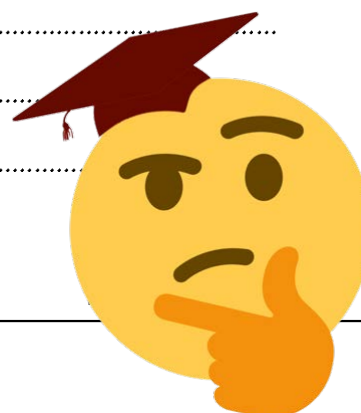
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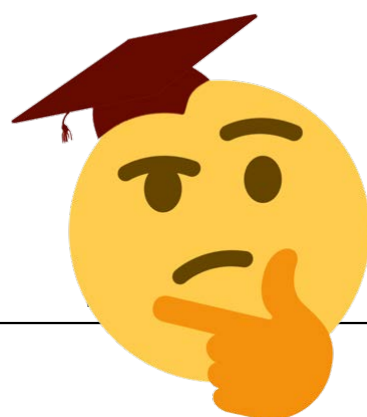
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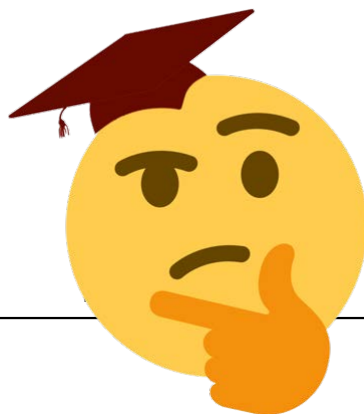
(iii) Find the least tension in the string during the motion.

[3]



3 - (9231/21_Winter_2017_Q1) - Circular Motion

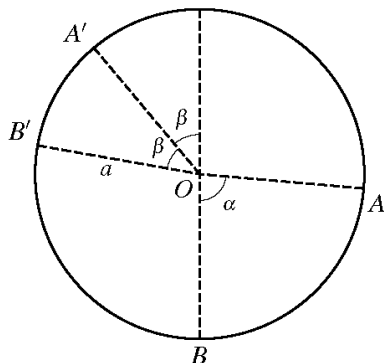
A particle P is moving in a circle of radius 0.8 m. At time t s its velocity is $(8 - pt + t^2) \text{ m s}^{-1}$, where p is a constant. The magnitude of the transverse component of the acceleration of P when $t = 2$ is zero. Find the magnitude of the radial component of the acceleration of P when $t = 2$. [4]



4 - (9231/21_Winter_2017_Q11) - Circular Motion, Bivariate Data

Answer only **one** of the following two alternatives.

EITHER

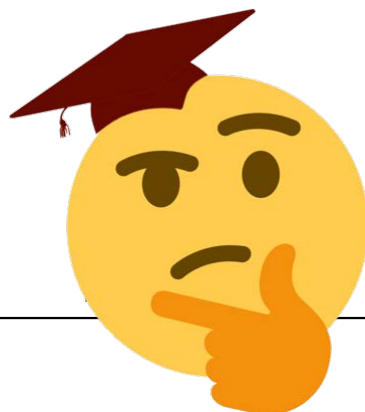


A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a . The centre of the sphere is O . The points A and A' are on the inner surface of the sphere, on opposite sides of the vertical through O ; the radius OA makes an angle α with the downward vertical and the radius OA' makes an angle β with the upward vertical. The point B is on the inner surface of the sphere, vertically below O . The point B' is on the inner surface of the sphere and such that OB' makes an angle 2β with the upward vertical through O (see diagram). It is given that $\cos \alpha = \frac{1}{16}$.

- (i) P is projected from A with speed u along the surface of the sphere downwards towards B . Subsequently it loses contact with the sphere at A' . Show that $u^2 = \frac{1}{8}ag(1 + 24 \cos \beta)$. [5]

- (ii) P is now projected from B with speed u along the surface of the sphere towards B' . Subsequently it loses contact with the sphere at B' . Find $\cos \beta$. [6]

- (iii) In part (i), the reaction of the sphere on P when it is initially projected at A is R . Find R in terms of m and g . [3]



OR

A large number of people attended a course to improve the speed of their logical thinking. The times taken to complete a particular type of logic puzzle at the beginning of the course and at the end of the course are recorded for each person. The time taken, in minutes, at the beginning of the course is denoted by x and the time taken, in minutes, at the end of the course is denoted by y . For a random sample of 9 people, the results are summarised as follows.

$$\Sigma x = 45.3 \quad \Sigma x^2 = 245.59 \quad \Sigma y = 40.5 \quad \Sigma y^2 = 195.11 \quad \Sigma xy = 218.72$$

Ken attended the course, but his time to complete the puzzle at the beginning of the course was not recorded. His time to complete the puzzle at the end of the course was 4.2 minutes.

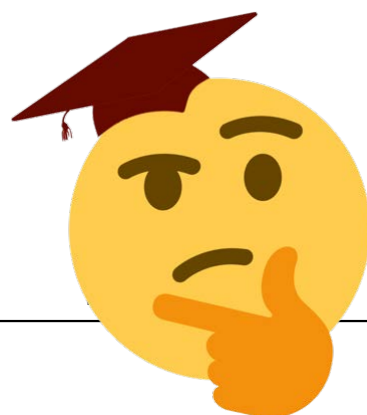
- (i) By finding, showing all necessary working, the equation of a suitable regression line, find an estimate for the time that Ken would have taken to complete the puzzle at the beginning of the course. [5]

The values of $x - y$ for the sample of 9 people are as follows.

$$0.2 \quad 0.8 \quad 0.5 \quad 1.0 \quad 0.2 \quad 0.6 \quad 0.2 \quad 0.5 \quad 0.8$$

The organiser of the course believes that, on average, the time taken to complete the puzzle decreases between the beginning and the end of the course by more than 0.3 minutes.

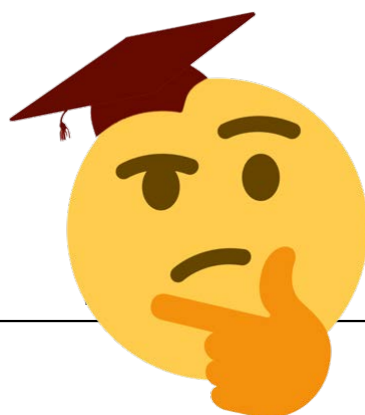
- (ii) Stating suitable hypotheses and assuming a normal distribution, test the organiser's belief at the $2\frac{1}{2}\%$ significance level. [9]



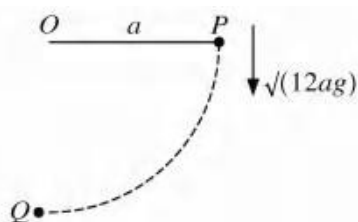
A particle P is moving in a fixed circle of radius 0.8 m . At time $t \text{ s}$ its velocity is $(t^2 - t + 2) \text{ m s}^{-1}$. Find the magnitudes of the radial and the transverse components of the acceleration of P when $t = 2$.
[3]

Radial component

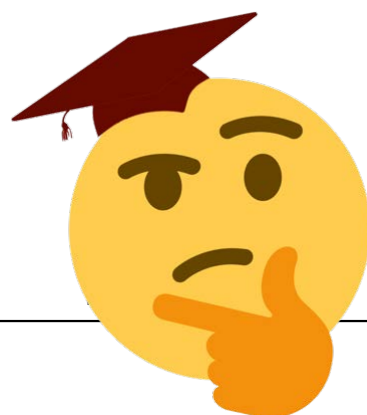
Transverse component



6 - (9231/23_Summer_2018_Q5) - *Momentum And Impulse, Circular Motion*



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is held with the string taut and horizontal. It is projected downwards with speed $\sqrt{12ag}$. At the lowest point of its motion, P collides directly with a particle Q of mass km which is at rest (see diagram). In the collision, P and Q coalesce. The tension in the string immediately after the collision is half of its value immediately before the collision. Find the possible values of k . [11]



7 - (9231/21_Summer_2018_Q11) - Circular Motion, Bivariate Data

Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is held so that the string is taut, with OP horizontal. The particle is projected downwards with speed $\sqrt{\left(\frac{2}{5}ag\right)}$ and begins to move in a vertical circle. The string breaks when its tension is equal to $\frac{11}{5}mg$.

- (i) Show that the string breaks when OP makes an angle θ with the downward vertical through O , where $\cos \theta = \frac{3}{5}$. Find the speed of P at this instant. [6]
- (ii) For the subsequent motion after the string breaks, find the distance OP when the particle P is vertically below O . [6]

OR

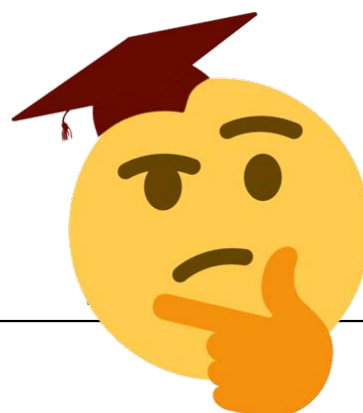
The regression line of y on x , obtained from a random sample of 6 pairs of values of x and y , has equation

$$y = 0.25x + k,$$

where k is a constant. The values from the sample are shown in the following table.

x	4	5	7	8	10	14
y	5	8	p	7	p	9

- (i) Find the value of p and the value of k . [6]



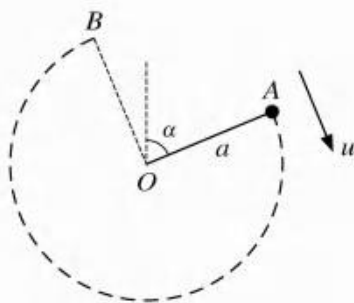
[2]

[illegible]

(iii) Test, at the 5% significance level, whether there is evidence of positive correlation between the variables. [4]



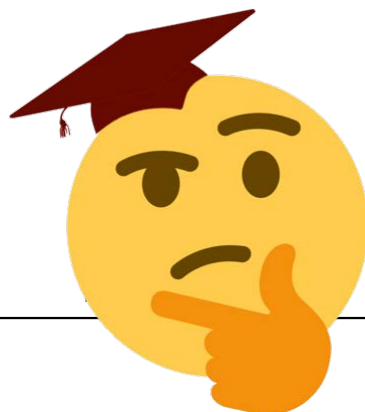
8 - (9231/22_Winter_2018_Q3) - Circular Motion



A particle of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The point A is such that $OA = a$ and OA makes an angle α with the upward vertical, where $\tan \alpha = \frac{12}{5}$. The particle is projected downwards from A with speed u perpendicular to the string and moves in a vertical plane (see diagram). The string becomes slack after the string has rotated through 270° from its initial position, with the particle now at the point B .

(i) Show that $u^2 = 2ag$. [5]

(ii) Find the maximum tension in the string as the particle moves from A to B . [4]



9 - (9231/21_Winter_2018_Q11) - Circular Motion, X2 Test

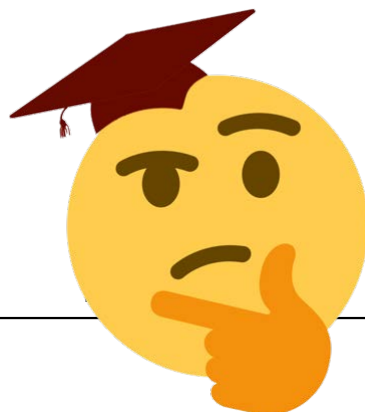
Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a . The centre of the sphere is O and the point C is on the inner surface of the sphere, vertically below O . The points A and B on the inner surface of the sphere are the ends of a diameter of the sphere. The diameter AOB makes an acute angle α with the vertical, where $\cos \alpha = \frac{4}{5}$, with A below the horizontal level of B . The particle is projected from A with speed u , and moves along the inner surface of the sphere towards C . The normal reaction forces on the particle at A and C are in the ratio 8 : 9.

(i) Show that $u^2 = 4ag$. [6]

(ii) Determine whether P reaches B without losing contact with the inner surface of the sphere. [6]



OR

A machine is used to produce metal rods. When the machine is working efficiently, the lengths, x cm, of the rods have a normal distribution with mean 150 cm and standard deviation 1.2 cm. The machine is checked regularly by taking random samples of 200 rods. The latest results are shown in the following table.

Interval	$146 \leq x < 147$	$147 \leq x < 148$	$148 \leq x < 149$	$149 \leq x < 150$
Observed frequency	1	2	23	52

$150 \leq x < 151$	$151 \leq x < 152$	$152 \leq x < 153$	$153 \leq x < 154$
69	36	15	2

As a first check, the sample is used to calculate an estimate for the mean.

(i) Show that an estimate for the mean from this sample is close to 150 cm. [2]

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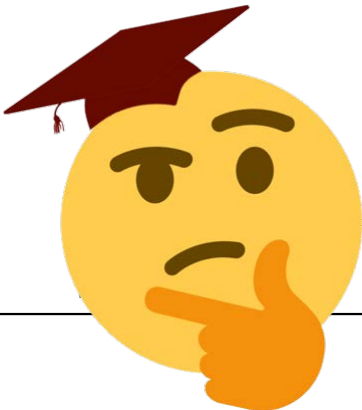
As a second check, the results are tested for goodness of fit of the normal distribution with mean 150 cm and standard deviation 1.2 cm. The relevant expected frequencies, found using the normal distribution function given in the List of Formulae (MF10), are shown in the following table.

Interval	$x < 147$	$147 \leq x < 148$	$148 \leq x < 149$	$149 \leq x < 150$
Observed frequency	1	2	23	52
Expected frequency	1.24	8.32	30.94	59.50

$150 \leq x < 151$	$151 \leq x < 152$	$152 \leq x < 153$	$153 \leq x$
69	36	15	2
59.50	30.94	8.32	1.24

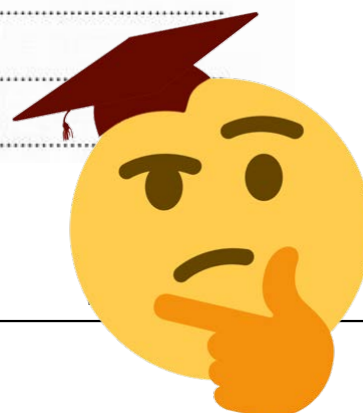
(ii) Show how the expected frequency for $151 \leq x < 152$ is obtained. [3]

(iii) Test, at the 5% significance level, the goodness of fit of the normal distribution to the results. [7]



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is moving in a complete vertical circle about O . The points A and B are on the circle, at opposite ends of a diameter, and such that OA makes an acute angle α with the upward vertical through O . The speed of P as it passes through A is $\frac{3}{2}\sqrt{ag}$. The tension in the string when P is at B is four times the tension in the string when P is at A .

[6]

This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small red graduation cap icon with a yellow tassel hanging from it. The paper appears to be a template for writing or drawing.

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(ii) Find the tension in the string when P is at B . [2]

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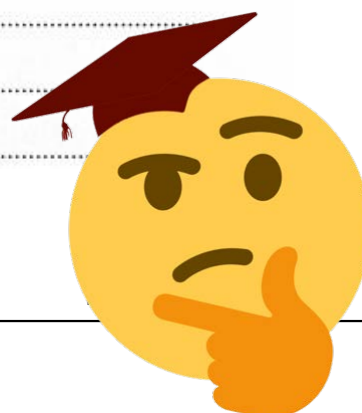
12 - (9231/21_Summer_2019_Q11) - *Momentum And Impulse, Circular Motion, Inference Using Normal And T-distributions*

Answer only **one** of the following two alternatives.

EITHER

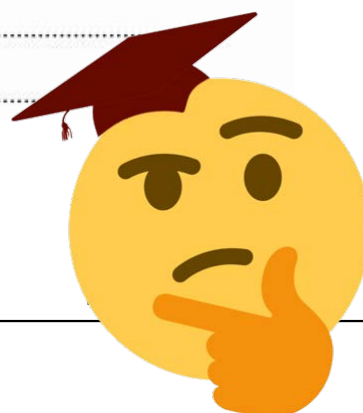
A particle P , of mass m , is able to move in a vertical circle on the smooth inner surface of a sphere with centre O and radius a . Points A and B are on the inner surface of the sphere and AOB is a horizontal diameter. Initially, P is projected vertically downwards with speed $\sqrt{\frac{21}{2}ag}$ from A and begins to move in a vertical circle. At the lowest point of its path, vertically below O , the particle P collides with a stationary particle Q , of mass $4m$, and rebounds. The speed acquired by Q , as a result of the collision, is just sufficient for it to reach the point B .

- (i) Find the speed of P and the speed of Q immediately after their collision. [7]

[illegible]

(ii) Find $\cos \theta$.

[5]

[illegible]

OR

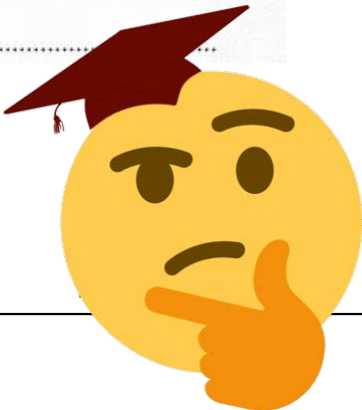
A farmer grows two different types of cherries, Type *A* and Type *B*. He assumes that the masses of each type are normally distributed. He chooses a random sample of 8 cherries of Type *A*. He finds that the sample mean mass is 15.1 g and that a 95% confidence interval for the population mean mass, μ g, is $13.5 \leq \mu \leq 16.7$.

(i) Find an unbiased estimate for the population variance of the masses of cherries of Type *A*. [3]

The farmer now chooses a random sample of 6 cherries of Type *B* and records their masses as follows.

12.2 13.3 16.4 14.0 13.9 15.4

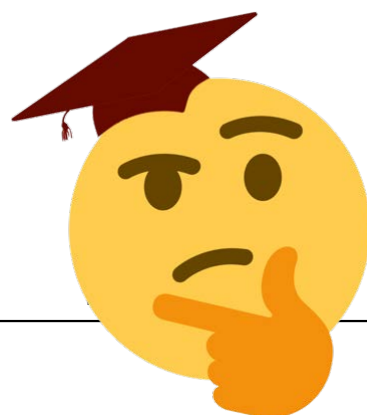
(ii) Test at the 5% significance level whether the mean mass of cherries of Type *B* is less than the mean mass of cherries of Type *A*. You should assume that the population variances for the two types of cherry are equal. [9]



[illegible]

13 - (9231/21_Winter_2019_Q1) - *Circular Motion*

A particle P is moving in a circle of radius 2 m. At time t seconds, its velocity is $(t-1)^2 \text{ m s}^{-1}$. At a particular time T seconds, where $T > 0$, the magnitude of the radial component of the acceleration of P is 8 m s^{-2} . Find the magnitude of the transverse component of the acceleration of P at this instant. [5]

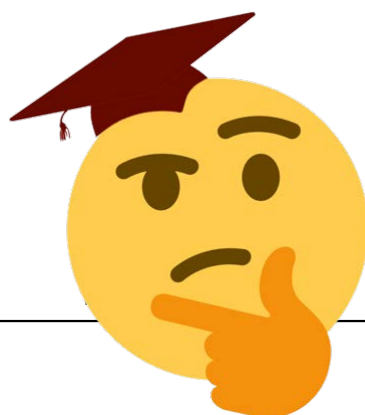


14 - (9231/21_Winter_2019_Q4) - *Circular Motion*

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O and P is held with the string taut and horizontal. The particle P is projected vertically downwards with speed $\sqrt{2ag}$ so that it begins to move along a circular path. The string becomes slack when OP makes an angle θ with the upward vertical through O .

(i) Show that $\cos \theta = \frac{2}{3}$. [5]

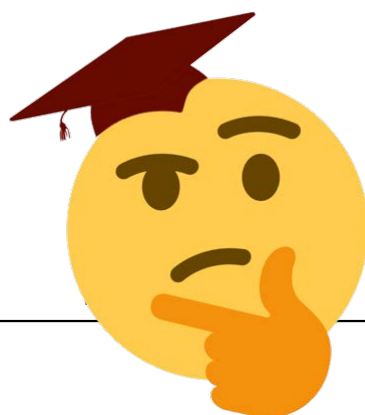
(ii) Find the greatest height, above the horizontal through O , reached by P in its subsequent motion. [4]



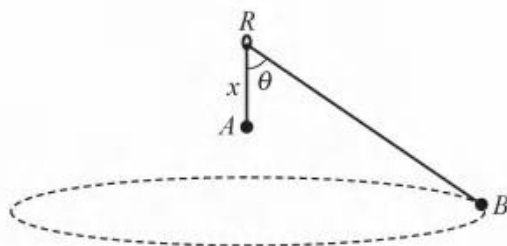
15 – (9231/33_Summer_2020_Q1) – *Circular Motion*

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O on a smooth horizontal plane. The particle P moves in horizontal circles about O . The tension in the string is $4mg$.

Find, in terms of a and g , the time that P takes to make one complete revolution. [2]



16 – (9231/31_Summer_2020_Q2) – Circular Motion



A light inextensible string of length a is threaded through a fixed smooth ring R . One end of the string is attached to a particle A of mass $3m$. The other end of the string is attached to a particle B of mass m . The particle A hangs in equilibrium at a distance x vertically below the ring. The angle between AR and BR is θ (see diagram). The particle B moves in a horizontal circle with constant angular speed $2\sqrt{\frac{g}{a}}$.

Show that $\cos \theta = \frac{1}{3}$ and find x in terms of a .

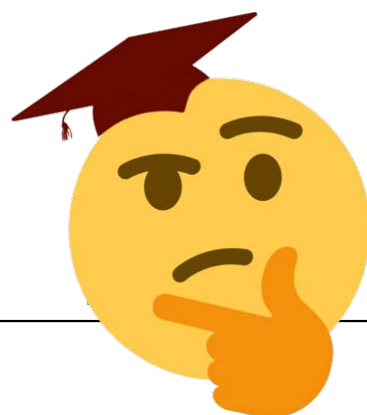
[5]



17 – (9231/33_Summer_2020_Q3) – *Circular Motion*

A particle Q of mass m is attached to a fixed point O by a light inextensible string of length a . The particle moves in complete vertical circles about O . The points A and B are on the path of Q with AB a diameter of the circle. OA makes an angle of 60° with the downward vertical through O and OB makes an angle of 60° with the upward vertical through O . The speed of Q when it is at A is $2\sqrt{ag}$.

Given that T_A and T_B are the tensions in the string at A and B respectively, find the ratio $T_A:T_B$. [6]



18 – (9231/31_Summer_2020_Q7) – *Circular Motion*

A hollow cylinder of radius a is fixed with its axis horizontal. A particle P , of mass m , moves in part of a vertical circle of radius a and centre O on the smooth inner surface of the cylinder. The speed of P when it is at the lowest point A of its motion is $\sqrt{\frac{7}{2}ga}$.

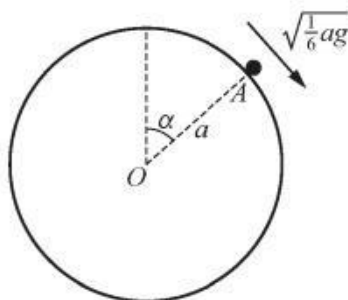
The particle P loses contact with the surface of the cylinder when OP makes an angle θ with the upward vertical through O .

(a) Show that $\theta = 60^\circ$. [5]

(b) Show that in its subsequent motion P strikes the cylinder at the point A . [5]



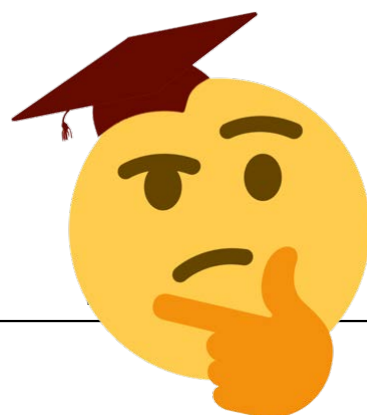
19 - (9231/32_Winter_2020_Q1) - Circular Motion



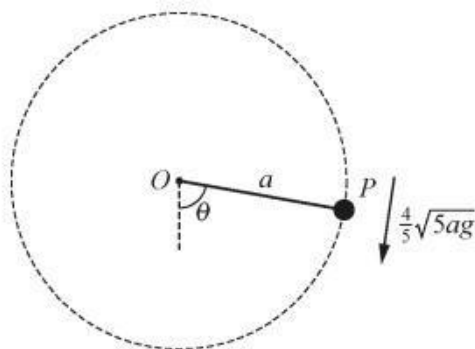
A fixed smooth solid sphere has centre O and radius a . A particle of mass m is projected downwards with speed $\sqrt{\frac{1}{6}ag}$ from the point A on the surface of the sphere, where OA makes an angle α with the upward vertical through O (see diagram). The particle moves in part of a vertical circle on the surface of the sphere. It loses contact with the sphere at the point B , where OB makes an angle β with the upward vertical through O .

Given that $\cos \alpha = \frac{2}{3}$, find the value of $\cos \beta$.

[5]



20 - (9231/31_Winter_2020_Q2) - Circular Motion



A particle P is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is held with the string taut and making an angle θ with the downward vertical. The particle P is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

Find the value of $\cos \theta$.

[5]



21 – (9231/31_Winter_2020_Q3) – *Circular Motion*

One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O . The other end of the string is attached to a particle of mass m . The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O . The length of the string during this motion is $(k+1)a$.

- (a) Find the value of k . [4]

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- (b) Find the value of $\cos \theta$. [2]

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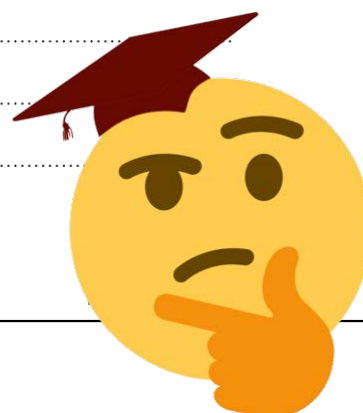
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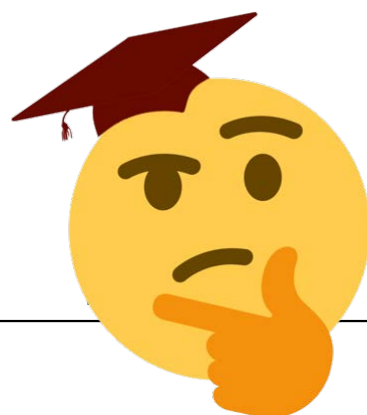


22 – (9231/31_Summer_2021_Q2) – Circular Motion

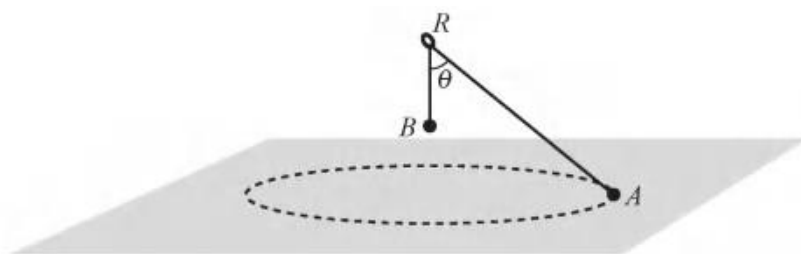
A hollow hemispherical bowl of radius a has a smooth inner surface and is fixed with its axis vertical. A particle P of mass m moves in horizontal circles on the inner surface of the bowl, at a height x above the lowest point of the bowl. The speed of P is $\sqrt{\frac{8}{3}ga}$.

Find x in terms of a .

[6]

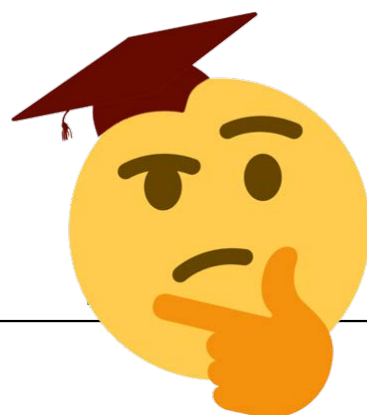


23 – (9231/33_Summer_2021_Q3) – Circular Motion

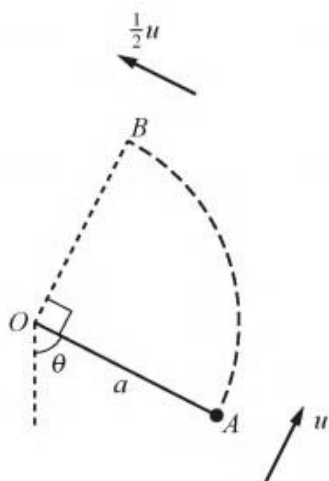


Particles A and B , of masses $3m$ and m respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R . Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles on a smooth horizontal surface with speed $\frac{2}{5}\sqrt{ga}$. The angle between AR and BR is θ (see diagram). The normal reaction between A and the surface is $\frac{12}{5}mg$.

- (a) Find $\cos \theta$. [3]
- (b) Find, in terms of a , the distance of B below the ring. [3]



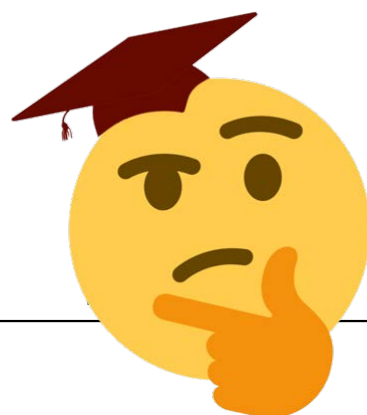
24 – (9231/33_Summer_2021_Q4) – Circular Motion



A particle of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle is initially held with the string taut at the point A , where OA makes an angle θ with the downward vertical through O . The particle is then projected with speed u perpendicular to OA and begins to move upwards in part of a vertical circle. The string goes slack when the particle is at the point B where angle AOB is a right angle. The speed of the particle when it is at B is $\frac{1}{2}u$ (see diagram).

Find the tension in the string at A , giving your answer in terms of m and g .

[8]



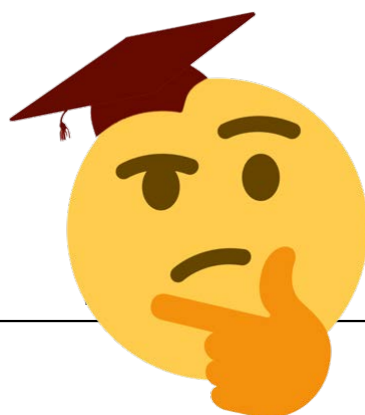
25 – (9231/31_Summer_2021_Q5) – Circular Motion

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle completes vertical circles with centre O . The points A and B are on the path of P , both on the same side of the vertical through O . OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O .

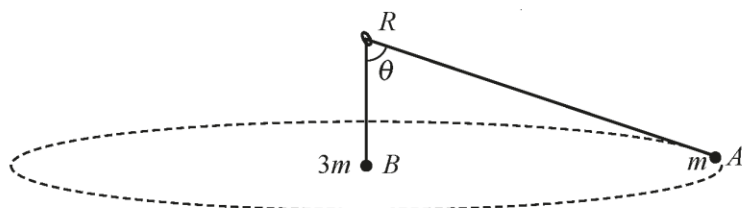
The speed of P when it is at A is u and the speed of P when it is at B is $\sqrt{ag}$. The tensions in the string at A and B are T_A and T_B respectively. It is given that $T_A = 7T_B$.

Find the value of θ and find an expression for u in terms of a and g .

[8]



26 – (9231/32_Winter_2021_Q3) – Circular Motion



Particles A and B , of masses m and $3m$ respectively, are connected by a light inextensible string of length a that passes through a fixed smooth ring R . Particle B hangs in equilibrium vertically below the ring. Particle A moves in horizontal circles with speed v . Particles A and B are at the same horizontal level. The angle between AR and BR is θ (see diagram).

- (a) Show that $\cos \theta = \frac{1}{3}$. [2]

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- (b) Find an expression for v in terms of a and g . [4]

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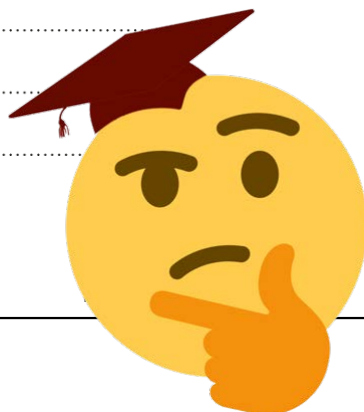
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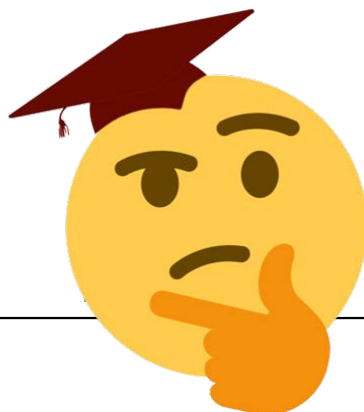


27 – (9231/31_Winter_2021_Q6) – Circular Motion

A particle P , of mass m , is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P moves in complete vertical circles about O with the string taut. The points A and B are on the path of P with AB a diameter of the circle. OA makes an angle θ with the downward vertical through O and OB makes an angle θ with the upward vertical through O . The speed of P when it is at A is $\sqrt{5ag}$.

The ratio of the tension in the string when P is at A to the tension in the string when P is at B is $9 : 5$.

- (a) Find the value of $\cos \theta$. [6]
- (b) Find, in terms of a and g , the greatest speed of P during its motion. [2]

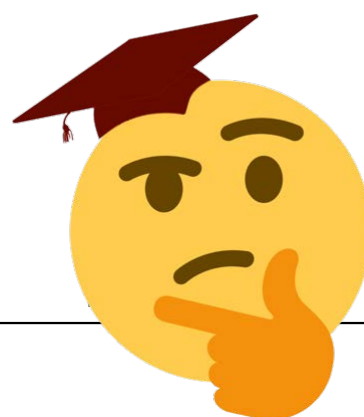


28 – (9231/31_Summer_2022_Q2) – Circular Motion

One end of a light inextensible string of length a is attached to a fixed point O . A particle of mass m is attached to the other end of the string. The particle is held at the point A with the string taut. The angle between OA and the downward vertical is equal to α , where $\cos \alpha = \frac{4}{5}$. The particle is projected from A , perpendicular to the string in an upwards direction, with a speed $\sqrt{3ga}$. It then moves along a circular path in a vertical plane. The string first goes slack when it makes an angle θ with the upward vertical through O .

Find the value of $\cos \theta$.

[5]



29 – (9231/33_Summer_2022_Q4) – Circular Motion

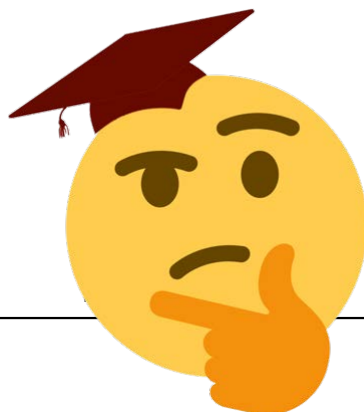
One end of a light inextensible string of length a is attached to a fixed point O . A particle of mass m is attached to the other end of the string and is held with the string taut at the point A . At A the string makes an angle θ with the upward vertical through O . The particle is projected perpendicular to the string in a downward direction from A with a speed u . It moves along a circular path in the vertical plane.

When the string makes an angle α with the downward vertical through O , the speed of the particle is $2u$ and the magnitude of the tension in the string is 10 times its magnitude at A .

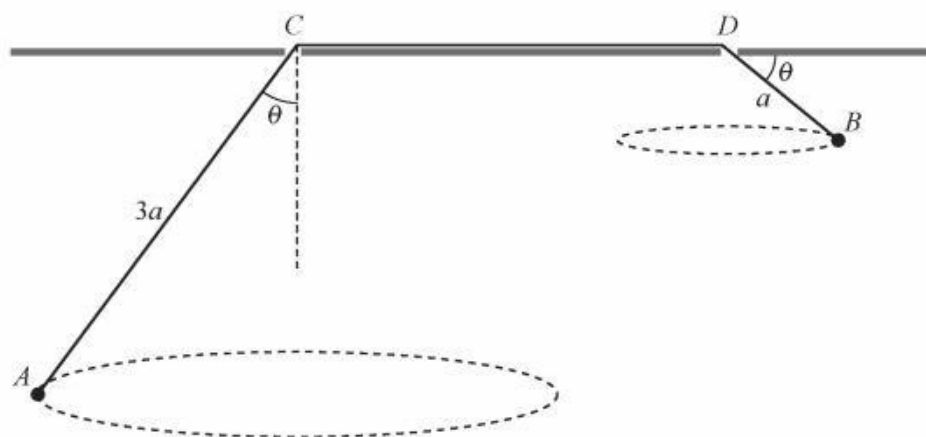
It is given that $u = \sqrt{\frac{1}{3}ga}$.

(a) Find, in terms of m and g , the magnitude of the tension in the string at A . [6]

(b) Find the value of $\cos \alpha$. [2]



30 - (9231/31_Summer_2022_Q5) - Circular Motion



A light inextensible string AB passes through two small holes C and D in a smooth horizontal table where $AC = 3a$ and $DB = a$. A particle of mass m is attached at the end A and moves in a horizontal circle with angular velocity ω . A particle of mass $\frac{3}{4}m$ is attached to the end B and moves in a horizontal circle with angular velocity $k\omega$. AC makes an angle θ with the downward vertical and DB makes an angle θ with the horizontal (see diagram).

Find the value of k .

[7]

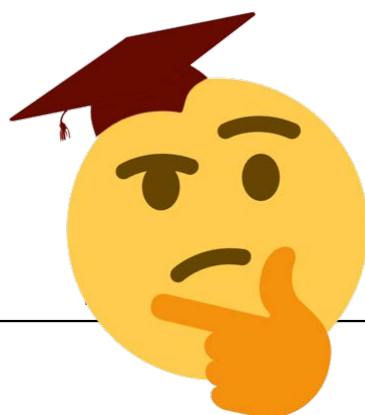


31 - (9231/31_Winter_2022_Q1) - *Circular Motion*

A particle of mass 2 kg is attached to one end of a light inextensible string of length 0.6 m. The other end of the string is attached to a fixed point on a smooth horizontal surface. The particle is moving in a circular path on the surface. The tension in the string is 20 N.

Find how many revolutions the particle makes per minute.

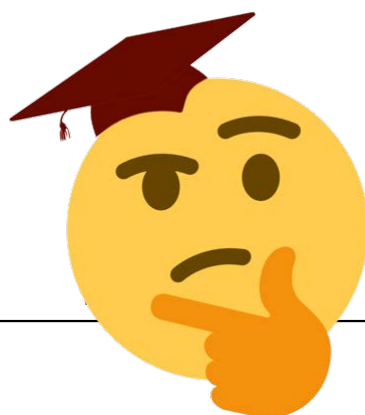
[3]



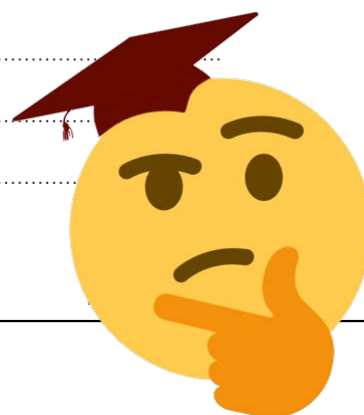
32 - (9231/32_Winter_2022_Q1) - *Circular Motion*

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The string is held taut with OP making an angle α with the downward vertical, where $\cos \alpha = \frac{2}{3}$. The particle P is projected perpendicular to OP in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane.

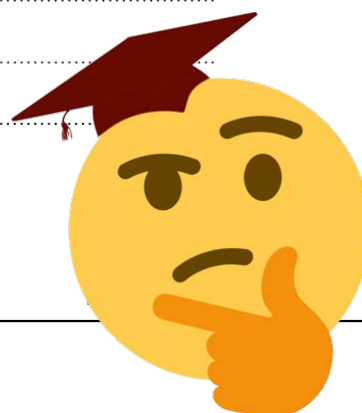
Find the cosine of the angle between the string and the upward vertical when the string first becomes slack. [4]



A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The string is held taut with OP horizontal. The particle P is projected vertically downwards with speed $\sqrt{\frac{1}{3}ag}$ and starts to move in a vertical circle. P passes through the lowest point of the circle and reaches the point Q where OQ makes an angle θ with the downward vertical. At Q the speed of P is $\sqrt{kag}$ and the tension in the string is $\frac{11}{6}mg$.



(b) In the subsequent motion, find the greatest height reached by P above the level of the lowest point of the circle. [4]



34 – (9231/31_Summer_2023_Q3) – Circular Motion, Further Work On Distributions

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is held at the point A , where OA makes an angle θ with the downward vertical through O , and with the string taut. The particle P is projected perpendicular to OA in an upwards direction with speed u . It then starts to move along a circular path in a vertical plane. The string goes slack when P is at B , where angle AOB is 90° and the speed of P is $\sqrt{\frac{4}{5}ag}$.

- (a) Find the value of $\sin \theta$. [2]

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- (b) Find, in terms of m and g , the tension in the string when P is at A . [5]

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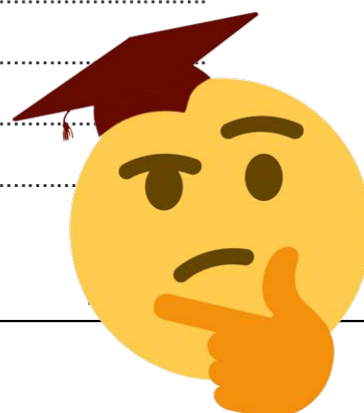
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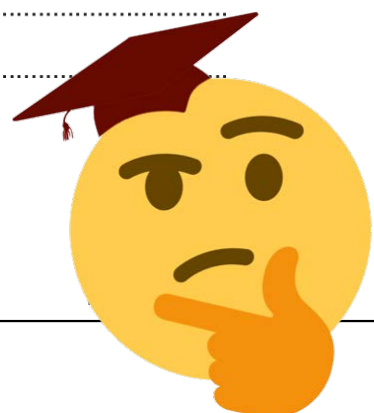
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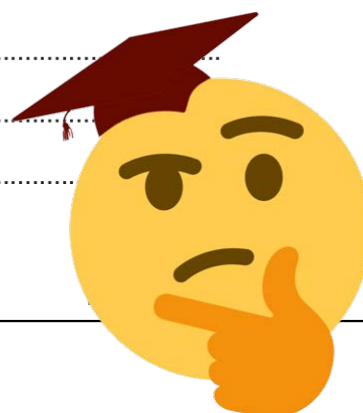
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[illegible]

A light elastic string of natural length a and modulus of elasticity λmg has one end attached to a fixed point O on a smooth horizontal surface. When a particle of mass m is attached to the free end of the string, it moves with speed v in a horizontal circle with centre O and radius x . When, instead, a particle of mass $2m$ is attached to the free end of the string, this particle moves with speed $\frac{1}{2}v$ in a horizontal circle with centre O and radius $\frac{3}{4}x$.

(a) Find x in terms of a . [5]

[illegible]

[2]

This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small, stylized illustration of a red graduation cap (mortarboard) with a yellow tassel hanging from it. The rest of the page is empty.

36 – (9231/31_Summer_2023_Q7) – Circular Motion, Further Work On Distributions

At time t s, a particle P is projected with speed 40 m s^{-1} at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The greatest height achieved by P during its flight is H m and the corresponding time is T s.

(a) Obtain expressions for H and T in terms of θ . [2]

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During the time between $t = T$ and $t = 3$, P descends a distance $\frac{1}{4}H$.

(b) Find the value of θ . [4]

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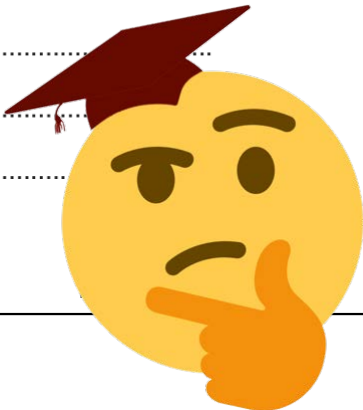
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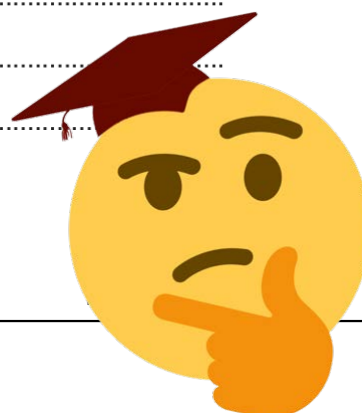
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[3]

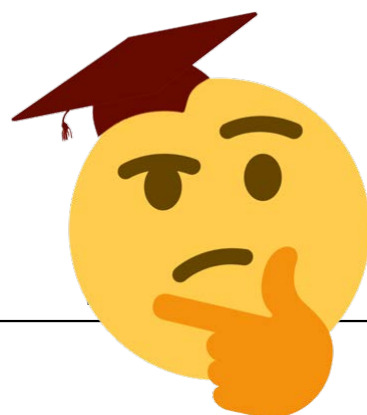
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37 - (9231/33_Summer_2023_Q1) - Circular Motion, Simple Harmonic Motion, Hook's Law

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is held at the point A , where OA makes an angle α with the downward vertical through O , and with the string taut. The particle P is projected perpendicular to OA in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane. The string goes slack when P is at B , where OB makes an angle θ with the upward vertical.

Given that $\cos \alpha = \frac{4}{5}$, find the value of $\cos \theta$.

[4]

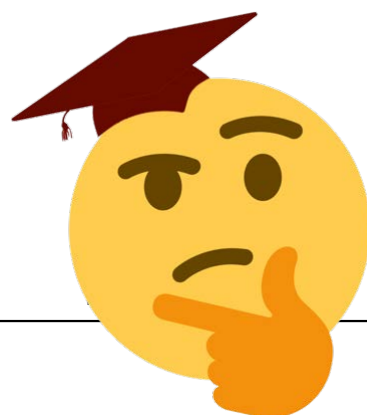


38 – (9231/33_Summer_2023_Q2) – Circular Motion, Further Work On Distributions

One end of a light elastic string, of natural length a and modulus of elasticity λmg , is attached to a fixed point O . The string lies on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected in the direction OP . When the length of the string is $\frac{4}{3}a$, the speed of P is $\sqrt{2ag}$. When the length of the string is $\frac{5}{3}a$, the speed of P is $\frac{1}{2}\sqrt{2ag}$.

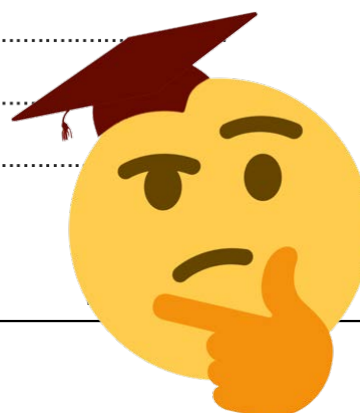
Find the value of λ .

[4]

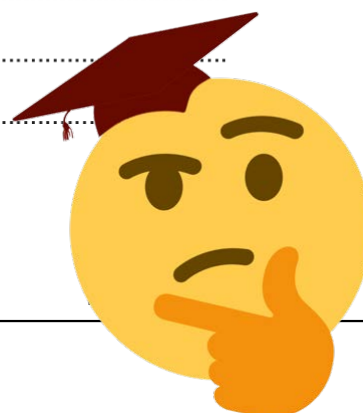


One end of a light elastic string, of natural length $12a$ and modulus of elasticity kmg , is attached to a fixed point O . The other end of the string is attached to a particle of mass m . The particle moves with constant speed $\frac{3}{2}\sqrt{3ag}$ in a horizontal circle with centre at a distance $12a$ below O . The string is inclined at an angle θ to the downward vertical through O .

(a) Find, in terms of a , the extension of the string. [5]

This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small, stylized illustration of a red graduation cap (mortarboard) with a tassel, resting on a yellow circular base. The rest of the page is empty.

[3]

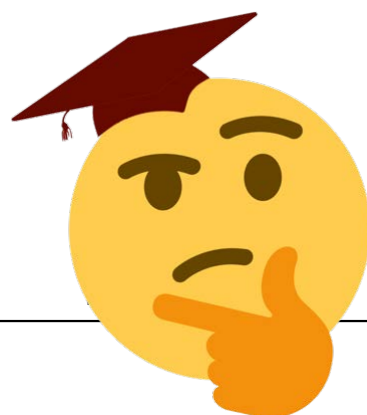
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40 – (9231/32_Winter_2023_Q1) – Circular Motion

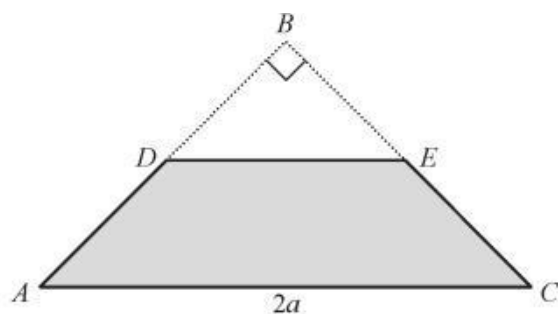
One end of a light inextensible string of length a is attached to a fixed point O . The other end of the string is attached to a particle of mass m . The string is taut and makes an angle θ with the downward vertical through O , where $\cos \theta = \frac{2}{3}$. The particle moves in a horizontal circle with speed v .

Find v in terms of a and g .

[4]



41 – (9231/32_Winter_2023_Q3) – Circular Motion

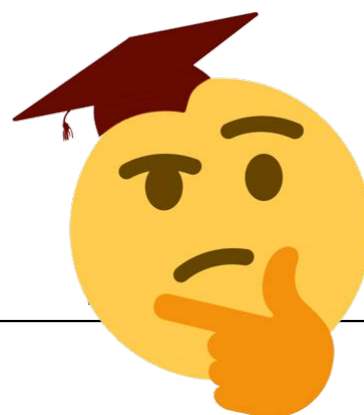


A uniform lamina is in the form of an isosceles triangle ABC in which $AC = 2a$ and angle $ABC = 90^\circ$. The point D on AB is such that the ratio $DB:AB = 1:k$. The point E on CB is such that DE is parallel to AC . The triangle DBE is removed from the lamina (see diagram).

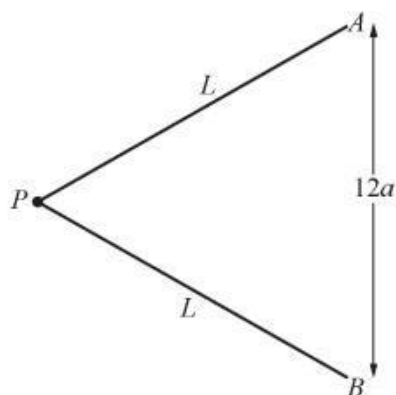
- (a) Find, in terms of k , the distance of the centre of mass of the remaining lamina $ADEC$ from the midpoint of AC . [4]

When the lamina $ADEC$ is freely suspended from the vertex A , the edge AC makes an angle θ with the downward vertical, where $\tan \theta = \frac{5}{18}$.

- (b) Find the value of k . [3]

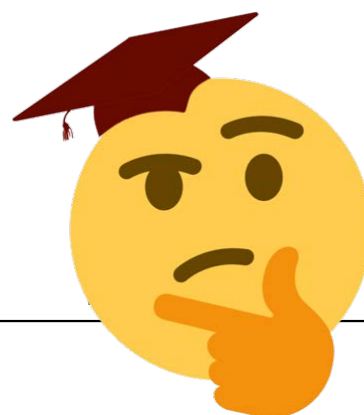


42 – (9231/33_Winter_2023_Q4) – Circular Motion

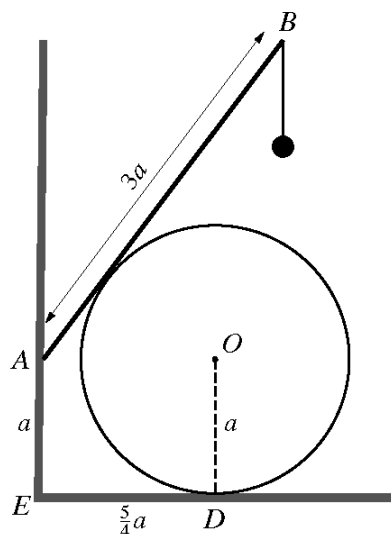


A light elastic string has natural length $8a$ and modulus of elasticity $5mg$. A particle P of mass m is attached to the midpoint of the string. The ends of the string are attached to points A and B which are a distance $12a$ apart on a smooth horizontal table. The particle P is held on the table so that $AP = BP = L$ (see diagram). The particle P is released from rest. When P is at the midpoint of AB it has speed $\sqrt{80ag}$.

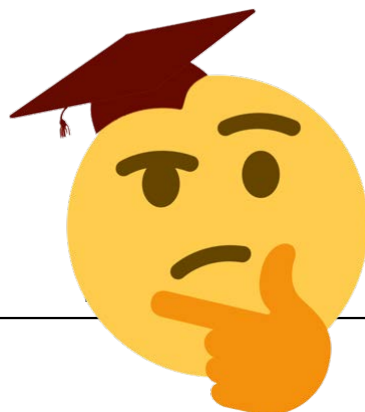
- (a) Find L in terms of a . [5]
- (b) Find the initial acceleration of P in terms of g . [3]



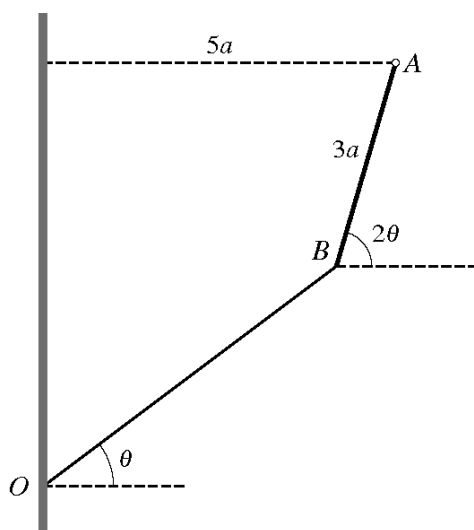
1 - (9231/21_Summer_2017_Q2) - Equilibrium Of A Rigid Body Under Coplanar Forces



A uniform smooth disc with centre O and radius a is fixed at the point D on a horizontal surface. A uniform rod of length $3a$ and weight W rests on the disc with its end A in contact with a rough vertical wall. The rod and the disc lie in a vertical plane that is perpendicular to the wall. The wall meets the horizontal surface at the point E such that $AE = a$ and $ED = \frac{5}{4}a$. A particle of weight kW is hung from the rod at B (see diagram). The coefficient of friction between the rod and the wall is $\frac{1}{8}$ and the system is in limiting equilibrium. Find the value of k . [8]



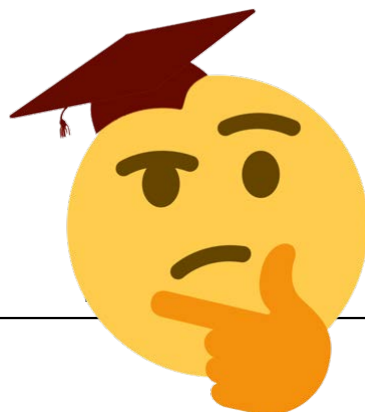
2 - (9231/23_Summer_2017_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces



A uniform rod AB of length $3a$ and weight W is freely hinged to a fixed point at the end A . The end B is below the level of A and is attached to one end of a light elastic string of natural length $4a$. The other end of the string is attached to a point O on a vertical wall. The horizontal distance between A and the wall is $5a$. The string and the rod make angles θ and 2θ respectively with the horizontal (see diagram). The system is in equilibrium with the rod and the string in the same vertical plane. It is given that $\sin \theta = \frac{3}{5}$ and you may use the fact that $\cos 2\theta = \frac{7}{25}$.

(i) Find the tension in the string in terms of W .

[3]



(ii) Find the modulus of elasticity of the string in terms of W . [4]

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(iii) Find the angle that the force acting on the rod at A makes with the horizontal. [3]

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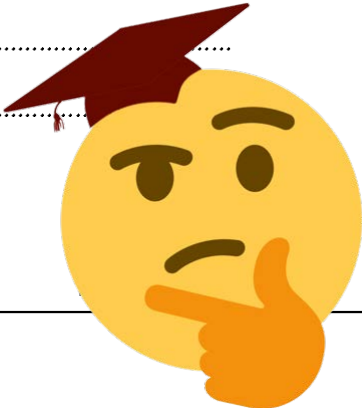
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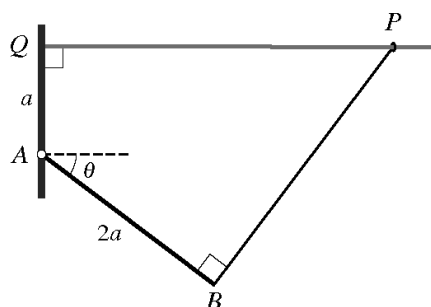
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3 - (9231/21_Winter_2017_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces



A small ring P of weight W is free to slide on a rough horizontal wire, one end of which is attached to a vertical wall at Q . The end A of a thin uniform rod AB of length $2a$ and weight $\frac{5}{2}W$ is freely hinged to the wall at the point A which is a distance a vertically below Q . A light elastic string of natural length $2a$ has one end attached to the ring P and the other end attached to the rod at B . The string is at right angles to the rod and A , B , P and Q lie in a vertical plane. The system is in limiting equilibrium with AB making an angle θ with the horizontal, where $\sin \theta = \frac{3}{5}$ (see diagram).

- (i) Find the tension in the string in terms of W . [2]

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- (ii) Find the coefficient of friction between the ring and the wire. [2]

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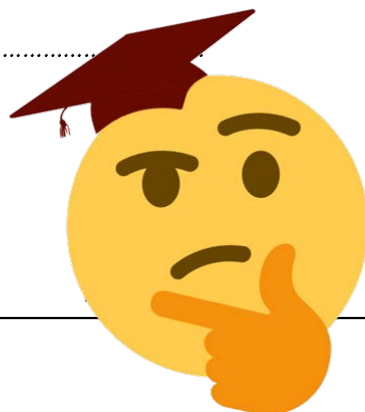
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- (iii) Find the magnitude of the resultant force on the rod at the hinge in terms of W . [3]

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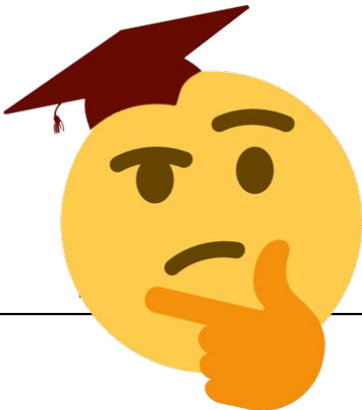


(iv) Find the modulus of elasticity of the string in terms of W . [3]

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4 - (9231/21_Summer_2018_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

A uniform rod AB has length $2a$ and weight W . The end A rests on rough horizontal ground and the end B rests against a smooth vertical wall. The rod is in a vertical plane that is perpendicular to the wall. The angle between the rod and the horizontal is θ . A particle of weight $5W$ hangs from the rod at the point C , with $AC = xa$, where $0 < x < 1$.

- (i) By taking moments about A , show that the magnitude of the normal reaction at B is $\frac{W(5x+1)}{2 \tan \theta}$. [3]

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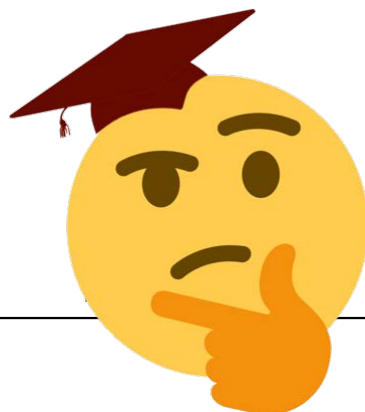
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The particle of weight $5W$ is now moved a distance a up the rod, so that $AC = (x+1)a$. This results in the magnitude of the normal reaction at B being double its previous value. The system remains in equilibrium with the rod at angle θ with the horizontal.

- (ii) Show that $x = \frac{4}{5}$. [3]

The coefficient of friction between the rod and the ground is $\frac{2}{3}$.

- (iii) Given that the rod is about to slip when the particle of weight $5W$ is in its second position, find the value of $\tan \theta$. [5]



5 - (9231/23_Summer_2018_Q4) - *Equilibrium Of A Rigid Body Under Coplanar Forces*

A uniform rod AB has length $2a$ and weight W . The end A rests on rough horizontal ground and the end B rests against a smooth vertical wall. The angle between the rod and the horizontal is θ , where $\tan \theta = \frac{4}{3}$. One end of a light inextensible rope is attached to a point C on the rod. The other end is attached to a point where the vertical wall and the horizontal ground meet. The rope is taut and perpendicular to the rod. The rope and rod are in a vertical plane perpendicular to the wall.

- (i) Show that $AC = \frac{18}{25}a$. [2]

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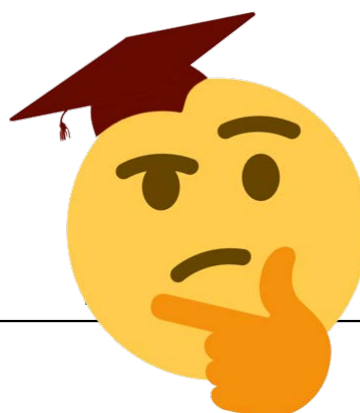
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The magnitude of the frictional force at A is equal to one quarter of the magnitude of the normal reaction force at A .

