ESAT PRACTICE TEST SERIES

ESAT Mock Test 3

Math 1 + Physics + Math 2

Duration: 2 hours (3 x 40 min)

Correct answers, fixed wording and concise worked solutions

ThrivingScholars

Worked solutions and verified answer key

This booklet contains the corrected student-facing answer key and concise worked solutions for ESAT Mock Test 3. The fixes applied are: Math 1 Q3 wording; Physics Q4, Q13, Q22 and Q26; Math 2 Q1, Q3, Q18 and Q19.

Math 1

1 C · 2 B · 3 E · 4 C · 5 D · 6 D · 7 E · 8 A · 9 A · 10 D · 11 D · 12 C · 13 B
14 A · 15 B · 16 E · 17 E · 18 E · 19 D · 20 E · 21 D · 22 D · 23 C · 24 D · 25 D · 26 A
27 C

Physics

1 D · 2 E · 3 B · 4 A · 5 E · 6 D · 7 F · 8 E · 9 A · 10 C · 11 D · 12 F · 13 E
14 H · 15 C · 16 F · 17 A · 18 H · 19 D · 20 A · 21 D · 22 F · 23 E · 24 B · 25 H · 26 G
27 C

Math 2

1 C · 2 A · 3 C · 4 C · 5 C · 6 B · 7 D · 8 E · 9 D · 10 F · 11 D · 12 B · 13 A
14 D · 15 E · 16 G · 17 D · 18 G · 19 C · 20 C · 21 C · 22 G · 23 E · 24 D · 25 A · 26 B
27 A

ESAT MOCK TEST 3

Math 1

27 questions · Answers placed beside each question for fast review.

MATH 1

Question 1

Answer **C**

There are 8 consecutive integers. If the sum of the first three is 63, what is the largest one?

- **A** 17
- **B** 25
- **C** 27
- **D** 30
- **E** 31

SHORT SOLUTION

Let the first integer be n. Then $n+(n+1)+(n+2)=63$, so $3n+3=63$, giving $n=20$. The largest of the eight integers is 27.

MATH 1

Question 2

Answer **B**

Joshua is 10 years old and his father is 42 years old. After how many years, his father's age is three times Joshua's age?

- **A** 5
- **B** 6
- **C** 7
- **D** 8
- **E** 9

SHORT SOLUTION

After t years, $42+t=3(10+t)$. Hence $42+t=30+3t$, so $t=6$.

MATH 1

Question 3

Answer **E**

N is a 7-digit number consists of 2 or 3. There are more 2s than 3s in N. If N is divisible by both 3 and 4, what is the remainder of N when it is divided by 7?

- **A** 1
- **B** 2
- **C** 3
- **D** 4
- **E** 5

SHORT SOLUTION

Let k be the number of 3s. The digit sum is $14+k$, so divisibility by 3 gives $k=1$. Divisibility by 4 forces the last two digits to be 32. Thus $N=2222232$, and $N\equiv 5 \pmod 7$.

MATH 1

Question 4

Answer **C**

The preliminary round of a violin competition is divided into four rounds. According to the rule, the average score of a percipient of these four rounds must be not less than 96 points in order to enter the finals. Shaelyn's scores in the first three rounds are 95, 97, and 94. What is the minimum score that she needs to get on the fourth round in order to enter the finals? (Hope Cup Math Competition, China)

- **A** 96
- **B** 97
- **C** 98
- **D** 99
- **E** 100

SHORT SOLUTION

The total score needed is $4\times 96=384$. The first three scores total $95+97+94=286$, so the fourth score must be at least $384-286=98$.

MATH 1

Question 5

Answer D

A haunted house has 8 windows. In how many ways can George the Ghost enter the house by one window and leave by a different window?

- **A** 28
- **B** 32
- **C** 40
- **D** 56
- **E** 63

SHORT SOLUTION

There are 8 choices for the entry window and then 7 different choices for the exit window, giving $8 \times 7 = 56$.

MATH 1

Question 6

Answer D

Jackie bought a bottle (30 oz) of 90% alcohol from pharmacy to make homemade disinfectant spray. How many ounces of water should he add in order to dilute it into 75% alcohol? (Hint: The amount of alcohol remains the same.)

- **A** 3
- **B** 4
- **C** 5
- **D** 6
- **E** 7

SHORT SOLUTION

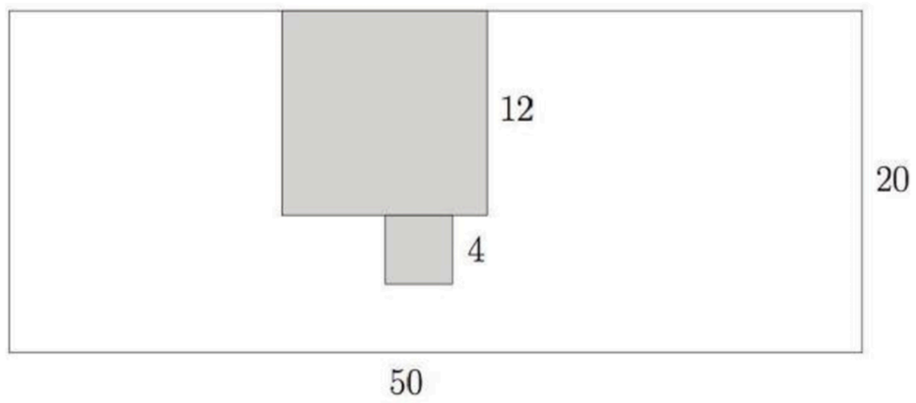
The amount of alcohol is $30 \times 0.90 = 27$ oz. If the final concentration is 75%, then $0.75T = 27$, so $T = 36$. Water added: $36 - 30 = 6$ oz.

MATH 1

Question 7

Answer E

Cut a 12×12 square and a 4×4 square from a 50×20 rectangle paper. What is the perimeter of the remaining part?



A 128

B 140

C 152

D 160

E 172

SHORT SOLUTION

The original rectangle perimeter is $2(50 + 20) = 140$. The 12×12 notch increases the perimeter by 24, and the attached 4×4 cut-out adds another 8. Total perimeter $= 140 + 32 = 172$.

MATH 1

Question 8

Answer **A**

Jasmine keeps adding numbers one by one from the following list

1, 3, 5, 7, $\cdots$.

as $1+3=4$, $1+3+5=9$, $1+3+5+7=16$, etc. At one point, she got 379 and realized that one number is missed in the addition. What is the sum of digits of the missing number?

A 3

B 6

C 8

D 9

E 11

SHORT SOLUTION

The sum of the first n odd numbers is n^2. Since 379 is just below $20^2=400$, the missing number is $400-379=21$. The digit sum is $2+1=3$.

MATH 1

Question 9

Answer **A**

$\overline{ab}$ and $\overline{cd}$ are two 2-digit numbers. If

$\overline{ab}-\overline{cd}=24$

and

$\overline{1ab}-\overline{cd1}=15$,

what is $a+d$?

A 5

B 8

C 10

D 11

E 12

SHORT SOLUTION

Using place value, $\overline{ab}=10a+b$ and $\overline{cd}=10c+d$. Solving the two given equations gives $a+d=5$.

MATH 1

Question 10

Answer **D**

The speed of a train is 15 meters per second. If it takes the train 10 seconds to pass a lamp post, how long does it take the train to completely pass a 300-meter long tunnel, in seconds? (Hint: first figure out the length of the train based on the information related to the lamppost)

- **A** 10
- **B** 20
- **C** 25
- **D** 30
- **E** 35

SHORT SOLUTION

The train length is $15 \times 10 = 150$ m. To pass the tunnel completely, it travels $300 + 150 = 450$ m. Time $= 450/15 = 30$ s.

MATH 1

Question 11

Answer **D**

How many pairs of positive integers (x, y) satisfy that $\frac{x}{3} = \frac{4}{y}$?

- **A** 3
- **B** 4
- **C** 5
- **D** 6
- **E** 7

SHORT SOLUTION

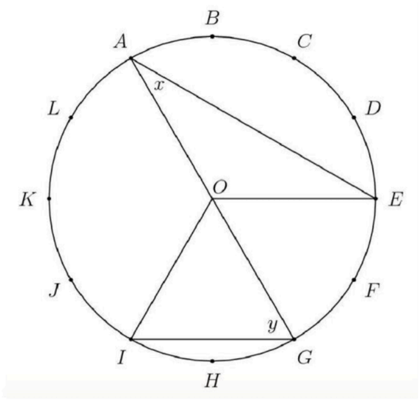
From $x/3 = 4/y$, we get $xy = 12$. The positive factor pairs are $(1, 12), (2, 6), (3, 4), (4, 3), (6, 2), (12, 1)$, so there are 6 pairs.

MATH 1

Question 12

Answer C

The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the sum of the angles x and y?



A 75

B 80

C 90

D 120

E 150

SHORT SOLUTION

The circle is divided into 12 equal arcs, so each arc is $30°$. Using central and inscribed angle relationships from the diagram gives $x + y = 90°$.

MATH 1

Question 13

Answer B

If for whole numbers x, y and z,

$360 = 2^x \times 3^y \times 5^z$,

what is $x + y + z$? Here $a^n = a \times a \times \cdots \times a$.

For example, $100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$.

- **A** 5
- **B** 6
- **C** 7
- **D** 8
- **E** 9

SHORT SOLUTION

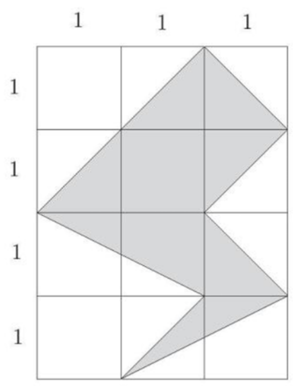
Prime factorise: $360 = 36 \times 10 = 2^2 \times 3^2 \times 2 \times 5 = 2^3 \times 3^2 \times 5^1$. Hence $x + y + z = 3 + 2 + 1 = 6$.

MATH 1

Question 14

Answer A

What is the area of the shaded region in the following 3×4 grid?



A 5

B 6

C 7

D 8

E 9

SHORT SOLUTION

Counting the shaded polygon by subtracting the surrounding right triangles from the 3×4 rectangle gives shaded area 5.

MATH 1

Question 15

Answer B

Randomly choose 3 different numbers from the set $\{1,\ 3,\ 5,\ 7,\ 9\}$. What is the probability that the three selected numbers could be side lengths of a triangle?

