

Week 1: Pre-AMC Level

Problem 1 If a 5-digit number $\overline{1X2X3}$ is divisible by 9, what is X ?
(Hint: Recall the divisibility test of 9).

Problem 2 If we define

$$a \star b = a^2 - b^2,$$

what is $5 \star 4$?

Problem 3 Jack tosses two fair dice. What is the probability that the sum of two values is no less than 9?

Recall that

$$\text{Probability} = \frac{\text{Number of valid cases}}{\text{Number all possible cases}}.$$

(A) $\frac{1}{4}$ (B) $\frac{5}{18}$ (C) $\frac{1}{3}$ (D) $\frac{5}{12}$ (E) $\frac{2}{3}$

Problem 4 How many 4-digit integers $\overline{abcd}$ which satisfy that

- (1) The thousands digit a and the tens digit c are even numbers;
- (2) The hundreds digit b and unit digit d are odd numbers.
- (3) This number is divisible by 5 ?

(A) 80 (B) 100 (C) 125 (D) 250 (E) 500

Problem 5 If

$$\frac{m}{10} = \frac{6}{n} = \frac{2}{5},$$

what is $m + n$?

(A) 10 (B) 13 (C) 14 (D) 16 (E) 19

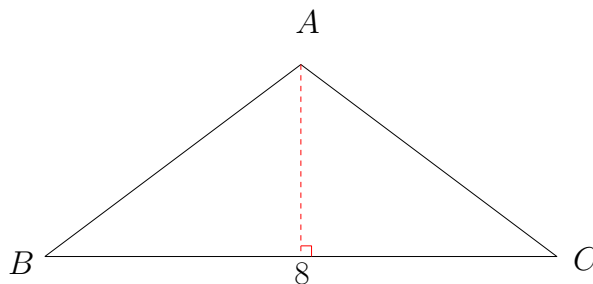
Problem 6 [Modification of AMC8] Nancy has 100 marbles. She gives 20% of them to her friend Pedro. Then Nancy gives 25% of what is left to another friend, Ben. Finally, Nancy gives 10% of what is now left in the bag to her brother Jimmy. What percentage of her original bag of marbles does Nancy have left for herself?

- (A) 20 (B) $33\frac{1}{3}$ (C) 38 (D) 45 (E) 54

Problem 7 What is the sum of all positive 2-digit integers have a remainder of 1 when divided by 9 and a remainder of 2 when divided by 2?

- (A) 29 (B) 56 (C) 84 (D) 119 (E) 164

Problem 8 $\triangle ABC$ is an isosceles triangle. $AB = AC$. If $BC = 8$ and the area of $\triangle ABC$ is 12, what is the perimeter of ABC ?



- (A) 12 (B) 16 (C) 18 (D) 19 (E) 20

Problem 9 What is the smaller angle between the hour hand and minute hand of a clock at 5 : 50 pm?

- (A) 100° (B) 125° (C) 150 (D) 155 (E) 165

Problem 10 The average age of 5 people in a room is 40 years. An 36-year-old person leaves the room. What is the average age of the four remaining people?

- (A) 39 (B) 40 (C) 41 (D) 43 (E) 45

Problem 11 In how many ways can we write 127 as the sum of two prime numbers?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

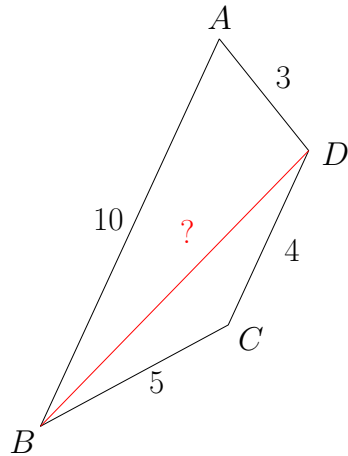
Problem 12 If for some positive integer n ,

$$\left(1 + \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \left(1 + \frac{1}{4}\right) \cdots \left(1 + \frac{1}{n}\right) = 50,$$

what is n ?

- (A) 20 (B) 39 (C) 65 (D) 99 (E) 100

Problem 13 In quadrilateral $ABCD$, $AB = 10$, $BC = 5$, $CD = 4$ and $AD = 3$. If the length of BD is an integer, what is BD ?



