TMUA MOCK TEST 2
Solution Book

Paper 1 Styled

- Graphs
- Trigonometry
- Logarithms

ThrivingScholars

Question 1

Find the **greatest sum**, $x + y$, of the real values x and y that satisfy the simultaneous equations:

$$\log_2\left(\frac{x^2}{y}\right) = 6 \quad \text{and} \quad 12 - \log_2 x = (\log_2 x)(\log_2 y)$$

Options:

A. $16 + \frac{\sqrt{2}}{2}$

B. $32 + \frac{\sqrt{2}}{4}$

C. 16

D. 20

E. 32

F. $\frac{1 + 128\sqrt{2}}{512}$

G. 120

H. 256

$\log_2 x^2 - \log_2 y = 6 \iff 2\log_2 x - \log_2 y = 6$

Let $a = \log_2 x$ and $b = \log_2 y$ so from the first equation $2a - b = 6$ A

From $12 - \log_2 x = (\log_2 x)(\log_2 y)$, $12 - a = ab \iff a(b+1) = 12$ B

From A: $b = 2a - 6$

Substituting this into B: $a(2a - 5) = 12$

$2a^2 - 5a - 12 = 0$

$(2a + 3)(a - 4) = 0$

$a = -\frac{3}{2}$ or $a = 4$

If $a = -\frac{3}{2}$, $b = -3 - 6 = -9$

If $a = 4$, $b = 8 - 6 = 2$

From $\left(-\frac{3}{2}, -9\right)$: $\log_2 x = -\frac{3}{2} \iff x = 2^{-\frac{3}{2}} = \frac{1}{2\sqrt{2}}$ and $\log_2 y = -9 \iff y = 2^{-9} = \frac{1}{512}$

From $(4,2)$: $\log_2 x = 4 \iff x = 2^4 = 16$ and $\log_2 y = 2 \iff y = 2^2 = 4$

The maximum value of $x + y = 20$.

The correct answer is D

Question 2

Find the **non-zero solution** to the equation:

$$\frac{3^{(16^x)}}{81^{(4^x)}} = \frac{1}{27}$$

Options:

A. $\log_4 3$

B. $2\log_4 3$

C. 1

D. 2

E. $\log_3 4$

F. $2\log_3 4$

$$\frac{3^{16^x}}{81^{4^x}} = \frac{1}{27}$$
$$\frac{3^{(4^2)^x}}{(3^4)^{4^x}} = 3^{-3}$$
$$\frac{3^{4^{2x}}}{3^{4^{x+1}}} = 3^{-3}$$
$$3^{(4^{2x}-4^{x+1})} = 3^{-3}$$
$$4^{2x} - 4^{x+1} = -3$$
$$4^{2x} - 4 \times 4^x + 3 = 0$$

Let $a = 4^x$

$$a^2 - 4a + 3 = 0$$
$$(a-1)(a-3) = 0$$

$a = 1 \Leftrightarrow 4^x = 1 \Leftrightarrow x = 0$ but non-zero solution required

$a = 3 \Leftrightarrow 4^x = 3 \Leftrightarrow x = \log_4 3$

The correct answer is A

Question 3

You are given:

$$\log_a(xy^2z) = 6$$
$$\log_a(x^2yz^4) = 9$$
$$\log_a(x^5y^7z^5) = 25$$

Where x, y, z are **real numbers greater than 1**.

What is the value of a?

Options:

A. 1

B. x

C. y

D. z

E. There is not enough information to determine if a is any of the other options.

$\log_a(xy^2z) = 6$ A
$\log_a(x^2yz^4) = 9$ B
$\log_a(x^5y^7z^5) = 25$ C

3A + B:
$3\log_a(xy^2z) + \log_a(x^2yz^4) = 3 \times 6 + 9$
$\log_a(x^3y^6z^3) + \log_a(x^2yz^4) = 27$
$\log_a(x^5y^7z^7) = 27$ D

D – C:
$\log_a(x^5y^7z^7) - \log_a(x^5y^7z^5) = 27 - 25$
$\log_a\left(\frac{x^5y^7z^7}{x^5y^7z^5}\right) = 2$
$\log_a z^2 = 2$
$2\log_a z = 2$
$\log_a z = 1$
$a^1 = z$
The answer is D

Question 4

The curve

$$y = ax^2 + 3$$

passes through the points

$$(2,\ \log_2 p) \quad \text{and} \quad (-1 + \log_2 p,\ 11)$$

where p is a **positive real number**.

Find the value of p.