A $\frac{1}{5}$

B $\frac{3}{10}$

C $\frac{1}{3}$

D $\frac{2}{5}$

E $\frac{1}{2}$

SHORT SOLUTION

There are \b$\binom{5}{3} = 10$ triples. The valid triangle triples are $(3, 5, 7), (3, 7, 9), (5, 7, 9)$, so the probability is $3/10$.

MATH 1

Question 16

Answer **E**

Marley practices exactly one sport each day of the week. She runs three days a week but never on two consecutive days. On Monday she plays basketball and two days later golf. She swims and plays tennis, but she never plays tennis the day after running or swimming. Which day of the week does Marley swim?

- **A** Sunday
- **B** Tuesday
- **C** Thursday
- **D** Friday
- **E** Saturday

SHORT SOLUTION

Monday is basketball and Wednesday is golf. Applying the non-consecutive running rule and the condition that tennis is not immediately after running or swimming leaves Saturday as the swimming day.

MATH 1

Question 17

Answer **E**

Three toy cars run around a round track. It takes them 8, 10 and 12 seconds to finish one full circle respectively. If three cars start together from the same point and run toward the same direction, after how many seconds will they get back to the starting point together for the first time?

- **A** 20
- **B** 40
- **C** 60
- **D** 80
- **E** 120

SHORT SOLUTION

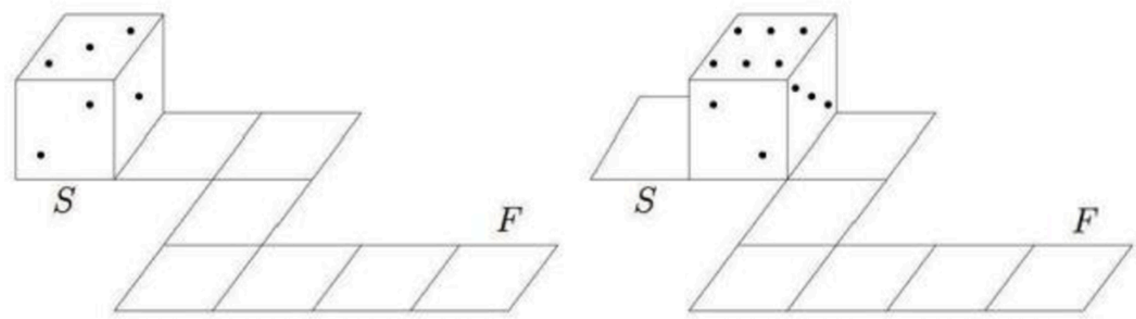
The cars return together after the least common multiple of $8, 10, 12$, which is 120 seconds.

MATH 1

Question 18

Answer **E**

Opposite faces of a die always add to 7. A die rolls on a circuit as represented below. At the starting point (S) the top face is 3. Which will be the top face at the final point (F)?



A 2

B 3

C 4

D 5

E 6

F 7

G 8

H 9

SHORT SOLUTION

Track the die through the path, using opposite faces summing to 7 after each roll. The final top face is 6.

MATH 1

Question 19

Answer **D**

How many numbers x satisfy that $(x^2-2)^2=4$?

- **A** 0
- **B** 1
- **C** 2
- **D** 3
- **E** 4
- **F** 5
- **G** 6
- **H** 7

SHORT SOLUTION

$(x^2-2)^2=4$ gives $x^2-2=\pm 2$. Thus $x^2=4$ or $x^2=0$, giving $x=-2,0,2$. There are 3 real numbers.

MATH 1

Question 20

Answer **E**

For how many two-digit numbers, if you switch the tens and ones digits (e.g., 18 → 81), the new 2-digit number is 18 more than the original one?

- **A** 3
- **B** 4
- **C** 5
- **D** 6
- **E** 7

SHORT SOLUTION

Let the number be $10a+b$. Reversing digits gives $10b+a$. The condition gives $10b+a=10a+b+18$, so $b-a=2$. There are 7 possible tens digits.

MATH 1

Question 21

Answer D

[AMC 8 Problem] Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."

- It is prime.
- It is even.
- It is divisible by 7.
- One of its digits is 9.

This information allows Malcolm to determine Isabella's house number. What is its units digit?

A 4

B 6

C 7

D 8

E 9

SHORT SOLUTION

The number must have exactly three of the four properties. The number 98 is even, divisible by 7, has a digit 9, and is not prime. Its units digit is 8.

MATH 1

Question 22

Answer D

In the addition problem shown, m, n, p, and q represent positive digits. When the problem is completed correctly, the value of $m + n + p + q$ is (Canadian Math Competition)

A 16

B 18

C 22

D 24

E 30

SHORT SOLUTION

From the units column, $3 + 2 + q$ ends in 2, so $q = 7$ with carry 1. From the tens column, $6 + p + 8 + 1$ ends in 4, so $p = 9$. From the hundreds column, $n + 7 + 5 + 2$ ends in 0, so $n = 6$ with carry $m = 2$. Therefore $m + n + p + q = 2 + 6 + 9 + 7 = 24$.

MATH 1

Question 23

Answer C

If two numbers x and y satisfy that $(x+1)^2+(y-5)^2=0$, what is $x+y$?

- **A** -1
- **B** 0
- **C** 4
- **D** 6
- **E** 7

SHORT SOLUTION

A sum of squares is zero only if both squares are zero. Thus $x+1=0$ and $y-5=0$, so $x=-1, y=5$, and $x+y=4$.

MATH 1

Question 24

Answer D

Abel tosses two coins and Ben tosses one coin. What is the probability that they get the equal number of heads?

Recall that

$$\text{Probability} = \frac{\text{Number of valid cases}}{\text{Number all possible cases}}.$$

- **A** $\frac{1}{8}$
- **B** $\frac{1}{4}$
- **C** $\frac{1}{3}$
- **D** $\frac{3}{8}$
- **E** $\frac{2}{5}$

SHORT SOLUTION

There are 8 equally likely outcomes. Equal heads occurs when Abel gets 0 heads and Ben gets 0 heads, or when Abel gets 1 head and Ben gets 1 head: $1+2=3$ cases. Probability $=3/8$.

MATH 1

Question 25

Answer **D**

x, y, z and w are four numbers. If the average of the following three numbers $x-y,\ y-z,\ z-w$ is 10 and $w=5$. What is x?

A 15

B 20

C 25

D 35

E there is no enough information

SHORT SOLUTION

The average of $x-y, y-z, z-w$ is 10, so their sum is 30. The sum telescopes:
$(x-y)+(y-z)+(z-w)=x-w=30$. Since $w=5$, $x=35$.

MATH 1

Question 26

Answer **A**

What is the tens digit of 7^{2020}?

A 0

B 1

C 2

D 3

E 4

SHORT SOLUTION

The last two digits of powers of 7 cycle: $07, 49, 43, 01$. Since 2020 is divisible by 4, 7^{2020} ends in 01, so the tens digit is 0.

MATH 1

Question 27

Answer **C**

Two trains 300 miles apart are traveling toward each other along the same track. One train goes 70 miles per hour; the other runs at 80 miles per hour. If both of them leave their stations at 7:00am, when will they meet?

A 8 : 00 am

B 8 : 30 am

C 9 : 00 am

D 9 : 30 am

E 10 : 00 am

SHORT SOLUTION

The trains approach each other at $70 + 80 = 150$ miles per hour. Time $= 300/150 = 2$ hours, so they meet at 9:00 am.

ESAT MOCK TEST 3

Physics

27 questions · Answers placed beside each question for fast review.

PHYSICS

Question 1

Answer **D**

Which of the following statements are correct in relation to Newton's 3rd law?

- 1. For every action there is an equal and opposite reaction.
- 2. According to Newton's 3rd law, there are no isolated forces.
- 3. Rockets cannot accelerate in deep space because there is nothing to generate an equal and opposite force.

A Only 1

B Only 2

C Only 3

D 1 and 2

E 2 and 3

F 1 and 3

SHORT SOLUTION

Statement 1 is the common formulation of Newton's third law. Statement 2 presents a consequence of the application of Newton's third law.

Statement 3 is false: rockets can still accelerate because the products of burning fuel are ejected in the opposite direction from which the rocket needs to accelerate.

PHYSICS

Question 2

Answer **E**

Which of the following statements are correct?

- 1. Positively charged objects have gained electrons.
- 2. Electrical charge in a circuit over a period of time can be calculated if the voltage and resistance are known.
- 3. Objects can be charged by friction.

A Only 1

B Only 2

C Only 3

D 1 and 2

E 2 and 3

F 1 and 3

G 1, 2 and 3

SHORT SOLUTION

Positively charged objects have lost electrons. $\text{Charge} = \text{Current} \times \text{Time} = \frac{\text{Voltage}}{\text{Resistance}} \times \text{Time}$.

Objects can become charged by friction as electrons are transferred from one object to the other.

PHYSICS

Question 3

Answer **B**

Which of the following statements is true?

A The gravitational force between two objects is independent of their mass.

B Each planet in the solar system exerts a gravitational force on the Earth.

C For satellites in a geostationary orbit, acceleration due to gravity is equal and opposite to the lift from engines.

D Two objects that are dropped from the Eiffel tower will always land on the ground at the same time if they have the same mass.

E All of the above.

F None of the above.

SHORT SOLUTION

Each body of mass exerts a gravitational force on another body with mass. This is true for all planets as well.

Gravitational force depends on the masses of both objects. Satellites stay in orbit due to centripetal force that acts tangentially to gravity (not because of the thrust from their engines). Two objects will only land at the same time if they also have the same shape or they are in a vacuum; otherwise air resistance gives different terminal velocities.

PHYSICS

Question 4

Answer **A**

Which of the following best defines an electrical conductor?

A Conductors allow electrical charge to pass through them easily because they contain mobile charge carriers and have low resistance.

B Conductors are usually made from non-metals and they conduct electrical charge in multiple directions.

C Conductors are usually made from metals and they conduct electrical charge in one fixed direction.

D Conductors are usually made from non-metals and they conduct electrical charge in one fixed direction.

E Conductors allow the passage of electrical charge with zero resistance because they contain freely mobile charged particles.

F Conductors allow the passage of electrical charge with maximal resistance because they contain charged particles that are fixed and static.

