CollegeBoard
Advanced Placement Program

AP® Calculus BC
2008 Scoring Guidelines
Form B

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2008 SCORING GUIDELINES (Form B)

Question 1

A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time $t \ge 0$ with

$$\frac{dx}{dt} = \sqrt{3t} \text{ and } \frac{dy}{dt} = 3\cos\left(\frac{t^2}{2}\right).$$

The particle is at position $(1, 5)$ at time $t = 4$.

(a) Find the acceleration vector at time $t = 4$.

(b) Find the y-coordinate of the position of the particle at time $t = 0$.

(c) On the interval $0 \le t \le 4$, at what time does the speed of the particle first reach 3.5 ?

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le 4$.

(a) $a(4) = \langle x''(4), y''(4)\rangle = \langle 0.433, -11.872\rangle$	1 : answer
(b) $y(0) = 5 + \int_4^0 3\cos\left(\frac{t^2}{2}\right) dt = 1.600$ or 1.601	3 : { 1 : integrand; 1 : uses $y(4) = 5$; 1 : answer
(c) Speed $= \sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)} = 3.5$ The particle first reaches this speed when $t = 2.225$ or 2.226.	3 : { 1 : expression for speed; 1 : equation; 1 : answer
(d) $\int_0^4 \sqrt{3t + 9\cos^2\left(\frac{t^2}{2}\right)}\, dt = 13.182$	2 : { 1 : integral; 1 : answer

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Question 2

For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

(a) How many kilometers does the car travel during the first 2 hours?

(b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

(a) $\int_0^2 r(t)\,dt = 206.370$ kilometers

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) $\frac{dg}{dt} = \frac{dg}{dx} \cdot \frac{dx}{dt}; \quad \frac{dx}{dt} = r(t)$

$$\left.\frac{dg}{dt}\right|_{t=2} = \left.\frac{dg}{dx}\right|_{x=206.370} \cdot r(2)$$

$$= (0.050)(120) = 6 \text{ liters/hour}$$

$3: \begin{cases} 2: \text{uses chain rule} \\ 1: \text{answer with units} \end{cases}$

(c) Let T be the time at which the car's speed reaches 80 kilometers per hour.

Then, $r(T) = 80$ or $T = 0.331453$ hours.

At time T, the car has gone

$x(T) = \int_0^T r(t)\,dt = 10.794097$ kilometers

and has consumed $g(x(T)) = 0.537$ liters of gasoline.

$4: \begin{cases} 1: \text{equation } r(t) = 80 \\ 2: \text{distance integral} \\ 1: \text{answer} \end{cases}$

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Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t) = 16 + 2\sin\left(\sqrt{t+10}\right)$ for $0 \le t \le 120$ minutes.

(a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

(b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

(c) The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

(d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

(a) $\frac{(0+7)}{2}\cdot 8 + \frac{(7+8)}{2}\cdot 6 + \frac{(8+2)}{2}\cdot 8 + \frac{(2+0)}{2}\cdot 2$
$= 115 \text{ ft}^2$

1 : trapezoidal approximation

(b) $\frac{1}{120}\int_0^{120} 115v(t)\,dt$
$= 1807.169$ or $1807.170 \text{ ft}^3/\text{min}$

3 :
- 1 : limits and average value constant
- 1 : integrand
- 1 : answer

(c) $\int_0^{24} 8\sin\left(\frac{\pi x}{24}\right)dx = 122.230$ or 122.231 ft^2

2 :
- 1 : integral
- 1 : answer

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is
$\frac{1}{20}\int_{40}^{60} C\cdot v(t)\,dt = 2181.912$ or $2181.913 \text{ ft}^3/\text{min}$.

Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds $2100 \text{ ft}^3/\text{min}$.

3 :
- 1 : volumetric flow integral
- 1 : average volumetric flow
- 1 : answer with reason

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Question 4

Let f be the function given by $f(x) = kx^2 - x^3$, where k is a positive constant. Let R be the region in the first quadrant bounded by the graph of f and the x-axis.

(a) Find all values of the constant k for which the area of R equals 2.

(b) For $k > 0$, write, but do not evaluate, an integral expression in terms of k for the volume of the solid generated when R is rotated about the x-axis.

(c) For $k > 0$, write, but do not evaluate, an expression in terms of k, involving one or more integrals, that gives the perimeter of R.