Options:

A. $\frac{1}{32}$

B. $\frac{1}{16}$

C. $\frac{1}{4}$

D. 1

E. 4

F. 16

G. 32

$y = ax^2 + 3 \qquad (2, \log_2 p) \qquad (-1 + \log_2 p, 11)$

Using $(2, \log_2 p)$: $\log_2 p = 4a + 3$ A

Using $(-1 + \log_2 p, 11)$: $11 = a(-1 + \log_2 p)^2 + 3 \Leftrightarrow a(-1 + \log_2 p)^2 = 8$ B

Substituting A into B:

$a(-1 + 4a + 3)^2 = 8$

$a(2a + 1)^2 = 2$

$4a^3 + 4a^2 + a - 2 = 0$

Using the factor theorem with $a = \frac{1}{2}$:

$\frac{4}{8} + \frac{4}{4} + \frac{1}{2} - 2 = \frac{1}{2} + 1 + \frac{1}{2} - 2 = 0$ so $(2a - 1)$ is a factor

$(2a - 1)(2a^2 + ka + 2) \equiv 4a^3 + 4a^2 + a - 2$

Equating coefficients of a^2: $2k - 2 = 4 \Leftrightarrow k = 3$

$(2a - 1)(2a^2 + 3a + 2) = 0$

For the quadratic factor, the discriminant $\Delta = 9 - 16$ shows that there are no further real solutions and the only real solution is $a = \frac{1}{2}$.

Substituting this into A: $\log_2 p = 2 + 3 = 5$ so $p = 2^5 = 32$

The correct answer is G

Question 5

Put the following expressions in order **from least to greatest**.

Which expression has the **smallest value**?

(i) $4 - \log_{10} \pi$

(ii) $2 + \log_{10} \sqrt{\pi}$

(iii) $\sqrt{3 + \log_{10} \pi}$

(iv) $\dfrac{5}{\sqrt{\log_{10} \pi}}$

Options:

A. $4 - \log_{10} \pi$

B. $2 + \log_{10} \sqrt{\pi}$

C. $\sqrt{3 + \log_{10} \pi}$

D. $\dfrac{5}{\sqrt{\log_{10} \pi}}$

This can be done by estimating the values using the approximation $\pi^2 \approx 10$

$$4 - \log_{10} \pi \approx 4 - \log_{10} 10^{\frac{1}{2}} = 4 - \frac{1}{2} = 3\frac{1}{2}$$

$$2 + \log_{10} \sqrt{\pi} \approx 2 + \log_{10} 10^{\frac{1}{4}} = 2 + \frac{1}{4} = 2\frac{1}{4}$$

$$\frac{5}{\sqrt{\log_{10} \pi}} \approx \frac{5}{\sqrt{\log_{10} 10^{\frac{1}{2}}}} = \frac{5}{\sqrt{\frac{1}{2}}} = 5\sqrt{2}$$

$$\sqrt{3 + \log_{10} \pi} \approx \sqrt{3 + \log_{10} 10^{\frac{1}{2}}} = \sqrt{3\frac{1}{2}}$$

The correct order is $\sqrt{3 + \log_{10} \pi}, 2 + \log_{10} \sqrt{\pi}, 4 - \log_{10} \pi, \dfrac{5}{\sqrt{\log_{10} \pi}}$

Question 6

Let $f(x)$ be a **quadratic function** in x.

Given that the graph of $y = f(x)$:

- Passes through the point $(-1, -3)$
- Has a **vertex at** $(1,\ 5)$

Which of the following is an expression for $f(x)$?

Options:

A. $-x^2 + 4x + 3$

B. $x^2 - 4x + 3$

C. $2x^2 - 4x + 3$

D. $-2x^2 - 4x + 3$

E. $-x^2 - 4x + 3$

F. $-2x^2 + 4x + 3$

G. $4x - 2x^2$

H. $4x - x^2$

Using completed square form: $y = a(x-1)^2 + 5$

$(-1,-3)$: $-3 = 4a + 5$

$a = -2$

$y = -2(x-1)^2 + 5 = -2x^2 + 4x + 3$

The correct answer is F

Question 7

Find the **complete set of values** of x, with

$$-\pi \le x \le \pi,$$

for which

$$(1 - 2\cos 2x)(1 + 2\sin x) \le 0.$$

Options:

A.

$$-\pi \le x \le -\frac{5\pi}{6}, \quad -\frac{\pi}{6} \le x \le \frac{\pi}{6}, \quad \frac{5\pi}{6} \le x \le \pi$$

B.

$$-\pi \le x \le \frac{5\pi}{6}, \quad \frac{5\pi}{6} \le x \le \pi$$

C.

$$-\frac{5\pi}{6} \le x \le -\frac{\pi}{6}, \quad \frac{\pi}{6} \le x \le \frac{5\pi}{6}$$

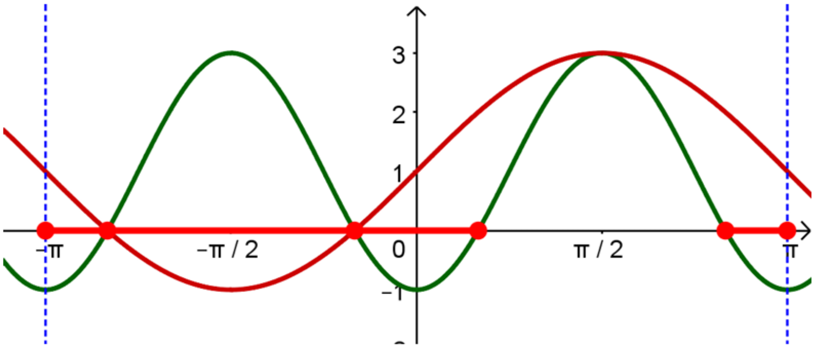
D.