SHORT SOLUTION

A conductor is a material that allows charge to move easily because it contains mobile charge carriers. Ordinary conductors have low resistance, not zero resistance.

PHYSICS

Question 5

Answer **E**

An 800 kg compact car delivers 20% of its power output to its wheels. If the car has a mileage of 30 miles/gallon and travels at a speed of 60 miles/hour, how much power is delivered to the wheels? 1 gallon of petrol contains 9×10^8 J.

A 10 kW

B 20 kW

C 40 kW

D 50 kW

E 100 kW

SHORT SOLUTION

First, calculate the rate of petrol consumption:

$$\frac{\text{Speed}}{\text{Consumption}} = \frac{60 \text{ miles/hour}}{30 \text{ miles/gallon}} = 2 \text{ gallons/hour}$$

Therefore, the total power is:

$$2 \text{ gallons} = 2 \times 9 \times 10^8 = 18 \times 10^8 \text{ J}$$

$$1 \text{ hour} = 60 \times 60 = 3600 \text{ s}$$

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \frac{18 \times 10^8}{3600}$$

$$P = \frac{18}{36} \times 10^6 = 5 \times 10^5 \text{ W}$$

Since efficiency is 20%, the power delivered to the wheels is

$$5 \times 10^5 \times 0.2 = 10^5 \text{ W} = 100 \text{ kW}.$$

PHYSICS

Question 6

Answer D

Which of the following statements about beta radiation are true?

- I. After a beta particle is emitted, the atomic mass number is unchanged.
- 2. Beta radiation can penetrate paper but not aluminium foil.
- 3. A beta particle is emitted from the nucleus of the atom when an electron changes into a neutron.

A I only

B 2 only

C I and 3

D I and 2

E 2 and 3

F I, 2 and 3

SHORT SOLUTION

Beta radiation is stopped by a few millimetres of aluminium, but not by paper. In β-radiation, a neutron changes into a proton plus an emitted electron, so the atomic mass number remains unchanged.

PHYSICS

Question 7

Answer F

A car with a weight of 15,000 N is travelling at a speed of 15 $\mathrm{m\,s^{-1}}$ when it crashes into a wall and is brought to rest in 10 milliseconds. Calculate the average braking force exerted on the car by the wall. Take $g = 10\,\mathrm{m\,s^{-2}}$.

A $1.25 \times 10^4\,\mathrm{N}$

B $1.25 \times 10^5\,\mathrm{N}$

C $1.25 \times 10^6\,\mathrm{N}$

D $2.25 \times 10^4\,\mathrm{N}$

E $2.25 \times 10^5\,\mathrm{N}$

F $2.25 \times 10^6\,\mathrm{N}$

SHORT SOLUTION

Firstly, calculate the mass of the car:

$$\text{mass} = \frac{\text{Weight}}{g} = \frac{15000}{10} = 1500\ \text{kg}.$$

Then using $v = u + at$ where $v = 0\ \mathrm{m\,s^{-1}}$, $u = 15\ \mathrm{m\,s^{-1}}$ and $t = 10 \times 10^{-3}\ \mathrm{s} = 0.01\ \mathrm{s}$.

$$a = \frac{0 - 15}{0.01} = 1500\ \mathrm{m\,s^{-2}}.$$

$$F = ma = 1500 \times 1500 = 2\,250\,000\ \mathrm{N}.$$

PHYSICS

Question 8

Answer **E**

Which of the following statements are correct?

- I. Electrical insulators are usually metals e.g. copper.
- 2. The flow of charge through electrical insulators is extremely low.
- 3. Electrical insulators can be charged by rubbing them together.

A Only 1

B Only 2

C Only 3

D I and 2

E 2 and 3

F I and 3

G I, 2 and 3

SHORT SOLUTION

Electrical insulators offer high resistance to the flow of charge. Insulators are usually non-metals; metals conduct charge very easily. Since charge does not flow easily to even out, they can be charged with friction.

PHYSICS

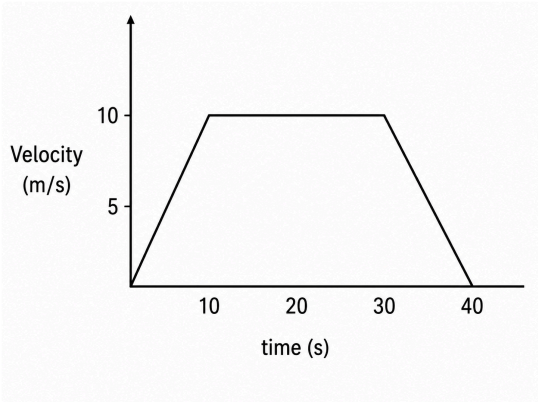
Question 9

Answer **A**

The graph below represents a car's movement. At $t = 0$ the car's displacement was 0 m.

Which of the following statements are NOT true?

- 1. The car is reversing after $t = 30$.
- 2. The car moves with constant acceleration from $t = 0$ to $t = 10$.
- 3. The car moves with constant speed from $t = 10$ to $t = 30$.



A 1 only

B 2 only

C 3 only

D 1 and 3

E 1 and 2

F 2 and 3

G 1, 2 and 3

SHORT SOLUTION

The car accelerates for the first 10 seconds at a constant rate and then decelerates after $t = 30$ seconds. It does not reverse, as the velocity is not negative. Therefore, only statement I is not true.

PHYSICS

Question 10

Answer **C**

A 1,000 kg rocket is launched during a thunderstorm and reaches a constant velocity 30 seconds after launch. Suddenly, a strong gust of wind acts on it for 5 seconds with a force of 10,000 N in the direction of movement. What is the resulting change in velocity?

- **A** $0.5\ \mathrm{m\,s^{-1}}$
- **B** $5\ \mathrm{m\,s^{-1}}$
- **C** $50\ \mathrm{m\,s^{-1}}$
- **D** $500\ \mathrm{m\,s^{-1}}$
- **E** $5000\ \mathrm{m\,s^{-1}}$
- **F** More information needed

SHORT SOLUTION

Using $F = ma$, where the unknown acceleration is $a = \dfrac{\Delta v}{\Delta t}$.

Hence: $\dfrac{F}{m} = \dfrac{\Delta v}{\Delta t}$.

Given $F = 10{,}000\ \mathrm{N}$, $m = 1{,}000\ \mathrm{kg}$ and $\Delta t = 5\ \mathrm{s}$.

So $\dfrac{10{,}000}{1{,}000} = \dfrac{\Delta v}{5}$.

Therefore $\Delta v = 10 \times 5 = 50\ \mathrm{m\,s^{-1}}$.

PHYSICS

Question 11

Answer **D**

A 0.5 tonne crane lifts a 0.01 tonne wardrobe by 100 cm in 5,000 milliseconds. Calculate the average power developed by the crane. Take $g = 10\,\mathrm{m\,s^{-2}}$.

A 0.2 W

B 2 W

C 5 W

D 20 W

E 50 W

F More information needed

SHORT SOLUTION

This question tests conversion to SI units and selection of relevant values.

0.01 tonnes = 10 kg; 100 cm = 1 m; 5,000 ms = 5 s.

$P = \frac{\text{Work Done}}{t} = \frac{F \times d}{t}$

The force is the weight of the wardrobe: $10 \times g = 10 \times 10 = 100\,\mathrm{N}$.

Thus, $P = \frac{100 \times 1}{5} = 20\,\mathrm{W}$.

PHYSICS

Question 12

Answer **F**

A 20 V battery is connected to a circuit consisting of a 1 Ω and 2 Ω resistor in parallel. Calculate the overall current of the circuit.

A 6.67 A

B 8 A

C 10 A

D 12 A

E 20 A

F 30 A

SHORT SOLUTION

Recall the resistance of a parallel circuit: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$.

Thus, $\frac{1}{R_T} = 1 + \frac{1}{2} = \frac{3}{2}$ and therefore $R = \frac{2}{3}\,\Omega$.

Using Ohm's Law: $I = \frac{V}{R} = \frac{20\,\mathrm{V}}{\frac{2}{3}\,\Omega} = 20 \times \frac{3}{2} = 30\,\mathrm{A}$.

PHYSICS

Question 13

Answer **E**

Which of the following statements is correct?

Assume the light ray crosses the boundary at a non-zero angle to the normal.

A. The speed of light changes when it enters water.

B. The speed of light changes when it leaves water.

C. The direction of light changes when it enters water.

D. The direction of light changes when it leaves water.

E. All of the above.

F. None of the above.

SHORT SOLUTION

At a non-zero angle to the normal, light changes speed when it passes between air and water and also changes direction due to refraction. Hence all four statements are true under the stated assumption.

PHYSICS

Question 14

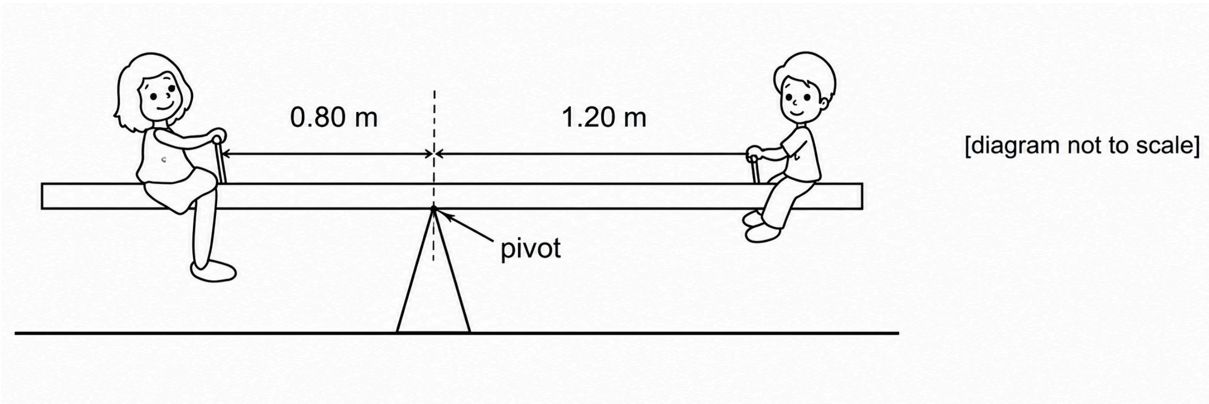
Answer **H**

A plank of non-uniform density which has a mass of 15 kg is used to make a see-saw. A pivot is placed under the centre of the plank as shown on the diagram.

