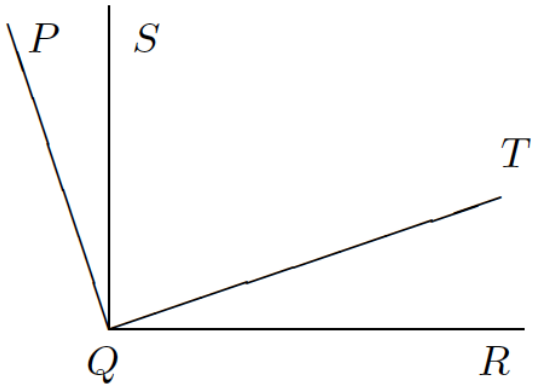
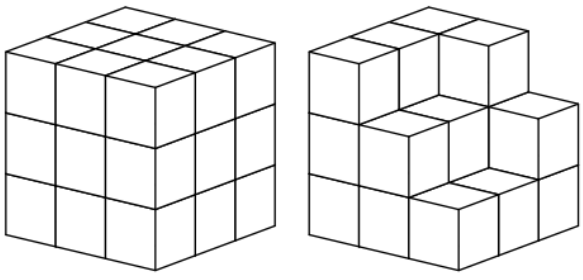
Week 6: Pre-AMC Level

Problem 1 Angle ∠PQR is equal to 138°. Also, ∠PQT = ∠SQR = 90° . The size of ∠SQT =



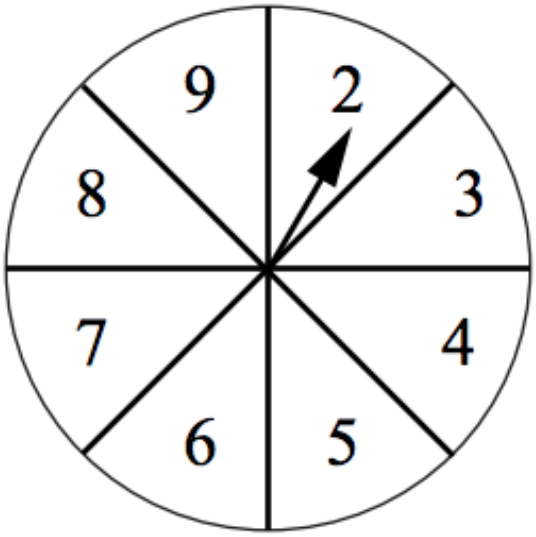
(A) 21 **(B)** 24 **(C)** 42 **(D)** 48 **(E)** 64

Problem 2 Natalie wanted to build the same cube as Diana had (fig. 1, left). However, Natalie ran out of small cubes and built only the part of the cube, as you can see in the fig. 2 (right). How many small cubes must be added to the fig. 2 to form fig. 1? (Math Kangaroo Problem)



(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

Problem 3 A circular spinner is divided into 8 equal sections, as shown. An arrow is attached to the center of the spinner. The arrow is spun once. What is the probability that the arrow stops in a section containing a prime number that is odd? (Canadian Math Competition)



(A) $\frac{1}{8}$ **(B)** $\frac{2}{8}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{4}{8}$ **(E)** $\frac{7}{8}$

Problem 4 Hector's father's current age is 5 times his age. Three years ago, the father's age is 8 times Hector's age. What is the sum of digits of father's current age? (Hope Cup Math competition)

(A) 3 **(B)** 4 **(C)** 7 **(D)** 8 **(E)** 9

Problem 5 Each of a, b, c, and d is a positive integer and is greater than 3. If

$$\frac{1}{a-2} = \frac{1}{b+2} = \frac{1}{c+1} = \frac{1}{d-3}$$

then which ordering of these four numbers is correct? (Canadian Math Competition)

(A) $a < b < c < d$ **(B)** $c < b < a < d$ **(C)** $b < a < c < d$

(D) $d < a < c < b$ **(E)** $b < c < a < d$

Problem 6 Emily writes down the largest two-digit prime such that each of its digits is prime. Krish writes down the smallest two-digit prime such that each of its digits is prime.Kirsten subtracts Krish's number from Emily's number.
What answer does Kirsten obtain?

