TMUA PRACTICE TEST SERIES

TMUA Mock Test 1

Algebra, Sequences, Functions & Geometry

- 20 multiple-choice questions
- Recommended time: 75 minutes
- Non-calculator
- Paper 1 styled topic revision set

SOLUTION BOOK

ThrivingScholars

VERIFIED ANSWER KEY

TMUA Mock Test 1 — Algebra, Sequences, Functions & Geometry

Complete worked solutions for all 20 questions. The answer key, option highlighting and worked solutions are generated from the same verified question set.

Answer key

Q1 **B**	Q2 **D**	Q3 **B**	Q4 **C**	Q5 **D**	Q6 **D**	Q7 **D**	Q8 **B**	Q9 **D**	Q10 **C**
Q11 **D**	Q12 **B**	Q13 **B**	Q14 **D**	Q15 **C**	Q16 **A**	Q17 **E**	Q18 **C**	Q19 **C**	Q20 **D**

Algebra & Functions

Questions 1–5 · expansions, coefficients, algebraic structure and surds

Q1

Algebra & Functions

Answer B

Question 1

The expansion of $(a - bx)^c$ is

$$4 - px + 108x^2 - qx^3 + rx^4$$

where a, b, c, p, q, r are positive real constants.

Find the value of $p + q + r$.

- **A** $81 - 84\sqrt{2}$
- **B** $81 + 132\sqrt{2}$
- **C** $132\sqrt{2} - 81$
- **D** $81 + 84\sqrt{2}$

WORKED SOLUTION

Since the expansion has degree 4, $c = 4$. Thus

$$(a - bx)^4 = a^4 - 4a^3bx + 6a^2b^2x^2 - 4ab^3x^3 + b^4x^4.$$

Comparing constant terms gives $a^4 = 4$, so $a = \sqrt{2}$.

Comparing the x^2 coefficients,

$$6a^2b^2 = 108.$$

Since $a^2 = 2$, $12b^2 = 108$, hence $b = 3$.

Therefore

$$p = 4a^3b = 24\sqrt{2}, \qquad q = 4ab^3 = 108\sqrt{2}, \qquad r = b^4 = 81.$$

$$p + q + r = \boxed{81 + 132\sqrt{2}}.$$

The correct answer is B.

Q2

Algebra & Functions

Answer **D**

Question 2

The coefficient of x^{11} in $(2 + x^2 + x^3)^8$ is equal to 28 times the coefficient of x^2 in $(2 + ax)^6$.

Find all possible values of a.

A $\pm 2\sqrt{7}$

B $\pm \frac{1}{2}$

C $\pm \frac{1}{4}$

D ± 1

WORKED SOLUTION

To obtain x^{11}, if x^2 is chosen from k factors and x^3 from j factors, then

$$2k + 3j = 11.$$

The non-negative solutions are $(k, j) = (4, 1)$ and $(1, 3)$.

$$\frac{8!}{4!1!3!}2^3 = 2240, \qquad \frac{8!}{1!3!4!}2^4 = 4480.$$

Hence the coefficient of x^{11} is 6720.

The coefficient of x^2 in $(2 + ax)^6$ is

$$\binom{6}{2}2^4a^2 = 240a^2.$$

Therefore

$$6720 = 28(240a^2) \Rightarrow a^2 = 1.$$

$$\boxed{a = \pm 1}.$$

The correct answer is D.

Q3

Algebra & Functions

Answer **B**

Question 3

The coefficient of x^2 in $(2+bx)^4$ is 2 times the coefficient of x^3 in $(1+bx)^6$.

Given $b \neq 0$, find b.

- **A** $\frac{8}{15}$
- **B** $\frac{3}{5}$
- **C** $\frac{3}{20}$
- **D** $\frac{6}{5}$

WORKED SOLUTION

The coefficient of x^2 in $(2+bx)^4$ is

$$\binom{4}{2}2^2b^2 = 24b^2.$$

The coefficient of x^3 in $(1+bx)^6$ is

$$\binom{6}{3}b^3 = 20b^3.$$

Thus

$$24b^2 = 2(20b^3) = 40b^3.$$

Since $b \neq 0$,

$$\boxed{b = \frac{3}{5}}.$$

The correct answer is B.