A boy of mass 35 kg sits at one end of the plank with his centre of gravity 1.20 m from the pivot. The see-saw balances when a woman of mass 60 kg sits on the plank on the other side of the pivot. Her centre of gravity is 0.80 m from the pivot.

Where is the centre of gravity of the plank and what is the magnitude of the force between the pivot and the plank?

(The gravitational field strength g is 10 N kg^-1.)



A 0.40 m on left of pivot; 100 N

B 0.40 m on left of pivot; 1100 N

C at the pivot; 100 N

D at the pivot; 1100 N

E 0.20 m on right of pivot; 100 N

F 0.20 m on right of pivot; 1100 N

G 0.40 m on right of pivot; 100 N

H 0.40 m on right of pivot; 1100 N

SHORT SOLUTION

Assume the centre of mass is at x metres to the right of the pivot.

Moments about pivot (CW positive):

$$35g(1.20) + Wx - 60g(0.8) = 0$$

$$Wx = 60g(0.8) - 35g(1.2)$$

$$Wx = 600(0.8) - 350(1.2) = 480 - 420 = 60$$

$$x = \frac{60}{W} = \frac{60}{150} = \frac{2}{5} = 0.40\,\mathrm{m}$$

Newton’s Second Law (vertical equilibrium on the plank):

$$R - W - 35g - 60g = 0$$

$$R = 150 + 350 + 600 = 1100\,\mathrm{N}$$

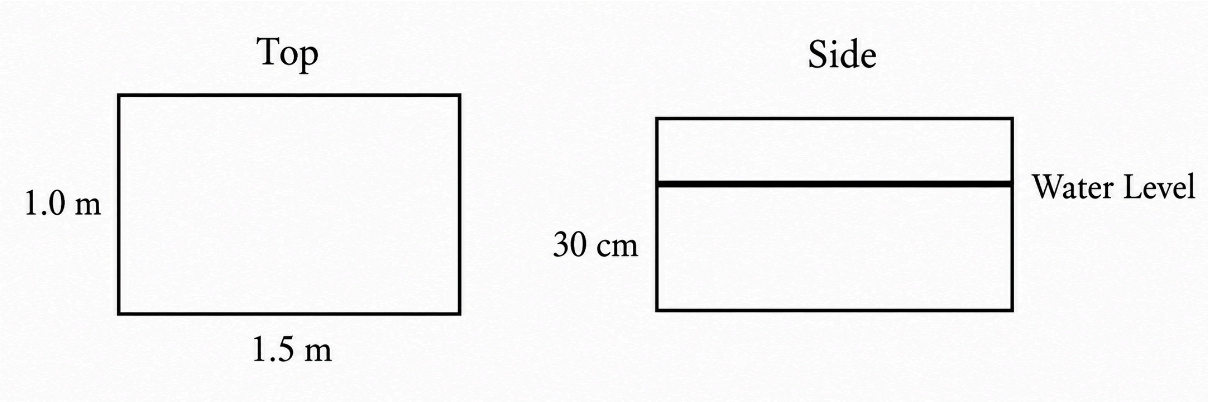
PHYSICS

Question 15

Answer C

A bath that is 1.0 m by 1.5 m is filled with water to a depth of 30 cm. The water in the bath is 40°C and it has been heated from the mains water at 20°C using a heating element that heats water on demand. It takes 3 minutes to run the bath and the heating element has an efficiency of 80%. The density of water, ρ, is 1000 kg m^{-3}, and the specific heat capacity, C_p, can be approximated as 4000 J kg^{-1} K^{-1}.

What power is used by the heating element?



A 2500 kW

B 2000 kW

C 250 kW

D 200 kW

E 100 kW

F 360 kW

G 3600 kW

H 1000 kW

SHORT SOLUTION

Answer: C

Work out how much energy is used to heat the water, then divide by the time taken to fill the bath to find the power used without accounting for losses. Then take account of the heating element efficiency to find the total power used.

Find the mass of water:

$m = \rho V$

$V = 1 \times 1.5 \times 0.3 = 0.45 \text{ m}^3$

$\therefore m = 1000 \times 0.45 = 450 \text{ kg}$

Energy: $E = mc_p \Delta T$

$E = 450 \times 4000 \times 20 = 3.6 \times 10^7 \text{ J}$

$P = \frac{\Delta E}{\Delta t} = \frac{3.6 \times 10^7}{60 \times 3} = 2 \times 10^5 \text{ W} = 200 \text{ kW}$

$P = \text{Efficiency} \times P_{\text{tot}}$

$\therefore P_{\text{tot}} = \frac{P}{\text{Efficiency}} = \frac{200}{0.8} = 250 \text{ kW}$

PHYSICS

Question 16

Answer **F**

A 5 kW power source is used to generate a current through a circuit with a resistance of $100\,\Omega$. The charge of an electron, e, is 1.6×10^{-19} C. How many electrons pass through the circuit in 16 seconds?

A 4×10^{21}

B 5×10^{20}

C $4\sqrt{3} \times 10^{19}$

D 5×10^{18}

E $5\sqrt{2} \times 10^{21}$

F $5\sqrt{2} \times 10^{20}$

G $8\sqrt{2} \times 10^{22}$

H $8\sqrt{2} \times 10^{21}$

SHORT SOLUTION

Answer: F

The current is the flow of charge in the circuit. Work out the current and use it to find the total charge and consequently the total number of electrons that flow in a given time.

$$P = I^2 R$$

$$I^2 = \frac{P}{R} = \frac{5000}{100} = 50 \text{ A}^2$$

$$I = \sqrt{50} = 5\sqrt{2} \text{ A}$$

Remember where Q is charge: $I = \dfrac{\Delta Q}{\Delta t}$

$$\therefore \Delta Q = \Delta t \times I = 16 \times 5\sqrt{2} \text{ C}$$

Number of electrons $= \dfrac{\Delta Q}{e}$

$$= \frac{16 \times 5\sqrt{2}}{1.6 \times 10^{-19}} = \frac{16 \times 5\sqrt{2}}{16 \times 10^{-20}} = 5\sqrt{2} \times 10^{20}$$

PHYSICS

Question 17

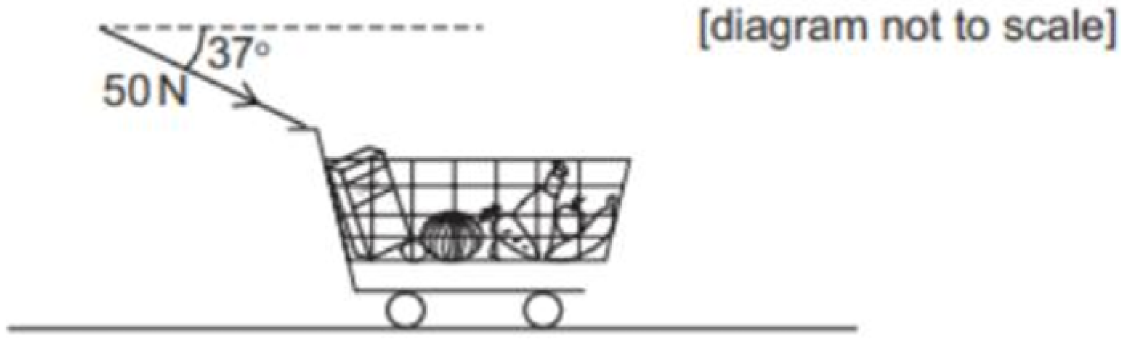
Answer **A**

A shopper pushes a supermarket trolley a distance of 15 m in a straight line across a level, horizontal surface. The shopper applies a constant force of 50 N at an angle of 37° below the horizontal. The total weight of the trolley and its contents is 350 N.

[diagram not to scale]

What is the magnitude of the total vertical force that the surface exerts on the trolley and how much work is done by the pushing force?

(You may use the approximations $\sin 37° = 0.60$; $\cos 37° = 0.80$.)



- **A** vertical force 380 N; work done 600 J
- **B** vertical force 380 N; work done 750 J
- **C** vertical force 390 N; work done 450 J
- **D** vertical force 390 N; work done 750 J

SHORT SOLUTION

Question 17: A

Newtons Second law upwards on trolley:

$$R - 350 - 50\sin(37) = 0$$
$$R - 350 - 50(0.6) = 0$$
$$R = 350 + 30 = 380\,\mathrm{N}$$

Work done = Force × Distance:

$$W = 50\cos(37) \times 15$$
$$W = 50(0.80) \times 15$$
$$W = 40 \times 15 = 600\,\mathrm{J}$$

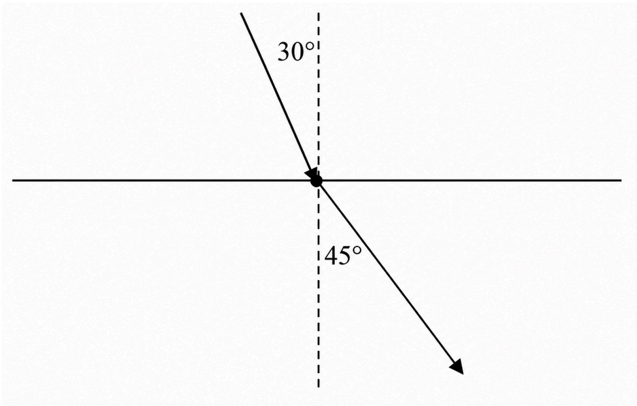
PHYSICS

Question 18

Answer **H**

A wave approaches the boundary between two mediums at an angle of 30° to the normal. Upon entering the second medium it is refracted such that it now travels at 45° to the normal. The wave travels at $5\,\mathrm{m\,s^{-1}}$ in the first medium.

What velocity does the wave travel at in the second medium?