- (ii) Show that the tension in the rope is $\frac{1}{4}W$. [6]

- (iii) Find expressions, in terms of W , for the magnitudes of the normal reaction forces at A and B . [2]



6 - (9231/21_Winter_2018_Q4) - *Equilibrium Of A Rigid Body Under Coplanar Forces*

A uniform rod AB of length $4a$ and weight W is smoothly hinged to a vertical wall at the end A . The rod is held at an angle θ above the horizontal by a light elastic string. One end of the string is attached to the point C on the rod, where $AC = 3a$. The other end of the string is attached to a point D on the wall, with D vertically above A and such that angle $ACD = 2\theta$. A particle of weight $\frac{1}{2}W$ is attached to the rod at B . It is given that $\tan \theta = \frac{8}{15}$.

(i) Show that the tension in the string is $\frac{17}{12}W$. [4]

(ii) Find the magnitude and direction of the reaction at the hinge. [5]

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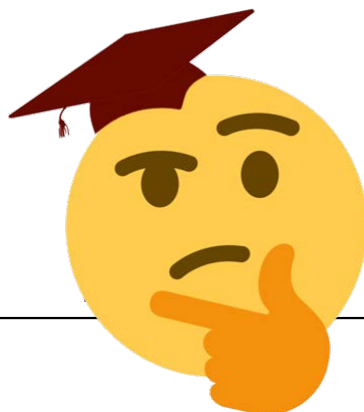
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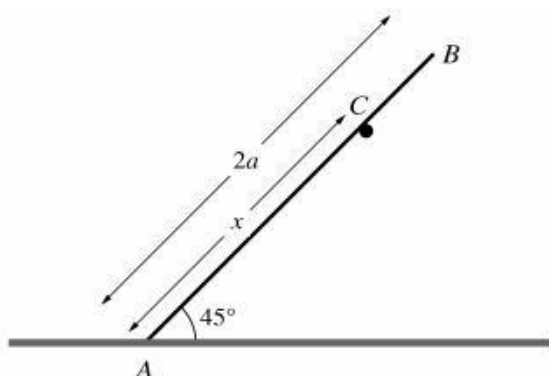
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(iii) Given that the natural length of the string is $2a$, find its modulus of elasticity. [2]



7 - (9231/22_Winter_2018_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces



A uniform rod AB of length $2a$ and weight W rests against a smooth horizontal peg at a point C on the rod, where $AC = x$. The lower end A of the rod rests on rough horizontal ground. The rod is in equilibrium inclined at an angle of 45° to the horizontal (see diagram). The coefficient of friction between the rod and the ground is μ . The rod is about to slip at A .

(i) Find an expression for x in terms of a and μ . [5]

(ii) Hence show that $\mu \geq \frac{1}{3}$. [2]

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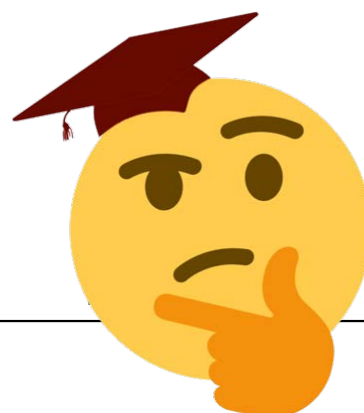
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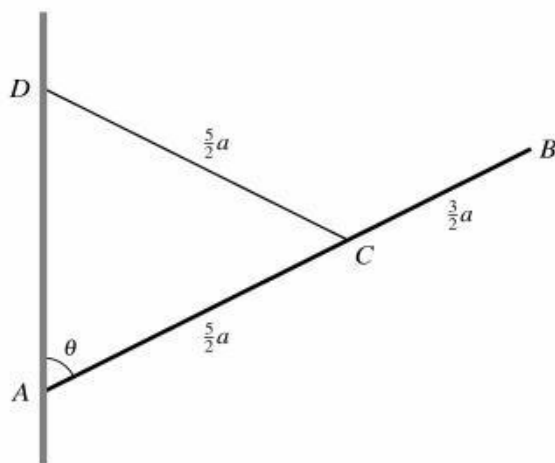
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(iii) Given that $x = \frac{3}{2}a$, find the value of μ and the magnitude of the resultant force on the rod at A . [4]

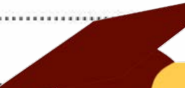


8 - (9231/21_Summer_2019_Q4) - *Equilibrium Of A Rigid Body Under Coplanar Forces*

A uniform rod AB of length $4a$ and weight W rests with the end A in contact with a rough vertical wall. A light inextensible string of length $\frac{5}{2}a$ has one end attached to the point C on the rod, where $AC = \frac{5}{2}a$. The other end of the string is attached to a point D on the wall, vertically above A . The vertical plane containing the rod AB is perpendicular to the wall. The angle between the rod and the wall is θ , where $\tan \theta = 2$ (see diagram). The end A of the rod is on the point of slipping down the wall and the coefficient of friction between the rod and the wall is μ .

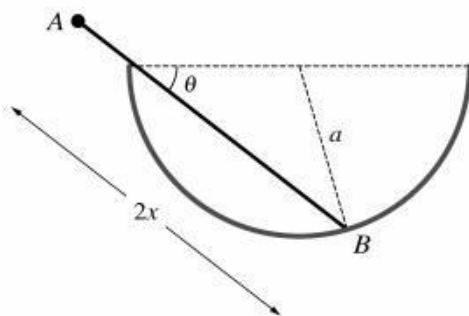
Find, in either order, the tension in the string and the value of μ .

[10]



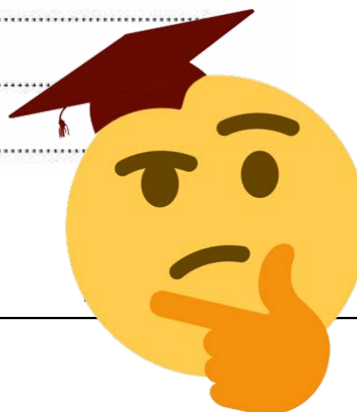
A yellow 'Thinking Face' emoji with a red graduation cap on its head. The emoji has a thoughtful expression with one eyebrow raised and a hand to its chin. It is positioned in the top right corner of the page.

9 - (9231/23_Summer_2019_Q5) - *Equilibrium Of A Rigid Body Under Coplanar Forces*



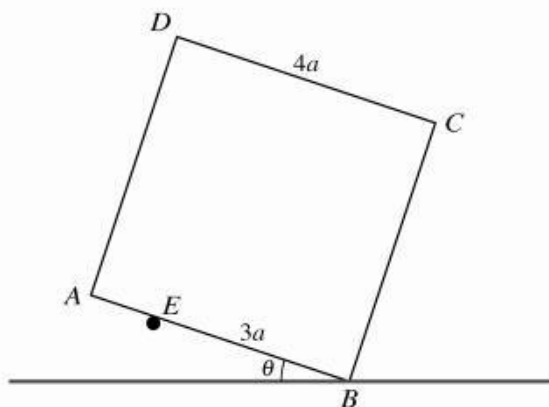
A uniform rod AB of length $2x$ and weight W rests on the smooth rim of a fixed hemispherical bowl of radius a . The end B of the rod is in contact with the rough inner surface of the bowl. The coefficient of friction between the rod and the bowl at B is $\frac{1}{3}$. A particle of weight $\frac{1}{4}W$ is attached to the end A of the rod. The end B is about to slip upwards when AB is inclined at an angle θ to the horizontal, where $\tan \theta = \frac{3}{4}$ (see diagram).

- (i) By resolving parallel to the rod, show that the normal component of the reaction of the bowl on the rod at B is $\frac{3}{4}W$. [5]



This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

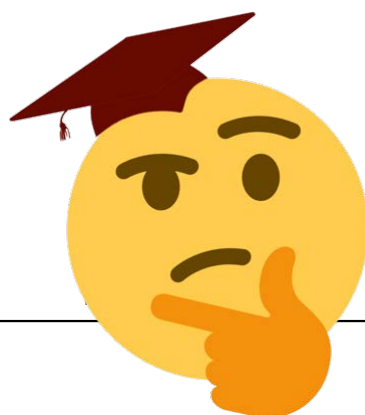


10 - (9231/21_Winter_2019_Q2) - Equilibrium Of A Rigid Body Under Coplanar Forces

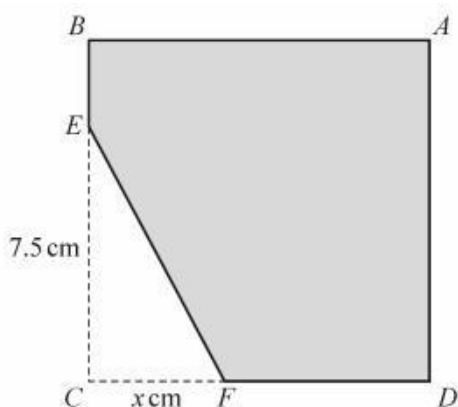
A uniform square lamina $ABCD$ of side $4a$ and weight W rests in a vertical plane with the edge AB inclined at an angle θ to the horizontal, where $\tan \theta = \frac{1}{3}$. The vertex B is in contact with a rough horizontal surface for which the coefficient of friction is μ . The lamina is supported by a smooth peg at the point E on AB , where $BE = 3a$ (see diagram).

(i) Find expressions in terms of W for the normal reaction forces at E and B . [5]

(ii) Given that the lamina is about to slip, find the value of μ . [3]



11 - (9231/31_Summer_2020_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

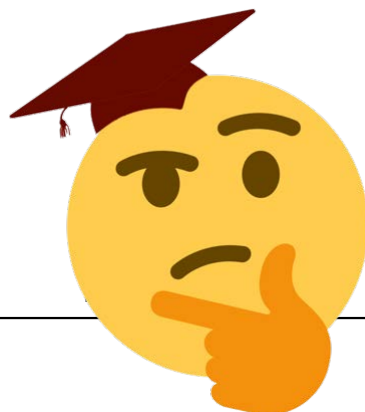


A uniform square lamina $ABCD$ has sides of length 10 cm. The point E is on BC with $EC = 7.5$ cm, and the point F is on DC with $CF = x$ cm. The triangle EFC is removed from $ABCD$ (see diagram). The centre of mass of the resulting shape $ABEFD$ is a distance $\bar{x}$ cm from CB and a distance $\bar{y}$ cm from CD .

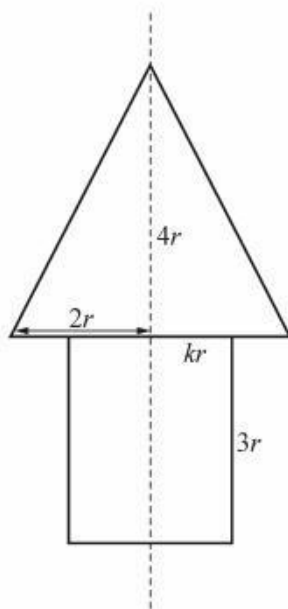
- (a) Show that $\bar{x} = \frac{400 - x^2}{80 - 3x}$ and find a corresponding expression for $\bar{y}$. [4]

The shape $ABEFD$ is in equilibrium in a vertical plane with the edge DF resting on a smooth horizontal surface.

- (b) Find the greatest possible value of x , giving your answer in the form $a + b\sqrt{2}$, where a and b are constants to be determined. [3]



12 - (9231/33_Summer_2020_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

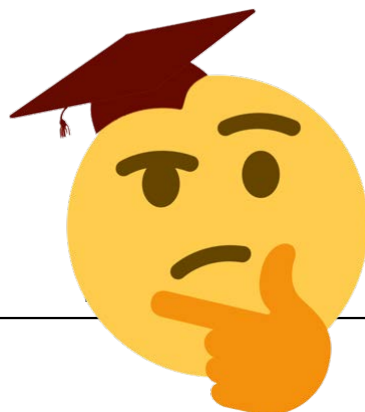


A uniform solid circular cone, of vertical height $4r$ and radius $2r$, is attached to a uniform solid cylinder, of height $3r$ and radius kr , where k is a constant less than 2. The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram). The cone and the cylinder are made of the same material.

- (a) Show that the distance of the centre of mass of the combined solid from the vertex of the cone is $\frac{(99k^2 + 96)r}{18k^2 + 32}$. [4]

The point C is on the circumference of the base of the cone. When the combined solid is freely suspended from C and hanging in equilibrium, the diameter through C makes an angle α with the downward vertical, where $\tan \alpha = \frac{1}{8}$.

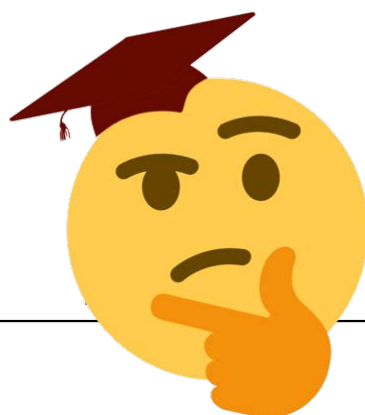
- (b) Given that the centre of mass of the combined solid is within the cylinder, find the value of k . [4]



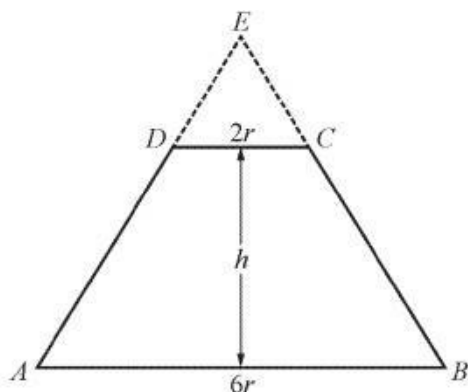
13 - (9231/32_Winter_2020_Q3) - *Equilibrium Of A Rigid Body Under Coplanar Forces*

An object consists of a uniform solid circular cone, of vertical height $4r$ and radius $3r$, and a uniform solid cylinder, of height $4r$ and radius $3r$. The circular base of the cone and one of the circular faces of the cylinder are joined together so that they coincide. The cone and the cylinder are made of the same material.

- (a) Find the distance of the centre of mass of the object from the end of the cylinder that is not attached to the cone. [4]
- (b) Show that the object can rest in equilibrium with the curved surface of the cone in contact with a horizontal surface. [3]

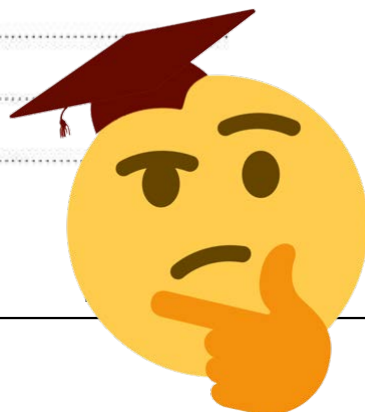


14 - (9231/33_Winter_2020_Q4) - *Equilibrium Of A Rigid Body Under Coplanar Forces*



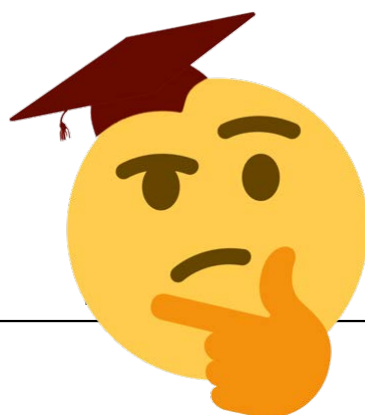
The diagram shows the cross-section $ABCD$ of a uniform solid object which is formed by removing a cone with cross-section DCE from the top of a larger cone with cross-section ABE . The perpendicular distance between AB and DC is h , the diameter AB is $6r$ and the diameter DC is $2r$.

- (a) Find an expression, in terms of h , for the distance of the centre of mass of the solid object from AB . [4]

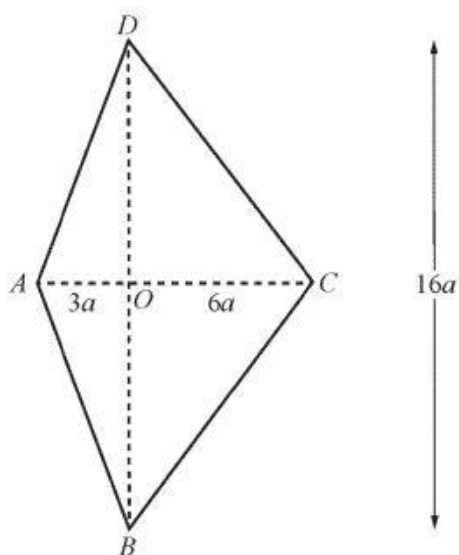


The object is freely suspended from the point B and hangs in equilibrium. The angle between AB and the downward vertical through B is θ .

(b) Given that $h = \frac{13}{4}r$, find the value of $\tan \theta$. [2]



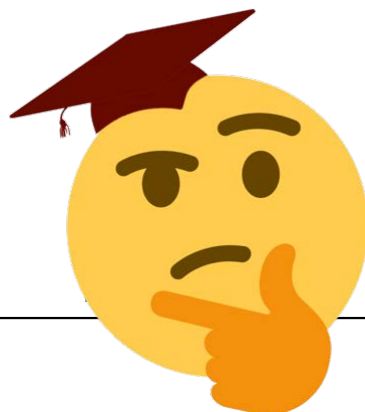
15 - (9231/33_Summer_2021_Q1) - Equilibrium Of A Rigid Body Under Coplanar Forces



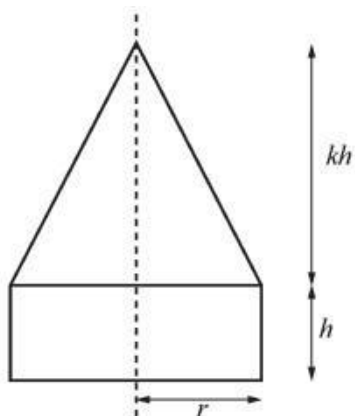
A uniform lamina $ABCD$ consists of two isosceles triangles ABD and BCD . The diagonals of $ABCD$ meet at the point O . The length of AO is $3a$, the length of OC is $6a$ and the length of BD is $16a$ (see diagram).

Find the distance of the centre of mass of the lamina from DB .

[3]



16 – (9231/31_Summer_2021_Q4) – *Equilibrium Of A Rigid Body Under Coplanar Forces*

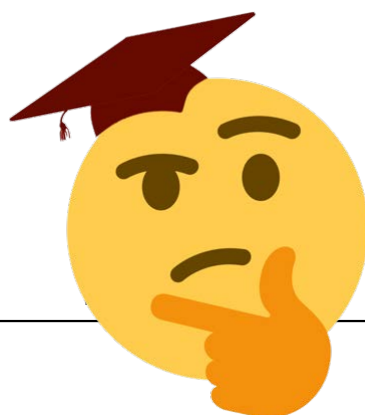


A uniform solid circular cone has vertical height kh and radius r . A uniform solid cylinder has height h and radius r . The base of the cone is joined to one of the circular faces of the cylinder so that the axes of symmetry of the two solids coincide (see diagram, which shows a cross-section). The cone and the cylinder are made of the same material.

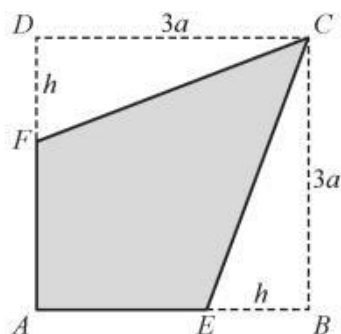
- (a) Show that the distance of the centre of mass of the combined solid from the base of the cylinder is $\frac{h(k^2 + 4k + 6)}{4(3 + k)}$. [4]

The solid is placed on a plane that is inclined to the horizontal at an angle θ . The base of the cylinder is in contact with the plane. The plane is sufficiently rough to prevent sliding. It is given that $3h = 2r$ and that the solid is on the point of toppling when $\tan \theta = \frac{4}{3}$.

- (b) Find the value of k . [3]



17 - (9231/31_Winter_2021_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

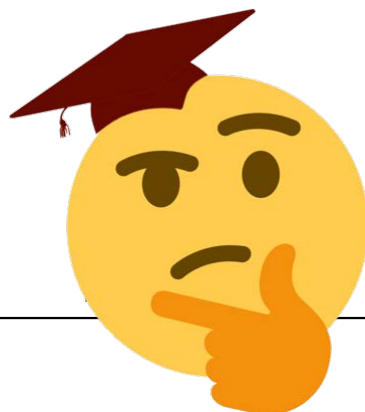


A uniform lamina $AECF$ is formed by removing two identical triangles BCE and CDF from a square lamina $ABCD$. The square has side $3a$ and $EB = DF = h$ (see diagram).

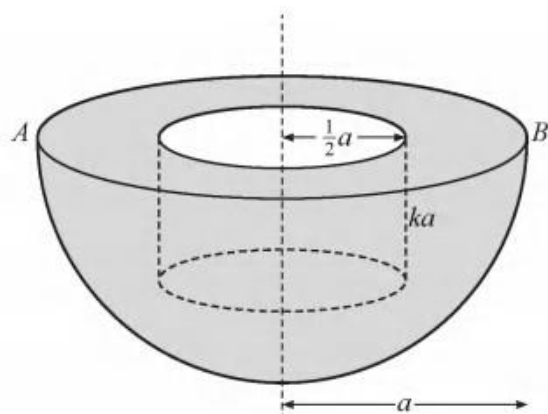
- (a) Find the distance of the centre of mass of the lamina $AECF$ from AD and from AB , giving your answers in terms of a and h . [5]

The lamina $AECF$ is placed vertically on its edge AE on a horizontal plane.

- (b) Find, in terms of a , the set of values of h for which the lamina remains in equilibrium. [3]



18 - (9231/32_Winter_2021_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

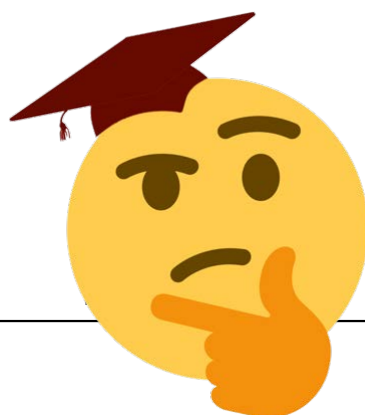


An object is formed by removing a solid cylinder, of height ka and radius $\frac{1}{2}a$, from a uniform solid hemisphere of radius a . The axes of symmetry of the hemisphere and the cylinder coincide and one circular face of the cylinder coincides with the plane face of the hemisphere. AB is a diameter of the circular face of the hemisphere (see diagram).

- (a) Show that the distance of the centre of mass of the object from AB is $\frac{3a(2-k^2)}{2(8-3k)}$. [4]

When the object is freely suspended from the point A , the line AB makes an angle θ with the downward vertical, where $\tan \theta = \frac{7}{18}$.

- (b) Find the possible values of k . [3]

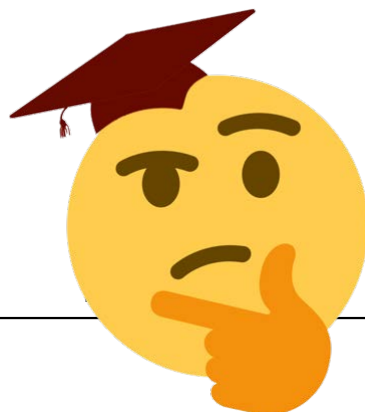


19 – (9231/33_Summer_2022_Q1) – *Equilibrium Of A Rigid Body Under Coplanar Forces*

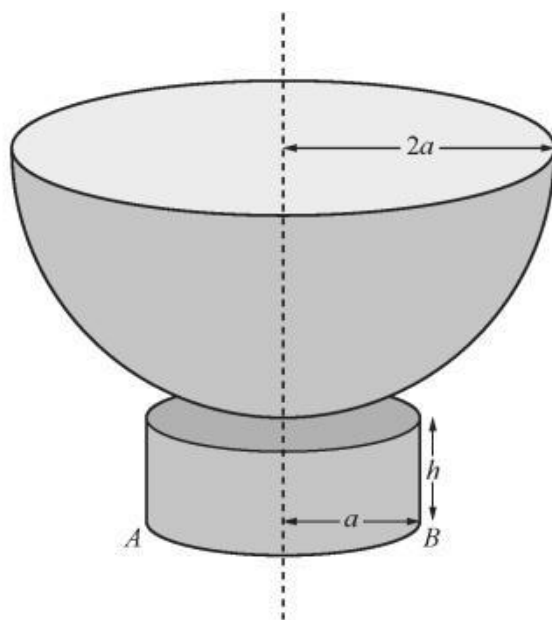
A uniform lamina $OABC$ is a trapezium whose vertices can be represented by coordinates in the x - y plane. The coordinates of the vertices are $O(0, 0)$, $A(15, 0)$, $B(9, 4)$ and $C(3, 4)$.

Find the x -coordinate of the centre of mass of the lamina.

[4]



20 - (9231/31_Summer_2022_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

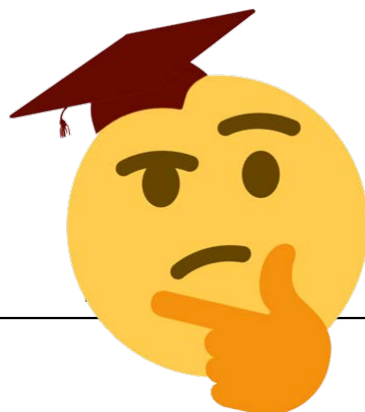


An object is composed of a hemispherical shell of radius $2a$ attached to a closed hollow circular cylinder of height h and base radius a . The hemispherical shell and the hollow cylinder are made of the same uniform material. The axes of symmetry of the shell and the cylinder coincide. AB is a diameter of the lower end of the cylinder (see diagram).

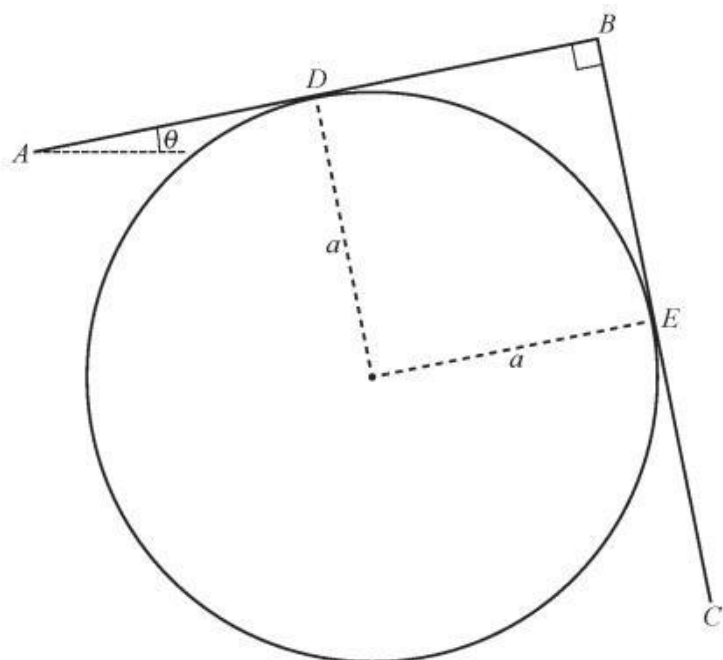
- (a) Find, in terms of a and h , an expression for the distance of the centre of mass of the object from AB . [4]

The object is placed on a rough plane which is inclined to the horizontal at an angle θ , where $\tan \theta = \frac{2}{3}$. The object is in equilibrium with AB in contact with the plane and lying along a line of greatest slope of the plane.

- (b) Find the set of possible values of h , in terms of a . [4]



21 – (9231/33_Summer_2022_Q7) – *Equilibrium Of A Rigid Body Under Coplanar Forces*

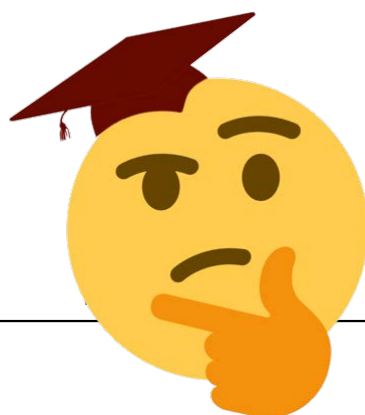


A uniform cylinder with a rough surface and of radius a is fixed with its axis horizontal. Two identical uniform rods AB and BC , each of weight W and length $2a$, are rigidly joined at B with AB perpendicular to BC . The rods rest on the cylinder in a vertical plane perpendicular to the axis of the cylinder with AB at an angle θ to the horizontal. D and E are the midpoints of AB and BC respectively and also the points of contact of the rods with the cylinder (see diagram). The rods are about to slip in a clockwise direction. The coefficient of friction between each rod and the cylinder is μ .

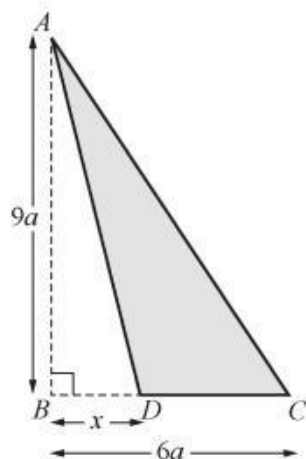
The normal reaction between AB and the cylinder is R and the normal reaction between BC and the cylinder is N .

(a) Find the ratio $R : N$ in terms of μ . [6]

(b) Given that $\mu = \frac{1}{3}$, find the value of $\tan \theta$. [3]



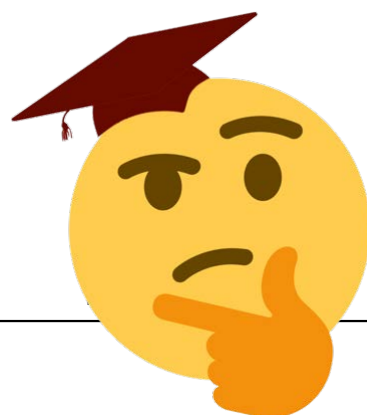
22 - (9231/32_Winter_2022_Q2) - Equilibrium Of A Rigid Body Under Coplanar Forces

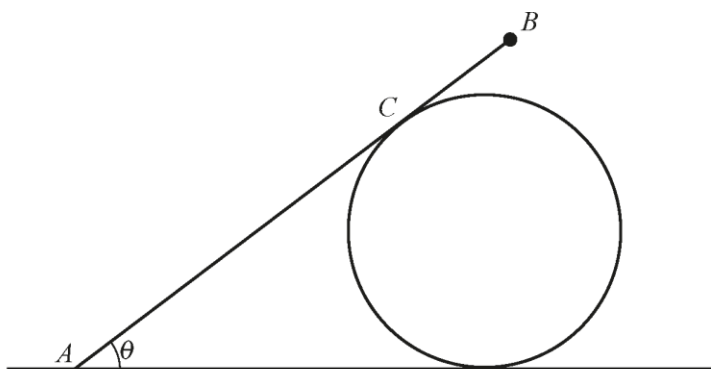


A uniform lamina is in the form of a triangle ABC in which angle B is a right angle, $AB = 9a$ and $BC = 6a$. The point D is on BC such that $BD = x$ (see diagram). The region ABD is removed from the lamina. The resulting shape ADC is placed with the edge DC on a horizontal surface and the plane ADC is vertical.

Find the set of values of x , in terms of a , for which the shape is in equilibrium.

[6]

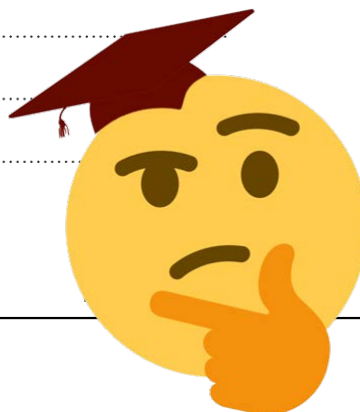


23 - (9231/31_Winter_2022_Q3) - *Equilibrium Of A Rigid Body Under Coplanar Forces*

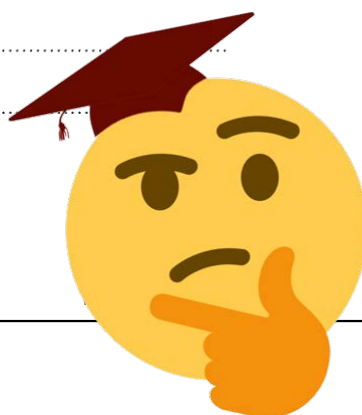
A smooth cylinder is fixed to a rough horizontal surface with its axis of symmetry horizontal. A uniform rod AB , of length $4a$ and weight W , rests against the surface of the cylinder. The end A of the rod is in contact with the horizontal surface. The vertical plane containing the rod AB is perpendicular to the axis of the cylinder. The point of contact between the rod and the cylinder is C , where $AC = 3a$. The angle between the rod and the horizontal surface is θ where $\tan \theta = \frac{3}{4}$ (see diagram). The coefficient of friction between the rod and the horizontal surface is $\frac{6}{7}$.

A particle of weight kW is attached to the rod at B . The rod is about to slip. The normal reaction between the rod and the cylinder is N .

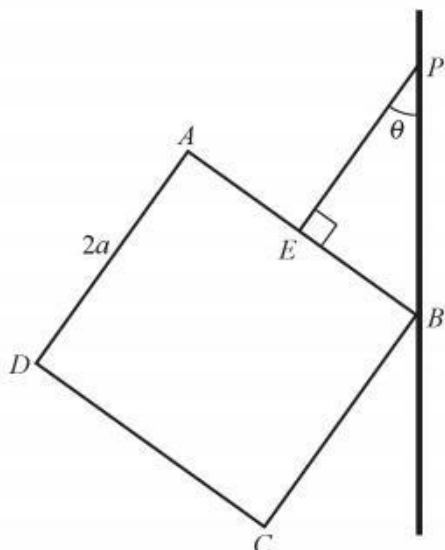
(a) Show that $N = \frac{8}{15}W(1+2k)$. [2]

[illegible]

[5]

[illegible]

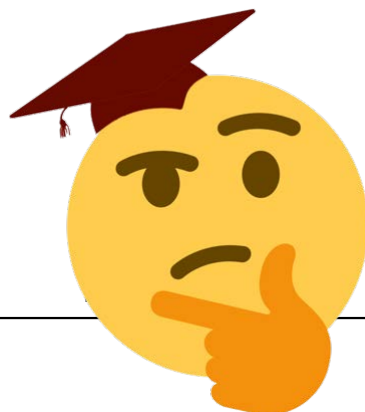
24 - (9231/31_Winter_2023_Q3) - Equilibrium Of A Rigid Body Under Coplanar Forces



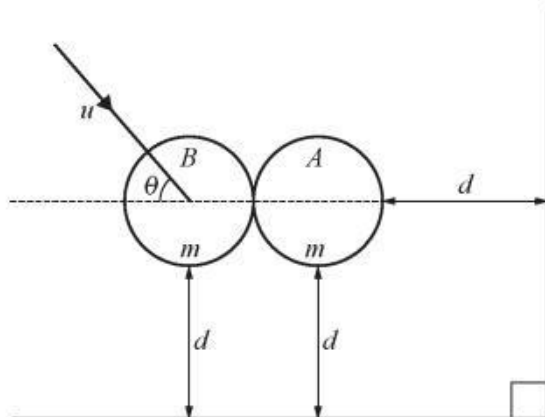
A uniform square lamina of side $2a$ and weight W is suspended from a light inextensible string attached to the midpoint E of the side AB . The other end of the string is attached to a fixed point P on a rough vertical wall. The vertex B of the lamina is in contact with the wall. The string EP is perpendicular to the side AB and makes an angle θ with the wall (see diagram). The string and the lamina are in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the lamina is $\frac{1}{2}$.

Given that the vertex B is about to slip up the wall, find the value of $\tan \theta$.

[8]

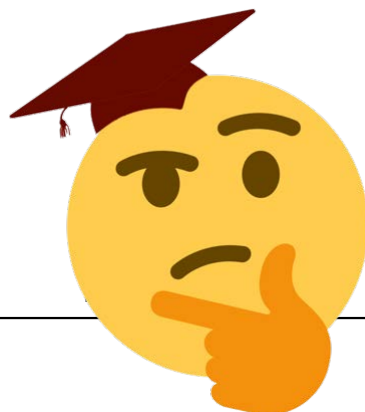


25 - (9231/32_Winter_2023_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

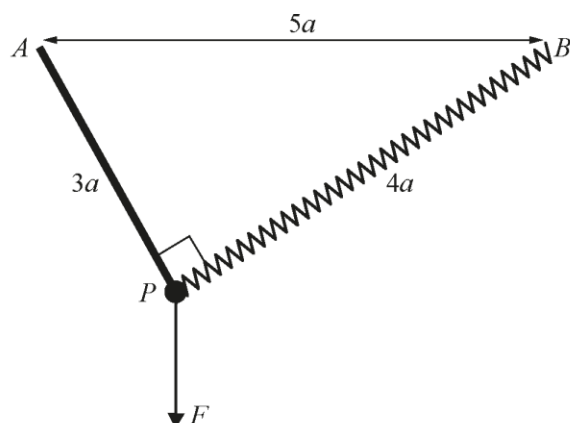


Two smooth vertical walls meet at right angles. The smooth sphere A , with mass m , is at rest on a smooth horizontal surface and is at a distance d from each wall. An identical smooth sphere B is moving on the horizontal surface with speed u at an angle θ with the line of centres when the spheres collide (see diagram). After the collision, the spheres take the same time to reach a wall. The coefficient of restitution between the spheres is $\frac{1}{2}$.

- (a) Find the value of $\tan \theta$. [4]
- (b) Find the percentage loss in the total kinetic energy of the spheres as a result of this collision. [3]



26 - (9231/32_Winter_2023_Q7) - *Equilibrium Of A Rigid Body Under Coplanar Forces*

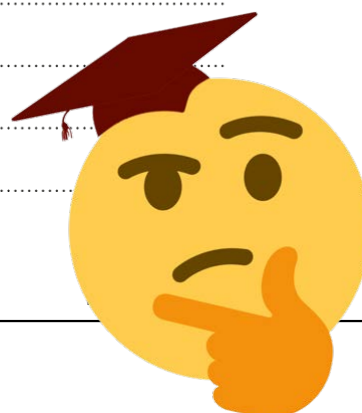


A particle P of mass m is attached to one end of a light rod of length $3a$. The other end of the rod is able to pivot smoothly about the fixed point A . The particle is also attached to one end of a light spring of natural length a and modulus of elasticity kmg . The other end of the spring is attached to a fixed point B . The points A and B are in a horizontal line, a distance $5a$ apart, and these two points and the rod are in a vertical plane.

Initially, P is held in equilibrium by a vertical force F with the stretched length of the spring equal to $4a$ (see diagram). The particle is released from rest in this position and has a speed of $\frac{6}{5}\sqrt{2ag}$ when the rod becomes horizontal.

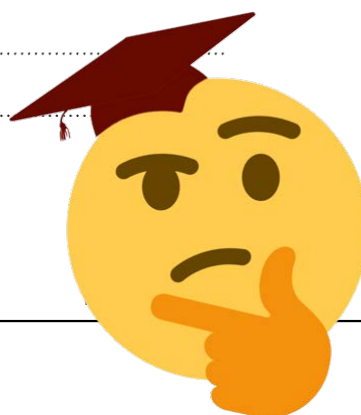
(a) Find the value of k .

[5]

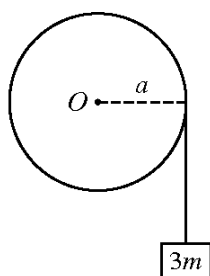


[2]

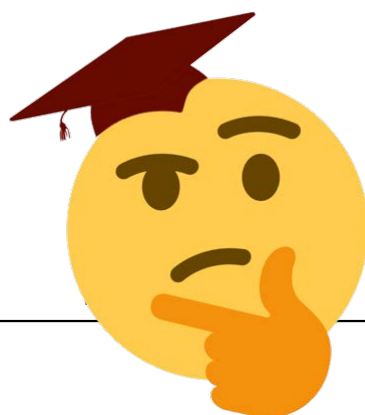
[2]



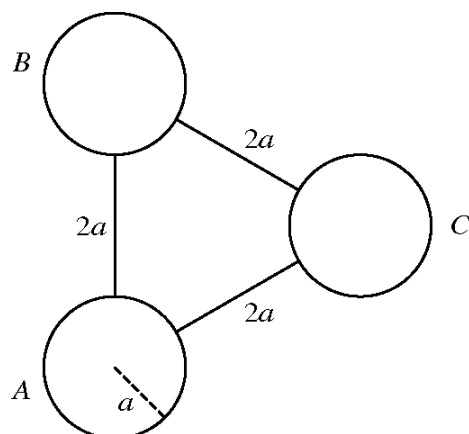
1 - (9231/23_Summer_2017_Q1) - Rotation Of A Rigid Body



A uniform disc with centre O , mass m and radius a is free to rotate without resistance in a vertical plane about a horizontal axis through O . One end of a light inextensible string is attached to the rim of the disc and wrapped around the rim. The other end of the string is attached to a block of mass $3m$ (see diagram). The system is released from rest with the block hanging vertically. While the block is in motion, it experiences a constant vertical resisting force of magnitude $0.9mg$. Find the tension in the string in terms of m and g . [5]



2 - (9231/21_Summer_2017_Q4) - Rotation Of A Rigid Body

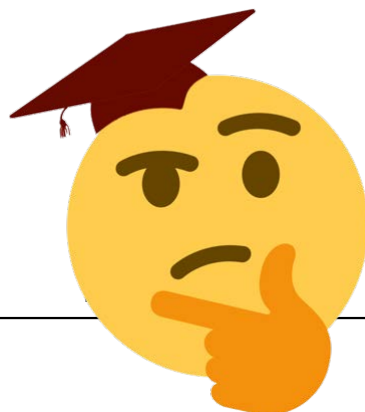


Three identical uniform discs, A , B and C , each have mass m and radius a . They are joined together by uniform rods, each of which has mass $\frac{1}{3}m$ and length $2a$. The discs lie in the same plane and their centres form the vertices of an equilateral triangle of side $4a$. Each rod has one end rigidly attached to the circumference of a disc and the other end rigidly attached to the circumference of an adjacent disc, so that the rod lies along the line joining the centres of the two discs (see diagram).

- (i) Find the moment of inertia of this object about an axis l , which is perpendicular to the plane of the object and through the centre of disc A . [6]

The object is free to rotate about the horizontal axis l . It is released from rest in the position shown, with the centre of disc B vertically above the centre of disc A .

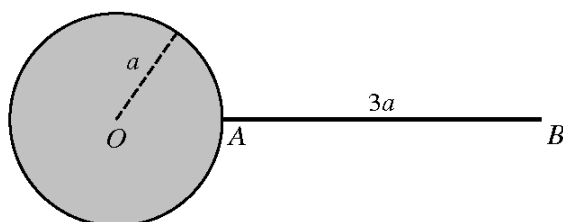
- (ii) Write down the change in the vertical position of the centre of mass of the object when the centre of disc B is vertically below the centre of disc A . Hence find the angular velocity of the object when the centre of disc B is vertically below the centre of disc A . [4]



3 - (9231/23_Summer_2017_Q11) - Rotation Of A Rigid Body, Bivariate Data

Answer only **one** of the following two alternatives.

EITHER



The diagram shows a uniform thin rod AB of length $3a$ and mass $8m$. The end A is rigidly attached to the surface of a sphere with centre O and radius a . The rod is perpendicular to the surface of the sphere. The sphere consists of two parts: an inner uniform solid sphere of mass $\frac{3}{2}m$ and radius a surrounded by a thin uniform spherical shell of mass m and also of radius a . The horizontal axis l is perpendicular to the rod and passes through the point C on the rod where $AC = a$.

- (i) Show that the moment of inertia of the object, consisting of rod, shell and inner sphere, about the axis l is $\frac{289}{15}ma^2$. [6]

The object is free to rotate about the axis l . The object is held so that CA makes an angle α with the downward vertical and is released from rest.

- (ii) Given that $\cos \alpha = \frac{1}{6}$, find the greatest speed achieved by the centre of the sphere in the subsequent motion. [6]

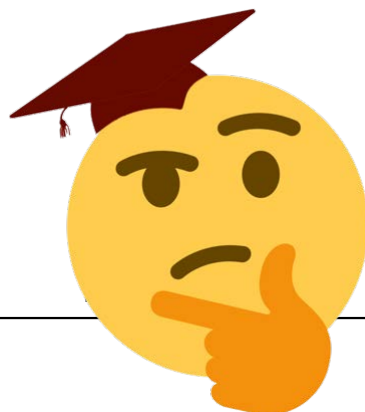
OR

The times taken to run 200 metres at the beginning of the year and at the end of the year are recorded for each member of a large athletics club. The time taken, in seconds, at the beginning of the year is denoted by x and the time taken, in seconds, at the end of the year is denoted by y . For a random sample of 8 members, the results are shown in the following table.

Member	A	B	C	D	E	F	G	H
x	24.2	23.8	22.8	25.1	24.5	24.0	23.8	22.8
y	23.9	23.6	22.8	24.5	24.2	23.5	23.6	22.7

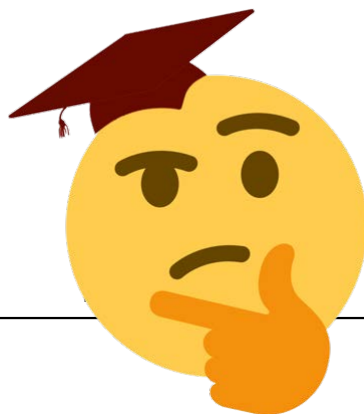
$$[\Sigma x = 191, \quad \Sigma x^2 = 4564.46, \quad \Sigma y = 188.8, \quad \Sigma y^2 = 4458.4, \quad \Sigma xy = 4510.99.]$$

- (i) Find, showing all necessary working, the equation of the regression line of y on x . [4]

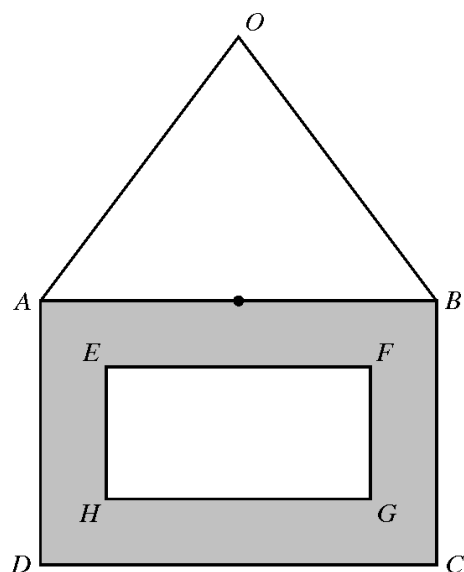


The athletics coach believes that, on average, the time taken by an athlete to run 200 metres decreases between the beginning and the end of the year by more than 0.2 seconds.

- (ii) Stating suitable hypotheses and assuming a normal distribution, test the coach's belief at the 10% significance level. [8]

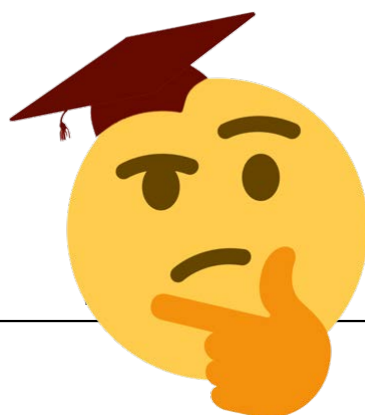


4 - (9231/21_Winter_2017_Q5) - Rotation Of A Rigid Body

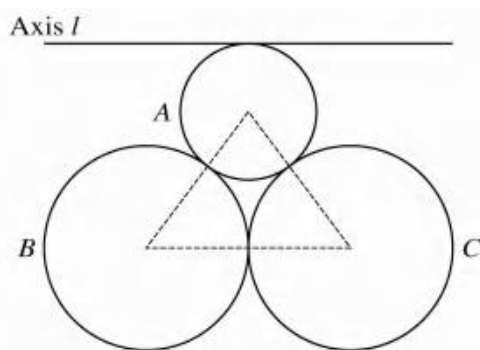


A uniform picture frame of mass m is made by removing a rectangular lamina $EFGH$ in which $EF = 4a$ and $FG = 2a$ from a larger rectangular lamina $ABCD$ in which $AB = 6a$ and $BC = 4a$. The side EF is parallel to the side AB . The point of intersection of the diagonals AC and BD coincides with the point of intersection of the diagonals EG and FH . One end of a light inextensible string of length $10a$ is attached to A and the other end is attached to B . The frame is suspended from the mid-point O of the string. A small object of mass $\frac{11}{12}m$ is fixed to the mid-point of AB (see diagram).

- (i) Show that the moment of inertia of the system, consisting of frame and small object, about an axis through O perpendicular to the plane of the frame, is $\frac{169}{3}ma^2$. [7]
- (ii) Show that small oscillations of the system about this axis are approximately simple harmonic and state their period. [5]

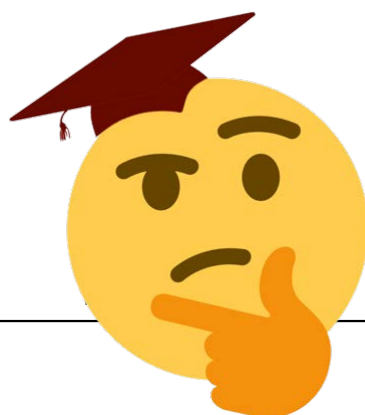


5 - (9231/21_Summer_2018_Q5) - Rotation Of A Rigid Body



Three thin uniform rings A , B and C are joined together, so that each ring is in contact with each of the other two rings. Ring A has radius $2a$ and mass $3M$; rings B and C each have radius $3a$ and mass $2M$. The rings lie in the same plane and the centres of the rings are at the vertices of an isosceles triangle. The object consisting of the three rings is free to rotate about the horizontal axis I which is tangential to ring A , in the plane of the object and perpendicular to the line of symmetry of the object (see diagram).

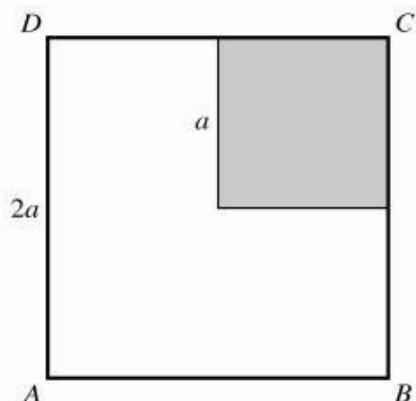
- (i) Show that the moment of inertia of the object about the axis I is $180Ma^2$. [7]
- (ii) Show that small oscillations of the object about the axis I are approximately simple harmonic, and state the period. [5]



6 - (9231/23_Summer_2018_Q11) - Rotation Of A Rigid Body

Answer only **one** of the following two alternatives.

EITHER

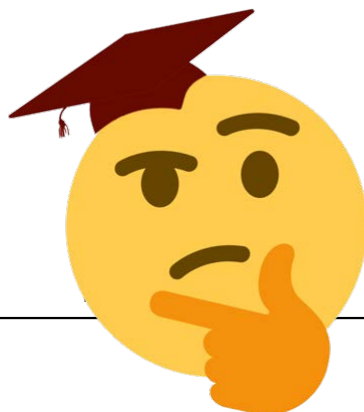


An object is formed from a square frame $ABCD$ with a square lamina attached in one corner of the frame. The frame consists of four identical thin rods, each of mass M and length $2a$. The lamina has mass kM and edges of length a . It has one vertex at C and adjacent sides in contact with CB and CD (see diagram).

- (i) Show that the moment of inertia of the object about an axis I through A perpendicular to the plane of the object is $\frac{2}{3}Ma^2(7k + 20)$. [8]

The object is released from rest with the edge AB horizontal and D vertically above A . The object rotates freely about the fixed axis I . The angular speed of the object is $\frac{1}{2}\sqrt{\left(\frac{5g}{a}\right)}$ when D is first vertically below A .

- (ii) Find the value of k . [6]



OR

A scientist carries out an experiment to investigate the quantity X , which takes the values 0, 1, 2, 3, 4, 5 or 6. He believes that the values taken by X follow a binomial distribution. He conducts 250 trials. His results are summarised in the following table.

x	0	1	2	3	4	5	6
Observed frequency	22	83	72	53	17	3	0

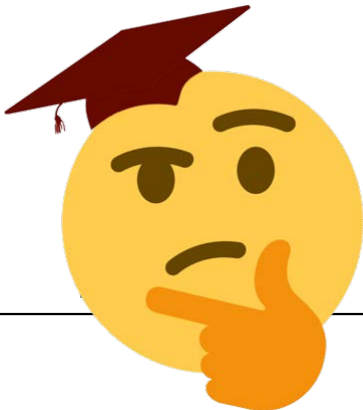
(i) Show that unbiased estimates of the mean and variance for these results are 1.876 and 1.266 respectively, correct to 3 decimal places. By evaluating the mean and variance of the distribution $B(6, 0.313)$, explain why X could have this distribution. [4]

The expected frequencies corresponding to the distribution $B(6, 0.313)$ are shown in the following table.

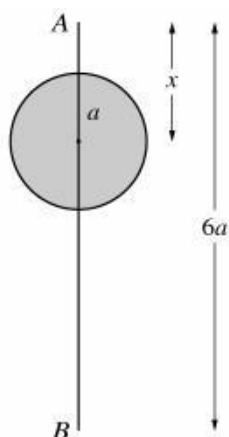
x	0	1	2	3	4	5	6
Observed frequency	22	83	72	53	17	3	0
Expected frequency	26.3	71.9	81.8	49.7	17.0	3.1	0.2

(ii) Show how the expected frequency for $x = 4$ is calculated. [2]

(iii) Test at the 5% significance level whether the scientist’s belief is correct. [8]



7 - (9231/21_Winter_2018_Q3) - Rotation Of A Rigid Body

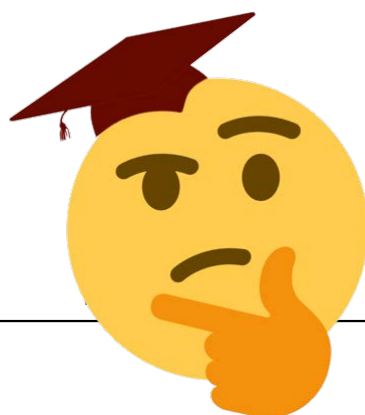


A uniform disc, of radius a and mass $2M$, is attached to a thin uniform rod AB of length $6a$ and mass M . The rod lies along a diameter of the disc, so that the centre of the disc is a distance x from A (see diagram).

- (i) Find the moment of inertia of the object, consisting of disc and rod, about a fixed horizontal axis l through A and perpendicular to the plane of the disc. [4]

The object is free to rotate about the axis l . The object is held with AB horizontal and is released from rest. When AB makes an angle θ with the vertical, where $\cos \theta = \frac{3}{5}$, the angular speed of the object is $\sqrt{\left(\frac{2g}{5a}\right)}$.

- (ii) Find the possible values of x . [5]



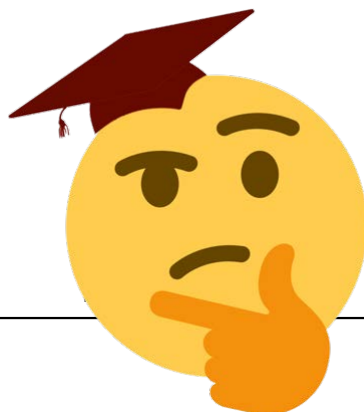
8 - (9231/22_Winter_2018_Q5) - Rotation Of A Rigid Body

An object is formed from a uniform circular disc, of radius $2a$ and mass $3M$, and a uniform rod AB , of length $3a$ and mass kM , where k is a constant. The centre of the disc is O . The end B of the rod is rigidly joined to a point on the circumference of the disc so that OBA is a straight line. The fixed horizontal axis l is in the plane of the object, passes through A and is perpendicular to AB .

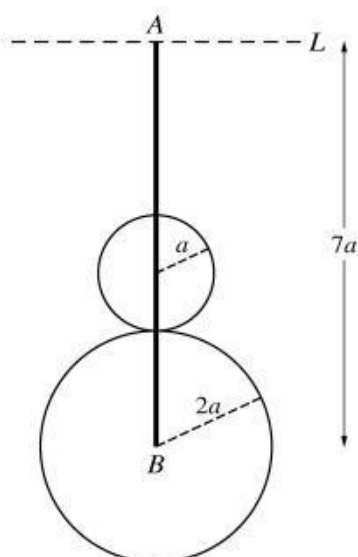
- (i) Show that the moment of inertia of the object about the axis l is $3Ma^2(26+k)$. [5]

The object is free to rotate about l .

- (ii) Show that small oscillations of the object about l are approximately simple harmonic. Given that the period of these oscillations is $4\pi\sqrt{\left(\frac{a}{g}\right)}$, find the value of k . [6]



9 - (9231/23_Summer_2019_Q4) - Rotation Of A Rigid Body



An object consists of two hollow spheres which touch each other, together with a thin uniform rod AB . The rod passes through small holes in the surfaces of the spheres. The rod is fixed to the spheres so that it passes through the centre of the smaller sphere. The end B of the rod is at the centre of the larger sphere. The larger sphere has radius $2a$ and mass M , the smaller sphere has radius a and mass kM , and the rod has length $7a$ and mass $5M$. A fixed horizontal axis L passes through A and is perpendicular to AB (see diagram).

- (i) Find the moment of inertia of the object, consisting of the rod and two spheres, about L . [6]

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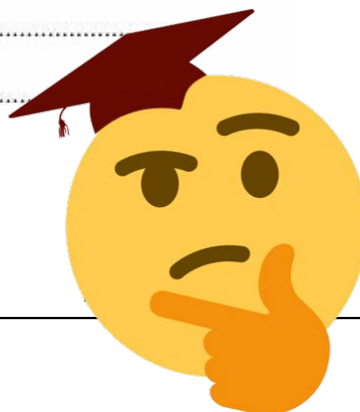
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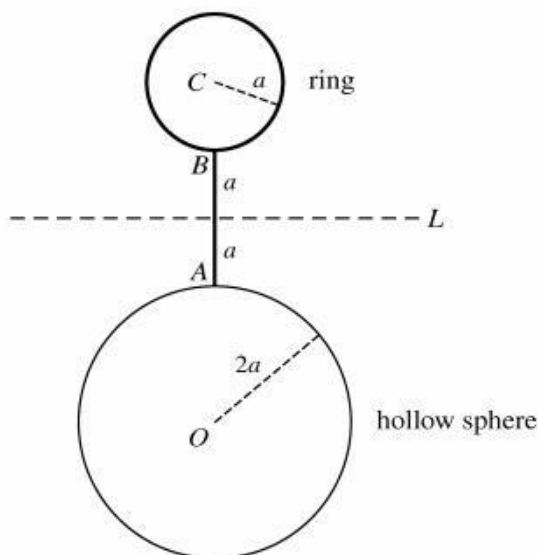
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[5]

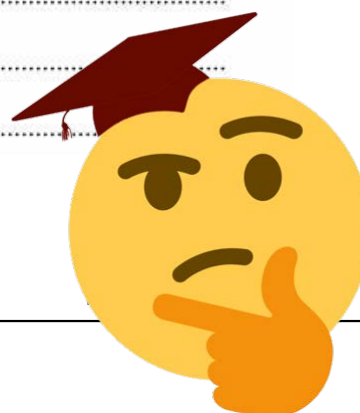
This image shows a blank sheet of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page. In the bottom right corner, there is a stylized illustration of a red pencil pointing towards the center of the page. The pencil has a yellow eraser and a sharp lead tip. The overall appearance is that of a clean, unused piece of stationery or a template for writing.

10 - (9231/21_Summer_2019_Q5) - *Rotation Of A Rigid Body, Simple Harmonic Motion*

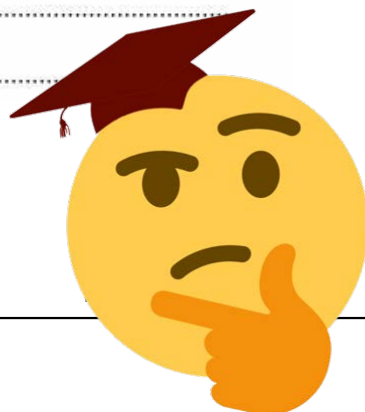


A thin uniform rod AB has mass kM and length $2a$. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere with centre O , mass kM and radius $2a$. The end B of the rod is rigidly attached to the circumference of a uniform ring with centre C , mass M and radius a . The points C, B, A, O lie in a straight line. The horizontal axis L passes through the mid-point of the rod and is perpendicular to the rod and in the plane of the ring (see diagram). The object consisting of the rod, the ring and the hollow sphere can rotate freely about L .