$$-\pi \le x \le -\frac{5\pi}{6}, \quad -\frac{\pi}{6} \le x \le \frac{\pi}{6}, \quad x = \frac{\pi}{2}, \quad \frac{5\pi}{6} \le x \le \pi$$

E.

$$-\pi \le x \le -\frac{5\pi}{6}, \quad x = \frac{\pi}{2}, \quad \frac{5\pi}{6} \le x \le \pi$$

$(1 - 2\cos 2x)(1 + 2\sin x) \le 0$



The product of $(1 - 2\cos 2x)$ and $(1 + 2\sin x)$ will be negative or 0 for all of $-\pi \le x \le \frac{\pi}{6}$, positive for $\frac{\pi}{6} < x < \frac{5\pi}{6}$ and negative again for $\frac{5\pi}{6} \le x \le \pi$

So $(1 - 2\cos 2x)(1 + 2\sin x) \le 0$ for $-\pi \le x \le \frac{\pi}{6}$ and $\frac{5\pi}{6} \le x \le \pi$

The correct answer is B.

Question 8

Find the **fraction of the interval**

$$0 \leq \theta \leq 2\pi$$

for which the inequality

$$(2\cos(\frac{\theta}{2}) + 1)(\sin\theta + \cos\theta) \leq 0$$

is true.

Options:

A. $\frac{1}{6}$

B. $\frac{7}{12}$

C. $\frac{3}{4}$

D. $\frac{1}{4}$

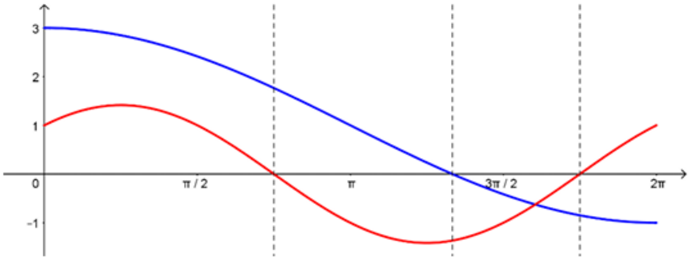
E. $\frac{1}{2}$

F. $\frac{5}{12}$

G. $\frac{5}{6}$

H. $\frac{2}{3}$

The graph of $y = 2\cos\frac{\theta}{2} + 1$ can be sketched directly
For $y = \sin\theta + \cos\theta$, this can be rewritten using
$R\sin(x+\alpha) = R\sin x \sin\alpha + R\cos x\cos\alpha$
$\sin\theta + \cos\theta = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$



$2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$
$\frac{7\pi}{12} + \frac{\pi}{4} = \frac{10\pi}{12}$

$\frac{3\pi}{4} \leq x \leq \frac{4\pi}{3}, \frac{7\pi}{4} \leq x \leq 2\pi$

Fraction $\frac{10\pi}{12} \div 2\pi = \frac{5}{12}$

$\frac{4\pi}{3} - \frac{3\pi}{4} = \frac{7\pi}{12}$

The correct answer is F

Question 9

Find the **number of solutions** of the equation

$$x \tan x = \sin x \quad \text{with} \quad -\pi \leq x \leq \pi.$$

Options:

A. 0

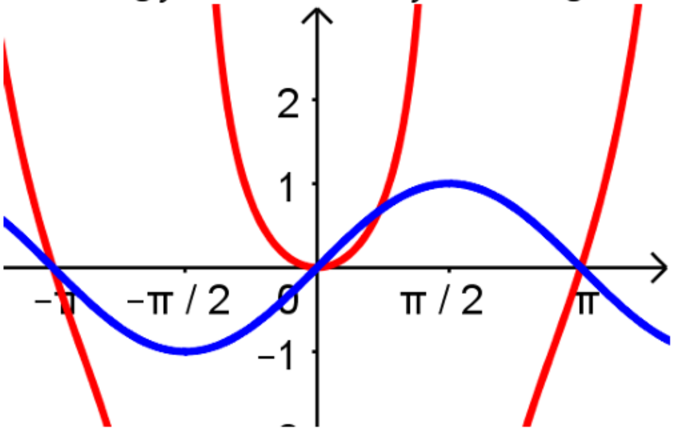
B. 1

C. 2

D. 3

E. 4

Sketching $y = x \tan x$ and $y = \sin x$ gives



There are 4 real solutions (being careful to include $x = \pm\pi$
The correct answer is E.

Question 10

How many **real roots** does the equation

$$3x^5 - 10x^3 - 225x + 10 = 0$$

have?