A $\frac{8}{\sqrt{3}}\,\mathrm{m\,s^{-1}}$

B $8\sqrt{3}\,\mathrm{m\,s^{-1}}$

C $\frac{8}{\sqrt{2}}\,\mathrm{m\,s^{-1}}$

D $10\sqrt{3}\,\mathrm{m\,s^{-1}}$

E $\frac{5}{2}\,\mathrm{m\,s^{-1}}$

F $10\sqrt{2}\,\mathrm{m\,s^{-1}}$

G $\frac{5}{\sqrt{2}}\,\mathrm{m\,s^{-1}}$

H $5\sqrt{2}\,\mathrm{m\,s^{-1}}$

SHORT SOLUTION

Question 18: H

$$n = \frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$n = \frac{\sin 30}{\sin 45}$$

Remember: $\sin 30 = \frac{1}{2}, \quad \sin 45 = \frac{1}{\sqrt{2}}$

$$\therefore\ n = \frac{1}{2} \div \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$v_2 = \frac{v_1}{n} = 5 \div \frac{\sqrt{2}}{2} = \frac{10}{\sqrt{2}}$$

$$\therefore\ v_2 = 5\sqrt{2}\,\mathrm{m\,s^{-1}}$$

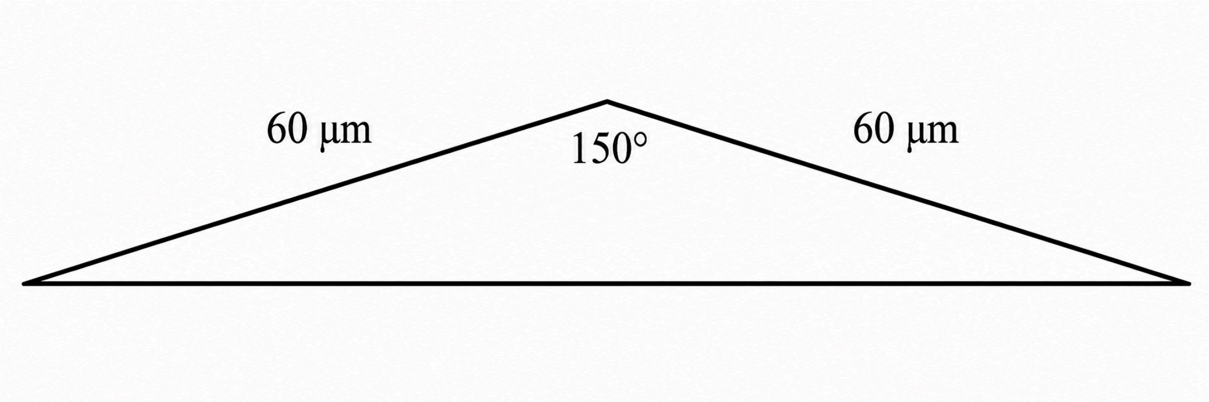
PHYSICS

Question 19

Answer **D**

A strip of conductive material is plated onto a silicon surface to allow an electronic signal to be transmitted. A cross section of the plated material is given below, and the resistance is measured over a 5 cm length of the strip to be 1.0 Ω.

What is the resistivity of the material?



A $6.0 \times 10^{-7}\,\Omega\,\mathrm{m}$

B $3.6 \times 10^{-9}\,\Omega\,\mathrm{m}$

C $2.4 \times 10^{-8}\,\Omega\,\mathrm{m}$

D $1.8 \times 10^{-8}\,\Omega\,\mathrm{m}$

E $2.0 \times 10^{-4}\,\Omega\,\mathrm{m}$

F $3.6 \times 10^{-10}\,\Omega\,\mathrm{m}$

G $5.4 \times 10^{-11}\,\Omega\,\mathrm{m}$

H $4.2 \times 10^{-11}\,\Omega\,\mathrm{m}$

SHORT SOLUTION

Work out the area of the cross section of the strip and use that to find the resistivity.

resistivity, $\rho = \text{resistance} \times \dfrac{\text{cross sectional area}}{\text{length}} = \dfrac{RA}{l}$

Recall area of triangle: $A = \frac{1}{2}ab\sin C$.

$A = \frac{1}{2}(6 \times 10^{-5})^2 \sin(150^\circ)$

Remember $\sin\theta$ is symmetrical about 90° so $\sin(150^\circ) = \sin(30^\circ) = \frac{1}{2}$.

$\therefore A = \frac{1}{2}(6 \times 10^{-5})^2 \sin(30^\circ)$

$A = \frac{1}{2}(6 \times 10^{-5})^2 \times \frac{1}{2}$

$A = \frac{1}{4} \times 36 \times 10^{-10} = 9 \times 10^{-10}\ \mathrm{m}^2$

$\rho = \dfrac{RA}{l} = \dfrac{1 \times 9 \times 10^{-10}}{0.05} = 9 \times 10^{-10} \times 20 = 1.8 \times 10^{-8}\ \Omega\,\mathrm{m}$

Note: This is approximately the resistivity of copper.

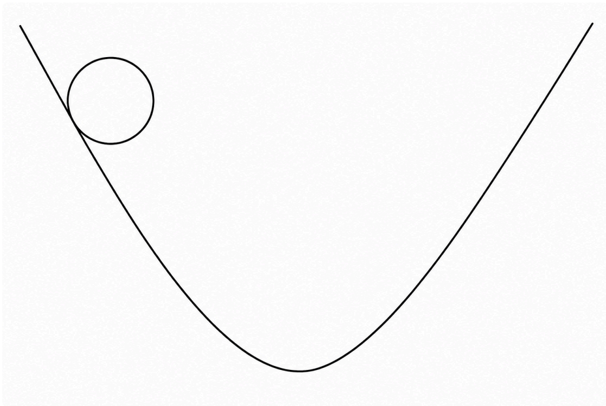
PHYSICS

Question 20

Answer **A**

A ball rolls down and up a dip which can have its side profile modelled by the equation $y = 2x^2$. The point $(0, 0)$ corresponds to the bottom of the dip, and y is the height of the surface at horizontal distance x away from this centre.

What is the angle of the contact force acting on the ball when it is 3 m away from the lowest point horizontally? (Give the smallest angle between the direction of the force and the horizontal.)



- **A** $\tan^{-1}\left(\frac{1}{12}\right)$
- **B** $\tan^{-1}(12)$
- **C** $\tan^{-1}(8)$
- **D** $\tan^{-1}\left(\frac{1}{8}\right)$
- **E** $\sin^{-1}\left(\frac{1}{12}\right)$
- **F** $\sin^{-1}(12)$
- **G** $\sin^{-1}(8)$
- **H** $\sin^{-1}\left(\frac{1}{8}\right)$

SHORT SOLUTION

Question 20: A

You can evaluate the angle by looking at either $x = -3$ or $x = 3$ as the dip is symmetrical. Use differentiation to find the gradient at either point then use the gradient to work out the angle between the force and horizontal.

Find gradient at $x = 3$:

$$y = 2x^2$$
$$\frac{dy}{dx} = 4x$$
$$\Rightarrow \frac{dy}{dx} = 4 \times 3 = 12$$

Direction of the force is normal to the surface the ball is rolling on:

$$m' = -\frac{1}{m} = -\frac{1}{12}$$

Find the angle with the horizontal of this gradient:

$$\tan\theta = \frac{1}{12}$$
$$\theta = \tan^{-1}\left(\frac{1}{12}\right)$$

Note: This is approximately $5°$ but trig function is given as the paper is non-calculator.

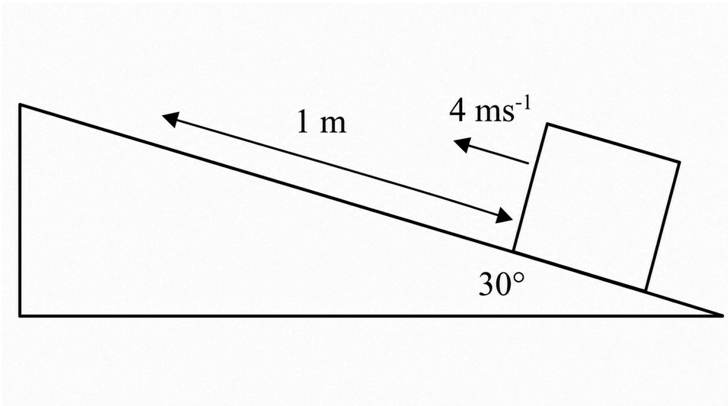
PHYSICS

Question 21

Answer **D**

A block of mass 2 kg sliding up a slope has an initial velocity of $4\,\mathrm{m\,s^{-1}}$. The slope is angled at $30°$ to the horizontal and the block moves 1 m before it stops. (Take $g = 10\,\mathrm{N\,kg^{-1}}$).

What is the average frictional force on the block as it moves up the slope?



A 2 N

B 1 N

C 3 N

D 6 N

E 4 N

F 5 N

G 0 N

H 7 N

SHORT SOLUTION

Question 21: D

Do an energy balance for the conversion of kinetic energy into gravitational potential energy and work done against friction.

Change in height:

$$h = s\sin\theta = 1\sin 30 = 10 \times \frac{1}{2} = 0.5\,\mathrm{m}$$

Energy balance:

$$KE = GPE + WD$$

$$\frac{1}{2}mv^2 = mgh + F_f s$$

$$\frac{1}{2} \times 2 \times 4^2 = 2 \times 10 \times 0.5 + F_f \times 1$$

$$16 = 10 + F_f$$

$$F_f = 6\,\mathrm{N}$$

PHYSICS

Question 22

Answer **F**

An object is pushed with a force of 3 N for 5 s and then a second force of 4 N for 4 s. The forces act in the same direction and push an object of mass 5 kg. At the end of the 9 s period, what is the total change in momentum?

- **A** $16\,\mathrm{kg\,m\,s^{-1}}$
- **B** $32\,\mathrm{kg\,m\,s^{-1}}$
- **C** $8\,\mathrm{kg\,m\,s^{-1}}$
- **D** $12\,\mathrm{kg\,m\,s^{-1}}$
- **E** $20\,\mathrm{kg\,m\,s^{-1}}$
- **F** $31\,\mathrm{kg\,m\,s^{-1}}$
- **G** $4\,\mathrm{kg\,m\,s^{-1}}$
- **H** $24\,\mathrm{kg\,m\,s^{-1}}$

SHORT SOLUTION

The impulse equals the change in momentum. The two impulses act in the same direction, so

$\Delta p = 3 \times 5 + 4 \times 4 = 15 + 16 = 31\,\mathrm{kg\,m\,s^{-1}}$.

PHYSICS

Question 23

Answer **E**

Light travels at a speed of $2.5 \times 10^8\,\mathrm{m\,s^{-1}}$ in a material. What is the critical angle for total internal reflection when light approaches an air boundary from within the material? (The speed of light in air is $3 \times 10^8\,\mathrm{m\,s^{-1}}$).