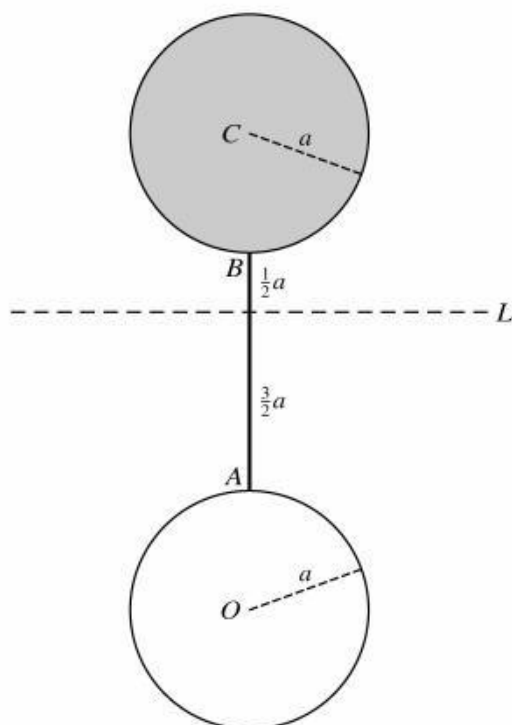
- (i) Show that the moment of inertia of the object about L is $\frac{3}{5}(8k + 3)Ma^2$. [6]



(ii) Find the set of possible values of k and the period of these oscillations in terms of k . [6]

[illegible]

11 - (9231/21_Winter_2019_Q5) - Rotation Of A Rigid Body, Simple Harmonic Motion

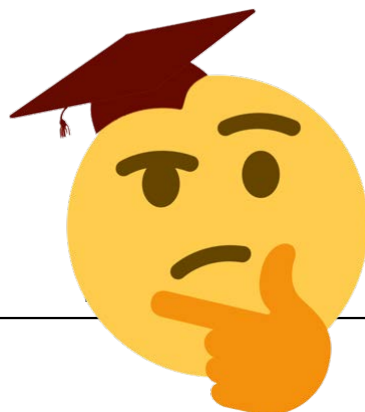


A thin uniform rod AB has mass λM and length $2a$. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere (spherical shell) with centre O , mass $3M$ and radius a . The end B of the rod is rigidly attached to the surface of a uniform solid sphere with centre C , mass $5M$ and radius a . The rod lies along the line joining the centres of the spheres, so that $CBAO$ is a straight line. The horizontal axis L is perpendicular to the rod and passes through the point of the rod that is a distance $\frac{1}{2}a$ from B (see diagram). The object consisting of the rod and the two spheres can rotate freely about L .

- (i) Show that the moment of inertia of the object about L is $\left(\frac{408 + 7\lambda}{12}\right)Ma^2$. [6]

The period of small oscillations of the object about L is $5\pi\sqrt{\left(\frac{2a}{g}\right)}$.

- (ii) Find the value of λ . [6]



A particle P moves on a straight line in simple harmonic motion. The centre of the motion is O , and the amplitude of the motion is 2.5 m. The points L and M are on the line, on opposite sides of O , with $OL = 1.5$ m. The magnitudes of the accelerations of P at L and at M are in the ratio 3 : 4.

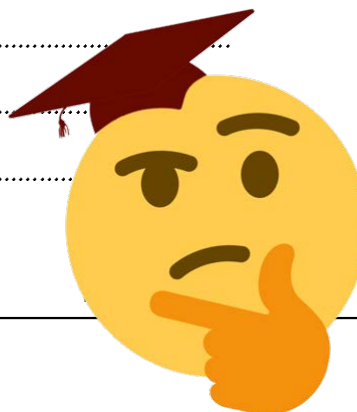
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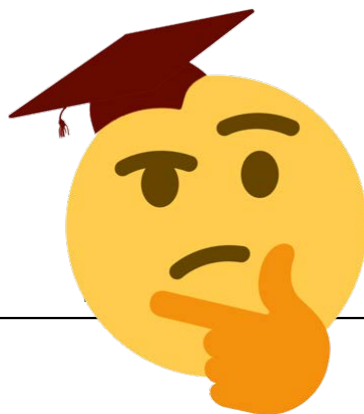
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(ii) Find the period of the motion. [5]

This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small, stylized illustration of a yellow smiley face wearing a red graduation cap with a tassel. The rest of the page is empty, providing space for writing or drawing.

(iii) Find the speed of P when it passes through L .

[2]



2 - (9231/21_Summer_2017_Q11) – Simple Harmonic Motion, X2 Test

Answer only **one** of the following two alternatives.

EITHER

A particle P of mass $3m$ is attached to one end of a light elastic spring of natural length a and modulus of elasticity kmg . The other end of the spring is attached to a fixed point O on a smooth plane that is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{2}{3}$. The system rests in equilibrium with P on the plane at the point E . The length of the spring in this position is $\frac{5}{4}a$.

- (i) Find the value of k .
- [3]

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The particle P is now replaced by a particle Q of mass $2m$ and Q is released from rest at the point E .

- (ii) Show that, in the resulting motion, Q performs simple harmonic motion. State the centre and the period of the motion.
- [6]

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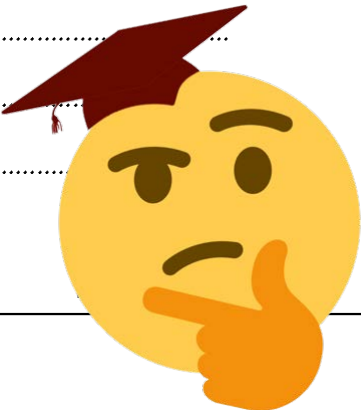
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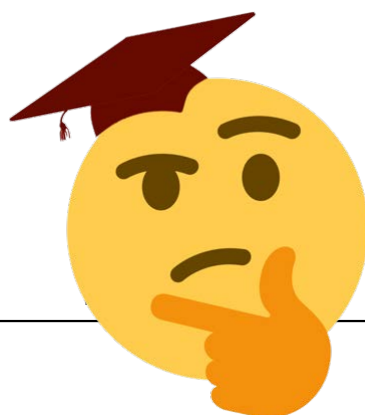
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(iii) Find the least tension in the spring and the maximum acceleration of Q during the motion. [5]



OR

A shop is supplied with large quantities of plant pots in packs of six. These pots can be damaged easily if they are not packed carefully. The manager of the shop is a statistician and he believes that the number of damaged pots in a pack of six has a binomial distribution. He chooses a random sample of 250 packs and records the numbers of damaged pots per pack. His results are shown in the following table.

Number of damaged pots per pack (x)	0	1	2	3	4	5	6
Frequency	48	69	78	32	22	1	0

(i) Show that the mean number of damaged pots per pack in this sample is 1.656. [1]

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The following table shows some of the expected frequencies, correct to 2 decimal places, using an appropriate binomial distribution.

Number of damaged pots per pack (x)	0	1	2	3	4	5	6
Expected frequency	36.01	82.36	a	39.89	b	1.74	0.11

(ii) Find the values of a and b , correct to 2 decimal places [5]

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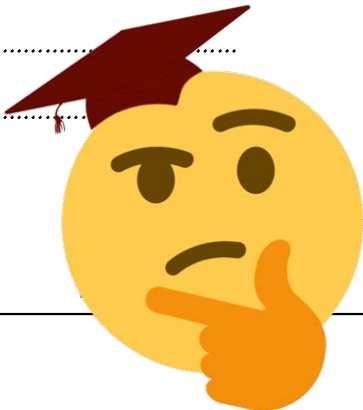
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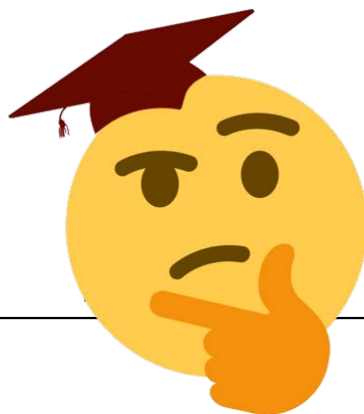
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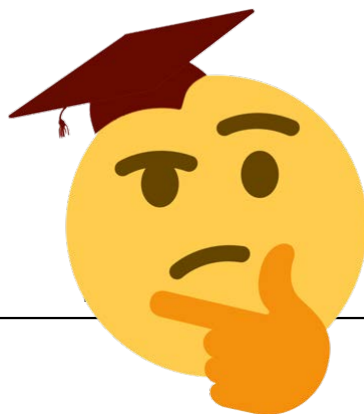


- (iii) Use a goodness-of-fit test at the 1% significance level to determine whether the manager's belief is justified. [8]



3 - (9231/21_Winter_2017_Q2) - Simple Harmonic Motion

The piston in a large engine rises and falls in simple harmonic motion. When the piston is 1.6 m below its highest level, the rate of change of its height is $\frac{3}{5}\pi$ metres per second. When the piston is 0.2 m below its highest level, the rate of change of its height is $\frac{1}{4}\pi$ metres per second. Find the amplitude and period of the motion. [7]



4 - (9231/21_Summer_2018_Q2) - *Simple Harmonic Motion*

A particle P moves on a straight line in simple harmonic motion. The centre of the motion is O . The points A and B are on the line, on opposite sides of O , with $OA = 1.6$ m and $OB = 1.2$ m. The ratio of the speed of P at A to its speed at B is 3 : 4.

- (i) Find the amplitude of the motion. [4]

The maximum speed of P during its motion is $\frac{1}{3}\pi$ m s⁻¹.

- (ii) Find the period of the motion. [2]

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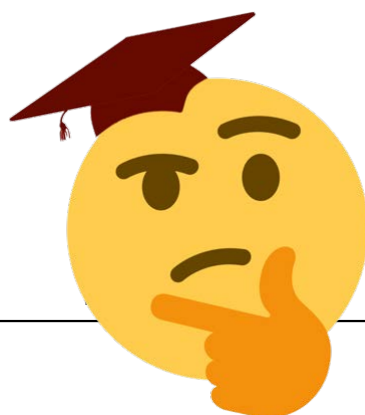
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- (iii) Find the time taken for P to travel directly from A to B . [3]



5 - (9231/23_Summer_2018_Q3) - Simple Harmonic Motion

A particle P moves on the positive x -axis in simple harmonic motion. The centre of the motion is a distance d m from the origin O , where $0 < d < 6.5$. The points A and B are on the positive x -axis, with $OA = 6.5$ m and $OB = 7.5$ m. The magnitude of the acceleration of P when it is at B is twice the magnitude of the acceleration of P when it is at A .

(i) Find d . [2]

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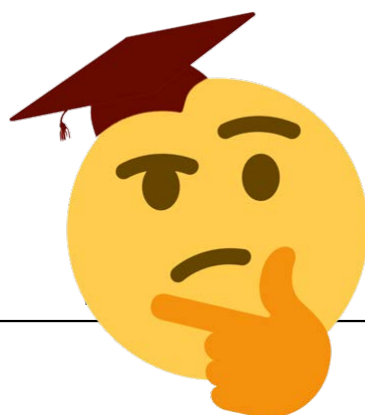
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The period of the motion is π s and the maximum acceleration of P during the motion is 10 m s^{-2} .

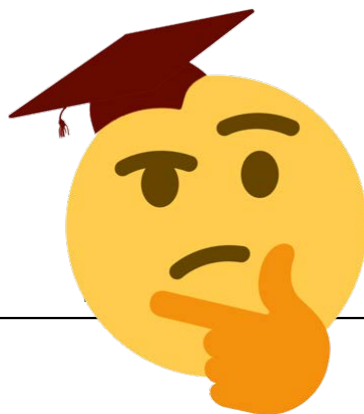
(ii) Find the speed of P when it is 7 m from O . [5]

(iii) Find the time taken by P to travel directly from A to B . [3]



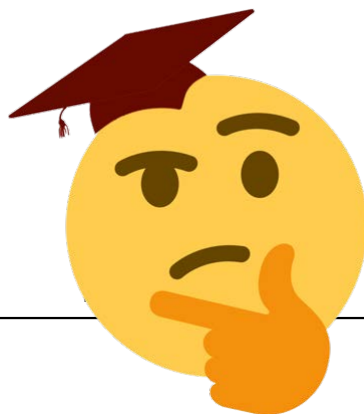
6 - (9231/21_Winter_2018_Q1) - Simple Harmonic Motion

A particle P oscillates in simple harmonic motion between the points A and B , where $AB = 6$ m. The period of the motion is $\frac{1}{2}\pi$ s. Find the speed of P when it is 2 m from B . [3]



7 - (9231/22_Winter_2018_Q1) - Simple Harmonic Motion

The point O is on the fixed horizontal line l . Points A and B on l are such that $OA = 0.1$ m and $OB = 0.5$ m, with A between O and B . A particle P oscillates on l in simple harmonic motion with centre O . The kinetic energy of P when it is at A is twice its kinetic energy when it is at B . Find the amplitude of the motion. [3]



8 - (9231/21_Winter_2018_Q5) - Simple Harmonic Motion

The fixed points A and B are on a smooth horizontal surface with $AB = 2.6$ m. One end of a light elastic spring, of natural length 1.25 m and modulus of elasticity λ N, is attached to A . The other end is attached to a particle P of mass 0.4 kg. One end of a second light elastic spring, of natural length 1.0 m and modulus of elasticity 0.6λ N, is attached to B ; its other end is attached to P . The system is in equilibrium with P on the surface at the point E .

(i) Show that $AE = 1.4$ m.

[4]

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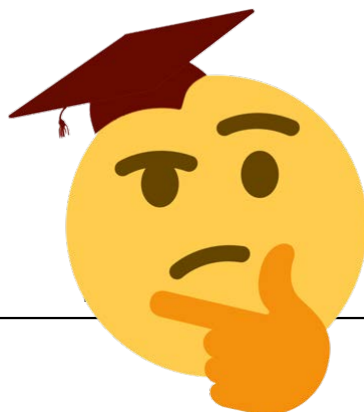
The particle P is now displaced slightly from E , along the line AB .

(ii) Show that, in the subsequent motion, P performs simple harmonic motion.

[5]

(iii) Given that the period of the motion is $\frac{1}{7}\pi$ s, find the value of λ .

[3]



9 - (9231/22_Winter_2018_Q11) - Simple Harmonic Motion, Inference Using Normal And T-distributions

Answer only **one** of the following two alternatives.

EITHER

One end of a light elastic spring, of natural length 0.8 m and modulus of elasticity 40 N, is attached to a fixed point O . The spring hangs vertically, at rest, with particles of masses 2 kg and M kg attached to its free end. The M kg particle becomes detached from the spring, and as a result the 2 kg particle begins to move upwards.

- (i) Show that the 2 kg particle performs simple harmonic motion about its equilibrium position with period $\frac{2}{3}\pi$ s. State the distance below O of the centre of the oscillations. [7]

The speed of the 2 kg particle is 0.4 m s^{-1} when its displacement from the centre of oscillation is 0.06 m.

- (ii) Find the amplitude of the motion. [3]

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- (iii) Deduce the value of M . [4]

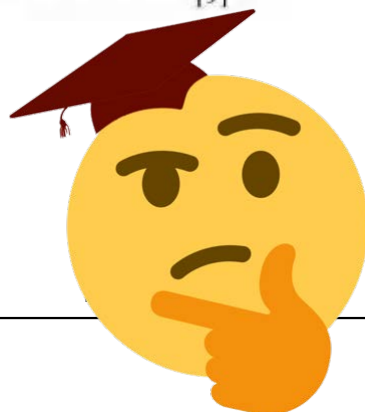
OR

In a particular country, large numbers of ducks live on lakes A and B . The mass, in kg, of a duck on lake A is denoted by x and the mass, in kg, of a duck on lake B is denoted by y . A random sample of 8 ducks is taken from lake A and a random sample of 10 ducks is taken from lake B . Their masses are summarised as follows.

$$\Sigma x = 10.56 \quad \Sigma x^2 = 14.1775 \quad \Sigma y = 12.39 \quad \Sigma y^2 = 15.894$$

A scientist claims that ducks on lake A are heavier on average than ducks on lake B .

- (i) Test, at the 10% significance level, whether the scientist's claim is justified. You should assume that both distributions are normal and that their variances are equal. [9]



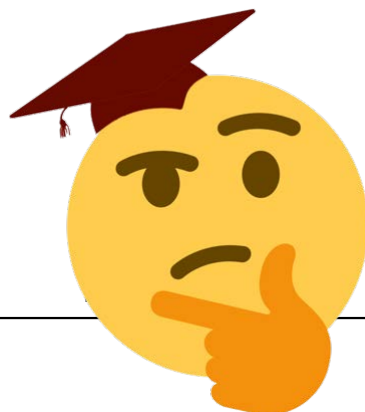
A second random sample of 8 ducks is taken from lake A and their masses are summarised as

$$\Sigma x = 10.24 \quad \text{and} \quad \Sigma (x - \bar{x})^2 = 0.294,$$

where $\bar{x}$ is the sample mean. The scientist now claims that the population mean mass of ducks on lake A is greater than p kg. A test of this claim is carried out at the 10% significance level, using only this second sample from lake A . This test supports the scientist's claim.

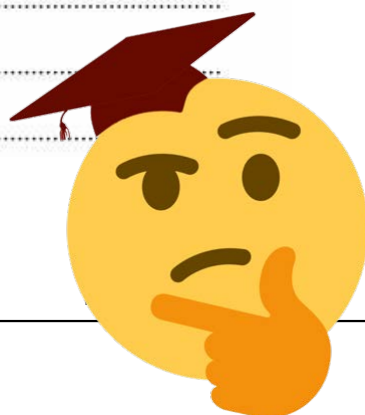
(ii) Find the greatest possible value of p .

[5]

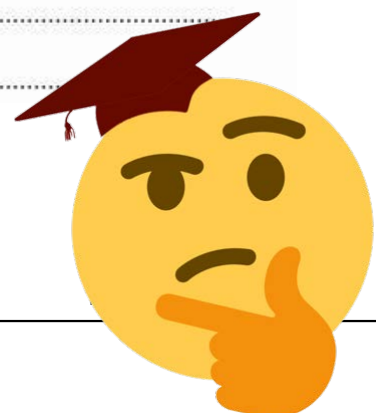


A particle P moves on a straight line in simple harmonic motion. The centre of the motion is O . The points A and B are on the line on opposite sides of O such that $OA = 3.5$ m and $OB = 1$ m. The speed of P when it is at B is twice its speed when it is at A . The maximum acceleration of P is 1 m s^{-2} .

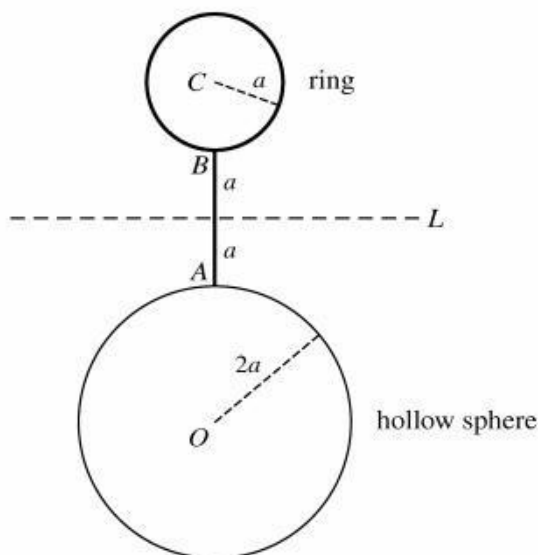
[4]

This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small, stylized illustration of a red graduation cap with a gold tassel hanging from it. The background is white, and the lines are evenly spaced across the page.

[4]

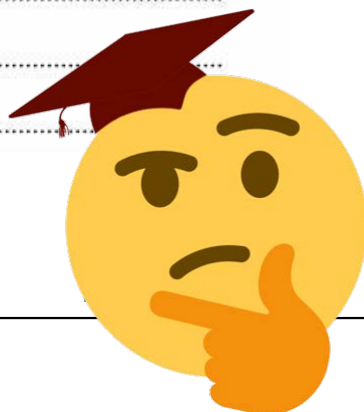
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11 - (9231/21_Summer_2019_Q5) - *Rotation Of A Rigid Body, Simple Harmonic Motion*

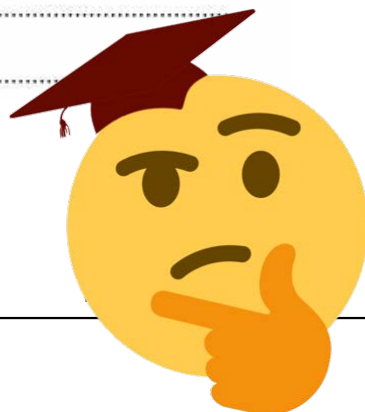


A thin uniform rod AB has mass kM and length $2a$. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere with centre O , mass kM and radius $2a$. The end B of the rod is rigidly attached to the circumference of a uniform ring with centre C , mass M and radius a . The points C, B, A, O lie in a straight line. The horizontal axis L passes through the mid-point of the rod and is perpendicular to the rod and in the plane of the ring (see diagram). The object consisting of the rod, the ring and the hollow sphere can rotate freely about L .

- (i) Show that the moment of inertia of the object about L is $\frac{3}{5}(8k + 3)Ma^2$. [6]

[illegible]

(ii) Find the set of possible values of k and the period of these oscillations in terms of k . [6]

[illegible]

12 - (9231/23_Summer_2019_Q11) - Simple Harmonic Motion, Inference Using Normal And T-distributions

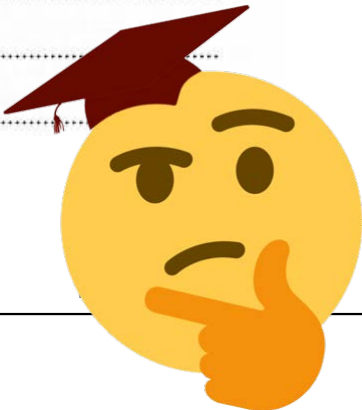
Answer only **one** of the following two alternatives.

EITHER

A light spring has natural length a and modulus of elasticity kmg . The spring lies on a smooth horizontal surface with one end attached to a fixed point O . A particle P of mass m is attached to the other end of the spring. The system is in equilibrium with $OP = a$. The particle is projected towards O with speed u and comes to instantaneous rest when $OP = \frac{3}{4}a$.

- (i) Use an energy method to show that $k = \frac{16u^2}{ag}$. [2]

- (ii) Show that P performs simple harmonic motion and find the period of this motion, giving your answer in terms of u and a . [4]



This image shows a blank sheet of white paper designed for handwriting practice. It features multiple horizontal rows, each defined by two dashed lines. The paper is otherwise empty, with no text or markings. In the bottom right corner, there is a decorative graphic element made of overlapping red and yellow shapes, resembling stylized leaves or petals.



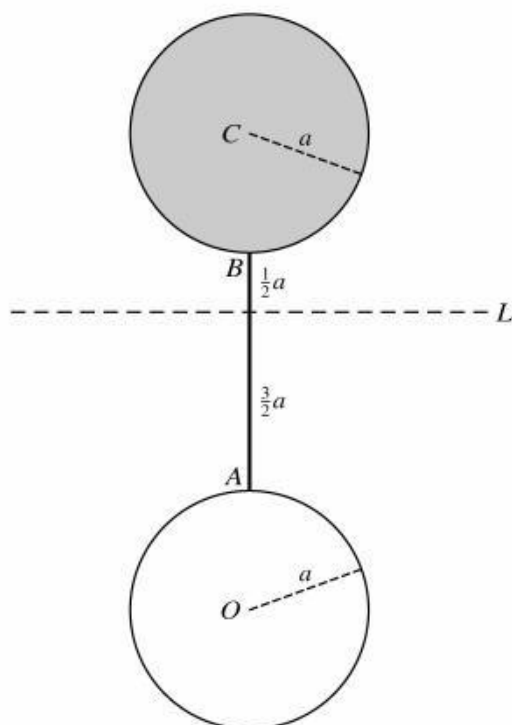
A company produces packets of sweets. Two different machines, *A* and *B*, are used to fill the packets. The manager decides to assess the performance of the two machines. He selects a random sample of 50 packets filled by machine *A* and a random sample of 60 packets filled by machine *B*. The masses of sweets, x kg, in packets filled by machine *A* and the masses of sweets, y kg, in packets filled by machine *B* are summarised as follows.

A test at the $\alpha\%$ significance level provides evidence that the mean mass of sweets in packets filled by machine A is less than the mean mass of sweets in packets filled by machine B. Find the set of possible values of α . [12]

[illegible]



13 - (9231/21_Winter_2019_Q5) - Rotation Of A Rigid Body, Simple Harmonic Motion

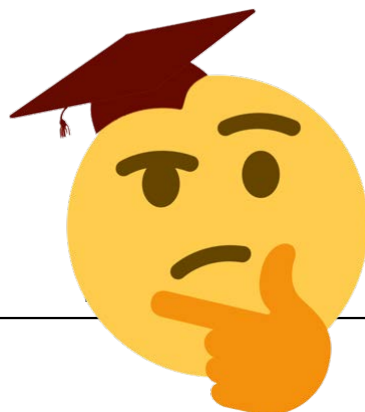


A thin uniform rod AB has mass λM and length $2a$. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere (spherical shell) with centre O , mass $3M$ and radius a . The end B of the rod is rigidly attached to the surface of a uniform solid sphere with centre C , mass $5M$ and radius a . The rod lies along the line joining the centres of the spheres, so that $CBAO$ is a straight line. The horizontal axis L is perpendicular to the rod and passes through the point of the rod that is a distance $\frac{1}{2}a$ from B (see diagram). The object consisting of the rod and the two spheres can rotate freely about L .

- (i) Show that the moment of inertia of the object about L is $\left(\frac{408 + 7\lambda}{12}\right)Ma^2$. [6]

The period of small oscillations of the object about L is $5\pi\sqrt{\left(\frac{2a}{g}\right)}$.

- (ii) Find the value of λ . [6]



14 - (9231/21_Winter_2019_Q11) - Simple Harmonic Motion, X2 Test

Answer only **one** of the following two alternatives.

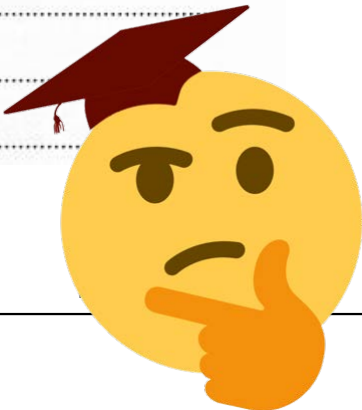
EITHER

The points A and B are a distance 1.2 m apart on a smooth horizontal surface. A particle P of mass $\frac{2}{3}\text{ kg}$ is attached to one end of a light spring of natural length 0.6 m and modulus of elasticity 10 N . The other end of the spring is attached to the point A . A second light spring, of natural length 0.4 m and modulus of elasticity 20 N , has one end attached to P and the other end attached to B .

(i) Show that when P is in equilibrium $AP = 0.75\text{ m}$. [3]

The particle P is displaced by 0.05 m from the equilibrium position towards A and then released from rest.

(ii) Show that P performs simple harmonic motion and state the period of the motion. [6]



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(iii) Find the speed of P when it passes through the equilibrium position. [2]

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(iv) Find the speed of P when its acceleration is equal to half of its maximum value. [3]

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OR

The number of puncture repairs carried out each week by a small repair shop is recorded over a period of 40 weeks. The results are shown in the following table.

Number of repairs in a week	0	1	2	3	4	5	≥ 6
Number of weeks	6	15	9	6	3	1	0

- (i) Calculate the mean and variance for the number of repairs in a week and comment on the possible suitability of a Poisson distribution to model the data. [3]

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Records over a longer period of time indicate that the mean number of repairs in a week is 1.6. The following table shows some of the expected frequencies, correct to 3 decimal places, for a period of 40 weeks using a Poisson distribution with mean 1.6.

Number of repairs in a week	0	1	2	3	4	5	≥ 6
Expected frequency	8.076	12.921	10.337	5.513	2.205	a	b

- (ii) Show that $a = 0.706$ and find the value of the constant b . [3]

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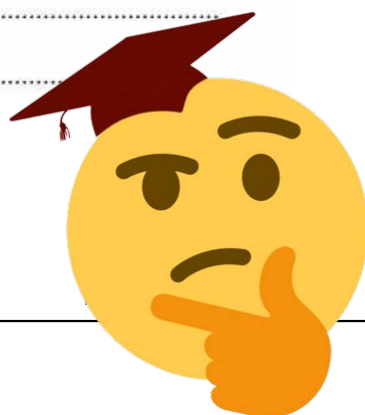
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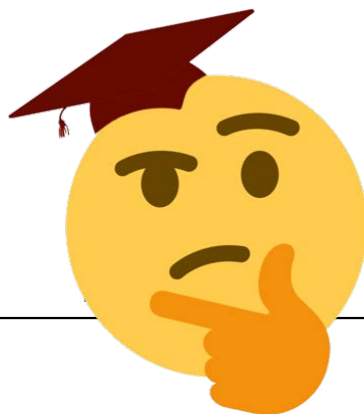
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15 - (9231/31_Summer_2020_Q3) - *Simple Harmonic Motion*

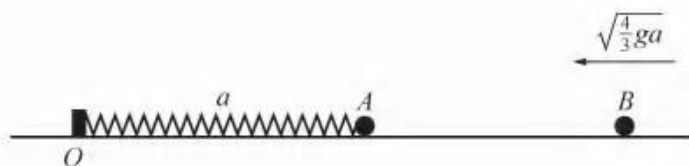
One end of a light elastic spring, of natural length a and modulus of elasticity $5mg$, is attached to a fixed point A . The other end of the spring is attached to a particle P of mass m . The spring hangs with P vertically below A . The particle P is released from rest in the position where the extension of the spring is $\frac{1}{2}a$.

(a) Show that the initial acceleration of P is $\frac{3}{2}g$ upwards. [3]

(b) Find the speed of P when the spring first returns to its natural length. [4]



16 - (9231/33_Summer_2020_Q7) - Simple Harmonic Motion



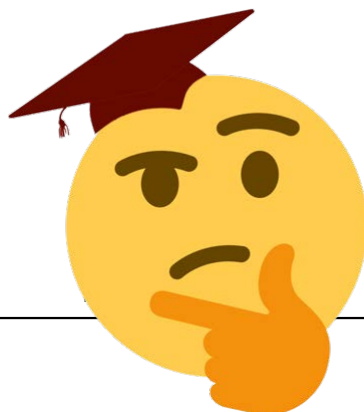
One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O . The other end of the spring is attached to a particle A of mass km , where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a . A second particle B , of mass m , is moving towards A with speed $\sqrt{\frac{4}{3}ga}$ along the line of the spring from the opposite direction to O (see diagram).

The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

(a) Find the value of k . [6]

At the point C the horizontal surface becomes rough, with coefficient of friction μ between the combined particle and the surface. The deceleration of the combined particle at C is $\frac{9}{20}g$.

(b) Find the value of μ . [4]

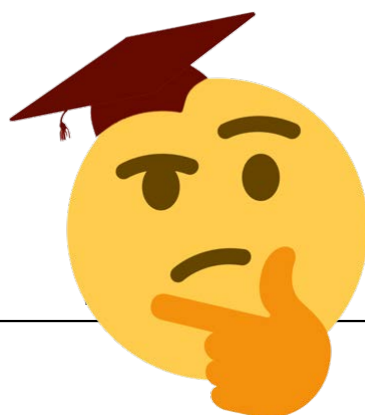


17 - (9231/33_Winter_2021_Q3) - *Simple Harmonic Motion*

A light elastic string has natural length a and modulus of elasticity $12mg$. One end of the string is attached to a fixed point O . The other end of the string is attached to a particle of mass m . The particle hangs in equilibrium vertically below O . The particle is pulled vertically down and released from rest with the extension of the string equal to e , where $e > \frac{1}{3}a$. In the subsequent motion the particle has speed $\sqrt{2ga}$ when it has ascended a distance $\frac{1}{3}a$.

Find e in terms of a .

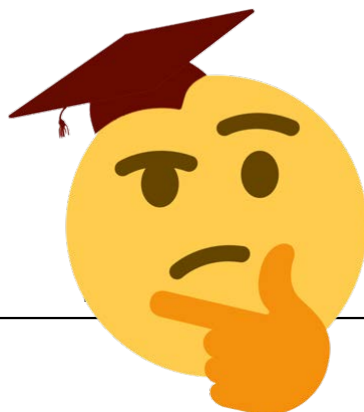
[6]



18 - (9231/32_Winter_2021_Q7) - Simple Harmonic Motion

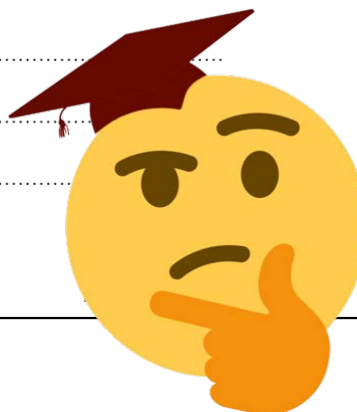
One end of a light inextensible string of length a is attached to a fixed point O . The other end of the string is attached to a particle P of mass m . The particle P is held vertically below O with the string taut and then projected horizontally. When the string makes an angle of 60° with the upward vertical, P becomes detached from the string. In its subsequent motion, P passes through the point A which is a distance a vertically above O .

- (a) The speed of P when it becomes detached from the string is V . Use the equation of the trajectory of a projectile to find V in terms of a and g . [4]
- (b) Find, in terms of m and g , the tension in the string immediately after P is initially projected horizontally. [4]

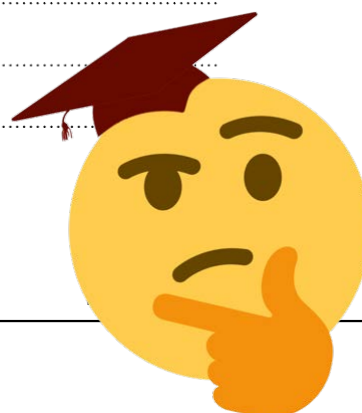


A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The string is held taut with OP horizontal. The particle P is projected vertically downwards with speed $\sqrt{\frac{1}{3}ag}$ and starts to move in a vertical circle. P passes through the lowest point of the circle and reaches the point Q where OQ makes an angle θ with the downward vertical. At Q the speed of P is $\sqrt{kag}$ and the tension in the string is $\frac{11}{6}mg$.

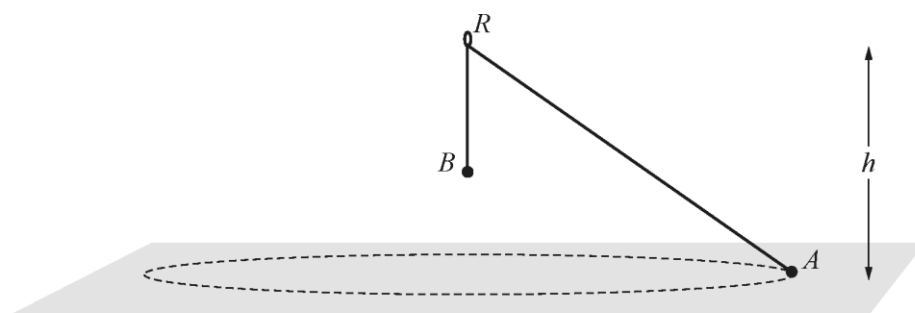
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- This image shows a blank sheet of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page. In the bottom right corner, there is a small, stylized illustration of a yellow smiley face with a red graduation cap (mortarboard) on its head. The cap has a black tassel hanging from it. The smiley face itself is simple, with two dots for eyes and a curved line for a mouth. The entire illustration is located in the lower right quadrant of the page.



(b) In the subsequent motion, find the greatest height reached by P above the level of the lowest point of the circle. [4]

This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small, stylized illustration of a black graduation cap (mortarboard) resting on a yellow rolled-up diploma. The rest of the page is empty and ready for writing.

20 – (9231/32_Winter_2022_Q6) – Simple Harmonic Motion



A light inextensible string is threaded through a fixed smooth ring R which is at a height h above a smooth horizontal surface. One end of the string is attached to a particle A of mass m . The other end of the string is attached to a particle B of mass $\frac{6}{7}m$. The particle A moves in a horizontal circle on the surface. The particle B hangs in equilibrium below the ring and above the surface (see diagram).

When A has constant angular speed ω , the angle between AR and BR is θ and the normal reaction between A and the surface is N .

When A has constant angular speed $\frac{3}{2}\omega$, the angle between AR and BR is α and the normal reaction between A and the surface is $\frac{1}{2}N$.

(a) Show that $\cos \theta = \frac{4}{9} \cos \alpha$. [5]

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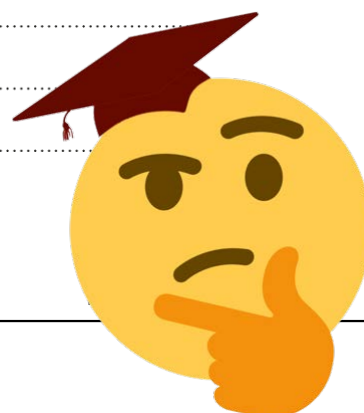
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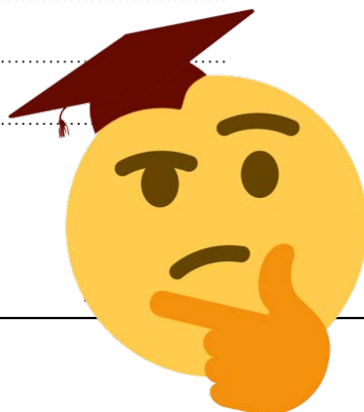
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21 – (9231/31_Summer_2023_Q1) – Simple Harmonic Motion, Hook’s Law

One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O . The other end of the string is attached to a particle P of mass m . The string hangs with P vertically below O . The particle P is pulled vertically downwards so that the extension of the string is $2a$. The particle P is then released from rest.

(a) Find the speed of P when it is at a distance $\frac{3}{4}a$ below O . [3]

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(b) Find the initial acceleration of P when it is released from rest. [2]

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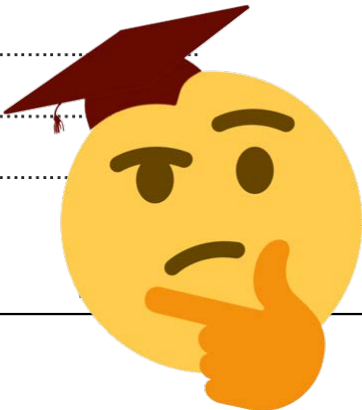
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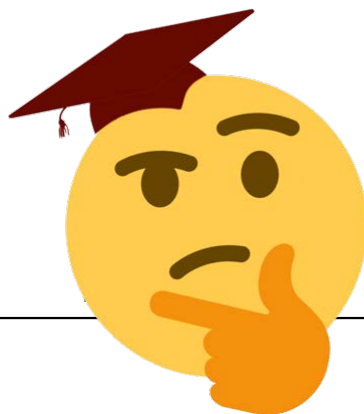


22 – (9231/33_Summer_2023_Q1) – Circular Motion, Simple Harmonic Motion, Hook's Law

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is held at the point A , where OA makes an angle α with the downward vertical through O , and with the string taut. The particle P is projected perpendicular to OA in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane. The string goes slack when P is at B , where OB makes an angle θ with the upward vertical.

Given that $\cos \alpha = \frac{4}{5}$, find the value of $\cos \theta$.

[4]

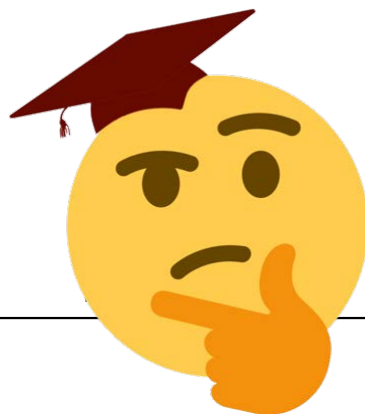


1 - (9231/31_Summer_2020_Q1) - *Projectile Motion*

A particle P is projected with speed u at an angle of 30° above the horizontal from a point O on a horizontal plane and moves freely under gravity. The particle reaches its greatest height at time T after projection.

Find, in terms of u , the speed of P at time $\frac{2}{3}T$ after projection.

[5]

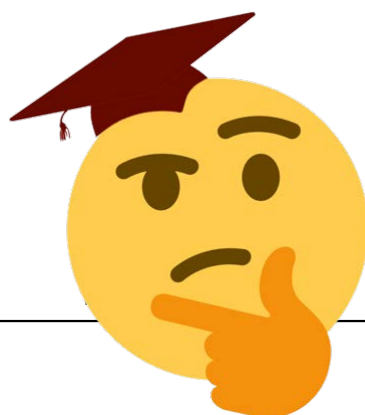


2 - (9231/33_Summer_2020_Q6) - Projectile Motion

A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The direction of motion of P makes an angle α above the horizontal when P first reaches three-quarters of its greatest height.

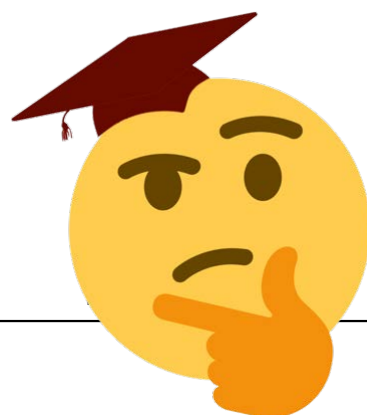
(a) Show that $\tan \alpha = \frac{1}{2} \tan \theta$. [6]

(b) Given that $\tan \theta = \frac{4}{3}$, find the horizontal distance travelled by P when it first reaches three-quarters of its greatest height. Give your answer in terms of u and g . [4]



(c) Find the other value of α for which P would pass through the point Q .

[4]



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5 - (9231/31_Summer_2021_Q7) - Projectile Motion

A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is 100 ms^{-1} at an angle θ above the horizontal, where $\tan \theta = \frac{4}{3}$. The two times at which P 's height above the plane is H m differ by 10 s.

(a) Find the value of H . [5]

(b) Find the magnitude and direction of the velocity of P one second before it strikes the plane. [4]



6 - (9231/33_Summer_2021_Q7) - Projectile Motion

A particle P is projected with speed u at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

- (a) Use the equation of the trajectory given in the List of formulae (MF19), together with the condition $y = 0$, to establish an expression for the range R in terms of u , θ and g . [2]

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- (b) Deduce an expression for the maximum height H , in terms of u , θ and g . [2]

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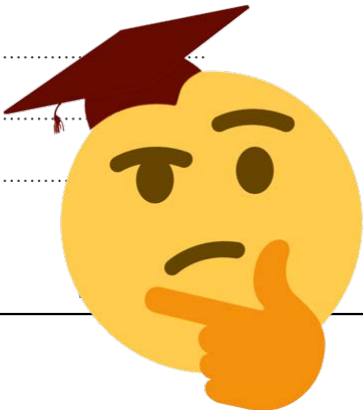
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It is given that $R = \frac{4H}{\sqrt{3}}$.

(c) Show that $\theta = 60^\circ$. [1]

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It is given also that $u = \sqrt{40}\text{ ms}^{-1}$.

(d) Find, by differentiating the equation of the trajectory or otherwise, the set of values of x for which the direction of motion makes an angle of less than 45° with the horizontal. [4]

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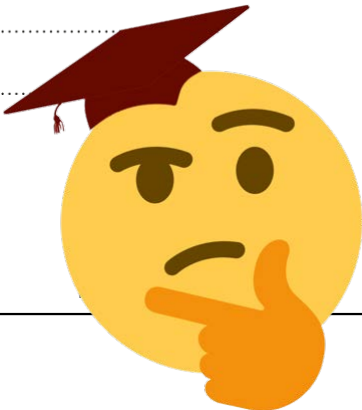
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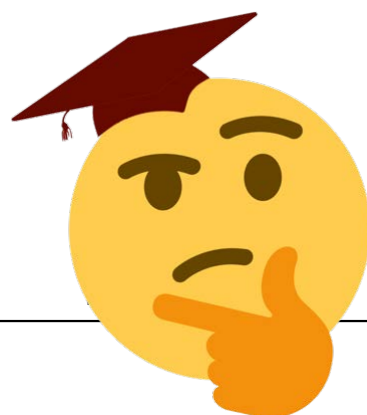


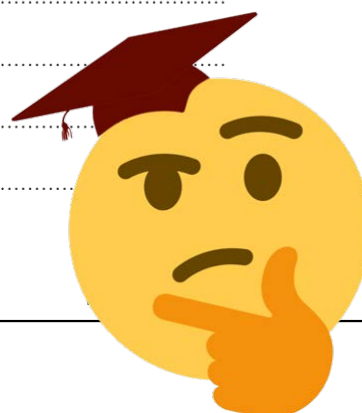
7 - (9231/33_Summer_2022_Q3) - *Projectile Motion*

A particle P is projected with speed 25 m s^{-1} at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. After 2 s the speed of P is 15 m s^{-1} .

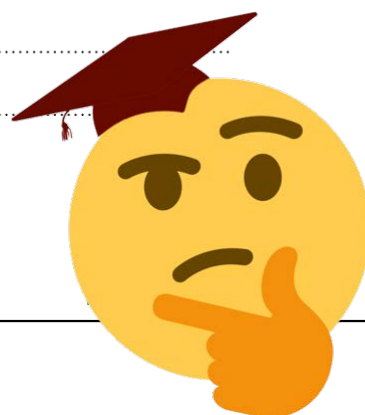
(a) Find the value of $\sin \theta$. [5]

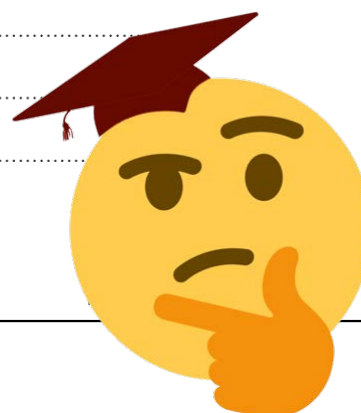
(b) Find the range of the flight. [3]



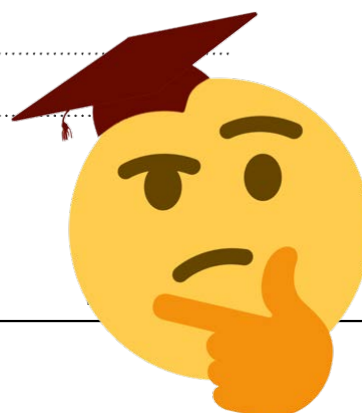
$$y = x \tan \theta - \frac{gx^2}{2u^2}(1 + \tan^2 \theta). \quad [4]$$
[illegible]

(b) Given that one possible value of $\tan \theta$ is $\frac{4}{3}$, find the other possible value of $\tan \theta$. [5]



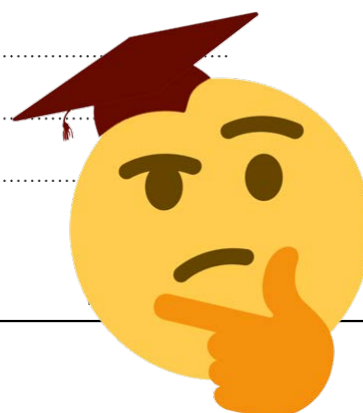
A decorative graphic in the bottom right corner featuring a maroon graduation cap with a gold tassel, positioned above a gold, cloud-like shape.

[7]

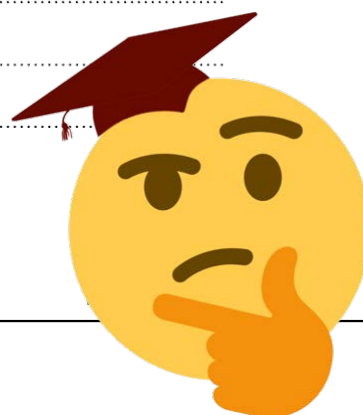
This image shows a blank sheet of white paper with horizontal dotted lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. In the bottom right corner, there is a small illustration of a red pencil pointing towards the left. The rest of the page is empty, providing space for writing practice.

A ball of mass 2 kg is projected vertically downwards with speed 5 ms^{-1} through a liquid. At time $t\text{ s}$ after projection, the velocity of the ball is $v\text{ ms}^{-1}$ and its displacement from its starting point is $x\text{ m}$. The forces acting on the ball are its weight and a resistive force of magnitude $0.2v^2\text{ N}$.

(a) Find an expression for v in terms of t . [6]

[illegible]

[1]



11 - (9231/31_Winter_2023_Q5) - *Projectile Motion*

A particle P is projected with speed $u \text{ ms}^{-1}$ at an angle θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. During its flight P passes through the point which is a horizontal distance $3a$ from O and a vertical distance $\frac{3}{8}a$ above the horizontal plane. It is given that $\tan \theta = \frac{1}{3}$.

- (a) Show that $u^2 = 8ag$. [2]

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A particle Q is projected with speed $V \text{ ms}^{-1}$ at an angle α above the horizontal from O at the instant when P is at its highest point. Particles P and Q both land at the same point on the horizontal plane at the same time.

- (b) Find V in terms of a and g . [7]

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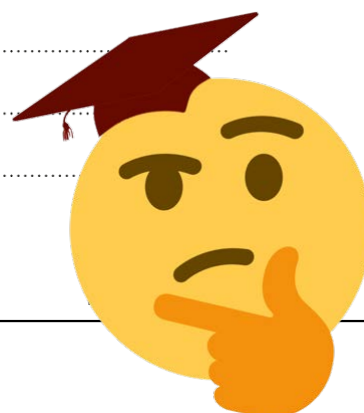
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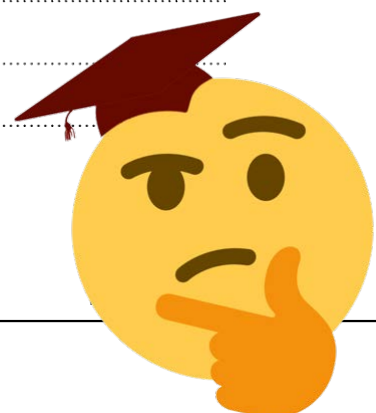
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This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small, stylized illustration of a red graduation cap with a gold tassel hanging from it. The rest of the page is empty.

12 – (9231/31_Winter_2023_Q6) – Projectile Motion

A particle P of mass m is attached to one end of a light inextensible rod of length $3a$. An identical particle Q is attached to the other end of the rod. The rod is smoothly pivoted at a point O on the rod, where $OQ = x$. The system, of rod and particles, rotates about O in a vertical plane.

At an instant when the rod is vertical, with P above Q , the particle P is moving horizontally with speed u . When the rod has turned through an angle of 60° from the vertical, the speed of P is $2\sqrt{ag}$, and the tensions in the two parts of the rod, OP and OQ , have equal magnitudes.

- (a) Show that the speed of Q when the rod has turned through an angle of 60° from the vertical is $\frac{2x}{3a-x}\sqrt{ag}$. [2]

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- (b) Find x in terms of a . [5]

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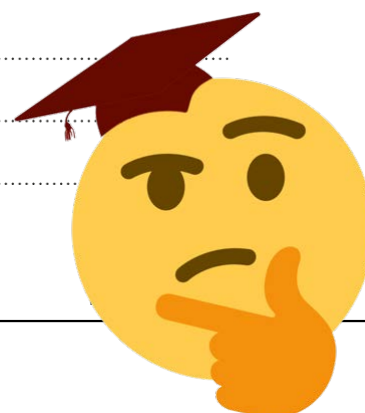
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(c) Find u in terms of a and g .

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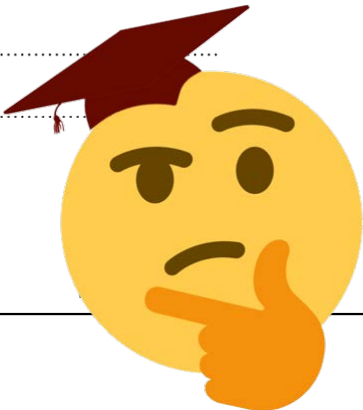
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13 - (9231/32_Winter_2023_Q6) - *Projectile Motion*

A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

- (a) Derive the equation of the trajectory of P in the form

$$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha. \quad [3]$$

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During its flight, P must clear an obstacle of height h m that is at a horizontal distance of 32 m from the point of projection. When $u = 40\sqrt{2} \text{ ms}^{-1}$, P just clears the obstacle. When $u = 40 \text{ ms}^{-1}$, P only achieves 80% of the height required to clear the obstacle.

- (b) Find the two possible values of h . [6]

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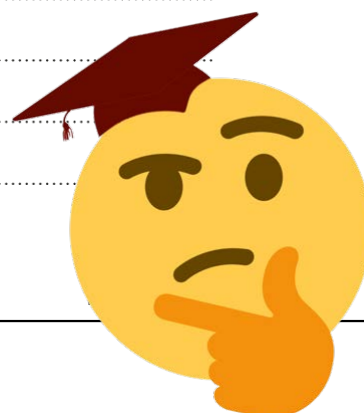
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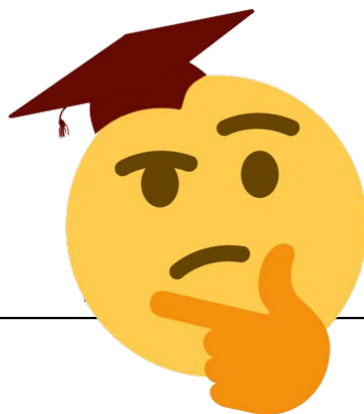


1 - (9231/33_Summer_2020_Q2) - Linear Motion Under Variable Force

A particle Q of mass m kg falls from rest under gravity. The motion of Q is resisted by a force of magnitude mkv N, where v ms⁻¹ is the speed of Q at time t s and k is a positive constant.

Find an expression for v in terms of g , k and t .

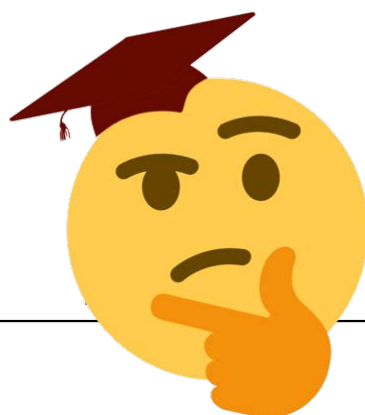
[6]



2 - (9231/31_Summer_2020_Q5) - Linear Motion Under Variable Force

A particle P is moving along a straight line with acceleration $3ku - kv$ where v is its velocity at time t , u is its initial velocity and k is a constant. The velocity and acceleration of P are both in the direction of increasing displacement from the initial position.

- (a) Find the time taken for P to achieve a velocity of $2u$. [3]
- (b) Find an expression for the displacement of P from its initial position when its velocity is $2u$. [5]

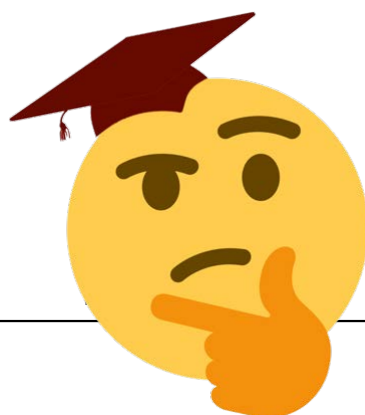


3 - (9231/31_Winter_2020_Q7) - *Linear Motion Under Variable Force*

A particle P moving in a straight line has displacement x m from a fixed point O on the line at time t s. The acceleration of P , in m s^{-2} , is given by $\frac{200}{x^2} - \frac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and P has velocity 10 m s^{-1} directed towards O .

(a) Show that the velocity $v \text{ m s}^{-1}$ of P is given by $v = \frac{10(1-2x)}{x}$. [5]

(b) Show that x and t are related by the equation $e^{-40t} = (2x-1)e^{2x-2}$ and deduce what happens to x as t becomes large. [5]



4 - (9231/32_Winter_2020_Q7) - Linear Motion Under Variable Force

A particle P of mass m kg moves in a horizontal straight line against a resistive force of magnitude mkv^2 N, where v ms⁻¹ is the speed of P after it has moved a distance x m and k is a positive constant. The initial speed of P is u ms⁻¹.

- (a) Show that $x = \frac{1}{k} \ln 2$ when $v = \frac{1}{2}u$. [4]

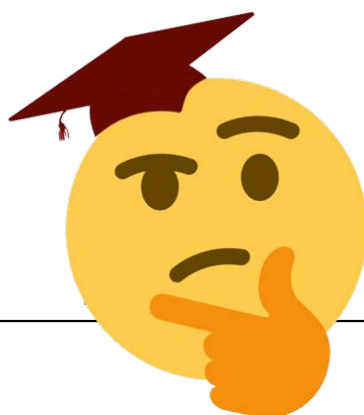
Beginning at the instant when the speed of P is $\frac{1}{2}u$, an additional force acts on P . This force has magnitude $\frac{5m}{v}$ N and acts in the direction of increasing x .

- (b) Show that when the speed of P has increased again to u ms⁻¹, the total distance travelled by P is given by an expression of the form

$$\frac{1}{3k} \ln \left(\frac{A - ku^3}{B - ku^3} \right),$$

stating the values of the constants A and B .

[7]

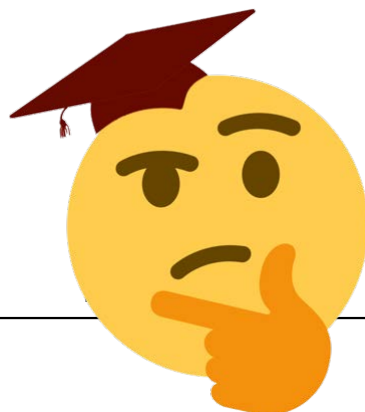


5 - (9231/31_Summer_2021_Q1) - *Linear Motion Under Variable Force*

A particle P of mass 1 kg is moving along a straight line against a resistive force of magnitude $\frac{10\sqrt{v}}{(t+1)^2}\text{ N}$, where $v\text{ m s}^{-1}$ is the speed of P at time $t\text{ s}$. When $t = 0$, $v = 25$.

Find an expression for v in terms of t .

[5]

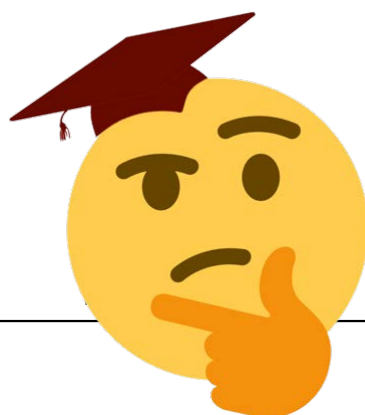


6 - (9231/33_Summer_2021_Q5) - *Linear Motion Under Variable Force*

A particle P of mass m kg is projected vertically upwards from a point O , with speed 20 ms^{-1} , and moves under gravity. There is a resistive force of magnitude $2mv$ N, where $v\text{ ms}^{-1}$ is the speed of P at time t s after projection.

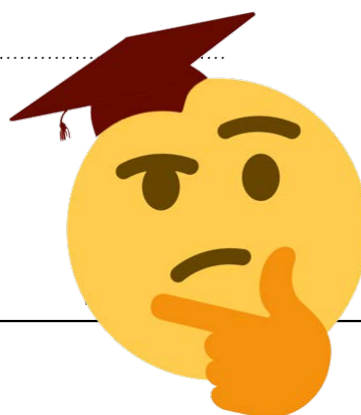
(a) Find an expression for v in terms of t , while P is moving upwards.

[6]



(b) Find an expression for x in terms of t , while P is moving upwards. [2]

(c) Find, correct to 3 significant figures, the greatest height above O reached by P . [2]

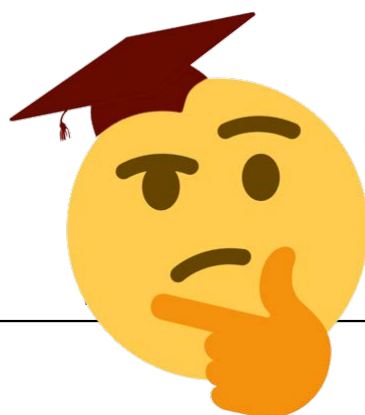


7 - (9231/31_Summer_2022_Q3) - Linear Motion Under Variable Force

A particle P is moving in a horizontal straight line. Initially P is at the point O on the line and is moving with velocity 25 m s^{-1} . At time $t \text{ s}$ after passing through O , the acceleration of P is $\frac{4000}{(5t+4)^3} \text{ m s}^{-2}$ in the direction PO . The displacement of P from O at time t is $x \text{ m}$.

Find an expression for x in terms of t .

[5]

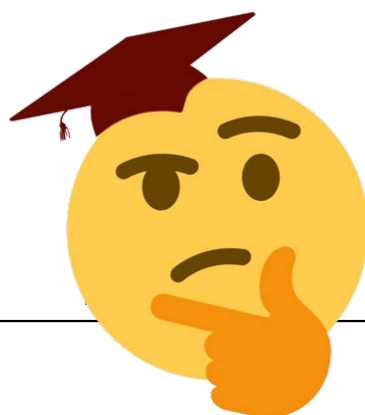


8 - (9231/33_Summer_2022_Q5) - *Linear Motion Under Variable Force*

A particle P of mass 4 kg is moving in a horizontal straight line. At time $t\text{ s}$ the velocity of P is $v\text{ ms}^{-1}$ and the displacement of P from a fixed point O on the line is $x\text{ m}$. The only force acting on P is a resistive force of magnitude $(4e^{-x} + 12)e^{-x}\text{ N}$. When $t = 0$, $x = 0$ and $v = 4$.