(A) 43 **(B)** 45 **(C)** 48 **(D)** 50 **(E)** 53

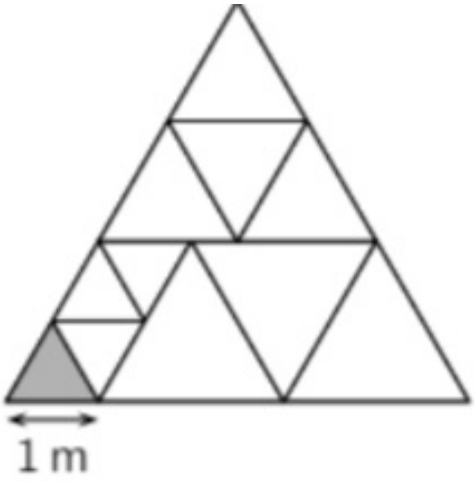
Problem 7 Last night, $\frac{5}{8}$ of the students in the dorm looked at Facebook, $\frac{3}{8}$ looked at Twitter, and $\frac{1}{4}$ looked at both. What fraction of the students looked at neither? Express your answer as a common fraction.

(A) $\frac{1}{3}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{5}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{8}$

Problem 8 A bookmark-maker sells bookmarks for \$1 each or 7 for \$6 What is the smallest amount, in dollars, you could pay for 2017 of her bookmarks? (British Math Competition)

(A) 1728 **(B)** 1729 **(C)** 1730 **(D)** 1731 **(E)** 1732

Problem 9 A big triangle is divided into equilateral triangles as shown in the figure below. The side of the shaded is 1 m. What is the perimeter of the big triangle?



(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

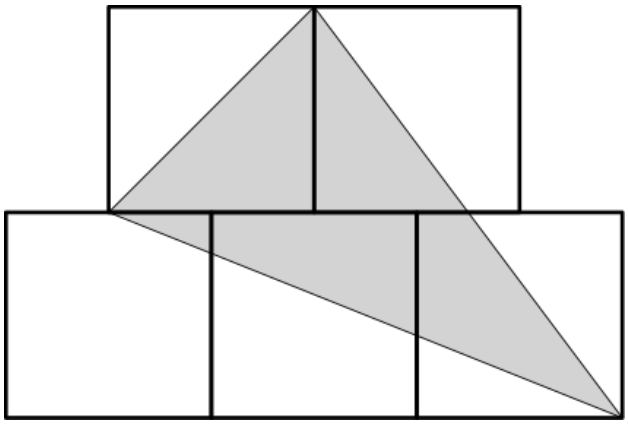
Problem 10 How many different 3-digit numbers consist of 3 distinct digits from $\{0, 1, 2, 9, 5\}$ and are divisible by 2, 3 and 5?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

Problem 11 The average score of students in team A and team B altogether is 18 in an AMC 8 contest. Team A has 20 students with an average score of 14. If the average score of students in class B is 23, how many students in team B?

(A) 12 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 18

Problem 12 The diagram shows 5 identical squares of side length 2 arranged symmetrically. What fraction of the diagram's area is shaded?



(A) $\frac{3}{5}$ **(B)** $\frac{7}{20}$ **(C)** $\frac{7}{10}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{4}{5}$

Problem 13 How many 7-digit palindromes (numbers that read the same backward as forward) can be formed using the digits 2, 2, 3, 3, 5, 5, 5? For example, one such number is 2355532. (AMC 10 Problem)

(A) 6 **(B)** 12 **(C)** 24 **(D)** 36 **(E)** 48

Problem 14 A standard, six-sided die is rolled three times. What is the probability that the three rolls are either all the same or all different?