- **A** 30°
- **B** 45°
- **C** 60°
- **D** $\sin^{-1}(\frac{4}{5})$
- **E** $\sin^{-1}(\frac{5}{6})$
- **F** $\sin^{-1}(\frac{3}{5})$
- **G** $\sin^{-1}(\frac{2}{3})$
- **H** $\sin^{-1}(\frac{4}{7})$

SHORT SOLUTION

Find the refractive index and then the critical angle.

$$n = \frac{c}{v}$$

$$n = \frac{3 \times 10^8}{2.5 \times 10^8} = \frac{6}{5} = 1.2$$

For the critical angle, $\sin C = \dfrac{1}{n}$.

$$\sin C = \frac{1}{1.2} = \frac{5}{6}$$

$$C = \sin^{-1}\left(\frac{5}{6}\right).$$

PHYSICS

Question 24

Answer **B**

A spring is used to support a mass of 2 kg. The spring's spring constant, k, is $50\,\mathrm{N\,m^{-1}}$. How much elastic potential energy is stored in the spring due to the mass it is supporting? (Take $g = 10\,\mathrm{N\,kg^{-1}}$.)

- **A** 1 J
- **B** 4 J
- **C** 20 J
- **D** 25 J
- **E** 8 J
- **F** 15 J
- **G** 14 J
- **H** 30 J

SHORT SOLUTION

Work out the extension using Hooke's law and then the elastic potential energy.

$W = F = kx$

$mg = kx$

$2 \times 10 = 50x \Rightarrow x = \dfrac{20}{50} = 0.4\,\mathrm{m}$

$E = \frac{1}{2}Fx = \frac{1}{2}mgx$

$E = \frac{1}{2} \times 2 \times 10 \times 0.4 = 4\,\mathrm{J}$

PHYSICS

Question 25

Answer **H**

A charge of 500 C is unloaded over a 30 s period. The voltage in the circuit is 90 V. What is the average power obtained?

- **A** 600 W
- **B** 450
- **C** 250 W
- **D** 300 W
- **E** 100 W
- **F** 1800 W
- **G** 1000 W
- **H** 1500 W

SHORT SOLUTION

Find the current from charge and time, then compute power.

$$I = \frac{\Delta Q}{\Delta t}$$

$$I = \frac{500}{30} = \frac{50}{3}\ \mathrm{A}$$

$$P = IV$$

$$\therefore P = \frac{50}{3} \times 90 = 1500\ \mathrm{W}$$

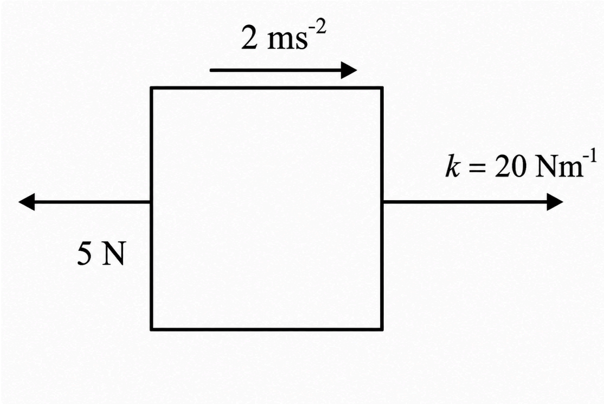
PHYSICS

Question 26

Answer **G**

A block of mass 4 kg is pulled with a spring of spring constant, $k = 20\,\mathrm{N\,m^{-1}}$. The acceleration of the block is $2\,\mathrm{m\,s^{-2}}$ and the force due to friction, F_f, is 5 N.

What is the extension in the spring?



A 22 cm

B 35 cm

C 8 cm

D 12 cm

E 5 cm

F 40 cm

G 65 cm

H 6 cm

SHORT SOLUTION

Resolve horizontally. The spring force must both overcome friction and accelerate the block:

$F_s - 5 = 4 \times 2$, so $F_s = 13\,\mathrm{N}$.

Using Hooke's law, $F_s = kx$, so $13 = 20x$. Hence $x = 0.65\,\mathrm{m} = 65\,\mathrm{cm}$.

PHYSICS

Question 27

Answer **C**

A cylindrical wire is made of a material with a resistivity of $8 \times 10^{-8}\,\Omega\,\mathrm{m}$. It has a radius of 0.5 mm, and a length of 1 km. If the voltage in the wire is 400 V, what is the current?

A $\frac{10\pi}{3}$ A

B $\frac{2\pi}{5}$ A

C $\frac{5\pi}{4}$ A

D $\frac{6\pi}{25}$ A

E $\frac{25\pi}{3}$ A

F $\frac{13\pi}{4}$ A

G $\frac{17\pi}{4}$ A

H $\frac{11\pi}{5}$ A

SHORT SOLUTION

Question 27: C

Find the resistance using the resistivity and dimensions of the wire. Then solve with $V = IR$.

resistivity, $\rho = \dfrac{RA}{l}$

$\therefore R = \dfrac{\rho l}{A}$

$A = \pi \times (0.5 \times 10^{-3})^2 = \dfrac{\pi}{4} \times 10^{-6}\,\mathrm{m}^2$

$R = \dfrac{8 \times 10^{-8} \times 1000}{\frac{\pi}{4} \times 10^{-6}} = \dfrac{320}{\pi}\,\Omega$

$I = \dfrac{V}{R}$

$\therefore I = 400 \div \dfrac{320}{\pi} = \dfrac{5\pi}{4}\,\mathrm{A}$

ESAT MOCK TEST 3

Math 2

27 questions · Answers placed beside each question for fast review.

MATH 2

Question 1

Answer **C**

Rearrange $y^4 - 4y^3 + 6y^2 - 4y + 2 = x^5 + 7$ to make y the subject, taking the principal fourth-root branch.

A $y = 1 + (x^5 + 7)^{1/4}$

B $y = -1 + (x^5 + 7)^{1/4}$

C $y = 1 + (x^5 + 6)^{1/4}$

D $y = -1 + (x^5 + 6)^{1/4}$

SHORT SOLUTION

Complete the binomial pattern:

$y^4 - 4y^3 + 6y^2 - 4y + 1 = (y-1)^4$.

So $(y-1)^4 + 1 = x^5 + 7$, hence $(y-1)^4 = x^5 + 6$. Taking the principal fourth root gives $y = 1 + (x^5 + 6)^{1/4}$.

MATH 2

Question 2

Answer **A**

The aspect ratio of my television screen is 4:3 and the diagonal is 50 inches. What is the area of my television screen?

A 1,200 inches^2

B 1,000 inches^2

C 120 inches^2

D 100 inches^2

E More information needed.

SHORT SOLUTION

Question 2: A

Let the width of the television be $4x$ and the height be $3x$.

By Pythagoras,

$$(4x)^2 + (3x)^2 = 50^2$$

Simplify:

$$25x^2 = 2500 \Rightarrow x = 10$$

Therefore, the screen is 30 inches by 40 inches, so the area is 1200 inches^2.

MATH 2

Question 3

Answer C

Rearrange the equation $\sqrt{1+3x^{-2}} = y^5+1$ to make x the subject, taking the positive square-root branch.

A $x = \dfrac{y^{10}+2y^5}{3}$

B $x = \dfrac{3}{y^{10}+2y^5}$

C $x = \sqrt{\dfrac{3}{y^{10}+2y^5}}$

D $x = \sqrt{\dfrac{y^{10}+2y^5}{3}}$

E $x = \sqrt{\dfrac{y^{10}+2y^5+2}{3}}$

F $x = \sqrt{\dfrac{3}{y^{10}+2y^5+2}}$

SHORT SOLUTION

Square both sides:

$1+3x^{-2} = (y^5+1)^2 = y^{10}+2y^5+1$.

Thus $3x^{-2} = y^{10}+2y^5$, so $x^2 = \dfrac{3}{y^{10}+2y^5}$. Taking the positive branch gives $x = \sqrt{\dfrac{3}{y^{10}+2y^5}}$.

MATH 2

Question 4

Answer C

Solve $3x-5y=10$ and $2x+2y=13$.

A $(x,y) = \left(\frac{19}{16}, \frac{85}{16}\right)$

B $(x,y) = \left(\frac{85}{16}, -\frac{19}{16}\right)$

C $(x,y) = \left(\frac{85}{16}, \frac{19}{16}\right)$

D $(x,y) = \left(-\frac{85}{16}, -\frac{19}{16}\right)$

E No solutions possible.

SHORT SOLUTION

The easiest way is to double the first equation and triple the second to get:

$6x-10y=20$ and $6x+6y=39$.

Subtract the first from the second to give: $16y=19$.

Therefore, $y=\frac{19}{16}$.

Substitute back into the first equation to give $x=\frac{85}{16}$.

MATH 2

Question 5

Answer **C**

The two inequalities $x + y \leq 3$ and $x^3 - y^2 < 3$ define a region on a plane. Which of the following points is inside the region?

- **A** (2, 1)
- **B** (2.5, 1)
- **C** (1, 2)
- **D** (3, 5)
- **E** (1, 2.5)
- **F** None of the above.

SHORT SOLUTION

This is fairly straightforward; the first inequality is the easier one to work with: options B, D and E violate it, so we just need to check A and C in the second inequality.

C: $1^3 - 2^2 < 3$, but A: $2^3 - 1^2 > 3$.

MATH 2

Question 6

Answer **B**

How many times do $y = x + 4$ and $y = 4x^2 + 5x + 5$ intersect?

- **A** 0
- **B** 1
- **C** 2
- **D** 3
- **E** 4

SHORT SOLUTION

Whilst this can be done graphically, it is quicker to do algebraically. Intersections occur where the curves have the same coordinates.

Thus: $x + 4 = 4x^2 + 5x + 5$.

Simplify: $4x^2 + 4x + 1 = 0$.

Factorise: $(2x + 1)(2x + 1) = 0$.

Thus, the two graphs only intersect once at $x = -\frac{1}{2}$.

MATH 2

Question 7

Answer **D**

How many times do $y = x^3$ and $y = x$ intersect?

A 0

B 1

C 2

D 3

E 4

SHORT SOLUTION

Answer: D

It's better to do this algebraically as the equations are easy to work with and you would need to sketch very accurately to get the answer. Intersections occur where the curves have the same coordinates. Thus: $x^3 = x$.