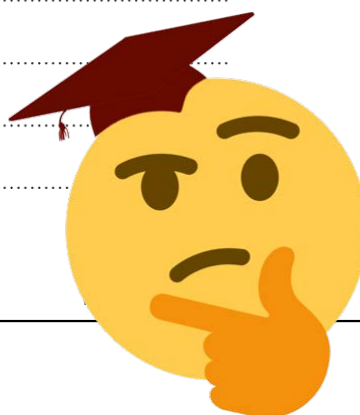
(a) Show by integration that $v = \frac{1 + 3e^x}{e^x}$. [4]

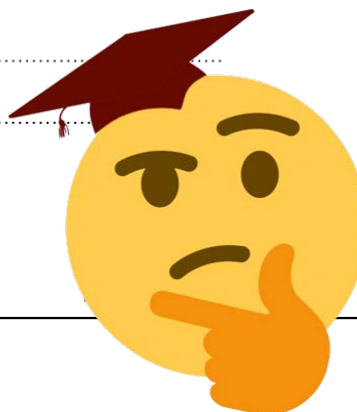
(b) Find an expression for x in terms of t . [4]



A particle P of mass 5 kg moves along a horizontal straight line. At time $t\text{ s}$, the velocity of P is $v\text{ m s}^{-1}$ and its displacement from a fixed point O on the line is $x\text{ m}$. The forces acting on P are a force of magnitude $\frac{500}{v}\text{ N}$ in the direction OP and a resistive force of magnitude $\frac{1}{2}v^2\text{ N}$. When $t = 0$, $x = 0$ and $v = 5$.

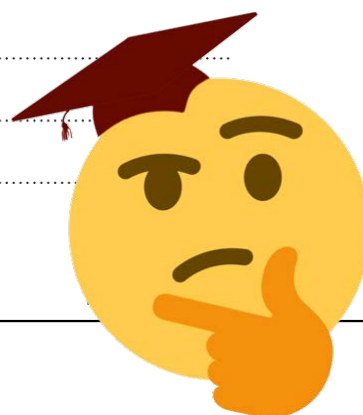
(a) Find an expression for v in terms of x . [6]

[illegible]



A particle of mass 0.5 kg moves along a horizontal straight line. Its velocity is $v \text{ ms}^{-1}$ at time $t \text{ s}$. The forces acting on the particle are a driving force of magnitude 50 N and a resistance of magnitude $2v^2 \text{ N}$. The initial velocity of the particle is 3 ms^{-1} .

(a) Find an expression for v in terms of t . [7]

[illegible]

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(b) Deduce the limiting value of v .

[1]

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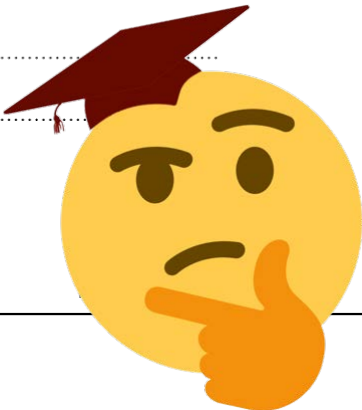
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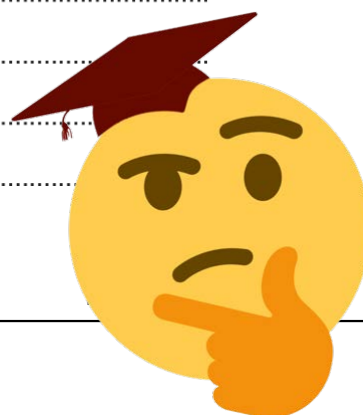
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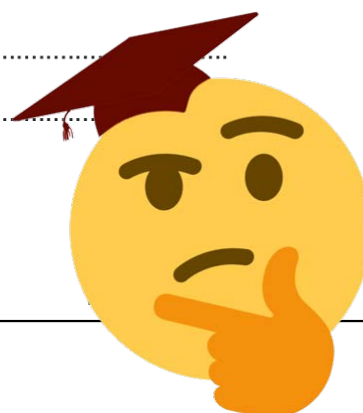


A particle P moving in a straight line has displacement x m from a fixed point O on the line and velocity v ms⁻¹ at time t s. The acceleration of P , in ms⁻², is given by $6v\sqrt{v+9}$. When $t = 0$, $x = 2$ and $v = 72$.

(a) Find an expression for v in terms of x . [4]

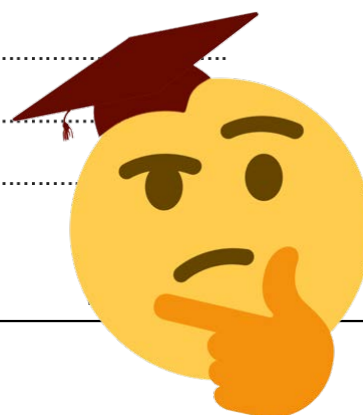
This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. In the bottom right corner, there is a small, stylized illustration of a yellow smiley face wearing a red graduation cap with a tassel. The overall appearance is that of a clean, unused piece of stationery or a worksheet.

[5]

[illegible]

A particle of mass m kg falls vertically under gravity, from rest. At time t s, P has fallen x m and has velocity v m s⁻¹. The only forces acting on P are its weight and a resistance of magnitude $kmgv$ N, where k is a constant.

(a) Find an expression for v in terms of t , g and k . [5]

This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small, stylized illustration of a yellow smiley face wearing a red graduation cap with a tassel. The rest of the page is empty.

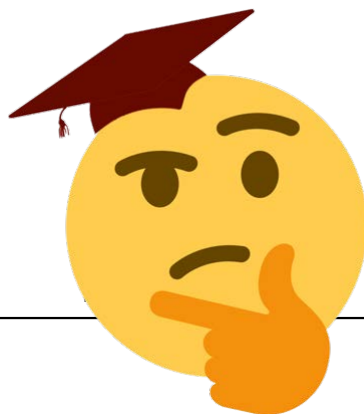
This image shows a blank sheet of white paper with horizontal blue ruling lines. In the bottom right corner, there is a small graphic of a red graduation cap with a yellow tassel hanging from it. The rest of the page is empty.



13 - (9231/32_Winter_2023_Q2) - *Linear Motion Under Variable Force*

A particle P of mass 0.5 kg moves in a straight line. At time $t\text{ s}$ the velocity of P is $v\text{ ms}^{-1}$ and its displacement from a fixed point O on the line is $x\text{ m}$. The only forces acting on P are a force of magnitude $\frac{150}{(x+1)^2}\text{ N}$ in the direction of increasing displacement and a resistive force of magnitude $\frac{450}{(x+1)^3}\text{ N}$. When $t = 0$, $x = 0$ and $v = 20$.

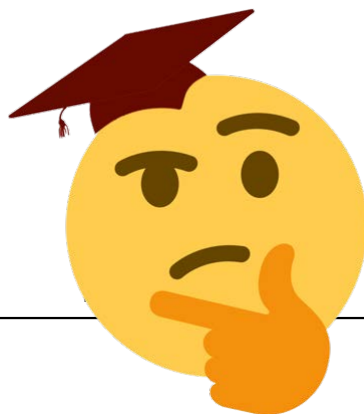
Find v in terms of x , giving your answer in the form $v = \frac{Ax+B}{(x+1)}$, where A and B are constants to be determined. [6]



1 - (9231/31_Winter_2020_Q1) - *Hook's Law*

A particle P of mass m is placed on a fixed smooth plane which is inclined at an angle θ to the horizontal. A light spring, of natural length a and modulus of elasticity $3mg$, has one end attached to P and the other end attached to a fixed point O at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.

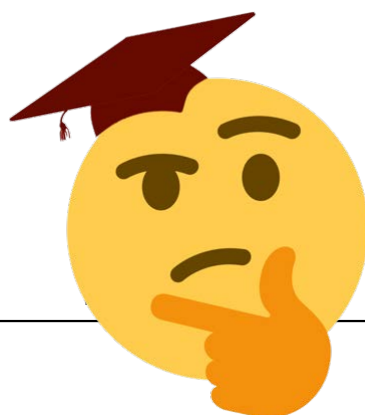
Find, in terms of a and θ , an expression for the greatest extension of the spring in the subsequent motion. [3]



2 - (9231/32_Winter_2020_Q6) - Hook's Law

One end of a light elastic string, of natural length a and modulus of elasticity k , is attached to a particle P of mass m . The other end of the string is attached to a fixed point Q . The particle P is projected vertically upwards from Q . When P is moving upwards and at a distance $\frac{4}{3}a$ directly above Q , it has a speed $\sqrt{2ga}$. At this point, its acceleration is $\frac{7}{3}g$ downwards.

Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by P . [8]

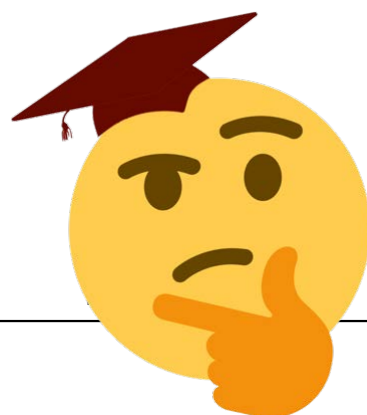


3 - (9231/33_Summer_2021_Q2) - *Hook's Law*

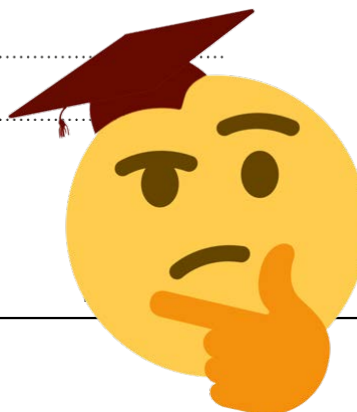
One end of a light elastic string of natural length 0.8 m and modulus of elasticity 36 N is attached to a fixed point O on a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of O . The particle is projected with speed $\sqrt{2}\text{ ms}^{-1}$ directly down the plane from the position where OP is equal to the natural length of the string.

Find the maximum extension of the string during the subsequent motion.

[5]



One end of a light elastic string, of natural length a and modulus of elasticity kmg , is attached to a fixed point A . The other end of the string is attached to a particle P of mass $4m$. The particle P hangs in equilibrium a distance x vertically below A .

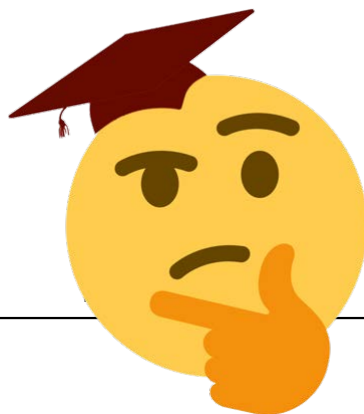


5 - (9231/31_Winter_2021_Q1) - *Hook's Law*

One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O on a smooth horizontal plane. A particle P of mass m is attached to the other end of the string and moves in a horizontal circle with centre O . The speed of P is $\sqrt{\frac{4}{3}ga}$.

Find the extension of the string.

[4]



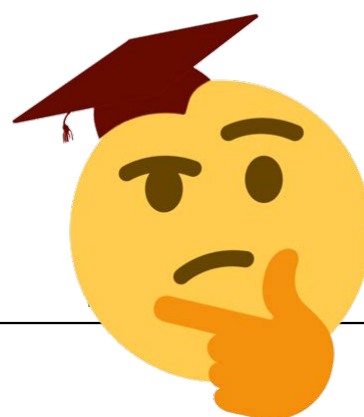
6 - (9231/32_Winter_2021_Q2) - Hook's Law

A light spring AB has natural length a and modulus of elasticity $5mg$. The end A of the spring is attached to a fixed point on a smooth horizontal surface. A particle P of mass m is attached to the end B of the spring. The spring and particle P are at rest on the surface.

Another particle Q of mass km is moving with speed $\sqrt{4ga}$ along the horizontal surface towards P in the direction BA . The particles P and Q collide directly and coalesce. In the subsequent motion the greatest amount by which the spring is compressed is $\frac{1}{5}a$.

Find the value of k .

[6]

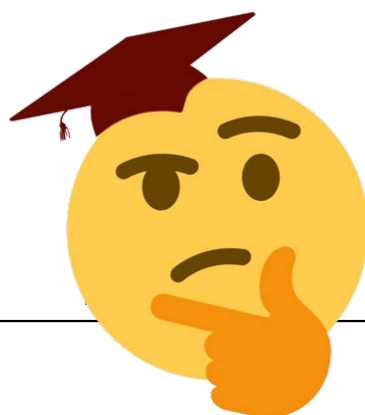


7 - (9231/31_Winter_2021_Q3) - Hook's Law

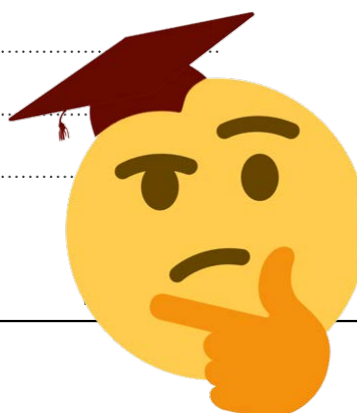
A light elastic string has natural length a and modulus of elasticity $12mg$. One end of the string is attached to a fixed point O . The other end of the string is attached to a particle of mass m . The particle hangs in equilibrium vertically below O . The particle is pulled vertically down and released from rest with the extension of the string equal to e , where $e > \frac{1}{3}a$. In the subsequent motion the particle has speed $\sqrt{2ga}$ when it has ascended a distance $\frac{1}{3}a$.

Find e in terms of a .

[6]



A diagram showing a rod of length 1.0 m pivoted at point A. A horizontal force of 7.5 N is applied at point B, perpendicular to the rod. The rod makes an angle θ with the horizontal.

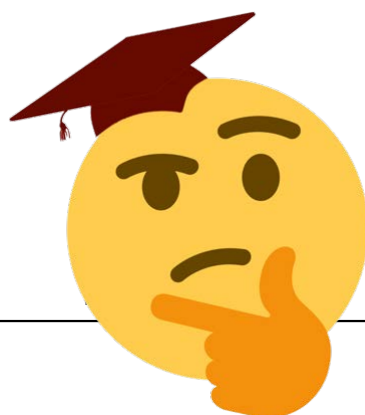


9 - (9231/33_Summer_2022_Q2) - *Hook's Law*

A particle P of mass m is attached to one end of a light elastic string of natural length a and modulus of elasticity $\frac{4}{3}mg$. The other end of the string is attached to a fixed point O on a rough horizontal surface. The particle is at rest on the surface with the string at its natural length. The coefficient of friction between P and the surface is $\frac{1}{3}$. The particle is projected along the surface in the direction OP with a speed of $\frac{1}{2}\sqrt{ga}$.

Find the greatest extension of the string during the subsequent motion.

[5]



10 – (9231/31_Winter_2022_Q2) – *Hook's Law*

A light elastic string has natural length a and modulus of elasticity $4mg$. One end of the string is fixed to a point O on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected along the surface in the direction OP . When the length of the string is $\frac{5}{4}a$, the speed of P is v . When the length of the string is $\frac{3}{2}a$, the speed of P is $\frac{1}{2}v$.

- (a) Find an expression for v in terms of a and g . [4]

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- (b) Find, in terms of g , the acceleration of P when the stretched length of the string is $\frac{3}{2}a$. [2]

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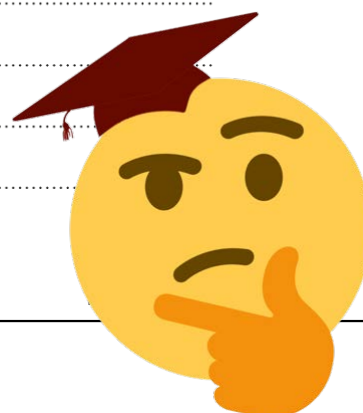
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11 – (9231/31_Summer_2023_Q1) – *Simple Harmonic Motion, Hook's Law*

One end of a light elastic string, of natural length a and modulus of elasticity $3mg$, is attached to a fixed point O . The other end of the string is attached to a particle P of mass m . The string hangs with P vertically below O . The particle P is pulled vertically downwards so that the extension of the string is $2a$. The particle P is then released from rest.

- (a) Find the speed of P when it is at a distance $\frac{3}{4}a$ below O . [3]

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- (b) Find the initial acceleration of P when it is released from rest. [2]

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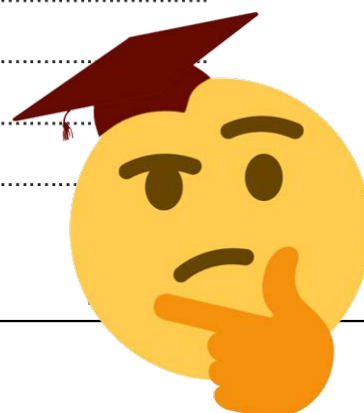
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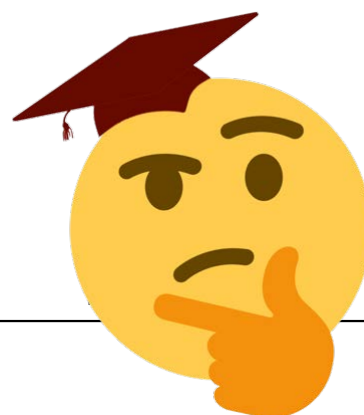


12 – (9231/33_Summer_2023_Q1) – Circular Motion, Simple Harmonic Motion, Hook's Law

A particle P of mass m is attached to one end of a light inextensible string of length a . The other end of the string is attached to a fixed point O . The particle P is held at the point A , where OA makes an angle α with the downward vertical through O , and with the string taut. The particle P is projected perpendicular to OA in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane. The string goes slack when P is at B , where OB makes an angle θ with the upward vertical.

Given that $\cos \alpha = \frac{4}{5}$, find the value of $\cos \theta$.

[4]



ANSWERS



1 - (9231/21_Summer_2017_Q1) - Momentum And Impulse

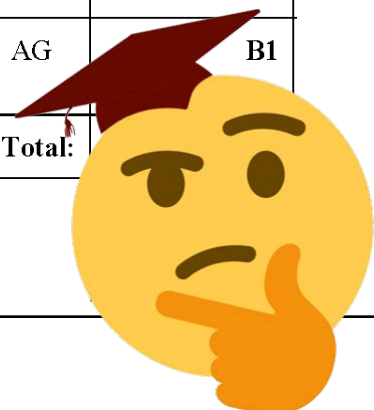
$0.08 \times (300 - v) = 1000 \times 0.02$ (AEF)	M1 A1
$v = 300 - 250 = 50 \text{ [m s}^{-1}\text{]}$	A1
Total:	3

2 - (9231/21_Summer_2017_Q3) - Momentum And Impulse

(i)	$3mv_A + mv_B = 3mu, v_B - v_A = eu$ (AEF)	M1
	$v_A = \frac{1}{4}(3 - e)u, v_B = \frac{3}{4}(1 + e)u$	A1, A1
	Total:	3
(ii)	$v_B' = -\frac{3}{4}v_B [= - (9/16)(1 + e)u]$ (AEF)	B1
	$[3mV_A +] mV_B = 3mv_A + mv_B' [V_B = 3(9 - 7e)u/16]$	M1
	$V_B [-V_A] = -e(v_B' - v_A) [V_B = e(21 + 5e)u/16]$	M1
	EITHER: $[4V_A =] (3 - e)v_A + (1 + e)v_B' = 0$ $\frac{1}{4}(3 - e)^2 - (9/16)(1 + e)^2 = 0$ (AEF)	(M1 A1)
	OR: $3(9 - 7e) = e(21 + 5e)$	(M1 A1)
	$5e^2 + 42e - 27 = 0, e = 3/5 \text{ or } 0.6$	M1 A1
	Total:	7

3 - (9231/23_Summer_2017_Q3) - Momentum And Impulse

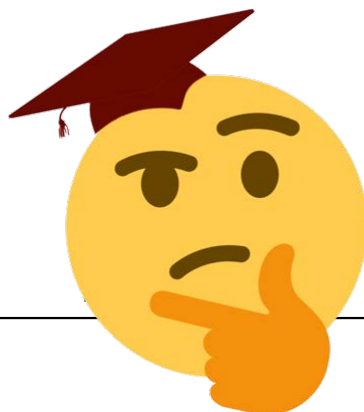
(i)	$mv_A + mv_B = mu$ (AEF)	*M1
	$v_B - v_A = \frac{2}{3}u$	*M1
	$v_B = 5u/6$	A1
	$w_B = \frac{1}{3}v_B = 5u/18$ AG	B1
	Total:	



(ii)	$v_A = u/6$	DA1
	<i>EITHER:</i> $(d-x)/v_A = d/v_B + x/w_B$ (AEF)	(M1 A1
	$6(d-x) = 1.2d + 3.6x$	M1 A1)
	<i>OR:</i> $x_A = (d/v_B) v_A = (6d/5u) u/6 = 0.2d$	(M1
	$t_2 = (0.8d)/(v_A + w_B) = 9d/5u$	M1 A1
	$y_A = v_A t_2 = 0.3d$ or $y_B = w_B t_2 = 0.5d$	A1)
	<i>OR2:</i> $x_A = (d/v_B) v_A = (6d/5u) u/6 = 0.2d$	(M1
	$(0.8d-x)/v_A = x/w_B$ or $0.8d/(v_A + w_B) = x/w_B$	M1 A1
	$4.8d - 6x = 3.6x$ or $1.8d = 3.6x$	A1)
	$x = \frac{1}{2}d$	A1
Total:		6

4 - (9231/21_Winter_2017_Q3) - Momentum And Impulse

(i)	$mv_A + kmv_B = 2mu + kmu$ (AEF)	M1	Use conservation of momentum for A & B (allow omission of m in all momentum eqns)
	$v_B - v_A = \frac{1}{2}(2u - u) [= \frac{1}{2}u]$	M1	Use Newton's restitution law with consistent LHS signs
	$v_B = u(2k+5)/2(k+1)$ or $u(k+5/2)/(k+1)$ (AEF)	A1	Combine to find v_B
	$[v_A = u(k+4)/2(k+1)] v_B > 4u/3$ if $k < 7/2$	M1 A1	Find inequality for k from speeds of B and C after 1st collision
		5	
(ii)	$kmv_B + mv_C = kmv_B + m(4u/3)$ (AEF) $[2w_B + v_C = 2(3u/2) + 4u/3 = 13u/3 \text{ when } k=2]$	M1	Use conservation of momentum for B & C
	$v_C - w_B = \frac{1}{2}(v_B - 4u/3) [= u/12]$ $(k+1)w_B = (k - \frac{1}{2})v_B + 2u$	M1	Use Newton's restitution law with consistent LHS signs Combine to find w_B
	$3w_B = (3/2)v_B + 2u$ with $v_B = 3u/2$, so $w_B = 17u/12$	*A1	when $k=2$
	$v_A = u$, $v_A < w_B$	DB1	Verify no further collisions between A and B
	<i>EITHER:</i> $(k+1)v_C = (3k/2)v_B + (2-k)(2u/3)$ $3v_C = 3v_B$ with $v_B = 3u/2$ so $v_C = 3u/2 > w_B$	(DB1)	<i>EITHER:</i> Find v_C and verify no further collisions between B and C
	<i>OR:</i> B and C cannot meet again since they move apart after colliding (AEF)	(DB1)	<i>OR:</i> State explicitly that no further collisions between B and C
		5	



5 - (9231/21_Summer_2018_Q1) - Momentum And Impulse

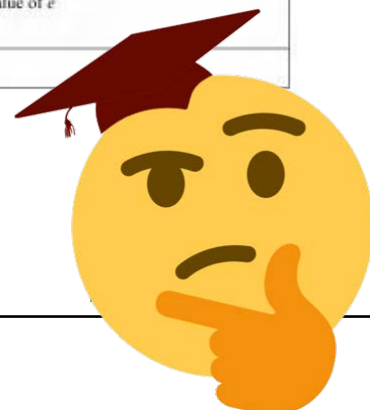
<i>EITHER:</i> $m(250 - 70) = 450 \times 0.04$	MI A1	Find m from change in momentum = Ft
$m = \frac{18}{180} = 0.1$	A1	(can allow M1 if $250 + 70$ used; similarly below)
<i>OR:</i> $a = (250 - 70)/0.04 [-4500]$	(M1)	Find deceleration a (ignore sign)
$m = \frac{450}{a} = 0.1$	(M1 A1)	Find m from $F = ma$
	3	

6 - (9231/23_Summer_2018_Q2) - Momentum And Impulse

(i)	$4mv_A + mv_B = 4mu$ (AEF)	MI	Use momentum (allow m omitted)
	$v_B - v_A = eu$	MI	Use Newton's law (M0 if LHS signs inconsistent)
	$v_A = (1/5)(4 - e)u$ AG	A1	Combine to verify/find speeds of A and B after collision
	$v_B = (4/5)(1 + e)u$	A1	
		4	
(ii)	$v_B' = [-] \frac{3}{5}e v_B [- = -(3/5)e(1 + e)u]$	MI	Relate vel. v_B' of B after colln. with wall to v_B
	$(1/5)(4 - e)u = (3/5)e(1 + e)u$	MI	Equate speeds of A and B (ignore sign of v_B' for both M1s)
	$3e^2 + 4e - 4 = 0, e = 2/3$	A1	Solve resulting quadratic for e , [implicitly] rejecting root -2
		3	
(iii)	<i>EITHER:</i> $4mw_A + mw_B = 3mv_A [- = 2mu]$ (AEF) $w_B - w_A = e \times 2v_A = (4/3)v_A$ or $(8/9)u$ (AEF) $[w_A = \frac{1}{5}v_A = (2/9)u, w_B = (5/3)v_A$ or $(10/9)u$ <i>OR:</i> Momentum before colln. is $3mv_A = 2mu > 0$, so after colln. momentum or speed of $B > 0$, so B collides again with barrier	MI A1	Use momentum (allow m omitted) with $v_A = -v_B' [- = \frac{2}{3}u]$ Use Newton's law (M0 if LHS signs inconsistent) Combine to find velocity of B after final collision Allow any similar valid argument Allow $v_A > 0$ by [implicit] inspection
		2	

7 - (9231/21_Summer_2018_Q3) - Momentum And Impulse

(i)	$mv_A = mu - mku$ (AEF)	MI	Use conservation of momentum (m may be omitted)
	$v_A = -e(u + ku)$	MI	Use Newton's restitution law
	$-e(1 + k) = 1 - k, e = \frac{(k-1)}{(k+1)}$ AG	A1	Combine to find/verify e
		3	
(ii)	$KE_i - KE_f = 0.6 KE_i$ or $0.4 KE_i = KE_f$	MI	Attempt to relate initial and final KEs
	$0.4 \times \frac{1}{2}mu^2(1 + k^2) = \frac{1}{2}mv_A^2 = \frac{1}{2}mu^2(1 - k)^2$	MI	Substitute for KEs ($\frac{1}{2}m$ may be omitted)
	$0.6k^2 - 2k + 0.6 = 0$ or $3k^2 - 10k + 3 = 0$ (AEF)	A1	Simplify to quadratic eqn in k
	$(3k - 1)(k - 3) = 0$ so solutions are 3 and $\frac{1}{3}$	A1	Find both solutions of quadratic eqn
	[Reject $k = \frac{1}{3}$] so $k = 3$	A1	(Implicitly) reject one solution to find possible value of k since $0 < e < 1$ or $v_A < 0$
	$e = \frac{2}{4} = \frac{1}{2}$ (FT on k)	A1✓	Find single corresponding value of e
		6	



8 - (9231/23_Summer_2018_Q5) - Momentum And Impulse, Circular Motion

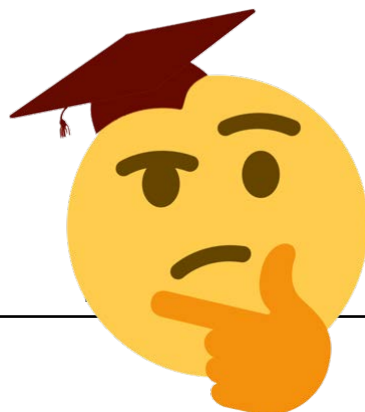
$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mgo$	M1	Find v_1^2 at lowest point from consvn. of energy (M0 if no m)
$v_1^2 = (12 + 2) ag = 14 ag$	A1	
$Mv_2 = mv_1$ with $M = (1 + k) m$	M1	Find new v_2 from consvn. of momentum
$v_2 = v_1 / (1 + k) [= \sqrt{(14 ag) / (1 + k)}]$	A1	
$T_1 = mv_1^2/a + mg = (14 + 1) mg = 15mg$	M1 A1	Find tension T_1 just before collision by using $F = ma$ radially
$T_2 = Mv_2^2/a + Mg$ $= (1 + k) \{14 / (1 + k)^2 + 1\} mg$	M1	Find tension T_2 just after collision by using $F = ma$ radially (M1 needs M , not m , throughout)
or $\{14 / (1 + k) + (1 + k)\} mg$ (AEF)	A1	
$14 + (1 + k)^2 = 15 (1 + k)^2$	M1	Equate T_2 and $\frac{1}{2} T_1$ to give any eqn in k
$2k^2 - 11k + 15 = 0, k = 2.5$ or 3 (M1 dep)	M1 A1	Solve resulting quadratic for k (M1 dep on all previous M1s, and requires <i>quadratic</i> eqn)
	11	

9 - (9231/21_Winter_2018_Q2) - Momentum And Impulse

(i)	$5mv_A + 2mv_B = 5mu - 4mu = mu$ (AEF)	M1	Use momentum (allow m omitted)
	$v_B - v_A = e(u + 2u) = 3eu$	M1	Use Newton's law (M0 if LHS signs inconsistent)
	$v_A = (u/7) (1 - 6e)$	A1	Combine to find/verify speeds of A and B after colln.
	$v_B = (u/7) (1 + 15e)$ AG	A1	(ignore signs)
		4	
(ii)	$(u/7) (1 - 6e) = -\frac{1}{2}u, e = \frac{3}{4}$ or 0.75	M1A1	Combine to find e from $v_A = -\frac{1}{2}u$ (M0 if dirn. of motion not reversed)
		2	
(iii)	$KE_A = \frac{1}{2} \times 5m \{u^2 - (\frac{1}{2}u)^2\}$ and $KE_B = \frac{1}{2} \times 2m \{(2u)^2 - (7u/4)^2\}$ [$= (15/8)mu^2$] [$= (15/16)mu^2$]	M1A1	Find loss of KE for A and B
	$KE_A / KE_B = (15/8) / (15/16) = 2:1$ or $2/1$ or 2	A1	Combine to find ratio
		3	

10 - (9231/22_Winter_2018_Q2) - Momentum And Impulse

(i)	$2mv_A + mv_B = 2mu$ (AEF)	M1	Use momentum (allow m omitted)
	$v_B - v_A = \frac{2}{5}u$	M1	Use Newton's law (M0 if LHS signs inconsistent)
	$v_A = 4u/9, v_B = 10u/9$	A1, A1	Combine to find speeds of A and B after collision
		4	



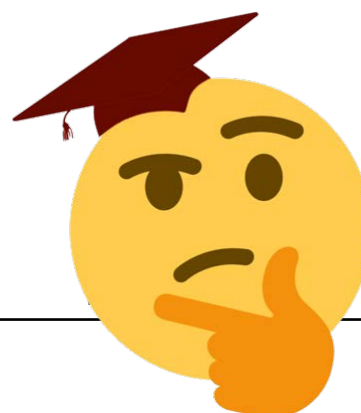
(ii)	$w_B = [-] \frac{1}{2} v_B [= \pm 5 \text{ u} / 9]$	M1	Relate speed w_B of B after colln. with wall to v_B (ignore sign)
	<i>EITHER:</i> $(d-x) / v_A = d/v_B + x/w_B$ (AEF)	M1	<i>EITHER:</i> Equate times in terms of dist. x from wall to 3rd colln.
	$(d-x)/4 = d/10 + x/5, x = d/3$	M1A1	Substitute for speeds to solve for x
	$t = (d-x) / v_A = (2 d / 3) / (4 \text{ u} / 9) = 3d / 2u$	A1	and hence find reqd. time t
	<i>OR:</i> $x_A = (d/v_B) v_A = (9d/10u) / (4u/9) = 2 d / 5$	(M1	<i>OR:</i> Find dist. x_A moved by A when B reaches wall
	$t = d/v_B + (d-x_A) / (v_A + w_B)$ $= d / (10 \text{ u} / 9) + (3 d / 5) / (4 \text{ u} / 9 + 5 \text{ u} / 9)$ $= 9d / 10u + 3d / 5u$	M1A1	Find t by adding times to and from wall (or equivalent method)
	$= 3d / 2u$	A1)	
		5	

11 - (9231/23_Summer_2019_Q1) - Momentum And Impulse

Impulse = $0.2 \times (250 - 40) = 42$	M1 A1	Find impulse from change in momtm. (may be implied) (if sign of 40 wrong here or below, can allow M1)
$T = 42 / 1200 = 7/200$ or 0.035 [s]	M1 A1	Find time T from impulse = Ft
Alternative method for question 1		
$a = 1200 / 0.2 = 6000$	M1 A1	Find acceleration a from $F = ma$ (may be implied)
$T = (250 - 40) / 6000 = 7/200$ or 0.035 [s]	M1 A1	Find time T from $v = u + at$
Alternative method for question 1		
K.E. = $\frac{1}{2} \times 0.2 \times (250^2 - 40^2) = 6090$	M1	Find loss of K.E. (may be implied; ignore sign of K.E.)
Work = $1200 \times \frac{1}{2} (250 + 40) T = 174\,000 T$	M1 A1	Find work done from $F \times \frac{1}{2} (u + v) t$ (A1 for both)
$T = 6090 / 174\,000 = 7/200$ or 0.035 [s]	A1	Equate to find T
	4	

12 - (9231/21_Summer_2019_Q3) - Momentum And Impulse

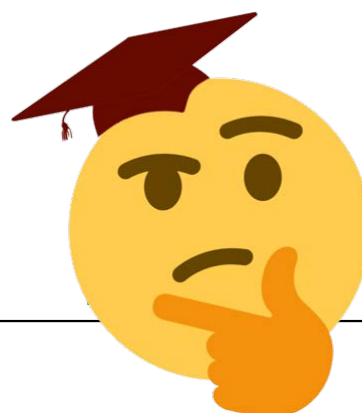
(i)	$2mv_A + 4mv_B = 4mu - 4mu$ [$v_A + 2v_B = 0$] (AEF)	M1	Use conservation of momentum for A and B (m may be omitted)
	$v_B - v_A = e(2u + u)$ [$v_B - v_A = 3eu$]	M1	Use Newton's restitution law with consistent LHS signs
	$v_A = -2eu$ and $v_B = eu$	A1	Combine to find v_A and v_B (A0 if directions unclear)
		3	
(ii)	$[4mv_B'] + mv_C = 4mv_B - (4/3) mu$ (AEF)	M1	Use conservation of momentum for B & C (m may be omitted)
	$v_C [-v_B'] = e(v_B + 4u/3)$	M1	Use Newton's restitution law
	$4v_B - (4/3) u = ev_B + 4eu/3, 4e - 4/3 = e^2 + 4e/3$	M1	Combine to find quadratic equation for e using $v_B' = 0$
	$3e^2 - 8e + 4 = 0, e = 2/3$ [$v_C = 4u/3$]	A1	Find value of e , (implicitly) rejecting $e = 2$
		4	



(iii)	For A: Loss = $\frac{1}{2} 2m(2u)^2 - \frac{1}{2} 2m(4/3u)^2 = (20/9) mu^2$ For B: Loss = $\frac{1}{2} 4m u^2 = \frac{1}{2} mu^2$ For C: Loss = 0	M1	$v_A = -(4/3)u$, $[v_B = (2/3)u]$, $v_C = (4/3)u$ One correct
		M1	Other two correct
	$E_{\text{initial}} - E_{\text{final}} \text{ or } L_1 + L_2 = (38/9) mu^2$	A1	Hence find loss in KE
	Alternative method for question 3(iii)		
	$E_{\text{initial}} = \frac{1}{2} 2m(2u)^2 + \frac{1}{2} 4mu^2 + \frac{1}{2} m(4u/3)^2$ $= 4mu^2 + 2mu^2 + (8/9) mu^2 = (62/9) mu^2$	M1	Find initial KE of 3 particles in terms of m and u
	$E_{\text{final}} = \frac{1}{2} 2mv_A^2 + \frac{1}{2} 4mv_B^2 + \frac{1}{2} mv_C^2$ $= (16/9) mu^2 + (8/9) mu^2 = (24/9) mu^2$	M1	Find final KE of 3 particles in terms of m and u
	$E_{\text{initial}} - E_{\text{final}} \text{ or } L_1 + L_2 = (38/9) mu^2$	A1	Hence find loss in KE
	Alternative method for question 3(iii)		
	$L_1 = \frac{1}{2} 2m(2u)^2 + \frac{1}{2} 4mu^2 - \frac{1}{2} 2mv_A^2 - \frac{1}{2} 4mv_B^2$ $= 4mu^2 + 2mu^2 - (16/9) mu^2 - (8/9) mu^2 = (30/9) mu^2$	M1	Find losses in KE in both collisions in terms of m and u
	$L_2 = \frac{1}{2} 4mv_B^2 + \frac{1}{2} m(4u/3)^2 - [\frac{1}{2} 4mv_B'^2] - \frac{1}{2} mv_C^2$ $= (8/9) mu^2 + (8/9) mu^2 - (8/9) mu^2 = (8/9) mu^2$	M1	
	$E_{\text{initial}} - E_{\text{final}} \text{ or } L_1 + L_2 = (38/9) mu^2$	A1	Hence find loss in KE
		3	

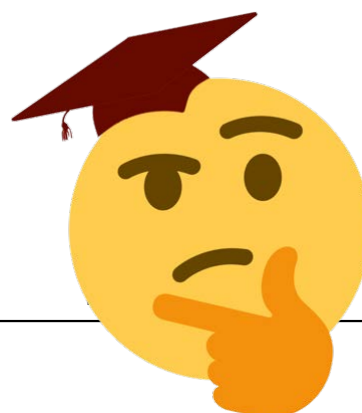
13 - (9231/23_Summer_2019_Q3) - Momentum And Impulse

(i)	$3mv_A + mv_B = 3mu$ [or $3v_A + v_B = 3u$]	(AEF)	M1	Use conservation of momentum for A and B (correct masses)
	$v_B - v_A = eu$		M1	Use Newton's restitution law with consistent LHS signs
	$v_A = \frac{1}{4}(3 - e)u$ [$v_B = \frac{1}{4}(1 + e)u$]		A1	Combine to find speed of A
	$w_B + v_C = v_B$ and $v_C - w_B = ev_B$	(AEF)	M1	Use conservation of momentum for B and C and Newton's restitution law with consistent LHS signs
	$w_B = \frac{1}{2}(1 - e)v_B$ and $v_C = \frac{1}{2}(1 + e)v_B$ $w_B = (3/8)(1 - e^2)u$ and $v_C = (3/8)(1 + e)^2u$ aef		A1 A1	Combine to find w_B and v_C in terms of v_B (may be implied) and with v_B replaced (note: $v_C = \frac{3}{8}e^2u + \frac{3}{4}eu + \frac{3}{8}u$)
			6	
(ii)	$\frac{1}{4}(3 - e) = (3/8)(1 + e)^2$		M1	Find equation in e by equating their v_A and their v_C
	$3e^2 + 8e - 3 = 0 = (3e - 1)(e + 3)$		M1	Simplify and solve resulting quadratic eqn for e ,
	$e = 1/3$		A1	(implicitly) rejecting $e = -3$
			3	



14 – (9231/21_Summer_2019_Q11) – Momentum And Impulse, Circular Motion, Inference Using Normal And T-distributions

E(i)	$\frac{1}{2}mu_P^2 = \frac{1}{2}m(2\sqrt{ag/2}) + mga \quad [u_P^2 = (25/2)ag]$	M1 A1	u_P is speed of P at lowest point, v_Q is speed of Q immediately after collision. Apply conservation of energy at lowest point (A0 if no m)
	$\frac{1}{2}4mv_Q^2 = 4mag$	M1	Find speed v_Q at lowest point by conservation of energy (A0 if no m)
	$v_Q = \sqrt{(2ag)} \text{ or } 1.41\sqrt{(ag)} \text{ or } 4.47\sqrt{a}$	A1	
	$mu_P = [\pm] mv_P + 4mv_Q$	M1	Find v_P using conservation of momentum (m may be omitted)
	$v_P = [\pm] (-5/\sqrt{2} + 4\sqrt{2})\sqrt{(ag)}$	A1	
	$ v_P = (3/\sqrt{2})\sqrt{(ag)} \text{ or } 2.12\sqrt{(ag)} \text{ or } 0.671\sqrt{a} \quad (\text{AEF})$	A1	Hence find speed of P
		7	
E(ii)	V_P is speed of P when it loses contact $\frac{1}{2}mV_P^2 = \frac{1}{2}mv_P^2 - mga(1 + \cos \alpha)$ $[V_P^2 = (9/2)ag - 2ga(1 + \cos \alpha) = (5/2 - 2 \cos \alpha)ag]$	M1 A1	Apply conservation of energy at D (A0 if no m)
	$[R_D] = mV_P^2/a - mg \cos \alpha = 0 \quad [V_P^2 = ag \cos \alpha]$	M1 A1	Apply $F = ma$ radially at D with reaction = 0
	$(5/2 - 2 \cos \alpha)ag = ag \cos \alpha, \cos \alpha = 5/6 \text{ or } 0.833$	A1	Combine to find $\cos \alpha$
		5	
O(i)	$ts_A / \sqrt{8} = \frac{1}{2}(16.7 - 13.5) [= 1.6]$	M1	Relate s_A to semi-width of confidence interval
	$t_{7,0.975} = 2.365 \quad (\text{to } 3 \text{ s.f.})$	A1	State or use correct tabular t value
	$[s_A = \sqrt{8} \times 1.6 / 2.365 = 1.9135], s_A^2 = 3.66[16]$	A1	Hence find unbiased estimate of A 's population variance
		3	
O(ii)	$H_0: \mu_A = \mu_B, H_1: \mu_A > \mu_B \quad (\text{AEF})$	B1	State hypotheses (B0 for $\bar{x} \dots$)
	$[\bar{x}_A = 15.1], \bar{x}_B = 85.2 / 6 = 14.2$	B1	Find sample mean for B
	$s_B^2 = (1221.06 - 85.2^2/6) / 5$ $= 561/250 \text{ or } 2.244 \text{ or } 1.498^2 \text{ (all to } 3 \text{ s.f.)}$	M1	Estimate or imply population variance for B (allow biased here: $1.87 \text{ or } 1.367^2$)
	$s^2 = (7s_A^2 + 5s_B^2) / 12 = 3.0709 \text{ or } 1.752^2$	M1 A1	Estimate (pooled) common variance (s_B^2 not needed explicitly)
	$t_{12,0.95} = 1.782$	B1	State or use correct tabular t value
	$[\cdot] t = (\bar{x}_A - \bar{x}_B) / (s \sqrt{(1/8 + 1/6)}) = 0.95$ $t < 1.78 \text{ so [accept } H_0]$	M1 A1	Find value of t (or can compare $\bar{x}_A - \bar{x}_B = 0.9$ with 1.69) Correct conclusion
	mean mass of B not less than mean mass of $A \quad (\text{AEF})$	B1	
		9	
			SCI: Implicitly taking s_A^2, s_B^2 as unequal population variances (may also earn first B1 B1 M1) $z = (\bar{x}_A - \bar{x}_B) / \sqrt{(s_A^2/8 + s_B^2/6)}$ $= 0.9 / \sqrt{(0.8317)} = 0.987$ $z < 1.645 \text{ so}$
			DepSCI: mean mass of B not less than mean mass of $A \quad (\text{AEF})$ Comparison with $z_{0.95}$ and conclusion (FT on z) (can earn at most 5/9)



15 - (9231/21_Winter_2019_Q3) - Momentum And Impulse

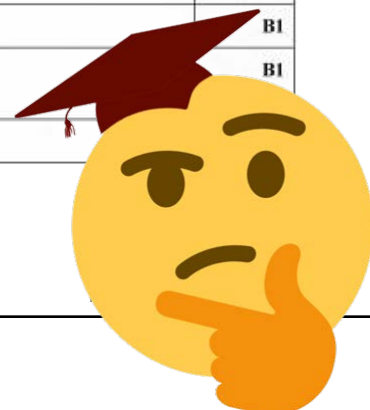
(i)	$5mv_A + 5mv_B = 5mu [v_A + v_B = u] \text{ and } v_B - v_A = e u$	M1	Use consvn. of momentum for A and B and use Newton's restitution law with consistent LHS signs. AEF
	$v_A = \frac{1}{2} (1 - e) u$	A1	Combine to verify speed of A . AG
	$v_B = \frac{1}{2} (1 + e) u$	A1	Find speed of B
		3	
(ii)	$5mv_B' + 3mv_C = 5mv_B \quad [5v_B' + 3v_C = 5v_B]$ $v_C - v_B' = e v_B$	M1	Use consvn. of momentum for B and C and use Newton's restitution law with consistent LHS signs. AEF
	$v_B' = (1/8) (5 - 3e) v_B \quad [v_C = (1/8) (5 + 5e) v_B]$	A1	Combine to find v_B' (v_C not reqd as B, C cannot collide again)
	$\frac{1}{2} (1 - e) u \leq (1/8) (5 - 3e) \times \frac{1}{2} (1 + e) u$	M1	Find condition on e using $v_A \leq v_B'$
	$3e^2 - 10e + 3 \leq 0$	A1	Simplify to a quadratic inequality
	$\frac{1}{3} \leq e$	A1	Solve to give a lower bound on e
	$\frac{1}{3} \leq e \leq 1$	A1	Non-strict inequality
		6	

16 - (9231/33_Summer_2020_Q5) - Momentum And Impulse

(a)	Let w be speed of A along line of centres after collision	M1	
	$\leftarrow mw = -mu \cos \alpha + mu \sin \alpha$		
	$w - 0 = e(u \cos \alpha + u \sin \alpha)$	M1	
	Rearrange: $\sin \alpha (u - eu) = \cos \alpha (u + eu)$	M1	
	$\tan \alpha = \frac{1+e}{1-e}$. AG	A1	
		4	
(b)	$\tan \alpha = 2 \Rightarrow e = \frac{1}{3}$	B1	
	$w = \frac{1}{3} u \left(\frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}} \right) = \frac{u}{\sqrt{5}}$	M1	
	Speed = $\sqrt{w^2 + (u \sin \alpha)^2}$	M1	
	$= \sqrt{\frac{u^2}{5} + \frac{4u^2}{5}} = u$	A1	
		4	

17 - (9231/31_Summer_2020_Q6) - Momentum And Impulse

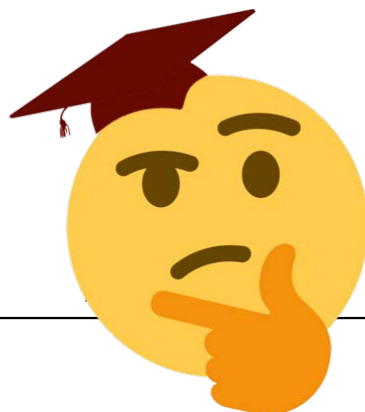
(a)	Let components of velocity (parallel to plane and perpendicular) after impact be (x, y)		
	$y = v \cos \alpha = eu \sin \alpha$	B1	
	$x = v \sin \alpha = u \cos \alpha$	B1	
	Divide: $\tan \alpha = \frac{1}{e \tan \alpha} : \tan^2 \alpha = \frac{1}{e}$	B1	



(b)	$v^2 = \frac{1}{3}u^2$	B1
	$\left(\frac{u \cos \alpha}{\sin \alpha}\right)^2 = \frac{1}{3}u^2$	M1
	$(\tan \alpha)^2 = 3$	M1
	$\alpha = 60^\circ$	A1
	$e = \frac{1}{3}$	A1
Alternative method for 6(b)		
	KE after impact $= \frac{1}{2}m(x^2 + y^2) = \frac{1}{2}m((u \cos \alpha)^2 + e^2(u \sin \alpha)^2)$	M1
	From (a) $\sin \alpha = 1/\sqrt{1+e}$ and $\cos \alpha = \sqrt{e}/\sqrt{1+e}$	B1
	KE $= \frac{1}{2}mu^2 \left(\frac{e}{1+e} + \frac{e^2}{1+e} \right) = \frac{1}{2}mu^2 e$	A1
	This is equal to $\frac{1}{3} \times \frac{1}{2}mu^2$ so $e = \frac{1}{3}$	M1
	$\tan \alpha = \sqrt{3}, \alpha = 60^\circ$	A1
		5

18 - (9231/32_Winter_2020_Q2) - Momentum And Impulse

Speeds v and w after collision $2mv + mw = 2mu \cos \alpha$	M1	Momentum equation with m . Correct masses, allow sin instead of cos
$w - v = eu \cos \alpha$	M1	Restitution, with consistent signs
$v = \frac{1}{3}u \cos \alpha (2 - e) = \frac{1}{3}u \cdot \frac{3}{5} \left(2 - \frac{1}{3} \right) = \frac{1}{3}u$	A1	
Square of speed of A $= \left(\frac{1}{3}u \right)^2 + (u \sin \alpha)^2$	M1	Uses correct speed perpendicular to motion
$= \left(\frac{1}{3}u \right)^2 + \left(\frac{4}{5}u \right)^2$ Speed $= \frac{13}{15}u$ ($= 0.867u$)	A1	
	5	

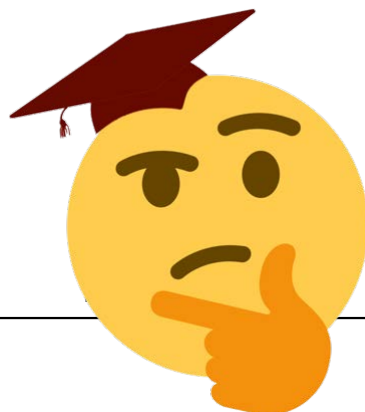


19 - (9231/32_Winter_2020_Q4) - Momentum And Impulse

(a)	$\uparrow N \cos \theta = mg$	B1	
	$\leftarrow N \sin \theta = mr \sin \theta \omega^2$	B1	
	$\cos \theta = \frac{mg}{N}$ so $\cos \theta = \frac{g}{\omega^2 r}$	B1	AG
		3	
(b)	$\cos \theta = \frac{r-x}{r} = \frac{g}{\omega^2 r}$	B1	Using trig of situation: must involve x
	In new situation: $r-4x = r \times \frac{g}{4\omega^2 r}$	M1	Using new situation with $4x$ and 2ω seen
	$r-x = 4(r-4x)$	M1	Combining
	$x = \frac{1}{5}r$	A1	
		4	

20 - (9231/31_Winter_2020_Q6) - Momentum And Impulse

(a)	$mu = mv + 2mv$	B1	Momentum equation (with m)
	$v-w = eu$	B1	Restitution with consistent signs
	$v = \frac{u}{3}(e+1)$ $w = \frac{u}{3}(1-2e)$	B1	Both correct.
		3	
(b)	Perpendicular to plane: $y = ev \sin \theta$ Parallel to plane: $x = v \cos \theta$	B1	Both
	Speed of B = $\sqrt{x^2 + y^2} = \sqrt{v^2 \left(\left(\frac{4}{5} \right)^2 + \left(\frac{2}{3} \cdot \frac{3}{5} \right)^2 \right)} = \frac{2}{\sqrt{5}}v$	M1	Speed of B
	KE of B = $\frac{1}{2} \cdot 2m \cdot \frac{4}{5} \cdot \frac{u^2}{9} (e+1)^2$	M1	KE of B in terms of u , $\frac{1}{2}$ and $2m$ needed
	KE of A = $\frac{1}{2} m \cdot \frac{u^2}{9} (1-2e)^2$ So $\frac{1}{2} m \cdot \frac{u^2}{9} (1-2e)^2 = \frac{5}{32} \cdot \frac{1}{2} \cdot 2m \cdot \frac{4}{5} \cdot \frac{u^2}{9} (e+1)^2$	M1 A1	Relate the two KEs
	$4(1-2e)^2 = (e+1)^2$ or $15e^2 - 18e + 3 = 0$	M1	Rearrange and simplify to quadratic
	$1+e = \pm 2(1-2e)$ $e = \frac{1}{5}, 1$	A1	Both values
		7	

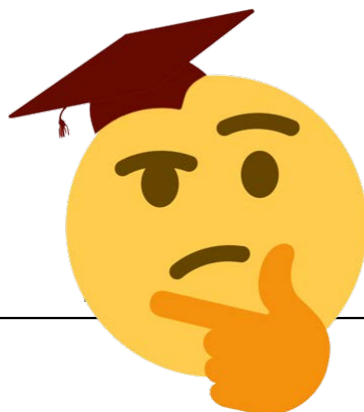


21 - (9231/31_Summer_2021_Q6) - Momentum And Impulse

(a)	Along line of centres, speeds v_1 and v_2 $mv_1 + mv_2 = mu \cos \alpha - mu \cos \beta$	M1	Momentum (condone missing masses).
	$v_2 - v_1 = eu(\cos \beta + \cos \alpha)$	M1	Restitution.
	Both correct, masses seen.	A1	
	$v_1 = 0$ so A has no speed along line of centres: moves perpendicular to line of centres	A1	AG.
		4	
(b)	$(v_2 = \frac{1}{2}u \cos \alpha = u \cos \beta)$ KE of B after collision is $\frac{1}{2}m(v_2^2 + (u \sin \beta)^2)$ KE of A after collision = $\frac{1}{2}m(u \sin \alpha)^2$	M1	Both components.
	Add both KEs and equate to $\frac{3}{4}mu^2$	M1	
	Simplify to equation in $\sin \alpha$	M1	
	$\sin \alpha = \frac{1}{\sqrt{2}}, \alpha = 45^\circ$	A1	
		4	

22 - (9231/33_Summer_2021_Q6) - Momentum And Impulse

(a)	Let velocities of A and B along line of centres after collision be v_1 and v_2 . $mv_1 + kmv_2 = mu \cos \theta$	M1	Momentum, must include m , allow cos/sin mix.
	$v_2 - v_1 = \frac{1}{3}u \cos \theta$	M1	Restitution, consistent signs, correct way up.
	Solve: $v_2 = \frac{4u \cos \theta}{3(1+k)}$	A1	AG shown convincingly.
		3	
(b)	$v_1 = \frac{(3-k)u \cos \theta}{3(1+k)}$	B1	Or equivalent, may be unsimplified.
	Use velocity of A with both components.	B1	$v_1^2 + (u \sin \theta)^2$ seen.
	$\frac{1}{2}kmv_2^2 + \frac{1}{2}m(v_1^2 + (u \sin \theta)^2) = \frac{3}{10} \times \frac{1}{2}mu^2$	M1	KE after = 30% KE before (all terms present). M0 if incorrect masses.
	Substitute from part (a) and for θ .	M1	Eliminate trigonometric terms, must be KE equation, in terms of k only.
	$(3-k)^2 + 16k = 2(1+k)^2, k^2 - 6k - 7 = 0$	M1	Obtain simplified quadratic equation in k .
	$k = 7$	A1	
		6	

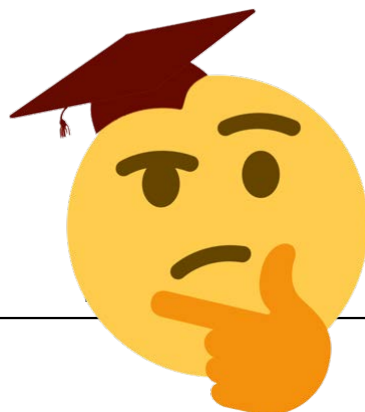


23 - (9231/32_Winter_2021_Q1) - Momentum And Impulse

(a)	Velocity: $\rightarrow u \cos \alpha$	B1	
	$\uparrow u \sin \alpha - gT$	B1	Allow 10 for g. Must be T.
		2	
(b)	$\frac{u \cos \alpha}{u \sin \alpha - gT} = -\frac{\sin \alpha}{\cos \alpha}$ oe	M1 FT	Allow missing minus sign on RHS for M1. FT from (a).
	$T = \frac{u}{g \sin \alpha}$	A1	
		2	
(c)	$\sin \alpha < 1$ giving $T > \frac{u}{g}$	B1	AG
		1	

24 - (9231/31_Winter_2021_Q2) - Momentum And Impulse

(a)	Separate variables and integrate: $\frac{dv}{v} = \left(\frac{1-2t^2}{t} \right) dt$ so $\ln v = \ln t - t^2 + c$	M1 A1	Must include logs. Condone missing modulus.
	$ v = Ate^{-t^2}$, $-v = Ate^{-t^2}$, $v = -Ate^{-t^2}$	A1	CAO.
		3	
(b)	$a = \frac{-Ate^{-t^2}(1-2t^2)}{t} = -Ae^{-t^2}(1-2t^2)$	M1	Substituting their answer to part (a) into given formula
	$t=1, a=5$ ($A=5e$)	M1	Use initial condition.
	Force = $5me^{1-t^2}(2t^2-1)$	A1	Use N2L, correct work only.
Alternative method for question 2(b)			
	$a = \frac{v(1-2t^2)}{t}$ substitute $t=1, a=5$ so $v=-5$	M1	Use initial condition. Use N2L, correct work only.
	Substituting in their answer to part (a) so ($A=5e$)	M1	
	Force = $5me^{1-t^2}(2t^2-1)$	A1	
		3	

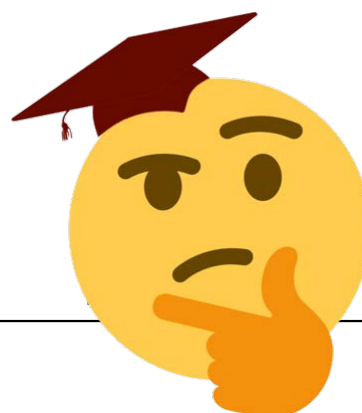


25 - (9231/31_Winter_2021_Q5) - Momentum And Impulse

At A: $\uparrow u \sin \theta - 8g \rightarrow u \cos \theta$	M1	Both.
$\tan \alpha = \frac{u \sin \theta - 8g}{u \cos \theta}$	A1	
At B: $\uparrow u \sin \theta - 32g \rightarrow u \cos \theta$	M1	Both.
$\tan \beta = \frac{u \sin \theta - 32g}{u \cos \theta}$	A1	
$\frac{u \sin \theta - 8g}{u \cos \theta} \times \frac{u \sin \theta - 32g}{u \cos \theta} = -1$	B1	Perpendicular directions, so $\tan \alpha \times \tan \beta = -1$.
$u^2 - 320u + 25600 = 0$	M1	Simplify to a quadratic in u .
$u = 160$	A1	
	7	

26 - (9231/32_Winter_2021_Q5) - Momentum And Impulse

(a)	Let speeds of A and B along line of centres after collision be v_1 and v_2 $mv_1 + \frac{3}{2}mv_2 = -\frac{3}{2}mu \cos 60^\circ + mu \left(= \frac{u}{4} \right)$	M1	Momentum with masses correct.
	$v_2 - v_1 = -\frac{2}{3}(-u \cos 60^\circ - u) (= u)$	M1	Restitution, with consistent signs on LHS.
	$\left(v_1 = -\frac{1}{2}u \right) \quad v_2 = \frac{1}{2}u$	A1	
	Perpendicular to line of centres, speed of B is $u \sin 60^\circ = \frac{\sqrt{3}}{2}u$	B1	
	Direction of B is now 60° above line of centres.	M1	
	Angle of deflection is 60° .	A1 FT	FT (120° – their direction of B angle)
		6	
(b)	KE before = $\frac{1}{2}mu^2 + \frac{1}{2} \cdot \frac{3m}{2}u^2 = \frac{5}{4}mu^2$	B1	
	KE after = $\frac{1}{2}m\left(\frac{u}{2}\right)^2 + \frac{1}{2} \cdot \frac{3m}{2}\left[\left(\frac{u}{2}\right)^2 + \left(\frac{\sqrt{3}u}{2}\right)^2\right] \left(= \frac{7}{8}mu^2 \right)$	B1 FT	FT only their speeds from (a)
	Loss in KE = $\frac{3}{8}mu^2$	B1	
		3	

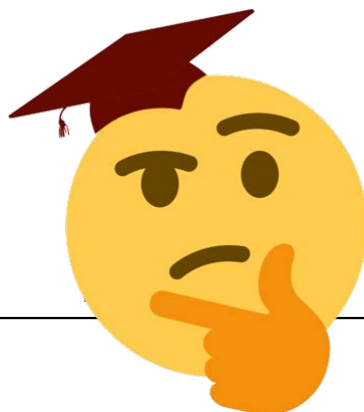


27 - (9231/32_Winter_2021_Q6) - Momentum And Impulse

(a)	$2v \frac{dv}{dx} = 8x - \frac{128}{x^3}$	M1	Separate variables and integrate, + c not required for M1.
	$v^2 = 4x^2 + 64x^{-2} + c$	A1	OE.
	$x = 8, v = -15$ and $c = -32$	M1	Use initial condition.
	$v^2 = \frac{4}{x^2}(x^4 - 8x^2 + 16)$ or $4x^2 - 32 + \frac{64}{x^2}$	A1	Correct expression for v^2 , AEF.
	$v^2 = \frac{4}{x^2}(x^2 - 4)^2$ giving $v = -\frac{2}{x}(x^2 - 4)$	A1	Convincingly shown, e.g. v is negative initially, AG.
		5	
(b)	$\frac{1}{2} \ln(x^2 - 4) = -2t (+A)$	M1	Use $v = \frac{dx}{dt}$ and integrate.
	$t = 0, x = 8, A = \frac{1}{2} \ln 60$	DM1	Use initial condition.
	$\frac{1}{2} \ln \left(\frac{x^2 - 4}{60} \right) = -2t$ giving $\frac{x^2 - 4}{60} = e^{-4t}$	M1	Remove log.
	$x = \sqrt{4 + 60e^{-4t}}$	A1	CAO
		4	