(a) For $x \ge 0$, $f(x) = x^2(k - x) \ge 0$ if $0 \le x \le k$

$$\int_0^k \left(kx^2 - x^3\right) dx = \left(\frac{k}{3}x^3 - \frac{1}{4}x^4\right)\Bigg|_{x=0}^{x=k} = \frac{k^4}{12}$$

$$\text{Area} = \frac{k^4}{12} = 2; \quad k = \sqrt[4]{24}$$

$$4 : \begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{value of integral} \\ 1 : \text{answer} \end{cases}$$

(b) $$\text{Volume} = \pi \int_0^k \left(kx^2 - x^3\right)^2 dx$$

$$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$$

(c) $$\text{Perimeter} = k + \int_0^k \sqrt{1 + \left(2kx - 3x^2\right)^2}\, dx$$

$$3 : \begin{cases} 1 : \int_0^k \sqrt{1 + (f'(x))^2}\, dx \\ 1 : \text{uses } f'(x) = 2kx - 3x^2 \text{ in integrand} \\ 1 : \text{answer} \end{cases}$$

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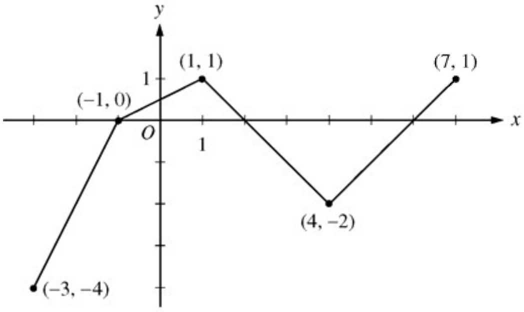
Question 5

Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.



Graph of g'

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

(a) g' changes from increasing to decreasing at $x = 1$;
g' changes from decreasing to increasing at $x = 4$.

Points of inflection for the graph of $y = g(x)$ occur at $x = 1$ and $x = 4$.

$2 : \begin{cases} 1 : x\text{-values} \\ 1 : \text{justification} \end{cases}$

(b) The only sign change of g' from positive to negative in the interval is at $x = 2$.

$$g(-3) = 5 + \int_2^{-3} g'(x)\,dx = 5 + \left(-\frac{3}{2}\right) + 4 = \frac{15}{2}$$
$$g(2) = 5$$
$$g(7) = 5 + \int_2^{7} g'(x)\,dx = 5 + (-4) + \frac{1}{2} = \frac{3}{2}$$

The maximum value of g for $-3 \le x \le 7$ is $\frac{15}{2}$.

$3 : \begin{cases} 1 : \text{identifies } x = 2 \text{ as a candidate} \\ 1 : \text{considers endpoints} \\ 1 : \text{maximum value and justification} \end{cases}$

(c) $$\frac{g(7) - g(-3)}{7 - (-3)} = \frac{\frac{3}{2} - \frac{15}{2}}{10} = -\frac{3}{5}$$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

(d) $$\frac{g'(7) - g'(-3)}{7 - (-3)} = \frac{1 - (-4)}{10} = \frac{1}{2}$$

No, the MVT does *not* guarantee the existence of a value c with the stated properties because g' is not differentiable for at least one point in $-3 < x < 7$.

$2 : \begin{cases} 1 : \text{average value of } g'(x) \\ 1 : \text{answer "No" with reason} \end{cases}$

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Question 6

Let f be the function given by $f(x) = \dfrac{2x}{1+x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Does the series found in part (a), when evaluated at $x = 1$, converge to $f(1)$? Explain why or why not.

(c) The derivative of $\ln\left(1+x^2\right)$ is $\dfrac{2x}{1+x^2}$. Write the first four nonzero terms of the Taylor series for $\ln\left(1+x^2\right)$ about $x = 0$.

(d) Use the series found in part (c) to find a rational number A such that $\left|A - \ln\left(\frac{5}{4}\right)\right| < \frac{1}{100}$. Justify your answer.

(a) $$\frac{1}{1-u} = 1 + u + u^2 + \cdots + u^n + \cdots$$
$$\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + \cdots + \left(-x^2\right)^n + \cdots$$
$$\frac{2x}{1+x^2} = 2x - 2x^3 + 2x^5 - 2x^7 + \cdots + (-1)^n 2x^{2n+1} + \cdots$$

3 : { 1 : two of the first four terms; 1 : remaining terms; 1 : general term }

(b) No, the series does not converge when $x = 1$ because when $x = 1$, the terms of the series do not converge to 0.

1 : answer with reason

(c) $$\ln\left(1+x^2\right) = \int_0^x \frac{2t}{1+t^2}\,dt$$
$$= \int_0^x \left(2t - 2t^3 + 2t^5 - 2t^7 + \cdots\right) dt$$
$$= x^2 - \frac{1}{2}x^4 + \frac{1}{3}x^6 - \frac{1}{4}x^8 + \cdots$$

2 : { 1 : two of the first four terms; 1 : remaining terms }

(d) $$\ln\left(\frac{5}{4}\right) = \ln\left(1 + \frac{1}{4}\right) = \left(\frac{1}{2}\right)^2 - \frac{1}{2}\left(\frac{1}{2}\right)^4 + \frac{1}{3}\left(\frac{1}{2}\right)^6 - \frac{1}{4}\left(\frac{1}{2}\right)^8 + \cdots$$

Let $A = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^4 = \frac{7}{32}$.

Since the series is a converging alternating series and the absolute values of the individual terms decrease to 0,

$$\left|A - \ln\left(\frac{5}{4}\right)\right| < \left|\frac{1}{3}\left(\frac{1}{2}\right)^6\right| = \frac{1}{3}\cdot\frac{1}{64} < \frac{1}{100}.$$

3 : { 1 : uses $x = \frac{1}{2}$; 1 : value of A; 1 : justification }