$x^3 - x = 0$

Thus: $x(x^2 - 1) = 0$

Spot the 'difference between two squares': $x(x+1)(x-1) = 0$

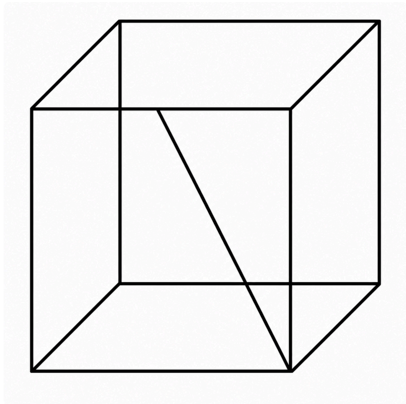
Thus there are 3 intersections: at $x = 0, 1$ and -1.

MATH 2

Question 8

Answer **E**

A cube has unit length sides. What is the length of a line joining a vertex to the midpoint of the opposite side?



A $\sqrt{2}$

B $\sqrt{\frac{3}{2}}$

C $\sqrt{3}$

D $\sqrt{5}$

E $\frac{\sqrt{5}}{2}$

SHORT SOLUTION

Answer: E

Note that the line is the hypotenuse of a right angled triangle with one side unit length and one side of length $\frac{1}{2}$.

By Pythagoras, $\left(\frac{1}{2}\right)^2 + 1^2 = x^2$.

Thus, $x^2 = \frac{1}{4} + 1 = \frac{5}{4}$.

$x = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$.

MATH 2

Question 9

Answer **D**

Find the value of

$$\int_1^4 \frac{3-2x}{x\sqrt{x}}\,dx$$

A $-\frac{13}{2}$

B $-\frac{85}{16}$

C $-\frac{13}{8}$

D -1

E $-\frac{1}{4}$

F $\frac{7}{4}$

G 7

SHORT SOLUTION

Answer: D

$$\int_1^4 \frac{3-2x}{x\sqrt{x}}\,dx$$

(in terms of powers of x and split up the fraction)

$= \int_1^4 \left(\frac{3}{x^{3/2}} - \frac{2}{x^{1/2}}\right)dx = \int_1^4 \left(3x^{-3/2} - 2x^{-1/2}\right)dx$

$= \left[\frac{3x^{-1/2}}{-1/2} - \frac{2x^{1/2}}{1/2}\right]_1^4$

$= \left[-6x^{-1/2} - 4x^{1/2}\right]_1^4$

$= \left(-6 \cdot \frac{1}{2} - 4 \cdot 2\right) - \left(-6 \cdot 1 - 4 \cdot 1\right)$

$= -11 - (-10)$

$= -1$

MATH 2

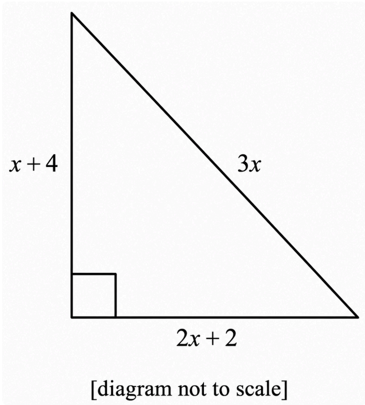
Question 10

Answer F

The diagram shows a right-angled triangle, with sides of length $x+4$, $2x+2$ and $3x$, all in cm.

[diagram not to scale]

What is the area, in cm^2, of the triangle?



A 10

B 12

C 28

D 36

E 40

F 54

G 70

SHORT SOLUTION

Using Pythagoras' Theorem, $(3x)^2 = (x+4)^2 + (2x+2)^2$.

Thus $9x^2 = x^2 + 8x + 16 + 4x^2 + 8x + 4 \Rightarrow 4x^2 - 16x - 20 = 0 \Rightarrow x^2 - 4x - 5 = 0$.

Factorising: $(x-5)(x+1) = 0 \Rightarrow x = -1, 5$. Since $x > 0$, take $x = 5$.

Area $= \frac{1}{2}(x+4)(2x+2) = \frac{1}{2}(9)(12) = 54 \text{ cm}^2$.

MATH 2

Question 11

Answer **D**

Which of the following is a simplification of $4 - \dfrac{x(3x+1)}{x^2(3x^2-2x-1)}$?

A $\dfrac{12x^3-8x^2-7x-1}{x(3x-1)(x-1)}$

B $\dfrac{4x^2+4x-1}{x(x+1)}$

C $\dfrac{4x^2+4x+1}{x(x+1)}$

D $\dfrac{4x^2-4x-1}{x(x-1)}$

E $\dfrac{4x^2-4x+1}{x(x-1)}$

F $\dfrac{12x^3-8x^2-x+1}{x(3x-1)(x-1)}$

SHORT SOLUTION

In the denominator, $3x^2-2x-1 \equiv (3x+1)(x-1)$.

Considering the whole expression: $4 - \dfrac{x(3x+1)}{x^2(3x+1)(x-1)}$.

Cancel $(3x+1)$: $4 - \dfrac{1}{x(x-1)}$.

Combine over the common denominator: $\dfrac{4x(x-1)-1}{x(x-1)} = \dfrac{4x^2-4x-1}{x(x-1)}$.

MATH 2

Question 12

Answer **B**

The ball for a garden game is a solid sphere of volume 192 cm^3.

For the children's version of the game the ball is a solid sphere made of the same material, but the radius is reduced by 25%.

What is the volume, in cm^3, of the children's ball?

A 48

B 81

C 96

D 108

E 144

SHORT SOLUTION

$V = \frac{4}{3}\pi r^3 \Rightarrow V \propto r^3$

$V_{\text{child}} = \frac{4}{3}\pi r_{\text{child}}^3$

$= \frac{4}{3}\pi(\frac{3}{4}r)^3 = (\frac{3}{4})^3 V$

$= \frac{27}{64} \times 192 = 27 \times 3 = 81\,\text{cm}^3$

MATH 2

Question 13

Answer **A**

Given that

$9^{2x-1} \times \frac{1}{27^x} = 81^x$

what is the value of x?

A $-\frac{2}{3}$

B $-\frac{2}{5}$

C $-\frac{1}{3}$

D $-\frac{1}{4}$

E $-\frac{1}{5}$

SHORT SOLUTION

Convert all bases to 3.

$(3^2)^{2x-1} \times \frac{1}{(3^3)^x} = (3^4)^x$

$3^{4x-2} \times 3^{-3x} = 3^{4x}$

$3^{x-2} = 3^{4x}$

Equating exponents: $x - 2 = 4x$

$3x = -2$

$x = -\frac{2}{3}$

MATH 2

Question 14

Answer **D**

PR and *QS* are the diagonals of a rhombus *PQRS*.

$PR = (3x + 2)$ cm

$QS = (8 - 2x)$ cm

The area of $PQRS$ is $11\,\mathrm{cm}^2$.

What is the difference, in cm, between the two possible lengths of PR?

A $2\frac{2}{3}$

B $4\frac{1}{2}$

C $5\frac{1}{3}$

D 8

SHORT SOLUTION

Area of rhombus = product of diagonals / 2

$\frac{(3x+2)(8-2x)}{2} = 11$

$12x - 3x^2 + 8 - 2x = 11$

$3x^2 - 10x + 3 = 0$

$(3x - 1)(x - 3) = 0 \Rightarrow x = \frac{1}{3}, 3$

Possible PR lengths: $3(3) + 2 = 11$

$3(\frac{1}{3}) + 2 = 3$

$11 - 3 = 8\,\mathrm{cm}$ difference

MATH 2

Question 15

Answer E

[diagram not to scale]

The diagram shows two congruent right-angled triangles PQR and TSR with right angles at Q and S, respectively.

$PQ = TS = 3\text{ cm}$

$QR = SR = 4\text{ cm}$

PRT is a straight line.

What is the length, in cm, of QS?

Math 2 Question 15 diagram

A 4

B $3\sqrt{2}$

C 5.2

D $4\sqrt{2}$

E 6.4

SHORT SOLUTION

Notice that the triangle PQR is similar to QUR. This can be deduced by noticing that the angles are the same. The ratio of lengths in PQR is 3:4:5 Therefore QU = 4 x ⅘ = 3.2cm QS = 2 x QU = 6.4cm

MATH 2

Question 16

Answer G

The total of three numbers p, q and r is 375.

The ratio $p : q$ is $5 : 7$.

The ratio $q : r$ is $4 : 11$.

What is the value of $p + r$?

- **A** 16
- **B** 60
- **C** 97
- **D** 125
- **E** 144
- **F** 231
- **G** 291
- **H** 315

SHORT SOLUTION

q can be written as 75p.

Therefore r can be written as 75p114=7720p

p+q+r=p+75p+7720p=254p=375

Therefore, p=60 and it follows that r=231

60+231=291

MATH 2

Question 17

Answer **D**

The straight line P has equation $3y - 2x = 12$ and intercepts the y-axis at the point $(0, p)$.

The straight line Q is parallel to P, passes through the point $(6, -1)$ and intercepts the y-axis at the point $(0, q)$.

What is the value of $p - q$?

A −9

B −7

C 1

D 9

E 14

F 17

SHORT SOLUTION

To find the value of p, substitute x = 0 and y = p into 3y - 2x = 12

3p = 12

p = 4

The equation of P can be rearranged to y=23x+4

Therefore the gradient of P and Q is 23

Since Q passes through (6, -1), the equation of Q is:

y-(-1)=23(x-6)

y=23x-5

Substitute x = 0, y = q:

q = -5

p - q = 4 - (-5) = 9

MATH 2

Question 18

Answer G

The vertices of a rectangle have coordinates:

P(4, 5) Q(4, 8) R(10, 8) S(10, 5)

PQRS is transformed by a clockwise rotation of 90° about P followed by a reflection in the x-axis.

What are the coordinates of the final position of R?

- **A** (-8, -10)
- **B** (-7, -1)
- **C** (-4, 1)
- **D** (-1, 11)
- **E** (1, -11)
- **F** (4, -1)
- **G** (7, 1)
- **H** (8, 10)

SHORT SOLUTION

Relative to $P = (4, 5)$, point $R = (10, 8)$ has vector $(6, 3)$.

A clockwise $90°$ rotation sends $(x, y) \mapsto (y, -x)$, so $(6, 3) \mapsto (3, -6)$.

The rotated image of R is $(4, 5) + (3, -6) = (7, -1)$. Reflecting in the x-axis gives $(7, 1)$.

MATH 2

Question 19

Answer **C**

Box A contains exactly 10 balls, of which 6 are red and 4 are blue.