28 - (9231/31_Winter_2021_Q7) - Momentum And Impulse

(a)	$u \cos \alpha = v \cos \beta$	M1	
	$eu \sin \alpha = v \sin \beta$	M1	
	Divide: $\tan \beta = e \tan \alpha$	A1	AG. Must see divide OE.
		3	
(b)	$v \sin \beta = w \cos \gamma \quad (= eu \sin \alpha)$	M1	
	$ev \cos \beta = w \sin \gamma \quad (= eu \cos \alpha)$	M1	
	Divide: $\tan \gamma = 1 / \tan \alpha: \gamma = 90^\circ - \alpha$	*A1	
	After second rebound, direction of motion is parallel to initial path.	DB1	
		4	
(c)	Final KE = $\frac{1}{2} m \left((eu \sin \alpha)^2 + (eu \cos \alpha)^2 \right) \left(= \frac{1}{2} me^2 u^2 \right)$	M1	Energy expression in terms of u .
	So $\frac{1}{2} me^2 u^2 = \frac{1}{9} \times \frac{1}{2} mu^2$ giving $e = \frac{1}{3}$	A1	
	Part (a) gives $\tan(90 - \alpha) = e \tan \alpha$	M1	
	So $\tan \alpha = \sqrt{3}$	A1	
		4	

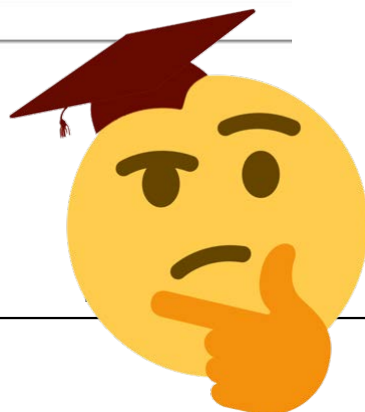


29 – (9231/31_Summer_2022_Q6) – Momentum And Impulse

(a)	Let v and w be speeds after collision: $mv + kmw = mu \cos \alpha$	M1	Momentum along line of centres.
	$w - v = \frac{1}{2}u \cos \alpha$	M1	NEL consistent signs.
	Add to give $\frac{3u \cos \alpha}{2(1+k)}$	A1	AG Convincing working.
	Substitute back or re-solve: $v = \left \frac{(2-k)u \cos \alpha}{2(1+k)} \right $	A1	Accept without modulus sign.
		4	
(b)	$\sqrt{(u \sin \alpha)^2 + \left(\frac{(2-k)u \cos \alpha}{2(1+k)} \right)^2}$	B1	For speed of A (SOI).
	Equal KE after collision: $\frac{1}{2}km \left(\frac{3u \cos \alpha}{2(1+k)} \right)^2 = \frac{1}{2}m \left((u \sin \alpha)^2 + \left(\frac{(2-k)u \cos \alpha}{2(1+k)} \right)^2 \right)$ $[9k(\cos \alpha)^2 = 4(1+k)^2(\sin \alpha)^2 + (2-k)^2(\cos \alpha)^2]$	M1	Equate KEs.
	Use $\tan \alpha = \frac{2}{3}$: $16(1+2k+k^2) + 9(4-4k+k^2) = 81k$	M1	
	$25k^2 - 85k + 52 = 0$ leading to $(5k-4)(5k-13) = 0$	M1	Obtain quadratic and attempt to solve.
	$k = \frac{4}{5}$ or $\frac{13}{5}$	A1	
		5	

30 – (9231/33_Summer_2022_Q6) – Momentum And Impulse

(a)	Let v be speed of rebound from 1 st collision: Energy loss: $\frac{1}{2}mv^2 = \frac{2}{5} \times \frac{1}{2}mu^2$, $v^2 = \frac{2}{5}u^2$	B1	Energy loss.
	$v \cos \alpha = u \cos \theta$ $v \sin \alpha = ev \sin \theta$	B1	Both.
	Combine to form equation in e only: $\frac{2}{5} = \frac{1}{5} + e^2 \times \frac{4}{5}$	M1	$v^2 = (u \cos \theta)^2 + (ev \sin \theta)^2$
	$e = \frac{1}{2}$	A1	
		4	
(b)	$\tan \alpha = e \tan \theta$, so $\tan \alpha = 1$, $\alpha = 45^\circ$	B1	
	For 2 nd collision $w \cos \beta = v \cos(180 - 60 - \alpha)$ $w \sin \beta = ev \sin(180 - 60 - \alpha)$	M1	Both. May be implied by the A1.
	$\tan \beta = e \tan(120 - \text{their } \alpha)$	M1	Divide to find β .
	$\beta = 61.8^\circ$	A1	
		4	

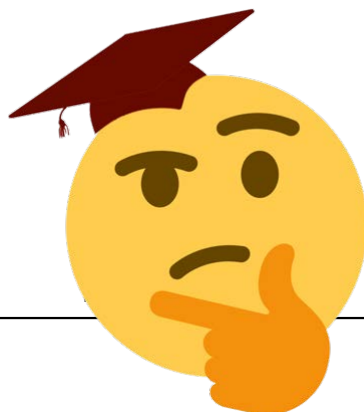


31 – (9231/31_Summer_2022_Q7) – Momentum And Impulse

(a)	For Q: $x = u \cos \beta T$	B1	
	For P: $x = \frac{35}{2} \cos \alpha (T+1)$	B1	
	Collision, so $\frac{35}{2} \cos \alpha (T+1) = u \cos \beta T$	M1	Equate and attempt to rearrange.
	$\frac{35}{2} \times \frac{3}{5} (T+1) = u \times \frac{2}{\sqrt{5}} T$ $4uT = 21\sqrt{5} (T+1)$	A1	AG Shown convincingly.
		4	
(b)	Vertical motion to collision: For Q: $y = u \sin \beta T - \frac{1}{2} g T^2$ For P: $y = \frac{35}{2} \sin \alpha (T+1) - \frac{1}{2} g (T+1)^2$	M1 A1	M1 for both expressions, one correct.
	Equate: $u \times \frac{1}{\sqrt{5}} T - \frac{1}{2} g T^2 = \frac{35}{2} \times \frac{4}{5} (T+1) - \frac{1}{2} g (T+1)^2$ $14(T+1) - \frac{1}{2} g (T^2 + 2T + 1 - T^2) = \frac{21}{4} (T+1)$	M1	Equate and attempt to solve
	$16T + 36 = 21T + 21, \quad 15 = 5T$ $T = 3$	A1	
		4	
(c)	$x = 42$	B1	
	$ y = 24$	M1	
	$y = -24$ (or 24 m below O)	A1	Correct sign or in words.
		3	

32 – (9231/32_Winter_2022_Q3) – Momentum And Impulse

In equilibrium, $\frac{16}{3} \frac{Mge}{a} = 4Mg, \quad e = \frac{3}{4}a$	B1	
In subsequent motion, Loss in GPE = gain in EPE + gain in KE	M1	Energy equation with GPE and KE terms correct and at least one EPE term. Dimensionally correct.
$\frac{6Mga}{4} = \frac{1}{2} \frac{16}{3} \frac{Mg}{a} \left(a^2 - \left(\frac{3a}{4} \right)^2 \right) + \frac{1}{2} 6Mv^2$	B1	EPE correct.
	A1	All correct.
$\frac{3Mga}{2} = \frac{8Mg}{3a} \frac{7}{16} a^2 + 3Mv^2$ etc	M1	Attempt to find v in terms of a and g .
$\frac{3ga}{3} = 3v^2, \quad v = \frac{1}{3} \sqrt{ga}$	A1	
	6	

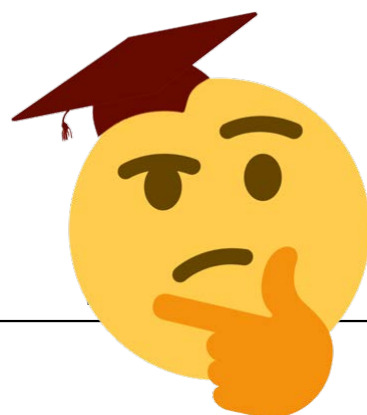


33 - (9231/31_Winter_2022_Q4) - Momentum And Impulse

(a)	$m \frac{dv}{dt} = 50 - 2v^2 \quad \frac{dv}{dt} = 4(25 - v^2)$	B1	N2L
	$\frac{1}{10} \int \frac{1}{5-v} + \frac{1}{5+v} dv = \int 4 dt$	M1	Separate variables and use partial fractions.
	$\frac{1}{10} (-\ln(5-v) + \ln(5+v)) = 4t + A$	M1 A1	Integrate into log terms. (Note: formula on MF19).
	Use $t=0, v=3$ to give $A = \frac{1}{10} \ln 4$	M1	Use initial condition.
	$4t = \frac{1}{10} \ln \frac{5+v}{4(5-v)}$ leading to $\frac{5+v}{20-4v} = e^{40t}$	M1	Rearrange to make v the subject.
	$v = \frac{5(4 - e^{-40t})}{4 + e^{-40t}}$	A1	
		7	
(b)	As $t \rightarrow \infty, v \rightarrow 5$	B1	
		1	

34 - (9231/31_Winter_2022_Q6) - Momentum And Impulse

(a)	Let speed of A after collision be $\rightarrow v_A$ and speed of B perpendicular to line of centres be $\downarrow v$ Along line of centres: $mu - km \frac{5}{8} u \cos \alpha = mv_A$	M1	Momentum.
	NEL: $0 - v_A = v \left(\frac{5}{8} u \cos \alpha + u \right)$	M1	NEL
	So $u - \frac{5}{8} ku \cos \alpha = -\frac{2}{3} \left(\frac{5}{8} u \cos \alpha + u \right)$	M1	Solve.
	Substitute for cos, to give $k=4$	A1	
		4	
(b)	$v_B = \frac{5}{8} u \sin \alpha = \frac{3}{8} u$	B1	Velocity perpendicular to line of centres
	$v_A = -u$	B1 FT	
	KE before = $\frac{1}{2} mu^2 + \frac{1}{2} km \left(\frac{5}{8} u \right)^2 = \frac{1}{2} mu^2 + \frac{25}{32} mu^2 = \frac{41}{32} mu^2$ KE after = $\frac{1}{2} mv_A^2 + \frac{1}{2} kmv_B^2 = \frac{1}{2} mu^2 + 2m \frac{9}{64} u^2 = \frac{25}{32} mu^2$	M1	NOTE: KE before and after for A is unchanged. Both.
	Loss = $mu^2 \left(\frac{41}{32} - \frac{25}{32} \right) = \frac{1}{2} mu^2$	A1	
		4	

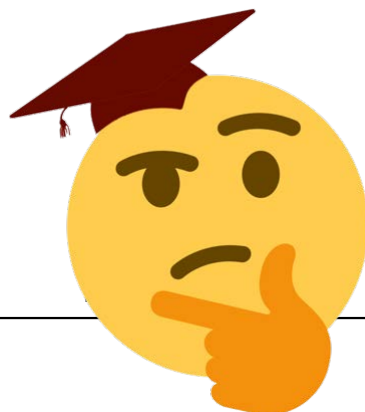


35 - (9231/32_Winter_2022_Q7) - Momentum And Impulse

(a)	Let v, w be speeds of A and B along line of centres after collision $mv + \frac{1}{2}mw = mu \cos \alpha - \frac{1}{2}m \cdot 2u \cos \beta$	M1	Momentum: masses correct, opposite signs on RHS.
	$w - v = e(2u \cos \beta + u \cos \alpha)$	M1	NEL: LHS signs must be consistent with momentum equation, same sign for both terms on RHS.
	$\alpha + \beta = 90^\circ$, so $\cos \beta = \sin \alpha$ Use this fact and solve to find w	M1	Solve to find an expression of the correct form.
	$w = \frac{2}{3}u \left(\frac{1}{4} \sin \alpha + \frac{13}{8} \cos \alpha \right)$	A1	
		4	
(b)	Perpendicular to line of centres, speed of B is $2u \sin \beta = 2u \cos \alpha$	B1	
	After, velocity of B makes angle α with line of centres, so $\tan \alpha = \frac{2u \cos \alpha}{w}$	B1	
	$\frac{\sin \alpha}{\cos \alpha} = \frac{2u \cos \alpha}{\frac{2}{3}u \left(\frac{1}{4} \sin \alpha + \frac{13}{8} \cos \alpha \right)}$ giving	M1*	Obtain homogeneous equation in $\cos$ and $\sin$ or an equation in $\tan$
	$3(\cos \alpha)^2 = \frac{1}{4}(\sin \alpha)^2 + \frac{13}{8} \sin \alpha \cos \alpha$ $2(\tan \alpha)^2 + 13 \tan \alpha - 24 = 0, (2 \tan \alpha - 3)(\tan \alpha + 8) = 0$	DM1	Obtain quadratic and solve to find values of $\tan \alpha$
	$\tan \alpha = \frac{3}{2}$	A1	
		5	

36 - (9231/31_Summer_2023_Q2) - Momentum And Impulse

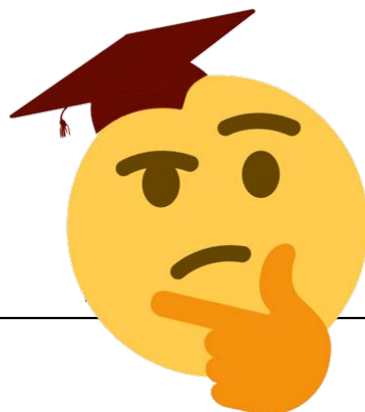
Parallel to wall Perpendicular to wall	$v \cos \theta = u \cos \alpha$ $v \sin \theta = eu \sin \alpha$	M1	Both
Dividing, $e = \frac{1}{2 \tan \alpha}$		A1	AEF
KE reduced by 20%, so $\frac{1}{2}mu^2 (\cos^2 \alpha + e^2 \sin^2 \alpha) = \frac{4}{3} \times \frac{1}{2}mu^2$		M1	Dimensionally correct equation in u or v , but not both. Must have either α or θ , but not both. Must see $\frac{4}{3}$ on the correct side of the equation.
Eliminate e : $\cos \alpha = \frac{4}{5}$		A1	
$e = \frac{2}{3}$		A1	



Alternative method for question 2		
Parallel to wall $v \cos \theta = u \cos \alpha$ Perpendicular to wall $v \sin \theta = ev \sin \alpha$	M1	Both
$\left[\sin(\theta) = \frac{\sqrt{5}}{5}, \cos(\theta) = \frac{2\sqrt{5}}{5} \right] \quad u \sin(\alpha) = \frac{\sqrt{5}v}{5e}, u \cos(\alpha) = \frac{2\sqrt{5}v}{5}$	A1	
$u^2 = [u^2 \cos^2(\alpha) + u^2 \sin^2(\alpha)] = \frac{4v^2}{5} + \frac{v^2}{5e^2}$	A1	AEF, e.g. $\frac{v^2}{5} \left(4 + \frac{1}{e^2} \right)$
$\frac{1}{2}mv^2 = \left[\frac{4}{5} \times \frac{1}{2}mu^2 \right] = \frac{2}{5}m \frac{v^2}{5} \left(4 + \frac{1}{e^2} \right)$	M1	Dimensionally correct equation in v . Must have either α or θ , but not both. Must see $\frac{4}{5}$ or $\frac{2}{5}$ on the correct side of the equation.
$e = \frac{2}{3}$	A1	
	5	

37 - (9231/33_Summer_2023_Q4) - Momentum And Impulse

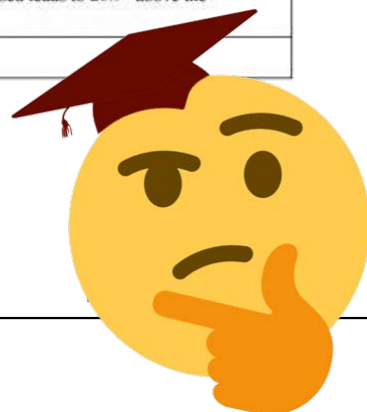
(a)	Let speeds of A and B along line of centres after collision be V_A and V_B	M1	Allow sign errors, allow missing m .
	$V_A + V_B = u \cos 30^\circ \quad (1)$		
	$-V_A + V_B = eu \cos 30^\circ \quad (2)$	M1	Signs on LHS must be consistent with (1).
	Speeds perpendicular to line of centres after collision are $u \sin 30^\circ$ and $2u$ Moving in same direction, so $\frac{V_A}{u \sin 30^\circ} = \frac{V_B}{2u} \quad (3)$	B1	SOI $V_B = 4V_A$
	Use $V_B = 4V_A$ in (1); $5V_A = u \cos 30^\circ$ From (2); $3V_A = eu \cos 30^\circ$ then Combine to find equation in e only.	M1	A complete method to find equation in e only
	$e = \frac{3}{5}$	A1	



(a)	Alternative method for question 4(a)		
	Let speeds of A and B along line of centres after collision be V_A and V_B .	M1	
	$V_A + V_B = u \cos 30^\circ$ (1)		Allow sign errors, allow missing m .
	$-V_A + V_B = eu \cos 30^\circ$ (2)	M1	Signs on LHS must be consistent with (1).
	Speeds perpendicular to line of centres after collision are $u \sin 30^\circ$ and $2u$ Moving in same direction, so $\frac{V_A}{u \sin 30^\circ} = \frac{V_B}{2u}$ (3)	B1	SO1 $V_B = 4V_A$
	Solve (1) and (2): $V_A = \frac{1}{2}u(1-e) \cos 30^\circ$, $V_B = \frac{1}{2}u(1+e) \cos 30^\circ$ Substitute in (3) to find equation in e only.	M1	Note: $V_A = \frac{u}{10}\sqrt{3}$, $V_B = \frac{4u}{10}\sqrt{3}$
	$e = \frac{3}{5}$	A1	
		5	
(b)	KE after = $\frac{1}{2}m \left(V_A^2 + \left(\frac{u}{2} \right)^2 \right) + \frac{1}{2}m ((2u)^2 + V_B^2)$	B1	Correct expression for KE for one of the spheres, after collision, with both components.
	KE for A after = $\frac{7}{50}mu^2$ or KE for B after = $\frac{56}{25}mu^2$ or KE loss for A = $\frac{9}{25}mu^2$ or KE gain for B = $\frac{6}{25}mu^2$	B1	Implied by total KE after = $\frac{119}{50}mu^2$.
	Total loss in KE = $\frac{3}{25}mu^2$	B1	Term $\frac{1}{2}m(2u)^2$ may be omitted from KE of B before and after.
		3	

38 - (9231/33_Summer_2023_Q7) - Momentum And Impulse

(a)	For aircraft, $d = 5T$	B1	
	For ball, $\uparrow 4 = u \sin \theta T - \frac{1}{2} \times 10 \times T^2$	B1	To point of collision.
	For ball, $\rightarrow u \cos \theta T = d + 8 = 5T + 8$	B1	
	Eliminate u : $4 = \frac{u^4}{5}T - \frac{1}{2} \times 10 \times T^2$, $u = \frac{5(4 + 5T^2)}{4T}$ and $u = \frac{5(5T + 8)}{3T}$ $3(4 + 5T^2) = 4(5T + 8)$	*M1	Dependent on LHS of second B1 being 4, expression involving only T
	$3T^2 - 4T - 4 = 0$	DM1	Dependent on previous M1. Obtain and solve 3-term quadratic.
	$T = 2$	A1	Single correct answer.
		6	Note $d = 8$ used leads to $T = \frac{2}{3}$ B1B1B0M1M1A0
(b)	$\tan^{-1} \frac{(u \sin \theta - 10T)}{u \cos \theta}$	M1	OE Accept 'tan ='
	$\tan^{-1} \frac{8}{9}$	A1	OE
	Direction is 41.6° below the horizontal	A1	CAO Note: $d = 8$ used leads to 20.9° above the horizontal.
		3	

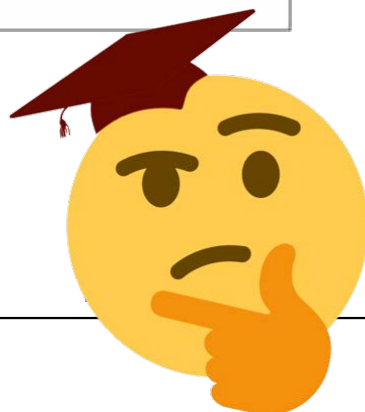


39 – (9231/31_Winter_2023_Q1) – Momentum And Impulse

(a)	PCLM: $mv = -mu \cos 60^\circ + 2mu \cos \theta$ $v = -\frac{1}{2}u + 2u \cos \theta = v = \frac{1}{2}u(4 \cos \theta - 1)$	B1	First line must be seen. AG
		1	
(b)	KE of A = $\frac{1}{2}m(v^2 + (u \sin 60^\circ)^2)$	B1	
	KE of A = 2 × KE of B, so $\frac{1}{2}m(v^2 + (u \sin 60^\circ)^2) = 2 \times \frac{1}{2} \times 2m(u \sin \theta)^2$	M1	
	$\left(\frac{1}{2}u(4 \cos \theta - 1)\right)^2 + \frac{3}{4}u^2 = 4u^2(\sin \theta)^2$ $8 \cos^2 \theta - 2 \cos \theta - 3 = 0$	M1	Use result of (a) and rearrange. Obtain 3-term quadratic.
	$\cos \theta = \frac{3}{4}, -\frac{1}{2}$ but angle is acute, so $\cos \theta = \frac{3}{4}$	A1	
		4	
(c)	NEL: $v = e(u \cos 60^\circ + u \cos \theta)$	M1	
	$v = u, u = e\left(\frac{1}{2}u + \frac{3}{4}u\right)$ $e = \frac{4}{5}$	A1	
		2	

40 – (9231/32_Winter_2023_Q5) – Momentum And Impulse

(a)	At A: $R_A - mg \cos \alpha = \frac{mv_A^2}{a}$	B1	
	At B: $-R_B + mg \cos \theta = \frac{mv_B^2}{a}$	B1	
	Energy: $\frac{1}{2}mv_A^2 - \frac{1}{2}mv_B^2 = mga(\cos \alpha + \cos \theta)$	M1	All terms present, allow sign errors, cos/sin mix.
		A1	AEF
	Eliminate velocities, use $R_B = \frac{1}{6}R_A$ and substitute angle values $\frac{1}{2}a(R_A - mg \cos \alpha) - \frac{1}{2}a(-R_B + mg \cos \theta) = mga(\cos \alpha + \cos \theta)$ $6R_B - \frac{3}{5}mg + R_B - \frac{4}{5}mg = 2mg \times \frac{7}{5}$ $R_B = \frac{3}{5}mg$	M1	Any equivalent working, leading to a dimensionally correct equation in R_B and mg .
		A1	
(b)		6	
	From first equation in (a), $6 \times \frac{3}{5}mg - \frac{3}{5}mg = \frac{mv_A^2}{a}$	M1	
	$v_A^2 = 3ag$, so $k = 3$	A1	CAO
		2	

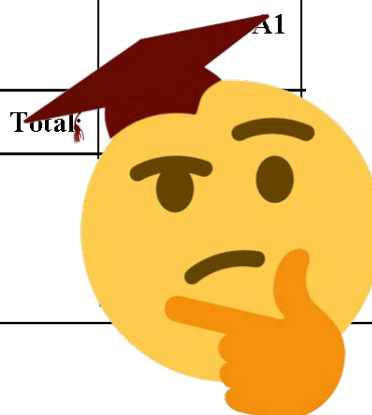


1 - (9231/21_Summer_2017_Q5) - Circular Motion

(i)	$\frac{1}{2}mv_1^2 = \frac{1}{2}mu^2 + mga \cos \alpha$ $v_1^2 = ag + 2ag \cos \alpha, v_1 = \sqrt{ag(1 + 2 \cos \alpha)}$ AG	M1 A1
	Total:	2
(ii)	$T_A + mg \cos \alpha = m(\sqrt{ag})^2/a, T_A = mg(1 - \cos \alpha)$ $\frac{1}{2}mv_2^2 = \frac{1}{2}mu^2 + mga \cos \alpha - mg \frac{2}{3}a \cos 60^\circ$	M1 A1
	or $\frac{1}{2}mv_1^2 - mg \frac{2}{3}a \cos 60^\circ$	M1 A1
	$v_2^2 = ag + 2ag \cos \alpha - \frac{2}{3}ag = ag(\frac{1}{3} + 2 \cos \alpha)$	A1
	$T_C + mg \cos 60^\circ = m v_2^2 / \frac{2}{3}a [= 3m v_2^2 / 2a]$ [$T_C = 3mg \cos \alpha$]	M1 A1
	$mg(1 - \cos \alpha) = 3mg(\frac{1}{3} + 2 \cos \alpha)/2 - \frac{1}{2}mg$	M1 A1
	$1 - \cos \alpha = 3 \cos \alpha, \cos \alpha = \frac{1}{4}$	A1
	Total:	10

2 - (9231/23_Summer_2017_Q5) - Circular Motion

(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos \alpha + \sin \alpha)$ $v^2 = u^2 + 2ag(4/5 + 3/5)$ $= u^2 + 14ag/5$ AG	M1 A1
	Total:	2
(ii)	$T_P = mu^2/a - mg \cos \alpha$	B1
	$T_Q = mv^2/a + mg \sin \alpha$	B1
	$v^2 = 2u^2 - ag(2 \cos \alpha + \sin \alpha) = 2u^2 - 11ag/5$ (AEF)	M1 A1
	$u = \sqrt{(5ag)} \text{ or } 2.24\sqrt{(ag)} [v^2 = 39ag/5]$	A1
	Total:	5
(iii)	$T_{min} + mg = mV^2/a$	B1
	$\frac{1}{2}mV^2 = \frac{1}{2}mu^2 - mga(1 - \cos \alpha)$ or $\frac{1}{2}mv^2 - mga(1 + \sin \alpha)$	M1
	$[V^2 = (5 - 2/5)ag = (39/5 - 16/5)ag = 23ag/5]$ $T_{min} = 23mg/5 - mg = 18mg/5 \text{ or } 3.6mg$	A1
	Total:	



3 - (9231/21_Winter_2017_Q1) - Circular Motion

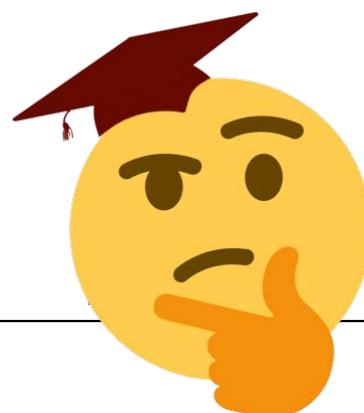
$-p + 2t = 0, p = 4$	MI A1	Find p by equating transverse acceln. to 0 at $t = 2$
$a_R = (8 - pt + t^2)^2 / 0.8$	MI	Find radial acceleration a_R at $t = 2$ in terms of p from v^2/r
$= 4^2 / 0.8 = 20 \text{ [m s}^{-2}\text{]}$	A1	Evaluate with $p = 4$
	4	

4 - (9231/21_Winter_2017_Q11) - Circular Motion, Bivariate Data

A(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(\cos \alpha + \cos \beta)$	M1 A1	Find v^2 at A' from conservation of energy (A0 if no m)
$mv^2/a = mg \cos \beta$ $u^2 = ag \cos \beta + 2ag(1/16 + \cos \beta)$	B1	Use $F = ma$ radially at A' with $R_{A'} = 0$ Use $\cos \alpha = 1/16$ and eliminate v^2 to verify u^2	
$= (1/8) ag(1 + 24 \cos \beta)$	AG	M1 A1	
		5	

A(ii)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \cos 2\beta)$	B1	Find u^2 at B' from conservation of energy (B0 if no m)
	$mv^2/a = mg \cos 2\beta$ $u^2 = ag \cos 2\beta + 2ag(1 + \cos 2\beta)$	B1	Use $F = ma$ radially at B' with $R_{B'} = 0$ Eliminate u^2 to find u^2
	$= ag(2 + 3 \cos 2\beta)$	M1	
	$1 + 24 \cos \beta = 8(2 + 3 \cos 2\beta)$ $= 16 + 48 \cos^2 \beta - 24$ $16 \cos^2 \beta - 8 \cos \beta - 3 = 0$	M1	Combine eqns for u^2 Formulate and solve quadratic to find $\cos \beta$
	$\cos \beta = 3/4$ [rejecting $-1/4$]	M1 A1	
		6	
A(iii)	$u^2 = (19/8) ag$ or $2.375 ag$ $R = mu^2/a + mg \cos \alpha$ $= (19/8 + 1/16) mg$	B1	Find u^2 using value of $\cos \beta$ Use $F = ma$ radially at A to find reaction R at A
	$= (39/16) mg$ or $2.44 mg$	M1 A1	
		3	

B(i)	e.g. $S_{xy} = 218.72 - 45.3 \times 40.5/9 = 14.87$ or 1.652 $S_{xx} = 245.59 - 45.3^2/9 = 17.58$ or 1.953 or $S_{yy} = 195.11 - 40.5^2/9 = 12.86$ or 1.429		Find reqd. values
EITHER: $b_1 = S_{xy} / S_{xx} = 14.87 / 17.58 = 0.84585 = 0.846$ $(y - 40.5/9) = b_1(x - 45.3/9)$ $(y - 4.5) = 0.846(x - 5.033)$	(MI A1)	Find gradient in $y - \bar{y} = b_1(x - \bar{x})$ to 3 s.f. Find eqn. of regression line to 3 s.f.	
$y = 0.846x + 0.24257 = 0.846x + 0.243$	MI A1		
$x = 4.68$	A1	Find x when $y = 4.2$	
OR: $b_2 = S_{xy} / S_{yy} = 14.87 / 12.86 = 1.1563 = 1.16$ $(x - 45.3/9) = b_2(y - 40.5/9)$ $(x - 5.033) = 1.16(y - 4.5)$	(MI A1)	Find gradient in $x - \bar{x} = b_2(y - \bar{y})$ to 3 s.f. Find eqn. of regression line to 3 s.f.	
$x = 1.16y - 0.170[04]$	MI A1		
$x = 4.69$	A1	Find x when $y = 4.2$ (A0 for $x = 4.70$)	
	5		



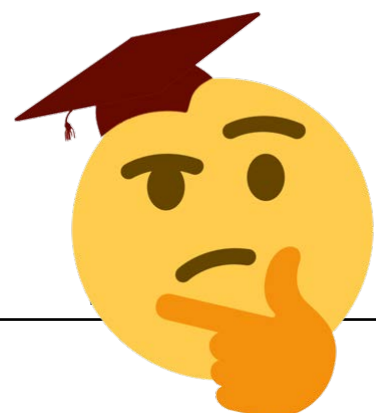
B(ii)	$H_0: \mu_x - \mu_y = 0.3, H_1: \mu_x - \mu_y > 0.3$ (AEF)	B1	State hypotheses (B0 for $\bar{x} \dots$)
	$\bar{d} = 4.8 / 9$ or $8/15$ or 0.533 (where $d = x - y$)	B1	Find sample mean
	$s^2 = (3.26 - 4.8^2/9) / 8$ $= 7/80$ or 0.0875 or 0.296^2	M1 A1	Estimate population variance (allow biased here: $7/90$ or 0.0778 or 0.279^2)
	$t = (\bar{d} - 0.3) / (s/\sqrt{9}) = 2.37$	M1 A1	Find value of t
	$t_{8, 0.975} = 2.306$ or 2.31	B1	State or use correct tabular t -value (or can compare $\bar{d}$ with $0.3 + t_{8, 0.975} s/\sqrt{9} = 0.527$)
	Reject H_0 if $t >$ tabular value $2.37 [\pm 0.1] > 2.31$ so accept belief (AEF)	M1	Valid method for reaching conclusion Correct conclusion, from correct values
	[of increase of more than 0.3] (AEF)	A1	SC: Wrong (hypothesis) test can earn only B1 for hypotheses
		9	

5 - (9231/23_Summer_2018_Q1) - Circular Motion

$a_R = (2^2 - 2 + 2)^2 / 0.8 = 4^2 / 0.8 = 20 \text{ [m s}^{-2}\text{]}$	M1 A1	Find radial acceleration a_R at $t = 2$ from v^2/r
$a_T = 2t - 1 = 3 \text{ [m s}^{-2}\text{]}$	B1	Find transverse acceleration a_T at $t = 2$ by differentiation
	3	

6 - (9231/23_Summer_2018_Q5) - Momentum And Impulse, Circular Motion

$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 + mga$	M1	Find v_1^2 at lowest point from consvn. of energy (M0 if no m)
$v_1^2 = (12 + 2) ag = 14 ag$	A1	
$Mv_2 = mv_1$ with $M = (1 + k) m$	M1	Find new v_2 from consvn. of momentum
$v_2 = v_1 / (1 + k) [= \sqrt{(14 ag) / (1 + k)}]$	A1	
$T_1 = mv_1^2/a + mg = (14 + 1) mg = 15mg$	M1 A1	Find tension T_1 just before collision by using $F = ma$ radially
$T_2 = Mv_2^2/a + Mg$ $= (1 + k) \{14 / (1 + k)\} mg$	M1	Find tension T_2 just after collision by using $F = ma$ radially (M1 needs M , not m , throughout)
or $\{14 / (1 + k) + (1 + k)\} mg$ (AEF)	A1	
$14 + (1 + k)^2 = 15 (1 + k)2$	M1	Equate T_2 and $\frac{1}{2} T_1$ to give any eqn in k
$2k^2 - 11k + 15 = 0, k = 2.5$ or 3 (M1 dep)	M1 A1	Solve resulting quadratic for k (M1 dep on all previous M1s, and requires <i>quadratic</i> eqn)
	11	



7 - (9231/21_Summer_2018_Q11) - Circular Motion, Bivariate Data

A(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga \cos \theta$ and	M1	Use conservation of energy (A0 if no m) and
	$T = mv^2/a + mg \cos \theta$ $T = mu^2/a + 3mg \cos \theta$	M1 A1	find tension T at P_1 by using $F = ma$ (A1 for both correct) Combine to verify $\cos \theta$
	$(11/5)mg = 2mg/5 + 3mg \cos \theta$, $\cos \theta = 3/5$	AG	M1 A1 by substituting $T = (11/5)mg$ and $u = \sqrt{(2ag/5)}$
	$v^2 = u^2 + 2ag \cos \theta = (2/5 + 6/5)ag = 8ag/5$ $v = \sqrt{(8ag/5)}$ or $\sqrt{(1.6ag)}$ or $4\sqrt{a}$	B1 (AEF)	Find v at P_1 (can assume $\cos \theta = 3/5$)
		6	
A(ii)	$t = (a \sin \theta) / (v \cos \theta) = 4a/3v$ [$= \frac{1}{3}\sqrt{a}$]	M1 A1	Find time t to P_2 by considering horizontal motion
	$h = (v \sin \theta) t + \frac{1}{2}gt^2$ $= 16a/15 + 5a/9 = 73a/45$ or $1.62a$	M1 A1	Find ht. fallen at P_2 by considering vertical motion
	$OP_2 = h + a \cos \theta = 20a/9$ or $2.22a$	M1 A1	Find OP_2
		6	
B(i)	$\Sigma x = 48$, $\Sigma y = 29 + 2p$, $\Sigma xy = 242 + 17p$, $\Sigma x^2 = 450$, $[\Sigma y^2 = 219 + 2p^2]$ $S_{xy} = 242 + 17p - 48 \times (29 + 2p)/6 = 10 + p$ $S_{xx} = 450 - 48^2/6 = 66$ $[S_{yy} = 219 + 2p^2 - (29 + 2p)^2/6 = (8p^2 - 116p + 473)/6]$	M1 A1	Find required values
	$0.25 = S_{xy} / S_{xx} = (10 + p)/66$, $p = 6.5$	M1 A1	Find p from gradient in eqn. of regression line
	$(29 + 2p)/6 = 0.25 \times 48/6 + k$, $k = 7 - 2 = 5$	M1 A1	Find k from means and regression line
		6	
B(ii)	$[\Sigma y = 42$, $\Sigma y^2 = 303.5$, $S_{yy} = 303.5 - 42^2/6 = 9.5]$ $r = S_{xy} / \sqrt{(S_{xx}S_{yy})} = 16.5 / \sqrt{(66 \times 9.5)} = 0.659$	M1 *A1	Find correlation coefficient r
		2	
B(iii)	$H_0: \rho = 0$, $H_1: \rho > 0$	B1	State both hypotheses (B0 for $r \dots$)
	<i>EITHER:</i> $r_{0.05} = 0.729$	*B1	State or use correct tabular one-tail r -value
	Accept H_0 if $ r < \text{tab. } r\text{-value}$ (AEF)	M1	State or imply valid method for conclusion
	<i>OR:</i> $t_r = r\sqrt{(n-2)/(1-r^2)} = 1.75$, $t_{0.05} = 2.132$	(*B1)	(Rarely seen)
	Accept H_0 if $ t_r < \text{tab. } t\text{-value}$ (AEF)	(M1)	
	No positive correlation (AEF)	A1	Correct conclusion (dep *A1, *B1)
		4	

8 - (9231/22_Winter_2018_Q3) - Circular Motion

(i)	$\frac{1}{2}mv_B^2 = \frac{1}{2}mu^2 - mga(\sin \alpha - \cos \alpha)$ $[v_B^2 = u^2 - (14/13)ag]$	M1A1	Find speed v_B at B by conservation of energy (A0 if no m) [$\sin \alpha = 12/13$, $\cos \alpha = 5/13$]
	$[T_B =] mv_B^2/a - mg \sin \alpha = 0$ [$v_B^2 = (12/13)ag$]	B1	Equate tension T_B at B to zero by using $F = ma$ radially
	$u^2 = (3 \sin \alpha - 2 \cos \alpha) ag$	M1	Combine to verify u^2
	$= (36/13 - 10/13) ag = 2 ag$ AG	A1	
		5	



(ii)	<i>EITHER:</i> $\frac{1}{2}mv_C^2 = \frac{1}{2}mu^2 + mga(1 + \cos \alpha)$ or $\frac{1}{2}mv_B^2 + mga(1 + \sin \alpha) [v_C^2 = 62ag/13]$	M1	Find v_C^2 at lowest point C by conservation of energy
	$T_{\max} = mv_C^2/a + mg$	B1	Find tension $T_{\max}$ at lowest point from $F = ma$ radially
	$= 62mg/13 + mg$	M1	Combine to find $T_{\max}$
	$= 75mg/13$ or 5.77 mg or 57.7 m	A1	
	<i>OR:</i> $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos \alpha + \cos \theta)$ $[V^2 = 2ag(18/13 + \cos \theta)]$	(M1)	Find V^2 at general point by conservation of energy (where OP is e.g. at θ to downward vertical)
	$T = mv^2/a + mg \cos \theta = (36/13 + 3 \cos \theta) mg$	B1	Find tension T at general point from $F = ma$ radially
	$T_{\max} = (36/13 + 3) mg$	M1	Combine to find $T_{\max}$ at lowest point where $\theta = 0$
	$= 75mg/13$ or 5.77 mg or 57.7 m	A1)	
		4	

9 - (9231/21_Winter_2018_Q11) - Circular Motion, X2 Test

A(i)	$\frac{1}{2}mv_C^2 = \frac{1}{2}mu^2 + mga(1 - \cos \alpha) [v_C^2 = u^2 + (2/5) ag]$	M1A1	Find speed v_C at C by conservation of energy (A0 if no m)
	$R_A = mg \cos \alpha + mv_C^2/a$	B1	Find reaction R_A at A by using $F = ma$
	$R_C = mg + mv_C^2/a$	B1	Find reaction R_C at C by using $F = ma$
	$8 \{ag + u^2 + (2/5) ag\} = 9 \{(4/5) ag + u^2\}$	M1	Verify u^2 using $R_C / R_A = 9/8$
	$u^2 = (8 + 16/5 - 36/5) ag = 4ag$ AG	A1	
		6	
A(ii)	$\frac{1}{2}mv_B^2 = \frac{1}{2}mu^2 - 2mga \cos \alpha$ or $\frac{1}{2}mv_C^2 - mga(1 + \cos \alpha) [v_C^2 = (22/5) ag]$	M1	Find speed v_B at B by conservation of energy (A0 if no m)
	$v_B^2 = (4 - 16/5) ag$ or $(22/5 - 18/5) ag = (4/5) ag$	*A1	
	≥ 0 so P reaches height of B (AEF; dep *A1)	A1	(or valid argument based on R_A)
	$R_A = m v_B^2/a - mg \cos \alpha = 4mg/5 - 4mg/5 = 0$	M1**A1	Find reaction R_A at B by using $F = ma$
	≥ 0 so P still [just] in contact at B (AEF; dep **A1)	A1	
		6	
B(i)	$\bar{x} = (1/200) \sum x f(x) = 30\,064 / 200 = 150.3[2]$	M1A1	Find sample mean to 4 s.f. using mid-interval values (B1 for $29\,964 / 200 = 149.8[2]$ or $30\,164 / 200 = 150.8[2]$)
		2	



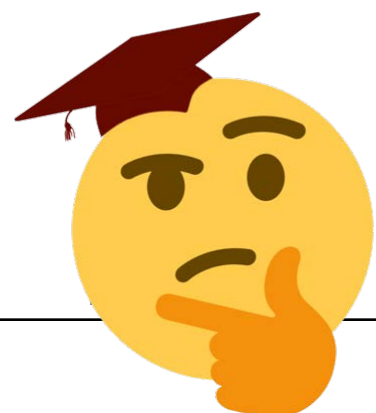
B(ii)	$P(151 \leq X < 152)$ $= P((151 - 150)/1.2 \leq Z < (152 - 150)/1.2)$ $= P(0.833 \leq Z < 1.667)$ $= 0.9522 - 0.7975 [= 0.1547] \text{ (to 4 s.f.)}$	M1A1	Find $P(151 \leq X < 152)$
	$E_{151} = 190.44 - 159.5 \text{ or } 200 \times 0.1547 = 30.94$ AG	A1	and hence expected frequency for $151 \leq x < 152$
		3	
B(iii)	H_0 : [Normal] distribution fits data (AEF)	B1	State (at least) null hypothesis in full
	O_i : 3 23 52 69 36 17	M1	Combine values consistent with all exp. values ≥ 5
	E_i : <u>9.56</u> 30.94 59.50 59.50 30.94 <u>9.56</u>		
	$\chi^2 = 4.501 + 2.038 + 0.945 + 1.517 + 0.828 + 5.790$	M1	Find value of χ^2 from $\Sigma (E_i - O_i)^2 / E_i$ [or $\Sigma O_i^2 / E_i - n$]
	$= 15.6 \text{ (to 3 s.f.)}$	A1	
	No. n of cells: 8 7 6 5 $\chi_{0.05}^2$: 14.07 12.59 <u>11.07</u> 9.488	B1	State or use consistent tabular value $\chi_{n-1, 0.05}^2$ (to 3 s.f.) [FT on number, n , of cells used to find χ^2]
	Accept H_1 if $\chi^2 > \text{tabular value}$ (AEF)	M1	State or imply valid method for conclusion
	$15.6 [\pm 0.1] > 11.1$ so distrn. doesn't fit [data] (AEF)	A1	Conclusion (requires both values correct)
		7	

10 - (9231/21_Summer_2019_Q1) - Circular Motion

(i)	$a_{\theta} = 2 (d\theta/dt)^2 = 2 (2 \sin 2t)^2 = 2 (2 \sin \pi/3)^2$	M1	Verify radial acceleration a_{θ} at $t = \pi/6$ from $r\omega^2$
	$= 2 (\sqrt{3})^2 = 6 \text{ [m s}^{-2}\text{]}$ AG	A1	
		2	
(ii)	$a_r = 2 d^2\theta/dt^2 = 2 (4 \cos 2t) = 2 (4 \cos \pi/3)$	M1	Find transverse acceleration a_r at $t = \pi/6$ by differentiation
	$= 4 \text{ [m s}^{-2}\text{]}$	A1	
		2	

11 - (9231/23_Summer_2019_Q2) - Circular Motion

(i)	$T_A = m v_A^2 / a - mg \cos \alpha$ [= $m (9ag/4) / a - mg \cos \alpha$]	B1	Find tension T_A at A from $F = ma$ radially
	$T_B = m v_B^2 / a + mg \cos \alpha$	B1	Find tension T_B at B from $F = ma$ radially
	$\frac{1}{2} m v_B^2 = \frac{1}{2} m v_A^2 + 2mga \cos \alpha$ [$v_B^2 = ag (9/4 + 4 \cos \alpha)$]	M1 A1	Apply conservation of energy at B (A0 if no m)
	$v_B^2 = 4v_A^2 - 5ag \cos \alpha = v_A^2 + 4ga \cos \alpha$	M1	Combine using $T_B = 4 T_A$ and $v_A^2 = 9ag/4$ to verify $\cos \alpha$
	$9ag \cos \alpha = 3v_A^2 = 27ag/4$, $\cos \alpha = 3/4$ AG	A1	
		6	
(ii)	$T_B = 4 T_A = (9 - 4 \cos \alpha) mg = 6mg$ or $v_B^2 = 21ag/4$, $T_B = (21/4 + \cos \alpha) mg = 6mg$	M1 A1	Find T_B
		2	



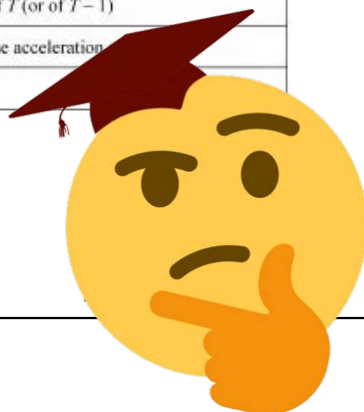
12 - (9231/21_Summer_2019_Q11) - Momentum And Impulse, Circular Motion, Inference Using Normal And T-distributions

E(i)	$\frac{1}{2}mu_P^2 = \frac{1}{2}m(2\sqrt{ag/2}) + mga \quad [u_P^2 = (25/2)ag]$	M1 A1	u_P is speed of P at lowest point, v_Q is speed of Q immediately after collision. Apply conservation of energy at lowest point (A0 if no m)
	$\frac{1}{2}4mv_Q^2 = 4mag$	M1	Find speed v_Q at lowest point by conservation of energy (A0 if no m)
	$v_Q = \sqrt{2ag}$ or $1.41\sqrt{ag}$ or $4.47\sqrt{a}$	A1	
	$mu_P = [\pm] mv_P + 4mv_Q$	M1	Find v_P using conservation of momentum (m may be omitted)
	$v_P = [\pm](-5\sqrt{2} + 4\sqrt{2})\sqrt{ag}$	A1	
	$ v_P = (3\sqrt{2})\sqrt{ag}$ or $2.12\sqrt{ag}$ or $0.671\sqrt{a}$ (AEF)	A1	Hence find speed of P
		7	
E(ii)	V_P is speed of P when it loses contact $\frac{1}{2}mV_P^2 = \frac{1}{2}mv_P^2 - mga(1 + \cos \alpha)$ $[V_P^2 = (9/2)ag - 2ga(1 + \cos \alpha) = (5/2 - 2 \cos \alpha)ag]$	M1 A1	Apply conservation of energy at D (A0 if no m)
	$[R_D] = mV_P^2/a - mg \cos \alpha = 0 \quad [V_P^2 = ag \cos \alpha]$	M1 A1	Apply $F = ma$ radially at D with reaction = 0
	$(5/2 - 2 \cos \alpha)ag = ag \cos \alpha, \cos \alpha = 5/6$ or 0.833	A1	Combine to find $\cos \alpha$
		5	

O(i)	$ts_A/\sqrt{8} = \frac{1}{2}(16.7 - 13.5) [= 1.6]$	M1	Relate s_A to semi-width of confidence interval
	$t_{7,0.975} = 2.365$ (to 3 s.f.)	A1	State or use correct tabular t value
	$[s_A = \sqrt{8} \times 1.6 / 2.365 = 1.9135], s_A^2 = 3.66[16]$	A1	Hence find unbiased estimate of A 's population variance
		3	
O(ii)	$H_0: \mu_A = \mu_B, H_1: \mu_A > \mu_B$ (AEF)	B1	State hypotheses (B0 for $\bar{x} \dots$)
	$[\bar{x}_A = 15.1], \bar{x}_B = 85.2/6 = 14.2$	B1	Find sample mean for B
	$s_B^2 = (1221.06 - 85.2^2/6) / 5$ $= 561/250$ or 2.244 or 1.498^2 (all to 3 s.f.)	M1	Estimate or imply population variance for B (allow biased here: 1.87 or 1.367^2)
	$s^2 = (7s_A^2 + 5s_B^2) / 12 = 3.0709$ or 1.752^2	M1 A1	Estimate (pooled) common variance (s_B^2 not needed explicitly)
	$t_{12,0.95} = 1.782$	B1	State or use correct tabular t value
	$[\cdot] t = (\bar{x}_A - \bar{x}_B) / (s \sqrt{1/8 + 1/6}) = 0.95$ $t < 1.78$ so [accept H_0]	M1 A1	Find value of t (or can compare $\bar{x}_A - \bar{x}_B = 0.9$ with 1.69) Correct conclusion
	mean mass of B not less than mean mass of A (AEF)	B1	
		9	
			SCI: Implicitly taking s_A^2, s_B^2 as unequal population variances (may also earn first B1 B1 M1) $z = (\bar{x}_A - \bar{x}_B) / \sqrt{(s_A^2/8 + s_B^2/6)}$ $= 0.9 / \sqrt{(0.8317)} = 0.987$ $z < 1.645$ so
			DepSCI: mean mass of B not less than mean mass of A (AEF) Comparison with $z_{0.95}$ and conclusion (FT on z) (can earn at most 5/9)

13 - (9231/21_Winter_2019_Q1) - Circular Motion

$(T-1)^4/2 = 8$	M1 A1	Equate radial acceln. to 8 at $t = T$ from v^2/r
$T = 3$ (or $T-1 = 2$)	A1	Hence find positive value of T (or of $T-1$)
$a_T = 2(T-1) = 4$ [m s ⁻²]	M1 A1	Find magnitude of transverse acceleration
	5	



14 - (9231/21_Winter_2019_Q4) - Circular Motion

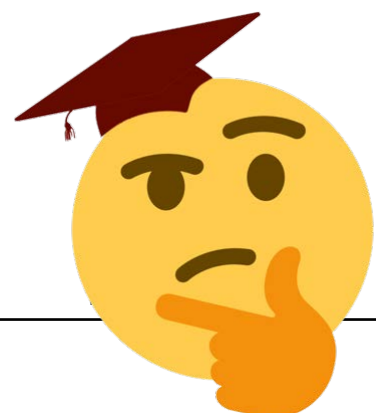
(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga \cos \theta$	M1	Use conservation of energy to slack point P_1
	$mv^2/a - mg \cos \theta = 0$	M1 A1	Equate tension at P_1 to 0 by using $F = ma$ A1 if both eqns correct, with m included. AG
	$v^2 = 2ag - 2ag \cos \theta = ag \cos \theta$	M1	Combine to verify $\cos \theta$ using $u = \sqrt{2ag}$
	$\cos \theta = 2/3$	A1	
		5	
(ii)	$v_V = v \sin \theta = \sqrt{2ag/3} (\sqrt{5/3})$ or $v_V^2 = (10/27) ag$	M1	Find vertical speed v_V at P_1
	$h = v_V^2/2g = (5/27) a$ or $0.185 a$	M1 A1	Find height risen above P_1 by considering vertical motion
	$h + a \cos \theta = (23/27) a$ or $0.852 a$	A1	Find total height risen above level of O
		4	

15 - (9231/33_Summer_2020_Q1) - Circular Motion

$T = 4mg = m a \omega^2$ so $\omega^2 = \frac{4g}{a}$	B1
Time per revn = $\frac{2\pi}{\omega} = \pi \sqrt{\frac{a}{g}}$	B1
	2

16 - (9231/31_Summer_2020_Q2) - Circular Motion

For A: $T = 3mg$ For B: $\uparrow T \cos \theta = mg$	M1
Equate: $3mg \cos \theta = mg$ $\cos \theta = \frac{1}{3}$	A1
$\rightarrow T \sin \theta = m r \omega^2$ with $r = (a - x) \sin \theta$	M1
Equate: $3mg = m(a - x)\omega^2$	A1
$x = \frac{a}{4}$	A1
	5



17 - (9231/33_Summer_2020_Q3) - Circular Motion

$T_A - mg \cos 60 = m \frac{4ag}{a} \quad (T_A = \frac{9}{2} mg)$	B1
$T_B + mg \cos 60 = m \frac{v^2}{a}$	B1
Energy: $\frac{1}{2} m (4ag - v^2) = mg \cdot 2a \cos 60$ $v^2 = 2ag$	M1A1
$T_B = \frac{3mg}{2}$	A1
Ratio is 3 : 1	A1
	6

18 - (9231/31_Summer_2020_Q7) - Circular Motion

(a)	$(N +)mg \cos \theta = \frac{mv^2}{a}$	B1
	$\frac{1}{2} mv^2 - \frac{1}{2} m \frac{7ag}{2} = -mg(a + a \cos \theta)$	M1A1
	Loses contact when $N = 0$, so combine and simplify	M1
	$\cos \theta = \frac{1}{2} \therefore \theta = 60^\circ$ AG	A1
		5

(b)	When P is vertically below O , its horizontal displacement is $a \sin 60$, so time $T = \frac{a \sin 60}{v \cos 60} = a\sqrt{3}/v = \sqrt{\frac{6a}{g}}$	M1
	From (a), $v^2 = \frac{1}{2} ag$	A1
	Vert: $h = \frac{v\sqrt{3}}{2} T - \frac{1}{2} g T^2$	M1
	$\frac{3}{2} a - 3a = -\frac{3a}{2}$	A1
	This corresponds to the point A	A1
Alternative method for question 7(b)		
	$y = x\sqrt{3} - \frac{4x^2}{a}$	M1A1
	Coordinates of A : $x = \frac{1}{2} a\sqrt{3}$, $y = -\frac{3}{2} a$	B1
	Substitute coordinates into $y = x\sqrt{3} - \frac{4x^2}{a}$ and show that these satisfy this equation	M1A1
		5



19 - (9231/32_Winter_2020_Q1) - Circular Motion

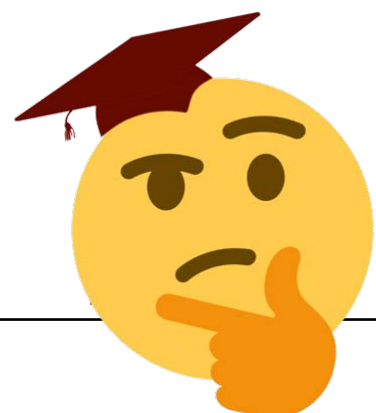
At B, $mg \cos \beta = \frac{mv^2}{a}$; ($v^2 = ag \cos \beta$)	B1	
$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(\cos \alpha - \cos \beta)$	M1A1	Energy equation with 4 terms and correct dimensions ($v^2 = \frac{3ga}{2} - 2ag \cos \beta$)
Substitute for u , $\cos \alpha$ and v : $ag \cos \beta = \frac{ag}{6} + 2ag\left(\frac{2}{3} - \cos \beta\right)$	M1	Eliminate to find $\cos \beta$
$\cos \beta = \frac{1}{2}$	A1	
	5	

20 - (9231/31_Winter_2020_Q2) - Circular Motion

At top, tension = 0, so $mg = \frac{mv^2}{a}$ ($v^2 = ag$)	B1	
$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga(1 + \cos \theta)$	M1 A1	Energy equation
Substitute for u and v : $ag = \frac{16}{25}5ag - 2ag(1 + \cos \theta)$	M1	Eliminate
$\cos \theta = \frac{1}{10}$	A1	
	5	

21 - (9231/31_Winter_2020_Q3) - Circular Motion

(a)	$T = 4mg \frac{ka}{a}$	B1	Use Hooke's law
	$T \sin \theta = \left(\frac{mrg}{a}\right) = m(k+1)a \sin \theta \frac{g}{a}$	M1	N2L horizontally. Must see T and k .
	$T = mg(k+1)$	A1	
	Equate: $k = \frac{1}{3}$	A1	
		4	
(b)	$\uparrow T \cos \theta = mg$	M1	
	$\left(T = \frac{4}{3}mg\right) \cos \theta = \frac{mg}{\frac{4}{3}mg} = \frac{3}{4}$	A1	
		2	



22 - (9231/31_Summer_2021_Q2) - Circular Motion

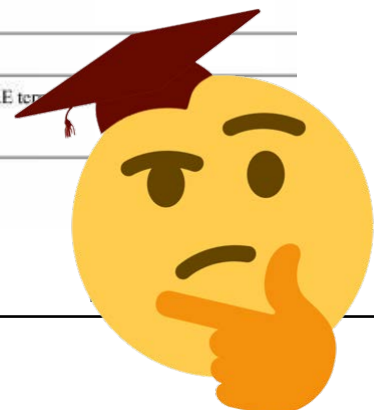
$\uparrow R \cos \theta = mg$	B1	
$\rightarrow R \sin \theta = \frac{mv^2}{r}$	B1	
$r = a \sin \theta$	B1	
$8 \cos \theta = 3(1 - (\cos \theta)^2)$	M1	Quadratic equation in $\cos \theta$.
$\cos \theta = \frac{1}{3}$	A1	
$x = \frac{2}{3}a$	A1	
	6	

23 - (9231/33_Summer_2021_Q3) - Circular Motion

(a)	For B: $T = mg$	B1	May be embedded.
	For A: $R + T \cos \theta = 3mg$	M1	All 3 terms, allow sign errors, allow sin/cos mix.
	Use given R to obtain $\cos \theta = \frac{3}{5}$	A1	
		3	
(b)	$T \sin \theta = \frac{3mv^2}{r}$	M1	May be seen in part (a), allow sin/cos mix.
	$r = AR \sin \theta$	B1	Or equivalent.
	[Combine to give $AR = \frac{3a}{4}$, so] $BR = \frac{1}{4}a$	A1	
		3	

24 - (9231/33_Summer_2021_Q4) - Circular Motion

$\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = mg(a \cos \theta + a \sin \theta)$	*M1	Energy equation, with 2 KE terms and a two-part GPE term, allow cos/sin mix.
$\frac{3}{4}u^2 = 2ag(\cos \theta + \sin \theta)$	A1	
At B, tension in string is zero, so $mg \sin \theta = \frac{m\left(\frac{u}{2}\right)^2}{a}$ ($u^2 = 4ag \sin \theta$)	B1	N2L.
Eliminate u^2	DM1	
$\tan \theta = 2$ OE	A1	
At A, $T - mg \cos \theta = \frac{mu^2}{a}$	B1	N2L.
$T = \frac{9\sqrt{5}}{5}mg$ ($= 4.02mg$)	M1 A1	Substitute to find T.
Alternative method for question 4		
$\frac{1}{2}mu^2 - \frac{1}{2}m\left(\frac{u}{2}\right)^2 = mg(a \cos \theta + a \sin \theta)$	*M1	Energy equation, with 2 KE terms, allow cos/sin mix.
$\frac{3}{4}u^2 = 2ag(\cos \theta + \sin \theta)$	A1	



At B, tension in string is zero, so $mg \sin \theta = \frac{m \left(\frac{u}{2} \right)^2}{a}$ ($u^2 = 4ag \sin \theta$)	B1	N2L
Eliminate θ : $u^2 = \frac{8ag\sqrt{5}}{5}$	DM1 A1	
At A, $T - mg \cos \theta = \frac{mu^2}{a}$	B1	N2L
$T = \frac{9\sqrt{5}}{5} mg$ ($= 4.02mg$)	M1 A1	Substitute to find T .
	8	

25 - (9231/31_Summer_2021_Q5) - Circular Motion

$T_A - mg \cos \theta = \frac{mu^2}{a}$	B1	
$T_B + mg \cos \theta = \frac{mag}{a}$	B1	
$T_A = 7T_B$ so $mg \cos \theta + \frac{mu^2}{a} = 7 \left(-mg \cos \theta + \frac{mag}{a} \right)$ $u^2 = ag(7 - 8 \cos \theta)$	M1	Use given relationship and combine.
Energy: $\frac{1}{2} mu^2 - \frac{1}{2} mag = mg(a \cos \theta + a \cos \theta)$ So $u^2 = ag(4 \cos \theta + 1)$	M1 A1	Energy equation.
Equate expressions for u^2	M1	
$\cos \theta = \frac{1}{2}$, $\theta = 60^\circ$	A1	CAO
$u = \sqrt{3ga}$	A1	CAO
	8	

26 - (9231/32_Winter_2021_Q3) - Circular Motion

(a)	$T = 3mg$ and $T \cos \theta = mg$	M1	Must see both of these separately.
	Combining, $\cos \theta = \frac{1}{3}$	A1	At least one step of working, AG.
		2	
(b)	$(\cos \theta = \frac{a-x}{x}, \text{ where } x = AR)$ $AR = \frac{3}{4}a$ or $BR = \frac{1}{4}a$ or radius = $\frac{a}{\sqrt{2}}$	B1	$\left(\sin \theta = \frac{\sqrt{8}}{3} \right)$
	$T \sin \theta = \frac{mv^2}{r}$	M1	
	Combining to find an equation in v^2 , a and g only.	DM1	
	$v^2 = 2ga$, $v = \sqrt{2ga}$	A1	
		4	



