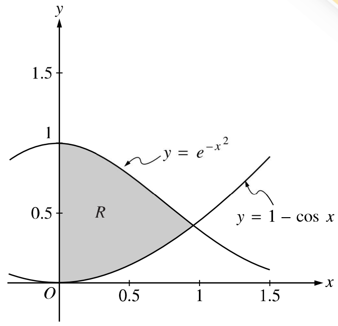
AP Calculus BC
2000 Scoring Guidelines

The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

AP Calculus AB–1 / BC–1



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Region R

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

1 : Correct limits in an integral in (a), (b), or (c).

(a) Area $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right) dx$

$= 0.590$ or 0.591

2 { 1 : integrand
1 : answer

(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2 \right) dx$

$= 0.55596\pi = 1.746$ or 1.747

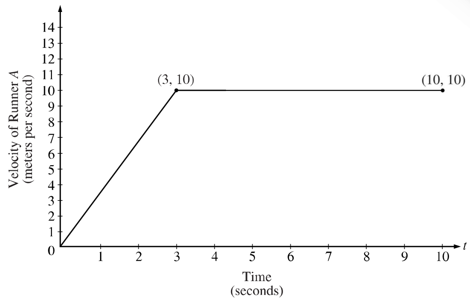
3 { 2 : integrand and constant
< −1> each error
1 : answer

(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$

$= 0.461$

3 { 2 : integrand
< −1> each error
Note: 0/2 if not of the form
$k \int_c^d (f(x) - g(x))^2 \, dx$
1 : answer

AP Calculus AB–2 / BC–2



Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

(a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

(b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

(a) Runner A: velocity $= \dfrac{10}{3} \cdot 2 = \dfrac{20}{3}$

$= 6.666$ or 6.667 meters/sec

Runner B: $v(2) = \dfrac{48}{7} = 6.857$ meters/sec

2 { 1 : velocity for Runner A; 1 : velocity for Runner B }

(b) Runner A: acceleration $= \dfrac{10}{3} = 3.333$ meters/sec^2

Runner B: $a(2) = v'(2) = \left.\dfrac{72}{(2t+3)^2}\right|_{t=2}$

$= \dfrac{72}{49} = 1.469$ meters/sec^2

2 { 1 : acceleration for Runner A; 1 : acceleration for Runner B }

(c) Runner A: distance $= \dfrac{1}{2}(3)(10) + 7(10) = 85$ meters

Runner B: distance $= \displaystyle\int_0^{10} \frac{24t}{2t+3}\,dt = 83.336$ meters

4 {
2 : distance for Runner A
  1 : method
  1 : answer
2 : distance for Runner B
  1 : integral
  1 : answer
}

(units) meters/sec in part (a), meters/sec^2 in part (b), and meters in part (c), or equivalent.

1: units

AP Calculus BC–3

The Taylor series about $x = 5$ for a certain function f converges to $f(x)$ for all x in the interval of convergence. The nth derivative of f at $x = 5$ is given by $f^{(n)}(5) = \dfrac{(-1)^n n!}{2^n (n+2)}$, and $f(5) = \dfrac{1}{2}$.

(a) Write the third-degree Taylor polynomial for f about $x = 5$.

(b) Find the radius of convergence of the Taylor series for f about $x = 5$.

(c) Show that the sixth-degree Taylor polynomial for f about $x = 5$ approximates $f(6)$ with error less than $\dfrac{1}{1000}$.

(a) $f'(5) = \dfrac{-1!}{2(3)},\ f''(5) = \dfrac{2!}{4(4)},\ f'''(5) = \dfrac{-3!}{8(5)}$

$$P_3(f,5)(x) = \frac{1}{2} - \frac{1}{6}(x-5) + \frac{1}{16}(x-5)^2 - \frac{1}{40}(x-5)^3$$

3 : $P_3(f,5)(x)$

<−1> each error or missing term

Note: <−1> max for improper use of extra terms, equality or +…

(b) $a_n = \dfrac{f^{(n)}(5)}{n!} = \dfrac{(-1)^n}{2^n (n+2)}$

$$\lim_{n\to\infty}\left|\frac{\dfrac{(-1)^{n+1}(x-5)^{n+1}}{2^{n+1}(n+3)}}{\dfrac{(-1)^n(x-5)^n}{2^n(n+2)}}\right| = \lim_{n\to\infty}\frac{1}{2}\left(\frac{n+2}{n+3}\right)|x-5|$$

$$= \frac{|x-5|}{2} < 1$$

The radius of convergence is 2.

4 {
1 : general term
1 : sets up ratio test
1 : computes the limit
1 : applies ratio test to get radius of convergence
}

(c) The Taylor series about $x = 5$ for the function f, when evaluated at $x = 6$, is an alternating series with absolute value of terms decreasing to 0. The error in approximating $f(6)$ with the 6th degree Taylor polynomial at $x = 6$ is less than the first omitted term in the series.

$$\left|f(6) - P_6(f,5)(6)\right| \le \frac{1}{2^7(9)} = \frac{1}{1152} < \frac{1}{1000}$$

2 {
1 : error bound $< \dfrac{1}{1000}$
1 : refers to an alternating series and indicates the error bound is found from the next term
}

AP Calculus BC–4

A moving particle has position $(x(t), y(t))$ at time t. The position of the particle at time $t = 1$ is (2,6) a. the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

(a) Find the acceleration vector at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t), y(t))$ have a slope of 8?