Box B contains exactly 15 balls, of which 3 are red and 12 are blue.

All the balls are identical in every respect, apart from colour.

One of the two boxes is chosen at random by tossing two fair coins, as follows:

"If both coins show heads, box A is selected. Otherwise box B is selected."

One ball is then randomly taken from the selected box.

What is the probability that a red ball is taken?

A $\frac{9}{400}$

B $\frac{3}{25}$

C $\frac{3}{10}$

D $\frac{2}{5}$

E $\frac{1}{2}$

F $\frac{4}{5}$

G $\frac{323}{400}$

SHORT SOLUTION

Box A is selected only if both coins show heads, so $P(A) = \frac{1}{4}$ and $P(B) = \frac{3}{4}$.

Therefore

$$P(\text{red}) = \frac{1}{4} \cdot \frac{6}{10} + \frac{3}{4} \cdot \frac{3}{15} = \frac{3}{20} + \frac{3}{20} = \frac{3}{10}.$$

MATH 2

Question 20

Answer **C**

Question 20:

A list of five numbers has mean x, median y and range z.

A sixth number is added to the list. This sixth number is greater than x.

Which of the following statements must be true?

- 1 The median of the six numbers cannot be one of the numbers in the list.
- 2 The mean of the six numbers is greater than x.
- 3 The range of the six numbers is greater than z.

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3

SHORT SOLUTION

Question 20: C

- Statement 1 is False as the middle two numbers could be equal.
- Statement 2 is True. Since a number larger than the mean is added, the mean will increase.
- Statement 3 is False. For the range to increase, the new number must be the new largest number; this is not necessarily stated.

MATH 2

Question 21

Answer **C**

An arithmetic progression has first term a and common difference d.

The sum of the first 5 terms is equal to the sum of the first 8 terms.

Which one of the following expresses the relationship between a and d?

A $a = -\frac{38}{3}d$

B $a = -7d$

C $a = -6d$

D $a = 6d$

E $a = 7d$

F $a = \frac{38}{3}d$

SHORT SOLUTION

One approach is to use the formula for the sum of the first 5 and the first 8 terms:

$$S_n = \tfrac{1}{2}n\big(2a + (n-1)d\big).$$

We have

$$S_5 = \tfrac{1}{2} \times 5\,(2a + 4d) = \tfrac{5}{2}(2a + 4d),$$

$$S_8 = \tfrac{1}{2} \times 8\,(2a + 7d) = 4(2a + 7d).$$

These are equal, so

$$\tfrac{5}{2}(2a + 4d) = 4(2a + 7d).$$

Multiply both sides by 2 and expand/collect terms:

$$5(2a + 4d) = 8(2a + 7d) \;\Rightarrow\; 10a + 20d = 16a + 56d \;\Rightarrow\; -6a = 36d \;\Rightarrow\; a = -6d.$$

So the answer is $a = -6d$.

MATH 2

Question 22

Answer **G**

Consider the simultaneous equations

$3x^2 + 2xy = 4$

$x + y = a$

where a is a real constant.

Find the complete set of values of a for which the equations have two distinct real solutions for x.

- **A** There are no values of a
- **B** $-2 < a < 2$
- **C** $-1 < a < 1$
- **D** $a = 0$
- **E** $a < -1$ or $a > 1$
- **F** $a < -2$ or $a > 2$
- **G** All real values of a

SHORT SOLUTION

We can solve the equations to find x by substituting for y. The second equation gives $y = a - x$, so the first equation becomes

$$3x^2 + 2x(a - x) = 4.$$

This rearranges to

$$x^2 + 2ax - 4 = 0,$$

whose discriminant is

$$(2a)^2 - 4 \cdot 1 \cdot (-4) = 4a^2 + 16.$$

This is always positive for all real a (since $4a^2 \geq 0$), so there are always two distinct real roots. Hence the correct answer is: all real values of a.

MATH 2

Question 23

Answer **E**

Find the shortest distance between the two circles with equations:

$(x+2)^2+(y-3)^2=18$

$(x-7)^2+(y+6)^2=2$

A 0

B 4

C 16

D $2\sqrt{2}$

E $5\sqrt{2}$

SHORT SOLUTION

The first circle has centre $(-2,3)$ and radius $\sqrt{18}=3\sqrt{2}$.

The second circle has centre $(7,-6)$ and radius $\sqrt{2}$.

It would help to draw a quick sketch of the situation.

The distance between the centres is $\sqrt{((-2)-7)^2+(3-(-6))^2}=\sqrt{81+81}=9\sqrt{2}$.

Subtracting the two radii ($3\sqrt{2}$ and $\sqrt{2}$) for the parts of the line lying within the circles gives $9\sqrt{2}-3\sqrt{2}-\sqrt{2}=5\sqrt{2}$. The correct answer is $5\sqrt{2}$.

This can also be proved by taking an arbitrary point on each circle, finding the distance between them and then minimising this, but that is more complicated than needed.

MATH 2

Question 24

Answer **D**

The function f is defined by $f(x) = x^3 + ax^2 + bx + c$.

a, b, c take the values 1, 2 and 3 with no two of them being equal and not necessarily in this order.

The remainder when $f(x)$ is divided by $(x+2)$ is R.

The remainder when $f(x)$ is divided by $(x+3)$ is S.

What is the largest possible value of $R - S$?

A -26

B 5

C 7

D 17

E 29

SHORT SOLUTION

For this question, we make use of the remainder theorem: the remainder when $f(x)$ is divided by $(x-a)$ is $f(a)$.

In this case, we take $a = -2$ and $a = -3$, so

$f(-2) = R$

$f(-3) = S$

Substituting these into the given formula for $f(x)$ gives

$f(-2) = -8 + 4a - 2b + c = R$

$f(-3) = -27 + 9a - 3b + c = S$

We want the largest possible value of $R - S$, so we subtract these two equations to get

$19 - 5a + b = R - S$.

To get the largest possible value, we want a to be as small as possible and b to be as large as possible, so we take $a = 1$, $b = 3$ (and then $c = 2$), giving $R - S = 19 - 5 + 3 = 17$.

Thus the correct answer is 17.

MATH 2

Question 25

Answer **A**

The non-zero constant k is chosen so that the coefficients of x^6 in the expansions of $(1+kx^2)^7$ and $(k+x)^{10}$ are equal.

What is the value of k?

A $\frac{1}{6}$

B 6

C $\frac{\sqrt{6}}{6}$

D $\sqrt{6}$

E $\frac{\sqrt{30}}{30}$

F $\sqrt{30}$

SHORT SOLUTION

Question 25: A

We can expand these two expressions using the binomial theorem, focusing on the term involving x^6:

$(1+kx^2)^7 = \cdots + \binom{7}{3}1^4(kx^2)^3 + \cdots = \cdots + \frac{7\times6\times5}{3\times2\times1}k^3x^6 + \cdots = \cdots + 35k^3x^6 + \ldots$

and

$(k+x)^{10} = \cdots + \binom{10}{6}k^4x^6 + \cdots = \cdots + \frac{10\times9\times8\times7}{4\times3\times2\times1}k^4x^6 + \cdots = \cdots + 210k^4x^6 + \ldots$

We therefore need $35k^3 = 210k^4$, so $k = \frac{35}{210} = \frac{5}{30} = \frac{1}{6}$.

Thus the correct answer is $\frac{1}{6}$.

MATH 2

Question 26

Answer B

Find the complete set of values of the constant c for which the cubic equation

$$2x^3 - 3x^2 - 12x + c = 0$$

has three distinct real solutions.

- **A** $-20 < c < 7$
- **B** $-7 < c < 20$
- **C** $c > 7$
- **D** $c > -7$
- **E** $c < 20$
- **F** $c < -20$

SHORT SOLUTION

Question 26: B

Sketch the cubic by finding its turning points and the y-intercept. For $y = 2x^3 - 3x^2 - 12x + c$, differentiate:

$\frac{dy}{dx} = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$.

Hence the stationary points are at $x = 2$ and $x = -1$ with coordinates $(-1,\ 7+c)$ and $(2,\ -20+c)$.

For $2x^3 - 3x^2 - 12x + c = 0$ to have three distinct real solutions, the turning points must lie on opposite sides of the x-axis. Since $-20 + c < 7 + c$, we need $-20 + c < 0$ and $7 + c > 0$.

Thus $c < 20$ and $c > -7$. Combining gives \(-7

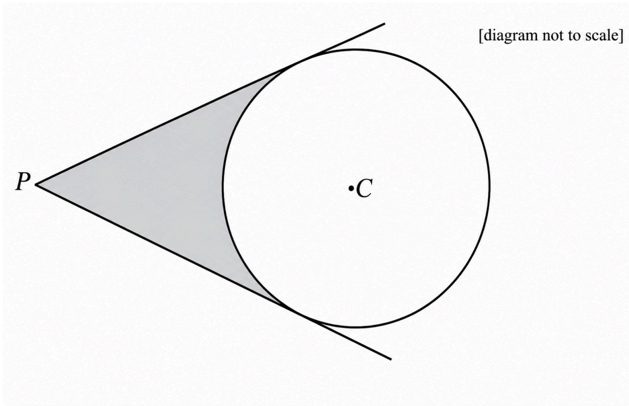
MATH 2

Question 27

Answer A

Tangents are drawn from a point P to a circle of radius 10 cm. The centre of the circle is C and the distance PC is 20 cm.

Which one of the following is an expression for the shaded area in square centimetres?



A $\frac{100}{3}(3\sqrt{3} - \pi)$

B $\frac{100}{3}(3\sqrt{5} - \pi)$

C $\frac{50}{3}(6\sqrt{3} - \pi)$

D $\frac{50}{3}(6\sqrt{5} - \pi)$

E $\frac{50}{3}(\sqrt{3} - 2\pi)$

SHORT SOLUTION

sin $\theta = 10/x = 10/20 = 1/2$

$\theta = 30°$

Area of the triangle:

$2 \times (\frac{1}{2} \times x \cos\theta \times 10)$

= $x\sqrt{3}/2 \times 10$

= $5x\sqrt{3}$

= $100\sqrt{3}$

Area of circular sector within triangles = $120/360 \times \pi \times 10^2$

= $100/3\pi$

Therefore shaded area = $100\sqrt{3} - 100/3\pi$

= $\frac{100}{3}(3\sqrt{3} - \pi)$