Options:

A. 1

B. 2

C. 3

D. 4

E. 5

$3x^5 - 10x^3 - 225x + 10 = 0$

$\frac{dy}{dx} = 15x^4 - 30x^2 - 225$

$15x^4 - 30x^2 - 225 = 0 \Leftrightarrow x^4 - 2x^2 - 15 = 0$

$(x^2 + 3)(x^2 - 5) = 0$

$x^2 = 5, x = \pm\sqrt{5}$

$x = \sqrt{5}, y = 10 - 200\sqrt{5}$ which is < 0

$x = -\sqrt{5}, y = 200\sqrt{5} + 10$ which is > 0

There are only two turning points so there must be three places that the graph of $y = 3x^5 - 10x^3 - 225x + 10$ crosses the x axis.

There are three real solutions.

The correct answer is C.

Question 11

Into how many **regions** is the plane divided when the following curves are drawn?

$$y = x^3 - x^2, \quad y = x^3 - 4x, \quad y = x^2$$

Options:

A. 11

B. 10

C. 9

D. 8

E. 7

. This question does not require an accurate sketch, just a general idea of the shape of each curve and the points of intersection for each possible pair of curves.
Let curve A be $y = x^3 - x^2$, curve B be $y = x^3 - 4x$ and curve C be $y = x^2$
Intersection points:

A and B: $x^3 - x^2 = x^3 - 4x \Leftrightarrow x^2 - 4x = 0 \Leftrightarrow x(x-4) = 0$

Intersect at $(0,0)$ and $(4,48)$

A and C: $x^3 - x^2 = x^2 \Leftrightarrow x^3 - 2x^2 = 0 \Leftrightarrow x^2(x-2) = 0$

Intersect at $(0,0)$ and $(2,4)$

B and C: $x^3 - 4x = x^2 \Leftrightarrow x^3 - x^2 - 4x = 0 \Leftrightarrow x(x^2 - x - 4) = 0$

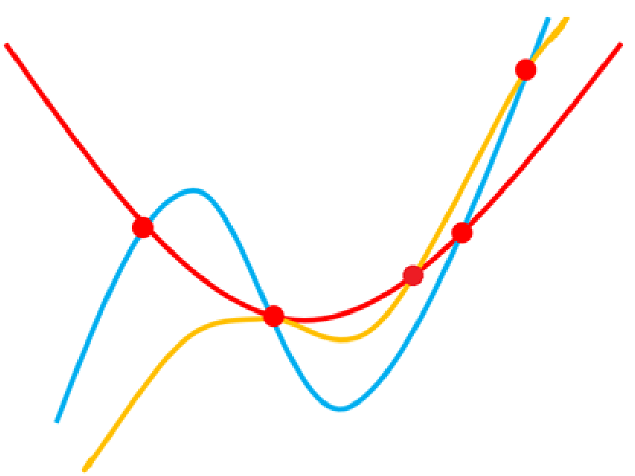
Intersect when $x = 0$, $x = \frac{1+\sqrt{17}}{2}$ and $x = \frac{1-\sqrt{17}}{2}$

There is no need to actually calculate the y coordinate for each of these, just to know where it lies in relation to the other points.

Since $4 < \sqrt{17} < 5$, $\frac{5}{2} < \frac{1+\sqrt{17}}{2} < 3$ and $\frac{25}{4} < \left(\frac{1+\sqrt{17}}{2}\right)^2 < 9$ i.e. this point is above $(2,4)$ but below $(4,48)$

$-2 < \frac{1-\sqrt{17}}{2} < -\frac{3}{2}$ and $\frac{9}{4} < \left(\frac{1+\sqrt{17}}{2}\right)^2 < 4$ i.e. this point is above $(0,0)$ but below $(2,4)$

A rough plot of these points and a sketch of the curves following their general shape allows the number of regions to be counted easily.



There are 10 distinct regions.
The correct answer is (b)

Question 12

The equation in x:

$$x^3 + 6x^2 - 63x + k = 0$$

has **three real solutions** when:

Options:

A. $-108 < k < 392$

B. $-392 < k < 108$

C. $108 < k < 392$

D. $-392 < k < 0$

E. $0 < k < 108$

$x^3 + 6x^2 - 63x + k = 0$

$y = x^3 + 6x^2 - 63x + k$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 12x - 63$

$3x^2 + 12x - 63 = 0$

$x^2 + 4x - 21 = 0$

$(x + 7)(x - 3) = 0$

$x = -7, y = k + 392$

$x = 3, y = k - 108$

$k + 392 > 0 \Leftrightarrow k > -392$

$k - 108 < 0 \Leftrightarrow k < 108$

$-392 < k < 108$

The correct answer is (b)