(d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

(a) acceleration vector $= (x''(t), y''(t)) = \left(\frac{2}{t^3}, -\frac{2}{t^3}\right)$

$$(x''(3), y''(3)) = \left(\frac{2}{27}, -\frac{2}{27}\right)$$

2 $\begin{cases} 1: \text{components of acceleration vector as a function of } t \\ 1: \text{acceleration vector at } t = 3 \end{cases}$

(b) $(x(t), y(t)) = \left(t + \frac{1}{t} + C_1,\ 2t - \frac{1}{t} + C_2\right)$

$(2, 6) = (x(1), y(1)) = (2 + C_1, 1 + C_2)$

$C_1 = 0,\ C_2 = 5$

$(x(3), y(3)) = \left(3 + \frac{1}{3}, 6 - \frac{1}{3} + 5\right) = \left(\frac{10}{3}, \frac{32}{3}\right)$

3 $\begin{cases} 1: \text{antidifferentiation} \\ 1: \text{uses initial condition at } t = 1 \\ 1: \text{position at } t = 3 \end{cases}$

Note: max 1/3 [1–0–0] if no constants of integration

(c) $\dfrac{dy}{dx} = \dfrac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 8$

$2 + \frac{1}{t^2} = 8\left(1 - \frac{1}{t^2}\right);\quad t^2 = \frac{9}{6}$

$t = \sqrt{\frac{3}{2}}$

2 $\begin{cases} 1: \frac{dy}{dx} = 8 \text{ as equation in } t \\ 1: \text{solution for } t \end{cases}$

(d) $\displaystyle\lim_{t\to\infty} \frac{dy}{dx} = \lim_{t\to\infty} \frac{2 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = 2$

– or –

Since $x(t) \to \infty$ as $t \to \infty$, the slope of the line is

$$\lim_{t\to\infty} \frac{y(t)}{x(t)} = \lim_{t\to\infty} \frac{2t - \frac{1}{t} + 5}{t + \frac{1}{t}} = 2$$

2 $\begin{cases} 1: \text{considers limit of } \frac{dy}{dx} \text{ or } \frac{y(t)}{x(t)} \\ 1: \text{answer} \end{cases}$

Note: 0/2 if no consideration of limit

AP Calculus AB–5 / BC–5

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$$\frac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

2 $\begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) When $x = 1$, $y^2 - y = 6$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3,\ y = -2$$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x-1)$

4 $\begin{cases} 1 : \ y^2 - y = 6 \\ 1 : \ \text{solves for } y \\ 2 : \ \text{tangent lines} \end{cases}$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x\left(2y - x^2\right) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$$-\frac{1}{4}x^5 = 6$$

$$x = \sqrt[5]{-24}$$

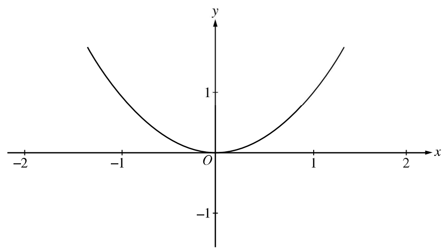
3 $\begin{cases} 1 : \ \text{sets denominator of } \dfrac{dy}{dx} \text{ equal to 0} \\ 1 : \ \text{substitutes } y = \dfrac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \quad\ \text{into the equation for the curve} \\ 1 : \ \text{solves for } x\text{-coordinate} \end{cases}$

AP Calculus BC–6

Consider the differential equation given by $\frac{dy}{dx} = x(y-1)^2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eleven points indicated.

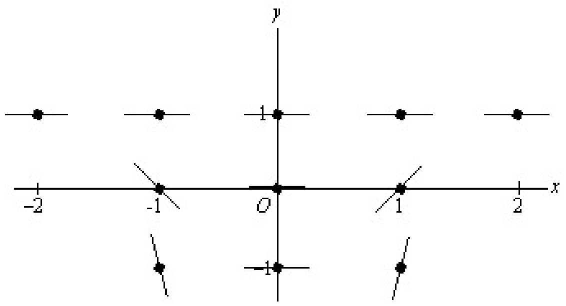
(b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown below.



(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -1$.

(d) Find the range of the solution found in part (c).

(a)



2 $\begin{cases} 1: \text{ zero slope at 7 points with } y = 1 \text{ and } x = 0 \\ 1: \text{ negative slope at } (-1,0) \text{ and } (-1,-1) \text{ positive slope at } (1,0) \text{ and } (1,-1) \text{ steeper slope at } y=-1 \text{ than } y = 0 \end{cases}$

(b) The graph does not have slope 0 where $y = 1$.

– or –

The slope field shown suggests that solutions are asymptotic to $y = 1$ from below, but the graph does not exhibit this behavior.

1 : reason

(c) $\frac{1}{(y-1)^2}\,dy = x\,dx$

$$-\frac{1}{y-1} = \frac{1}{2}x^2 + C$$

$$\frac{1}{2} = 0 + C; \quad C = \frac{1}{2}$$

$$-\frac{1}{y-1} = \frac{1}{2}\left(x^2+1\right); \quad y = 1 - \frac{2}{x^2+1}$$

5 $\begin{cases} 1: \text{ separates variables} \\ 1: \text{ antiderivatives} \\ 1: \text{ constant of integration} \\ 1: \text{ uses initial condition } f(0) = -1 \\ 1: \text{ solves for } y \\ \quad 0/1 \text{ if } y \text{ is linear} \end{cases}$

Note: max 2/5 [1-1-0-0-0] if no constant of integration

Note: 0/5 if no separation of variables

(d) range is $-1 \le y < 1$

1 : answer

0/1 if –1 not in range