Q4

Algebra & Functions

Answer C

Question 4

Find the coefficient of x in

$$(1-x)^0-(1-x)^1+(1-x)^2-(1-x)^3+\cdots+(1-x)^{50}.$$

A 51

B -51

C -25

D 25

WORKED SOLUTION

The coefficient of x in $(1-x)^n$ is $-n$. Including the alternating signs gives

$$1-2+3-4+\cdots+49-50.$$

Pairing consecutive terms,

$$(1-2)+(3-4)+\cdots+(49-50).$$

There are 25 pairs, each equal to -1. Therefore

$$\boxed{-25}.$$

The correct answer is C.

Q5

Algebra & Functions

Answer D

Question 5

Find the value of

$$\sqrt{16+8\sqrt{3}+3}-\sqrt{12-4\sqrt{3}+1}.$$

A $\sqrt{6+12\sqrt{3}}$

B $\sqrt{3}-5$

C 3

D $5-\sqrt{3}$

WORKED SOLUTION

Since

$$(4+\sqrt{3})^2=16+8\sqrt{3}+3,$$

we have $\sqrt{16+8\sqrt{3}+3}=4+\sqrt{3}$.

Also,

$$(2\sqrt{3}-1)^2=12-4\sqrt{3}+1,$$

and $2\sqrt{3}-1>0$, so the second radical is $2\sqrt{3}-1$.

$$4+\sqrt{3}-(2\sqrt{3}-1)=\boxed{5-\sqrt{3}}.$$

The correct answer is D.

Coordinate Geometry

Questions 6–13 · circles, tangents, loci, reflections and area

Q6

Coordinate Geometry

Answer D

Question 6

The circles

$$(x-r)^2+(y-2)^2=r^2$$

and

$$(x+r)^2+(y+r)^2=4r^2$$

touch precisely once when:

A $r=\sqrt{3}-1$

B $r=\frac{8}{5}$

C $r=\frac{2}{1+\sqrt{5}}$

D $r=\frac{1+\sqrt{5}}{2}$

WORKED SOLUTION

The centres are $(r, 2)$ and $(-r, -r)$, with radii r and $2r$. Their centre distance is

$$d=\sqrt{(2r)^2+(r+2)^2}=\sqrt{5r^2+4r+4}.$$

For external tangency, $d=r+2r=3r$. Hence

$$5r^2+4r+4=9r^2 \quad\Longrightarrow\quad r^2-r-1=0.$$

Because a radius is positive,

$$r=\frac{1+\sqrt{5}}{2}.$$

Internal tangency would require $d=2r-r=r$, which gives $r^2+r+1=0$, so there is no positive real value from that case.

$$\boxed{r=\frac{1+\sqrt{5}}{2}}.$$

The correct answer is D.

Q7

Coordinate Geometry

Answer D

Question 7

Find the value(s) of a such that the turning point of

$$y = x^2 + 2ax + 1$$

is closest to the origin.

A $a = \sqrt{2}$

B $a = \pm\sqrt{2}$

C $a = \frac{\sqrt{2}}{2}$

D $a = \pm\frac{1}{\sqrt{2}}$

WORKED SOLUTION

Complete the square:

$$y = (x + a)^2 + 1 - a^2.$$

The turning point is $(-a, 1 - a^2)$. Its squared distance from the origin is

$$D^2 = a^2 + (1 - a^2)^2 = a^4 - a^2 + 1.$$

Let $t = a^2$. Then

$$D^2 = t^2 - t + 1 = \left(t - \frac{1}{2}\right)^2 + \frac{3}{4}.$$

This is minimized when $t = \frac{1}{2}$, so

$$\boxed{a = \pm\frac{1}{\sqrt{2}}}.$$

The correct answer is D.

Q8

Coordinate Geometry

Answer **B**

Question 8

The line $y = 2x + c$ intersects the circle $x^2 + y^2 = 9$ at two points A and B. M is the midpoint of chord AB.

The equation of the locus of M, as c varies between $-3\sqrt{5}$ and $3\sqrt{5}$, is:

- **A** $x^2 + y^2 = 2$
- **B** $y = -\frac{1}{2}x$
- **C** $y = \frac{1}{2}x$
- **D** $y = x^2$

WORKED SOLUTION

The midpoint of a chord lies on the perpendicular from the centre of the circle to that chord. The circle has centre $(0, 0)$.