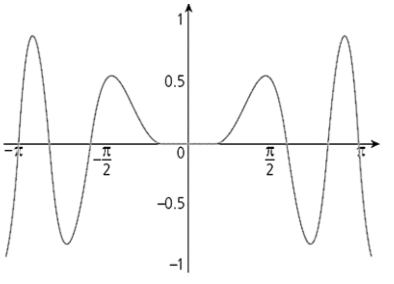
Question 13

Which of the graphs below shows a sketch of the function

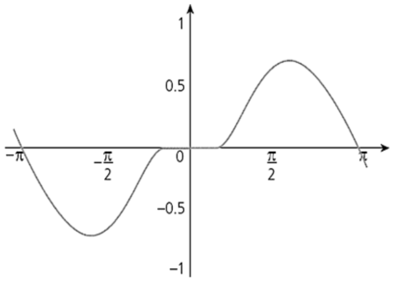
$$y = e^{-\frac{1}{x^2}} \cos(x^2) \quad \text{for } -\pi \le x \le \pi?$$

Options:

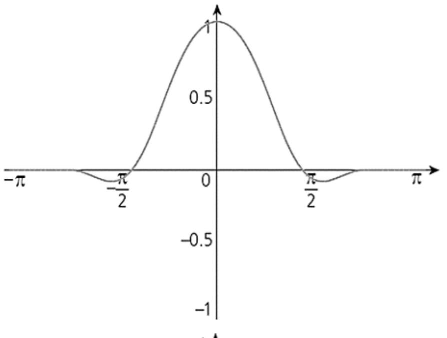
A.



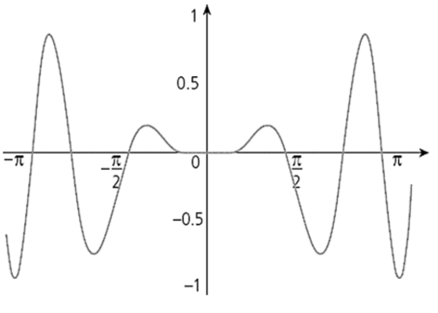
B.



C.



D.



The correct graph is **D**. The function $y = e^{-\frac{1}{x^2}} \cos(x^2)$ is defined for all $x \neq 0$, and as $x \to 0$, the exponential term $e^{-1/x^2} \to 0$ extremely fast, which causes the function to approach zero smoothly at the center. At the same time, $\cos(x^2)$ introduces rapid oscillations that increase in frequency as $|x|$ grows. This means the graph should have a smooth peak at $x = 0$, be symmetric about the y-axis, and show small, fast oscillations that fade out toward the edges. Among the four options, **only Graph D** shows all these features: a smooth central peak, symmetry, and faint oscillations that match the damping effect of the exponential factor.

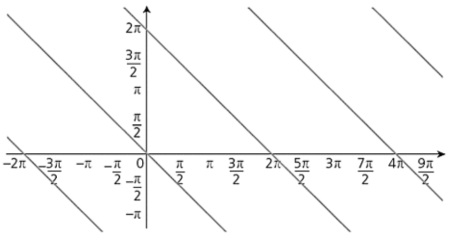
Question 14

The graph of all the points (x, y) in the **xy-plane** that satisfy the equation

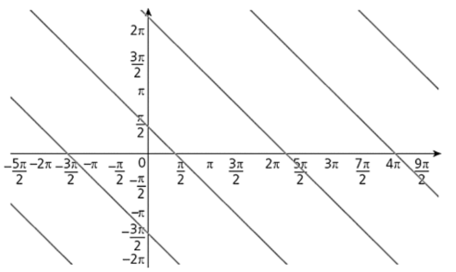
$$\sin(x + y) = 1$$

is shown in:

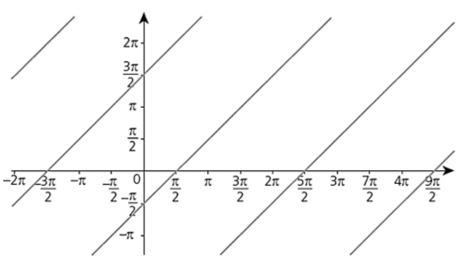
A.



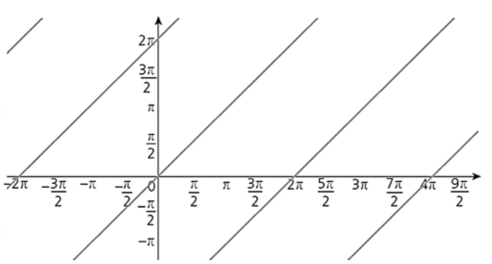
B.



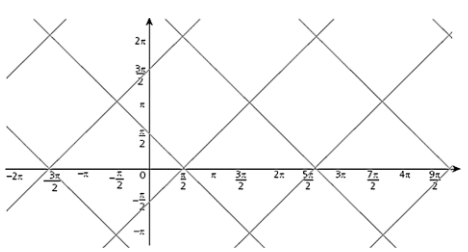
C.



D.



E.



The correct answer is **E** because $\sin(x + y) = 1$ only when $x + y = \frac{\pi}{2} + 2n\pi$, which represents a set of diagonal lines with slope –1 at specific intervals. This means the graph should show only those individual lines where the sine function reaches its maximum. Among the options, only graph E displays this correct pattern of evenly spaced diagonal lines, making it the right choice.