- (A) 7 (B) 8 (C) 9 (D) 10
(E) there is no enough information

Problem 14 What is the unit digit of 7^{2020} ? Here

$$7^n = \underbrace{7 \times 7 \times 7 \cdots 7}_n.$$

(Hint: check unit digits of 7 , 7^2 , 7^3 , $\cdots$ and find the pattern)

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

Problem 15 Here x , y , z and u represent four different numbers.

$$\begin{array}{r} x \quad y \\ + \quad z \quad x \\ \hline u \quad x \end{array} \qquad \begin{array}{r} x \quad y \\ - \quad z \quad x \\ \hline x \end{array}$$

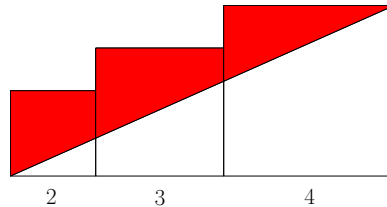
$$x + y + z + u =$$

- (A) 12 (B) 14 (C) 16 (D) 18 (E) 20

Problem 16 Billy, a seasonal worker in the town of Cowra, collected 8 buckets of cherries on his first day. Each day after that he increased the number of buckets he picked by 2 buckets per day. According to this pattern, how many buckets will he collect in the first 20 days?

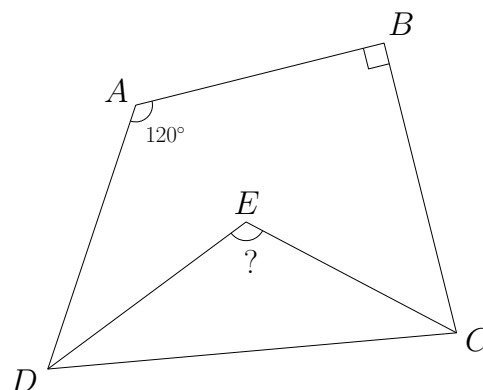
- (A) 308 (B) 500 (C) 540 (D) 580 (E) 600

Problem 17 Three squares are lined up horizontally as shown. Their side lengths are 2, 3 and 4 respectively. A straight line is drawn from the top right corner of the largest square to the bottom left hand corner of the smallest square. What is the area of the shaded region?



- (A) 10 (B) 11 (C) 13 (D) 16 (E) 18

Problem 18 In quadrilateral $ABCD$, $\angle A = 120^\circ$ and $\angle B = 90^\circ$. ED and EC bisect $\angle ADC$ and $\angle BCD$ respectively. What is $\angle E$?
(Hint: what $\angle ADC + \angle BCD$?)



- (A) 90 (B) 95 (C) 100 (D) 105 (E) 110

Problem 19 In a mathematics contest with ten problems, a student gains 6 points for a correct answer and loses 1 points for an incorrect answer. If Olivia answered every problem and her score was 32, how many correct answers did she have?

- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Problem 20 Emily and Owen going to library that is one mile from their house. They leave home simultaneously. Emily rides her bicycle to the library at a constant speed of 12 miles per hour. Owen walks to the library at a constant speed of 5 miles per hour. How many minutes before Owen does Emily arrive?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Answers

1. 6 2. 9 3. B 4. B 5. E
6. E
7. D (Hint: You may first list all positive 2-digit integers have a remainder of 1 when divided by 9. Then pick those that have a remainder of 2 when divided by 5.)
8. C (Hint: Use are to find the height on BC . Then apply Pythagorean Theorem to find AB and AC .)
9. B 10. C
11. A (Hint: Consider parity of prime numbers.)
12. D (Hint: $1 + \frac{1}{2} = \frac{3}{2}$, $1 + \frac{1}{3} = \frac{4}{3}$, $\dots$ Then consider cancellation.)
13. B (Hint: Recall that sum of two sides of a triangle is always greater than the third one.)
14. A (Hint: Check unit digits of $7, 7^2, 7^3, \dots$, and find the pattern.)
15. D 16. C (Hint: Use Gauss's formula.)
17. B (Hint: Use cut and paste technique to find the area.)
18. D (Hint: What is $\angle ADC + \angle BCD$?)
19. C 20. E