Every chord AB lies on a line of slope 2, so the perpendicular through the origin has slope $-\frac{1}{2}$. Therefore every midpoint M satisfies

$$\boxed{y = -\frac{1}{2}x}.$$

The stated range of c restricts which part of this line is traced; the equation of the locus is still $y = -\frac{1}{2}x$.

The correct answer is B.

Q9

Coordinate Geometry

Answer D

Question 9

The tangent to

$$x^2 + y^2 + 6x + 2y + 2 = 0$$

at $(-5, 1)$ passes through $(-3, 3)$. The other tangent from $(-3, 3)$ touches the circle at:

A $(-3, 2\sqrt{2})$

B $(-3, 1)$

C $(-1, 2\sqrt{2})$

D $(-1, 1)$

WORKED SOLUTION

Complete the square:

$$(x+3)^2 + (y+1)^2 = 8.$$

The centre is $(-3, -1)$. The external point $(-3, 3)$ lies on the vertical line through the centre, so the two points of tangency are mirror images in $x = -3$.

Reflecting $(-5, 1)$ in $x = -3$ gives

$$\boxed{(-1, 1)}.$$

The correct answer is D.

Q10

Coordinate Geometry

Answer C

Question 10

The circle

$$x^2 + y^2 + 2ax + 2by = c$$

encloses $(1, 1)$ precisely when:

- **A** $c > 2(1 - a - b)$
- **B** $a^2 + b^2 < 1$
- **C** $c > 2(a + b + 1)$
- **D** $a^2 + 2ab + b^2 < c$

WORKED SOLUTION

Complete the square:

$$(x + a)^2 + (y + b)^2 = c + a^2 + b^2.$$

For $(1, 1)$ to lie strictly inside the circle,

$$(1 + a)^2 + (1 + b)^2 < c + a^2 + b^2.$$

Expanding and cancelling $a^2 + b^2$ gives

$$2 + 2a + 2b < c.$$

Hence

$$\boxed{c > 2(a + b + 1)}.$$

The correct answer is C.

Q11

Coordinate Geometry

Answer D

Question 11

Which of the following is a tangent to the circle

$$(x-2)^2+(y-3)^2=4?$$

A $x=3+\sqrt{2}$

B $y=5+\frac{\sqrt{2}}{2}-x$

C $y=2(2-\sqrt{2})$

D $y=x+1-2\sqrt{2}$

WORKED SOLUTION

The circle has centre $(2,3)$ and radius 2.

For option D,

$$y=x+1-2\sqrt{2}$$

or

$$x-y+1-2\sqrt{2}=0.$$

The distance from $(2,3)$ to this line is

$$\frac{|2-3+1-2\sqrt{2}|}{\sqrt{1^2+(-1)^2}}=\frac{2\sqrt{2}}{\sqrt{2}}=2.$$

This equals the radius, so the line is tangent.

$$\boxed{y=x+1-2\sqrt{2}}.$$

The correct answer is D.

Q12

Coordinate Geometry

Answer B

Question 12

The reflection of (a, a) in the line

$$y = \frac{1}{a}x, \qquad a \neq 0$$

is:

A $\left(\frac{a(a-1)^2}{a^2+1}, \frac{a^2(a^2+3)}{a^2+1}\right)$

B $\left(\frac{a(a^2+2a-1)}{a^2+1}, \frac{-a(a^2-2a-1)}{a^2+1}\right)$

C $\left(\frac{a(a-1)^2}{a^2+1}, \frac{-a(a-1)^2}{a^2+1}\right)$

D $\left(\frac{a(a+1)^2}{a^2+1}, \frac{a^2(a+1)^2}{a^2+1}\right)$

WORKED SOLUTION

A direction vector for the line is $\begin{pmatrix} a \\ 1 \end{pmatrix}$. Reflection in this line has matrix

$$\frac{1}{a^2+1}\begin{pmatrix} a^2-1 & 2a \\ 2a & 1-a^2 \end{pmatrix}.$$

Applying this to $\begin{pmatrix} a \\ a \end{pmatrix}$ gives

$$x' = \frac{a(a^2+2a-1)}{a^2+1}, \qquad y' = \frac{-a(a^2-2a-1)}{a^2+1}.$$

Hence the image is

$$\boxed{\left(\frac{a(a^2+2a-1)}{a^2+1}, \frac{-a(a^2-2a-1)}{a^2+1}\right)}.$$

The correct answer is B.