Question 15

Find the fraction of the interval $0 \leq \theta \leq 2\pi$ for which the inequality

$$(\cos\theta + \sin\theta)\,(\frac{\sqrt{3}}{2} + \cos\frac{\theta}{2})\,(\cos^2\theta - 1) \leq 0$$

is satisfied.

Options:

A. $\frac{1}{4}$
B. $\frac{5}{12}$
C. $\frac{1}{2}$
D. $\frac{2}{3}$
E. $\frac{3}{4}$
F. $\frac{5}{6}$

$\cos\theta + \sin\theta$ can be rewritten as $R\sin(x + \alpha)$
$R\sin\alpha = 1, R\cos\alpha = 1, R^2 = 2 \Leftrightarrow R = \sqrt{2}$. $\tan\alpha = -1 \Leftrightarrow \alpha = \frac{\pi}{4}$
$\cos\theta + \sin\theta \equiv \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$
$\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) < 0$ for $\frac{3\pi}{4} < x < \frac{7\pi}{4}$
$\frac{\sqrt{3}}{2} + \cos\frac{1}{2}\theta = 0$ when $\cos\frac{1}{2}\theta = -\frac{\sqrt{3}}{2}$
i.e. $\frac{1}{2}\theta = \frac{5\pi}{6} \Leftrightarrow \theta = \frac{5\pi}{3}$ (Note: no other values in $0 \leq \theta \leq 2\pi$ give $\cos\frac{1}{2}\theta = -\frac{\sqrt{3}}{2}$)
For $0 \leq \theta \leq \frac{5\pi}{3}$, $\frac{\sqrt{3}}{2} + \cos\frac{1}{2}\theta > 0$
$\cos^2\theta - 1 \leq 0$ for all θ

	0 to $\frac{3\pi}{4}$	$\frac{3\pi}{4}$ to $\frac{5\pi}{3}$	$\frac{5\pi}{3}$ to $\frac{7\pi}{4}$	$\frac{7\pi}{4}$ to 2π
$\cos\theta + \sin\theta$	+	−	−	+
$\frac{\sqrt{3}}{2} + \cos\frac{1}{2}\theta$	+	+	−	−
$\cos^2\theta - 1$	−	−	−	−
	−	+	−	+

$(\cos\theta + \sin\theta)\left(\frac{\sqrt{3}}{2} + \cos\frac{1}{2}\theta\right)(\cos^2\theta - 1) \leq 0$ for $0 \leq \theta \leq \frac{3\pi}{4}$ and $\frac{5\pi}{3} \leq \theta \leq \frac{7\pi}{4}$

$$\frac{3\pi}{4} + \left(\frac{7\pi}{4} - \frac{5\pi}{3}\right) = \frac{9\pi}{12} + \frac{21\pi}{12} - \frac{20\pi}{12} = \frac{10\pi}{12} = \frac{5\pi}{6}$$

Then,

$$\frac{\frac{5\pi}{6}}{2\pi} = \frac{5}{12}$$

The correct answer is **B**.

Question 16

Find the value of

$$\sum_{r=0}^{100} \cos\left(\frac{2r+1}{4}\pi\right)$$

Options:

A. 0

B. $\frac{\sqrt{2}}{2}$

C. $\sqrt{2}$

D. $2\sqrt{2}$

E. $100\sqrt{2}$

F. $101\sqrt{2}$

$$\sum_{r=0}^{100} \cos\left(\frac{2r+1}{4}\pi\right) = \cos\frac{\pi}{4} + \cos\frac{3\pi}{4} + \cos\frac{5\pi}{4} + \cos\frac{7\pi}{4} + \cdots + \cos\frac{201\pi}{4}$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \cdots + \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

The values occur in groups of four. Each group of four sum to zero $\left(\frac{\mathbf{1}}{\sqrt{\mathbf{2}}} - \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} - \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} + \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} = \mathbf{0}\right)$.

Since there are $\mathbf{101} = \mathbf{25} \times \mathbf{4} + \mathbf{1}$ terms to be added, the last term must be the first term of the next group of four, $\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$.

The answer is B.

Question 17

Find the number of solutions of the equation

$$x^2 \cos 4x = 8 \sin 4x$$

in the interval $0 \leq x \leq \pi$.

Options:

A. 0

B. 1

C. 2

D. 3

E. 4

F. 5

When $\cos 4x = 0$, $\sin 4x \neq 0$, so the solutions of the equation $x^2 \cos 4x = 8 \sin 4x$ are the same as the solutions of the equation $x^2 = 8 \tan 4x$.
The graphs of both $y = x^2$ and $y = 8 \tan 4x$ pass through the origin, but the gradient of the tangent to $y = x^2$ at $(0, 0)$ is zero, whereas the gradient of the tangent to $y = 8 \tan 4x$ at $(0, 0)$ is positive ($\frac{d}{dx}(8 \tan 4x) = 8 \times 4 \sec^2 4x = 32$ when $x = 0$). So, as x increases from zero, the graph of $y = x^2$ is below the graph of $y = 8 \tan 4x$ until it passes the first vertical asymptote; moreover, since, between successive asymptotes and above the x-axis, both graphs have increasing gradient, with $y = 8 \tan 4x$ tending to infinity before reaching the next asymptote, it follows that the graphs intersect precisely once.
On the graph of $y = \tan x$ the asymptotes are at $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{5\pi}{2}$, etc.. Thinking of $y = 8 \tan 4x$ as a one-way stretch parallel to the x-axis scale factor $\frac{1}{4}$ (followed by a one-way stretch parallel to the y-axis, scale factor 8), the asymptotes on this graph, in the interval $0 \leq x \leq \pi$, are at $x = \frac{\pi}{8}$, $x = \frac{3\pi}{8}$, $x = \frac{5\pi}{8}$ and $x = \frac{7\pi}{8}$.
By the reasoning given above, the equation $x^2 = 8 \tan 4x$, and therefore the equation $x^2 \cos 4x = 8 \sin 4x$, has precisely one solution on each of $\left[0, \frac{\pi}{8}\right)$, $\left(\frac{\pi}{8}, \frac{3\pi}{8}\right)$, $\left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ and $\left(\frac{5\pi}{8}, \frac{7\pi}{8}\right)$. (There is no solution in $(\frac{7\pi}{8}, \pi]$, since on this interval the graph of $y = 8 \tan 4x$ is still below the x-axis.) There are four solutions in total.
The answer is E.

Question 18

x satisfies the simultaneous equations:

$$\sqrt{2}\sin 3x - 2\cos 3x = 1 - \sqrt{2}$$

$$\sin 3x + 2\sqrt{2}\cos 3x = \frac{1}{2}(4 + \sqrt{2})$$

where $-180° \leq x \leq 180°$.

Find the sum of the possible values of x.

Options:

A. 45°

B. 90°

C. 135°

D. 180°

E. 225°

F. 0°

Subtracting $\sqrt{2}$ times the first equation from the second equation gives:

$4\sqrt{2}\cos 3x = \frac{1}{2}(4+\sqrt{2}) - \sqrt{2}(1-\sqrt{2}) = 2 + \frac{\sqrt{2}}{2} - \sqrt{2} + 2 = \frac{8-\sqrt{2}}{2} \Rightarrow \cos 3x = k$

Since $\cos(-3x) \equiv \cos 3x$ for all values of x, any solution, $x = a$, to any equation of the form $\cos 3x = k$, for some real number k, in the interval $-180° \leq x \leq 180°$, must be paired with a second solution, $x = -a$, and therefore adding all of the solutions must equal zero. There is no need to find the value of $\cos 3x$ nor solve the resulting equation.

Adding $\sqrt{2}$ times the first equation to the second equation gives:

$3\sin 3x = \sqrt{2}(1-\sqrt{2}) + \frac{1}{2}(4+\sqrt{2}) = \sqrt{2} - 2 + 2 + \frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2} \Leftrightarrow \sin 3x = \frac{\sqrt{2}}{2}$

So, $\sin 3x = \frac{\sqrt{2}}{2}$ on $-540° \leq 3x \leq 540°$, then

$3x = -315°, -225°, 45°, 135°, 405°$, or $495°$, so $x = -105°, -75°, 15°, 45°, 135°$, or $165°$.

Adding these up gives $-105 - 75 + 15 + 45 + 135 + 165 = 180°$.

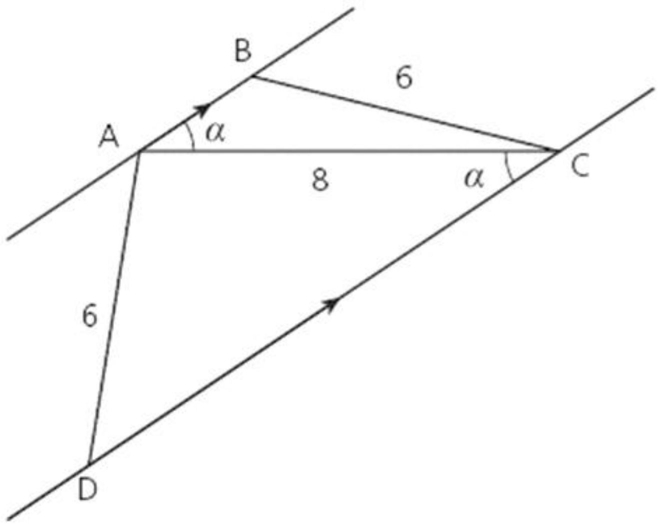
The answer is D.

Question 19

ABCD is a trapezium with side AB parallel to side CD.
The lengths of sides AD and BC are both 6 units.
The diagonal AC is of length 8 units and makes an angle of α with both sides AB and CD, as shown in the diagram.

It is also given that the area of triangle ACD is five times the area of triangle ABC.

Find the value of $\cos\alpha$.