27 - (9231/31_Winter_2021_Q6) - Circular Motion

(a)	At A $T_A - mg \cos \theta = m \times \frac{5ag}{a}$	B1	N2L
	At B $T_B + mg \cos \theta = m \times \frac{v^2}{a}$	B1	N2L
	$\frac{1}{2} m \times 5ag - \frac{1}{2} mv^2 = mga \times 2 \cos \theta$	M1	Energy equation with correct number of terms.
	$v^2 = 5ag - 4ga \cos \theta$	A1	Accept multiplied by m and/or divided by a .
	Use ratio of tensions = 9 : 5	M1	Use ratio and simplify to an expression in $\cos \theta$.
	$\cos \theta = \frac{2}{5}$	A1	CAO
		6	
(b)	Greatest speed at lowest point $-\frac{1}{2} m \times 5ag + \frac{1}{2} mV^2 = mga \times (1 - \cos \theta)$	M1	Energy equation including lowest point, correct number of terms.
	$V = \sqrt{\frac{3 \log}{5}}$	A1 FT	Ft their $\cos \theta$ from part (a).
		2	

28 - (9231/31_Summer_2022_Q2) - Circular Motion

$-\frac{1}{2} mv^2 + \frac{1}{2} m \times 3ga = mga(\cos \theta + \cos \alpha)$	M1 A1	Energy equation.
$(T +)mg \cos \theta = \frac{m}{a} v^2$	B1	N2L
$ag \cos \theta = 3ga - 2ga \cos \theta - 2ga \times \frac{4}{5}$	M1	Combine to find $\cos \theta$.
$\cos \theta = \frac{7}{15}$	A1	
	5	



29 - (9231/33_Summer_2022_Q4) - Circular Motion

(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga(\cos\theta + \cos\alpha)$	M1	Energy equation with all necessary terms, GPE terms must be resolved, allow sin/cos mix, allow sign error.
	$\frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2 = mga(\cos\theta + \cos\alpha)$	A1	$2u$ may be substituted later. Implied by $\frac{3}{2} \times \frac{2}{3} ag = g(\cos\theta + \cos\alpha)$
	At A, $T + mg \cos\theta = \frac{m}{a}u^2$	B1	N2L
	Also, $10T - mg \cos\alpha = \frac{m}{a}4u^2$	B1	N2L and use of tension (10T)
	Use all three (two N2L and energy) equations to find T in terms of m and g only.	M1	Might see $9T - mg(\cos\theta + \cos\alpha) = \frac{3m}{a} \times \frac{2}{3} ga$ $(\cos\theta + \cos\alpha) = 1$ $(10\cos\theta + \cos\alpha) = 4$
	$T = \frac{1}{3}mg$	A1	
		6	
(b)	Substitute back, $10 \times \frac{1}{3}mg - mg \cos\alpha = \frac{4m}{a} \times \frac{2}{3}ga$	M1	Any appropriate method to obtain $\cos\alpha$
	$\cos\alpha = \frac{2}{3}$	A1	
		2	

30 - (9231/31_Summer_2022_Q5) - Circular Motion

For A: $T \sin\theta = mr\omega^2$	M1	N2L horizontal.
$r = 3a \sin\theta$	B1	Correct expression for radius.
$T = m \times 3a\omega^2$	A1	
Similarly, for B: $T \cos\theta = \frac{3}{4}m \times r \times k^2\omega^2$	M1	N2L horizontal
$T = \frac{3}{4}mak^2\omega^2$	A1	
$m \times 3a\omega^2 = \frac{3}{4}mak^2\omega^2$	M1	Equate expressions for T .
$k^2 = 4, \quad k = 2$	A1	



Alternative method for question 5		
For A: $T \cos \theta = mg$, $T \sin \theta = mr\omega^2$	M1	N2L horizontal and vertical.
$r = 3a \sin \theta$	B1	Correct expression for radius.
$T = m \times 3a\omega^2 = \frac{5}{4}mg$, $\omega^2 = \frac{5}{12} \times \frac{g}{a}$	A1	Combine to obtain expression for ω^2 .
Similarly, for B: $T \cos \theta = \frac{3}{4}m \times r \times k^2 \omega^2$	M1	N2L horizontal.
$T = \frac{3}{4}mak^2 \omega^2$	A1	
$\frac{5}{4}mg = \frac{3}{4}mak^2 \times \frac{5g}{12a}$	M1	Substitute for T and ω .
$k^2 = 4$, $k = 2$	A1	
	7	

31 - (9231/31_Winter_2022_Q1) - Circular Motion

Use $F = ma$: $20 = \frac{2 \times v^2}{0.6}$ OR $20 = 2 \times 0.6\omega^2$	M1	
$v^2 = 6$ OR $\omega^2 = \frac{50}{3}$	A1	
Number of revolutions per min = $\frac{60v}{0.6 \times 2\pi}$ OR $\frac{60\omega}{2\pi}$ so 39(.0) revolutions	A1 FT	38.9848,...
	3	

32 - (9231/32_Winter_2022_Q1) - Circular Motion

When string goes slack, $mg \cos \beta = \frac{m}{a}v^2$, $v^2 = ag \cos \beta$	B1	N2L May include T , but B1 not awarded until $T = 0$.
$\frac{1}{2}m \cdot 3ag - \frac{1}{2}mv^2 = mg(a \cos \alpha + a \cos \beta)$	B1	Energy equation.
So $u^2 - ag \cos \beta = 2ag \left(\cos \beta + \frac{2}{3} \right)$	M1	Combine.
$\cos \beta = \frac{u^2 - \frac{4}{3}ag}{3ag} = \frac{5}{9}$	A1	
	4	



33 – (9231/33_Winter_2022_Q5) – Circular Motion

(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga \cos \theta$ $\left[kag = \frac{1}{3}ag + 2ag \cos \theta \right]$	B1	Energy equation.
	$T - mg \cos \theta = \frac{mv^2}{a}$ So $\frac{11}{6}mg - mg \cos \theta = \frac{m}{a}kag, \frac{11}{6} - \cos \theta = k$	B1	N2L at B.
	Solve simultaneously.	M1	
	$k = \frac{4}{3}, \cos \theta = \frac{1}{2}$	A1	Both.
		4	
(b)	Initial speed $u = \sqrt{kag} \sin \theta$	B1	
	Use $v^2 = u^2 + 2as: 0 = (\sqrt{kag} \sin \theta)^2 - 2gs$	M1	
	$s = \frac{1}{2}a$	A1	
	Height above lowest point $= s + a - a \cos \theta = \frac{1}{2}a + a - \frac{1}{2}a = a$	A1 FT	
		4	

34 – (9231/31_Summer_2023_Q3) – Circular Motion, Further Work On Distributions

(a)	At B, $mg \sin \theta = \frac{mv^2}{5a}$	M1	Allow cos instead of sin for M1. Do not award until tension = 0 used. Mass must be seen. No sign error.
	$\sin \theta = \frac{4}{5}$	A1	
		2	
(b)	At A, $T - mg \cos \theta = \frac{mv^2}{a}$	B1	
	Energy $\frac{1}{2}mv^2 - \frac{1}{2}m \times \frac{4ag}{5} = mga(\cos \theta + \sin \theta)$	M1 A1	Energy equation with 4 terms, dimensionally correct. Mass must be present, allow sign errors. Must see $\frac{1}{2}$ in the KE terms.
	Solve to find T	M1	Complete method leading to an expression in mg for T .
	$T = \frac{21}{5}mg$	A1	CWO
		5	

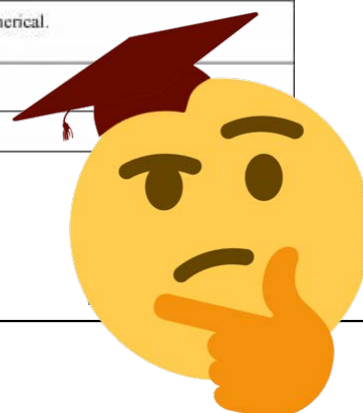


35 – (9231/31_Summer_2023_Q5) – Circular Motion, Further Work On Distributions

(a)	Hooke's law, $T_1 = \frac{\lambda mg}{a}(x-a)$ or $T_2 = \frac{\lambda mg}{a}\left(\frac{3}{4}x-a\right)$	B1	
	Also, $T_1 = \frac{mv^2}{x}$ and equate $\frac{mv^2}{x} = \frac{\lambda mg}{a}(x-a)$	M1	$v^2 = \frac{\lambda gx(x-a)}{a}$ Dimensionally correct terms.
	Similarly: $\frac{\lambda mg\left(\frac{3x}{4}-a\right)}{a} = \frac{2m\left(\frac{1}{2}v\right)^2}{\frac{3}{4}x}$	M1	$v^2 = \frac{3\lambda gx}{2a}\left(\frac{3}{4}x-a\right)$ Must have $\frac{1}{2}v$ and $\frac{3x}{4}$ on the RHS. Their dimensionally correct T_2 .
	Equate expressions for v^2 and solve for x in terms of a .	M1	
	$x = 4a$	A1	WWW
		5	SC B3 for answer of $4a$ using λ instead of λmg .
(b)	$\lambda = \frac{a}{xg(x-a)}v^2$ or $\lambda = \frac{2a}{3xg\left(\frac{3}{4}x-a\right)}v^2$ and substitute $x = 4a$, $v = \sqrt{12ag}$	M1	FT their expression for x .
	1	A1	CAO
		2	

36 – (9231/31_Summer_2023_Q7) – Circular Motion, Further Work On Distributions

(a)	$H = 80\sin^2\theta$ or $\frac{800\sin^2\theta}{g}$	B1	
	$T = 4\sin\theta$ or $\frac{40\sin\theta}{g}$	B1	
		2	
(b)	Between $t = T$ and $t = 3$ $\uparrow \frac{1}{4}H = \frac{1}{2} \times 10 \times (3-T)^2$	M1 A1	No extra terms.
	Use results from part (a) $\frac{1}{4}80\sin^2\theta = 5(3-4\sin\theta)^2$ $4\sin^2\theta - 8\sin\theta + 3 = 0$	M1	Substitute their expressions for H and T from part (a) and obtain a quadratic equation in $\sin\theta$ with no more than three terms.
	$\sin\theta = \frac{1}{2}$, $\theta = 30^\circ$	A1	Single answer. NFWW.
	Alternative method for question 7 part (b)		
	$\frac{3}{4}H = y(3) = 40 \times 3 \sin\theta - \frac{1}{2} \times 10 \times 3^2$	M1 A1	$120 \sin\theta - 45$
	Use results from (a): $\frac{3}{4}80\sin^2\theta = 120 \sin\theta - 45$ $4\sin^2\theta - 8\sin\theta + 3 = 0$	M1	Substitute their expressions for H and T from part (a) and obtain a quadratic equation in $\sin\theta$ with no more than three terms.
	$\sin\theta = \frac{1}{2}$, $\theta = 30^\circ$	A1	Single answer. NFWW.
		4	
(c)	When $t = 3$ speeds $\rightarrow 40\cos\theta$ and $\uparrow 40\sin\theta - 10 \times 3$	B1	
	Square and add to find square of speed: $v^2 = (20\sqrt{3})^2 + (-10)^2$	M1	Must be numerical.
	$v^2 = 1300$, $v = 10\sqrt{13}$ [$= 36.1$]	A1	
		3	



37 - (9231/33_Summer_2023_Q1) - Circular Motion, Simple Harmonic Motion, Hook's Law

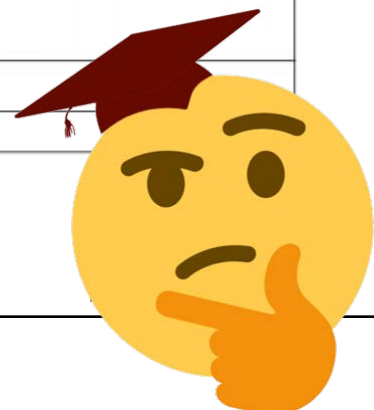
$\frac{1}{2}m \cdot 3ag - \frac{1}{2}mv^2 = mga(\cos \alpha + \cos \theta)$	M1	Energy equation, 4 terms, dimensionally correct, mass must be present, allow sign errors, allow sin in both terms on RHS
$mg \cos \theta = \frac{mv^2}{a}$	B1	N2L, may include tension initially but not awarded until tension = 0 used
$\frac{3}{2}mag - \frac{1}{2}mag \cos \theta = mga\left(\frac{4}{5} + \cos \theta\right)$ $\frac{3}{2}\cos \theta = \frac{7}{10}$	M1	Dependent on tension = 0 and on an energy equation, eliminate v^2 .
$\cos \theta = \frac{7}{15}$	A1	If no m in energy equation and no further errors, award SCB2 for correct final answer
	4	

38 - (9231/33_Summer_2023_Q2) - Circular Motion, Further Work On Distributions

$\frac{1}{2}m\left(v^2 - \frac{v^2}{4}\right) = \frac{1}{2}\frac{\lambda mg}{a}\left(\left(\frac{2}{3}a\right)^2 - \left(\frac{1}{3}a\right)^2\right)$	M1	Kinetic energy = elastic potential energy, 4 terms, dimensionally correct, allow sign errors.
$\frac{1}{2}m\left(2ag - \frac{1}{2}ag\right) = \frac{1}{2}\frac{\lambda mg}{a}\left(\left(\frac{2}{3}a\right)^2 - \left(\frac{1}{3}a\right)^2\right)$	A1	With v substituted.
Solve $\left[\frac{3}{4}v^2 = \lambda g\left(\frac{3}{9}a\right)\right]$	M1	Solve to find value for λ dependent on energy equation with 3 or 4 terms
$\lambda = \frac{9}{2}$	A1	SCB2 for $\lambda = \frac{9}{2}mg$ if given λ not used
	4	

39 - (9231/33_Summer_2023_Q5) - Circular Motion

(a)	$\uparrow T \cos \theta = mg$	B1	
	$\rightarrow T \sin \theta = \left[\frac{mv^2}{r}\right] = m \times \frac{27ag}{4r}$	B1	
	$r = 12a \tan \theta$ used	M1	
	Divide: $\tan \theta = \frac{27}{4 \times 12 \tan \theta}$, so $\tan \theta = \frac{3}{4}$	M1	Finds value for $\tan \theta$ OE. Reduces to equation in θ or x , no k .
	$r = 9a$, extension of string = $3a$	A1	
Alternative method for question 5(a)			
	Let L be stretched length of string. $\uparrow T \cos \theta = mg$	B1	Or $T \times \frac{12a}{L} = mg$
	$\rightarrow T \sin \theta = \left[\frac{mv^2}{r}\right] = m \times \frac{27ag}{4r}$	B1	
	$r = L \sin \theta$ used	M1	
	Use $\cos \theta = \frac{12a}{L}$ and $\sin \theta = \frac{(L^2 - 144a^2)^{0.5}}{L}$ and eliminate T .	M1	
	$[L^2 - 144a^2 = 81a^2]$ $L = 15a$, extension of string = $3a$	A1	
		5	



(b)	Hooke's law: $T = \frac{kmg(L-12a)}{12a}$	B1	
	Eliminate T : $\frac{kmg(L-12a)}{12a} = \frac{mgL}{12a}$	M1	
	$k = \frac{L}{L-12a} = 5$	A1	
		3	

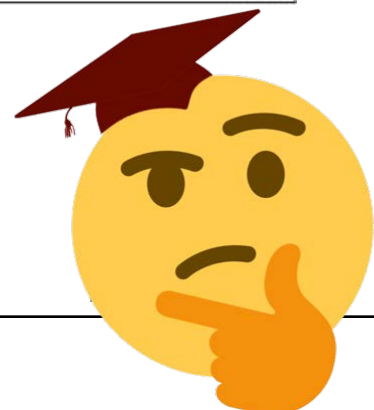
40 - (9231/32_Winter_2023_Q1) - Circular Motion

$\uparrow T \cos \theta = mg$	B1	
$\rightarrow T \sin \theta = \frac{mv^2}{a \sin \theta}$	B1	
Eliminate T and substitute for θ	M1	
$v = \sqrt{\frac{5}{6} ag}$	A1	
	4	

41 - (9231/32_Winter_2023_Q3) - Circular Motion

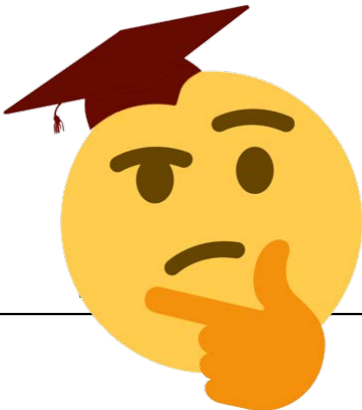
(a)	Mass is proportional to area	B1	At least two areas correct, at least one distance correct												
	<table><tr><th></th><th>Area</th><th>Centre of mass from AC</th></tr><tr><td>ABC</td><td>$\frac{1}{2} \times 2a \times a$</td><td>$\frac{1}{3}a$</td></tr><tr><td>BDE</td><td>$\frac{1}{2} \times 2 \left(\frac{a}{k}\right)^2$</td><td>$a - \frac{2}{3} \left(\frac{a}{k}\right)$</td></tr><tr><td>ADEC</td><td>$-\frac{1}{2} \times 2 \left(\frac{a}{k}\right)^2 + a^2$</td><td>$\bar{x}$</td></tr></table>		Area	Centre of mass from AC	ABC	$\frac{1}{2} \times 2a \times a$	$\frac{1}{3}a$	BDE	$\frac{1}{2} \times 2 \left(\frac{a}{k}\right)^2$	$a - \frac{2}{3} \left(\frac{a}{k}\right)$	ADEC	$-\frac{1}{2} \times 2 \left(\frac{a}{k}\right)^2 + a^2$	$\bar{x}$		
		Area	Centre of mass from AC												
	ABC	$\frac{1}{2} \times 2a \times a$	$\frac{1}{3}a$												
	BDE	$\frac{1}{2} \times 2 \left(\frac{a}{k}\right)^2$	$a - \frac{2}{3} \left(\frac{a}{k}\right)$												
ADEC	$-\frac{1}{2} \times 2 \left(\frac{a}{k}\right)^2 + a^2$	$\bar{x}$													
$\bar{x} \left(-\frac{1}{2} \times 2 \left(\frac{a}{k}\right)^2 + a^2 \right) = \frac{1}{3} a \times a^2 - \left(a - \frac{2}{3} \left(\frac{a}{k}\right) \right) \times \left(\frac{a}{k}\right)^2$	M1A1	Moments equation, dimensionally correct, correct number of terms.													
$\bar{x} = \frac{a(k^2 + k - 2)}{3k(k+1)}$	A1	Allow unsimplified single fraction $\frac{a(k^3 - 3k + 2)}{3k(k^2 - 1)}$.													
	4														

(b)	$\tan \theta = \frac{\bar{x}}{a} \left[= \frac{5}{18} \right]$	B1	With <i>their</i> $\bar{x}$ from part (a).
	So $18(k^2 + k - 2) = 5(3k^2 + 3k)$ $k^2 + k - 12 = 0$	M1	Obtain a polynomial in k only, e.g. $k^3 - 13k + 12 = 0$, may be implied.
	$(k + 4)(k - 3) = 0$, $k = 3$ only	A1	CWO
		3	



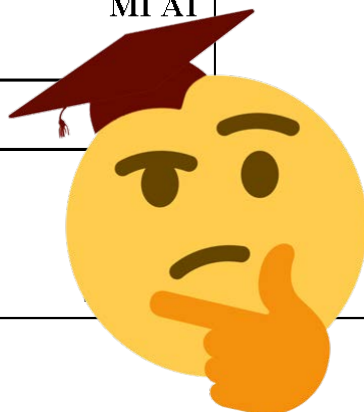
42 - (9231/33_Winter_2023_Q4) - Circular Motion

(a)	Energy equation: gain in KE = loss in EPE	B1	At least one correct EPE.
	$\frac{1}{2}m \times 80ag = \frac{1}{2} \times \frac{5mg}{8a} \left((2L - 8a)^2 - (4a)^2 \right)$	M1	3-term energy equation.
	$4L^2 - 8aL - 20a^2 = 0$	A1	All correct.
	$L = 10a$		
		5	
(b)	At P: $2T \cos \theta = m \times \text{acceleration}$	M1	
	Hooke's law: $T = \frac{5mg}{8a} \times 12a = \frac{15}{2}mg$	M1	
	Solve, acceleration = $12g$	A1	
		3	



1 - (9231/21_Summer_2017_Q2) - Equilibrium Of A Rigid Body Under Coplanar Forces

Take moments for rod about some point such as: A: $R_P \times AP - kW \times 3a \cos \theta = W \times (3a/2) \cos \theta$ $[R_P \times 3a/4 - kW \times 9a/5 = W \times 9a/10]$ so $15 R_P - 36 kW = 18 W$] O: $F_A \times (5a/4) - kW \times (3a \cos \theta - 5a/4)$ $= -W \times (5a/4 - (3a/2) \cos \theta)$ $[F_A \times 5a/4 - kW \times 11a/20 = -W \times 7a/20]$ so $25 F_A - 11 kW = -7 W$] P: $R_A \times AP \sin \theta + F_A \times AP \cos \theta - kW \times (3a - AP) \cos \theta$ $= W \times (3a/2 - AP) \cos \theta$ $[R_A \times 3a/5 + F_A \times 9a/20 - kW \times 27a/20 = W \times 9a/20]$ so $12 R_A + 9 F_A - 27 kW = 9 W$] B: $R_P \times (3a - AP) - R_A \times 3a \sin \theta - F_A \times 3a \cos \theta$ $= W \times (3a/2) \cos \theta$ $[R_P \times 9a/4 - R_A \times 12a/5 - F_A \times 9a/5 = W \times 9a/10]$ so $45 R_P - 48 R_A - 36 F_A = 18 W$] C: $R_P \times (3a/2 - AP) - R_A \times (3a/2) \sin \theta - F_A \times (3a/2) \cos \theta + kW \times (3a/2) \cos \theta = 0$ $[R_P \times 3a/4 - R_A \times 6a/5 - F_A \times 9a/10 + kW \times 9a/10 = 0]$ so $15 R_P - 24 R_A - 18 F_A + 18 kW = 0$] F: $R_P \cos \theta \times (3a - AP) \cos \theta - R_P \sin \theta \times AP \sin \theta$ $- F_A \times 3a \cos \theta = W \times (3a/2) \cos \theta$ $[(3/5)R_P \times 27a/20 - (4/5)R_P \times 3a/5]$ $- F_A \times 9a/5 = W \times 9a/10$ so $81 R_P - 48 R_P - 180 F_A = 90 W$]	M1 A1
Find two more indep. eqns, e.g. resolution of forces on rod: Horizontally: $R_A = R_P \sin \theta [= 4R_P/5]$ Vertically: $F_A + (k+1)W = R_P \cos \theta [= 3R_P/5]$ Along AB: $R_A \cos \theta = F_A \sin \theta + (k+1)W \sin \theta$ Normal to AB: $R_P = R_A \sin \theta + F_A \cos \theta + (k+1)W \cos \theta$	B1 B1
$F_A = +R_A/8$ or $-R_A/8$ as appropriate	B1
$[\sin \theta = 4/5, \cos \theta = 3/5, \tan \theta = 4/3]$	M1
$F_A \downarrow: [R_P = 6W, F_A = 3W/5, R_A = 24W/5], k = 2$ $F_A \uparrow: [R_P = 30W/17, F_A = 3W/17, R_A = 24W/17], k = 4/17$	M1 A1
Total:	

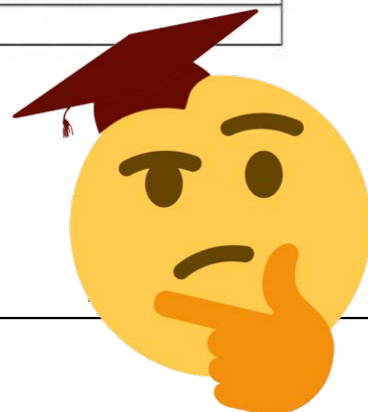


2 - (9231/23_Summer_2017_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(i)	$T \times 3a \sin \theta = W \times 1.5a \cos 2\theta$	M1 A1
	$T = 7W/30 \text{ or } 0.233W$	A1
	Total:	3
(ii)	$OB = (5a - 3a \cos 2\theta) / \cos \theta = 26a/5 \text{ or } 5.2a$	M1 A1
	$T = \lambda(OB - 4a)/4a$	M1
	$= \lambda(6a/5)/4a = 3\lambda/10, \lambda = 7W/9 \text{ or } 0.778W$	A1
	Total:	4
(iii)	$X = T \cos \theta [= 14W/75 \text{ or } 0.187W]$	M1
	$Y = T \sin \theta + W [= 57W/50 \text{ or } 1.14W]$ $\phi = \tan^{-1}(Y/X) = \tan^{-1}(171/28)$	M1
	$= 80.7^\circ \text{ or } 1.41 \text{ radians}$	B1
	Total:	3

3 - (9231/21_Winter_2017_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(i)	$T \times 2a = (5W/2) \times a \cos \theta$	M1	Take moments for rod about A
	$T = \frac{1}{2} (5W/2) \times (4/5) = W$	A1	to find tension T
		2	
(ii)	$T \sin \theta = \mu(W + T \cos \theta)$	M1	Use $F_f = \mu R_f$ at P
	$(3/5)W = \mu(1 + 4/5)W = \mu(9/5)W, \mu = 1/3$	A1	to find μ
		2	
(iii)	<i>EITHER:</i> $[\pm] X = T \sin \theta = 3W/5 \text{ or } 0.6W$ $[\pm] Y = 5W/2 - T \cos \theta \text{ or } 5W/2 + W - R_P$	(B1)	<i>EITHER:</i> Find horizontal component X of force at A Find vertical component Y of force at A
	$= 17W/10 \text{ or } 1.7W$	(B1)	
	<i>OR:</i> $[\pm] X = (5W/2) \sin \theta = 3W/2 \text{ or } 1.5W$	(B1)	<i>OR:</i> Find component X of force at A along BA
	$[\pm] Y = (5W/2) \cos \theta - T = W$	(B1)	Find component Y of force at A perp. to BA
	$R_A^2 = X^2 + Y^2 = 13W^2/4, R_A = \frac{1}{2}W\sqrt{13} \text{ or } 1.80W$	B1FT	Find magnitude of resultant force R_A at A (FT on X, Y)
		3	
(iv)	$PB = (a + 2a \sin \theta) / \cos \theta = 5a/4 + 3a/2 = 11a/4 \text{ or } 2.75a$ $\text{or } x = 3a/4 \text{ or } 0.75a$	B1	Find length PB or extension x of string
	$T = \lambda(PB - 2a)/2a, \lambda = 8W/3 \text{ or } 2.67W$	M1 A1	Find modulus λ using Hooke's Law
		3	

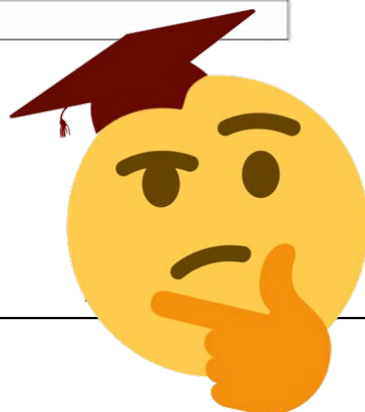


4 - (9231/21_Summer_2018_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(i)	$R_B \cdot 2a \sin \theta = W a \cos \theta + 5W x a \cos \theta$	MI A1	Moments at A
	$R_B = W \frac{(5x+1)}{2 \tan \theta}$ AG	A1	Simplify to give R_B
		3	
(ii)	$R_B' = W \frac{(5x+5+1)}{2 \tan \theta}$ (AEF)	B1	Find new R_B' by moments at A
	$5x+6 = 2(5x+1), x = \frac{4}{5}$ AG	MI A1	Find/verify x from $R_B' = 2 R_B$
		3	
(iii)	$R_A = 6W, F_A' = R_B' \text{ or } 2 R_B \left[= \frac{10W}{2 \tan \theta} = 4W \right]$	B1, B1	Find R_A and new F_A' by resolutions (may be earned earlier)
	$\frac{2}{3} = \frac{F_A'}{R_A}$	M1	Relate R_A and F_A' using $\mu = \frac{F}{R}$
	$= \frac{\left(\frac{5W}{\tan \theta} \right)}{6W} \text{ so } \tan \theta = \frac{5}{4} \text{ or } 1.25$	MI A1	Find $\tan \theta$ SC Allow M1 M1 for ground smooth and wall rough (max $\frac{2}{5}$)
		5	

5 - (9231/23_Summer_2018_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(i)	$AC = AD \cos \theta$ and $AD = 2a \cos \theta \left[= 6a/5 \right]$ so $AC = 2a \cos^2 \theta = 2a (3/5)^2 = 18a/25$ AG	MI A1	Find AC (D denotes other end of rope from C)
		2	
(ii)	A: $R_B \times 2a \sin \theta - W \times a \cos \theta - T \times AC = 0$ [$R_B \times 8a/5 - W \times 3a/5 - T \times 18a/25 = 0$ so $40 R_B - 15 W - 18 T = 0$]	MI A1	Take moments for rod about one chosen point (Note that a vertical resolution will then give T, earning 6/6)
	B: $F_A \times 2a \sin \theta - R_A \times 2a \cos \theta + W \times a \cos \theta + T \times (2a - AC) = 0$ [$F_A \times 8a/5 - R_A \times 6a/5 + W \times 3a/5 + T \times 32a/25 = 0$ so $15 W + 32 T = 30 R_A - 40 F_A = 20 R_A$]		
	C: $F_A \times AC \sin \theta - R_A \times AC \cos \theta + R_B \times (2a - AC) \sin \theta - W \times (a - AC) \cos \theta = 0$ [$F_A \times 72a/125 - R_A \times 54a/125 + R_B \times 128a/125 - W \times 21a/125 = 0$ so $128 R_B - 21 W = 54 R_A - 72 F_A = 36 R_A$]		
	D: $R_A \times 2a \cos \theta - R_B \times 2a \sin \theta - W \times a \cos \theta = 0$ [$R_A \times 6a/5 - R_B \times 8a/5 - W \times 3a/5 = 0$ so $6 R_A - 8 R_B - 3 W = 0$]		
	G: $F_A \times a \sin \theta - R_A \times a \cos \theta + R_B \times a \sin \theta + T \times (a - AC) = 0$ [$F_A \times 4a/5 - R_A \times 3a/5 + R_B \times 4a/5 + T \times 7a/25 = 0$ so $20 F_A - 15 R_A + 20 R_B + 7 T = 0$]		(G is mid-point of AB)
	Horizontally: $R_B - F_A = T \sin \theta \left[= 4T/5 \right]$	B1	Find two more indep. eqns, e.g. resolution of forces on rod
(iii)	Vertically: $R_A - W = T \cos \theta \left[= 3T/5 \right]$	B1	(a second moment eqn. may be used)
	$T = W/4$ AG	MI A1	Find or verify T using $F_A = \frac{1}{4} R_A, \sin \theta = 4/5, \cos \theta = 3/5$
		6	
(iii)	$R_A = 23W/20 \text{ or } 1.15 W \left[F_A = 23W/80 = 0.2875 W \right]$	B1	Find R_A, R_B (can assume $T = W/4$)
	$R_B = 39W/80 \text{ or } 0.487_{(5)} W$	B1	
		2	

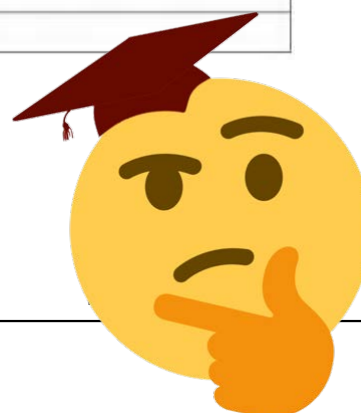


6 - (9231/21_Winter_2018_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(i)	$T \times 3a \sin 2\theta = W \times 2a \cos \theta + \frac{1}{2} W \times 4a \cos \theta$	M1A1	Take moments for rod about A
	$6a T \sin \theta \cos \theta = 4a W \times a \cos \theta$	M1	Verify tension T
	$T = 2W/3 \sin \theta = 2W/3(8/17) = 17W/12 \text{ AG}$	A1	using $\sin \theta = 8/17, \cos \theta = 15/17$
		4	
(ii)	$\text{EITHER: } X = T \cos \theta = (5/4) W \text{ or } 1.25W$	B1	Find horizontal component X of force at A
	$Y = W + \frac{1}{2} W - T \sin \theta = (5/6) W \text{ or } 0.833W$	B1	Find vertical component Y of force at A
	$F = \sqrt{X^2 + Y^2} = (5\sqrt{13}/12) W \text{ or } 1.50W$	B1	Find magnitude of F
	Upward force at angle to horizontal of $\tan^{-1} Y/X = \tan^{-1} 2/3 = 33.7^\circ \text{ or } 0.588 \text{ radians}$	M1A1	Find direction of F (AEF; A0 if direction unclear)
	OR: $R_{\parallel} = 1.5 W \sin \theta + T \cos 2\theta = (305/12 \times 17) W \text{ or } 1.45W$	(B1)	Find component $R_{\parallel}$ parallel to AB of force at A
	$R_{\perp} = 1.5 W \cos \theta - T \cos 2\theta = (5/34) W \text{ or } 0.147W$	B1	Find component $R_{\perp}$ normal to AB of force at A
	$F = \sqrt{R_{\parallel}^2 + R_{\perp}^2} = (5\sqrt{13}/12) W \text{ or } 1.50W$	B1	Find magnitude of F
	Upward force at angle to AB of $\tan^{-1} R_{\perp}/R_{\parallel} = \tan^{-1} 6/61 = 5.6^\circ \text{ or } 0.098 \text{ radians}$	M1A1	Find direction of F (AEF; A0 if direction unclear)
		5	
(iii)	$T = \lambda (CD - 2a)/2a$	M1	Find modulus λ using Hooke's Law
	$CD = 3a \text{ so } \lambda = 2T = 17W/6 \text{ or } 2.83W$	A1	
		2	

7 - (9231/22_Winter_2018_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(i)	$A: R_C \times x - W \cos 45^\circ \times a = 0$ $B: F_A \cos 45^\circ \times 2a - R_A \cos 45^\circ \times 2a - R_C \times (2a - x) + W \cos 45^\circ \times a = 0$ $C: F_A \cos 45^\circ \times x - R_A \cos 45^\circ \times x + W \cos 45^\circ \times (x - a) = 0$ $G: F_A \cos 45^\circ \times a - R_A \cos 45^\circ \times a + R_C \times (x - a) = 0$ $D: R_A \cos 45^\circ \times a - R_C \cos 45^\circ \times x \cos 45^\circ - R_C \cos 45^\circ \times (x - a) \cos 45^\circ = 0$	B1	Take moments for rod about one chosen point ($F_A \times$ may be replaced by μR_A and $\cos 45^\circ$ by e.g. $1/\sqrt{2}$) (A single resolution along the rod will then suffice since no R_C) (G is mid-point of AB) (D is on ground below G)
	Horizontally: $F_A - R_C \cos 45^\circ = 0$	B1	Find two more indep. eqns, e.g. resolution of forces on rod
	Vertically: $R_A + R_C \cos 45^\circ - W = 0$	B1	(a second moment eqn. may be used)
	Along rod: $F_A \cos 45^\circ + R_A \cos 45^\circ - W \cos 45^\circ = 0$	(B1)	
	Perp. to rod: $F_A \cos 45^\circ - R_A \cos 45^\circ - R_C + W \cos 45^\circ = 0$	B1)	
	$[R_A = W/(1+\mu), F_A = \mu W/(1+\mu), R_C = \mu W\sqrt{2}/(1+\mu)]$ $x = a(1+\mu)/2\mu \text{ or } \frac{1}{2}a(1+1/\mu)$	M1A1	Combine to find x (using $F_A = \mu R_A$ and $\cos 45^\circ = 1/\sqrt{2}$)
		5	
	$a(1+\mu)/2\mu \leq 2a \text{ so } \mu \geq \frac{1}{2} \text{ AG}$	M1A1	Verify μ using $x \leq 2a$
		2	
(iii)	$a(1+\mu)/2\mu = 3a/2 \text{ so } \mu = \frac{1}{2}$	M1A1	Find μ when $x = 3a/2$ using result in (i)
	$F_A = W/3, R_A = 2W/3 [R_C = (\sqrt{2})W/3]$ $N_A = \sqrt{F_A^2 + R_A^2} = (\sqrt{5}/3)W \text{ or } 0.745W$	M1A1	Find F_A, R_A and hence magnitude of resultant force N_A at A
		4	

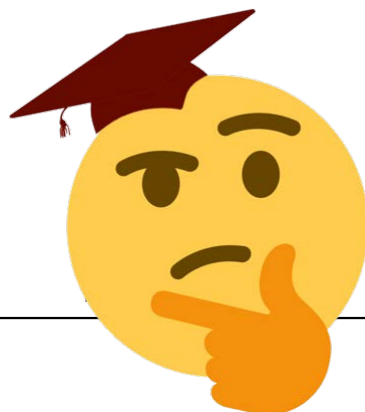


8 - (9231/21_Summer_2019_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

$A: T \times 5a/2 \sin(\pi - 2\theta) - W \times 2a \sin \theta = 0$ $C: F_A \times 5a/2 \sin \theta - R_A \times 5a/2 \cos \theta - W \times \frac{1}{2}a \sin \theta = 0$ $B: F_A \times 4a \sin \theta - R_A \times 4a \cos \theta - W \times 2a \sin \theta + T \times 3a/2 \sin(\pi - 2\theta) = 0$ $G: F_A \times 2a \sin \theta - R_A \times 2a \cos \theta - T \times \frac{1}{2}a \sin(\pi - 2\theta) = 0$ (G is mid-point of AB) $D: R_A \times 5a \cos \theta - W \times 2a \sin \theta = 0$	MI A1	Take moments for rod about one chosen point [$\sin \theta = 2/\sqrt{5}$, $\cos \theta = 1/\sqrt{5}$, $\sin(\pi - 2\theta) = \sin 2\theta = 4/5$]
$R_A = T \sin \theta$	B1	Find two more independent equations
$F_A = W - T \cos \theta$	B1	e.g. resolution of forces on rod (a second moment equation may be used)
$F_A = \mu R_A$	B1	Relate F_A and R_A (may be implied)
$T = (2W \sin \theta) / (5/2 \sin 2\theta) = 2W / (5 \cos \theta)$	MI	Find T by any method (e.g. from moments about A)
$= 2W/\sqrt{5}$ or $(2\sqrt{5}/5) W$ or $0.894 W$	A1	
$F_A = (3/5) W$ and $R_A = (4/5) W$ $\mu = \frac{3}{4}$ or 0.75	MI A1	Find or imply F_A and R_A by any method (e.g. from resolutions) and hence μ
	A1	
	10	

9 - (9231/23_Summer_2019_Q5) - Equilibrium Of A Rigid Body Under Coplanar Forces

(i)	$R_B \cos \theta + F_B \sin \theta = W \sin \theta + \frac{1}{4} W \sin \theta$ $F_B = \frac{1}{2} R_B$ $(4/5 + \frac{1}{2} \times 3/5) R_B = (1 + \frac{1}{4})(3/5) W$, $R_B = \frac{3}{4} W$	*MI A1	Resolve forces along rod AB ($\sin \theta = 3/5$, $\cos \theta = 4/5$)
		B1	Relate F_B and R_B (may be implied)
		DM1 A1	Combine using $\tan \theta = \frac{3}{4}$ to verify R_A
		5	
(ii)	$R_C = F_B \cos \theta - R_B \sin \theta + W \cos \theta + \frac{1}{4} W \cos \theta$ $= (\frac{1}{2} \times \frac{3}{4} \times 4/5) W - (\frac{3}{4} \times 3/5) W + (5/4 \times 4/5) W$ $= (1/5 - 9/20 + 1) W = \frac{3}{4} W$	*MI A1	Resolve forces perpendicular to rod AB, where C denotes rim Substitute to find R_C
		DM1	
		A1	
		4	
(iii)	$A: R_C \times AC + (R_B \sin \theta - F_B \cos \theta) \times 2x - W \cos \theta \times x$ $[= W(\frac{1}{2} \times 2x - \frac{3}{4} \times 8a/5 + 9/20 \times 2x - 1/5 \times 2x - 4x/5)]$ $C: (R_B \sin \theta - F_B \cos \theta) \times 2a \cos \theta - W \cos \theta \times CG + \frac{1}{4} W \cos \theta \times AC$ $[= W(9/20 \times 2a - 1/5 \times 2a - 8a/5 + x + \frac{1}{4} \times 2x - \frac{1}{4} \times 8a/5 \cos \theta)]$ $G: R_C \times CG - (R_B \sin \theta - F_B \cos \theta) \times x - \frac{1}{4} W \cos \theta \times x$ $[= W(\frac{1}{2} \times 8a/5 - \frac{3}{4} \times x - 9/20 \times x + 1/5 \times x - 1/5 \times x)]$ $B: R_C \times 2a \cos \theta - W \cos \theta \times x - \frac{1}{4} W \cos \theta \times 2x$ $[= W(\frac{1}{2} \times 2a - x - \frac{1}{4} \times 2x) \cos \theta]$	MI A1	Find total moments about any point, denoting rod's centre by G (taken on either side of C) where $AC = 2x - 2a \cos \theta = 2x - 8a/5$ $CG = x - AC = 2a \cos \theta - x = 8a/5 - x$
	$x = a$	A1	Substitute and equate to zero to find x
		3	

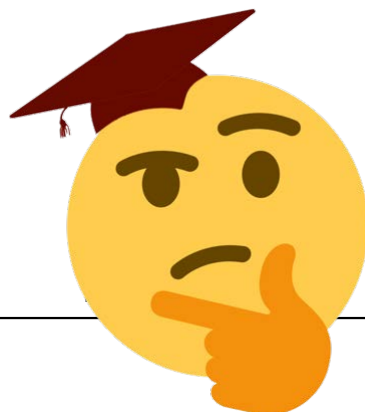


10 - (9231/21_Winter_2019_Q2) - Equilibrium Of A Rigid Body Under Coplanar Forces

(i)	$R_E \times 3a = W \cos \theta \times 2a - W \sin \theta \times 2a$ or $R_E \times 3a = W \times 2a (1 - \tan \theta) \cos \theta$ or $R_E \times 3a = 2\sqrt{2}W \sin\left(\frac{\pi}{4} - \theta\right)$	M1 A1	Take moments about B
	$R_E = 4W / 3\sqrt{10}$	A1	Find normal reaction at E. AEF
	$R_B = W - R_E \cos \theta = 3W/5$	M1 A1	Find normal reaction at B by resolving forces vertically
		5	
(ii)	$F_B = R_E \sin \theta = 2W/15$	M1 A1	Find friction at B by resolving forces horizontally
	$\mu = (2/15) / (3/5) = 2/9$	A1	Find μ from $F_B = \mu R_B$
		3	

11 - (9231/31_Summer_2020_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)		Area	Centre of mass from BC	Centre of mass from DC	M1
	Square	100	5	5	
	Triangle	$\frac{1}{2}x, 15/2$	$\frac{1}{3}x$	$\frac{5}{2}$	
	Shape $ABEFD$	$100 - \frac{15}{4}x$	$\bar{x}$	$\bar{y}$	
Take moments about BC :					
$\left(100 - \frac{15}{4}x\right)\sigma \cdot \bar{x} = 500\sigma - \frac{15}{4}x\sigma \cdot \frac{1}{3}x$					
(M1 for all terms present)					
	$\bar{x} = \frac{400 - x^2}{80 - 3x}$	AG			A1
	Take moments about DC :				M1
	$\left(100 - \frac{15}{4}x\right) \cdot \bar{y} = 100 \times 5 - \frac{15}{4}x \cdot \frac{5}{2}$				
	$\bar{y} = \frac{800 - 15x}{160 - 6x}$				A1
					4
(b)	Use condition: $\bar{x} \geq x$				B1
	$2x^2 - 80x + 400 \geq 0$				M1
	$x = 20 - 10\sqrt{2}$				A1
					3

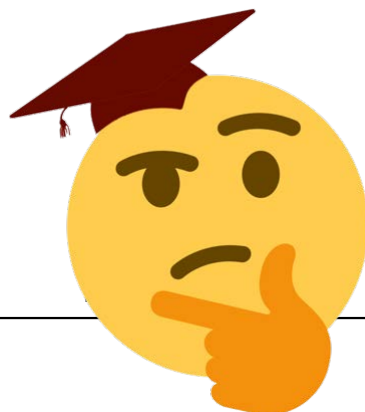


12 - (9231/33_Summer_2020_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)		Mass	Centre of mass from vertex of cone	B1
	Cylinder	$\pi(kr)^2 \cdot 3r\rho$	$\frac{11}{2}r$	
	Cone	$\frac{1}{3}\pi(2r)^2 \cdot 4r\rho$	$3r$	
	Combined	$\left(\pi(kr)^2 \cdot 3r + \frac{1}{3}\pi(2r)^2 \cdot 4r\right)\rho$	$\bar{x}$	
	Take moments about vertex: $\bar{x} \times \left(\pi(kr)^2 \cdot 3r + \frac{1}{3}\pi(2r)^2 \cdot 4r\right) = \frac{11r}{2} \times \pi(kr)^2 \cdot 3r + 3r \times \frac{1}{3}\pi(2r)^2 \cdot 4r$ leading to			M1A1
	$\bar{x} = \frac{(99k^2 + 96)r}{18k^2 + 32}$			A1
				4
(b)	$\tan \alpha = \frac{\bar{x} - 4r}{2r} \quad (= 1/8)$			M1
	$\bar{x} = \frac{17}{4}r$			A1
	Equate to answer in (a) and attempt to find k			M1
	$45k^2 = 80: \quad k = \frac{4}{3}$			A1
				4

13 - (9231/32_Winter_2020_Q3) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)		Volume	Centre of mass from base	B1	Distances correct
	Cone	$\frac{1}{3}\pi(3r)^2 \cdot 4r$	$4r + r$		
	Cylinder	$\pi(3r)^2 \cdot 4r$	$2r$		
	Combined	$\frac{4}{3}\pi(3r)^2 \cdot 4r$	$\bar{x}$		
	Taking moments about base of cylinder: $\bar{x} \cdot \frac{4}{3}\pi(3r)^2 \cdot 4r = \frac{1}{3}\pi(3r)^2 \cdot 4r \cdot 5r + \pi(3r)^2 \cdot 4r \cdot 2r$			M1 A1	Moments equation
	$\bar{x} = \frac{11}{4}r$			A1	
				4	
(b)	Condition: $OG \cos \theta < OA$ (where O is vertex of cone and OA is slant height of cone)			B1	Correct condition for equilibrium
	$\left(4r + \frac{5r}{4}\right) \times \frac{4}{5} < 5r$			M1	Expression in terms of r
	$21 < 25$ True			A1	Correct conclusion, with correct working
				3	

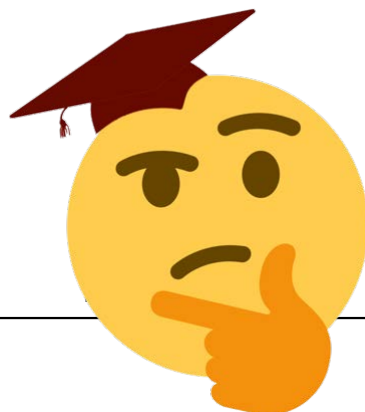


14 - (9231/33_Winter_2020_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)		Volume	Centre of mass from AB	B1	For $9h/8$ or $3h/8$ (unsimplified)
	Small cone	$\frac{1}{3}\pi r^2 \cdot \frac{h}{2}$	$h + \frac{1}{4} \cdot \frac{h}{2} \left(= \frac{9h}{8} \right)$		
	Large cone	$\frac{1}{3}\pi(3r)^2 \cdot \frac{3h}{2}$	$\frac{1}{4} \cdot \frac{3h}{2} \left(= \frac{3h}{8} \right)$		
	Object	$\frac{26}{6}\pi(r)^2 h$	$\bar{x}$		
Take moments about AB $\frac{13}{3}\pi r^2 h \bar{x} = \frac{27}{6}\pi r^2 h \cdot \frac{3h}{8} - \frac{1}{6}\pi r^2 h \cdot \frac{9h}{8}$				M1 A1	Moments equation: Allow use of relative masses 1, 26, 27
$\bar{x} = \frac{9h}{26}$				A1	
				4	
(b)	$\tan \theta = \frac{\bar{x}}{3r}$			M1	
	$(= \frac{3h}{26r})$ Use $h = \frac{13}{4}r$			A1	
	$\tan \theta = \frac{3}{8}$				
				2	

15 - (9231/33_Summer_2021_Q1) - Equilibrium Of A Rigid Body Under Coplanar Forces

	Area	Centre of mass from DB	B1	All distances correct. $ABCD$ can be split in other ways, for example ADC and ABC .
ABD	$24a^2$	$-a$		
BCD	$48a^2$	$2a$		
Combined	$72a^2$	$\bar{x}$		
Taking moments about DB : $72a^2\bar{x} = 24a^2 \times -a + 48a^2 \times 2a$ OR Taking moments about A : $72a^2\bar{x} = 24a^2 \times 2a + 48a^2 \times 5a$ OR Taking moments about G : $24a^2(\bar{x}+a) = 48a^2 \times (2a-\bar{x})$			M1	Moments equation with masses in correct ratio.
$\bar{x} = a$			A1	CWO
Alternative method for question 1				
ADC : distance of centre of mass from $BD = \frac{6a-3a}{3} = a$ ABC : distance of centre of mass from $BD = \frac{6a-3a}{3} = a$			B1	One calculation.
Second calculation or statement about symmetry			M1	
$\bar{x} = a$			A1	
			3	



16 - (9231/31_Summer_2021_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)		Volume	Centre of mass from base of cylinder	M1	
	Cone	$\frac{1}{3}\pi r^2 kh$	$h + \frac{kh}{4}$		
	Cylinder	$\pi r^2 h$	$\frac{h}{2}$		
	Combined	$\pi r^2 h \left(\frac{1}{3}k + 1\right)$	$\bar{x}$		
	Take moments about base: $\pi r^2 h \left(\frac{1}{3}k + 1\right) \bar{x} = \frac{1}{3}\pi r^2 kh \left(h + \frac{kh}{4}\right) + \pi r^2 h \frac{h}{2}$				A1 A1 2 terms correct. All terms correct.
	$\bar{x} = \frac{h(k^2 + 4k + 6)}{4(3 + k)}$			A1	AG. Shown convincingly.
				4	
(b)	$\tan \theta = \frac{r}{\bar{x}}$			M1	
	$\frac{4}{3} = \frac{6h(k + 3)}{h(k^2 + 4k + 6)}$ $2k^2 - k - 15 = 0$			M1	Equate to $\frac{4}{3}$ and simplify to quadratic.
	$k = 3$			A1	CAO. No other solutions.
				3	

17 - (9231/31_Winter_2021_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)		Area	Centre of mass from AD	M1	Attempt at moments with three terms.
	Square	$9a^2$	$\frac{3}{2}a$		
	CDF	$\frac{3}{2}ah$	a		
	BEC	$\frac{3}{2}ah$	$3a - \frac{1}{3}h$		
	Resulting AEFC	$9a^2 - 3ah$	$\bar{x}$		
Taking moments about AD:					
	$(9a^2 - 3ah) \bar{x} = \left(9a^2 \times \frac{3}{2}a\right) - \left(\frac{3}{2}ah \times a\right) - \left(\frac{3}{2}ah \times \left(3a - \frac{1}{3}h\right)\right)$			A1 A1	Two terms correct. All correct.
	$\bar{x} = \frac{27a^2 - 12ah + h^2}{6(3a - h)} \left(= \frac{9a - h}{6}\right)$			A1	AEF
	$\bar{y} = \bar{x}$			B1	By symmetry or equal to their $\bar{x}$.
				5	
(b)	For equilibrium, $\bar{x} \leq 3a - h$ $27a^2 - 12ah + h^2 \leq 6(3a - h)^2$			B1	Accept strict inequality.
	$27a^2 - 24ah + 5h^2 \geq 0$			M1	Homogeneous 3-term quadratic inequality.
	$h \leq \frac{9}{5}a$			A1	CAO.
				3	



18 - (9231/32_Winter_2021_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)		Volume	Centre of mass from AB	M1	Attempt at moments, 3 terms.
	Hemisphere	$\frac{2}{3}\pi a^3$	$\frac{3}{8}a$		
	Cylinder	$\pi ka\left(\frac{a}{2}\right)^2$	$\frac{ka}{2}$		
	Remainder	$\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2$	$\bar{x}$		
Taking moments about AB : $\left(\frac{2}{3}\pi a^3 - \pi ka\left(\frac{a}{2}\right)^2\right) \times \bar{x} = \left(\frac{2}{3}\pi a^3 \times \frac{3}{8}a\right) - \left(\pi ka\left(\frac{a}{2}\right)^2 \times \frac{ka}{2}\right)$				A1	Any 2 terms correct.
				A1	All correct.
$\bar{x} = \frac{3a(2-k^2)}{2(8-3k)}$				A1	Shown convincingly, AG.
				4	
(b)	$\tan \theta = \frac{\bar{x}}{a}$			B1	
	$\frac{3(2-k^2)}{2(8-3k)} = \frac{7}{18}$				
	$27k^2 - 21k + 2 = 0$				
	$k = \frac{2}{3}$ and $k = \frac{1}{9}$				
				M1	Rearrange to form quadratic.
				A1	Both answers correct.
				3	

19 - (9231/33_Summer_2022_Q1) - Equilibrium Of A Rigid Body Under Coplanar Forces

	Area	Distance from Oy	M1	Attempt at moments equation with all necessary terms. Other options possible for RHS of moments equation, for example: (1) OAC : 30×6 and ABC : 12×9 (2) OBC : 12×4 and OAB : 30×8 (3) Subtraction: $60 \times 7.5 - 6 \times 1 - 12 \times 13$
Triangle OCD	6	2		
Rectangle $DEBC$	24	6		
Triangle BAE	12	11		
Trapezium $OCBA$	42	$\bar{x}$		
where D is point with coordinates (3, 0) and E is a point with coordinates (9, 0).				
Parts that would give correct total area 42			B1	
Moments about Oy $42\bar{x} = 6 \times 2 + 24 \times 6 + 12 \times 11$ (=288)			A1	Correct equation.
$\bar{x} = \frac{288}{42} = 6.86$			A1	
			4	

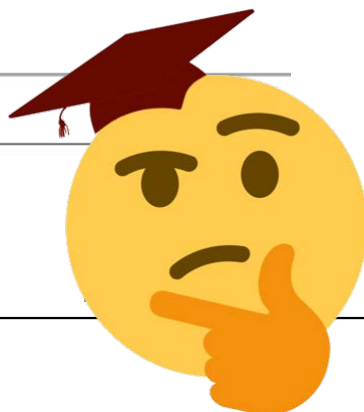


20 - (9231/31_Summer_2022_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)		Area	Centre of mass from AB	M1	Moments equation, condone missing ends of cylinder. One expression on the RHS correct.
	Cylinder	$2\pi ah + 2\pi a^2$	$\frac{1}{2}h$		
	Shell	$2\pi(2a)^2$	$h+a$		
	Moments about AB $\bar{x}(2\pi ah + 2\pi a^2 + 2\pi(2a)^2) = 2\pi(2a)^2 \times (h+a) + (2\pi ah + 2\pi a^2)\left(\frac{1}{2}h\right)$			A1 A1	One correct expression on RHS correct scores A1.
	$(2h+10a)\bar{x} = h^2 + ah + 8ah + 8a^2$ $\bar{x} = \frac{h^2 + 9ah + 8a^2}{2(h+5a)}$			A1	
				4	
(b)	$\tan \theta \leq \frac{a}{\bar{x}}$			B1	
	$\bar{x} = \frac{h^2 + 9ah + 8a^2}{2(h+5a)} \leq \frac{3}{2}a$ $h^2 + 6ah - 7a^2 \leq 0$			M1	Form inequality and rearrange to quadratic, condone equation.
	$(h-a)(h+7a) \leq 0$			M1	Attempt to solve, condone equation.
	$(-7a \leq) h \leq a$			A1	
				4	

21 - (9231/33_Summer_2022_Q7) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)	Frictional force = $\mu \times$ normal reaction at D and E	B1	$F_{AB} = \mu R$, $F_{BC} = \mu N$
	Moments about B, $Na - Ra = Wa(\sin \theta - \cos \theta)$ Moments about centre, $F_{AB}a + F_{BC}a = Wa(\cos \theta - \sin \theta)$ Moments about D, $F_{BC}a + Na = Wa(\cos \theta + \sin \theta)$ Moments about E, $Ra - F_{AB}a = Wa(\cos \theta + \sin \theta)$	B1	One moments equation about any point involving all relevant forces, resolved if necessary (AEF).
	Parallel to AB, $N - F_{AB} = W \sin \theta + W \sin \theta$ Perpendicular to AB, $F_{BC} + R = W \cos \theta + W \cos \theta$	B1	Two resolutions: all relevant terms, different frictional forces [Vertical: $R \cos \theta + F_{BC} \cos \theta + N \sin \theta = F_{AB} \sin \theta + W + W$ Horizontal: $F_{BC} \sin \theta + F_{AB} \cos \theta + R \sin \theta = N \cos \theta$] Alternative approach using two moments equations can earn the B1B1
	$N - R = \frac{1}{2}((1-\mu)N - (1+\mu)R)$	M1	Combine appropriate equations.
	$N\left(1 - \frac{1}{2}(1-\mu)\right) = R\left(1 - \frac{1}{2}(1+\mu)\right)$ $N\left(\frac{1}{2} + \frac{1}{2}\mu\right) = R\left(\frac{1}{2} - \frac{1}{2}\mu\right)$	M1	Collect terms to obtain ratio/fraction in terms of μ only (CWO), any equivalent simplified form.
	$R : N = 1 + \mu : 1 - \mu$	A1	
		6	
(b)	Divide resolution equations: $\tan \theta = \frac{N - \mu R}{\mu N + R}$	M1	Must include μ terms.
	Use $R = 2N$: $\tan \theta = \frac{\frac{1}{3}N}{\frac{7}{3}N}$	M1	FT their answer to part (a).
	$\tan \theta = \frac{1}{7}$	A1	
		3	

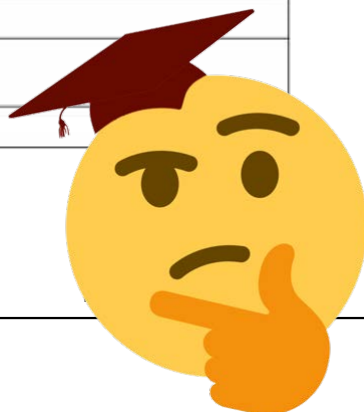


22 - (9231/32_Winter_2022_Q2) - Equilibrium Of A Rigid Body Under Coplanar Forces

	Area	Distance from AB		
ABC	$27a^2$	$2a$		
ABD	$\frac{9}{2}ax$	$\frac{1}{3}x$		
Shape ADC	$27a^2 - \frac{9}{2}ax$	$\bar{x}$		
Taking moments about AB $\bar{x} \times \left(27a^2 - \frac{9}{2}ax \right) = 27a^2 \times 2a - \frac{9}{2}ax \times \frac{1}{3}x$ $\bar{x} = \frac{54a^3 - \frac{3}{2}ax^2}{27a^2 - \frac{9}{2}ax}$			M1	Moments equation with 3 terms.
			A1	At least 2 terms correct.
			A1	All correct.
For equilibrium, $x \leq \bar{x}$, $54a^3 - \frac{3}{2}ax^2 \geq x \left(27a^2 - \frac{9}{2}ax \right)$			B1	Use correct condition: allow strict inequality. Can be implied by correct final answer $x \leq 3a$.
$54a^3 - 27ax + 3x^2 \geq 0$ $(x - 3a)(x - 6a) \geq 0$			M1	Simplify and attempt to solve a quadratic inequality or equation.
$(0 \leq) x \leq 3a$ [only]			A1	CAO
Alternative method for question 2				
Taking moments with B as origin.			M1	
$\bar{x} = \frac{1}{3}(0 + x + 6a) = 2a + \frac{1}{3}x$			A2	
For equilibrium, $x \leq \bar{x}$, so $x \leq 2a + \frac{1}{3}x$			B1	Allow strict inequality.
$(0 \leq) x \leq 3a$			M1	
			A1	
			6	

23 - (9231/31_Winter_2022_Q3) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)	Let F and R be friction and normal reaction at A Take moments about A , for rod $N \times 3a = W \times 2a \cos \theta + kW \times 4a \cos \theta$	M1	Correct terms, allow sign errors and cos/sin mix.
	$3N = (2 + 4k)W \times \frac{4}{5}$ $N = \frac{8}{15}W(1 + 2k)$	A1	At least one intermediate line of working.
		AG	
		2	
(b)	$\uparrow N \cos \theta + R = W + kW$	B1	Resolve (to include R) for rod.
	$\rightarrow F = N \sin \theta$ and $F = \frac{6}{7}R$	B1	Both.
	so $R = \frac{28}{75}W(1 + 2k)$ or $R = \frac{21}{45}W(1 + k)$	M1	Find R or N .
	Eliminate to find k	M1	Complete method.
	$k = \frac{1}{3}$	A1	
		5	