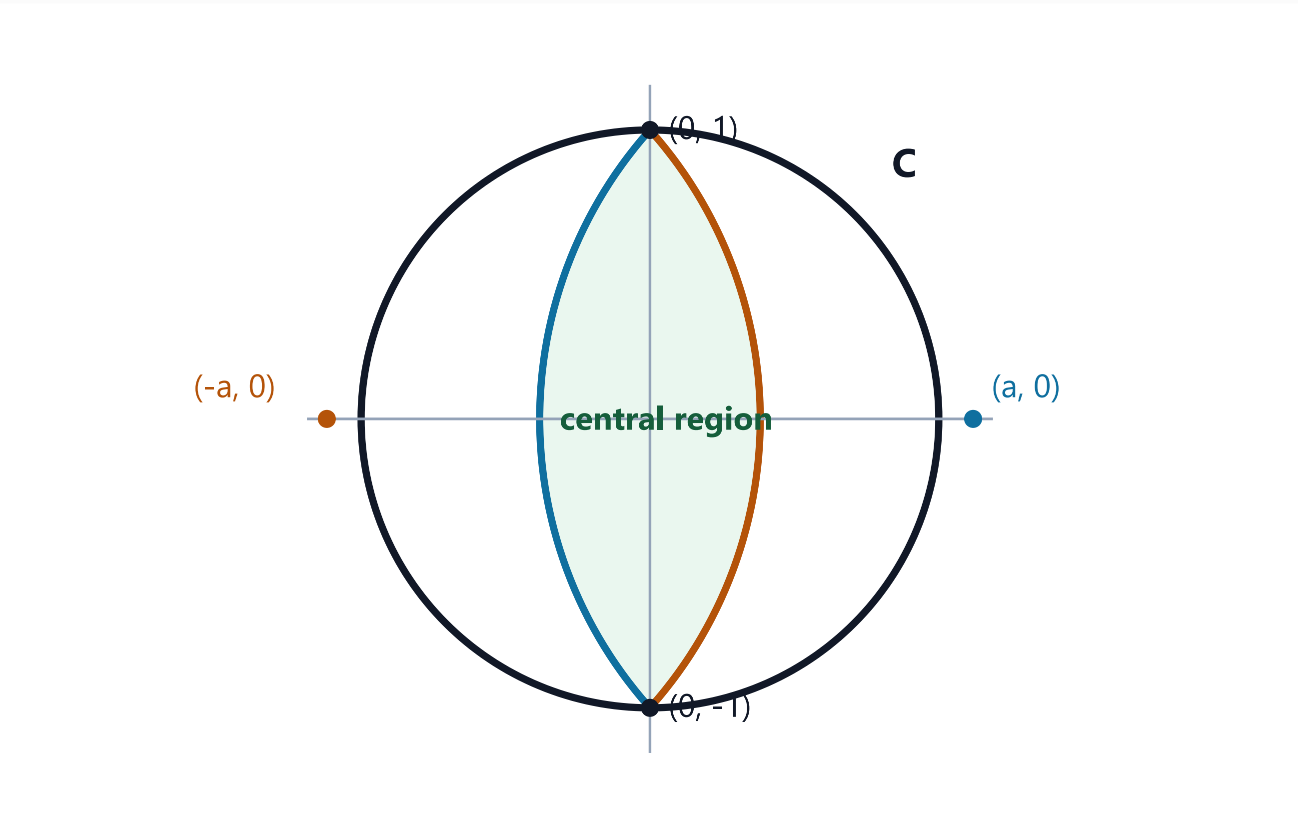
Q13

Coordinate Geometry

Answer **B**

Question 13

The circle C has equation $x^2 + y^2 = 1$. Two circles of equal radius r, with centres $(a, 0)$ and $(-a, 0)$, where $a > 0$, both pass through $(0, 1)$ and $(0, -1)$. This is shown in the diagram.



The two arcs divide C into three distinct regions. If the three regions have equal area, which equation is satisfied by a?

