

Week 4: Pre-AMC Level

Problem 1 [AMC8 Problem] Margie's car can go 32 miles on a gallon of gas, and gas currently costs \$4 per gallon. How many miles can Margie drive on \$20?

- (A) 64 (B) 128 (C) 160 (D) 320 (E) 640

Problem 2 If n is a positive integer satisfying that

$$n + (\text{the second largest divisor of } n) = 27,$$

how many divisors does n have?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Problem 3 If

$$\sqrt{16} - \sqrt{4} = \sqrt{100} - \sqrt{n},$$

what is the number n ?

- (A) 25 (B) 36 (C) 49 (D) 64 (E) 81

Problem 4 Edward has a pile of marbles. If he counts by 3, there are 2 left. If he counts by 4, there are 3 left. If he counts by 5, there are 4 left. Let N be the total number of marbles. If N is between 170 and 190, what is the product of all digits of N ?

- (A) 0 (B) 8 (C) 14 (D) 24 (E) 63

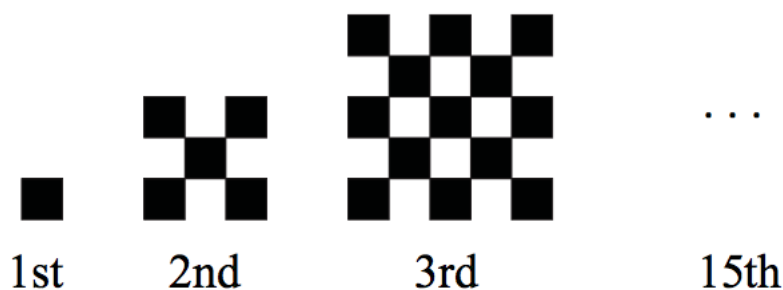
Problem 5 Square A has side length 10. The sum of areas of square B and square C equals the area of square A. If side lengths of square B and square C are both integers, what is the sum of their side lengths?

- (A) 10 (B) 11 (C) 12 (D) 14 (E) 15

Problem 6 Let N be the greatest 4-digit number whose digits have a product of 60. What is the sum of the digits of N ?

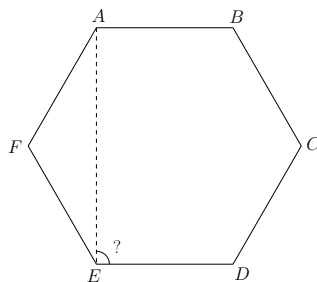
- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17

Problem 7 The diagram below shows a sequence of shapes made up of black and white floor tiles where each shape after the first has two more rows and two more columns than the one before it. How many black tiles would be required to create the 15th shape in the sequence? (British Math Competition)



- (A) 401 (B) 421 (C) 441 (D) 461 (E) 481

Problem 8 $ABCDEF$ is regular hexagon, i.e. all angles are the same and all sides are equal. What is the angle $\angle AED$?



- (A) 60° (B) 75° (C) 80° (D) 90° (E) 94°

Problem 9 Let $I = \frac{53}{25}$, $II = \frac{19}{8}$ and $III = \frac{25}{11}$. Which of the following is correct?

- (A) $I < II < III$ (B) $I < III < II$ (C) $II < III < I$
 (D) $II < I < III$ (E) $III < II < I$

Problem 10 For how many two-digit numbers, if you switch the tens and ones digits (e.g., $18 \rightarrow 81$), the new 2-digit number is 18 more than the original one?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Problem 11 [AMC 8 Problem] Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."

- (1) It is prime.
- (2) It is even.
- (3) It is divisible by 7.
- (4) One of its digits is 9.

This information allows Malcolm to determine Isabella's house number. What is its units digit?

- (A) 4 (B) 6 (C) 7 (D) 8 (E) 9

Problem 12 In the addition problem shown, m , n , p , and q represent positive digits. When the problem is completed correctly, the value of $m + n + p + q$ is (Canadian Math Competition)

$$\begin{array}{r} n\ 6\ 3 \\ 7\ p\ 2 \\ +\ 5\ 8\ q \\ \hline m\ 0\ 4\ 2 \end{array}$$

- (A) 16 (B) 18 (C) 22 (D) 24 (E) 30

Problem 13 If two numbers x and y satisfy that

$$(x + 1)^2 + (y - 5)^2 = 0,$$

what is $x + y$?

- (A) -1 (B) 0 (C) 4 (D) 6 (E) 7

Problem 14 Abel tosses two coins and Ben tosses one coin. What is the probability that they get the equal number of heads?

Recall that

$$\text{Probability} = \frac{\text{Number of valid cases}}{\text{Number all possible cases}}.$$

- (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{3}{8}$ (E) $\frac{2}{5}$

Problem 15 x , y , z and w are four numbers. If the average of the following three numbers

$$x - y, \quad y - z, \quad z - w$$

is 10 and $w = 5$. What is x ?

- (A) 15 (B) 20 (C) 25 (D) 35
(E) there is not enough information

Problem 16 What is the tens digit of

$$7^{2020}?$$

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

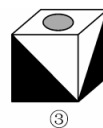
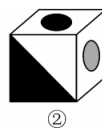
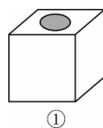
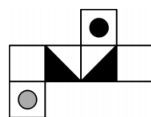
Problem 17 Two trains 300 miles apart are traveling toward each other along the same track. One train goes 70 miles per hour; the other runs at 80 miles per hour. If both of them leave their stations at 7:00am, when will they meet?

- (A) 8 : 00 *am* (B) 8 : 30 *am* (C) 9 : 00 *am* (D) 9 : 30 *am*
(E) 10 : 00 *am*

Problem 18 $ABCD$ is a rectangle with $AB = 10$ and $BC = 20$. If AB is increased by 20% and BC is decreased by 20%, the area of $ABCD$ has changed by what percent? (Below $-$ means decrease and $+$ means increase)?

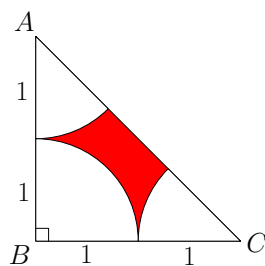
- (A) -20% (B) -4% (C) 0 (D) 4% (E) 20%

Problem 19 Which cube on the right can be unfolded into the two-dimensional figure on the left (so called *net*) (Hope Cup Math Competition, China)?



- (A) 1 (B) 2 (C) 3 (D) 4
(E) None of these

Problem 20 $\triangle ABC$ is a right triangle with $\angle B = 90^\circ$, $AB = BC = 2$. Draw three circles with radius 1 centered at A , B and C respectively. What is the area of the region (the red part) that lie within the triangle but outside three circles?



- (A) $\frac{1}{4}$ (B) $\frac{\pi}{10}$ (C) $\frac{1}{3}$ (D) $2 - \frac{\pi}{2}$ (E) $\frac{1}{2}$

Answers

1. C 2. D 3. D 4. E 5. D

6. B 7. B

8. D (Hint: The sum of 6 angles of a hexagon is $(6 - 2) \times 180 = 720$.
Also, $\angle AED = \angle FED - \angle FEA$)

9. B 10. E

11. D 12. D 13. C 14. D

15. D (Hint: What is $(x - y) + (y - z) + (z - w)$?)

16. A (Hint: Examine tens digits of $7, 7^2, 7^3, 7^4, 7^5, \dots$ and try to find a pattern.)

17. C 18. B 19. A

20. D (Hint: Use cut-and-paste. The red region = The right triangle
- (Three portions of a circle with radius 1).

If you put these three portions together, what fraction of a full circle do you get?)