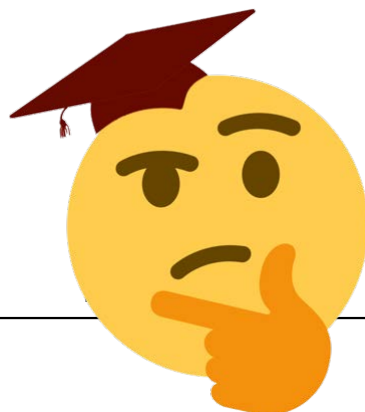


24 - (9231/31_Winter_2023_Q3) - Equilibrium Of A Rigid Body Under Coplanar Forces

Let N be normal reaction at B and F the frictional force acting downwards $\uparrow T \cos \theta = F + W$	B1	
$\rightarrow T \sin \theta = N$	B1	
Moments about B : $Ta = W \sin \theta \times a + W \cos \theta \times a$	M1A1	A moments equation with all relevant forces.
$F = \frac{1}{2}N$ used	M1	
$(\cos \theta + \sin \theta) \left(\cos \theta - \frac{1}{2} \sin \theta \right) = 1$ oe	M1	Combine to obtain equation in θ . Equation in trigonometric functions only.
$\frac{1}{2} \cos \theta \sin \theta = \frac{3}{2} (\sin \theta)^2$ $\sin \theta (\cos \theta - 3 \sin \theta) = 0$	M1	Solve trigonometric equation.
$\tan \theta = \frac{1}{3}$	A1	
	8	

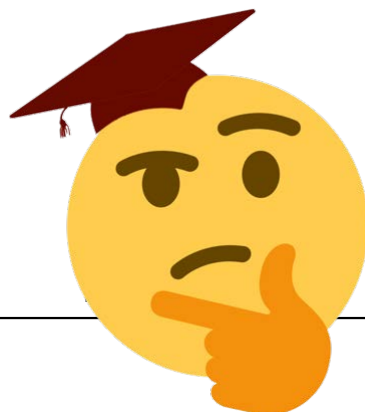
25 - (9231/32_Winter_2023_Q4) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)	After collision, A has velocity v_A towards wall on right and B has component of velocity towards lower wall of $u \sin \theta$. Same distance and time, so $v_A = u \sin \theta$.	B1	
	Along line of centres: PCLM: $mv_A + mv_B = mu \cos \theta$ NEL: $v_A - v_B = eu \cos \theta$	M1	Both, consistent signs, must be $\cos \theta$.
	$2 \sin \theta = (1 + e) \cos \theta$	M1	Eliminating v_A and v_B to find an equation in θ . Condone common factor of u . Note: $v_A = \frac{3}{4}u \cos \theta$ and $v_B = -\frac{1}{4}u \cos \theta$.
	$\tan \theta = \frac{3}{4}$	A1	
		4	
(b)	Final KE = $\frac{1}{2} m (v_A^2 + (u \sin \theta)^2 + v_B^2)$	B1	Both components of velocity of B needed.
	Loss = $\frac{1}{2} mu^2 - \frac{1}{2} m (v_A^2 + (u \sin \theta)^2 + v_B^2) = \frac{1}{2} mu^2 \left(1 - \frac{9}{25} - \frac{9}{25} - \frac{1}{25} \right)$	M1	v_A, v_B, θ substituted, ft <i>their</i> final KE with both components of velocity of B included.
	Loss = $\frac{3}{25} mu^2$ Percentage loss = 24%	A1	
		3	



26 - (9231/32_Winter_2023_Q7) - Equilibrium Of A Rigid Body Under Coplanar Forces

(a)	Gain in KE = $\frac{1}{2}m\left(\frac{6}{5}\sqrt{2ag}\right)^2$ and Gain in GPE = $mg \times 3a \sin \theta$	B1	
	Loss in EPE = $\frac{1}{2}\frac{kmg}{a}\left((3a)^2 - a^2\right)$	B1	
	Energy equation: $\frac{1}{2}m\left(\frac{6}{5}\sqrt{2ag}\right)^2 + mg \times 3a \sin \theta = \frac{1}{2}\frac{kmg}{a}\left((3a)^2 - a^2\right)$	M1	KE, GPE and at least one EPE present, allow sign errors, dimensionally correct.
		A1	All correct.
	$\frac{36}{25}mg + \frac{12}{5}mg = 4kmg$ $k = \frac{24}{25}$	A1	
		5	
(b)	In lower position, tension T in spring = $kmg \times \frac{3a}{a} = 3kmg = \frac{72}{25}mg$ Perpendicular to rod, $T = (F + mg) \cos \theta$	M1	Hooke's law.
	So, $(F + mg) \cos \theta = \frac{72}{25}mg$, $F = \frac{19}{5}mg$	A1	
	Alternative method for question 7(b)		
	In lower position, tension T in spring = $kmg \times \frac{3a}{a} = 3kmg = \frac{72}{25}mg$ $T' \cos \theta = T \sin \theta$ $T' \sin \theta + T \cos \theta = F + mg$	M1	Hooke's law, Eliminate T' .
	$F = \frac{19}{5}mg$	A1	
		2	
(c)	Let tension in rod = T' Parallel to rod, $T' = (F + mg) \sin \theta$	M1	
	$T' = \frac{96}{25}mg$	A1	
	Alternative method for question 7(c)		
	$T' \cos \theta = T \sin \theta$ $T' \sin \theta + T \cos \theta = F + mg$	M1	At least one equation seen with their T and/or F.
	$T' = \frac{96}{25}mg$	A1	
		2	



1 - (9231/23_Summer_2017_Q1) - Rotation Of A Rigid Body

<i>EITHER:</i> $T \times a = (\frac{1}{2} m a^2) \frac{d^2 \theta}{dt^2} \text{ or } (\frac{1}{2} m a) \frac{d^2 x}{dt^2}$	(B1)
$3mg - 0.9mg - T = 3m \times a \frac{d^2 \theta}{dt^2} \text{ or } 3m \frac{d^2 x}{dt^2}$	M1 A1)
<i>OR:</i> $T a \theta = \frac{1}{2} (\frac{1}{2} m a^2) (\frac{d\theta}{dt})^2$	(B1)
$(3mg - 0.9mg - T) a \theta = \frac{1}{2} 3m (a \frac{d\theta}{dt})^2$	M1 A1)
$6T = 2.1mg - T, T = 0.3 mg$	M1 A1
Total:	5

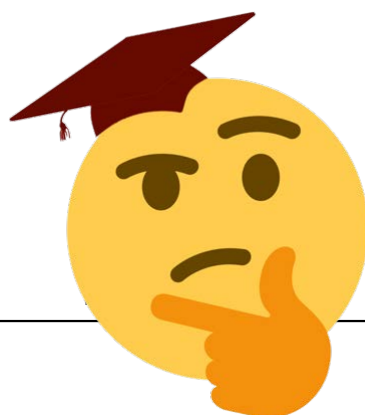
2 - (9231/21_Summer_2017_Q4) - Rotation Of A Rigid Body

(i)	$I_{\text{discs}} = \frac{1}{2} m a^2 + 2 \times \{ \frac{1}{2} m a^2 + m (4a)^2 \}$ [= $(\frac{1}{2} + 2 \times \{33/2\}) m a^2 = 67 m a^2/2$]	M1 A1
	$I_{AB} \text{ or } I_{AC} = \frac{1}{3} (\frac{1}{3} m) a^2 + (\frac{1}{3} m) (2a)^2$ [= $13 m a^2/9$]	(AEF) M1 A1
	$I_{BC} = \frac{1}{3} (\frac{1}{3} m) a^2 + (\frac{1}{3} m) (2a\sqrt{3})^2$ [= $37 m a^2/9$]	(AEF) A1
	$I = (67/2 + 37/9 + 2 \times 13/9) m a^2 = 81 m a^2/2$	A1
	Total:	6
(ii)	$h = 4a$	B1
	$\frac{1}{2} I \omega^2 = 4 m g h, \omega^2 = 64g/81a$	M1 A1 FT
	$\omega = (8/9) \sqrt{(g/a)} \text{ or } 0.889 \sqrt{(g/a)} \text{ or } 2.81/\sqrt{a}$	A1
	Total:	4



3 - (9231/23_Summer_2017_Q11) - Rotation Of A Rigid Body, Bivariate Data

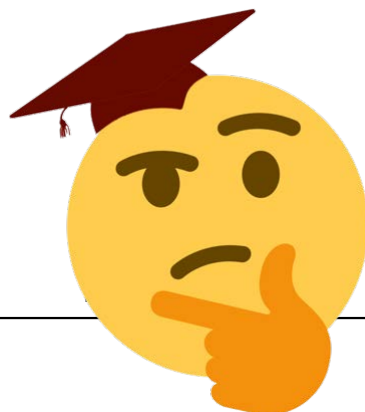
(a)(i)	$I_{AB} = \frac{1}{3} 8m (3a/2)^2 + 8m (a/2)^2$ or $8m (3a)^2/12 + 8m (a/2)^2$ [= $8 ma^2$]	M1 A1
	$I_{\text{shell}} = \frac{2}{3} ma^2 + m (2a)^2$ [= $(14/3) ma^2$]	M1
	$I_{\text{sphere}} = (2/5) (3m/2)a^2 + (3m/2) (2a)^2$ [= $(33/5) ma^2$]	M1 A1
	$I = (8 + 14/3 + 33/5) ma^2 = (289/15) ma^2$ AG	A1
	Total:	6
(a)(ii)	$\frac{1}{2} I \omega^2 = (5mg/2) 2a (1 - \cos \alpha) - 8mg (a/2) (1 - \cos \alpha)$ or $((21/2)mg \times 2a/21) (1 - \cos \alpha)$	M1 A1
	$= mga (1 - \cos \alpha) = 5mga/6$	A1
	$\omega^2 = 25g/289a$ or $0.0865 g/a$ or $\omega = (5/17) \sqrt{(g/a)}$ or $0.294 \sqrt{(g/a)}$	A1
	$v_{\text{max}} = 2a\omega = 2a \sqrt{(25g/289a)}$ $= \sqrt{(100ag/289)}$ or $(10/17)\sqrt{(ag)}$	M1
	or $0.588\sqrt{(ag)}$ or $1.86\sqrt{a}$ (AEF)	A1
	Total:	6
(b)(i)	e.g. $S_{xy} = 4510.99 - 191 \times 188.8/8 = 3.39$ or 0.424 $S_{xx} = 4564.46 - 191^2/8 = 4.335$ or 0.542 $b = S_{xy} / S_{xx} = 3.39/4.335 = 0.782$	M1 A1
	$(y - 188.8/8) = b (x - 191/8)$ $(y - 23.6) = 0.782 (x - 23.875), y = 0.782x + 4.93$	M1 A1
	Total:	4



(b)(ii)	$H_0: \mu_x - \mu_y = 0.2, H_1: \mu_x - \mu_y > 0.2$ (AEF)	B1
	$d_i: 0.3 \quad 0.2 \quad 0 \quad 0.6 \quad 0.3 \quad 0.5 \quad 0.2 \quad 0.1$	M1
	$\bar{d} = 2.2 / 8 = 0.275$	B1
	$s^2 = (0.88 - 2.2^2/8) / 7$	M1
	$[= 11/280 \text{ or } 0.0393 \text{ or } 0.198^2]$ $t_{7, 0.9} = 1.41[5]$	B1
	$t = (\bar{d} - 0.2) / (s/\sqrt{8}) = 1.07$	M1 A1
	[Accept H_0 .] No evidence for coach's belief (AEF)	B1 FT
Total:		8

4 - (9231/21_Winter_2017_Q5) - Rotation Of A Rigid Body

(i)	$A_{ABCD} = 24a^2, A_{EFGH} = 8a^2, A_{Frame} = 16a^2$ $M_{ABCD} = 3m/2 \text{ and } m_{EFGH} = m/2$	B1	Use areas to find masses M_{ABCD} and m_{EFGH}
	$I_{ABCD} = \frac{1}{12} M_{ABCD} ((3a)^2 + (2a)^2)$ $[= (13/3) M_{ABCD} a^2 = (13/2) ma^2]$	B1	Find MI of ABCD about axis at centre Q
	$I_{EFGH} = \frac{1}{12} m_{EFGH} ((2a)^2 + a^2)$ $[= (5/3) m_{EFGH} a^2 = (5/6) ma^2]$	B1	Find MI of EFGH about axis at centre Q
	$I_{ABCD} - I_{EFGH} + m \times (6a)^2 [= (121/2 - 113/6) ma^2$ $\text{or } (17/3 + 36) ma^2 = (125/3) ma^2]$	M1 A1	Find MI of frame about axis at O Result also follows from combining four rectangular parts
	$I_{Coin} = (11m/12) \times (4a)^2 [= (44/3) ma^2]$	B1	Find MI of small object about axis at O
	$I = (13/2 - 5/6 + 36 + 44/3) ma^2 = (169/3) ma^2$ AG	A1	Combine to verify MI of system about axis at O
		7	
(ii)	$[-] I d^2\theta/dt^2 \text{ or } [-] I \alpha$ $= mg = 6a \sin \theta + (11/12)mg \times 4a \sin \theta$ $\text{or } (23/12)mg = (116/23)a \sin \theta$ $[= (29/3) mga \sin \theta]$	M1 A1	Use eqn of circular motion to relate $d^2\theta/dt^2$ to $\sin \theta$, where θ is angle of QO with vertical
	$d^2\theta/dt^2 \text{ or } \alpha = -(29g/169a) \theta$ $\text{or } -(0.172 g/a) \theta$	M1 A1	Approximate $\sin \theta$ by θ to show SHM (M0 if wrong sign or $\cos \theta \approx \theta$ used)
	$T = 2\pi / \sqrt{(29g/169a)}$ $= 26\pi \sqrt{(a/29g)} \text{ or } 15.2 \sqrt{(a/g)} \text{ or } 4.80 \sqrt{a}$ (AEF)	A1	Find period T from $T = 2\pi/\omega$ (A1 requires some simplification)
		5	



5 - (9231/21_Summer_2018_Q5) - Rotation Of A Rigid Body

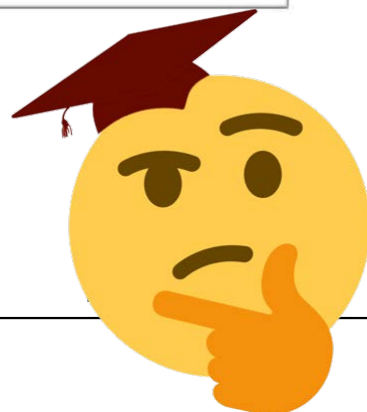
(i)	$I_A = \frac{1}{2} 3M(2a)^2$ [= 6 Ma^2]	*M1	Find MI of A // to axis l at A 's centre by $\perp$ axis theorem
	$I_A' = I_A + 3M(2a)^2$ [= 18 Ma^2] (dep *M1)	M1 A1	Find MI of A about axis l
	$I_B = \frac{1}{2} 2M(3a)^2$ [= 9 Ma^2]	**M1	Find MI of B (or C) // to axis l at its centre by $\perp$ axis theorem
	$I_B' = I_B + 2M(6a)^2$ [= 81 Ma^2] (dep **M1)	M1 A1	Find MI of B (or C) about axis l
	$I = (18 + 2 \times 81) Ma^2 = 180 Ma^2$ AG	A1	Verify MI of object about axis l (A0 if inadequate explanation)
	SC: $I_A' = \frac{1}{2} \{3M(2a)^2 + 3M(2a)^2\}$ [= 12 Ma^2]	(M1)	SC: Invalidly applying theorems in wrong order
	$I_B' = \frac{1}{2} \{2M(3a)^2 + 2M(6a)^2\}$ [= 45 Ma^2]	(M1)	(max $\frac{2}{7}$)
		7	

(ii)	$[-] I \frac{d^2\theta}{dt^2} = 3Mg \times 2a \sin \theta + 2 \times 2Mg \times 6a \sin \theta$ or $7Mg \times \frac{30a}{7} \sin \theta$ [= 30 $Mga \sin \theta$]	*M1 A1	Use eqn of circular motion to find $d^2\theta/dt^2$ where θ is angle of plane of object with vertical
	$\frac{d^2\theta}{dt^2} = -\left(\frac{g}{6a}\right) \theta$ or $-\left(\frac{0.167g}{a}\right) \theta$ (M1 dep *M1)	M1 A1	Approximate $\sin \theta$ by θ to show SHM (no $\therefore$ is M1 A0)
	$T = \frac{2\pi}{\sqrt{\left(\frac{g}{6a}\right)}}$ $= 2\pi \sqrt{\left(\frac{6a}{g}\right)}$ or $15.4 \sqrt{\left(\frac{a}{g}\right)}$ or $4.87 \sqrt{a}$ (AEF)	B1✓	Find period T from $T = 2\pi/\omega$ (FT on ω^2 ; requires some simplification)
		5	

6 - (9231/23_Summer_2018_Q11) - Rotation Of A Rigid Body

A(i)	$I_{\text{lamina}} = \frac{1}{2} kM((\frac{1}{2}a)^2 + (\frac{1}{2}a)^2)$ [= ($k/6$) Ma^2]	B1	Find or state MI of lamina about axis at its centre
	$I'_{\text{lamina}} = I_{\text{lamina}} + kM((3a/2)^2 + (3a/2)^2)$ [= (14 $k/3$) Ma^2]	M1 A1	Find MI of lamina about axis at A
	EITHER: $I_{AB} = \frac{1}{2} Ma^2 + Ma^2$ [= (4/3) Ma^2]	B1	Find or state MI of AB (or AD) about axis at A
	$I_{BC} = \frac{1}{2} Ma^2 + M(a^2 + (2a)^2)$ [= (16/3) Ma^2]	M1 A1	Find or state MI of BC (or DC) about axis at A
	OR: $I_{rod} = \frac{1}{2} Ma^2 + Ma^2$ [= (4/3) Ma^2]	(B1)	Find or state MI of any rod about axis at centre of frame
	$I_{\text{frame}} = 4 \times I_{rod} + 4M(a^2 + a^2)$ [= (40/3) Ma^2]	(M1 A1)	Find or state MI of frame about axis at A
	$I = (14k/3 + 2 \times 4/3 + 2 \times 16/3) Ma^2$ or $(14k/3 + 40/3) Ma^2$ [= $\frac{2}{3} Ma^2 (7k + 20)$] AG	A1	A0 if inadequate explanation
		8	

A(ii)	$\frac{1}{2} I \omega^2 = kMg \times 3a + 2Mg \times 2a + Mg \times 4a$ or $kMg \times 3a + 4Mg \times 2a$	M1 A2	Find ω^2 or angular speed ω when D below B by energy Award A1 if error in only term
	$\omega^2 = \{3(3k + 8) / (7k + 20)\} g/a$ $4 \times 3(3k + 8) = 5(7k + 20), k = 4$	A1	Find k by equating ω^2 to $\{1/2 \sqrt{(5g/a)}\}^2$
		M1 A1	
		6	



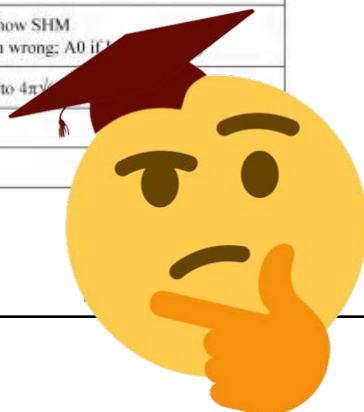
B(i)	$\bar{x} = 469/250 = 1.876$ and $\sigma^2 = (1195 - 469^2/250) / 249 = 1.266$	AG AG	B1	Verify mean and unbiased variance of data, showing method B0 if biased variance, but allow other marks (max 3/4)
	$6 \times 0.313 = 1.878$ and $6 \times 0.313 \times 0.687 = 1.29[0]$		MI A1	Find mean <i>np</i> and variance <i>npq</i> of binomial distribution
	Means and variances are similar for <i>X</i> and <i>B</i> (6, 0.313)		A1	Valid explanation (AEF; needs 1.878, 1.290 correct to 0.002)
			4	
B(ii)	$250 \times {}^6C_4 \times 0.313^4 \times 0.687^2 = 250 \times 0.06795 = 16.99$ Allow $49.7 \times ({}^6C_4 \times 0.313) / ({}^6C_3 \times 0.687)$		MI A1	Show how expected frequency for $x = 4$ is found (AEF for ${}^6C_4 = 15$)
			2	
B(iii)	H_0 : [Binomial] distribution fits data (AEF)		B1	State (at least) null hypothesis
	O_i : 22 83 72 53 <u>20</u> E_i : 26.3 71.9 81.8 49.7 <u>20.3</u>		*MI A1	Combine last 3 cells so that all exp. value ≥ 5
	$\chi^2 = 0.703 + 1.714 + 1.174 + 0.219 + 0.004 = 3.81$ (to 3 s.f.)		MI A1	Find value of χ^2 from $\Sigma (E_i - O_i)^2 / E_i$ [or $\Sigma O_i^2 / E_i - n$] (A1 dep *M1)
	No. <i>n</i> of cells: 7 <u>6</u> $\chi_{0.05}^2$: 11.07 9.488 <u>7.815</u>		B1✓	State or use consistent tabular value $\chi_{n-2, 0.05}^2$ (to 3 s.f.) [FT on number, <i>n</i> , of cells used to find χ^2]
	Accept H_0 if $\chi^2 <$ tabular value		MI	State or imply valid method for conclusion
	$3.81 [\pm 0.01] < 7.81[5]$ so distribution fits or scientist's belief is correct		A1	Conclusion (AEF; requires both values approx. correct)
			8	

7 - (9231/21_Winter_2018_Q3) - Rotation Of A Rigid Body

(i)	$I_{AB} = \frac{1}{2} M (3a)^2 + M (3a)^2$ or $(4/3) M (3a)^2$ [= 12 Ma^2]	B1	Find or state MI of rod <i>AB</i> about axis <i>l</i>
	$I_{disc} = \frac{1}{2} 2 M a^2 + 2 M x^2$ [= $Ma^2 + 2 M x^2$]	M1A1	Find MI of disc about axis <i>l</i>
	$I = 13 M a^2 + 2 M x^2$	A1	Find MI of object about axis <i>l</i>
		4	
(ii)	$\frac{1}{2} I \omega^2 = Mg \times 3a \cos \theta + 2Mg x \cos \theta$	M1A1	Find ω^2 or angular speed ω at angle θ by energy, with $\cos \theta = 3/5$
	$\omega^2 = 6(3a + 2x)g / 5(13 a^2 + 2x^2)$	A1	
	$6a(3a + 2x) = 2(13 a^2 + 2x^2)$ $x^2 - 3ax + 2a^2 = 0, x = a$ or $2a$	M1A1	Equate ω^2 to $2g/5a$ and solve quadratic for <i>x</i>
		5	

8 - (9231/22_Winter_2018_Q5) - Rotation Of A Rigid Body

(i)	$I_{disc} = \frac{1}{2} \times \frac{1}{2} 3M (2a)^2$ [= 3 Ma^2]	M1	Find MI of disc parallel to axis <i>l</i> at its centre using perpendicular axis theorem (M0 if theorem not used)
	$I_{disc} = I_{disc} + 3M (5a)^2 = 78 Ma^2$	M1A1	Find MI of disc about axis <i>l</i>
	$I_{rod} = \frac{1}{2} kM (3a/2)^2 + kM (3a/2)^2 = 3k Ma^2$	B1	Find or state MI of rod <i>AB</i> about axis <i>l</i>
	$I = (78 + 3k) Ma^2 = 3 Ma^2 (26 + k)$ AG	A1	Find MI of object about axis <i>l</i> (A0 if inadequate explanation)
	SC : $I_{disc} = \frac{1}{2} \{ \frac{1}{2} 3M (2a)^2 + 3M (5a)^2 \} = (81/2) Ma^2$	(M1)	SC : Invalidly applying theorems in wrong order to disc (max 2)
		5	
(ii)	$[-] \int d^2\theta/dt^2 = 3Mg \times 5a \sin \theta + (kMg \times (3a/2) \sin \theta$ [= 3 (5 + $\frac{1}{2} k$) $Mga \sin \theta$]	M1A1	Use eqn of circular motion to find $d^2\theta/dt^2$ where θ is angle of rod with vertical
	$d^2\theta/dt^2 = - \{ g (10 + k) / 2a(26 + k) \} \theta$ (AEF)	M1A1	Approximate $\sin \theta$ by θ to show SHM (M0 if $\cos \theta \approx \theta$ used or sign wrong; A0 if 1)
	$2\pi / \sqrt{\{ (10 + k) / 2(26 + k) \}} = 4\pi$	M1	Find <i>k</i> by equating period <i>T</i> to $4\pi\sqrt{\dots}$
	$2 (26 + k) = 4 (10 + k), k = 6$	A1	
		6	

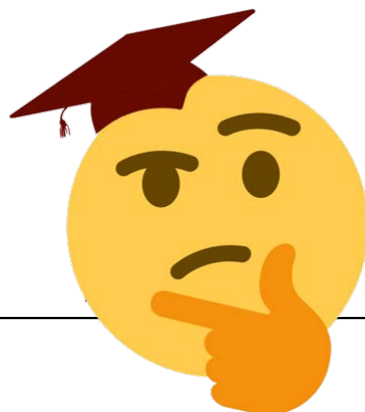


9 - (9231/23_Summer_2019_Q4) - Rotation Of A Rigid Body

(i)	$I_{AB} = \frac{1}{2} 5M(7a/2)^2 + 5M(7a/2)^2$ or $(4/3) 5M(7a/2)^2$ $[= (245/3) Ma^2]$	B1	Find or state MI of rod AB about axis at A
	$I_M = \frac{1}{2} M(2a)^2 + M(7a)^2$ $[= (155/3) Ma^2]$	M1 A1	M1 for one term correct, A1 for both terms correct
	$I_{BC} = \frac{1}{2} kM a^2 + kM(4a)^2$ $[= (50/3) kMa^2]$	M1 A1	M1 for one term correct, A1 for both terms correct
	$I = [(245/3 + 155/3 + 50k/3) Ma^2] = (50/3) (8 + k) Ma^2$	A1	Find MI of object about axis at A , simplified to 2 terms aef
		6	
(ii)	$\frac{1}{2} I \omega^2 = 5Mg(7a/2)(1 - \cos 60^\circ) + Mg(7a)(1 - \cos 60^\circ)$ $+ kMg(4a)(1 - \cos 60^\circ)$	*M1 A1	Find eqn. for ω^2 when AB vertical by energy (3 terms on RHS, same trig expression in each for M1) A1 for 2 terms correct on RHS
	$\frac{1}{2} (49 + 8k) / \frac{1}{2} (50/3) (8 + k) = 81/400$	A1	A1 for RHS and LHS all correct
	$\frac{1}{2} (49 + 8k) / \frac{1}{2} (50/3) (8 + k) = 81/400$	DM1	Equate ω^2 to $81g/400a$ to find k
	$4(49 + 8k) = 27(8 + k), k = 4$	A1	
	Alternative method for question 4(ii)		
	$(6 + k) Mg \times \{(49 + 8k)/2(6 + k)\} (1 - \cos 60^\circ)$ $= \frac{1}{2} (49 + 8k) Mga (1 - \cos 60^\circ) = \frac{1}{4} (49 + 8k) Mga$	*M1 A1 A1	M1 for mass $\times$ com $\times$ trig expression, A1 for 2 parts correct A1 all correct
	$\frac{1}{2} (49 + 8k) / \frac{1}{2} (50/3) (8 + k) = 81/400$	DM1	Equate ω^2 to $81g/400a$ to find k
	$4(49 + 8k) = 27(8 + k), k = 4$	A1	
		5	

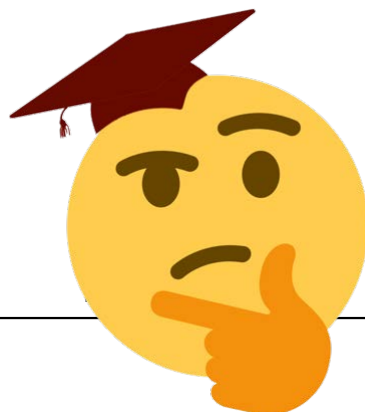
10 - (9231/21_Summer_2019_Q5) - Rotation Of A Rigid Body, Simple Harmonic Motion

(i)	$I_{rod} = \frac{1}{2} kMa^2$	B1	Find or state MI of rod AB about axis L
	$I_{sphere} = \frac{1}{2} kM(2a)^2 + kM(3a)^2$ $[= (35k/3) Ma^2]$	M1 A1	M1 for one term correct, A1 for both terms correct
	$I_{ring} = \frac{1}{2} \times Ma^2 + M(2a)^2$ $[= (9/2) Ma^2]$	M1 A1	M1 for one term correct, A1 for both terms correct
	$I = (k/3 + 35k/3 + 9/2) Ma^2 = (3/2) (8k + 3) Ma^2$ AG	A1	MI of object about axis L
		6	
(ii)	$[-] I d^2\theta/dr^2 = -kMg \times 3a \sin \theta + Mg \times 2a \sin \theta$ $[= -(3k - 2) Mga \sin \theta]$	M1 A1	Use equation of circular motion to find $d^2\theta/dr^2$ where θ is angle of rod with vertical
	$d^2\theta/dr^2 = -\{2g(3k - 2) / 3a(8k + 3)\} \theta$ (AEF)	M1	Approximate $\sin \theta$ by θ to give standard form of SHM equation (M0 if wrong sign or $\cos \theta \approx \theta$ used)
	SHM if $3k - 2 > 0, k > 2/3$	M1 A1	Find possible values of k
	$T = 2\pi \sqrt{\{3a(8k + 3) / 2g(3k - 2)\}}$ or $\pi \sqrt{\{6a(8k + 3) / g(3k - 2)\}}$	A1	Find period T
		6	



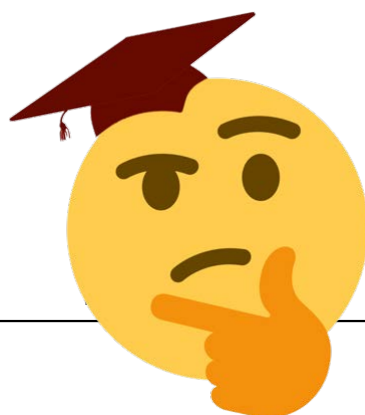
11 - (9231/21_Winter_2019_Q5) - Rotation Of A Rigid Body, Simple Harmonic Motion

(i)	$I_{rod} = \frac{1}{3} \lambda M a^2 + \lambda M (a/2)^2$ [$= (7/12) \lambda M a^2$]	B1	Find or state MI of rod AB about axis L
	$I_O = \frac{2}{3} 3M a^2 + 3M (5a/2)^2$ [$= (83/4) M a^2$]	M1 A1	Find MI of hollow sphere centre O about axis L
	$I_C = (2/5) 5M a^2 + 5M (3a/2)^2$ [$= (53/4) M a^2$]	M1 A1	Find MI of solid sphere centre C about axis L
	$I = (7\lambda/12 + 83/4 + 53/4) M a^2$ $= ((7\lambda + 408) / 12) M a^2$	A1	Verify MI of object about axis L . AG
		6	
(ii)	$[-] I d^2\theta/dt^2 = [-3Mg \times (5a/2) \sin \theta + 5Mg \times (3a/2) \sin \theta]$ $- \lambda Mg \times (a/2) \sin \theta$	M1 A1	Use eqn of circular motion to find $d^2\theta/dt^2$ where θ is angle of rod with vertical. AEF
	$d^2\theta/dt^2 = - \{6g\lambda / (7\lambda + 408)a\} \theta$	M1*	Approximate $\sin \theta$ by θ to give standard form of SHM eqn
	$T = 2\pi \sqrt{\{(7\lambda + 408)a / 6g\lambda\}} = 5\pi\sqrt{(2a/g)}$	DM1A1	Find possible values of λ by equating period T to $5\pi\sqrt{(2a/g)}$. AEF
	$\lambda = 6$	A1	
		6	



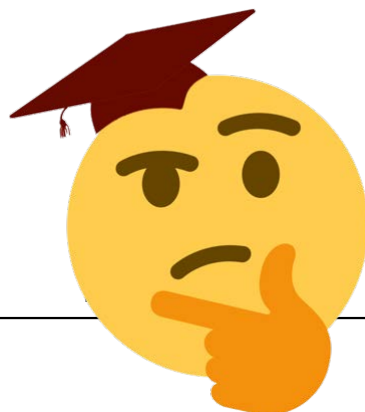
1 - (9231/23_Summer_2017_Q2) - Simple Harmonic Motion

(i)	$OL/OM = 3/4, OM = 2 \text{ [m]}$	M1 A1
	Total:	2
(ii)	<i>EITHER:</i> $\omega t_{LM} = \sin^{-1}(OL/2.5) + \sin^{-1}(OM/2.5)$ <i>or</i> $\omega t_{OL} = \sin^{-1}(OL/2.5), \omega(2 - t_{OL}) = \sin^{-1} OM/2.5$ so $2\omega = \sin^{-1} 0.6 + \sin^{-1} 0.8 [= 0.6435 + 0.9273]$	(*M1 A1)
	<i>OR:</i> $\omega t_{LM} = \cos^{-1}(-OM/2.5) - \cos^{-1}(OL/2.5)$ <i>or</i> $\omega t_{AL} = \cos^{-1}(OL/2.5), \omega(2 + t_{AL}) = \cos^{-1}(-OM/2.5)$ so $2\omega = \cos^{-1}(-0.8) - \cos^{-1} 0.6 [= 2.498 - 0.927]$ <i>or</i> $\pi - \cos^{-1} 0.8 - \cos^{-1} 0.6$	(*M1 A1)
	$\omega = \pi / 4 \text{ or } 0.785$ (M1 dep *M1)	DM1 A1
	$T = 2\pi / \omega = 8 \text{ [s]}$	B1 FT
	Total:	5
(iii)	$v_L = \omega \sqrt{(2.5^2 - 1.5^2)} = 2\omega = \pi / 2 \text{ or } 1.57 \text{ [m s}^{-1}\text{]}$	M1 A1
	Total:	2



2 - (9231/21_Summer_2017_Q11) - Simple Harmonic Motion, X2 Test

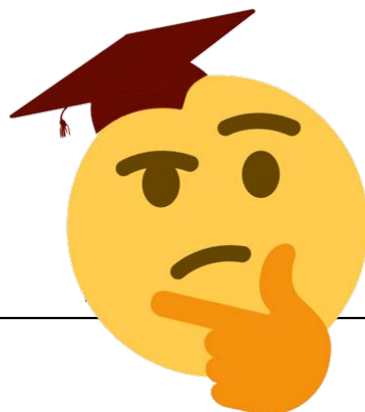
(a)(i)	$T = 3mg \sin \alpha \quad [= 2mg]$	B1
	$T = kmg (5a/4 - a) / a \quad [= \frac{1}{4} kmg]$	B1
	$k = 8$	B1
	Total:	3
(a)(ii)	<i>EITHER:</i> $\pm 2m \, d^2 OQ / dt^2 = 2mg \sin \alpha - kmg (OQ - a) / a$	(M1 A1
	$d^2 OQ / dt^2 = (4g/a) (7a/6 - OQ)$	A1
	$d^2 x / dt^2 = -(4g/a) x$ where $x = OQ - 7a/6$	A1)
	<i>OR:</i> $2mg \sin \alpha = kmg (e - a) / a, e = 7a/6$	(M1
	$\pm 2m \, d^2 x / dt^2 = 2mg \sin \alpha - kmg (e + x - a) / a$	M1 A1
	$d^2 x / dt^2 = -(4g/a) x$	A1)
	Centre is $7a/6$ (or $1.17 a$) from O	B1
	Period is $\pi \sqrt{a/g}$ or $0.993 \sqrt{a}$	B1
	Total:	6
(a)(iii)	$x_0 = 5a/4 - e = a/12$	B1
	$T_{min} = kmg (5a/4 - 2x_0 - a) / a = 2mg/3$	M1 A1
	$(d^2 x / dt^2)_{max} = [\pm] (4g/a) x_0 = [\pm] \frac{1}{3} g$	M1 A1
	Total:	5



(b)(i)	$x = (1/250) \sum x f(x) = 414/250 = 1.656 \text{ AG}$	B1
	Total:	1
(b)(ii)	$p = x/6 = 0.276, q = 0.724$	M1 A1
	$a = 250 {}^6C_2 q^4 p^2 = 78.49 \pm 0.01 \quad (\text{to 2 d.p.})$	A2
	$b = 250 {}^6C_4 q^2 p^4 = 11.41 \pm 0.01 \quad (\text{to 2 d.p.})$	A1
	Total:	5
(b)(iii)	H_0 : Distribution fits data <i>or</i> distribution is binomial (AEF)	B1
	$O_i:$ 48 69 78 32 23 $E_i:$ 36.01 82.36 78.49 39.89 <u>13.26</u> (± 0.01)	M1FT A1
	$\chi^2 = 3.992 + 2.167 + 0.003 + 1.561 + 7.154$	M1
	$= 14.9$	A1
	No. n of cells: 7 6 <u>5</u> 4 3 $\chi_{n-2, 0.99}^2:$ 15.09 13.28 <u>11.34</u> 9.210 6.635	B1FT
	Accept H_1 if $\chi^2 > \text{tabular value}$ $14.9 [\pm 0.1] > 11.34$ so distn. doesn't fit [data]	(AEF) M1
	<i>or</i> manager's belief not justified	(AEF) A1
	Total:	8

3 - (9231/21_Winter_2017_Q2) - Simple Harmonic Motion

$(3\pi/5)^2 = \omega^2 (a^2 - (a - 1.6)^2)$	(AEF)	M1 A1	Use $v^2 = \omega^2 (a^2 - x^2)$ in each posn. (M1 for either)
$(\pi/4)^2 = \omega^2 (a^2 - (a - 0.2)^2)$ $(3\pi/5)^2 (0.4a - 0.04) = (\pi/4)^2 (3.2a - 2.56)$		A1	Combine to find amplitude a and ω^2 (or ω)
$a = 2.6 \text{ [m]}$		M1 A1	(M1 for either)
$\omega^2 = (\pi/4)^2$ <i>or</i> $\omega = \pi/4$	(AEF)	A1	Find other unknown
$T = 2\pi / (\pi/4) = 8 \text{ [s]}$		B1FT	Find period T from $T = 2\pi/\omega$ ($\sqrt{\quad}$ on ω^2 or ω)
		7	



4 - (9231/21_Summer_2018_Q2) - Simple Harmonic Motion

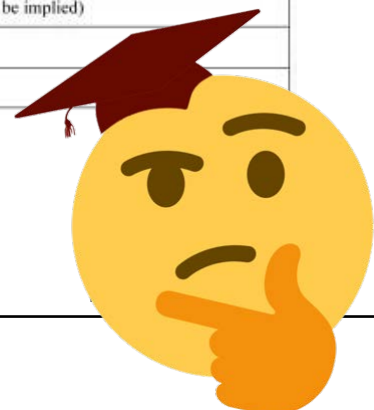
(i)	$v_A^2 = \omega^2(a^2 - 1.6^2)$ and $v_B^2 = \omega^2(a^2 - 1.2^2)$	B1	Use $v^2 = \omega^2(a^2 - x^2)$ at A and B (may be implied)
	$\frac{9}{16} = \frac{(a^2 - 1.6^2)}{(a^2 - 1.2^2)}$ or $\frac{(a^2 - 2.56)}{(a^2 - 1.44)}$	M1 A1	Find eqn. for a from ratio of v_A to v_B
	$7a^2 = 40.96 - 12.96$ or 10×2.8 , $a^2 = 4$, $a = 2$ [m]	A1	and hence a (error in $\frac{9}{16}$ will lose all A1s, so max $\frac{4}{11}$)
		4	
(ii)	$\omega = \frac{1}{3} \frac{\pi}{a} = \frac{\pi}{6}$, $T = \frac{2\pi}{\omega} = 12$ [s]	M1 A1	Find ω and hence period T from $v_{\max} = \omega a$ and $T = \frac{2\pi}{\omega}$
		2	
(iii)	$\omega^{-1} \sin^{-1}\left(\frac{1.6}{a}\right) + \omega^{-1} \sin^{-1}\left(\frac{1.2}{a}\right) [= \omega^{-1} \frac{\pi}{2}]$ or $\omega^{-1} \cos^{-1}\left(\frac{-1.6}{a}\right) - \omega^{-1} \cos^{-1}\left(\frac{1.2}{a}\right)$ or $\frac{1}{2} T - \omega^{-1} \cos^{-1}\left(\frac{1.6}{a}\right) - \omega^{-1} \cos^{-1}\left(\frac{1.2}{a}\right)$	M1	Find time from A to B from $x = a \sin \omega t$ or $a \cos \omega t$ All terms must be correct (FT on a , ω) for M1
	$= \omega^{-1}(0.9273 + 0.6435)$ or $\omega^{-1}(2.498 - 0.9273)$ or $6 - \omega^{-1}(0.6435 + 0.927)$ (AEF throughout)	A1	
	$= 1.771 + 1.229$ or $4.771 - 1.771$ or $6 - 1.229 - 1.771$,	A1	(OR use geometry of circular motion with radius a and angular velocity ω)
		3	

5 - (9231/23_Summer_2018_Q3) - Simple Harmonic Motion

(i)	$\omega^2(7.5 - d) = 2 \omega^2(6.5 - d)$, $d = 5.5$	M1 A1	Find d by relating accelns. at A and B (ω^2 may be omitted)
		2	
(ii)	$\omega = 2\pi/T = 2$	B1	Find ω from period T (may be implied)
	$a = 10/\omega^2 = 2.5$ [m] (ignore sign; FT on ω)	B1✓	Find amplitude a from max. acceleration 10
	$v^2 = \omega^2(a^2 - x^2)$, $x = 7 - d [= 1.5]$	M1, M1	Find speed v when $OP = 7$ (2nd M1 dep. on 1st M1)
	$= 2^2(2.5^2 - 1.5^2) = 16$, $v = 4$ [m s ⁻¹]	A1	
		5	
(iii)	$\omega^{-1} \sin^{-1}(x_B/a) - \omega^{-1} \sin^{-1}(x_A/a)$ or $\omega^{-1} \cos^{-1}(x_A/a) - \omega^{-1} \cos^{-1}(x_B/a)$	M1	Find time from A to B from $x = a \sin \omega t$ or $a \cos \omega t$ (all terms must be correct (FT on a , ω) for M1)
	$= \frac{1}{2} \sin^{-1}(2/2.5) - \frac{1}{2} \sin^{-1}(1/2.5)$ or $\frac{1}{2} \cos^{-1}(1/2.5) - \frac{1}{2} \cos^{-1}(2/2.5)$ $= \frac{1}{2}(0.9273 - 0.4115)$ or $0.4636 - 0.2058$ or $\frac{1}{2}(1.1593 - 0.6435)$ or $0.5796 - 0.3218$	A1	To 3 d.p., AEF throughout
	$= 0.258$ [s]	A1	
		3	

6 - (9231/21_Winter_2018_Q1) - Simple Harmonic Motion

$\omega = 2\pi/T = 4$	B1	Find ω from period T (may be implied)
$v = \omega \sqrt{(a^2 - x^2)} = 4 \sqrt{(3^2 - 1^2)} = 8\sqrt{2}$ or 11.3 [m s ⁻¹]	M1A1	Find speed v when $BP = 2$
	3	



7 - (9231/22_Winter_2018_Q1) - Simple Harmonic Motion

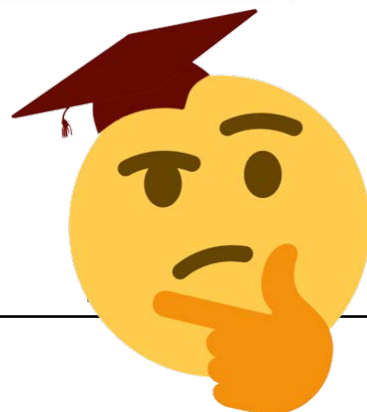
$v_A^2 = \omega^2 (a^2 - 0.1^2)$ and $v_B^2 = \omega^2 (a^2 - 0.5^2)$	B1	Use $v^2 = \omega^2 (a^2 - x^2)$ at <i>A</i> and <i>B</i> (may be implied)
$a^2 - 0.1^2 = 2 (a^2 - 0.5^2)$	M1	Find amplitude <i>a</i> from ratio 2 of $[\frac{1}{2} m] v_A^2$ to $[\frac{1}{2} m] v_B^2$
$a^2 = 0.5 - 0.01 = 0.49, a = 0.7$ [m]	A1	(taking ratio $\frac{1}{2}$ loses A1)
	3	

8 - (9231/21_Winter_2018_Q5) - Simple Harmonic Motion

(i)	$\lambda (AE - 1.25) / 1.25 =$	M1A1	Verify <i>AE</i> by equating equilibrium tensions
	$0.6\lambda (2.6 - AE - 1.0) / 1.0$ (AEF)	A1	SC: vertical motion can earn M1 only
	$AE - 1.25 = 1.2 - 0.75 AE, AE = 1.4$ AG	A1	
		4	
(ii)	$[\pm] m d^2x/dt^2 = -\lambda (0.15 + x) / 1.25 + 0.6\lambda (0.2 - x) / 1.0$ or $+\lambda (0.15 - x) / 1.25 - 0.6\lambda (0.2 + x) / 1.0$	M1A2	Apply Newton's law at $1.4 + x$ or $1.4 - x$ from <i>A</i> (lose A1 for one incorrect tension term)
	$d^2x/dt^2 = -(1.4\lambda / 0.4)x = -3.5\lambda x$	M1A1	Simplify to give SHM eqn. in standard form (A0 if no minus sign, or dirn. of acceln. is undefined) SC: B2 if result stated without derivation (max 2/5)
		5	
(iii)	$\omega = 2\pi / (\pi/7) [= 14] = \sqrt{(3.5\lambda)}$	M1A1	Find eqn. for λ using $T = 2\pi/\omega$ with FT on ω from SHM eqn.
	$\lambda = 14^2 / 3.5 = 56$	A1	Solve for λ
		3	

9 - (9231/22_Winter_2018_Q11) - Simple Harmonic Motion, Inference Using Normal And T-distributions

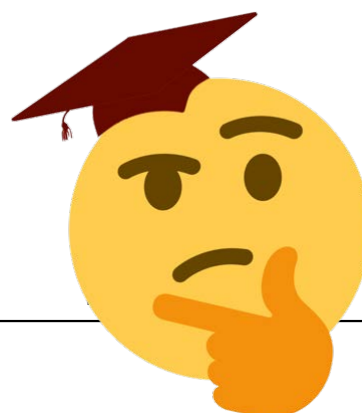
A(i)	EITHER: $40 e_0 / 0.8 = 2g, e_0 = 0.4$ [or $OP_0 = 1.2$] [m]	M1A1	Find extension e_0 [or OP_0] at equilibrium position P_0
	$2 d^2x/dt^2 = -40 (e_0 + x) / 0.8 + 2g$ or $= +40 (e_0 - x) / 0.8 - 2g$	M1A1	Use Newton's law at general point (e.g. <i>x</i> below or above P_0) (ignore LHS sign here only)
	OR: $2 d^2y/dt^2 = -40 (y - 0.8) / 0.8 + 2g$ $= -50 y + 60$	(M1A1)	Use Newton's law at general point in terms of $y = OP$ (ignore LHS sign here only)
	Take $x = y - 1.2$ to give	M1A1	Change variable to give standard form of SHM eqn
	$d^2x/dt^2 = -25x$	A1	Hence SHM (A0 if wrong sign or LHS unclear) (B1 only for stating SHM eqn. without proof)
	$T = 2\pi / \sqrt{(25)} = 2\pi/5$ AG	A1	Verify period <i>T</i> using $T = 2\pi/\omega$ with $\omega = \sqrt{(25)}$
	$OP_0 = 1.2$ [m]	B1	State OP_0 explicitly (may imply first M1 A1)
A(ii)	$0.4^2 = 25 (a^2 - 0.06^2)$	M1A1	Find amplitude <i>a</i> from $v^2 = \omega^2 (a^2 - x^2)$
	$a = \sqrt{(0.0064 + 0.0036)} = 0.1$ [m]	A1	
		3	
A(iii)	$40 e_1 / 0.8 = (2 + M) g,$ $[e_1 = 0.4 + 0.2 M]$ or $40(e_1 - e_0) / 0.8 = M g$ $[e_1 - e_0 = 0.2 M]$	M1A1	Find extension e_1 [or OP_1] at first equilibrium position P_1 or equate additional extension to <i>M</i> by Hooke's Law
	$e_1 - e_0 = a, M = a/0.2 = 0.5$	M1A1	Find <i>M</i> by relating e_0, e_1 and <i>a</i>
		4	



B(i)	$H_0: \mu_x = \mu_y, H_1: \mu_x > \mu_y$ (or in terms of μ_A, μ_B)	B1	State hypotheses (B0 for $\bar{x} \dots$)
	$s_x^2 = (14.1775 - 10.56^2/8) / 7 = 0.03404$ and $s_y^2 = (15.894 - 12.39^2/10) / 9 = 0.06031$ (to 3 s.f.)	M1A1	Estimate both population variances (allow biased here: 0.02979 and 0.05428)
	$s^2 = (7s_x^2 + 9s_y^2) / 16$ or $(14.1775 - 10.56^2/8 + 15.894 - 12.39^2/10) / 16$ $= 0.78109 / 16$ or 0.04882 or 0.22095 ² (to 3 s.f.)	M1A1	Find pooled estimate of common variance (M1 A1 for s_x^2 and s_y^2 may be implied here)
	$t_{0.05} = 1.337$ (to 3 s.f.)	*B1	State or use correct tabular t value
	$t = (1.32 - 1.239) / s\sqrt{(1/10 + 1/8)} = 0.773$	M1A1	Calculate value of t (or $-t$) (or can compare $\bar{y} - \bar{x} = 0.081$ with 0.140)
	$t < \text{tabular value}$ so claim not justified or A's not heavier than B's (AEF)	B1	Correct conclusion (FT on t , dep *B1)
	SC: $z = (1.32 - 1.239) / \sqrt{(s_x^2/8 + s_y^2/10)}$ $= 0.081 / \sqrt{(0.078)} = 0.799$	(B1)	SC: Implicitly taking s_x^2, s_y^2 as unequal popln. variances (may also earn first B1 M1 A1)
	$z < 1.282$ so claim is not justified (AEF)	B1	Comparison with $z_{0.05}$ and conclusion (FT on z ; max 5/9)
		9	
B(ii)	$\bar{x} = 1.28$ and $s^2 = 0.294 / 7 = 0.042$ (or 0.205^2)	M1	Find sample mean & estimate popn. var (allow M1 if $0.294 / 8$)
	$t = (1.28 - p) / \sqrt{(0.042/8)}$ (AEF)	M1A1	Find value of t
	$t > 1.415$ (< 1.40), $p < 1.28 - 0.1025$, $p_{\max} = 1.18$	M1A1	Find $p_{\max}$ by comparison with tabular value, here $t_{7, 0.9}$
		5	

10 - (9231/21_Summer_2019_Q2) - Simple Harmonic Motion

(i)	$v_B = 2v_A, v_B^2 = 4v_A^2, \omega^2(a^2 - 1^2) = 4\omega^2(a^2 - 3.5^2)$	M1	Find amplitude a : allow M1 for $v_A = 2v_B$
	$3a^2 = 48, a = [\pm] 4$ [m]	A1	
	$1 = a\omega^2, \omega = 1/\sqrt{a} [= 1/2]$	B1	Find ω from given maximum acceleration
	$v_O = a\omega = \sqrt{a} = 2$ [m s ⁻¹]	B1	Find speed v_O at O
		4	
(ii)	$\omega t_{AB} = \sin^{-1}(3.5/a) + \sin^{-1}(1/a)$ $= \sin^{-1} 0.875 + \sin^{-1} 0.25$	M1 A1	Find equation (AEF) for ωt_{AB} , combining t_A and t_B , using for example $x = a \sin \omega t$ allow sign errors for the M1
	$= 1.065 + 0.253$ (or $t_{AB} = 2.131 + 0.505$)	A1	or $x = a \cos \omega t$
	$t_{AB} = 2 \times 1.318 = 2.64$ [s]	A1	Hence find t_{AB}
	Alternative method for question 2(ii)		
	$\omega t_{AB} = \cos^{-1}(-1/a) - \cos^{-1}(3.5/a)$ $= \cos^{-1}(-0.25) - \cos^{-1} 0.875$ or $\pi - \cos^{-1} 0.25 - \cos^{-1} 0.875$ (AEF)	M1 A1	or $x = a \cos \omega t$
	$= 1.823 - 0.505$ (or $t_{AB} = 3.647 - 1.011$)	A1	
	$t_{AB} = 2 \times 1.318 = 2.64$ [s]	A1	Hence find t_{AB}
		4	



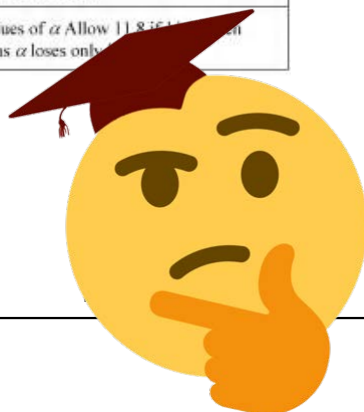
11 - (9231/21_Summer_2019_Q5) - Rotation Of A Rigid Body, Simple Harmonic Motion

(i)	$I_{rod} = \frac{1}{2} kMa^2$	B1	Find or state MI of rod AB about axis L .
	$I_{sphere} = \frac{2}{5} kM(2a)^2 + kM(3a)^2 \quad [= (35k/3) Ma^2]$	M1 A1	M1 for one term correct, A1 for both terms correct
	$I_{cog} = \frac{1}{2} \times Ma^2 + M(2a)^2 \quad [= (9/2) Ma^2]$	M1 A1	M1 for one term correct, A1 for both terms correct
	$I = (k/3 + 35k/3 + 9/2) Ma^2 = (3/2)(8k + 3) Ma^2 \quad \text{AG}$	A1	MI of object about axis L
		6	
(ii)	$[-] I d^2\theta/dt^2 = -kMg \times 3a \sin \theta + Mg \times 2a \sin \theta$ $[- = -(3k - 2) Mga \sin \theta]$	M1 A1	Use equation of circular motion to find $d^2\theta/dt^2$ where θ is angle of rod with vertical
	$d^2\theta/dt^2 = -[2g(3k - 2) / 3a(8k + 3)] \theta \quad \text{(AEF)}$	M1	Approximate $\sin \theta$ by θ to give standard form of SHM equation (M0 if wrong sign or $\cos \theta \approx \theta$ used)
	SHM if $3k - 2 > 0, k > 2/3$	M1 A1	Find possible values of k
	$T = 2\pi \sqrt{\{3a(8k + 3) / 2g(3k - 2)\}}$ or $\pi \sqrt{\{6a(8k + 3) / g(3k - 2)\}}$	A1	Find period T
		6	

12 - (9231/23_Summer_2019_Q11) - Simple Harmonic Motion, Inference Using Normal And T-distributions

E(i)	$\frac{1}{2} mu^2 = \frac{1}{2} kmg (\frac{1}{4} a)^2/a, k = 16u^2/ag$	AG	M1 A1 Verify k by using conservation of energy (M0 if ω is found and SHM formula $v = a\omega$ then used)
			2
E(ii)	$\pm m d^2x/dt^2 = kmg x/a$		*M1 Apply Newton's law at general point (e.g. $OP = a + x$), requires m
	$d^2x/dt^2 = -(kg/a) x$ or $-(16u^2/a^2) x [\omega = \sqrt{(kg/a)} \text{ or } 4u/a]$	A1	Derive standard SHM form (requires minus sign)
	Period is $2\pi\sqrt{(a/kg)} = 2\pi\sqrt{(a^2/16u^2)} = \pi a / 2u$	DM1 A1	Find period from $2\pi/\omega$
		4	
E(iii)	$[\frac{1}{2} mv^2 = \frac{1}{4} \times \frac{1}{2} mu^2] v^2 = \frac{1}{4} u^2 \text{ or } v = (\sqrt{3}/2) u$	M1	Relate v and u using given loss in energy
	$x = (\frac{1}{4} a) \sin \omega t, v = (\frac{1}{4} a) \omega \cos \omega t \text{ or } u \cos \omega t$	M1	Relate v and t
	$\cos \omega t = (\sqrt{3}/2) u / (\frac{1}{4} a) \omega = (\sqrt{3}/2)$	M1 A1	Combine to find $\cos \omega t$
	$t = (\pi/6) / (4u/a) = \pi a / 24u \text{ or } 0.0417 \pi a / u \text{ or } 0.131 a/u$	M1 A1	and hence t
		6	

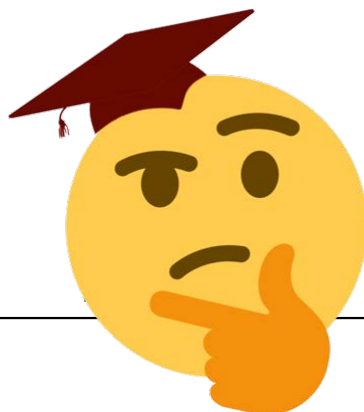
O	$\bar{x} - \bar{y} = 0.448 - 0.48 = [-] 0.032$	B1	Find difference in sample means (either sign; may be implied)
	$s_x^2 = (10.1 - 22.4^2/50) / 49 = 0.0648 / 49 \quad [= 0.001322]$	M1 A1	Estimate both population variances (may be implied)
	$s_y^2 = (16.3 - 28.8^2/60) / 59 = 2.476 / 59 \quad [= 0.04197]$	A1	(allow biased here: 0.001296 and 0.04127)
	$s_c^2 = s_x^2/50 + s_y^2/60$	M1	Estimate combined variance
	$= 0.0007259 \text{ or } 0.02694^2 \text{ (to 3 s.f. throughout)}$	A1	
	$z = 0.032 / s_c \quad \text{[or } 0.032 / s_p \sqrt{(1/50 + 1/60)}]$ $= 1.188 \quad \text{[or } 1.0895]$	M1 A1	Find value of z (either sign)
	$\Phi(z) = 0.8826 \quad \text{[or } 0.8620]$	A1	Find $\Phi(z)$
	$100 \times (1 - \Phi(z)) = 11.7 \text{ [or } 13.8]$	M1 A1	Find limiting value for α , based on one-tail test (M0 for basing on two-tail test)
	$\alpha > (\text{or } \gg) 11.7 \text{ [or } 13.8]$	A1	Find set of possible values of α Allow 11.8 if 11.7 given (Misreading $\alpha\%$ as α loses only 1 mark)



O	Alternative method for question 11O		
	$\bar{x} - \bar{y} = 0.448 - 0.48 = [-] 0.032$	B1	Find difference in sample means (either sign; may be implied)
	$s_x^2 = (10.1 - 22.4^2/50) / 49 = 0.0648 / 49$ [= 0.001322]	M1 A1	Estimate both population variances (may be implied)
	$s_y^2 = (16.3 - 28.8^2/60) / 59 = 2.476 / 59$ [= 0.04197]	A1	(allow biased here: 0.001296 and 0.04127)
	Assume equal [population] variances	B1	State assumption
	$s_p^2 = (49 s_x^2 + 59 s_y^2) / 108$ or $(0.0648 + 2.476) / 108$ = 0.02353 or 0.1534 ²	B1	(Find pooled estimate of common variance s_x^2 and s_y^2 not needed explicitly so may be implied by result)
	$z = 0.032 / s_p$ [or $0.032 / s_p \sqrt{(1/50 + 1/60)}$] = 1.188 [or 1.0895]	M1 A1	Find value of z (either sign)
	$\Phi(z) = 0.8826$ [or 0.8620]	A1	Find $\Phi(z)$
	$100 \times (1 - \Phi(z)) = 11.7$ [or 13.8]	M1 A1	Find limiting value for α , based on one-tail test (M0 for basing on two-tail test)
	$\alpha > (or \geq) 11.7$ [or 13.8]	A1	Find set of possible values of α Allow 11.8 if 11.74 seen (Misreading $\alpha\%$ as α loses only last A1)
		12	