(A) $\frac{1}{6}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{5}{12}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{11}{36}$

Problem 15 What is the sum of first digit and last digit of the smallest positive integer whose digits add to 2019?

(A) 4 **(B)** 6 **(C)** 8 **(D)** 9 **(E)** 12

Problem 16 What is the minimum positive integer n such that the following statement is correct?
"Randomly choose people. We can ALWAYS find at least three of them who were born in the same month."

(A) 13 **(B)** 20 **(C)** 24 **(D)** 25 **(E)** 36

Problem 17 2020 beads (black and white) are put on a string as follows. The first bead on the left is white. How many black beads are on the string? (Hope Cup Math Competition, China)

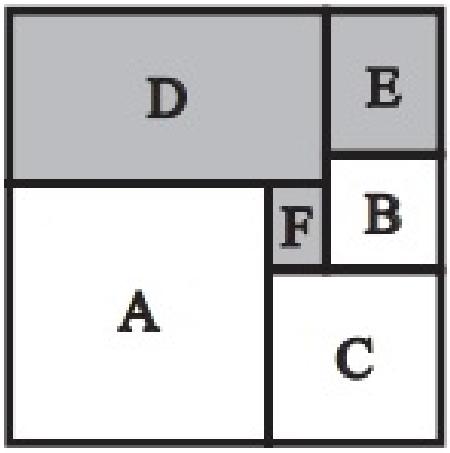
(A) 1340 **(B)** 1344 **(C)** 1346 **(D)** 1350 **(E)** 1352

Problem 18 In Pascal's Triangle, each number is the sum of the number just above it and to the left and the number just above it and to the right. So the middle number in Row 2 is 2 because $1 + 1 = 2$. What is the sum of the numbers in Row 8 of Pascal's Triangle? (Mathcounts Problem)

Row 0					1					
Row 1				1		1				
Row 2			1		2		1			
Row 3		1		3		3		1		
Row 4	1		4		6		4		1	
Row 5	1		5		10		10		5	1

(A) 64 **(B)** 128 **(C)** 256 **(D)** 512 **(E)** 1024

Problem 19 The square labeled A has area 81. The square labeled B has area 16. The square labeled C has area 36. These three squares along with the shaded, non-overlapping rectangles labeled D, E and F are arranged to form a large square as shown. What is the area of rectangle D?



(A) 40 **(B)** 50 **(C)** 60 **(D)** 66 **(E)** 68

Problem 20 A rabbit and a turtle race on a track of 1000 meters. The speed of the rabbit is 15 times the speed of the turtle. They start together.
As the rabbit is 100 meters away from the destination, it decides to take a nap since the turtle is still far behind.
However, when the rabbit wakes up, it finds that the turtle has just arrived at the destination.
How many meters has the turtle run during the time when the rabbit is taking the nap? (Hope Cup Math Competition, China)

(A) 860 **(B)** 880 **(C)** 900 **(D)** 920 **(E)** 940

Answers

1. C 2. C 3. C

4. D (Hint: One way is to use algebra, Let x = Hector's current age. Then set up an equation.)

5. E 6. D 7. B 8. B 9. D 10. C

11. D (Hint: You may let x be the number of students in team B and set up an equation)

12. B (Hint: You may use cut-and-paste technique. For example, put two 2×1 rectangles on two upper corners to form a 6×4 big rectangle. Here the side length = 2 is not really needed. You can use any number and will get the same fraction.)

13. A (Hint: 5 must be in the middle. You just need to arrange 2,3,5 on one side and the other side is symmetric.)

14. D 15. E 16. D

17. C (Hint: Note that each cycle (1 white, 1 black, l white, 2 black, 1 white, 3 black) has 9 beads.)

18. C (Hint: Add the sum of each row and find the pattern.)

19. D

20. E (Hint: How many meters has the turtle run right before the rabbit started to take the nap?)