A $3(a^2 + 1)\arctan\left(\frac{1}{a}\right) - 3a = \pi$

B $6(a^2 + 1)\arctan\left(\frac{1}{a}\right) - 6a = \pi$

C $6(a^2 + 1)\arctan\left(\frac{1}{a}\right) - 3a = \pi$

D $12(a^2 + 1)\arctan\left(\frac{1}{a}\right) - 6a = \pi$

WORKED SOLUTION

Each larger circle passes through $(0, 1)$, so its radius satisfies

$$r^2 = a^2 + 1.$$

Let

$$\theta = \arctan\left(\frac{1}{a}\right).$$

One quarter of the central region is a sector of radius r and angle θ, minus a right triangle with perpendicular sides a and 1. Its area is

$$\frac{1}{2}(a^2+1)\arctan\left(\frac{1}{a}\right)-\frac{a}{2}.$$

There are four congruent quarters, so the central region has area

$$2\left[(a^2+1)\arctan\left(\frac{1}{a}\right)-a\right].$$

The other two regions are mirror images. If all three regions have equal area, each has area $\frac{\pi}{3}$. Therefore

$$2\left[(a^2+1)\arctan\left(\frac{1}{a}\right)-a\right]=\frac{\pi}{3}.$$

Multiplying by 3,

$$\boxed{6(a^2+1)\arctan\left(\frac{1}{a}\right)-6a=\pi}.$$

The correct answer is B.

Sequences, Series & Algebraic Reasoning

Questions 14–20 · recurrences, remainders, factors, polynomial patterns and logarithms

Q14

Sequences & Series

Answer D

Question 14

The function f is defined on the positive integers by $f(1) = 2$. For $n \geq 1$,

$$f(n+1) = 5f(n) + 1 \quad \text{if } f(n) \text{ is odd,}$$

$$f(n+1) = \frac{1}{2}f(n) \quad \text{if } f(n) \text{ is even.}$$

Find $\sum_{r=1}^{100} f(r)$.

A 535

B 546

C 560

D 563

WORKED SOLUTION

Starting from 2, the sequence is

$$2 \to 1 \to 6 \to 3 \to 16 \to 8 \to 4 \to 2.$$

Thus the seven-term cycle is

$$2, 1, 6, 3, 16, 8, 4$$

with sum 40.

Since $100 = 14 \cdot 7 + 2$, the first 98 terms contribute

$$14 \cdot 40 = 560.$$

The final two terms are 2 and 1. Hence

$$\boxed{560 + 2 + 1 = 563}.$$

The correct answer is D.

Q15

Algebra & Functions

Answer C

Question 15

When $2x^2 + 3x - 15$ is multiplied by $(ax - 4)$, and the resulting product is divided by $x + 2$, the remainder is 182.

Find a.

A -9

B -5

C 5

D 9

WORKED SOLUTION

Let

$$F(x) = (ax - 4)(2x^2 + 3x - 15).$$

By the Remainder Theorem,

$$F(-2) = 182.$$

So

$$(-2a - 4)(8 - 6 - 15) = 182$$

$$(-2a - 4)(-13) = 182.$$

Therefore

$$26(a + 2) = 182 \Rightarrow a + 2 = 7 \Rightarrow \boxed{a = 5}.$$

The correct answer is C.

Q16

Algebra & Functions

Answer **A**

Question 16

Find the non-zero solution of

$$\frac{3^{16^x}}{81^{4^x}} = \frac{1}{27}.$$

A $\log_4 3$

B $\log_{16} 3$

C 1

D 2

WORKED SOLUTION

Since $81 = 3^4$ and $\frac{1}{27} = 3^{-3}$,

$$3^{16^x - 4(4^x)} = 3^{-3}.$$

Hence

$$16^x - 4(4^x) = -3.$$

Let $t = 4^x$. Since $16^x = t^2$,

$$t^2 - 4t + 3 = 0 \Rightarrow (t-1)(t-3) = 0.$$

Thus $4^x = 1$ or $4^x = 3$. The first gives $x = 0$, so the non-zero solution is

$$\boxed{x = \log_4 3}.$$

The correct answer is A.

Q17

Algebraic Reasoning

Answer **E**

Question 17

Let n be a positive integer. $x^2 + 2$ is a factor of

$$(x^4 - 5)^n - (x^2 + 1)^{n+1} + (x^2 + 4)^n (x^2 + 3)^n$$

for:

A odd n

B even n

C no n

D all n

E $n = 1$

WORKED SOLUTION

Modulo $x^2 + 2$,

$$x^2 \equiv -2, \qquad x^4 \equiv 4.$$

So the remainder is

$$(4 - 5)^n - (-2 + 1)^{n+1} + (-2 + 4)^n (-2 + 3)^n$$
$$= (-1)^n - (-1)^{n+1} + 2^n = 2(-1)^n + 2^n.$$

For divisibility,

$$2(-1)^n + 2^n = 0.$$

Dividing by 2,

$$(-1)^n + 2^{n-1} = 0.$$

Thus n must be odd, giving $2^{n-1} = 1$, so

$$\boxed{n = 1}.$$

The correct answer is E.