A. $\dfrac{\sqrt{14}}{3}$

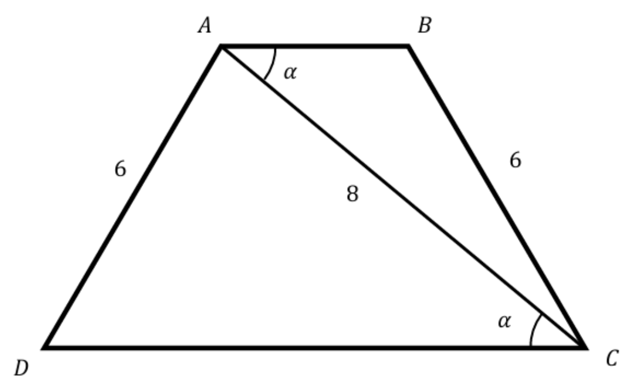
B. $\dfrac{3}{4}$

C. $\dfrac{3\sqrt{35}}{20}$

D. $\dfrac{\sqrt{42}}{7}$

E. $\dfrac{7\sqrt{42}}{48}$

F. $\dfrac{\sqrt{28}}{6}$



Area $ACD = 5 \times$ Area ABC

$\frac{1}{2} \times 8 \times CD \sin\alpha = 5 \times \frac{1}{2} \times 8 \times AB \sin\alpha$

$CD = 5AB$

Let $AB = x$ so $CD = 5x$

Cosine rule for triangle ABC: $\cos\alpha = \frac{x^2+64-36}{2\times8\times x} = \frac{x^2+28}{16x}$ (1)

Cosine rule for triangle ACD: $\cos\alpha = \frac{2x^2+64-36}{2\times8\times5x} = \frac{25x^2+28}{80x}$ (2)

Equating $\frac{x^2+28}{16x} = \frac{25x^2+28}{80x}$

$5(x^2+28) = 25x^2 + 28$

$5x^2 + 140 = 25x^2 + 28$

$x^2 = \frac{28}{5}$

Substituting back into (1) gives $\cos\alpha = \frac{\frac{28}{5}+28}{16\frac{\sqrt{28}}{\sqrt{5}}}$

Multiplying top and bottom by 5 gives $\cos\alpha = \frac{28+140}{16\sqrt{28}\sqrt{5}} = \frac{168}{32\sqrt{35}} = \frac{21}{4\sqrt{35}} = \frac{21\sqrt{35}}{140} = \frac{3\sqrt{35}}{20}$

The answer is C.

Question 20

What is the maximum value of

$$\left(\frac{1}{2-\sin(x+135°)}\right)^2$$

in the interval $90° \leq x \leq 270°$?

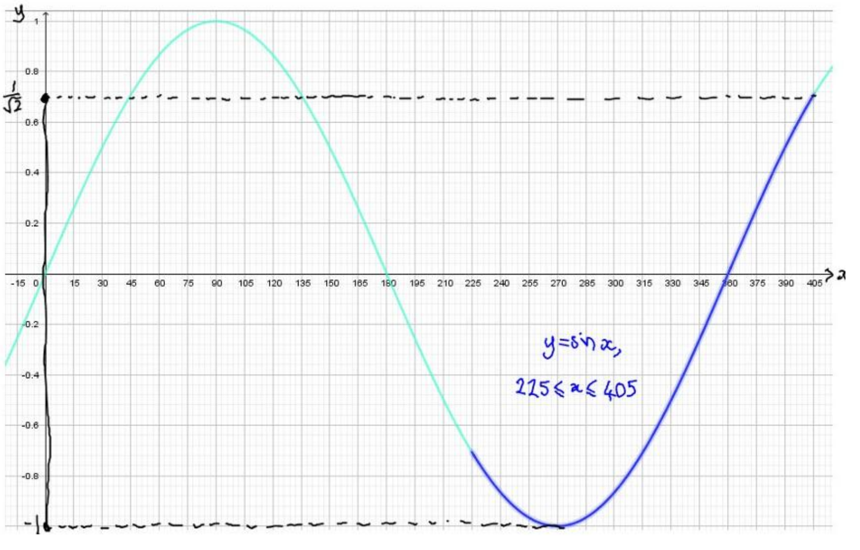
A. $\frac{1}{4}$

B. $\frac{1}{9}$

C. 2

D. $\frac{18+8\sqrt{2}}{49}$

E. 4



So $2-\frac{1}{\sqrt{2}} \leq 2-\sin(x+135°) \leq 3 \Leftrightarrow \frac{1}{3} \leq 0 \leq \frac{1}{2-\sin(x+135°)} \leq \frac{1}{2-\frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{2\sqrt{2}-1} \times \frac{2\sqrt{2}+1}{2\sqrt{2}+1} = \frac{4+\sqrt{2}}{7}$.

It follows that $\frac{1}{9} \leq \left[\frac{1}{2-\sin(x+135°)}\right]^2 \leq \left(\frac{4+\sqrt{2}}{7}\right)^2 = \frac{16+8\sqrt{2}+2}{49} = \frac{18+8\sqrt{2}}{49}$.

The answer is d).