13 - (9231/21_Winter_2019_Q5) - Rotation Of A Rigid Body, Simple Harmonic Motion

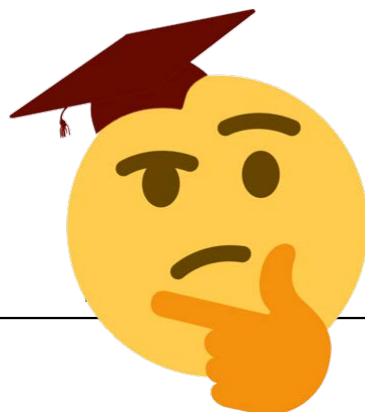
(i)	$I_{rot} = \frac{1}{3} \lambda M a^2 + \lambda M (a/2)^2$ [= (7/12) $\lambda M a^2$]	B1	Find or state MI of rod AB about axis L
	$I_O = \frac{2}{3} \lambda M a^2 + 3M (5a/2)^2$ [= (83/4) $M a^2$]	M1 A1	Find MI of hollow sphere centre O about axis L
	$I_C = (2/5) 5M a^2 + 5M (3a/2)^2$ [= (53/4) $M a^2$]	M1 A1	Find MI of solid sphere centre C about axis L
	$I = (7\lambda/12 + 83/4 + 53/4) M a^2$ = ((7 λ + 408) / 12) $M a^2$	A1	Verify MI of object about axis L. AG
		6	
(ii)	$[-] I d^2\theta/dt^2 = [-3Mg \times (5a/2) \sin \theta + 5Mg \times (3a/2) \sin \theta]$ = $\lambda Mg \times (a/2) \sin \theta$	M1 A1	Use eqn of circular motion to find $d^2\theta/dt^2$ where θ is angle of rod with vertical. AEF
	$d^2\theta/dt^2 = - \{6g\lambda / (7\lambda + 408)a\} \theta$	M1*	Approximate $\sin \theta$ by θ to give standard form of SHM eqn
	$T = 2\pi \sqrt{\{(7\lambda + 408)a / 6g\lambda\}} = 5\pi\sqrt{2a/g}$	DM1A1	Find possible values of λ by equating period T to $5\pi\sqrt{2a/g}$. AEF
	$\lambda = 6$	A1	
		6	



14 - (9231/21_Winter_2019_Q11) - Simple Harmonic Motion, X2 Test

A(i)	$10 (AP - 0.6) / 0.6 = 20 (1.2 - AP - 0.4) / 0.4$	M1 A1	Verify AP by equating equilibrium tensions. AEF
	$4 AP - 2.4 = 9.6 - 12 AP$ $AP = 0.75 \text{ [m]}$	A1	AG
		3	
A(ii)	$m \frac{d^2x}{dt^2} = -10 (0.15 + x) / 0.6 + 20 (0.05 - x) / 0.4$ or $m \frac{d^2x}{dt^2} = +10 (0.15 - x) / 0.6 - 20 (0.05 + x) / 0.4$	M1 A1 A1	Apply Newton's law at $0.75 + x$ or $0.75 - x$ from A (M1 requires LHS and 2 tensions: A1 for each correct tension)
	$\frac{1}{5} \frac{d^2x}{dt^2} = -(80 / 1.2)x, \frac{d^2x}{dt^2} = -100x$	M1 A1	Simplify to give SHM eqn. in standard form
	$T = 2\pi/\omega = 2\pi/10 = \pi/5$ or 0.628 [s]	DB1	State the period T with FT on ω from SHM eqn.
A(iii)	$(a = 0.75 - 0.7 = 0.05)$ $v_{\max} = \omega \times a$	M1	Find speed at equilibrium position from ωa
	$v_{\max} = 0.5 \text{ [m s}^{-1}\text{]}$	A1	
		2	
A(iv)	$x = a/2 = 0.025$	M1	Find value of x giving half max. acceln.
	$v = \omega \sqrt{(a^2 - x^2)} = 10 \sqrt{(0.05^2 - 0.025^2)}$	M1	
	$v = 0.433 \text{ [m s}^{-1}\text{]}$	A1	Find corresponding speed
		3	

B(i)	$\bar{x} = (1/40) \sum x f(x) = 68/40 = 1.7$	B1	Find mean of sample
	$(1/40) \sum x^2 f(x) = 178/40 = 4.45, \text{ Var} = 4.45 - 1.7^2 = 1.56$	B1	Find variance of sample
	Mean and variance are similar so Poisson may be suitable	B1	State valid comment
		3	
B(ii)	$a = 40 \times 1.6^5 e^{-1.6} / 5! = 40 \times 0.01764$	M1	AG
	$a = 0.706$	A1	Verify a from Poisson term
	$b = 40 - 39.758 = 0.242$	B1*	Find b
		3	
B(iii)	H_0 : Distribution fits/models data	B1	State (at least) null hypothesis in full
	O_i : 6 15 9 10 E_i : 8.076 12.921 10.337 8.666	DM1 A1	Combine values consistent with all exp. values ≥ 5 (FT on b , dep B1*)
	$\chi^2 = 0.5337 + 0.3345 + 0.1729 + 0.2053 = 1.25$	M1	Find value of χ^2 from $\sum (E_i - O_i)^2 / E_i$ [or $\sum O_i^2 / E_i - n$]
	$\chi^2 = 1.25$	A1	
	No. n of cells: 7 6 5 4 3 $\chi_{0.1, 0.9}^2$: 10.64 9.236 7.779 6.251 4.605	DB1	State or use consistent tabular value $\chi_{0.1, 0.9}^2$ (to 3 s.f.) [FT on number, n , of cells used to find χ^2]
	Accept H_0 if $\chi^2 < \text{tabular value}$ (using their values)	M1	AEF
	$1.25 [\pm 0.1] < 6.25$ so distrn. fits [data] or distrn. is a suitable model	A1	Conclusion (requires both values approx. correct). AEF
		8	



15 - (9231/31_Summer_2020_Q3) - Simple Harmonic Motion

(a)	$T - mg = m.a$	M1
	$T = 5mg, \frac{1}{2}a / a = \frac{5}{2}mg$	M1
	$a = \frac{3}{2}g$ (upwards) AG	A1
		3
(b)	Gain in KE = $\frac{1}{2}mv^2$ Gain in GPE = $\frac{1}{2}mga$	B1
	Loss in EPE = $\frac{1}{2} \frac{5mg \cdot \left(\frac{1}{2}a\right)^2}{a}$	B1
	$\frac{1}{2}mv^2 + \frac{1}{2}mga = \frac{1}{2} \frac{5mg \cdot \left(\frac{1}{2}a\right)^2}{a} \quad [\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}mga = \frac{5}{8}mga]$	M1
	$v = \frac{1}{2}\sqrt{ga}$	A1
		4

16 - (9231/33_Summer_2020_Q7) - Simple Harmonic Motion

(a)	Collision: $(k+1)mv = mu$ so $v = \frac{u}{k+1}$	B1
	Loss in KE = $\frac{1}{2}m(k+1)\left[v^2 - \frac{1}{4}v^2\right]$	B1
	Gain in EPE = $\frac{1}{2} \cdot 4mg/a \cdot \left(\frac{a}{4}\right)^2$	B1
	So, $\frac{1}{2} \cdot \frac{3}{4}m(k+1) \frac{u^2}{(k+1)^2} = \frac{mga}{8}$	M1
	Use value of u and solve	M1
	$k = 3$	A1
		6
(b)	$T + F = 4m \cdot \frac{9g}{20}$	M1
	$4mg \cdot \frac{1}{a} + F = \frac{9mg}{5}$	M1
	$\mu R = 4mg / 5$ and compare with $R = 4mg$	B1
	$\mu = \frac{1}{5}$	A1
		4

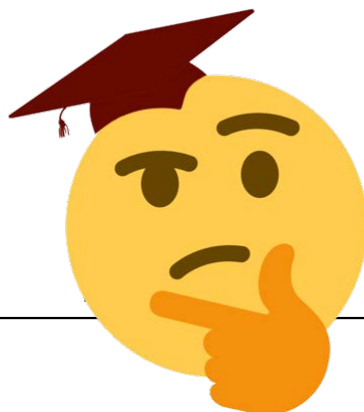


17 - (9231/33_Winter_2021_Q3) - Simple Harmonic Motion

Loss in EPE = $\frac{1}{2} \times \frac{12mge^2}{a} - \frac{1}{2} \times \frac{12mg}{a} \times \left(e - \frac{a}{3}\right)^2 \left(= \frac{2mg}{3} (6e - a) \right)$	B1	Either term correct.
Gain in KE = $\frac{1}{2}mv^2$ and Gain in GPE = $\frac{mga}{3}$	B1	
Gain in KE + Gain in GPE = Loss in EPE	M1	KE, GPE and at least one EPE term.
$\frac{1}{2}mv^2 + \frac{mga}{3} = \frac{2mg}{3}(6e - a)$	A1	All terms correct.
Simplify to a linear equation in e.	M1	
$e = \frac{1}{2}a$	A1	
	6	

18 - (9231/32_Winter_2021_Q7) - Simple Harmonic Motion

(a)	Coordinates of A: $x = a \sin 60$, $y = a - a \cos 60$	B1	
	$\frac{a}{2} = \frac{a\sqrt{3}}{2}\sqrt{3} - \frac{g \left(\frac{a\sqrt{3}}{2}\right)^2}{2V^2 \cdot \frac{1}{4}}$	M1	Substitute <i>their</i> (x, y) into correct trajectory equation.
	Rearrange to find V^2 .	M1	
	$V^2 = \frac{3}{2}ag$, $V = \sqrt{\frac{3}{2}ag}$	A1	
		4	
(b)	$\frac{1}{2}mu^2 - \frac{1}{2}mV^2 = mga(1 + \cos 60)$	M1	Energy equation.
	$u^2 = \frac{9}{2}ag$	A1	u is the speed at P.
	$T - mg = \frac{m}{a}u^2$	M1	N2L
	$T = \frac{11}{2}mg$	A1	
		4	

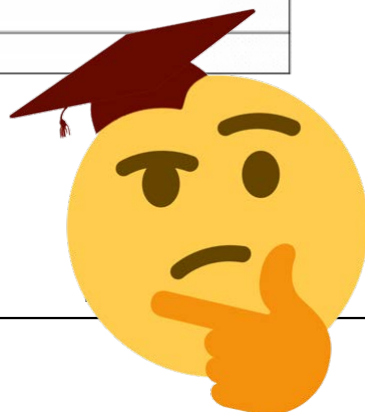


19 - (9231/31_Winter_2022_Q5) - Simple Harmonic Motion

(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgo \cos \theta$ $\left[kag = \frac{1}{3}ag + 2ag \cos \theta \right]$	B1	Energy equation.
	$T - mg \cos \theta = \frac{mv^2}{a}$ So $\frac{11}{6}mg - mg \cos \theta = \frac{m}{a}kag, \frac{11}{6} - \cos \theta = k$	B1	N2L at B.
	Solve simultaneously.	M1	
	$k = \frac{4}{3}, \cos \theta = \frac{1}{2}$	A1	Both.
		4	
(b)	Initial speed $\uparrow = \sqrt{kag} \sin \theta$	B1	
	Use $v^2 = u^2 + 2as; 0 = (\sqrt{kag} \sin \theta)^2 - 2gs$	M1	
	$s = \frac{1}{2}a$	A1	
	Height above lowest point $= s + a - a \cos \theta = \frac{1}{2}a + a - \frac{1}{2}a = a$	A1 FT	
		4	

20 - (9231/32_Winter_2022_Q6) - Simple Harmonic Motion

(a)	$T = \frac{6}{7}mg$	B1	May be implied.
	$T \sin \theta = mr\omega^2 = mh \tan \theta \times \omega^2$	B1	Allow r for radius.
	Radius of circle $= h \tan \theta$ [So $\omega^2 = \frac{6g}{7h} \cos \theta$]	B1	
	In second scenario, $\frac{9}{4}\omega^2 = \frac{6g}{7h} \cos \alpha$	M1	Second scenario, equivalent result.
	Equate, $\frac{6g}{7h} \cos \theta = \frac{4}{9} \times \frac{6g}{7h} \cos \alpha$ giving $\cos \theta = \frac{4}{9} \cos \alpha$	A1	Combine convincingly to obtain given result. AG
		5	
(b)	First scenario: $N + T \cos \theta = mg$		
	Second scenario, $\frac{1}{2}N + T \cos \alpha = mg$	B1	Both.
	Equate: $mg - \frac{6}{7}mg \cos \theta = 2mg - \frac{12}{7}mg \cos \alpha$	M1	$12 \cos \alpha - 6 \cos \theta = 7$
	$\cos \alpha = \frac{3}{4}$	A1	
	$N = \frac{5}{7}mg$	A1	
		4	

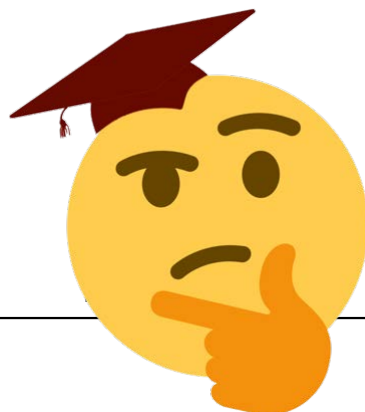


21 - (9231/31_Summer_2023_Q1) - Simple Harmonic Motion, Hook's Law

(a)	$\frac{3mg}{2a}(2a)^2$	B1	Correct EPE term seen
	$\frac{1}{2}mv^2 + mg \times \left(3a - \frac{3}{4}a\right) = \frac{3mg}{2a}(2a)^2$	M1	Dimensionally correct energy equation. Must have one KE, one EPE term and at least one GPE. Allow sign errors.
	$v = \sqrt{\frac{15}{2}ag} \quad [2.74\sqrt{ag}]$	A1	AEF
		3	
(b)	$T - mg = mA$ and $T = \frac{3mg}{a} \times 2a$	M1	N2L and Hooke's law
	Acceleration = $5g$ [upwards]	A1	Allow ± 50 or $\pm 5g$
		2	

22 - (9231/33_Summer_2023_Q1) - Circular Motion, Simple Harmonic Motion, Hook's Law

$\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(\cos\alpha + \cos\theta)$	M1	Energy equation, 4 terms, dimensionally correct, mass must be present, allow sign errors, allow sin in both terms on RHS
$mg \cos\theta = \frac{mv^2}{a}$	B1	N2L, may include tension initially but not awarded until tension = 0 used
$\frac{3}{2}mag - \frac{1}{2}mag \cos\theta = mga\left(\frac{4}{5} + \cos\theta\right)$ $\frac{3}{2}\cos\theta = \frac{7}{10}$	M1	Dependent on tension = 0 and on an energy equation, eliminate v^2 .
$\cos\theta = \frac{7}{15}$	A1	If no m in energy equation and no further errors, award SCB2 for correct final answer
	4	



1 - (9231/31_Summer_2020_Q1) - Projectile Motion

For greatest height, $T = \frac{u}{2g}$	B1
At $t = \frac{2T}{3}$, $\uparrow v_v = \frac{u}{2} - \frac{2Tg}{3} = \frac{u}{6}$	M1
$\rightarrow v_h = \frac{u\sqrt{3}}{2}$	A1
Speed = $\sqrt{v_v^2 + v_h^2} = \sqrt{\frac{u^2}{36} + \frac{3u^2}{4}}$	M1
$= \frac{\sqrt{7}}{3}u$	A1
	5

2 - (9231/33_Summer_2020_Q6) - Projectile Motion

(a)	Greatest height = $\frac{(u \sin \theta)^2}{2g}$	M1A1
	At $\frac{1}{4}$ greatest height, $\rightarrow v \cos \alpha = u \cos \theta$	M1
	At $\frac{1}{4}$ greatest height, $\uparrow (v \sin \alpha)^2 = (u \sin \theta)^2 - 2g \cdot \frac{3}{4} \frac{(u \sin \theta)^2}{2g}$	M1
	$v \sin \alpha = \frac{1}{2}u \sin \theta$	A1
	So $\tan \alpha = \frac{1}{2} \tan \theta$ AG	A1
		6

(b)	$\tan \alpha = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$ $\uparrow \frac{1}{2}u \sin \theta = u \sin \theta - gt$	M1
	$t = \frac{2u}{5g}$	A1
	$\rightarrow t = d / u \cos \theta$	M1
	$= \frac{6u^2}{25g}$	A1
		4

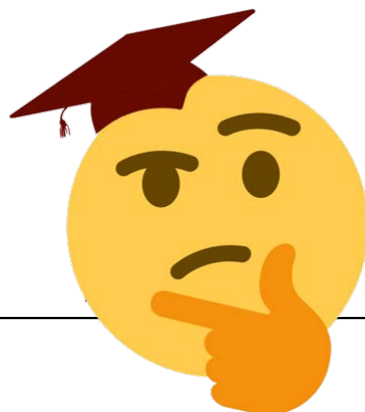


3 - (9231/31_Winter_2020_Q5) - Projectile Motion

(a)	$\rightarrow x = u \cos \alpha t$ $\uparrow y = u \sin \alpha t - \frac{1}{2}gt^2$	B1	Both
	Eliminate t : $y = u \sin \alpha \cdot \frac{x}{u \cos \alpha} - \frac{1}{2}g \left(\frac{x}{u \cos \alpha} \right)^2$	M1	Eliminate
	$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha$	A1	AG
		3	
(b)	Greatest height = $\frac{(u \sin \alpha)^2}{2g} = \frac{u^2}{4g}$	M1 A1	Accept alternative methods, for example differentiate expression in (a) and equate to 0.
	$t = u \sin 45 / g$ so $d = u \cos 45 \cdot u \sin 45 / g = \frac{u^2}{2g}$	A1	AG
		3	
(c)	Use greatest height displacements in trajectory equation $\frac{u^2}{4g} = \frac{u^2}{2g} \tan \alpha - \frac{gu^4}{2u^2 4g^2} \sec^2 \alpha$	M1	Use equation of trajectory (substitute coordinates of Q)
	$u^2 = 2u^2 \tan \alpha - \frac{u^2}{2} (1 + \tan^2 \alpha)$	M1	Use of $\sec^2 \alpha = (1 + \tan^2 \alpha)$
	$\tan^2 \alpha - 4 \tan \alpha + 3 = 0$	M1	Obtain a three-term quadratic in $\tan \alpha$
	$\tan \alpha = 1, 3$ so $\alpha = 71.6^\circ$	A1	Both solutions needed
		4	

4 - (9231/32_Winter_2020_Q5) - Projectile Motion

(a)	Quote trajectory equation from MF19 and use $\cos \theta = 1 / \sec \theta$ $y = x \tan \theta - \frac{gx^2}{2u^2} (1 + \tan^2 \theta)$	B1	Must include step with $\sec^2 \theta$ Allow derived from first principles AG
		1	
(b)	$16 = 20 - \frac{10 \times 100}{2u^2} (1 + 4)$	M1	Substitute into result (a)
	$u^2 = 625, (u = 25)$	A1	
	Use equation again: $30 = 18 \tan \theta - \frac{10 \times 324}{2 \times 625} (1 + (\tan \theta)^2)$	M1	
	$2.592(\tan \theta)^2 - 18 \tan \theta + 32.592 = 0$	A1	3 term quadratic. Alternatives include: $54t^2 - 375t + 679 = 0$, $324t^2 - 2250t + 4074 = 0$
	Discriminant = $324 - 4 \times 2.592 \times 32.592 = -13.91$	M1	Discriminant for alternatives: -6039 and -217404
	As this is less than 0, no real solutions for θ	A1	CWO
		6	



5 - (9231/31_Summer_2021_Q7) - Projectile Motion

(a)	At greatest height $0 = 100 \sin \theta - gt$	M1	
	$t = 8$	A1	
	Therefore times at height H are $t = 3$ (and $t = 13$)	B1	
	Substitute into $H = 100 \sin \theta t - \frac{1}{2}gt^2$	M1	
	$H = 195$	A1	
	Alternative method to question 7(a)		
	$\uparrow H = 100 \sin \theta t - \frac{1}{2}gt^2$	M1	
	And $H = 100 \sin \theta(t+10) - \frac{1}{2}g(t+10)^2$	A1	
	Subtract: $1000 \sin \theta = \frac{1}{2}g(20t+100)$	M1	
	$t = 3$	B1	
	$H = 195$	A1	
(a)	Alternative method to question 7(a)		
	$\uparrow H = 100 \sin \theta t - \frac{1}{2}gt^2$	B1	
	Difference between roots = $\frac{\sqrt{(100 \sin \theta)^2 - 2gH}}{\frac{1}{2}g}$	M1 A1	
	Equate to 10 and rearrange to find H	M1	
	$H = 195$	A1	
		5	
(b)	Time to required point = 15 s	B1	
	$\uparrow v = 100 \sin \theta - 10 \times 15 (= -70)$ $\rightarrow v = 100 \cos \theta = 60$	B1	Both components.
	Magnitude = 92.2	B1	
	Angle below horizontal = $\tan^{-1}(70/60) = 49.4^\circ$	B1	
		4	



6 - (9231/33_Summer_2021_Q7) - Projectile Motion

(a)	$y = 0$ in trajectory equation: $R \tan \theta - g \frac{R^2}{2u^2 (\cos \theta)^2} = 0$	M1	
	$(R =) \frac{2u^2 \sin \theta \cos \theta}{g}$ only	A1	Any equivalent single term expression, for example: $\frac{u^2 \sin 2\theta}{g}$, $\frac{2u^2 \tan \theta}{g \sec^2 \theta}$, at least one intermediate line of working, not just quoting a result. SC B1 using SUVAT.
		2	
(b)	$x = \text{their } \frac{u^2 \sin \theta \cos \theta}{g}$ and substitute in trajectory equation.	M1	Or use SUVAT.
	$H = \frac{u^2 (\sin \theta)^2}{2g}$	A1	Single term.
		2	
(c)	Use $R = \frac{4H}{\sqrt{3}}$ and simplify: $\tan \theta = \sqrt{3}$, $\theta = 60^\circ$	B1	AG
		1	
(d)	$\frac{dy}{dx} = \tan \theta - \frac{gx}{u^2 (\cos \theta)^2}$	M1	Differentiate with respect to x.
	$\tan \theta - \frac{x}{4(\cos \theta)^2} = \pm 1$ used	M1	Use $\frac{dy}{dx} = \pm 1$ as limiting case.
	$x = \sqrt{3} + 1$, $x = \sqrt{3} - 1$	A1	
	$\sqrt{3} - 1 < x < \sqrt{3} + 1$	A1	Strict inequality, exact values.
	Alternative method for question 7(d)		
(d)	$y = \sqrt{3}x - \frac{1}{2}x^2$, $\frac{dy}{dx} = \sqrt{3} - x$	M1	Differentiate with respect to x.
	$\frac{dy}{dx} = \pm 1$ used	M1	Use $\frac{dy}{dx} = \pm 1$ as limiting case.
	$x = \sqrt{3} + 1$, $x = \sqrt{3} - 1$	A1	
	$\sqrt{3} - 1 < x < \sqrt{3} + 1$	A1	Strict inequality, exact values.
	Alternative method for question 7(d)		
(d)	When moving at 45° to horizontal, $v_x = \pm v_y$	M1	Used, both cases considered.
	$v_x = \sqrt{40} \cos \theta$, $v_y = \sqrt{40} \sin \theta - 10t$ $t = \frac{1}{10}(\sqrt{30} - \sqrt{10})$, $t = \frac{1}{10}(\sqrt{30} + \sqrt{10})$	M1	
	$x = \sqrt{3} + 1$, $x = \sqrt{3} - 1$	A1	
	$\sqrt{3} - 1 < x < \sqrt{3} + 1$	A1	Strict inequality, exact values.
		4	

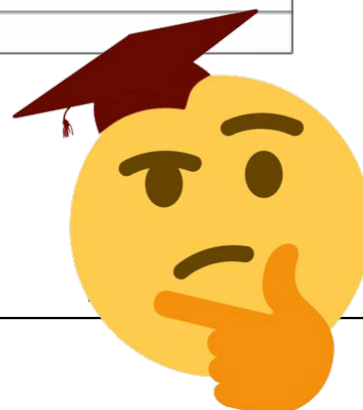


7 - (9231/33_Summer_2022_Q3) - Projectile Motion

(a)	Components of velocity : $\rightarrow 25 \cos \theta$ $\uparrow 25 \sin \theta - 2g$	B1	
	Speed = $\sqrt{(25 \cos \theta)^2 + (25 \sin \theta - 2g)^2}$	M1 A1	Expression for speed or square of speed.
	$(25 \cos \theta)^2 + (25 \sin \theta - 2g)^2 = 15^2$ $625 - 100g \sin \theta + 4g^2 = 225$	M1	Attempt to solve and find value for $\sin \theta$
	$\sin \theta = \frac{800}{1000} = \frac{4}{5}$	A1	
		5	
(b)	Time of flight = $\left(\frac{2 \times 25 \sin \theta}{g} \right) = 4 \text{ (s)}$	B1	
	Range = $\frac{2 \times 25 \sin \theta}{g} \times 25 \cos \theta$	M1	Any equivalent method.
	Range = 60 (m)	A1	CWO
	Alternative method for question 3(b)		
	$y = \frac{4}{3}x - \frac{1}{45}x^2$	B1	Equation of trajectory..
	Substitute $y = 0$ and solve	M1	
	60 (m)	A1	
		3	

8 - (9231/32_Winter_2022_Q5) - Projectile Motion

(a)	$\rightarrow x = u \cos \theta t$	B1	Result quoted from MF19 scores 0/4.
	$\uparrow y = u \sin \theta t - \frac{1}{2}gt^2$	B1	
	Eliminate t : $y = u \sin \theta \times \frac{x}{\cos \theta} - \frac{1}{2}g \left(\frac{x}{u \cos \theta} \right)^2$	M1	
	$y = x \tan \theta - \frac{gx^2}{2u^2} (1 + \tan^2 \theta)$	A1	Must be an intermediate line of working. AG
		4	
(b)	$20 = 30 \times \frac{4}{3} - 10 \times \frac{30^2}{2u^2} \times \left(1 + \left(\frac{4}{3} \right)^2 \right)$	M1	Substituting values correctly.
	$u^2 = 625, [u = 25]$	A1	
	Substitute back into trajectory equation. $20 = 30 \tan \theta - \frac{g30^2}{2.25^2} \sec^2 \theta = 30 \tan \theta - \frac{36}{5}(1 + \tan^2 \theta)$ $18 \tan^2 \theta - 75 \tan \theta + 68 = 0$	M1	Obtain a 3-term quadratic.
	One solution is $\frac{4}{3}, (3 \tan \theta - 4)(6 \tan \theta - 17) = 0$	M1	
	Giving $\tan \theta = \frac{17}{6}$	A1	
		5	

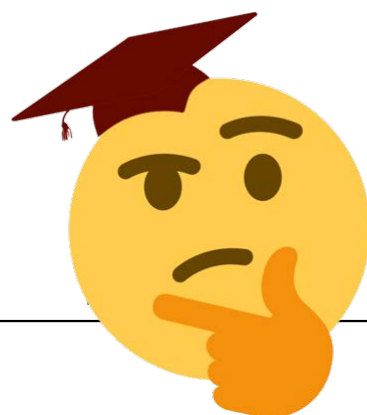


9 - (9231/31_Winter_2022_Q7) - Projectile Motion

(a)	$\uparrow 0 = V \sin 75^\circ t - \frac{1}{2} g t^2$	M1	
	$t = \frac{2V}{g} \sin 75^\circ$	A1	AG
		2	
(b)	Vertical component of velocity is unchanged.	B1	
		1	
(c)	Horizontally to wall, $\rightarrow 15 = V \cos 75^\circ; \left(t = \frac{15}{V \cos 75^\circ} \right)$	B1	
	$eV \cos 75^\circ$	B1	Speed after rebound.
	$T = \frac{15}{eV \cos 75^\circ}$	M1	Time back to O $\left(t = \frac{3}{5} T \right)$
	Vertically for whole flight: $t + T = \frac{2V}{g} \sin 75^\circ$	M1	
	$\frac{15}{V \cos 75^\circ} + \frac{15}{eV \cos 75^\circ} = \frac{2V}{g} \sin 75^\circ$	A1	
	$V^2 \cos 75^\circ \sin 75^\circ = 20g$	M1	
	Multiply by 2: $V^2 \sin 150^\circ = 40g$, $V^2 = 80g$ $V = 4\sqrt{5g} (= 8.94\sqrt{g})$	A1	
		7	

10 - (9231/31_Winter_2023_Q2) - Projectile Motion

(a)	$2 \frac{dv}{dt} = 2g - 0.2v^2$	B1	
	Separate variables and attempt to integrate: $\frac{dv}{0.1(100 - v^2)} = dt$	M1	Integrate to a ln term of the correct form.
	$\frac{1}{20} \ln \left(\frac{10+v}{10-v} \right) = 0.1t + c$	A1	
	$t = 0, v = 5, c = \frac{1}{20} \ln 3$	M1	Use initial condition.
	$2t = \ln \frac{10+v}{3(10-v)}, e^{2t} = \frac{10+v}{3(10-v)}$	M1	Rearrange, removing ln.
	$v = \frac{30 - 10e^{-2t}}{3 + e^{-2t}}$	A1	AEF
		6	
(b)	$v \rightarrow 10$	B1FT	FT from expression of correct form.
		1	

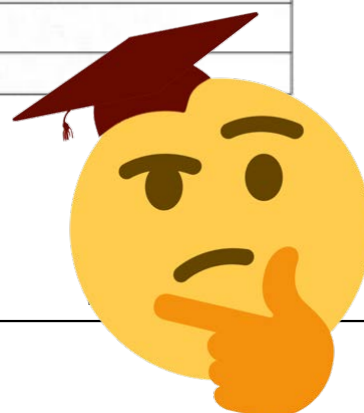


11 - (9231/31_Winter_2023_Q5) - Projectile Motion

(a)	Use equation of trajectory: $y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha$; $\frac{3}{8}a = 3a \times \frac{1}{3} - g \times \frac{(3a)^2}{2u^2} \left(1 + \frac{1}{9}\right)$	M1	
	$\frac{5}{8}a = \frac{5ga}{u^2}$, $u^2 = 8ga$	A1	At least one step of working. AG
		2	
(b)	For P, time of flight $T = \frac{2u \sin \theta}{g}$	B1	
	For P, range = $Tu \cos \theta$	B1	
	For Q, time of flight = $\frac{1}{2}T$, so range = $\frac{1}{2}TV \cos \alpha$ Equate: $V \cos \alpha = 2u \cos \theta$ (1)	B1	
	For Q vertically: $0 = V \sin \alpha \times \frac{1}{2}T - \frac{1}{2}gT^2$, so $T = \frac{4V \sin \alpha}{g}$	M1	
	Equate with result for P; $\frac{4V \sin \alpha}{g} = \frac{2u \sin \theta}{g}$ so $V \sin \alpha = \frac{1}{2}u \sin \theta$ (2)	A1	
	From (1) and (2): $V^2 = u^2 \left(4(\cos \theta)^2 + \frac{1}{4}(\sin \theta)^2\right)$	M1	
	$V^2 = u^2 \times \left(4 \times \frac{9}{10} + \frac{1}{4} \times \frac{1}{10}\right) = \frac{145}{40}u^2$ $V = \sqrt{29ag}$	A1	
		7	

12 - (9231/31_Winter_2023_Q6) - Projectile Motion

(a)	Angular speeds of P and Q are equal, so $\frac{v_Q}{x} = \frac{v_P}{3a-x}$	M1	
	$v_Q = \frac{2x\sqrt{ag}}{3a-x}$	A1	Shown convincingly; angular speeds equal stated. AG
		2	
(b)	For P: $T + mg \cos 60^\circ = \frac{m \times 4ag}{3a-x}$	B1	
	For Q: $T - mg \cos 60^\circ = \frac{mv_Q^2}{x}$	B1	
	Eliminate T: $-mg \cos 60^\circ + \frac{m \cdot 4ag}{3a-x} = mg \cos 60^\circ + \frac{mv_Q^2}{x}$	M1	
	$\frac{m \times 4ag}{3a-x} = 1 + \frac{mx \cdot 4ag}{(3a-x)^2}$ $4a(3a-x) = (3a-x)^2 + 4ax$, $x^2 + 2ax - 3a^2 = 0$	M1	Solve to find x . Obtain 3-term quadratic equation.
	$(x-a)(x+3a) = 0$, $x = a$	A1	
		5	



(c)	Energy changes from initial position: Gain in KE of P : $\frac{1}{2}m(4ag - u^2)$ Loss in KE of Q : $\frac{1}{2}m\left(\left(\frac{u}{2}\right)^2 - v_0^2\right)$ Loss in GPE of $P = mg(3a - x)(1 - \cos 60^\circ) (= mga)$ Gain in GPE of $Q = mgx(1 - \cos 60^\circ) \left(= \frac{1}{2}mga\right)$	B1	KEs correct.
		B1FT	GPEs correct.
	$\frac{1}{2}m(4ag - u^2) - \frac{1}{2}m\left(\left(\frac{u}{2}\right)^2 - v_0^2\right) = -mgx(1 - \cos 60^\circ) + mg(3a - x)(1 - \cos 60^\circ)$	M1	Energy equation.
	Simplify: $4ag - \frac{5}{4}u^2 + ag = ag$ $u^2 = \frac{16}{5}ag, \quad u = \frac{4}{5}\sqrt{5ag}$	A1	AEF
		4	

13 - (9231/32_Winter_2023_Q6) - Projectile Motion

(a)	$\rightarrow x = u \cos \alpha t$ $\uparrow y = u \sin \alpha t - \frac{1}{2}gt^2$	B1	Both correct.
	Eliminate t : $y = u \sin \alpha \times \frac{x}{u \cos \alpha} - \frac{1}{2}g\left(\frac{x}{u \cos \alpha}\right)^2$	M1	Eliminate.
	$y = x \tan \alpha - \frac{gx^2}{2u^2} \sec^2 \alpha \quad \text{AG}$	A1	
		3	
(b)	$\frac{4}{5}h = 32 \tan \alpha - \frac{g32^2}{2 \times 40^2} \sec^2 \alpha \quad [= 32 \tan \alpha - \frac{16}{5} \sec^2 \alpha]$	B1	
	$h = 32 \tan \alpha - \frac{g32^2}{2(40\sqrt{2})^2} \sec^2 \alpha \quad [= 32 \tan \alpha - \frac{8}{5} \sec^2 \alpha]$	B1	
	$32 \tan \alpha - \frac{8}{5} \sec^2 \alpha = \frac{5}{4} \left(32 \tan \alpha - \frac{16}{5} \sec^2 \alpha \right)$ $32t - \frac{8}{5}(1+t^2) = 40t - 4(1+t^2)$ $3t^2 - 10t + 3 = 0$	M1	Equate expressions for h and obtain a 3-term quadratic in $\tan \alpha$.
	$t = 3, \quad \frac{1}{3}$	A1	Both correct
	$h = 80$ $h = \frac{80}{9}$	M1	For using their value of t to work out one value of h .
		A1	Both correct
		6	



1 - (9231/33_Summer_2020_Q2) - Linear Motion Under Variable Force

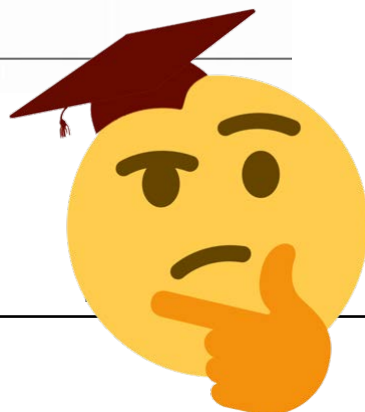
$\frac{dv}{g - kv} = dt$	M1
$-\frac{1}{k} \ln(g - kv) = t + A$	M1
$t = 0, v = 0: A = -\frac{1}{k} \ln g$	A1
$t = -\frac{1}{k} \ln \frac{g - kv}{g}$	M1
$v = \frac{g}{k} (1 - e^{-kt})$	M1A1
	6

2 - (9231/31_Summer_2020_Q5) - Linear Motion Under Variable Force

(a)	$\frac{dv}{3u - v} = k dt$	M1
	$-\ln(3u - v) = kt + d$ $t = 0, v = u: d = -\ln 2u$	M1
	$v = 2u: t = \frac{1}{k} \ln 2$	A1
		3
(b)	$v \frac{dv}{dx} = 3ku - kv \quad [\Rightarrow \frac{v dv}{3u - v} = k dx]$	B1
	$\frac{-(3u - v) + 3u}{3u - v} dv = k dx$ so $-v - 3u \ln(3u - v) = kx + c$	M1A1
	$x = 0, v = u: c = -u - 3u \ln 2u$	M1
	$v = 2u: x = \frac{u}{k} (3 \ln 2 - 1)$	A1
		5

3 - (9231/31_Winter_2020_Q7) - Linear Motion Under Variable Force

(a)	$v \frac{dv}{dx} = -\frac{100}{x^2} + \frac{200}{x^2}$ $\frac{v^2}{2} = \frac{50}{x^2} - \frac{200}{x} + A$ $x = 1, v = -10: A = 200$	M1 A1	Correct equation and attempt to integrate Correct
	$v^2 = \frac{100(2x - 1)^2}{x^2}$	M1	Rearrange to find v^2
	$v = \pm \frac{10(2x - 1)}{x}$ and take negative sign to meet initial condition, so $v = \frac{10(1 - 2x)}{x}$	A1	Convincingly shown (no mention of $\pm$ scores A0) AG
		5	



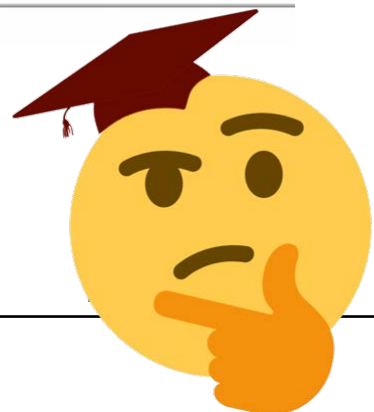
(b)	$\frac{x dx}{1-2x} = 10 dt$ $\frac{1}{2} \left(\frac{1}{1-2x} - 1 \right) dx = 10 dt$ $-\frac{1}{4} \ln 1-2x - \frac{x}{2} = 10t + B$	M1 A1	Rearrange and attempt to integrate
	$t = 0, x = 1: B = -\frac{1}{2}$	M1	Use initial condition
	$2x - 2 = -40t - \ln 1-2x $ so $e^{-40t} = (2x-1)e^{2x-2}$	A1	Convincingly shown, working required AG
	For large values of t , $x \rightarrow \frac{1}{2}$	B1	CAO
		5	

4 - (9231/32_Winter_2020_Q7) - Linear Motion Under Variable Force

(a)	$mv \frac{dv}{dx} = -kmv^2$	B1	N2L, with m
	$\ln v = -kx + c$	M1	Separate variables and integrate
	$x = 0, v = u: c = \ln u$	M1	Use initial condition
	$v = \frac{1}{2}u: \ln \frac{1}{2} = -kx,$ $x = \frac{1}{k} \ln 2$	A1	AG
		4	
(b)	$mv \frac{dv}{dx} = -mkv^2 + \frac{5m}{v}$	B1	N2L (allow missing m in this part)
	$\frac{v^2 dv}{5 - kv^3} = dx - \frac{1}{3k} \ln(5 - kv^3) = x(+d)$	M1A1	Separate variables and integrate
	Using (a) $-\frac{1}{3k} \ln(5 - kv^3) = x - \frac{1}{3k} \ln(5 - ku^3) - \frac{1}{k} \ln 2$	M1M1	Use condition. M0 if $v = \frac{1}{2}u, x = 0$ used unless $\frac{1}{k} \ln 2$ is added on later Rearrange dependent on ln solution
	$x = \frac{1}{3k} \ln \left(\frac{40 - ku^3}{5 - ku^3} \right)$	M1A1	Use $v = u$
		7	

5 - (9231/31_Summer_2021_Q1) - Linear Motion Under Variable Force

	$\frac{dv}{\sqrt{v}} = -\frac{10 dt}{(t+1)^2}$	M1	Separate variables.
	$2\sqrt{v} = \frac{10}{t+1} + A$	M1 A1	Attempt to integrate.
	$t = 0, v = 25, A = 0$	M1	Use correct initial condition.
	$v = \frac{25}{(t+1)^2}$	A1	CAO
		5	



6 - (9231/33_Summer_2021_Q5) - Linear Motion Under Variable Force

(a)	$m \frac{dv}{dt} = -mg - 2mv$	B1	Use of SUVAT implies 0 marks. N2L, must include m .
	$\ln(5+v) = -2t(+A)$	M1	Separate variables and integrate 3-term N2L, condone omission of constant.
	$\ln(5+v) = -2t+A$	A1 FT	FT only sign error in N2L.
	$t=0, v=20, A=\ln 25$	M1	Use correct initial condition.
	$2t = \ln\left(\frac{25}{5+v}\right), e^{2t} = \frac{25}{5+v}$	M1	Remove all logs.
	$v = 25e^{-2t} - 5$	A1	
Alternative method for question 5(a)			
	$m \frac{dv}{dt} = -mg - 2mv$	B1	N2L, must include m .
	$\frac{dv}{dt} + 2v = -g$: Integrating factor $= e^{2t}$	M1	
	$\frac{d(ve^{2t})}{dt} = -ge^{2t}, ve^{2t} = -\frac{g}{2}e^{2t} (+A)$	M1	Integrate both sides, condone omission of constant.
	$ve^{2t} = -\frac{g}{2}e^{2t} + A$	A1 FT	FT only sign error in N2L.
(a)	$t=0, v=20, A=25$	M1	Use correct initial condition.
	$ve^{2t} = -\frac{g}{2}e^{2t} + 25, v = 25e^{-2t} - 5$	A1	
		6	
(b)	$x = -\frac{25}{2}e^{-2t} - 5t(+B)$	M1	Use of SUVAT implies 0 marks. Integrate their expression from part (a).
	$t=0, x=0, B=\frac{25}{2}$ $x = \frac{25}{2}(1 - e^{-2t}) - 5t$	A1 FT	FT only expressions of the form $v = Pe^{kt} + Q$ for P, Q non-zero.
		2	
(c)	Greatest height when $v=0$, so $t = 0.8047\dots$ or $\frac{1}{2}\ln 5$	M1	Use of SUVAT in part (a) or part (b) implies 0 marks. Find value of t , may be embedded.
	$x = 5.98 \text{ m}$	A1	CWO
		2	

7 - (9231/31_Summer_2022_Q3) - Linear Motion Under Variable Force

$\frac{dv}{dt} = -\frac{4000}{(5t+4)^3}; v = -\frac{400}{(5t+4)^2} (+A)$	M1 A1	Integrate. Constant of integration needed for A1.
$t=0, v=25, A=25-25=0$	M1	Find constant.
$v = \frac{dx}{dt}; x = -80(5t+4)^{-1} + B$ $x=0, t=0, B=20$	M1	Integrate and find constant.
$x = \frac{-80}{5t+4} + 20 \left(= \frac{100t}{5t+4} \right)$	A1	
	5	



8 - (9231/33_Summer_2022_Q5) - Linear Motion Under Variable Force

(a)	$4v \frac{dv}{dx} = -(4e^{-x} + 12)e^{-x}$	B1	
	$\frac{1}{2}v^2 = \frac{1}{2}e^{-2x} + 3e^{-x} (+A)$	M1	Expression of the correct form.
	$v = 4, \quad x = 0, \quad A = \frac{9}{2}$	A1	
	$v^2 = e^{-2x} + 6e^{-x} + 9 = (3 + e^{-x})^2$ $v = 3 + e^{-x} = \frac{1+3e^x}{e^x}$	A1	AG Must see the factorisation. Condone lack of justification for taking positive square root.
		4	
(b)	$\frac{dx}{dt} = \frac{1+3e^x}{e^x}$ so $\int \frac{e^x}{3e^x+1} dx = \int 1 dt$ $\frac{1}{3} \ln(3e^x+1) = t (+B)$	M1* A1	Integration to obtain ln term Correct answer with constant of integration
	$t = 0, \quad x = 0, \quad B = \frac{1}{3} \ln 4$ $3t = \ln \frac{3e^x+1}{4}$	DM1	Find the constant and substitute into their general solution.
	$x = \ln \left(\frac{4}{3} e^{3t} - \frac{1}{3} \right)$	A1	OE
		4	

9 - (9231/32_Winter_2022_Q4) - Linear Motion Under Variable Force

(a)	$5v \frac{dv}{dx} = \frac{500}{v} - \frac{1}{2}v^2$	B1	Sight of m or 5 is required.
	$\frac{10v^2 dv}{1000 - v^3} = dx$	M1	Separate variables and attempt to integrate into a log term.
	$-\frac{10}{3} \ln(1000 - v^3) = x (+A)$	A1	
	$x = 0, v = 5, \quad A = -\frac{10}{3} \ln 875$	M1	Evaluate constant: correct initial condition used.
	$x = \frac{10}{3} \ln \frac{875}{1000 - v^3}$	M1	Make v the subject: correct use of logs.
	$v = \left[(1000 - 875e^{-0.3x}) \right]^{\frac{1}{3}}$	A1	$v = 5 \left[(8 - 7e^{-0.3x}) \right]^{\frac{1}{3}}$; A0 if $e^{\ln}$ terms.
		6	
(b)	Maximum value of v is 10	B1	No FT: result can be found from initial equation.
		1	



10 - (9231/33_Winter_2022_Q4) - Linear Motion Under Variable Force

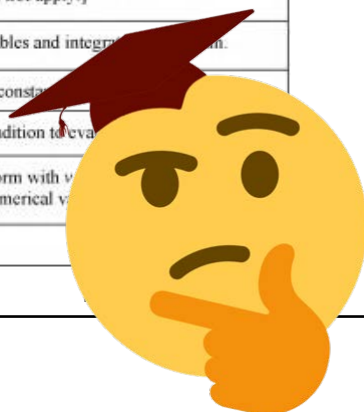
(a)	$m \frac{dv}{dt} = 50 - 2v^2 \quad \frac{dv}{dt} = 4(25 - v^2)$	B1	N2L
	$\frac{1}{10} \int \frac{1}{5-v} + \frac{1}{5+v} dv = \int 4 dt$	M1	Separate variables and use partial fractions.
	$\frac{1}{10} (-\ln(5-v) + \ln(5+v)) = 4t + A$	M1 A1	Integrate into log terms. (Note: formula on MF19).
	Use $t = 0, v = 3$ to give $A = \frac{1}{10} \ln 4$	M1	Use initial condition.
	$4t = \frac{1}{10} \ln \frac{5+v}{4(5-v)}$ leading to $\frac{5+v}{20-4v} = e^{40t}$	M1	Rearrange to make v the subject.
	$v = \frac{5(4 - e^{-40t})}{4 + e^{-40t}}$	A1	
		7	
(b)	As $t \rightarrow \infty, v \rightarrow 5$	B1	
		1	

11 - (9231/31_Summer_2023_Q6) - Linear Motion Under Variable Force

(a)	$v \frac{dv}{dx} = 6v\sqrt{v+9}$ and attempt to separate variables and integrate	M1	
	$2\sqrt{v+9} = 6x + A$	A1	
	$x = 2, v = 72 \quad A = 6$	M1	Use initial condition to find constant.
	$v = 9(x+1)^2 - 9$	A1	Correct, AEF.
		4	
(b)	$\left[\frac{dx}{dt} = 9(x^2 + 2x), \quad \frac{dx}{x(x+2)} = 9dt \right] \quad \frac{1}{2} \left(\frac{1}{x} - \frac{1}{x+2} \right) dx = 9dt$	M1	Separate variables and write in the form $\left(\frac{a}{x} - \frac{b}{x+c} \right) dx = dt$
	$\frac{1}{2} \ln \left(\frac{x}{x+2} \right) = 9t + B$	A1	Integrate, any correct form.
	$t = 0, x = 2 \quad B = \frac{1}{2} \ln \frac{1}{2}$	M1	Use initial condition to find constant.
	$18t = \ln \left(\frac{2x}{x+2} \right) \quad e^{18t} = \frac{2x}{x+2}$	M1	Take logarithms
	$x = \frac{2e^{18t}}{2 - e^{18t}} \quad \text{or} \quad x = \frac{2}{2e^{-18t} - 1}$	A1	Any correct form
		5	

12 - (9231/33_Summer_2023_Q6) - Linear Motion Under Variable Force

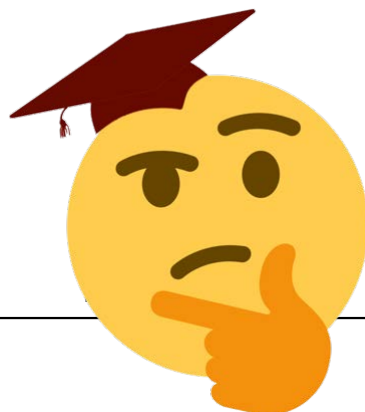
(a)	$m \frac{dv}{dt} = mg(1 - kv)$	B1	Mass must be seen at this point or earlier. [SUVAT does not apply.]
	$-\frac{1}{k} \ln(1 - kv) = gt + A$	M1	Separate variables and integrate correctly.
	$t = 0, v = 0 \quad [A = 0]$	A1	Correct, with constant.
	$v = \frac{1}{k} (1 - e^{-kgt})$	M1	Use initial condition to evaluate constant.
		A1	Any correct form with v and t . Final A0 if numerical value.
		5	



(b)	$k = 0.05$ and so $\frac{dx}{dt} = 20(1 - e^{-0.5t})$	*M1	Attempt to integrate if expression contains a term of the form be^{kt} .
	Integrate: $x = 20\left(t + 2e^{-0.5t}\right) + B$	A1	$x = \frac{1}{k}\left(t + \frac{1}{gk}e^{-kgt}\right) + B$
	$t = 0, x = 0 \Rightarrow B = -40$	DM1	Use initial condition to evaluate their constant.
	When $v = 12$, from part (a), $e^{-0.5t} = 1 - 0.05 \times 12 = 0.4$, $t = -2\ln 0.4$	M1	1.83...
	$x = -40\ln 0.4 + 40 \times 0.4 - 40 = 12.7$	A1	$40\ln \frac{5}{2} - 24$
Alternative method for question 6(b)			
	$v \frac{dv}{dx} = g(1 - kv)$ leading to $\left(1 - \frac{1}{1 - kv}\right) dv = -kg dx$	*M1	Separate variables and write in integrable form
	$v + \frac{1}{k} \ln(1 - kv) = -kgx + B$	DM1 A1	Dependent on previous M1. Attempt to integrate.
	$v = 0, x = 0 \Rightarrow B = 0$ and $k = 0.05$, $v = 12 \Rightarrow 12 + 20\ln 0.4 = -0.5x$	M1	Dependent on both previous M1s. Use initial condition to evaluate their constant and use $v = 12$
	$x = 12.7$	A1	$40\ln \frac{5}{2} - 24$
		5	

13 - (9231/32_Winter_2023_Q2) - Linear Motion Under Variable Force

$0.5v \frac{dv}{dx} = \frac{150}{(x+1)^2} - \frac{450}{(x+1)^3}$	M1	Allow sign errors.
Integrate: $0.5v^2 = -\frac{300}{x+1} + \frac{450}{(x+1)^2} + A$	M1A1	Correct powers, allow sign errors.
$x = 0, v = 20 \Rightarrow A = 50$	M1	Use initial condition.
Rearrange: $v^2 = \frac{100(x^2 - 4x + 4)}{(x+1)^2}$	A1	AEF
$v^2 = \frac{100(x-2)^2}{(x+1)^2}$ so $v = \pm \frac{10(x-2)}{(x+1)}$ From initial condition, sign must be negative, $v = \frac{20-10x}{x+1}$	A1	Signs dealt with convincingly.
	6	



1 - (9231/31_Winter_2020_Q1) - *Hook's Law*

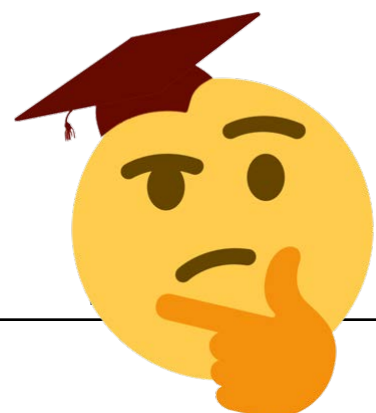
Gain in EPE = $\frac{1}{2} \frac{3mgx^2}{a}$	B1	EPE gain.
Loss in GPE = $mgx \sin \theta$ Equate	M1	Equate energies
$x = \frac{2}{3} a \sin \theta$	A1	Using forces scores B0M0A0
	3	

2 - (9231/32_Winter_2020_Q6) - *Hook's Law*

$T + mg = m \frac{7}{3} g$	M1	
$\frac{a}{3}$ With $T = k \frac{3}{a}$ giving $k = 4mg$	A1	AG
Let greatest height above Q be $\frac{4}{3}a + x$ Gain in GPE = mgx and Loss in KE = $\frac{1}{2} m 2ga$	B1	The length being found may be expressed as the total extension of the string or the greatest height above Q. GPE and KE
Gain in EPE = $\frac{1}{2} \frac{4mg}{a} \left(\left(x + \frac{a}{3} \right)^2 - \left(\frac{a}{3} \right)^2 \right)$	B1	EPE Note: initial EPE = $\frac{2mga}{9}$
$\frac{4mg}{2a} \left(x^2 + \frac{2ax}{3} + \frac{a^2}{9} - \frac{a^2}{9} \right) + mgx = mga$	M1 A1	Energy equation, correct number of terms
$2x^2 + \frac{7ax}{3} - a^2 = 0$	M1	Simplify to quadratic
$x = \frac{1}{3}a$ so greatest height is $\frac{5}{3}a$	A1	

3 - (9231/33_Summer_2021_Q2) - *Hook's Law*

$\frac{1}{2} \times \frac{36x^2}{0.8} (= 22.5x^2)$	B1	EPE correct.
Loss in GPE + loss in KE = gain in EPE $x mg \sin \alpha + \frac{1}{2} m \times 2 = \frac{1}{2} \times \frac{36x^2}{0.8}$	*M1	Energy equation with only GPE, EPE and KE terms, allow sign errors, allow missing g for M1 only, weight must be resolved (allow sin or cos).
All terms correct	A1	
$\left(\frac{6}{5}x + \frac{1}{5} = \frac{9}{4}x^2 \right)$ [leading to $45x^2 - 24x - 4 = 0$]	DM1	Simplify to 3-term quadratic and attempt to solve.
$(3x-2)(15x+2)=0$ $x = \frac{2}{3}$ only	A1	
	5	



4 - (9231/32_Summer_2021_Q3) - Hook's Law

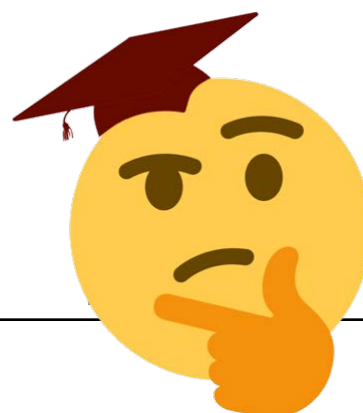
(a)	Use Hooke's Law: $4mg = \frac{kmg(x-a)}{a}$ leading to $k = \frac{4a}{x-a}$	B1	AG. Shown convincingly.
		1	
(b)	Gain in KE + gain in EPE = loss in GPE	B1	One correct EPE term seen.
	$\frac{1}{2} \times 6m \times \frac{ga}{9} + \frac{1}{2} \frac{kmg}{a} \left(\left(x + \frac{a}{3} - a \right)^2 - (x-a)^2 \right) = 6mg \times \frac{a}{3}$	M1 A1	All 3 types of energy included in energy equation. All terms correct.
	Simplify and substitute for k from part (a)	M1	
	Obtain linear equation in x and a	M1	
	$x = \frac{5}{3}a$	A1	($k = 6$)
		6	

5 - (9231/31_Winter_2021_Q1) - Hook's Law

$T = \frac{3mgx}{a}$	B1	Their tensions equated to obtain a quadratic equation, CAO.
$T = \frac{4mga}{3(a+x)}$	B1	
$9x^2 + 9ax - 4a^2 = 0$ leading to $(3x-a)(3x+4a) = 0$	M1	
$x = \frac{1}{3}a$	A1	
	4	

6 - (9231/32_Winter_2021_Q2) - Hook's Law

At the collision of P and Q: $(m+km)v = kmv$	M1	Momentum conserved, allow missing k on RHS.
So $v = \frac{k\sqrt{4ga}}{(1+k)}$	A1	
$EPE = \frac{1}{2} \times \frac{5mg}{a} \times \left(\frac{a}{5} \right)^2 \left(= \frac{mga}{10} \right)$	B1	
Loss in KE = Gain in EPE: $\frac{1}{2} m(k+1)v^2 = \frac{1}{2} \times \frac{5mg}{a} \times \left(\frac{a}{5} \right)^2$	M1	Energy equation, LHS correct, EPE dimensionally correct.
Substitute for v and rearrange to form quadratic equation in k $20k^2 = 1+k$	M1	
$k = \frac{1}{4}$	A1	
	6	



7 - (9231/31_Winter_2021_Q3) - Hook's Law

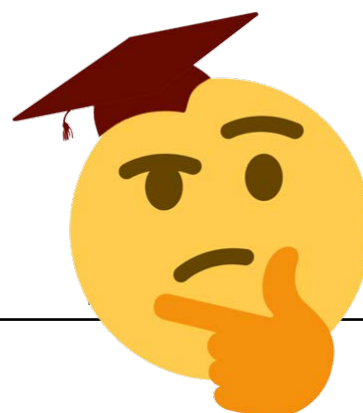
Loss in EPE = $\frac{1}{2} \times \frac{12mge^2}{a} - \frac{1}{2} \times \frac{12mg}{a} \times \left(e - \frac{a}{3}\right)^2 = \frac{2mg}{3}(6e - a)$	B1	Either term correct.
Gain in KE = $\frac{1}{2}mv^2$ and Gain in GPE = $\frac{mga}{3}$	B1	
Gain in KE + Gain in GPE = Loss in EPE	M1	KE, GPE and at least one EPE term.
$\frac{1}{2}mv^2 + \frac{mga}{3} = \frac{2mg}{3}(6e - a)$	A1	All terms correct.
Simplify to a linear equation in e.	M1	
$e = \frac{1}{2}a$	A1	
	6	

8 - (9231/31_Summer_2022_Q1) - Hook's Law

(a)	$T \cos \theta = 7.5, \quad T \sin \theta = 10$	B1	
	$T = \left(7.5^2 + 10^2\right)^{\frac{1}{2}} = 12.5 \text{ N}$	B1	
		2	
(b)	Hooke's law: $T = \frac{50x}{0.8}, \quad x = 0.2$	B1	
	$(x + 0.8) \sin \theta = 1 \times \frac{10}{12.5}$	M1	
	Vertical distance = $\frac{4}{5} = 0.8$	A1	
		3	

9 - (9231/33_Summer_2022_Q2) - Hook's Law

$\frac{1}{2} \times \frac{4}{3} \frac{mg}{a} x^2$	B1	EPE term correct
$\frac{1}{3} mgx$	B1	Work term correct
Loss in KE = gain in EPE + work done against friction $\frac{1}{2}mv^2 = \frac{1}{2} \times \frac{4}{3} \frac{mg}{a} x^2 + \frac{1}{3} mgx$	M1	Energy equation with 3 terms, allow sign error.
$\frac{1}{2} \times \frac{1}{4} ga = \frac{2}{3} \frac{g}{a} x^2 + \frac{1}{3} gx$ $16x^2 + 8ax - 3a^2 = 0$ $(4x - a)(4x + 3a) = 0$	M1	Obtain and attempt to solve a 3-term quadratic equation.
$x = \frac{1}{4}a$	A1	
	5	



10 – (9231/31_Winter_2022_Q2) – Hook's Law

(a)	Loss in KE = Gain in EPE, so $\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{2} \times \frac{4mg}{a} \left(\left(\frac{1}{2}a\right)^2 - \left(\frac{1}{4}a\right)^2 \right)$	B1	EPE terms correct.
		M1	All 4 terms and no extras.
	$\frac{3}{4}mv^2 = \frac{4mg}{a} \times \frac{3}{16}a^2$	M1	Simplify.
	$v^2 = ag, \quad v = \sqrt{ag}$	A1	
		4	
(b)	Hooke's law: tension = $\frac{4mg}{a} \times \frac{1}{2}a (= 2mg)$	M1	
	Acceleration = $\frac{2mg}{m} = 2g$	A1	Accept -2g.
		2	

11 – (9231/31_Summer_2023_Q1) – Simple Harmonic Motion, Hook's Law

(a)	$\frac{3mg}{2a}(2a)^2$	B1	Correct EPE term seen
	$\frac{1}{2}mv^2 + mg \times \left(3a - \frac{3}{4}a\right) = \frac{3mg}{2a}(2a)^2$	M1	Dimensionally correct energy equation. Must have one KE, one EPE term and at least one GPE. Allow sign errors.
	$v = \sqrt{\frac{15}{2}ag} \quad [2.74\sqrt{ag}]$	A1	AEF
		3	
(b)	$T - mg = mA$ and $T = \frac{3mg}{a} \times 2a$	M1	N2L and Hooke's law
	Acceleration = $5g$ [upwards]	A1	Allow ± 50 or $\pm 5g$
		2	

12 – (9231/33_Summer_2023_Q1) – Circular Motion, Simple Harmonic Motion, Hook's Law

$\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(\cos\alpha + \cos\theta)$	M1	Energy equation, 4 terms, dimensionally correct, mass must be present, allow sign errors, allow sin in both terms on RHS
$mg \cos\theta = \frac{mv^2}{a}$	B1	N2L, may include tension initially but not awarded until tension = 0 used
$\frac{3}{2}mag - \frac{1}{2}mag \cos\theta = mga\left(\frac{4}{5} + \cos\theta\right)$ $\frac{3}{2}\cos\theta = \frac{7}{10}$	M1	Dependent on tension = 0 and on an energy equation, eliminate v^2 .
$\cos\theta = \frac{7}{15}$	A1	If no m in energy equation and no further errors, award SCB2 for correct final answer
	4	