Q18

Algebraic Reasoning

Answer C

Question 18

a and b are non-zero integers.

When $x^2 - 2ax - a^2$ is divided by $x - b$, the remainder is 1.

The polynomial $4bx^2 - 6x - 10$ has $2x - a$ as a factor.

It follows that $a + b$ equals:

A -5

B -3

C -1

D 0

WORKED SOLUTION

By the Remainder Theorem,

$$b^2 - 2ab - a^2 = 1. \tag{1}$$

Since $2x - a$ is a factor of $4bx^2 - 6x - 10$, $x = \frac{a}{2}$ is a root:

$$4b\left(\frac{a}{2}\right)^2 - 6\left(\frac{a}{2}\right) - 10 = 0.$$

Hence

$$ba^2 - 3a - 10 = 0 \quad \Longrightarrow \quad a(ab - 3) = 10. \tag{2}$$

Thus a is a non-zero integer divisor of 10. Using $b = (3a + 10)/a^2$, the only divisor that gives a non-zero integer b and also satisfies (1) is

$$a = -2, \qquad b = 1.$$

Therefore

$$\boxed{a + b = -1}.$$

The correct answer is C.

Q19

Sequences & Series

Answer C

Question 19

The polynomial p_n is defined by

$$p_n(x) = (x - 2n + 1) + (2x - 2n + 3) + (3x - 2n + 5) + \cdots + (nx - 1).$$

Given $n \geq 2$, what is the remainder when $p_n(x)$ is divided by $p_{n-1}(x)$?

A $\frac{n^2-1}{2}$

B n

C -1

D 1

WORKED SOLUTION

The k-th term is $kx - 2n + (2k - 1)$, so

$$p_n(x) = \sum_{k=1}^{n} [kx - 2n + (2k - 1)].$$

Therefore

$$p_n(x) = \frac{n(n+1)}{2}x - n^2.$$

Similarly,

$$p_{n-1}(x) = \frac{n(n-1)}{2}x - (n-1)^2.$$

The root of p_{n-1} is

$$x = \frac{2(n-1)}{n}.$$

Since the divisor is linear, the remainder is

$$p_n\left(\frac{2(n-1)}{n}\right) = (n+1)(n-1) - n^2 = \boxed{-1}.$$

The correct answer is C.

Q20

Algebra & Functions

Answer D

Question 20

Given that x, y, z are positive real numbers,

$$4\log_z x = y, \qquad \log_x z = y, \qquad x + \log_y z = 0.$$

Which of the following is true?

- **A** Unique solution for x, but not for y and z.
- **B** Unique solutions for x and y, but infinitely many solutions for z.
- **C** Unique solutions for x and z, but infinitely many solutions for y.
- **D** Unique solution for x, y and z.

WORKED SOLUTION

Since

$$\log_x z = \frac{1}{\log_z x},$$

let $t = \log_z x$. The first two equations give

$$y = 4t, \qquad y = \frac{1}{t}.$$

Thus $4t^2 = 1$. Because $y > 0$, we must have $t > 0$, so

$$t = \frac{1}{2}, \qquad y = 2.$$

Also $\log_z x = \frac{1}{2}$, hence $z = x^2$. The third equation becomes

$$x + \log_2(x^2) = 0,$$

or equivalently

$$2\ln x + x\ln 2 = 0.$$

Define $F(x) = 2\ln x + x\ln 2$ for $x > 0$. Then

$$F'(x) = \frac{2}{x} + \ln 2 > 0,$$

so F is strictly increasing. Moreover, $F(x) \to -\infty$ as $x \to 0^+$, while $F(1) = \ln 2 > 0$. Therefore $F(x) = 0$ has exactly one positive solution. Numerically, $x \approx 0.7667$, so $z = x^2 \approx 0.5878$; both are valid logarithm bases.

Hence x, y and z are all uniquely determined.

The correct answer is D.